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STABILIZATION OF ELECTRON STREAMS IN TYPE III

SOLAR RADIO BURSTS

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ABSTRACT

We show that the electron streams that give rise to Type III solar radio bursts are stable and will not be decelerated while propagating out of the solar corona. The stabilization mechanism depends on the parametric oscillating two-stream instability. Radiation is produced near the fundamental and second harmonic of the local electron plasma frequency. Estimates of the emission at the second harmonic indicate that the wave spectra created by the oscillating two-stream instability can account for the observed intensities of Type III bursts.

I. Introduction

Sporadic Type III solar radio bursts are caused by particle streams accelerated in the vicinity of a chromospheric flare. These energetic particles propagate through the solar corona at a constant velocity between 0.3 - 0.6 c, and excite radiation at the fundamental and second harmonic of the local electron plasma frequency, ω_e. Recent observations at 1 AU using data from the radiometer and particle experiments on the IMP-6 satellite (Alvarez, Haddock, and Lin 1972; Lin 1973) demonstrate that the particle streams consist primarily of electrons with energies in the range 1-100 keV.

The interaction of such streams with the background coronal plasma produces unstable electrostatic waves with frequency near the electron plasma frequency and phase velocity near the beam velocity (Sturrock 1964; Kaplan and Tsytovich 1968; Zheleznyakov and Zaitsev 1970b; Smith and Fung 1971). According to current views, these electro-static waves are subsequently transformed into electromagnetic radiation at ω_e by scattering from ion density fluctuations, and at $2\omega_e$ by scattering from each other. A review of the observations and theory can be found in Smith (1970).

While the above qualitative picture is generally accepted, quantitative estimates of the radiation spectrum and intensity require a more precise nonlinear theory of the interaction of the beam with the plasma. To estimate the electromagnetic radiation and the spectrum emitted in Type III bursts, a knowledge of the plasma wave energy spectrum before transformation is required. This in turn depends on whether the beam is decelerated to form a plateau in velocity space, or whether instead non-linear effects cut off the plateau formation at an early stage and sta-bilize the beam.

Kaplan and Tsytovich (1968) calculated the effect of induced nonlinear scattering of the plasma waves by thermal fluctuations, and concluded that this scattering was fast enough to stabilize the beam before the formation of a plateau. However, as Zheleznyakov and Zaitsev (1970a) correctly pointed out, the calculation of the scattering time by Kaplan and Tsytovich was inaccurate. Zheleznyakov and Zaitsev calculated the correct time for the development of this nonlinear process and concluded that it was longer than the characteristic time for quasilinear deformation. Therefore, in contrast to the conclusion reached by Kaplan and Tsytovich, stabilization does not occur. In subsequent publications, Zheleznyakov and Zaitsev (1970b) and Zaitsev, Mityakov and Rapoport (1972) calculated the power emitted in Type III bursts, based on the hypothesis that nonlinear effects do not stabilize the beam.

Smith and Fung (1971) also considered the question of whether beam-induced plasma waves could be scattered by polarization clouds of ions to stabilize the beam. They concluded that this mechanism could not stabilize electron streams in the solar corona.

One of the most striking features of Type III bursts, however, is that the source region propagates at almost constant velocity from the inner corona to nearly 1 AU (Fainberg, Evans, and Stone 1972). In this paper we demonstrate that a nonlinear mechanism does exist that will prevent the deceleration of the electron stream. The mechanism that we consider is the "oscillating two-stream instability" (Nishikawa 1968a,b; Sanmartin 1970). This instability is driven by the fields generated through the beam-plasma instability, and transfers the energy in these fields to regions of phase space where the fields do not interact resonantly with the electron stream. The result is to prevent deceleration of the stream by its self-generated fields.

In the next section, we discuss the theory of the oscillating two-stream instability. In Section III, we calculate the conditions under which this nonlinear process can prevent plateau formation. In the last section, we use these results to estimate the power emitted in Type III bursts. However, we defer to a later paper a detailed consideration of the frequency spectrum and its temporal evolution.

II. The Oscillating Two-stream Instability

This instability has been discussed recently in the literature as an efficient mechanism for coupling energy from high-phase-velocity oscillations into shorter-wavelength oscillations which readily transfer energy to the ambient coronal plasma. A detailed mathematical treatment based on the fluid equations can be found in Nishikawa (1968a,b); an analysis based on the Vlasov-Maxwell equations has been given by Sanmartin (1970).

To our knowledge this instability has not yet been discussed with respect to astrophysical plasmas, so we present here a brief but self-contained derivation of the dispersion relation based on simple physical arguments and refer the mathematically interested reader to the original references.

Consider a plasma in the presence of a long-wavelength high-frequency field

$$E = E_o \cos (\omega_o t + k_o x)$$

such that the phase velocity of the wave $\omega_o/k_o \gg V_e$, the thermal velocity of the plasma.

In the absence of this oscillating field, the normal electrostatic mode of the plasma is the electron plasma oscillation with

wavenumber k and angular frequency $\omega_{ek} = \omega_e(1 + 3k^2 \lambda_D^2)^{\frac{1}{2}}$, and

the ion acoustic wave with the same wavenumber and with frequency

$\omega_A = kC_s/(1 + k^2\omega_D^2)^{\frac{1}{2}}$. (In the unperturbed plasma with equal ion and

electron temperatures, this mode is heavily damped and cannot properly

be considered a normal mode.) Here the ion sound speed $C_s \equiv [(\gamma_e KT_e +$

$\gamma_i KT_i)/M]^{\frac{1}{2}}$; the Debye length $\lambda_D \equiv (KT_e/4\pi n_e e^2)^{\frac{1}{2}}$; e is the unit

(positive) electronic charge; K is Boltzmann's constant; n_e , T_e ,

and T_i are respectively the electron density, electron temperature and

ion temperature of the coronal plasma; γ_e and γ_i are the adiabatic

indices of the electrons and ions; $\omega_e^2 = 4\pi n e^2/m$; and m , M are

the electron and ion masses, respectively. These normal modes obey the

equations

$$\frac{\partial^2}{\partial t^2} n_{ek} + \nu_1 \frac{\partial}{\partial t} n_{ek} + \omega_{ek}^2 n_{ek} = 0 ,$$

$$(1)$$

$$\frac{\partial^2}{\partial t^2} n_{ik} + \nu_2 \frac{\partial}{\partial t} n_{ik} + \omega_A^2 n_{ik} = 0 ,$$

where n_{ek} , n_{ik} are the spatially-Fourier-analyzed electron and ion

density fluctuations with wave number k , and ν_1 , ν_2 are the total

damping rates (collisional and collisionless) of these fluctuations

respectively. The presence of an external force due to a long wave-

length field oscillating at a frequency $\omega_o \sim \omega_e$ couples these two

modes. For simplicity in calculating the coupling forces we assume

$k_o = 0$. (The extension to finite k_o is straightforward and does not

influence our conclusions for the cases of interest here.)

The coupling force can be derived by simple arguments. Let

$n_i(x)$, $n_e(x)$ be the ion and electron density fluctuations. The driving

field tends to displace the electrons relative to the ions. This driver now produces high frequency charge fluctuations with a frequency ω_o. Then, because $n_e \simeq n_i$, we have

$$\delta n_e = \frac{\partial n_i}{\partial x} \Delta x = \frac{-\partial n_i}{\partial x} \frac{eE_o}{m\omega_o^2} \cos \omega_o t = ikn_{ik} \frac{eE_o \cos \omega_o t}{m \omega_o^2} \quad ,$$

where we have used a Fourier space transform to replace $\partial/\partial x$ by ik. The electron equation now becomes

$$\frac{\partial^2}{\partial t^2} n_{ek} + \nu_1 \frac{\partial}{\partial t} n_{ek} + \omega_{ek}^2 n_{ek} = \frac{ikn_{ik} eE_o \cos \omega_o t}{m} \quad . \qquad (2)$$

The coupling force on the ions is due to the gradient of the electric field pressure,

$$\frac{1}{8\pi} \frac{\partial}{\partial x} (E_o \cos \omega_o t + E(x))^2 \quad ,$$

where $E(x)$ is the self-consistent electric field. Using Poisson's equation, we have

$$\frac{\partial}{\partial x} \frac{(E_o \cos \omega_o t + E(x))^2}{8\pi} \simeq -eE_o \delta n_e \cos \omega_o t \quad .$$

The term $E^2(x)$ is neglected. The equation for the ion acoustic fluctuations becomes

$$\frac{\partial^2}{\partial t^2} n_{ik} + \nu_2 \frac{\partial}{\partial t} n_{ik} + \omega_A^2 n_{ik} = \frac{1}{M} \frac{\partial}{\partial x} [eE_o \delta n_e(x) \cos \omega_o t]$$

$$= - \frac{ikn_{ek} eE_o \cos \omega_o t}{M} \quad . \qquad (3)$$

The coupled equations (2) and (3) describe the instability. To derive the dispersion relation for the system, we Fourier transform equations (2) and (3) in time according to

$$
\begin{pmatrix} n_{ek}(t) \\ n_{ik}(t) \end{pmatrix} = \int_{-\infty}^{\infty} d\omega \, e^{-i\omega t} \begin{pmatrix} n_{ek}(\omega) \\ n_{ik}(\omega) \end{pmatrix}
$$

and obtain the system

$$
(\omega^2 - \omega_{ek}^2 + i\nu_1\omega) \, n_{ek}(\omega) + i\Omega_{ek}^2[n_{ik}(\omega-\omega_o) + n_{ik}(\omega+\omega_o)] = 0
$$

(4)

$$
(\omega^2 - \omega_A^2 + i\nu_2\omega) \, n_{ik}(\omega) - i\Omega_{ik}^2 \, [n_{ek}(\omega-\omega_o) + n_{ek}(\omega+\omega_o)] = 0
$$

where $(\Omega_{ek}^2, \Omega_{ik}^2) = keE_o/2 \, (1/m \, ; \, 1/M)$. We see that $n_{ik}(\omega)$ couples with $n_{ek}(\omega \pm \omega_o)$, and $n_{ek}(\omega \pm \omega_o)$ couples with $n_{ik}(\omega)$ and $n_{ik}(\omega \pm 2\omega_o)$.

We then obtain the (infinite) system of equations

$$
\left\{
\begin{array}{l}
(\omega^2 - \omega_A^2 + i\nu_2\omega)n_{ik}(\omega) - i\Omega_{ik}^2 n_{ek}(\omega-\omega_o) - i\Omega_{ik}^2 n_{ek}(\omega+\omega_o) = 0 \\[2ex]
i\Omega_{ek}^2 n_{ik}(\omega) + [(\omega-\omega_o)^2 - \omega_{ek}^2 + i\nu_1(\omega-\omega_o)]n_{ek}(\omega-\omega_o) + i\Omega_{ek}^2 n_{ik}(\omega-2\omega_o) = 0 \\[2ex]
i\Omega_{ek}^2 n_{ik}(\omega) + i\Omega_{ek}^2 n_{ik}(\omega+2\omega_o) \\[2ex]
\qquad\qquad + [(\omega+\omega_o)^2 - \omega_{ek}^2 + i\nu_1(\omega+\omega_o)]n_{ek}(\omega+\omega_o) = 0 \\[2ex]
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\vdots
\end{array}
\right.
$$

(5)

We shall be interested in Re $\omega \lesssim \omega_A \ll \omega_o$ and thus neglect $n_{ik}(\omega \pm 2\omega_o)$

as being off-resonant. This closes the infinite set (5), and the dispersion relation for the resonant normal modes of the coupled system is given by the secular equation

$$
\begin{vmatrix}
\omega^2 - \omega_A^2 + i\nu_2\omega & -i\Omega_{ik}^2 & -i\Omega_{ik}^2 \\
\\
i\Omega_{ek}^2 & (\omega-\omega_o)^2 - \omega_{ek}^2 + i\nu_1(\omega-\omega_o) & 0 \\
\\
i\Omega_{ek}^2 & 0 & (\omega+\omega_o)^2 - \omega_{ek}^2 + i\nu_1(\omega+\omega_o)
\end{vmatrix} = 0 \qquad (6)
$$

Assuming that $\omega_o \simeq \omega_{ek}$, and that the damping rates ν_1 , ν_2 are small, we may approximate

$$
(\omega \pm \omega_o)^2 - \omega_{ek}^2 + i\nu_1(\omega \pm \omega_o) \simeq \pm\, 2\omega_{ek}(\omega \pm \delta) \pm i\omega_o\nu_1 \quad,
$$

where $\delta \equiv \omega_o - \omega_{ek}$ is the "frequency mismatch" between the driver and the electron normal mode.

The secular equation (6) becomes simply

$$
(\omega^2 + i\omega\nu_2 - \omega_A^2)\,[(\omega + i\nu_1)^2 - \delta^2] + \frac{k^2 e^2 E_o^2}{4mM}\,\frac{\delta}{\omega_{ek}} = 0 \quad. \qquad (7)
$$

We now write $\omega = \omega_r + i\gamma$, where ω_r and γ are real. In the present analysis, $\gamma > 0$ implies instability, which is reflected in the growth of ion acoustic waves at frequency ω_r and electron plasma oscillations at $\omega_r \pm \omega_o$. From (7) we see that we may have $\gamma > 0$ under the following conditions:

> (i) $\delta < 0$, $\omega_r = 0$ (i.e., $\gamma \gg \omega_r$) ;
>
> (ii) $\delta > 0$, $\omega_r \neq 0$.

Case (ii) is the resonant three-wave interaction of the high-frequency driver with two lower-frequency normal modes, known as the "decay instability" (Tsytovich 1970, Davidson 1972). We shall be concerned with case (i), because in the case of Type III bursts the driver wave is simply the beam-excited plasma wave with $\omega_o \simeq \omega_e$, and therefore $\delta = \omega_o - \omega_{ek} \simeq \omega_e - \omega_{ek} < 0$. From Eq. (7), with $\delta < 0$, we outline below the basic features of the instability.

There is a threshold amplitude for the instability to occur, which is found by setting $\omega_r = \gamma = 0$ in Eq. (7) and minimizing with respect to δ. The result is

$$\frac{E_T^2}{8\pi \, n_e \, KT_e} \simeq 4 \, \frac{\nu_1}{\omega_{ek}} \tag{8}$$

where we have used the fact that $k^2\lambda_D^2 << 1$ and have assumed that $T_e \simeq T_i$.

Notice that ν_1 represents the total electron damping and, for the case of a Maxwellian plasma, can be written as

$$\nu_1 = \nu_c + \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \frac{\omega_e}{k^3\lambda_D^3} \exp\left[-\frac{1}{2k^2\lambda_D^2}\right] \tag{9}$$

where ν_c is the electron-ion collision frequency.

The growth rate for fields well above threshold is given by

$$\gamma_{1k} \simeq \omega_{ek}\left(\frac{m}{M}\right)^{1/3}(k^2\lambda_D^2)^{1/3}\left(\frac{E_o^2}{8\pi n_e KT_e}\right)^{1/3} , \tag{10}$$

where we have maximized γ_{1k} with respect to δ and neglected ν_1, ν_2 and ω_A compared with γ_{1k}.

Equation (10) describes the growth rate of the oscillating two-stream instability at very early times. We shall also be interested

in the radiation produced after the plasma has reached a quasi-steady
state and is near marginal stability. The growth rate then becomes

$$\gamma_{1k} = \frac{k^2 e^2}{4mM} (E_o^2 - E_T^2) \left(\frac{1}{2\omega_{ek}}\right)\left(\frac{1}{\omega_A^2 - \nu_1 \nu_2/2}\right) \quad .$$ (11)

The electron plasma waves produced as a result of the oscillating
two-stream instability have much lower phase velocities than the driver
wave. This can be seen from the energy and momentum conservation conditions
for the waves, plus the usual restriction that $k^2\lambda_D^2 \ll 1$. Denoting the
driver, electron and ion waves by (ω_o , k_o) , (ω_{ek_e}, k_e), and (ω_r, k_i) ,
the conservation relations are $\omega_o \simeq \omega_r + \omega_e$ and $k_o \simeq k_i + k_e$. Noting
that $\omega_o = \omega_{ek_o} \simeq \omega_e$, $k_o \simeq V_b/V_e\lambda_D$, $\omega_{ek_e} \simeq \omega_e$, $k_e \simeq \alpha\lambda_D^{-1}$,
$\omega_r \ll \gamma_{1k_e} \ll \omega_e$, $k_i = k_o - k_e \simeq - k_e$, with $\alpha \simeq .1 - .2$ and $V_b \simeq 20\ V_e$,
we find that the phase velocity of the growing wave is of order $\omega_e/k_e \simeq 5\text{-}10\ V_e$,
while the phase velocity of the driver is $\omega_e/k_o \simeq 20\ V_e$. Notice that the
excited ion waves appear only as solutions of the nonlinear dielectric
constant and disappear in the absence of the driving field.

Because of the above interaction, the energy of the stream-excited
plasma waves is transferred to a region of velocity space where these waves
do not interact resonantly with the stream particles. We stress, however,
that the energy transfer is not direct; the nonresonant waves are induced
through the driven bulk motion of the electrons relative to the ions, and
are not in random phase with the driver. Thus, the energy and momentum
relations above need only be satisfied as approximations. In the non-
resonant region of velocity space, the induced plasma waves can be Landau-
damped by the ambient plasma, creating energetic tails in the ambient
electron distribution. Some of the consequences of this tail formation

with respect to the Type III radiation will be examined in Section III below.

Before closing this section, we point out that the above theory, including such consequences as tail formation, has been confirmed by a series of computer simulations following both electrons and ions in their self-consistent orbits (Kruer and Dawson 1970, 1972; Kainer, Dawson, and Coffey 1972).

III. Nonlinear Effects on the Beam-Plasma Interaction.

We examine next the collisionless relaxation of an electron beam propagating in the solar corona under the following assumptions:

(i)   the interaction is one dimensional, i.e. all the plasma waves in the system propagate in the direction of the beam.

(ii)   the excitation of the plasma waves is considered under conditions that the beam particles can resonate with waves in a finite range of wave number k . This implies the usual restriction

$$\frac{\Delta V_b}{V_b} > \left(\frac{n_b}{n_e} \right)^{1/3} \tag{12}$$

for the validity of a quasilinear description, where ΔV_b is the velocity spread in the beam and n_b is the density of the beam.

(iii) The system is homogeneous.

For a discussion of these assumptions, see Smith (1970).

Under these assumptions the evolution of the beam will be described by the well-known set of quasilinear equations

$$\frac{\partial F_b(v,t)}{\partial t} = \frac{\partial}{\partial t}\left[D(v,\ t)\ \frac{\partial}{\partial v}\ F_b(v,\ t)\right] \tag{13}$$

$$\frac{\partial \mathscr{E}_{ok}(t)}{\partial t} = 2\ \gamma_{2k}\mathscr{E}_{ok}(t) \tag{14}$$

$$D(v,t) = \frac{16\pi^2 e^2}{m^2}\ \int_0^\infty dk\ \delta(\omega_{ek} - kv)\ \mathscr{E}_{ok}(t) \tag{15}$$

$$\gamma_{2k}(t) = \frac{\pi}{2}\ \omega_e\ \frac{\omega_e^2}{|k|^3}\ k\left[\frac{\partial}{\partial v}\ F_b(v,t)\right]_{v = \frac{\omega_{ek}}{k}} \tag{16}$$

where $\mathscr{E}_k(t)$ is the spectral energy density $|E_k|^2/8\pi$ at the wavenumber k . A detailed discussion of the above system of equations can be found in Davidson (1972). We give here only the features relevant to the problem under consideration. The unstable waves will have frequency ω_e and will be generated only in regions where $\gamma_{2k}(t) > 0$, which corresponds to phase velocities in the region (Fig. 1)

$$V_b - \Delta V_b < V_p < V_b\ . \tag{17}$$

At each instant of time half of the directed energy of the beam goes into electrostatic waves, and half into flattening the beam, i.e.,

$$\mathscr{E}(t) = \int dk\ \mathscr{E}_k(t) \simeq n_b m V_b\ \Delta V_b(t)\ . \tag{18}$$

The beam will flatten as in Fig. 2 within a time of the order $(n_e/n_b)\omega_e^{-1}$, after which all wave generation by the beam will cease, and the beam spread $\Delta V_b \simeq V_b$. Applying quasilinear theory to the case of the Type III bursts we find that the electron beam decelerates without

a distance of ~ 30 km. Kaplan and Tsytovich (1968) and Smith and Fung (1971) discussed the possibility that induced nonlinear scattering of plasma waves out of the beam direction would stabilize the beam. This process was shown to be inadequate for electron beams (Zheleznyakov and Zaitsev 1970a, Smith and Fung 1971).

Let us examine now the nonlinear effects introduced by the oscillating two-stream instability. As the unstable waves grow in the region resonant with the beam, the background plasma sees at time t a field oscillating with frequency $\omega_o \sim \omega_e$ and amplitude $E_o(t)^*$. When this field exceeds the threshold field given by Eq. (8), the oscillating two-stream instability starts pumping wave energy out of the mode k, into plasma oscillations with lower phase velocity (Fig. 3) and zero frequency (i.e., purely growing) ion density fluctuations. This effect removes plasma waves from resonance with the beam, and may cut off formation of the plateau. This will occur if the growth time of the new waves becomes shorter than the generation time of the resonant waves.

We can now calculate the conditions necessary for stabilizing the beam against quasilinear diffusion. Our calculation is similar to that of Tsytovitch and Shapiro (1965), and the reader is referred to that reference for more details. The procedure is straightforward. One finds an approximate solution to the coupled equations for the beam-plasma and oscillating two-stream instabilities. This solution permits an estimate of the variation of the beam distribution function δF_b.

The beam distribution function evolves according to Eqs. (13) and (15). In the resonant region, the evolution of the field energy is described by

$$\frac{\partial \mathcal{E}_{ok}}{\partial t} = 2\gamma_{2k}\, \mathcal{E}_{ok} - 2\gamma_{1k}\, \mathcal{E}_{Nk} \tag{19}$$

where $\mathcal{E}_{Nk}$ is the field energy excited in the nonresonant region by the oscillating two-stream instability. In the nonresonant region the field energy $\mathcal{E}_{Nk}$ is given by

$$\frac{\partial \mathcal{E}_{Nk}}{\partial t} = 2\gamma_{1k} \mathcal{E}_{Nk} \qquad . \qquad (20)$$

Although we are interested in the early stages in the development of the instabilities, we will use Eq. (11) for γ_{1k}. This is because the growth rate at marginal stability is smaller than that given by Eq. (10) well above threshold. In Eq. (11) we neglect E_T^2 compared with E_o^2. [The validity of this assumption can be confirmed from the computer simulations of Kainer, Dawson and Coffey (1972)]. The resulting set of equations (13), (15), (19) and (20) have the identical form to those solved by Tsytovich and Shapiro (1965). Consequently, the variation of F_b is given by

$$\left| \frac{\partial(\delta F_b)}{\partial v} \right| \simeq \mu \ell n \left[\frac{\mu \, n_b \, m \, V_b \, \Delta V_b}{\mathcal{E}_N(t = 0)} \right] \frac{\partial F_b}{\partial v} \qquad , \qquad (21)$$

where $\mathcal{E}_N(t=0)$ is the initial energy density of the waves in the nonresonant region and $\mu = \mu(k) = \gamma_{2k}/\gamma_{1k}$.

The beam will stabilize against plateau formation if

$$\left| \frac{\partial \delta F_b}{\partial v} \right| << \frac{\partial F_b}{\partial v} \qquad ,$$

which implies that

$$\mu \ell n \left[\frac{n_b \, m \, V_b \Delta V_b}{\mathcal{E}_N(t = 0)} \right] << 1 \qquad . \qquad (22)$$

Because the logarithmic dependence is very weak, we can replace it with

an upper bound, namely,

$$\ln\left(\frac{\mu n_b \, m \, V_b \, \Delta V_b}{\mathcal{E}_N(t=0)}\right) \ll \ln\left(\frac{n_b \, m \, V_b^2}{n_e KT_e} \, n_e \lambda_D^3\right)$$

$$\ll \ln\left(\frac{V_b^2}{V_e^2} \, \frac{n_e V_b^3}{\omega_e^3}\right) \simeq 30 \;,$$

where $V_e^2 \equiv KT_e/m$ and we have used $(V_b/V_e) \simeq 20$, $n_e \simeq 10^8 \; cm^{-3}$, and $\omega_e \simeq \cdot 10^8$. Therefore, we may approximate (22) by

$$\mu < 0.03 \;. \tag{23}$$

Because μ depends only on n_b/n_e and $\Delta V_b/V_b$ we can find the value of $\Delta V_b/V_b$ necessary for stabilization as a function of n_b/n_e. From Eq. (16) we find that

$$\gamma_{2k} = \frac{\pi}{2} \omega_e \left(\frac{n_b}{n_e}\right)\left(\frac{V_b}{\Delta V_b}\right)^2 \;. \tag{24}$$

In order to find γ_{1k}, we use Eq. (11) and replace $E_0^2/8\pi$ by the value $n_b \, m \, V_b \Delta V_b$ as given by Eq. (18). If we also assume that $\omega_A^2 \gg \nu_1 \nu_2/2$, then

$$\gamma_{1k} \simeq \left(\frac{n_b}{n_e}\right) \frac{V_b \, \Delta V_b}{V_e^2} \, \omega_{ek} \tag{25}$$

and

$$\mu \simeq \left(\frac{V_b}{\Delta V_b}\right)^2 \frac{V_e^2}{V_b \Delta V_b} \ll .03 \;. \tag{26}$$

This constitutes an extremely conservative estimate, because the instability threshold can be easily exceeded and the more appropriate growth rate will be that given by Eq. (10). In that case the beam is stable as long as

$$\frac{\Delta V_b}{V_b} \geq 33 \left(\frac{V_e}{V_b} \frac{n_b}{n_e} \right)^{2/7} \qquad . \qquad (27)$$

In Fig. 4, using $V_e \simeq 5 \times 10^8$ cm/s, $V_b \simeq 10^{10}$ cm/s, we have plotted $\Delta V_b/V_b$ as a function of n_b/n_e , assuming that equality holds in Eq. (27).

From Fig. 4 we can see that only if the ratio of the beam-to-plasma density is much larger than 10^{-4} is the oscillating two-stream instability not capable of stabilizing the beam against formation of a quasilinear plateau. However, for smaller density ratios the quasilinear velocity diffusion of the beam will cease early in the process and plasma wave energy will be concentrated in the regions of phase space shown in Fig. 3.

IV. Discussion

A detailed calculation of the radiation spectra produced by the oscillating two-stream instability will be presented in a future publication. Here we simply demonstrate that the observed power can be accounted for by this theory. For brevity we concentrate on the $2\omega_e$ radiation but similar considerations also apply to radiation at the fundamental.

As described previously, the oscillating two stream instability will create symmetric [i.e., $\mathscr{E}_N(k, \omega_e) \approx \mathscr{E}_N(-k, \omega_e)$] oscillation spectra in the non-resonant region (Fig. 3), predominantly in the direction of the beam. These electrostatic oscillations can collide head-on with each other and radiate a transverse wave at $2\omega_e$ (Fig. 5). The process is greatly enhanced by the fact that the electrostatic spectrum is an even function of the phase velocity. In contrast, the linear beam-plasma instability creates an asymmetric longitudinal spectrum, with phase velocities in the direction of the beam velocity. Subsequent emission at the

second harmonic requires intermediate scattering to produce plasma waves
with phase velocities antiparallel to the beam (Smith 1970).

The energy density of plasma waves, W , produced by the oscil-
lating two-stream instability will be determined by the mechanism that
stabilizes it. In the discussion below we consider two possibilities.
The first is that the oscillating two-stream instability stabilizes at
field energy densities close to the threshold given by eq. (8). At thresh-
old, γ_{1k} given by Eq. (11) is near zero and the system is stable. The
second possibility for stabilization arises because early in the develop-
ment of the oscillating two-stream instability the electric fields are
well above threshold. These plasma waves will be Landau-damped by particles
in the thermal plasma. This results in the formation of suprathermal tails,
which have been observed in the computer experiments of Kainer, Dawson,
and Coffey (1972). The instability will stabilize when the Landau damping
due to the tails becomes equal to or larger than the growth due to the oscil-
lating two-stream instability. A third possibility, which we shall not
consider further, is that resonance broadening effects characteristic of
strong turbulence can lead to stabilization. An analysis of the discussion
by Bezzerides and Weinstock (1972) indicates that one must extract a large
portion of the beam energy in order for resonance broadening to be important.
We do not find it necessary to do this and, consequently, shall not consider
effects of strong turbulence any further.

In general, regardless of the details of the stabilization mechanism
for the oscillating two-stream instability, the electromagnetic radiation
emitted at $2\omega_e$ is given by (Sturrock, Ball and Baldwin 1965)

$$J(\underline{k}_T,\omega) \sim 2\omega_e) = \frac{\sqrt{3}\pi^2 e^2}{m^2 c^3} \int d\underline{k} \frac{(k_T^2 - 2\underline{k}_T \cdot \underline{k})^2}{|\underline{k}_T - \underline{k}|^2} \left[1 - \frac{(\underline{k}_T \cdot \underline{k})^2}{k_T^2 k^2} \right] \mathcal{E}_N(\underline{k}_T - \underline{k}) \mathcal{E}_N(\underline{k}) , \quad (28)$$

where the radiated wave number is given by $k_T \sim \sqrt{3}\, \omega_e/c$ and is much smaller than the excited electrostatic wave numbers. Therefore Eq. (28) can be written as

$$J(\underline{k}_T \,,\, \omega \sim 2\omega_e) = \frac{4\sqrt{3}\ \pi^2\ e^2}{m^2\ c^3}\ k_T^2\ \cos^2 \theta\ \sin^2 \theta \int d\underline{k}\ \mathscr{E}_N(\underline{k})\ \mathscr{E}_N(-\underline{k}) \ . \qquad (29)$$

One can estimate the emission by taking the spectrum to be almost one dimensional with a peak at the wavenumber with the maximum growth rate, $k_M \equiv \alpha k_D$, where $\alpha \ll 1$ and $k_D \equiv \lambda_D^{-1}$. Using $\mathscr{E}_N(-k) \simeq \mathscr{E}_N(k)$ and noting that

$$W = \int d\underline{k}\ \mathscr{E}_N(k) \ , \qquad (30)$$

Eq. (29) gives

$$J(k_T \,,\, \omega \sim 2\omega_e) = \frac{4\sqrt{3}\ \pi^2\ e^2\ k_T^2}{m^2\ c^3}\ \frac{W^2}{\alpha k_D^3}\ \sin^2 \theta\ \cos^2 \theta \ ,$$

$$\simeq \frac{3\sqrt{3}\pi}{\alpha} \left(\frac{V_e}{c}\right)^5 \left(\frac{W}{n_e KT_e}\right)^2 n_e KT_e\ \omega_e\ \sin^2 \theta\ \cos^2 \theta \ . \qquad (31)$$

The flux at the Earth is

$$F \cong \frac{2\pi\ V_{rad}}{R^2} \left[\frac{J(\underline{k}_T \,,\, 2\omega_e)}{2\omega_e} \right] \frac{erg}{cm^2 sec\ Hz\ ster} \qquad (32)$$

where V_{rad} is the radiating volume and R the distance of the source from the Earth. We have assumed that there is no damping of the radiation.

Using as typical parameters $n_e \approx 8 \times 10^7$ cm^{-3}, $\omega_e \sim 5 \times 10^8$ s^{-1} , $V_e \sim 5 \times 10^8$ cm s^{-1} , $T_e \sim 2 \times 10^6$ K , $V_{rad} \sim 10^{28}$ cm^3 , $R \sim 2 \times 10^{13}$ cm, $\alpha \sim .1$, and defining

$$\lambda = \frac{W}{n_e KT_e} \quad ,$$

we find $\qquad F \sim 2 \times 10^{-9} \, \lambda^2 \, \dfrac{W}{m^2 Hz} \qquad$ at the 80 MHz level.

We should emphasize that we use these parameters only for illustrative purposes. A later, more detailed analysis will include a range of parameters extending from less than a solar radius to 1 A.U.

Observations give $F \sim 10^{-21} - 10^{-17}$ W/m^{2}Hz . Therefore in order to account for the observed flux we need $\lambda \sim 10^{-6} - 10^{-4}$. Because $n_b m V_b \Delta V_b / n_e KT_e \sim 10^{-3} - 10^{-2}$, only a small energy transfer is required to account for the observed power level. Therefore, it does not appear difficult to stabilize electron beams against quasi-linear diffusion, and at the same time to produce sufficient radiation to account for the observed Type III burst intensities.

The determination of the value of λ and the details of the radiation spectrum depend on the detailed nonlinear stabilization mechanism of the oscillating two-stream instability, which will be discussed in a future publication. However some further estimates are possible. If electrostatic fluctuations with $\lambda \sim 10^{-6} - 10^{-4}$ exist in the non resonance part of the spectrum, higher-energy electrons in the ambient thermal distribution will interact with them via Landau damping and form long suprathermal tails. The time scale for this interaction is determined by the diffusion coefficient and is approximately given at the 80 MHz level by

$$\tau \sim \frac{v_e^2}{\alpha^2 D} \sim \frac{\Delta V_{ph}}{\alpha^2 \omega_e \lambda V_{ph}} \sim 5 \times 10^{-8} \, \lambda^{-1} \, s \quad ,$$

where $\Delta V_{ph} \sim V_{ph}$ is the range of phase velocities over which the oscillating two-stream instability is unstable. Therefore for values of $\lambda \sim 10^{-6} - 10^{-4}$ the plasma can form non-Maxwellian tails within times shorter than 1 millisecond. One then finds the marginally stable situation shown in Fig. 6. Marginal stability analysis (Papadopoulos and Pongratz 1973) shows that the tails have a velocity dependence $\sim v^{-n}$, with $n \sim 1-3$. The existence of the tails will stabilize the oscillating two-stream instability, while the enhanced electrostatic fields in the region of the tails will prevent the re-development of the beam-plasma instability. In this way the role of the beam will be to form and sustain these power law tails in the coronal plasma, accompanied by an enhanced level of electrostatic fluctuations.

The level of fluctuations will be determined by the maximum of either the non-thermal equilibrium level due to the long-tails, or the level of fields in the non-resonant region necessary to prevent the beam plasma interaction from resuming. The value of λ due to non-thermal tails is

$$\lambda_o \gtrsim \lambda_t \left(\frac{V_b}{V_e}\right)^2 \quad ,$$

where λ_t is the level of thermal noise. The approximate equality results from the formation of a high temperature Maxwellian tail. However, a power law such as results from marginal stability analysis will produce an even greater enhancement. The value λ_1 to prevent the beam-plasma instability from resuming is given by

$$\lambda_1 = \mu \, E^2 / 8\pi n_e KT_e \quad ,$$

where $\mu = \gamma_{2k}/\gamma_{1k}$ from Eqs. (24) and (11) and E_T is given by Eq. (8) with ν_1 now determined by the tails. The value of $\lambda = \max (\lambda_o , \lambda_1)$. Notice that the value of $\lambda_o \sim 10^{-5}$ which is of the order required to explain the observations.

Similar considerations can be applied with respect to radiation at ω_e .

V. Summary

We have demonstrated that the oscillating two-stream instability, induced by the fields created from the interaction of the electron stream with the coronal plasma, can stabilize the beam against quasilinear diffusion. This interaction creates a marginally stable situation in which the beam propagates essentially unaffected while sustaining in the ambient plasma suprathermal tails and suprathermal enhanced plasma oscillations. These oscillations can then scatter from ambient density fluctuations, and from each other, producing the observed radio emission at the plasma frequency and its second harmonic.

FOOTNOTE

One may easily verify that for the parameters under consideration, the condition for a monochromatic pump spectrum

$$\delta\omega/\gamma_1 \simeq \frac{\partial\omega}{\partial k} \delta K(t)/\gamma_1 < 1$$

is easily satisfied within a couple of growth times of the beam-plasma instability.

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FIGURE CAPTIONS

1. Region of unstable wave generation in the beam plasma interaction.

2. Final form of the electron distribution function predicted by quasi-linear theory. Unstable wave generation has ceased.

3. Band of phase velocities excited by the oscillating two-stream instability.

4. Stability criterion of an electron beam as a function of $\Delta V_b/V_b$ and n_b/n_e, using $V_e = 5 \times 10^8$ cm/s and $V_b = 10^{10}$ cm/s.

5. Scattering of two electrostatic plasma waves at frequency ω_e radiating an electromagnetic wave at frequency $2\omega_e$.

6. Final form of the electron distribution function predicted by the parametric oscillating two-stream instability. Enhanced emission continues.
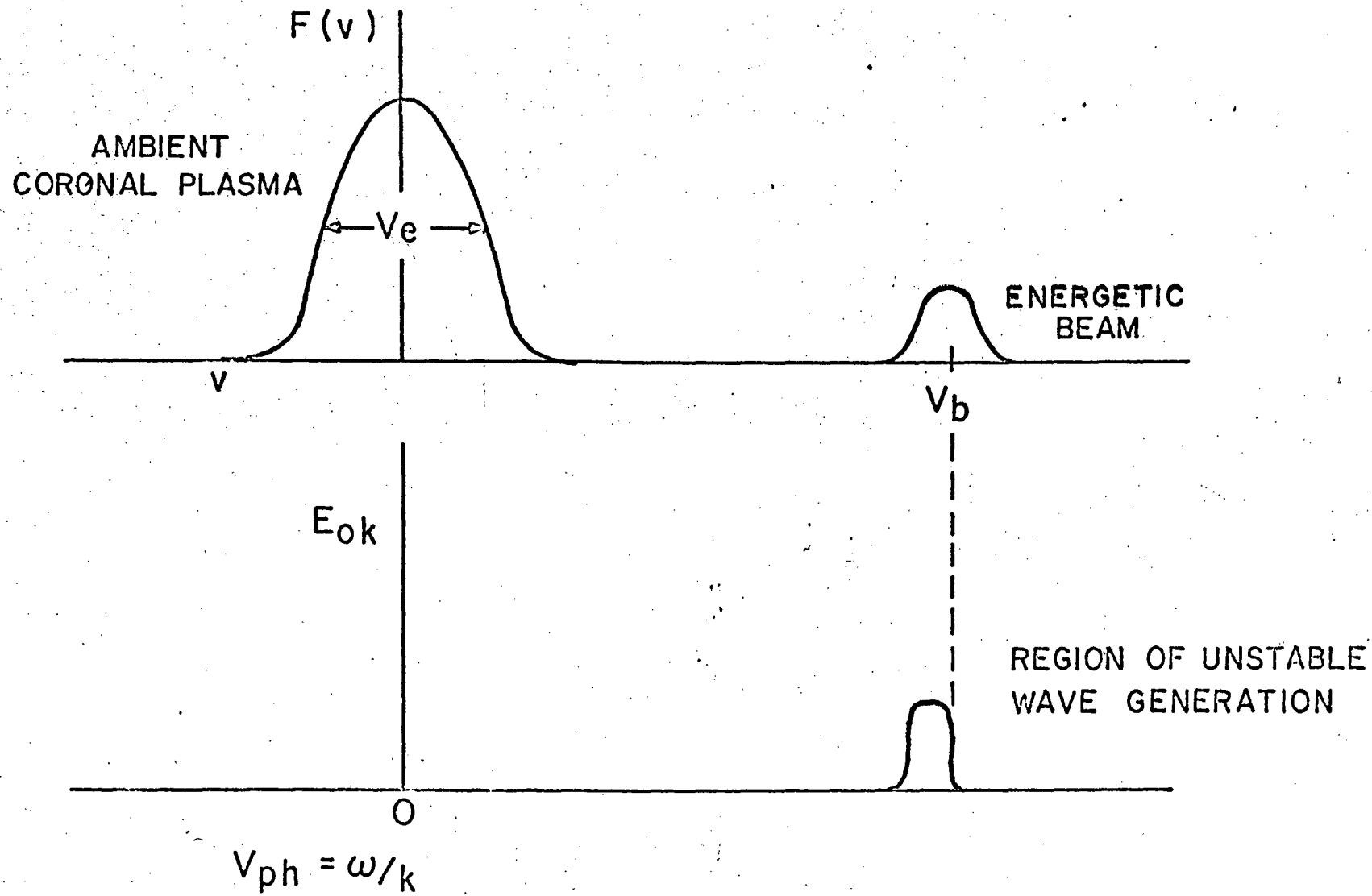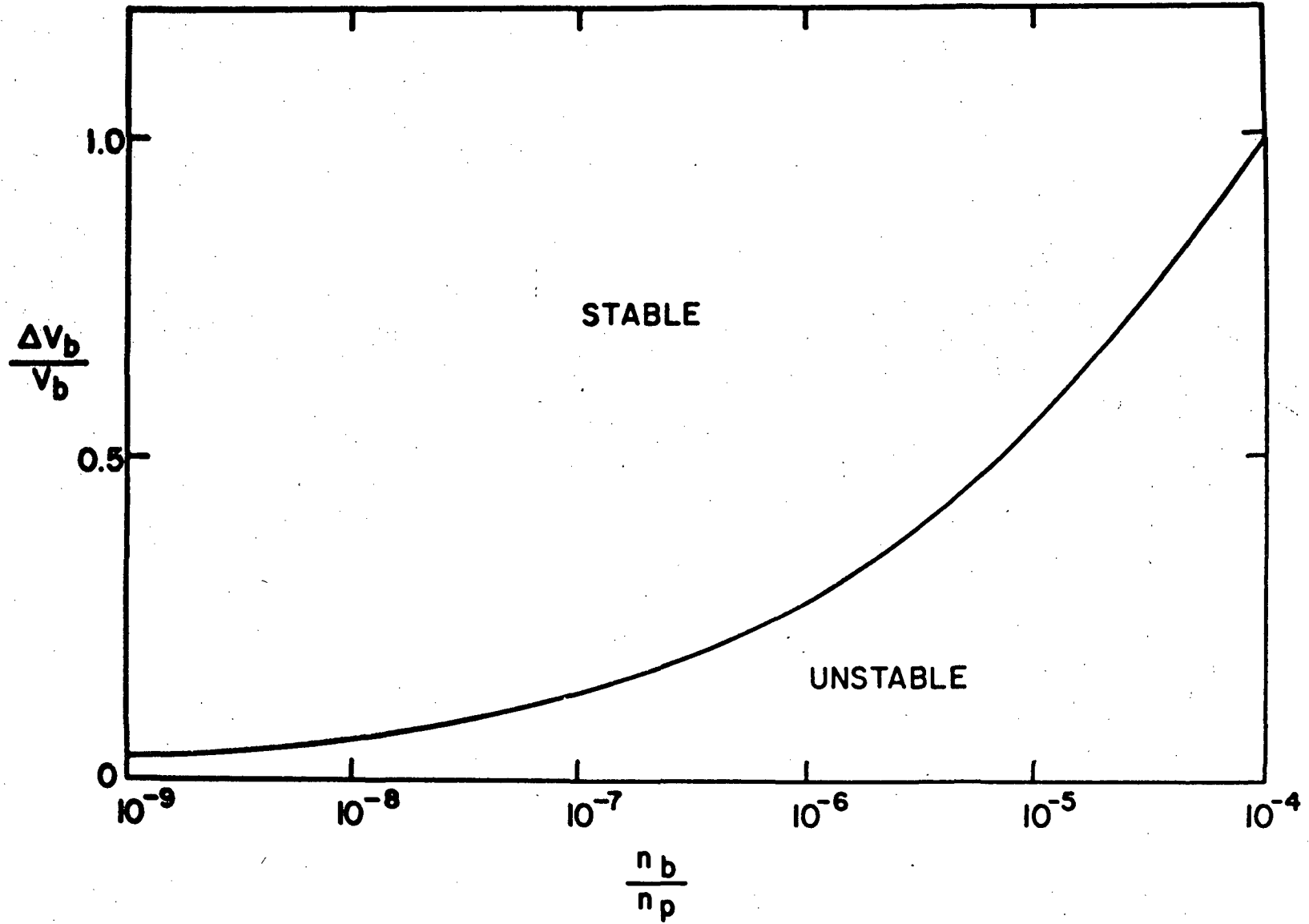
Figure 1
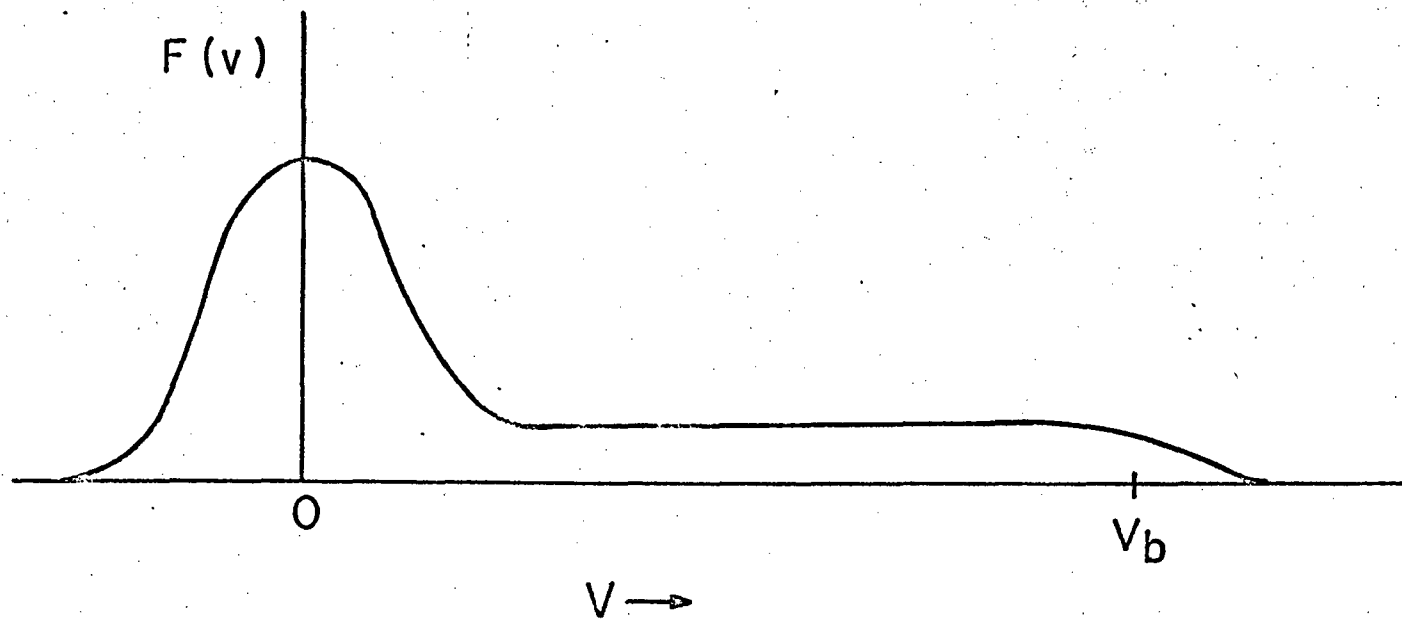
Figure 2

Figure 3

Figure 4

Figure 5

Figure 6