POLAR MOTION AND EARTH TIDES FROM BEACON EXPLORER C

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MAY 1973

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ABSTRACT

Seventeen months of laser tracking of the Beacon Explorer C spacecraft by a Goddard Space Flight Center laser system have been analyzed. The amplitude and phase of the solid-earth and ocean-tide perturbations of the orbit and the variation in latitude of the tracking station have been determined. From an analysis of the orbital inclination obtained from 6 hour data spans the tidal perturbations of the orbit were found to have a magnitude equivalent to a second degree Love number, $k_2$, of 0.245 with a phase lag of about 3.2 degrees. These numbers are in complete agreement with values obtained previously from a much shorter data span, although $k_2$ is lower than the value derived from seismic data. This discrepancy is probably due to the influence of the oceans on the satellite result. After removing the tidal perturbations the residuals in inclination were of order 0.04 arcseconds. This implies that the variation in latitude of the station was being determined during the 17 month period with an rms deviation of about 1.4 meters with respect to the smoothed Bureau International de l'Heure values. When polar motion is not modeled in the orbit determination program the fifteen meter variation in latitude of the station caused by the Chandler and annual wobbles of the earth on its axis are evident in the data.
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INTRODUCTION

For seventeen months during 1970 and 1971 a Goddard Space Flight Center laser tracking station in Greenbelt, Maryland participated in observing programs for the determination of geodynamic and geodetic parameters. In the first of these programs, (Ref. 1) the Beacon Explorer C (BE-C) satellite was used for the determination of the variation of latitude of the tracking station (Ref. 2) arising from polar motion. The results of this program showed that laser ranging measurements from a single tracking station to a single satellite could be used to monitor latitude variations to an accuracy of about 1 meter from 6 hours of tracking data. As part of the same experiment a measurement of the perturbation of the orbit by the solid-earth and ocean-tides was made (Ref. 3). In this analysis it was shown that the tidal perturbations of the BE-C satellite could be modeled almost exactly by a tidal potential function represented by a second degree spherical harmonic off-set slightly in axial symmetry (phase angle) from the disturbing body. The amplitude of the perturbation corresponded to a Love number for the solid-earth of 0.245 ±0.005, but because the value was significantly smaller than the value of about 0.29 obtained from seismic data, it was concluded that the oceans were substantially influencing the result. This had been suggested during the course of the analysis by Lambeck and Cazenave (Ref. 4), and that it was a complex solid-earth and ocean tidal perturbation that was really being observed.

The first observing program lasted five months. It was followed by an international laser and optical tracking experiment, known as ISAGEX (Ref. 5) which lasted 12 months. During this program the laser station at GSFC tracked the BE-C satellite on a number of occasions from which data it was possible to continue the determination of the variation of latitude of the tracking station and to extend the tidal perturbation analysis. The present paper describes the results of these investigations into the variation of latitude and the determination of the earth and ocean tides over the full period of the two experiments, seventeen months.

TECHNIQUE AND RESULTS

From the laser tracking site at GSFC a total of four consecutive passes of the BE-C satellite can be observed during any one day (see Fig. 1). The orbit of BE-C is approximately circular at a height of about 1000 km with an inclination to the equator of 41 degrees. Since the latitude of the Goddard tracking site is about 39 degrees the predominant motion of the satellite, as seen by the station, is from west to east. Consequently, the station is well positioned for making
Figure 1. A Typical BE-C Ground Track
accurate determinations of the orbital inclination of the satellite because on one or two passes each day the satellite can be observed as it moves through the position of maximum northerly latitude (see Fig. 1). At the position of maximum latitude the geocentric latitude is equal to the osculating inclination. The basic orbital arc length used in this analysis was 6 hours, composed of four consecutive passes of BE-C.

During the seventeen months there were 38 occasions when 4 consecutive passes were observed. Twenty-eight of these occurred during the first five months and it has been with the data during this earlier period that most of the analysis has been conducted. Although there were many other occasions when only one, two or perhaps, three passes of BE-C were obtained, none of these data have been used in the analysis. The primary reason for not using these data was that orbital arcs with less than four passes were rather weakly determined. In contrast, four passes provided very good determination of all the elements of the orbit, and in particular, of the orbital inclination. It is the variation in orbital inclination of BE-C that has been investigated here.

Initially, values of the latitude and longitude of the tracking station were adopted and a value of the height determined from the laser data. The technique used for this determination has been described fully in reference 1 but, briefly, it involves taking the 4-pass orbital arcs and recovering six orbit parameters for each arc and a single value of the station height from all the arcs. This value of height is, of course, dependent on the gravitational field model used in the determination of the orbit; and in this investigation it was the GEM 1 gravity field (ref. 6). Subsequently, this field was modified (GEM 1*) by incorporating a change in the values of the degree 19 and order 13 terms with which BE-C is resonant. This change is discussed later and in another paper (ref. 7). The final position adopted for the tracking station (GODLAS) was

Latitude: 39° 01' 13'' 88 north
Longitude: 283° 10' 18'' 50 east
Height: 9.29 meters

Using this position and the GEM 1 gravity field, orbits through the first 28 four-pass orbits (5 months) were determined. From these orbits an ephemeris of the satellite was computed (ref. 7) from which the values of the latitude at the maximum latitude positions were derived. Then an orbit was fitted through three weeks of the laser data toward the beginning of the data span. In contrast to the short 6-hour orbital arcs, in which errors and unmodeled effects in the gravity field were absorbed into the orbit parameters, this long-arc orbit
averaged through most of these errors and could therefore be used as a reference orbit. This "long-arc" was then projected through all 28 short arcs and the latitudes at the maximum latitude positions were derived from an ephemeris. The force models included in the orbit determination procedures were gravity, luni-solar gravity, solar radiation pressure and air drag. Since the long arc through three weeks of data was a least squares best fit solution it represented our best ability to compute the perturbations of BE-C due to the above effects. Thus, subtracting the long-arc maximum latitude values from the short arc maximum latitudes effectively removed the perturbations of gravity, the sun and moon, and air drag from the short-arc (6-hour) orbits. This approach is usually referred to as the "long minus short-arc technique" (for further discussion, see Refs. 1 and 7).

The residuals in inclination (short-arc minus long-arc maximum latitudes) therefore contained perturbations from unmodeled forces (such as tides) and any errors in the force models that were "removed" in the long minus short-arc technique.

The inclination residuals for the five month period approached 1 arcsecond on several occasions. They showed a very clear systematic variation with a period of about 85 days which has subsequently been shown to be caused principally by the solid-earth tides. The perturbations of the orbit due to the solid-earth tides, represented by two second degree spherical harmonics directed toward the sun and moon, were then computed. It was immediately evident that the perturbations computed using a value of about 0.25 for Love's number \( k_2 \) fitted the observed residual pattern extremely well. Several values of \( k_2 \) were used in the tidal computations and the results compared with the observed residuals in inclination.

Figure 2 shows the root mean square of the fit of the tidal model to the inclination residuals against Love's number, \( k_2 \). Although the residual pattern was largely removed when a value of \( k_2 = 0.245 \) was used, there remained a signature within the residuals that could not be removed by changing \( k_2 \). The next stage of complexity that was introduced into the tidal model was a phase lag. The axis of symmetry of the second degree tidal bulge was off-set from the direction of the disturbing body by a few degrees. For a phase lag of about three degrees a small decrease in the rms of the residuals was obtained but, more important, a residual pattern was emerging that was recognizable as a resonance perturbation.

The Beacon Explorer C satellite is resonant with terms of 19th degree and 13th order in the gravity field. These terms produce perturbations with periods of about 5.5 days, which is the approximate period of the signature in the residuals.
From these inclination residuals, using values of $k_2 = 0.245$ and a phase lag of 3.2 degrees, the amplitude and phase of a sinusoidal oscillation with the resonance period was derived. This amplitude and phase was then converted into new values of the resonant terms, $C_{19}^{13}$ and $S_{19}^{13}$ (Ref. 7). The original normalized values (GEM 1) were

$$C_{19}^{13} = -0.0285 \times 10^{-6}$$

$$S_{19}^{13} = -0.0454 \times 10^{-6}$$

and the new values (GEM 1*) were

$$C_{19}^{13} = -0.0240 \times 10^{-6}$$

$$S_{19}^{13} = -0.0557 \times 10^{-6}$$

Figure 2. Effect of Solid-Earth Tides on the Inclination of BE-C with Zero Phase Lag
With these new values of the resonant coefficients the whole analysis was repeated. Although it was re-calculated, the station height remained unchanged. The residuals about the tidal curve were smaller but the best value of $k_2$ remained unchanged. Figure 3 shows the effect of introducing the new resonance terms and a phase lag ($\phi$) on the rms of the residuals. Again, the optimum value of the phase, $\phi = 3.2^\circ$, was unaltered by modifying the resonance terms. Thus, the analysis of the first 5 months of BE-C data indicated (Ref. 3)

\[
k_2 = 0.245 \pm 0.005 \\
\phi = 3.2^\circ \pm 0.5^\circ
\]
together with improved resonance coefficients.

![Figure 3. Effect of Resonance and Tidal Phase Lag on the Inclination of BE-C for $k_2 = 0.245$. --- with Resonance, ---- Resonance Removed.](image)
During the following 12 months a total of 8 four-pass orbits of BE-C were obtained by the GODLAS station. Using the gravitational and tidal models determined from the earlier 5-month period an attempt was made to extrapolate the reference orbit forward through the new data to see if the tidal parameters already determined would satisfy this much longer period. Because of this greater period involved some improvements to the radiation pressure and air drag models were incorporated. These mainly concerned the modeling of the cross-sectional area of the satellite presented to the sun and to velocity vector. Prior to this analysis a fixed cross-sectional area of the satellite, independent of attitude, was used. This was replaced with a variable cross-section that described the complex shape of the BE-C satellite (see ref. 7).

With this improvement the maximum latitudes for the new data were derived, using the same station position, and the long-arc orbit extended for a full seventeen months. The agreement between the long arc maximum latitudes and the short arc maximum latitudes for the whole 17 month period was slightly better than for the 5-month period, having an rms of fit of 0.045 arcseconds. Figure 4 shows the variation in inclination for seventeen months as indicated in the data and computed in the long-arc reference orbit. It should be emphasized that the orbit adjustment was for only a 3-week data span and that this orbit was extrapolated backwards and forwards to cover the full 17 months. The agreement between the observation and the theory based on a second degree spherical harmonic expansion, using values of \( k_2 \) and \( \varphi \) already determined, is remarkable. Figure 4 confirms the results already obtained for \( k_2 \) from the five month data span and shows no indication of any change in the tidal parameters through the seventeen month period.

From seismic and other data, values of \( k_2 \) for the solid-earth can also be derived and all these approaches indicate a value near 0.30. A probable explanation of this difference in value of \( k_2 \) has been advanced by Lambeck and Cazenave (Ref. 4) who suggest that the satellite result is not a direct measure of \( k_2 \) for the solid-earth alone, but is a combination of a solid-earth and ocean-tide perturbation. They indicate that certain terms in the oceanic tidal potential function can produce perturbations of the satellite orbit that resemble closely the solid-earth perturbations and therefore modify the interpretation of the satellite results. If the seismic values are accepted as correct (Ref. 8) and had been applied in our computations then our result would correspond to an observation of the ocean tide perturbation on BE-C. The magnitude of this perturbation is then 15% of the expected solid-earth component with a phase difference of 180 degrees.
Figure 4. Tidal Perturbation of BE-C Inclination for $k_2 = 0.245$, $\varphi = 3.2^\circ$

- Laser data, --- Theory
Throughout this investigation we have modeled the rotation of the earth according to the announcements of the Bureau Internation de l'Heure (BIH). In particular, the position of the earth's pole of rotation given by the BIH has been used in the orbit calculations to compute the variation of latitude of the GODLAS tracking station from its nominal position. If polar motion had not been included in the calculations, the latitude of the tracking station would have been in error and this error would have been absorbed directly into the orbital inclination. Thus, the variation in orbital inclination over many months would have included a component due to the variation in latitude of the tracking station, that is, polar motion.

Using the GEM 1* gravity model and the tidal parameters, $k_2$ and $\phi$ already discussed, the variation in latitude of GODLAS has been derived and is shown in Figure 5. Effectively, no information on the pole position was used in the determination of the short 6-hour orbits, thus forcing the latitude variation into the orbital inclination. Figure 5 also shows the smoothed BIH latitude variation that had been used previously. The BIH variation is derived from astronomical measures obtained at 30 to 40 observatories around the world, averaged over a 5-day period to provide a raw pole position (5-day mean), and then smoothed through the 5-day means to provide the curve shown in Figure 5.

It is not immediately obvious that the variation of latitude obtained from the laser data is any better than the BIH smoothed. However, variations in the position of the pole over about 24 hours can be larger than a meter due to the forced diurnal nutation and thus the BIH curve must be smoothing through motions of the pole to which the 6-hour laser data could be sensitive. The rms of the laser data about the BIH curve is about 1.38 meters but the residuals do not appear to be randomly distributed. It is possible that the laser data contain information about the diurnal motions of the pole but we believe it more likely that there remain perturbations of the orbit, such as ocean-tides, that have not yet been fully removed from the inclination.

CONCLUSIONS

Seventeen months of range data from a Goddard Space Flight Center laser tracking station have been analyzed to provide determinations of the tidal perturbations of the BE-C satellite and the variation in latitude of the tracking station. The tidal perturbations of the orbital inclination have a clear and distinctive signature that can be represented by a second degree spherical harmonic to about 5%. The amplitude of the observed perturbation is, however, approximately 15% smaller than expected from seismic studies of the elastic deformation of the earth and we are forced to conclude that the observed perturbation is a result of the combined effects of both the solid-earth and the oceans.
Figure 5. Variation in Latitude of the Goddard Laser Tracking Station
• Determined from Laser Data, ———— BIH Smoothed Values
The variation in latitude of the station due to polar motion was determined with a standard deviation of 1.38 meters with respect to the smoothed BIH values. The results showed the large variation in latitude due to the Chandler and annual motions of the pole and demonstrated that a single laser tracking system can usefully monitor polar motion in its meridian.

An integral part of the technique that was used for both the tidal and polar motion analyses was the determination and projection of an orbital arc over a seventeen month period. Anticipated numerical difficulties in the integration of the orbit over time spans this large were avoided by choosing an appropriate step-size to limit the errors of truncation and round-off. Consequently, we believe our results are not limited by arithmetical errors but rather by errors in the force models, such as gravity and tides. This conclusion is strengthened by the fact that our orbital fit over seventeen months was as good as that over five months.

REFERENCES


