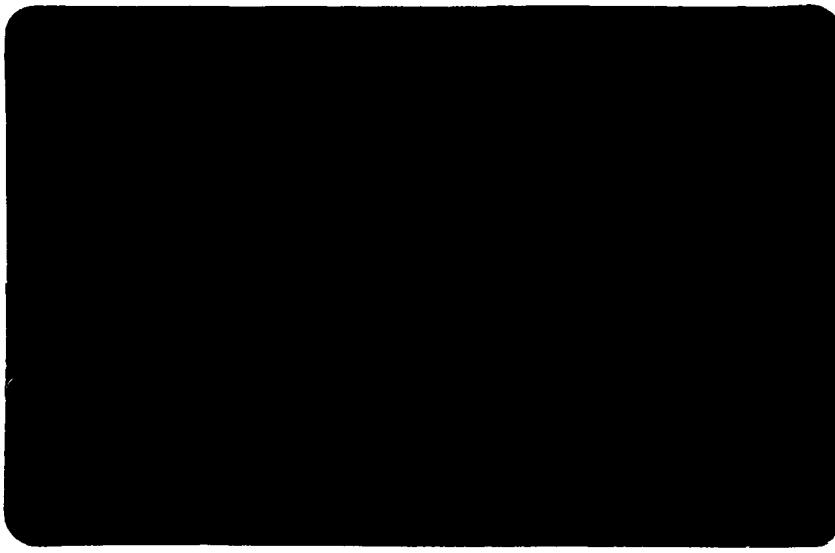N73-31333

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A RANDOM NUMBER GENERATOR FOR CONTINUOUS RANDOM VARIABLES

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ABSTRACT

A FORTRAN IV routine is given which may be used to generate random observations of a continuous real valued random variable. To use the procedure it is necessary to have as input only a table

$$\{(x_{(i)}, F(x_{(i)})); i = 0,1,2, \ldots k, k+1\} \text{ where } x_{(0)} < x_{(1)} < \ldots < x_{(k+1)}$$

and $F(x_{(1)}) \sim 0$, $F(x_{(k)}) \sim 1$.

A necessary part of any simulation study is the sampling of values of X , a random variable with cumulative distribution function F(X) . F may be specified either by a mathematical formula or empirically by a table of (X,F(X)) values. We consider here the special case where X is a real valued continuous random variable. Regardless of the specific form of F(X) , the probability distribution of the random variable Y = F(X) is the uniform distribution U[0,1] on the unit interval.

There are a great many satisfactory pseudorandom number generators to obtain values from U[0,1] , e.g. [2]

$$(1.1) \quad y_{n+1} = 31415926 \, y_n + 27182818 \pmod{2^{35}}.$$

The problem of sampling from F(X) may be looked at, then, as equivalent to finding an inversion formula

$$(1.2) \quad X = F^{-1}(Y) .$$

When it is not convenient to express $F^{-1}(Y)$ as a mathematical formula we may use an empirically obtained table $\{x_{(i)}, F(x_{(i)},) ;$ $x_{(1)} < x_{(2)} < \cdots < x_{(k)}, F(x_{(1)}) \sim 0 , F(X_{(k)}) \sim 1\}$ plus a good interpolation routine $F_I^{-1}(Y)$ to take random values from U[0,1] and

obtain their approximate inverses as random samples from "nearly F(X)" .

A random number generator is advocated which uses an interpolation based on the highly stable piecewise cubic interpolation formula of Akima [1] . The user reads in a table $\{x_{(i)}, F(x_{(i)})$; $x_{(0)} < x_{(1)} < \ldots < x_{(k)} < x_{(k+1)}\}$. The routine then computes and stores the coefficients of $X = F_I^{-1}(Y)$. Calls on the random number generator will first obtain a pseudorandom number y from $U[0,1]$. If $y \leq F(x_{(1)})$, the random number given is $x_{(1)}$, if $y \geq F(x_{(k)})$, the random number given is $x_{(k)}$. If $F(x_{(1)}) < y < F(x_{(k)})$, the random number given is $F_I^{-1}(y)$. A listing of the random number generator is given in Appendix I .

Naturally, the quality of the random number generator proposed is determined by the ability of F_I^{-1} to approximate a variety of cumulative distribution functions. In Appendix II , we show how the interpolation errors of $F_I^{-1}(Y)$ compare with those of linear interpolation in the case of the standard normal, Cauchy and double exponential distributions with location 0 and scale 1 . Exact values of $F^{-1}(Y)$ are given for $Y = .50(.02).98(.005).995$. Interpolated values of F_I^{-1} and linear interpolation are obtained at points between the mesh values and the relative errors computed and compared.

REFERENCES

1. Akima, H., A new method of interpolation and smooth curve fitting based on local procedures, J. A.C.M., 17(1970),589-602.

2. Knuth, Donald E., Seminumerical Algorithms, Addison-Wesley, Reading, Mass., 1969.

APPENDIX I

```
      C INTERPOLATION OF THE INVERSE NORML  DISTRIBUTION
            COMMON N,P1(50),P2(50),P3(50),P4(50),X(50)
      C N IS THE NUMBER OF INTERPOLATING KNOTS, N SHOULD BE LESS OR EQ TO 50
            N = 50
            CALL INIT
      C M IS THE NUMBER OF RANDOM NUMBERS TO BE GENERATED M SMALLER THAN 20
            M = 20
            CALL RNG(M)
             CALL EXIT
            END
```

```
      C INTERPOLATION OF THE INVERSE NORML  DISTRIBUTION
            COMMON N,P1(50),P2(50),P3(50),P4(50),X(50)
      C N IS THE NUMBER OF INTERPOLATING KNOTS, N SHOULD BE LESS OR EQ TO 50
```

```
         SUBROUTINE INIT
      C ******************
      C THIS SUBROUTINE GENERATES THE PARAMETERS OF THE CUBICS THAT I
      C    INTERPOLATE THE DATA KNOTS
      C    ARRAYS P1,P2,P3,P4 CONTAIN THE PARAMETERS
         COMMON N,P1(50),P2(50),P3(50),P4(50),X(50)
100      FORMAT (2F12.6)
200      FORMAT  (2X,10F12.6)
      C ******* READ DATA ***********************
         READ 100,((X(I),P4(I)),I=1,N)
      C ********** READ DATA FINISHES ***********
         N2 = N-2
         S1=(P4(2)-P4(1))/(X(2)-X(1))
         S2=(P4(3)-P4(2))/(X(3)-X(2))
         S3=(P4(3)-P4(4))/(X(3)-X(4))
         DO 10 I=3,N2
         S4 = (P4(I+2) -P4(I+1))/(X(I+2) - X(I+1))
         W1 = ABS(S2-S1)
         W3 = ABS(S4-S3)
         IF (W1*W3) 20,30,20
20       P3(I) = (W3*S2 + W1*S3)/(W1+W3)
         GO TO 40
30       P3(I) = .5*(S2+S3)
40       S1 = S2
         S2 = S3
10       S3 = S4
         DO 60 I=3,N2
         A= X(I+1) - X(I)
         P1(I) = (P3(I) + P3(I+1) - 2.0*(P4(I+1) - P4(I))/A)/(A*A)
         P2(I)=(3.0*(P4(I+1)-P4(I)) /A - 2.0*P3(I) - P3(I+1))/A
60       CONTINUE
         RETURN
         END
```

```fortran
      SUBROUTINE RNG(M)
      COMMON N,P1(50),P2(50),P3(50),P4(50),X(50)
      REAL*8 RN(20),I34,I35
100   FORMAT (I6,9F12.6)
C ****** RANDOM NUMBER GENERATOR ********
      I34 = 57721566
      I35 = 2.0**35
      DO 10 I=1,M
      I34 = ( 31415926 *I34 + 27182828  )
      I34 = (DMOD(I34,I35))
      RN(I) = I34/I35
      PRINT 100,I,RN(I)
      IF (RN(I).LE.X(3)) RN(I) = X(3)
10    IF (RN(I).GE.X(N-2)) RN(I)= X(N-2)
      IC = 1
      N3 = N-3
      DO 20 I =3,N3
      DO 30 J=1,M
      V = RN(J) - X(I)
      IF (RN(J)-X(I)) 40,50,50
50    IF (RN(J) - X(I+1)) 60,60,40
60    YRN  = P1(I)*V*V*V + P2(I)*V*V + P3(I)*V + P4(I)
C  YRN IS THE GENERATED RANDOM OBSERVATION
      PRINT 100,IC,RN(J),YRN
      IC = IC + 1
40    CONTINUE
      IF (IC.GT.M) GO TO 9
30    CONTINUE
20    CONTINUE
9     CONTINUE
      RETURN
      END
```

NORMAL DISTRIBUTION

F(X)=	X=	E(AKIMAS)=	E(LINEAR)=
0.50000	-0.00000		
0.50333	0.00833	0.00002	0.00007
0.50667	0.01666	0.00003	0.00007
0.51000	0.02500	0.00000	0.00000
0.51333	0.03334	0.00001	0.00013
0.51667	0.04168	0.00002	0.00014
0.52000	0.05002	0.00000	0.00000
0.52333	0.05837	0.00002	0.00020
0.52667	0.06672	0.00002	0.00021
0.53000	0.07508	0.00000	0.00000
0.53333	0.08345	0.00001	0.00027
0.53667	0.09182	0.00002	0.00028
0.54000	0.10020	0.00000	0.00000
0.54333	0.10858	0.00002	0.00034
0.54667	0.11698	0.00002	0.00035
0.55000	0.12538	0.00000	0.00000
0.55333	0.13379	0.00002	0.00041
0.55667	0.14222	0.00002	0.00042
0.56000	0.15065	0.00000	0.0
0.56333	0.15910	0.00002	0.00048
0.56667	0.16756	0.00002	0.00049
0.57000	0.17603	0.00000	0.0
0.57333	0.18451	0.00002	0.00055
0.57667	0.19301	0.00002	0.00056
0.58000	0.20152	0.00000	0.0
0.58333	0.21005	0.00002	0.00063
0.58667	0.21859	0.00002	0.00064
0.59000	0.22715	0.00000	0.00000
0.59333	0.23573	0.00002	0.00070
0.59667	0.24432	0.00002	0.00071
0.60000	0.25293	0.00000	0.00000
0.60333	0.26157	0.00002	0.00078
0.60667	0.27022	0.00002	0.00079
0.61000	0.27889	0.00000	0.00000
0.61333	0.28759	0.00002	0.00086
0.61667	0.29630	0.00002	0.00087
0.62000	0.30504	0.00000	0.00000
0.62333	0.31381	0.00002	0.00094
0.62667	0.32260	0.00002	0.00095
0.63000	0.33141	0.00000	0.00000
0.63333	0.34025	0.00002	0.00103
0.63667	0.34912	0.00002	0.00104
0.64000	0.35801	0.00000	0.00000
0.64333	0.36694	0.00002	0.00112
0.64667	0.37589	0.00003	0.00113
0.65000	0.38488	0.00000	0.0
0.65333	0.39389	0.00002	0.00121
0.65667	0.40294	0.00003	0.00122
0.66000	0.41202	0.00000	0.0
0.66333	0.42114	0.00002	0.00131
0.66667	0.43029	0.00003	0.00132
0.67000	0.43948	0.00000	0.00000
0.67333	0.44871	0.00002	0.00140
0.67667	0.45797	0.00003	0.00142
0.68000	0.46727	0.00000	0.00000
0.68333	0.47662	0.00003	0.00151
0.68667	0.48601	0.00003	0.00152
0.69000	0.49544	0.00000	0.00000

0.69333	0.50491	0.00003	0.00162
0.69667	0.51443	0.00003	0.00163
0.70000	0.52400	0.00000	0.00000
0.70333	0.53362	0.00003	0.00173
0.70667	0.54328	0.00004	0.00175
0.71000	0.55300	0.00000	0.00000
0.71333	0.56277	0.00003	0.00186
0.71667	0.57260	0.00004	0.00187
0.72000	0.58248	0.00000	0.00000
0.72333	0.59241	0.00003	0.00199
0.72667	0.60241	0.00004	0.00201
0.73000	0.61247	0.00000	0.0
0.73333	0.62259	0.00004	0.00212
0.73667	0.63277	0.00005	0.00214
0.74000	0.64302	0.00000	0.00000
0.74333	0.65334	0.00004	0.00227
0.74667	0.66373	0.00005	0.00229
0.75000	0.67419	0.00000	0.0
0.75333	0.68472	0.00004	0.00243
0.75667	0.69534	0.00006	0.00246
0.76000	0.70603	0.00000	0.00000
0.76333	0.71680	0.00005	0.00260
0.76667	0.72765	0.00006	0.00263
0.77000	0.73860	0.00000	0.00000
0.77333	0.74963	0.00005	0.00279
0.77667	0.76075	0.00007	0.00282
0.78000	0.77197	0.00000	0.00000
0.78333	0.78329	0.00005	0.00299
0.78667	0.79470	0.00008	0.00302
0.79000	0.80623	0.00000	0.00000
0.79333	0.81786	0.00006	0.00321
0.79667	0.82960	0.00009	0.00324
0.80000	0.84146	0.00000	0.00000
0.80333	0.85343	0.00007	0.00344
0.80667	0.86553	0.00010	0.00349
0.81000	0.87776	0.00000	0.0
0.81333	0.89012	0.00007	0.00371
0.81667	0.90262	0.00011	0.00375
0.82000	0.91526	0.00000	0.0
0.82333	0.92805	0.00008	0.00400
0.82667	0.94099	0.00013	0.00405
0.83000	0.95410	0.00000	0.00000
0.83333	0.96736	0.00009	0.00433
0.83667	0.98080	0.00015	0.00439
0.84000	0.99442	0.00000	0.00000
0.84333	1.00823	0.00009	0.00471
0.84667	1.02223	0.00016	0.00476
0.85000	1.03643	0.00000	0.0
0.85333	1.05085	0.00012	0.00511
0.85667	1.06548	0.00018	0.00518
0.86000	1.08035	0.00000	0.00000
0.86333	1.09546	0.00013	0.00560
0.86667	1.11083	0.00024	0.00569
0.87000	1.12646	0.00000	0.00000
0.87333	1.14237	0.00014	0.00616
0.87667	1.15858	0.00028	0.00626
0.88000	1.17509	0.00000	0.0
0.88333	1.19193	0.00018	0.00680
0.88667	1.20912	0.00033	0.00692
0.89000	1.22667	0.00000	0.00000
0.89333	1.24460	0.00019	0.00759

0.89667	1.26295	0.00044	0.00774
0.90000	1.28173	0.00000	0.0
0.90333	1.30097	0.00023	0.00852
0.90667	1.32071	0.00055	0.00871
0.91000	1.34097	0.00000	0.0
0.91333	1.36180	0.00026	0.00968
0.91667	1.38323	0.00072	0.00991
0.92000	1.40532	0.00000	0.0
0.92333	1.42812	0.00030	0.01115
0.92667	1.45168	0.00099	0.01146
0.93000	1.47608	0.00000	0.0
0.93333	1.50139	0.00034	0.01306
0.93667	1.52769	0.00142	0.01349
0.94000	1.55510	0.00000	0.0
0.94333	1.58372	0.00035	0.01568
0.94667	1.61370	0.00220	0.01630
0.95000	1.64521	0.00000	0.00000
0.95333	1.67844	0.00023	0.01946
0.95667	1.71363	0.00370	0.02039
0.96000	1.75108	0.00000	0.00000
0.96333	1.79115	0.00186	0.02539
0.96667	1.83433	0.00277	0.02697
0.97000	1.88121	0.00000	0.00000
0.97333	1.93264	0.00101	0.03603
0.97667	1.98975	0.00830	0.03921
0.98000	2.05419	0.00000	0.00000
0.98167	2.08990	0.00450	0.02640
0.98333	2.12849	0.00335	0.02805
0.98500	2.17054	0.00000	0.0
0.98667	2.21681	0.00174	0.03720
0.98833	2.26838	0.00562	0.04050
0.99000	2.32679	0.00000	0.00000
0.99167	2.39441	0.00661	0.06224
0.99333	2.47517	0.02432	0.07186
0.99500	2.57624	0.00000	0.00000

CAUCHY DISTRIBUTION

F(X)=	X=	E(AKIMAS)=	E(LINEAR)=
0.50000	0.0		
0.50333	0.01047	0.00002	0.00010
0.50667	0.02095	0.00002	0.00012
0.51000	0.03143	0.0	0.00000
0.51333	0.04191	0.00010	0.00032
0.51667	0.05241	0.00012	0.00034
0.52000	0.06291	0.00000	0.00000
0.52333	0.07344	0.00005	0.00054
0.52667	0.08397	0.00005	0.00056
0.53000	0.09453	0.00000	0.00000
0.53333	0.10510	0.00005	0.00076
0.53667	0.11570	0.00005	0.00078
0.54000	0.12633	0.00000	0.0
0.54333	0.13698	0.00005	0.00098
0.54667	0.14767	0.00005	0.00101
0.55000	0.15838	0.00000	0.00000
0.55333	0.16914	0.00005	0.00121
0.55667	0.17993	0.00006	0.00123
0.56000	0.19076	0.00000	0.00000
0.56333	0.20164	0.00005	0.00143
0.56667	0.21256	0.00006	0.00146
0.57000	0.22353	0.00000	0.00000
0.57333	0.23455	0.00006	0.00166
0.57667	0.24562	0.00006	0.00169
0.58000	0.25676	0.00000	0.00000
0.58333	0.26795	0.00005	0.00190
0.58667	0.27921	0.00006	0.00192
0.59000	0.29053	0.00000	0.0
0.59333	0.30192	0.00006	0.00213
0.59667	0.31338	0.00007	0.00216
0.60000	0.32492	0.00000	0.00000
0.60333	0.33654	0.00006	0.00237
0.60667	0.34824	0.00007	0.00240
0.61000	0.36002	0.00000	0.0
0.61333	0.37190	0.00006	0.00262
0.61667	0.38386	0.00008	0.00266
0.62000	0.39593	0.00000	0.0
0.62333	0.40809	0.00006	0.00287
0.62667	0.42036	0.00008	0.00291
0.63000	0.43274	0.00000	0.00000
0.63333	0.44523	0.00007	0.00313
0.63667	0.45784	0.00009	0.00318
0.64000	0.47056	0.00000	0.00000
0.64333	0.48342	0.00007	0.00340
0.64667	0.49640	0.00010	0.00344
0.65000	0.50953	0.00000	0.00000
0.65333	0.52279	0.00008	0.00367
0.65667	0.53620	0.00010	0.00372
0.66000	0.54975	0.00000	0.0
0.66333	0.56347	0.00008	0.00396
0.66667	0.57735	0.00011	0.00400
0.67000	0.59140	0.00000	0.00000
0.67333	0.60562	0.00009	0.00425
0.67667	0.62003	0.00012	0.00430
0.68000	0.63462	0.00000	0.0
0.68333	0.64941	0.00009	0.00456
0.68667	0.66440	0.00013	0.00462
0.69000	0.67960	0.00000	0.0

0.69333	0.69502	0.00010	0.00487
0.69667	0.71066	0.00014	0.00493
0.70000	0.72654	0.00000	0.00000
0.70333	0.74267	0.00011	0.00521
0.70667	0.75904	0.00015	0.00527
0.71000	0.77568	0.00000	0.00000
0.71333	0.79259	0.00012	0.00556
0.71667	0.80978	0.00017	0.00563
0.72000	0.82727	0.00000	0.00000
0.72333	0.84507	0.00013	0.00592
0.72667	0.86318	0.00019	0.00600
0.73000	0.88162	0.00000	0.0
0.73333	0.90040	0.00014	0.00631
0.73667	0.91955	0.00020	0.00640
0.74000	0.93906	0.00000	0.00000
0.74333	0.95897	0.00015	0.00672
0.74667	0.97927	0.00022	0.00681
0.75000	1.00000	0.00000	0.00000
0.75333	1.02117	0.00015	0.00716
0.75667	1.04279	0.00025	0.00726
0.76000	1.06489	0.00000	0.00000
0.76333	1.08749	0.00017	0.00761
0.76667	1.11061	0.00029	0.00774
0.77000	1.13428	0.00000	0.0
0.77333	1.15851	0.00019	0.00811
0.77667	1.18334	0.00031	0.00823
0.78000	1.20879	0.00000	0.00000
0.78333	1.23490	0.00021	0.00865
0.78667	1.26169	0.00034	0.00878
0.79000	1.28919	0.00000	0.0
0.79333	1.31745	0.00023	0.00922
0.79667	1.34650	0.00039	0.00937
0.80000	1.37638	0.00000	0.0
0.80333	1.40714	0.00025	0.00985
0.80667	1.43881	0.00044	0.01001
0.81000	1.47146	0.00000	0.00000
0.81333	1.50512	0.00028	0.01053
0.81667	1.53987	0.00049	0.01071
0.82000	1.57575	0.00000	0.0
0.82333	1.61283	0.00030	0.01128
0.82667	1.65120	0.00056	0.01148
0.83000	1.69091	0.00000	0.00000
0.83333	1.73205	0.00034	0.01211
0.83667	1.77471	0.00065	0.01234
0.84000	1.81899	0.00000	0.0
0.84333	1.86499	0.00037	0.01304
0.84667	1.91282	0.00076	0.01331
0.85000	1.96261	0.00000	0.00000
0.85333	2.01449	0.00041	0.01408
0.85667	2.06860	0.00089	0.01440
0.86000	2.12510	0.00000	0.0
0.86333	2.18419	0.00046	0.01526
0.86667	2.24604	0.00107	0.01564
0.87000	2.31086	0.00000	0.0
0.87333	2.37890	0.00051	0.01662
0.87667	2.45043	0.00128	0.01706
0.88000	2.52571	0.00000	0.00000
0.88333	2.60509	0.00057	0.01820
0.88667	2.68892	0.00157	0.01873
0.89000	2.77760	0.00000	0.00000
0.89333	2.87161	0.00062	0.02006

0.89667	2.97144	0.00196	0.02070
0.90000	3.07768	0.00000	0.00000
0.90333	3.19100	0.00068	0.02229
0.90667	3.31216	0.00250	0.02308
0.91000	3.44202	0.00000	0.0
0.91333	3.58160	0.00072	0.02502
0.91667	3.73205	0.00328	0.02601
0.92000	3.89474	0.00000	0.0
0.92333	4.07127	0.00072	0.02844
0.92667	4.26353	0.00445	0.02972
0.93000	4.47373	0.00000	0.00000
0.93333	4.70463	0.00061	0.03286
0.93667	4.95945	0.00631	0.03458
0.94000	5.24217	0.00000	0.0
0.94333	5.55776	0.00021	0.03882
0.94667	5.91236	0.00950	0.04123
0.95000	6.31374	0.00000	0.0
0.95333	6.77198	0.00097	0.04730
0.95667	7.30019	0.01549	0.05092
0.96000	7.91579	0.00000	0.0
0.96333	8.64274	0.00113	0.06036
0.96667	9.51440	0.01719	0.06638
0.97000	10.57886	0.00000	0.0
0.97333	11.90868	0.00677	0.08316
0.97667	13.61746	0.03465	0.09503
0.98000	15.89446	0.00000	0.0
0.98167	17.34325	0.00691	0.06051
0.98333	19.08113	0.02053	0.06658
0.98500	21.20479	0.00000	0.00000
0.98667	23.85944	0.00264	0.08326
0.98833	27.27147	0.03265	0.09517
0.99000	31.82018	0.00000	0.00000
0.99167	38.18890	0.02764	0.13328
0.99333	47.73947	0.08864	0.16661
0.99500	63.65543	0.00000	0.00000

DOUBLE EXP DISTRIBUTION

F(X)=	X=	E(AKIMAS)=	E(LINEAR)=
0.50000	2.07944		
0.50333	2.09951	0.00002	0.00223
0.50667	2.11971	0.00002	0.00225
0.51000	2.14005	0.00000	0.0
0.51333	2.16053	0.00002	0.00228
0.51667	2.18115	0.00002	0.00230
0.52000	2.20191	0.00000	0.0
0.52333	2.22281	0.00003	0.00233
0.52667	2.24387	0.00002	0.00234
0.53000	2.26507	0.00000	0.0
0.53333	2.28642	0.00003	0.00238
0.53667	2.30793	0.00002	0.00239
0.54000	2.32959	0.00000	0.0
0.54333	2.35140	0.00002	0.00244
0.54667	2.37338	0.00003	0.00245
0.55000	2.39552	0.00000	0.00000
0.55333	2.41783	0.00002	0.00249
0.55667	2.44030	0.00002	0.00250
0.56000	2.46294	0.00000	0.00000
0.56333	2.48575	0.00002	0.00255
0.56667	2.50874	0.00002	0.00255
0.57000	2.53191	0.00000	0.00000
0.57333	2.55526	0.00003	0.00260
0.57667	2.57879	0.00004	0.00263
0.58000	2.60250	0.00000	0.00000
0.58333	2.62641	0.00002	0.00268
0.58667	2.65050	0.00004	0.00269
0.59000	2.67479	0.00000	0.0
0.59333	2.69928	0.00003	0.00273
0.59667	2.72398	0.00004	0.00276
0.60000	2.74887	0.00000	0.0
0.60333	2.77398	0.00003	0.00281
0.60667	2.79929	0.00003	0.00282
0.61000	2.82483	0.00000	0.00000
0.61333	2.85058	0.00003	0.00288
0.61667	2.87655	0.00004	0.00290
0.62000	2.90275	0.00000	0.00000
0.62333	2.92918	0.00003	0.00295
0.62667	2.95585	0.00004	0.00297
0.63000	2.98276	0.00000	0.00000
0.63333	3.00991	0.00003	0.00304
0.63667	3.03730	0.00004	0.00305
0.64000	3.06495	0.00000	0.0
0.64333	3.09286	0.00004	0.00312
0.64667	3.12103	0.00005	0.00314
0.65000	3.14947	0.00000	0.00000
0.65333	3.17817	0.00004	0.00321
0.65667	3.20716	0.00005	0.00324
0.66000	3.23643	0.00000	0.00000
0.66333	3.26599	0.00003	0.00331
0.66667	3.29584	0.00006	0.00333
0.67000	3.32599	0.00000	0.0
0.67333	3.35645	0.00005	0.00340
0.67667	3.38721	0.00005	0.00343
0.68000	3.41830	0.00000	0.00000
0.68333	3.44972	0.00004	0.00351
0.68667	3.48146	0.00005	0.00353
0.69000	3.51355	0.00000	0.0

0.69333	3.54598	0.00004	0.00363
0.69667	3.57877	0.00006	0.00366
0.70000	3.61192	0.00000	0.00000
0.70333	3.64544	0.00005	0.00375
0.70667	3.67934	0.00006	0.00378
0.71000	3.71362	0.00000	0.00000
0.71333	3.74831	0.00005	0.00389
0.71667	3.78339	0.00007	0.00391
0.72000	3.81890	0.00000	0.0
0.72333	3.85483	0.00006	0.00402
0.72667	3.89119	0.00008	0.00406
0.73000	3.92800	0.00000	0.00000
0.73333	3.96527	0.00006	0.00418
0.73667	4.00300	0.00009	0.00421
0.74000	4.04122	0.00000	0.00000
0.74333	4.07993	0.00006	0.00434
0.74667	4.11915	0.00009	0.00438
0.75000	4.15888	0.00000	0.00000
0.75333	4.19915	0.00007	0.00452
0.75667	4.23997	0.00010	0.00456
0.76000	4.28135	0.00000	0.0
0.76333	4.32331	0.00007	0.00471
0.76667	4.36586	0.00011	0.00475
0.77000	4.40903	0.00000	0.0
0.77333	4.45282	0.00008	0.00491
0.77667	4.49727	0.00012	0.00496
0.78000	4.54238	0.00000	0.0
0.78333	4.58819	0.00009	0.00514
0.78667	4.63470	0.00013	0.00519
0.79000	4.68194	0.00000	0.0
0.79333	4.72995	0.00009	0.00539
0.79667	4.77873	0.00014	0.00545
0.80000	4.82831	0.00000	0.00000
0.80333	4.87874	0.00010	0.00567
0.80667	4.93002	0.00016	0.00573
0.81000	4.98219	0.00000	0.00000
0.81333	5.03529	0.00011	0.00597
0.81667	5.08935	0.00017	0.00604
0.82000	5.14440	0.00000	0.00000
0.82333	5.20047	0.00013	0.00631
0.82667	5.25762	0.00020	0.00639
0.83000	5.31587	0.00000	0.0
0.83333	5.37528	0.00014	0.00669
0.83667	5.43589	0.00023	0.00678
0.84000	5.49775	0.00000	0.00000
0.84333	5.56091	0.00015	0.00712
0.84667	5.62542	0.00026	0.00722
0.85000	5.69136	0.00000	0.00000
0.85333	5.75878	0.00016	0.00761
0.85667	5.82775	0.00031	0.00773
0.86000	5.89834	0.00000	0.00000
0.86333	5.97063	0.00019	0.00817
0.86667	6.04471	0.00036	0.00830
0.87000	6.12066	0.00000	0.00000
0.87333	6.19859	0.00021	0.00881
0.87667	6.27859	0.00043	0.00897
0.88000	6.36079	0.00000	0.0
0.88333	6.44530	0.00024	0.00957
0.88667	6.53227	0.00051	0.00976
0.89000	6.62183	0.00000	0.0
0.89333	6.71414	0.00027	0.01047

0.89667	6.80939	0.00063	0.01070
0.90000	6.90776	0.00000	0.00000
0.90333	7.00946	0.00031	0.01157
0.90667	7.11474	0.00080	0.01184
0.91000	7.22384	0.00000	0.00000
0.91333	7.33706	0.00035	0.01291
0.91667	7.45472	0.00104	0.01325
0.92000	7.57719	0.00000	0.00000
0.92333	7.70487	0.00040	0.01461
0.92667	7.83822	0.00139	0.01505
0.93000	7.97778	0.00000	0.0
0.93333	8.12415	0.00043	0.01682
0.93667	8.27803	0.00197	0.01741
0.94000	8.44024	0.00000	0.0
0.94333	8.61171	0.00042	0.01983
0.94667	8.79359	0.00296	0.02065
0.95000	8.98720	0.00000	0.00000
0.95333	9.19418	0.00023	0.02415
0.95667	9.41651	0.00491	0.02537
0.96000	9.65663	0.00000	0.00000
0.96333	9.91767	0.00215	0.03088
0.96667	10.20360	0.00387	0.03291
0.97000	10.51968	0.00000	0.00000
0.97333	10.87303	0.00131	0.04284
0.97667	11.27363	0.01056	0.04685
0.98000	11.73608	0.00000	0.0
0.98167	11.99712	0.00520	0.03088
0.98333	12.28305	0.00450	0.03291
0.98500	12.59913	0.00000	0.0
0.98667	12.95249	0.00184	0.04284
0.98833	13.35308	0.00732	0.04685
0.99000	13.81554	0.00000	0.00000
0.99167	14.36251	0.00786	0.07030
0.99333	15.03195	0.02895	0.08170
0.99500	15.89501	0.00000	0.00000