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## FOREWORD

This report is one of a series prepared by The Boeing Vertol Company, Philadelphia, Pennsylvania for the National Aeronautics and Space Administration, Ames Research Center, Moffett Field, California under contract NAS2-6598. The studies reported under Volumes I through IV and VIII through $X$ were jointly funded by NASA and the U.S. Army Air Mobility Research and Development Laboratory, Ames Directorate. Volumes V through VII were funded by the U. S. Air Force Flight Dynamics Laboratory, Wright Patterson Air Force Base, Ohio.

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## SUMMARY

This report documents the development of a real time mathematical model of a tilt rotor aircraft. This mathematical model is to be used in conjunction with the NASA Flight Simulator for Advanced Aircraft (FSAA) at Ames Research Center for evaluation of aircraft performance and handling qualities. In addition to developing the mathematical model, a parallel programming effort was conducted utilizing Boeing-Vertol's Hybrid Simulation Laboratory for the purpose of developing and evaluating model simplification.

The mathematical model is an eleven degree of freedom total force model. This model includes the basic six degree of freedom rigid body outer loop equations written about the instantaneous center of gravity with the inertial and aerodynamic terms included. The rotor is treated as a point source of forces and moments with appropriate response time lags and actuator dynamics. The wing has one vertical bending and one wing torsion degree of freedom. These structural degrees of freedom are treated on a "quasistatic"basis; i.e., the natural frequencies of vibration of the structure are much higher than the frequencies of the rigid body motion, and the coupling is in the aerodynamic terms. Each nacelle has an independent pitch degree of freedom about the wing pivot. The aerodynamics of the wing, tail, rotors, landing gear and fuselage are included. wing and tail mutual interference effects and turbine engine performance and dynamic responses are represented.

The control system elements represented include pilot command (longitudinal and lateral stick, pedals, nacelle position and rate, power), three-axis stability augmentation systems (SAS), thrust management system (includes rotor constant speed govenor) and a load alleviation system (LAS). The LAS system incorporates feedback to rotor cyclic and collective pitch for purposes of improving stability, blade load reduction, gust alleviation and increased damping of aeroelastic modes. Control system actuator dynamics are represented by appropriate second order systems.

The mathematical model was programmed on Boeing's hybrid computer. This program was real time and was used to evaluate model simplification and also to develop and optimize stability augmentation, control, and load alleviation systems.

The mathematical model was written to make it as flexible and as general as possible while still retaining the real time execution capability. This program is a valuable design tool for control system design, SAS optimization, and flying qualities evaluations and improvements. The model is capable of operating in all modes of $V / S T O L$ flight (forwards, backwards, and sidewards) with no restrictions. This mathematical model represents the Model 222 tilt rotor configuration as proposed in Boeing's 'Study of $V / S T O L$ Tilt Rotor Research Aircraft Program (Phase II)", dated January 1973.

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## NOMENCLATURE

| Symbol | Definition | Units |
| :---: | :---: | :---: |
| A | Rotor disc area (per rotor) | $f t^{2}$ |
| AR | Aspect ratio | N.D. |
| ${ }^{A_{D}}(u+5 v)$ | Coefficients of curve fit equation for wing drag coefficient as a function of angle of attack and surface deflection | -- |
| ${ }^{A_{N F}}(u+4 v)$ | Coefficients of curve fit equation for normal force coefficient with zero cyclic pitch | -- |
| $A^{A}(u+4 v)$ | Coefficients of curve fit equation for rotor power coefficient with zero cyclic pitch | -- |
| ${ }^{A_{P M}}(u+4 v)$ | Coefficients of curve fit equation for rotor pitching moment coefficient with zero cyclic pitch | -- |
| $A_{S F}(u+4 v)$ | Coefficients of curve fit equation for rotor side force coefficient with zero cyclic pitch | -- |
| ${ }^{A_{T}}(u+4 v)$ | Coefficients of curve fit equation for rotor thrust coefficient with zers cyclic pitch | -- |
| $A_{Y M}(u+4 v)$ | Coefficients of curve fit equation for ) rotor yawing coefficient with zero cyclic pitch | -- |
| Alc | Lateral cyclic angle in rotor wind axes | deg |
| ${ }^{\text {A }}$ ic | Lateral cyclic angle in swashplate axes | deg |
| ${ }^{\text {¹c }}$ | Lateral cyclic angle in swashplate axes resolved through swashplate phase angle | deg |
| $\bar{a}$ | Speed of sound or acceleration $\mathrm{ft} / \mathrm{sec}$ or | $\mathrm{ft} / \mathrm{sec}^{2}$ |
| a | Acceleration | $\mathrm{ft} / \mathrm{sec}^{2}$ |
| $\left(a_{g} / \mathrm{a}\right)$ | Ratio of lift curve slope in ground effect to lift curve slope out of ground effect | ND |


| Symbol | Definition | Units |
| :---: | :---: | :---: |
| $\mathrm{B}_{\mathrm{G}}$ | Percent brake pedal deflection | N.D. |
| B.L. | Aircraft butt line | Inches |
| ${ }^{\mathrm{B}} 1 \mathrm{c}$ | Longitudinal cyclic angle in rotor wind axes | deg |
| ${ }^{B}{ }^{\prime} \mathrm{C}$ | Longitudinal cyclic angle in swashplate axes | deg |
| $B_{l c}^{\prime \prime}$ | Longitudinal cyclic angle in swashplate axes resolved through swashplate phase angle | deg |
| b | Span of lifting surface (wing, tail,etc.) | feet |
| C | Chord | $f t$. |
| $C_{\text {D }}$ | Drag coefficient $=\frac{D}{q S}$ | ND |
| $\mathrm{C}_{\mathrm{D}}$ | Drag coefficient at zero lift | ND |
| $\Delta C_{D}$ | Drag coefficient increment | ND |
| $C_{\text {DS }}$ | Drag coefficient referred to rotor slipstream dynamic pressure $=$ $D / q_{S} S$ | ND |
| $\mathrm{C}_{L}$ | Lift coefficient = L/qS | ND |
| $\mathrm{C}_{\mathrm{L}_{\mathrm{O}}}$ | Average lift coefficient | ND |
| $\triangle C_{L}$ | Lift coefficient increment | ND |
| $\mathrm{C}_{L_{S}}$ | Lift coefficient referred to rotor slipstream dynamic pressure $=L / q_{S} S$ | ND |
| $\mathrm{C}_{\mathrm{L}_{\alpha}}$ | Lift curve slope | 1/rad |
| $\mathrm{C}_{\mathrm{L} \delta}$ | Lift increment due to flap deflection | 1/deg |
| $\mathrm{C}_{L}$ | Rolling moment coefficient $=$ L/q bs | ND |
| $\mathrm{C}_{\mathrm{L}}{ }_{S}$ | Rolling moment coefficient referred to rotor slipstream dynamic pressure $=$ $\mathrm{L} / \mathrm{q}_{\mathrm{S}} \mathrm{bS}$ | ND |

## NOMENCLATURE

| Symbol | Definition | Units |
| :---: | :---: | :---: |
| $C_{M}$ | Pitching moment coefficient $=\mathrm{M} / \mathrm{qSC}$ | ND |
| $\mathrm{C}_{\mathrm{M}_{\mathrm{O}}}$ | Wing pitching moment coefficient as a function of flap deflection; pitching moment coefficient of fuselage or nacelles at zero angle of attack | ND |
| $\triangle \mathrm{CM}$ | Pitching moment coefficient increment | ND |
| $\mathrm{C}_{M_{S}}$ | Pitching moment coefficient referred to rotor slipstream dynamic pressure $=$ $\mathrm{M} / \mathrm{q}_{\mathrm{S}} \mathrm{SC}$ |  |
| $\mathrm{C}_{\mathrm{M}_{\delta}}$ | Change in wing/body pitching moment coefficient as a function of flaperon deflection | ND |
| $\mathrm{C}_{\mathrm{N}}$ | Yawing moment coefficient $=\mathrm{N} / \mathrm{qSb}$ | ND |
| $\mathrm{C}_{\mathrm{N}}$ | Yawing moment coefficient of fuselage or nacelles at zero angle of attack | ND |
| $c_{\eta_{s}}$ | Yawing moment coefficient referred to rotor slipstream dynamic pressure = $\mathrm{N} / \mathrm{q}_{\mathrm{s}} \mathrm{Sb}$ | ND |
| $\mathrm{C}_{\mathrm{NF}}$ | Rotor normal force coefficient $=N F / \rho \pi \Omega^{2} R^{4}$ | ND |
| $\mathrm{C}_{\mathrm{NF}_{\mathrm{O}}}$ | Rotor normal force coefficient with zero cyclic pitch | ND |
| $\mathrm{CP}_{P}$ | Rotor power coefficient $=\frac{550 \mathrm{RHP}}{\rho \pi \Omega^{3} \mathbf{R}^{5}}$ | ND |
| $\mathrm{C}_{\mathrm{P}}$ | Rotor power coefficient with zero cyclic pitch | ND |
| $\mathrm{C}_{\mathrm{PM}}$ | Rotor hub pitching moment coefficient $=\mathrm{PM} / \mathrm{P} \pi \Omega^{2} \mathrm{R}^{5}$ | ND |
| $\mathrm{C}_{\mathrm{PM}}^{\mathrm{O}}$ | Rotor hub pitching moment coefficient with zero cyclic pitch | ND |
| $\mathrm{C}_{\text {SF }}$ | Rotor side force coefficient $=$ $\mathrm{SF} / \rho \pi \Omega^{2} \mathrm{R}^{4}$ | ND |

## NOMENCLATURE

Symbol
Definition
Units

ND

ND
ND

ND
rotor thrust coefficient referred to $T / q_{S} A$

Side force coefficient $=Y / q S$
Rotor yawing moment coefficient $\eta / \rho \pi \Omega^{2} R^{5}$
Rotor yawing moment coefficient with zero cyclic pitch
$C_{Y_{\alpha}}$
$C_{0}$
$C_{1} \quad$ Coefficient of equation that defines pitching moment coefficient as a function of flap deflection
$C_{2}$
Coefficient of equation that defines pitching moment coefficient as a function of flap deflection

D Rotor diameter
(D/T) Aircraft download to thrust ratio
$\mathrm{D}_{\mathrm{NF}_{1 \rightarrow 4}} \quad$ Coefficients in the equation for the cha cyclic angle
$D_{P_{1 \rightarrow 7}}$ Coefficients in the equation for the change in hub pitching moment coefficient with lateral cyclic angle
${ }^{\mathrm{D}} \mathrm{SF}_{1-4}$ in side force coefficient with lateral cyclic angle

## NOMENCLATURE

| Symbol | Definition | Units |
| :---: | :---: | :---: |
| $\mathrm{D}_{\text {ST }} \mathrm{n}$ | Damping coefficients of the landing gear oleo struts | lb/ft/sec |
| $\mathrm{D}_{\mathrm{YM}_{1 \rightarrow 7}}$ | Coefficients in the equation for the change in hub yawing moment coefficient with lateral cyclic angle | 1/deg |
| $\mathrm{dC}_{\mathrm{NF}} / \mathrm{dA}_{l c}$ | Change in rormal force coefficient with lateral cyclic angle | 1/deg |
| $\mathrm{dC}_{\mathrm{NF}} / \mathrm{dB}_{1 \mathrm{c}}$ | Change in normal force coeffictent with longitudinal cyclic angle | 1/deg |
| $\mathrm{dC}_{P M} / \mathrm{dA}_{1 c}$ | Change in hub pitching moment coefficient with lateral cyclic angle | 1/deg |
| $\mathrm{dC}_{\mathrm{PM}} / \mathrm{dB}_{1 \mathrm{c}}$ | Change in hub pitching moment coefficłent with longitudtnal cyclic angle | 1/deg |
| $\mathrm{dC}_{\mathrm{PM}} / \mathrm{dQ}$ | Change in hub pitching moment coefficient with pitch rate | 1/rad/sec |
| $\mathrm{dC}_{\mathrm{SF}} / \mathrm{dA}^{\text {l }}$ c | Change in side force coefficient with lateral cyclic angle | 1/deg |
| $\mathrm{dC}_{S F} / \mathrm{dB}_{1 \mathrm{c}}$ | Change in side force coefficient with longitudinal cyclic angle | 1/deg |
| $\mathrm{dC}_{\mathrm{YM}} / \mathrm{dA}^{\text {lc }}$ | Change in hub yawing moment coefficient with lateral cyclic angle | 1/deg |
| $\mathrm{dCYM} / \mathrm{dBlc}$ | Change in hub yawing moment coefficient with longitudinal cyclic angle | 1/deg |
| $\mathrm{dC}_{Y M} / \mathrm{dR}$ | Change in hub yawing moment coefficient with yaw rate | 1/rad/sec |
| $\mathrm{dC}_{\mathrm{M}} / \mathrm{dC}_{\mathrm{L}}$ | Change in wing pitching moment with lift coefficient | ND |
| $d \sigma / d B$ | Change in fuselage sidewash angle with sideslip angle | ND |
| EI | Product of modulus of elasticity and moment of inertia | $1 \mathrm{~b}-\mathrm{in}^{2}$ |
| $E I_{0}$ | Product of modulus of elastictty and moment of inertia at wing root | $1 b-i n^{2}$ |

## NOMENCLATURE

| Symbol | Definition | Units |
| :--- | :--- | :--- |
| $\mathrm{E}_{\mathrm{NF}_{\mathrm{l} \rightarrow 4}}$ | Coefficients in the equation for the <br> change in normal force coefficient with <br> longitudinal cyclic angle | $\mathrm{l} / \mathrm{deg}$ |
| $\mathrm{E}_{\mathrm{PM}_{l \rightarrow 7}}$ | Coefficients in the equation for the <br> change in hub pitching moment coeffi- <br> cient with longitudinal cyclic angle | l |

## NOMENCLATURE

| Symbol | Definition | Units |
| :---: | :---: | :---: |
| ${ }^{\mathrm{f}} \mathrm{e}_{u}$ | Leading edge umbrella drag | $f t^{2}$ |
| $\mathrm{f}_{\mathrm{NF}}$ | Multiplier on rotor normal force | ND |
| $f_{p}$ | Multiplier on rotor power | ND |
| $\mathrm{f}_{\mathrm{PM}}$ | Multiplier on rotor hub pitching moment | ND |
| $\mathrm{f}_{Q}$ | Multiplier on rotor torque | ND |
| ${ }^{\mathrm{f}} \mathrm{SF}$ | Multiplier on rotor side force | ND |
| $\mathrm{f}_{\mathrm{T}}$ | Multiplier on rotor thrust | ND |
| ${ }^{\text {YM }}$ | Multiplier on rotor hub yawtng moment | ND |
| G | Generalized moment | ft-1b |
| GEF | Ground effect factor $=\left[1-\frac{(\Delta \varepsilon)}{\varepsilon}\right]$ | ND |
| $\mathrm{G}_{\mathrm{Al}}^{\alpha}$ | Load alleviation system gain - change in lateral cyclic with angle of attack | deg/deg |
| $\mathrm{G}_{\mathrm{Al}}^{\beta}$ | Load alleviation system gain - change in lateral cyclic with angle of sideslip | deg/deg |
| $\mathrm{G}_{\mathrm{B}} \mathrm{l}_{\alpha}$ | Load alleviation system gain - change in longttudinal cyclic with angle of attack | deg/deg |
| $\mathrm{G}_{\mathrm{Gl}}$ | Governor gain deg/s | $/ \mathrm{rad} / \mathrm{sec}$ |
| $\mathrm{G}_{\mathrm{G} 2}$ | Governor gain deg/se | $\mathrm{rad} / \mathrm{sec}$ |
| $\mathrm{G}_{\mathrm{G} 3}$ | Governor gain d | /sec/deg |
| $\mathrm{G}_{\mathrm{p}}$ | Lateral directional SAS gain | hes/rad/sec |
| $\mathrm{G}_{\mathrm{pr}} 1$ | Lateral directional SAS gain | hes/rad/sec |
| $\mathrm{G}_{\mathrm{P} \delta_{S}}$ | Lateral directional SAS gain | hes/inch |
| $\mathrm{G}_{\mathrm{q}}$ | Longitudinal SAS gain | /rad/sec |
| $\mathrm{Gr}_{r}$ | Lateral directional SAS gain | hes/rad/sec |


| Symbol | Definition | Units |
| :---: | :---: | :---: |
| $\mathrm{G}_{\mathrm{r} 2}$ | Lateral directonal SAS gatn | inches/rad/sec |
| $\mathrm{G}_{r_{\delta r}}$ | Lateral directional SAS gain | inches/rad/sec |
| $\mathrm{G}_{\beta p}$ | Lateral directional SAS gain | inches/rad |
| $\mathrm{G}_{\beta r}$ | Lateral directional SAS gatn | inches/rad |
| $G_{\beta \delta r}$ | Lateral directional SAS gain | inches/inch |
| $\mathrm{G}_{\delta \mathrm{Bl}}$ | Longitudinal SAS gain | deg/inch |
| $\mathrm{G}_{\delta \mathrm{B} 2}$ | Longitudinal SAS gain | deg/inch |
| $\mathrm{G}_{\delta \mathrm{TH}}$ | Governor throttle gain | deg/inch |
| $\mathrm{G}_{\theta}$ | Longitudinal SAS gain | deg/rad/sec |
| $\mathrm{G}_{\phi}$ | Lateral-directional SAS gatn | inches/rad/sec |
| $\mathrm{G}_{\psi}$ | Lateral directional SAS gain | inch/inch |
| $G_{\psi \delta r}$ | Lateral directional SAS gatn | inch/inch |
| g | Gravitational constant | $\mathrm{ft} / \mathrm{sec}^{2}$ |
| H | Height | ft. |
| HP | Horsepower | -- |
| ${ }^{H} P M(u+4 v)$ | Coefficients in the equation for the change in hub pitching moment with pitch rate | -- |
| $\mathrm{H}^{\prime} \mathrm{w}^{\prime} \text { FUEL }$ | Horizontal distance between wing mass element center of gravity and fuel center of gravity | $f t$ |
| $H_{W^{\prime}}{ }^{\prime} F$ | Horizontal distance between wing mass element center of gravity and fixed nacelle center of gravity | ft |
| $H_{W^{\prime}}^{\prime}{ }^{\prime}$ | Horizontal distance between wing mass element center of gravity and fixed nacelle center of gravity | $f t$ |

## NOMENCLATURE

| Symbol | Definttion | Units |
| :---: | :---: | :---: |
| h | Height or angular momentum ft or | lb-ft-sec |
| $\mathrm{h}_{\mathrm{CG}}^{\mathrm{N}}$ | Angular momentum of nacelle about lb-ft aircraft center of gravity | $\mathrm{t}-\mathrm{sec}$ |
| $\mathrm{h}_{\mathrm{F}}$ | Distance from wing pivot plane to fuselage mass element center of gravity | ft |
| $\mathrm{h}_{\mathrm{P}}$ | Height of pivot above wing chord line or angular momentum of racelle about the pivot |  |
| $\mathrm{h}_{\mathrm{T}}$ | Landing gear oleo strut deflection during ground contact | $f t$ |
| $\mathrm{h}_{\mathrm{w}}$ | Distance from wing pivot plane to wing mass element center of gravity | $f t$ |
| $\mathrm{h}_{0}$ | Angular momentum of an element of mass about its own center of gravity | 1b-ft-sec |
| $\mathrm{h}_{1}$ | Wing vertical bending deflection | $f t$ |
| h/D | Rotor hub height to rotor diameter ratio | ND |
| $\mathrm{h}_{\theta}$ | Distance from aircraft center of gravity bottom of right main gear following a positive pitch rotation | $f t$ |
| $\mathrm{h}_{\phi}$ | Distance from aircraft center of gravity to bottom of right main gear following a positive roll | $f t$ |
| I | Mass moment of inertia | slug-ft ${ }^{2}$ |
| $\mathrm{I}_{\mathrm{Xx}}$ | Vehicle mass roll moment of inertia about center of gravity | slug-ft ${ }^{2}$. |
| $I_{x x_{0}}$ | Mass roll moment of inertia of aircraft components about their own center of gravity | $s \operatorname{lug}-\mathrm{ft}{ }^{2}$ |
| $I_{x \times}{ }^{(F)}$ | Mass roll moment of inertia of fuselage mass element about its center of gravity | slug-ft ${ }^{2}$ |

## NOMENCLATURE

| Symbol | Definition | Units |
| :---: | :---: | :---: |
| $I_{x x}(W)$ | Mass roll moment of inertia of wing mass element about its center of gravity | slug-ft ${ }^{2}$ |
| $I_{x x}^{\prime}$ | Mass roll moment of inertia of the tilting portion of each nacelle about its center of gravity | $s l u g-f t^{2}$ |
| $\mathrm{I}_{\mathrm{Yy}}$ | Vehicle mass pitch moment of inertia about center of gravity | slug-ft ${ }^{2}$ |
| $\mathrm{I}_{\mathrm{yy}}^{\mathrm{o}}$ | Mass pitch moment of inertla of aircraft components about their own center of gravity | slug-ft ${ }^{2}$ |
| $I_{Y Y}(F)$ | Mass pitch moment of inertia of fuselage mass element about its center of gravity | slug-ft ${ }^{2}$ |
| $I_{Y Y}^{(W)}$ | Mass pitch moment of inertia of wing mass element about its center of gravity | slug-ft ${ }^{2}$ |
| $I_{Y Y}^{\prime}$ | Mass pitch moment of inertia of the tilting portion of each nacelle about its center of gravity | slug-ft ${ }^{2}$ |
| $I_{x z}$ | Vehicle mass product of inertia about center of gravity | slug-ft ${ }^{2}$ |
| $I_{x z_{0}}$ | Mass product of inertia of atrcraft components about their own center of gravity | $s l u g-f t^{2}$ |
| $\begin{gathered} I(F) \\ x z \end{gathered}$ | Mass product of inertia of fuselage mass element about its center of gravity | slug-ft ${ }^{2}$ |
| $\begin{gathered} I(W) \\ x Z \end{gathered}$ | Mass product of inertia of wing mass element about its center of gravity | slug-ft ${ }^{2}$ |
| $I_{x z}^{\prime}$ | Mass product of inertia of the tilting portion of each nacelle about its center of gravity | slug-ft ${ }^{2}$ |

## NOMENCLATURE

| Symbol | Definition | Units |
| :---: | :---: | :---: |
| $\mathrm{I}_{\mathrm{zz}}$ | Vehicle mass yaw moment of inertia about center of gravity | slug-ft ${ }^{2}$ |
| $I_{z z_{0}}$ | Mass yaw moment of inertia of atrcraft components about their own center of gravity | slug-ft ${ }^{2}$ |
| $\begin{gathered} I\left(F^{r}\right) \\ Z Z \end{gathered}$ | Mass yaw moment of inertia of fuselage mass element about its center of gravity | slug-ft ${ }^{2}$ |
| $I_{z z}^{(W)}$ | Mass yaw moment of inertia of wing mass element about its center of gravity | slug-ft ${ }^{2}$ |
| $I_{z z}^{\prime}$ | Mass yaw moment of inertia of the tilting portion of each nacelle about its center of gravity | slug-ft ${ }^{2}$ |
| 1 | Incidence angle | deg or rad |
| 全 | Unit vector in i direction | -- |
| $J_{\text {xx }}$ | Dummy inertia $=\left(\mathrm{I}_{z z} \mathrm{II}_{\mathrm{Yy}}\right)$ | slug-ft ${ }^{2}$ |
| $J_{Y M}(u+4 v)$ | Coefficients of curve fit equation for rotor hub moment with hub yaw rate | -- |
| $J_{Y Y}$ | Dummy inertia $=\left(\mathrm{I}_{\mathrm{xx}}{ }^{-\mathrm{I}} \mathrm{zz}\right.$ ) | slug-ft ${ }^{2}$ |
| $J_{z z}$ | Dummy inertia $=\left(\mathrm{I}_{\mathbf{Y Y}}{ }^{-1} \mathbf{x x}\right.$ ) | slug-ft ${ }^{2}$ |
| 予 | Unit vector in $j$ direction | -- |
| $\mathrm{K}_{\mathrm{A}}^{\prime}$ | Wing slipstream correction factor | ND |
| $\frac{K_{D 1}}{T}-\frac{K_{D} 4}{T}$ | Coefficients of curve fit equation for wing download as a function of rotor height/diameter ratio | ND |
| $\underset{\mathrm{M}}{\mathrm{~K}_{\mathrm{M}}} \rightarrow \frac{\mathrm{~K}_{\mathrm{M} 4}}{\mathrm{~T}}$ | Coefficients of curve fit equation for wing pitching moment as a function of rotor height/diameter ratio | ND |
| $\mathrm{K}_{\text {K }}$ | Multiplier on slipstream rolling moment coefficient | ND |
| $\mathrm{K}_{72}$ | Multiplier on slipstream yawing moment coefficient | ND |

NOMENCLATURE

| Symbol | Definition | Units |
| :---: | :---: | :---: |
| $\mathrm{K}_{\mathrm{ST}}{ }_{\eta}$ | Landing gear spring constants | lb/ft |
| $\mathrm{K}_{\mathrm{WI}} \rightarrow \mathrm{K}_{\mathrm{Wl} 0}$ | Coefficients for wing bending equations |  |
| $\mathrm{K}_{\delta}{ }_{\text {B }}$ | Multiplier on longitudinal cyclic pitch available from longitudınal stick | inch/inch |
| $\mathrm{K}_{\delta} \mathrm{e}$ | Ratio between longitudinal stick motion and elevator deflection | deg/inch |
| $\mathrm{K}_{\delta} \mathrm{R}$ | Multiplier on longitudinal cyclic pitch available from pedal displacement | inch/inch |
| $\mathrm{K}^{\text {RUD }}$ | Ratio between pedal and rudder deflection | deg/Inch |
| $\mathrm{K}_{\delta \mathrm{s}}$ | Multiplier on longitudinal cyclic pitch and differential collective available from lateral stick | inch/inch |
| $\mathrm{K}_{\delta}{ }^{\text {s }}$ | Provision for lateral cyclic pitch * on lateral stick | deg/deg |
| $K_{\theta}$ | Wing stiffness | $\mathrm{ft}-\mathrm{lb} / \mathrm{rad}$ |
| K | Coefficient of fuselage drag coefficient equation to account for drag due to sideslip | $1 / \mathrm{rad}^{3}$ |
| $\mathrm{K}_{1}$ | Coefficient of fuselage drag coefficient equation | $1 / \mathrm{rad}^{2}$ |
| $\mathrm{K}_{2}$ | Coefficient of fuselage drag coefficient equation | 1/rad |
| $\mathrm{K}_{3}$ | Coefficient of fuselage lift coefficient equation | 1/rad |
| $\mathrm{K}_{4}$ | Coefficient of fuselage lift coefficient equation | $1 / \mathrm{rad}^{2}$ |
| $\mathrm{K}_{5}$ | Coefficient of fuselage pitching moment coefficient equation | 1/rad |

NOMENCLATUFE

| Symbol | Definition | Units |
| :---: | :---: | :---: |
| $\mathrm{K}_{6}$ | Coefficient of fuselage pitching moment coefficient equation | $1 / \mathrm{rad}{ }^{2}$ |
| $\mathrm{K}_{7}$ | Coefficient of fuselage side force coefficient equation | 1/rad |
| $\mathrm{K}_{8}$ | Coefficient of fuselage side force coefficient equation | 1/rad |
| $\mathrm{K}_{9}$ | Coefficient of fuselage yawing moment coefficient equation | 1/rad |
| $\mathrm{K}_{10}$ | Coefficient of fuselage yawing moment coefficient equation | $1 / \mathrm{rad}^{2}$ |
| K20 | Wing/body interference effects on $\mathrm{C}_{\mathrm{L} \beta}$ | 1/rad |
| $\mathrm{K}_{21}$ | Wing planform effects on $\mathrm{C}_{\mathrm{L} \beta}$ | 1/rad |
| $\mathrm{K}_{22}$ | Wing planform and lift effects on $C_{\eta \beta}$ | 1/rad |
| $\mathrm{K}_{30}$ | Coefficient of nacelle drag coeffictent equation | 1/rad |
| $\mathrm{K}_{31}$ | Coefficient of nacelle drag coefficient equation | $1 / \mathrm{rad}^{2}$ |
| $\mathrm{K}_{32}$ | Coefficient of nacelle lift coefficient equation | 1/rad |
| $\mathrm{K}_{34}$ | Coefficient of nacelle pitching moment coefficient equation | 1/rad |
| $\mathrm{K}_{35}$ | Coefficient of nacelle pitching moment coefficient equation | $1 / r a d^{2}$ |
| $\mathrm{K}_{36}$ | Coefficient of nacelle side force coefficient equation | 1/rad |
| $\mathrm{K}_{37}$ | Coefficient of nacelle side force coefficient equation | $1 / \mathrm{rad}^{2}$ |
| $\mathrm{K}_{38}$ | Coefficient of nacelle yawing moment coefficient equation | $1 / \mathrm{rad}$ |
| $\mathrm{K}_{39}$ | Coefficient of nacelle yawing moment coefficient equation | $1 / \mathrm{rad}^{2}$ |

## NOMENCLATURE

| Symbol | Definition | Units |
| :---: | :---: | :---: |
| $\mathrm{K}_{40}$ | Coefficient of nacelle yawing moment coefficient equation | $1 / \mathrm{rad}$ |
| $\mathrm{K}_{41}$ | Coefficient of nacelle yawng moment coefficient equation | $1 / \mathrm{rad}^{2}$ |
| $\mathrm{K}_{42}$ | Coefficient of fuselage lift coefficient equation | ND |
| $\underline{\hat{k}}$ | Unit vector in $k$ direction | -- |
| L | Rolling moment or nacelle shaft length | $f t-l b, f t$ |
| L | Rolling Moment | $f t-1 b$ |
| $\ell$ | Distance from nacelle pivot to nacelle center of gravity | ft |
| \&' | Horizontal distance from nacelle pivot to noted aircraft component center of gravity position - positive forward from pivot | $f t$ |
| $\ell^{A C}$ | Horizontal distance from horizontal tail quarter chord to wing aerodynamic center | $f t$ |
| ${ }^{\ell} \mathrm{F}$ | Horizontal distance from pivot to center of gravity of fuselage mass element | ft |
| $\ell_{0}$ | Wing root lift/foot | lb/ft: |
| $\ell_{\text {PA }}$ | Horizontal distance from pivot to center of gravity of pilots station positive forward from pivot | $f t$ |
| $\ell_{\text {w }}$ | Horizontal distance from pivot to wing mass element center of gravity |  |
| M | Pitching moment | ft-lb |
| n | Pitching moment | Et-1b |
| M/T | Pitching moment/rotor thrust | $f t-1 b / l b$ |
| m | Aircraft total mass | slugs |


| Symbol | Definition | Units |
| :---: | :---: | :---: |
| $\dot{\operatorname{H}}_{\mathrm{f}}$ | Mass of fuselage mass element | slugs |
| ${ }^{1 m_{\mathrm{N}}}$ | Mass of one nacelle | slugs |
| ${ }^{\text {c/w }}$ | Mass of wing mass element | slugs |
| N | Yawing moment | $f t-1 b$ |
| $\eta$ | Yawing moment | ft-1b |
| NF | Rotor normal force | 1b |
| ${ }^{\mathrm{N}} \mathrm{I}$ | Engine gas generator speed | rev/min |
| $\mathrm{N}_{1}$ IND | Engine gas generator indicator | -- |
| N ${ }_{\text {a }}$ | Engine gas generator speed at sea level standard, static conditions | $\mathrm{rev} / \mathrm{min}$ |
| $\mathrm{N}_{1} \theta$ IND | Referred engine gas generator speed indicator | -- |
| $\mathrm{N}_{\text {I I }}$ | Engine power turbine speed |  |
| $\stackrel{\text { N }}{\text { I I }}$ | Engine power turbine speed at sea level standard static conditions |  |
| P | Body axes roll rate | rad/sec |
| PC | Horizontal distance from wing leading edge to pivot location | ft. |
| $\mathrm{p}^{N}$ | Nacelle axes roll rate | rad/sec |
| PR | Nacelle wind axes roll rate | rad/sec |
| p | Body axes roll rate | rad/sec |
| Q | Body axes pitch rate or rotor torque | $\mathrm{rad} / \mathrm{sec}$ or $\mathrm{lb}-\mathrm{ft}$ |
| $Q_{\text {IND }}$ | Torque indicator | ND |
| $Q_{\text {MAX }}$ | Maximum engine torque available | lb-ft |
| $Q^{N}$ | Nacelle axes pitch rate | rad/sec |
| $Q^{\text {R }}$ | Nacelle wind axes pitch rate | r.ad/sec |

## NOMENCLATURE

| Symbol | Definftion | Units |
| :---: | :---: | :---: |
| Q* | Engine torque at sea level standard static condition | lb-ft |
| q | Body axes pitch rate or freestream rad dynamic pressure | sec or $\mathrm{lb} / \mathrm{ft}^{2}$ |
| $q_{s}$ | Dynamic pressure based on rotor <br> slipstream $=(q+T / A)$ | $1 \mathrm{~b} / \mathrm{ft}^{2}$ |
| R | Body axes yaw rate or rotor resultant force or rotor radius | rad/sec or lb or ft |
| RHP | Rotor horsepower | -- |
| $\mathrm{R}^{\mathrm{N}}$ | Nacelle axes yaw rate | rad/sec |
| $\mathrm{R}^{\mathrm{R}}$ | Nacelle wind axes yaw rate | rad/sec |
| r | Body axes yaw rate | rad/sec |
| $\underline{r}$ | Radius vector | -- |
| $r_{n}$ | Landing gear tire radius | ft. |
| S | Surface area | $f t^{2}$ |
| SF | Rotor side force | 1b |
| SHP | Shaft horsepower | -- |
| SHP* | Engine shaft horsepower at sea level standard static conditions |  |
| T | Rotor thrust | 1 b |
| TEA | Engine referred turbine inlet temperature | degrees |
| $\left(\mathrm{T}_{\mathrm{IGE}} / \mathrm{T}_{\text {OGE }}\right)$ | Ratio of the rotor thrust in ground effect to the thrust out of ground effect | -- |
| $\mathrm{T}_{1} \rightarrow \mathrm{~T}_{3}$ | Coefficients of curve fit equations for rotor/rotor interference | ND |
| t | Time | sec |

## NOMENCLATURE

| Symbol | Definition | Units |
| :---: | :---: | :---: |
| u | Body axes longitudinal component of velocity at aircraft center of gravity or rotor hub, wing, horizontal and vertical tail velocities referred to rotor shaft and local surface chord axes respectively. | $\mathrm{ft} / \mathrm{sec}$ |
| $u^{\prime}$ | Body axes longitudinal component of velocity at rotor hub and wing aerodynamic center | $\mathrm{ft} / \mathrm{sec}$ |
| ${ }^{\text {P }}$ PA | Body axes longitudinal component of velocity at pilots station | $\mathrm{ft} / \mathrm{sec}$ |
| V | Total velocity | $\mathrm{ft} / \mathrm{sec}$ |
| $\mathrm{V}_{\mathrm{t}}$ | Rotor tip speed | $\mathrm{ft} / \mathrm{sec}$ |
| $V^{\prime}$ | Resultant flow through rotor disc | $\mathrm{ft} / \mathrm{sec}$ |
| $\mathrm{V}_{*}$ | Non-dimensional rotor forward velocity | N.D. |
| V | Total Velocity Vector |  |
| v | Body axes lateral component of velocity at alrcraft center of gravity or rotor hub wing, horizontal and vertical tail velocities referred to rotor shaft and local surface chord axes respectively | $\mathrm{ft} / \mathrm{sec}$ |
| $v^{\prime}$ | Body axes lateral component of velocity at rotor hub and wing aerodynamic center | $\mathrm{ft} / \mathrm{sec}$ |
| $\mathrm{v}_{1}$ | Rotor induced velocity | $\mathrm{ft} / \mathrm{sec}$ |
| ${ }^{\text {P }}$ PA | Body axes lateral component of velocity at pilots station. | $\mathrm{ft} / \mathrm{sec}$ |
| ${ }^{*}$ * | Non-dimensional rotor induced velocity | N.D. |
| W.L. | Fuselage water line position | inches |
| W' | Weight of aircraft components | 1 l . |
| WDTIND | Fuel flow indicator |  |
| W | Body axes vertical component of velocity at aircraft center of gravity or rotor | $\mathrm{ft} / \mathrm{sec}$ |

Symbol Definition

Units
hub, wing, horizontal and vertical tail velocities referred to rotor shaft and local surface chord axes respectively
${ }^{W}$ PA $\quad$ Body axes vertical component of velocity at pilots station.
$X_{\text {subscript }}$ Longitudinal distance, measured positive ft. forward from nacelle pivot along body axes
$\Delta X_{\text {subscript }}$ Longitudinal force, measured positive for- $1 b$. ward along body axes

X Total longitudinal aerodynamic force at lb. center of gravity measured positive forward along body axes.
x sprscript Longitudinal force, measured positive subscript forward along body axes.
$\dot{X}_{\text {North }}$ Longitudinal ground track velocity ft/sec
$Y_{\text {subscript }}$ Lateral distance, measured positive ft. along right wing along body axes
$\Delta Y_{\text {subscript }}$ Lateral force, measured positive along right wing in body axes

Yaero Total lateral aerodynamic force at center of gravity measured positive along right wing in body axes

Y sprscript
Lateral force, measured positive along right wing in body axes
$\dot{Y}_{\text {East }}$ Lateral ground track velocity ft/sec
Zsubscript Vertical distance, measured positive down ft. nacelle pivot along body axes
$\Delta Z_{\text {subscript Vertical }}$ force, measured positive down lb. along body axes

Zaero Total vertical aerodynamıc force at center lb.

## NOMENCLATURE

| Symbol | Definition | Units |
| :---: | :---: | :---: |
|  | of gravity, measured positive down along body axes. |  |
| zsprscript <br> subscript | Vertical force, measured positive down along body axes | 1 b . |
| $\mathrm{Z}_{\text {down }}$ | Vertical ground track velocity | $\mathrm{ft} / \mathrm{sec}$ |
| z | Vertical distance from nacelle pivot to center of gravity of aircraft component, positive down from nacelle pivot along body axes. | ft. |
| $\alpha$ | Angle of attack | rad |
| $\beta$ | Angle of sideslip | rad |
| $\Delta_{W}^{\prime}{ }^{\prime}$ fuel | Vertical distance between wing fuel center of gravity and wing mass element center of gravity | ft. |
| $\Delta_{W^{\prime}}^{\prime} N F$ | Vertical distance between fixed nacelle center of gravity and wing mass element center of gravity. | ft. |
| $\Delta_{W^{\prime}}^{\prime}{ }^{\prime}$ | Vertical distance between wing center of gravity and wing mass element center of gravity. | $f t$. |
| $\delta$ | Control element (surface or stick) angular or linear displacement | deg. or in. |
| $\delta^{\prime}{ }_{C}$ | Vertical distance between cargo center of gravity and fuselage mass element center of gravity | ft. |
| $\delta^{\prime} \mathrm{CR}$ | Vertical distance between crew center of gravity and fuselage mass element center of gravity | ft. |
| $\delta^{\prime} \mathbf{F}^{\prime}$ | Vertical distance between fuselage center of gravity and fuselage mass element ceenter of. gravity. | ft. |
| $\delta^{\prime}{ }^{\text {HT }}$ | Vertical distance between horizontal tail center of gravity and fuselage mass element center of gravity | ft. |


| Symbol | NOMENCLATURE | UNITS |
| :---: | :---: | :---: |
|  | Definition |  |
| $\delta_{\mathrm{VT}}^{\prime}$ | Vertical distance between vertical tail center of gravity and fuselage mass element center of gravity | ft. |
| $\varepsilon$ | Wing or rotor downwash angle | rad |
| $\varepsilon_{0}$ | Wing downwash angle at zero wing angle of attach | rad |
| $\varepsilon_{i L R}$ | Rotor/rotor interference angle, left rotor on right rotor | rad |
| $\varepsilon_{i R L}$ | Rotor/rotor interference angle, right rotor on left rotor | rad |
| $\varepsilon_{\text {w }}$ | Wing on rotor interference | rad |
| $\zeta$ | Rotor sideslip angle or damping ratio | rad |
| $\zeta_{\text {wl }} \rightarrow \zeta_{w 4}$ | Wing damping ratio | N.D. |
| $H^{\prime}{ }^{\prime} \text { fuel }$ | Horizontal distance between wing fuel center of gravity and wing mass element center of gravity | ft. |
| $H_{W^{\prime}}^{\prime} N F$ | Horizontal distance between fixed nacelle center of gravity and wing mass element center of gravity | ft. |
| $H_{W^{\prime} w}^{\prime}$ | Horizontal distance between wing center of gravity and wing mass element center of gravity | ft. |
| $\eta_{c}^{1}$ | Horizontal distance between cargo center of gravity and fuselage mass element center of gravity | ft. |
| $\eta_{C R}^{\prime}$ | Horizontal distance between crew center of gravity and fuselage mass element center of gravity | $f t$. |
| $\eta_{F}^{\prime}$ | Horizontal distance between fuselage center of gravity and fuselage mass | $f t$ |


| Symbol | NOMENCLATURE | Units |
| :---: | :---: | :---: |
|  | Definition |  |
|  | element center of gravity |  |
| $\eta_{\mathrm{HT}}$ | Horizontal tail efficiency | N.D. |
| \#' HT | Horizontal distance between horizontal tail center of gravity and fuselage mass element center of gravity. | lb. |
| $\eta$ VT | Vertical tail efficiency factor | N.D. |
| $\eta^{\prime}$ | ```Horizontal distance between vertical tail center of gravity and fuselage mass element center of gravity``` | $f t$. |
| $\eta_{T R}$ | Transmission efficiency | N.D. |
| $\theta$ | Aircraft pitch or Euler angle or temperature ratio |  |
| $\theta_{t}$ | Wing twist angle | rad |
| $\theta 0.75$ | Rotor collective pitch angle at three quarter radius | deg. |
| $\lambda$ | Angle between the rotor shaft and a line drawn through the nacelle center of gravity from the pivot. | rad |
| $\mu$ | Rotor advance ratio $=\mathrm{V} / \Omega \mathrm{R}$ | N.D. |
| $\mu_{S}$ | Tire sliding coefficient of friction when sliding sidewards (for concrete) | N.D. |
| $\mu_{0}$ | Tire rolling coefficient of friction (for concrete) | N.D. |
| $\mu_{1}$ | Coefficient of rolling friction for brakes | N.D. |
| $\xi_{R 1} \rightarrow \xi$ | Terms of wing immersed area calculation | --. |
| $\pi$ | 3.14159 | - |
| $\rho$ | Ambient air density | $s l u g / f t^{3}$ |
| $\sigma$ | Fuselage sidewash angle | rad |


| Symbol | NOMENCLATURE | Units |
| :---: | :---: | :---: |
|  | Definition |  |
| $\sigma_{h}$ | Ambient density ratio | N.D. |
| $\tau$ | Angle between freestream velocity and rotor resultant force | rad |
| ${ }^{\tau} \mathrm{D}$ | Engine response time constant | sec. |
| $\tau_{E}$ | Engine response time constant | sec. |
| $\tau_{\text {HT }}$ | Horizontal tail effectiveness | rad/rad |
| $\tau_{\text {LAS }}$ | Load alleviation system time constant | sec |
| ${ }^{\tau}$ VT | Vertical tail effectiviness | $\mathrm{rad} / \mathrm{rad}$ |
| $\tau_{P}$ | Lateral directional SAS time constant | sec |
| $\tau_{r}$ | Lateral directional SAS time constant | sec |
| $\tau_{\phi}$ | Lateral directional SAS time constant | sec |
| $\tau_{\phi \delta S}$ | Lateral directional SAS time constant | sec |
| $\tau_{\psi}$ | Lateral dırectional SAS time constant | sec |
| $\tau_{\psi}{ }_{\delta r}$ | Lateral directional SAS time constant | sec |
| $\tau_{1}$ | Rotor thrust response time constant | sec |
| $\tau_{2}$ | Rotor thrust response time constant | sec |
| $\phi$ | Aircraft roll angle or Euler angle | rad |
| $\phi_{\mathrm{P}}$ | Rotor swashplate phase angle | rad |
| $\phi_{1} \rightarrow \phi_{5}$ | Functions in wing vertical bending equations | - |
| $\chi$ | Rotor wake skew angle | rad |
| $\psi$ | Aircraft yaw angle or Euler angle | rad |
| $\Omega$ | Rotor or engine rotational speed | $\mathrm{rad} / \mathrm{sec}$ |
| $\underline{\Omega}$ | Rotational speed vector | $\mathrm{rad} / \mathrm{sec}$ |
| $\omega$ | Natural frequency | $\mathrm{rad} / \mathrm{sec}$ |
| $\omega_{w 1} \rightarrow \omega_{w 3}$ | Wing natural frequency | $\mathrm{rad} / \mathrm{sec}$ |

## Subscripts

| A | Available |
| :---: | :---: |
| AC | Aerodynamic center |
| ACT | Actuator |
| AERO | Aerodynamic force |
| a | Aileron |
| B | Longitudinal stick |
| c | Cargo |
| CG | Center of gravity |
| CR | Crew |
| C/4 | Quarter chord |
| DUM | Dummy variable |
| E | Engine |
| EFF | Effective |
| e | Elevator or effective |
| F | Fuselage |
| FAC | Fuselage aerodynamic center |
| FUEL | Fuel in wing |
| FUEL $_{\text {CG }}$ | Fuel center of gravity |
| FUS | Fuselage |
| $F^{\prime}$ | Fuselage less landing gear |
| f | Flap |
| GLAS | Load alleviation system |
| GYRO | Gyroscopic |
| g | Ground or gust |
| HL | Left rotor hub |


| HR | Right rotor hub |
| :---: | :---: |
| HT | Horizontal tail |
| HTCG | Horizontal tail center of gravity |
| IGE | In ground effect |
| i | Immersed |
| L | Left wing or rotor |
| LAS | Load alleviation system |
| LE | Left engine |
| LG | Landing gear |
| L-L | Rotor lead-lag |
| LN | Left nacelle |
| LR | Left rotor |
| LRH | Left rotor hub |
| LT | Left wing tip |
| LW | Left wing |
| $\mathrm{LW}_{\mathrm{O}}$ | Left wing referred to freestream |
| MAX | Maximum |
| N | Nacelle or natural frequency |
| NF | Fixed portion of nacelle |
| NFCS | Fixed portion of nacelle center of gravity |
| NL | Left nacelle |
| NR | Right nacelle |
| NT | Tilting portion of nacelle |
| n | Landing gear index, $n=1$ left gear, $n=2$ right gear, $n=3$ nose gear |

Subscripts

| OGE | Out of ground effect |
| :--- | :--- |
| $P$ | Power, nacelle pivot, or rotor polar moment <br> of inertia |


| POWER | Power |
| :--- | :--- |
| PA | Pilot station |

$\mathrm{R} \quad$ Right wing, rotor or rudder pedal
RE $\quad$ Right engine

FEQ Required
RIGID Rigid
RN Right nacelle
RR Right rotor
RRH Right rotor hub
RT Right wing tip
RUD Rudder
RW Right wing
RW $\quad$ Right wing referred to freestream
S Rotor shaft, side, or lateral stick
SP Spoiler
STALL Stall
T Tail, total or wing tip
TH Throttle
VT Vertical tail
VTCG Vertical tail center of gravity
W
Wing
WAC Wing aerodynamic center
WCG Wing center of gravity

Subscripts

| x | Along the lonitudinal body axes, positive <br> forward |
| :--- | :--- |
| $\mathrm{y} \quad$Along the lateral body axes, positive out <br> right wing |  |
| zAlong the vertical body axes, positive <br> down |  |
| Denotes a vector quantity |  |

Superscripts
(c) Referes to cargo or payload weight
(CR) Refers to aircraft crew weight
F Fuselage
F' Fuselage less landing gear
HT Horizontal tail
( HT )
IGE In ground effect
LW Left wing
$\mathrm{N} \quad$ Nacelle
NL Left wing tip at pivot
NR Right wing tip at pivot
(p) Roll axes
(q) Pitch axes

RW Right wing
(r) Yaw axes

T
VT
(VT)

W

Total of horizontal and verlical tail
Vertical tail
Referes to vertical tail weight con ponent Wing

Superscripts

| (W'FUEL) | Refers to wing fuel weight |
| :---: | :---: |
| $\left(W_{f}{ }^{\prime}\right)$ | Refers to fuselage weight component |
| ( $\mathrm{W}^{\prime}{ }_{\mathrm{NF}}$ ) | Refers to weight of fixed portion of nacelle |
| ( $\mathrm{W}^{\prime}{ }_{W}$ ) | Refers to wing weight component |
| - | First derivative with respect to time; represents velocity |
| - • | Second derivative with respect to time; represents acceleration |
| " | Denotes an interim calculation or coeffiectent in local wind axes |
| '' | Denotes an interim calculation |
| - | Denotes average value |
| * | Denotes interim calculation or calculation in freestream wind axes |
| ' | Denotes an interim calculation |
| + | Denotes an interim calculation |
| $\wedge$ | Denotes an interim calculation |
| 11 | Absolute values |
| NOTES |  |

1. Some symbols not defined in this section, but used in this report, are defined in the section of the report they are used.
2. Alternate definitions, where applicable, for each symbol are given. Select the appropiate definition for each particular section
3. All distances are measured with respect to the nacelle pivot. Distances are posittve forward, down and to the right of the pivot. Forces are positive forward, down, and to the right.
4. $\Delta$ or $\delta$ preceeding a symbol generally denotes an incremental change.
1.0 INTRODUCTION

Piloted simulation is a useful and important tool in the design, development and test of new flight vehicles. Figure 1.1 shows a summary of some of these uses as they could be applied to the Model 222 tilt rotor aircraft.

As part of Contract NAS2-6598 Boeing Vertol developed a complex mathematical model of the Model 222 tilt rotor, intended primarily for use with the NASA Flight Simulator for Advanced Aircraft (FSAA) at Ames Research Center. The purpose of this report is to document the development of that mathematical model and to substantiate the methods which were uniquely developed for this purpose.

$$
1.0-1
$$

```
- Evaluation of Tilt Rotor Performance
- Maneuver Capability
- VTOL and STOL Takeoff and Landing Capability
- As a Tool to Evaluate Configuration Changes
- Changes in Cockpit Layout
- Changes in Tail Size
- Changes in Geometry
- Changes in SAS Configuration
- Changes in Elastic Characteristics
- As a Flight Test Support Tool
- Development of Emergency Techniques
- Familiarization of Flight Crews with Aircraft Characteristics Prior to Flight
- Correlation Studies
- Exploration of Flight-Discovered Phenomena
```

Figure No. 1.1. Summary of Uses for Piloted Simulation

### 2.0 GENERAL DESCRIPTION OF SIMULATION

The objective of this program was to develop a real time simulation program for a tilt rotor aircraft to be used at the NASAAmes simulation facility in conjunction with the Flight Simulator for Advanced Aircraft (FSAA) for evaluation of tilt rotor aircraft performance and handling characteristics throughout the flight envelope and identifying problem areas within the envelope.

The mathematical model developed under this contract includes the basic 6 degree of rigid body freedom outer loop equations written about the instantaneous center of gravity with all inertial and aerodynamic coupling terms included. Euler angles are used to properly orient the aircraft in space.

Rotor forces and moments are input to the equations from curve-fit data. The rotor data bank applies to the Boeing Model 222 tilt rotor. Calculation of the rotor forces and moments on-line for real time simulation is not practical because of the complexity of the programs required to represent the lag-flap coupling effects of the soft-in-plane hingless rotor. Analytical studies show that the lag-flap coupling has a large effect upon the phasing of the hub forces and moments of the rotor thereby altering the direct rotor effects on aircraft stability significantly. The rotor rotational degree of freedom is included to represent the effects of rotor inertia which are included in the representation of the thrust management system.

The effects of rotor-on-wing, wing-on-rotor, and rotor-onhorizontal tail are included in this program. The effects of rotor-on-wing are represented by calculation of the slipstream angle of attack of the portions of the aircraft operating in the rotor slipstream by momentum methods and resolving the associated forces and moments to body axes. Correlation with test data are shown in Section 6.5.2 to verify these interference effects. The effects of the rotor slipstream on the horizontal tail downwash are also calculated by momentum methods. The angle through which the flow through the rotor is turned is assumed to represent the change in tail downwash. Provisions are made to incorporate the upwash effects of the wing on the rotor. Lifting line theory should be used to compute these effects.

The effects on lateral/directional parameters caused by rotor wake skew on the wing are included by computing the change in immersed wing area during sidewards flight and sideslips.

Structural dynamics effects included consist of the first mode wing vertical bending and the first wing torsion mode. These wing structural modes have been included on a "quasistatic" basis; i.e. the natural frequencies of vibration of the structure are much higher than the frequencies of the rigid body motion, and the coupling is in the aerodynamic terms.

The aerodynamics of the fuselage, empennage, nacelles, wings and rotors are included in detail. The aerodynamics of the
wing and rotors are written separately for the left and right sides. The effects of the wing leading edge umbrellas are included, with provisions for the direct effects of wing download and pitching moment with the umbrellas open in slow flight. Ground effects are considered on the rotors, wing and horizontal tail. The effects of Mach number on the airplane are treated by application of the Prandtl-Glauert rule. The effects of Mach number on the rotor data have been included in the curve fit equations.

The control system elements represented include pilot command, three axis stability augmentation systems, a load alleviation system (LAS) and a thrust management system. Control system actuator dynamics are represented by appropriate first order and second order lags. The systems are assumed to be "tight" in that thresholds, biases and hysteresis loops are neglected.

Turbine engine performance with appropriate dynamic responses are included. Engine power is computed for the range of flight condition necessary to cover the flight envelope. A relatively simple engine dynamic response model modulates the power output in response to pilot control of throttle position.

Landing gear is represented by a spring-damper system without complex calculation of oleo strut response.

The effects of rotor tilt angle on the aircraft center of gravity and inertia are included. Forces and moments resulting from
acceleration of the nacelles during tilting maneuvers are calculated in the program.

An airframe representation/preprocessor calculation is included that enables the user to input the location of major structural elements of the aircraft in terms of water line, butt line and station line location. All lengths and inertias required by the equation are then calculated. This feature enables the user to quickly change the location of major elements to assess their impact on vehicle response. The rest of the input data required has been kept to a minimum to augment the programs' usefulness. Provisions have been included to provide a very flexible design tool which enables the istute user to perform a wide variety of studies. Figure 2.1 summarizes the salient features of the mathematical model inscribed in this document. It should be emphasized that his model has full flight envelope capability.

```
- Full Flight Envelope Capability with Total Force
    Representation
- 6 Rigid Body Degrees of Freedom
- Independent Nacelle Pitch Degree of Freedom
- 2 Elastic Degrees of Freedom
- 1 Rotor Rotational Degree of Freedom
- Includes the Aerodynamics of:
- Rotors
- Wings
- Rotor/Wing \& Wing/Rotor Interference
- Fuselage
- Landing Gear
- Tail Surfaces
- Engines
- Control System Elements:
- Pilot Command
- SAS
- Load Alleviation System (LAS)
- Thrust and Power Management System
- Aeroelastic Representation
- Wing Vertical Bending
- Wing Torsion
```

Figure 2.1. Salient Features of Math Model

### 3.0 SIGN CONVENTIONS

Standard aircraft sign conventions have been used throughout this report. Sign conventions are as follows:

Positive X axis forward
Positive $Y$ axis outward along the right wing.
Positive $Z$ axis downward perpendicular to the $X Y$ plane. Lift is positive along the negative $Z$ axis. Pitching moment is positive nose-up about the $Y$ axis. Sideforce is positive outward in the direction of the positive $Y$ axis.

Yawing moment is positive nose-right.
Kolling moment is positive right wing down.
Positive elevator deflection is trailing edge down
Positive rudder deflection is rudder-trailing-edge-left.
Positive aileron deflection is right-flaperon-trailing-edge-down.

Positive spoiler deflection is left-hand-spoiler-deflec-ted-upward.

Positive deflection of the pilot's stick and rudder pedals yields positive aircraft pitch, roll, and yaw moments from negative control deflections.

Rotor sign conventions are illustrated in Section 7.0

Special sign convention used in the derivations are noted in the appropriate section.

### 4.0 MODEL 222 TILT ROTCR AIRCRAFT DESCRIPTION

The Boeing Model 222 Tilt Rotor Research Aircraft, shown in Figure 4.1 uses two 26-foot diameter soft in-plane hingeless rotors of the same design that has already been demonstrated in the NASA/Ames 40 by 80 -foot tunnel. The soft in-plane rotor is mechanically simple and provides excellent flying qualities characteristics as well as freedom from aeroelastic problems. It is service proven on the FAA certified BO-105 helicopter. For transition, the rotors tilt from hover position (rotor disk horizontal) to cruise position (rotor disk vertical). Intermediate nacelle positions provide optimum performance capability for climb, descent and for STOL operations.

The Nodel 222 is powered by two modified Lycoming T53-L-13B turboshaft engines mounted in fixed (nontilting) nacelles at each wing tip. The rotors are interconnected by a cross shaft for single engine operation. The engine power available yields excellent single engine and temperature-altitude performance.

Fuselage and empennage are production (MU-2J) components, modified to accept the Model 222 wing and two production (OV-10) ejection seats. The retractable tricycle landing gear is also the existing MU-2J gear modified to provide increased energy absorption.

Collective and cyclic pitch of the rotors, together with nacelle tilt, provide high control power in hover. In the cruise mode, control is by conventional airplane elevators,
rudder, flaperons and spoilers. Leading-edge "umbreila" flaps and large deflection trailing-edge flaps reduce download and ground effect turbulences in hover. Operation of flaps, umbrellas and elevator as well as phasing out of the rotor controls is mechanically programed with nacelle tilt to relieve pilot workload.

A limited-authority stability augmentation system includes feedback from angle-of-attack, yaw angle, and dynamic pressure. In cruise flight it feeds back two axes of cyclic pitch to the rotor control. This provides increased static stability and reduces blade loads to increase fatigue margins. The feedback system is not required for either stability or structural integrity. This system permits easy variation of the stability characteristics of the aircraft.


### 5.0 EQUATIONS OF MOTION

This section presents the derivation of the airframe equations of motion and the simplifications that were made in order to obtain the final equations as presented in Appendix E. The treatment accounts for all six rigid-body degrees-of-freedom including the effects of the tilting nacelles and rotors. The principal features of the derivation are:

- Assumption of $X-Z$ plane of symmetry
- The basic equations are derived about the instantaneous center of gravity of the aircraft since the center of gravity is strongly dependent on nacelle incidence.
- Rotor and engine gyroscopic terms are included
- The wing elastic degrees of freedom do not couple inertially. The coupling occurs through the aerodynamic terms in the equations as discussed in Section 12.
- Wing aeroelastic effects are not included in the center of gravity calculations.


### 5.1 AXES SYSTEM

A set of right-handed orthogonal axes OXYZ is placed at the center of mass of the aircraft and is fixed in the aircraft such that $O X$ lies in the lateral plane of symmetry and is positive forward parallel to the fuselage water line zero. The remaining axes are placed as shown in Figure 5.1. The orientation of the aircraft is defined with respect to a


Figure 5.1. Axes Systems
set of earth-fixed axes $C X^{\prime} Y^{\prime} Z^{\prime}$. With the axes OXYZ initially parallel to $C X^{\prime} Y^{\prime} Z^{\prime}$, the aircraft is yawed to the right about $O$ through an angle $\psi$, then pitched up about $O Z$ through the angle $\theta$ and finally rolled right about $O X$ through the angle $\phi$.

If $\underline{V}$ and $\underline{\Omega}$ are the aircraft velocity and angular velocity vectors relative to the earth-fixed axes, the projections of these vectors on the moving axes are $U, V$, and $W$, for the components along $O X, O Y$ and $O Z$, and $P, Q$ and $R$ for the angular velocity components.

Thus

$$
\begin{align*}
& \underline{V}=U \hat{\underline{i}}+V \hat{i}+W \underline{\underline{k}}  \tag{5,1}\\
& \underline{\Omega}=P \underline{\hat{i}}+Q \hat{\dot{j}}+R \underline{\hat{k}} \tag{5.2}
\end{align*}
$$

where the unit vectors $\underline{\hat{i}}, \hat{i}$ and $\underline{\hat{k}}$ lie along $O X, O Y$ and $O Z$.

### 5.2 AIRCRAFT GROUND TRACK

The components of $\underline{V}$ relative to the earth-fixed axes are obtained in terms of $U, V, W$ and $\psi, \theta, \phi$ as, (See Reference 10 ),

$$
\begin{align*}
\frac{d X^{\prime}}{d t}= & U \cos \theta \cos \psi+V(\sin \phi \sin \theta \cos \psi-\cos \phi \sin \psi) \\
& +W(\cos \phi \sin \theta \cos \psi+\sin \theta \sin \psi) \\
{\frac{d Y^{\prime}}{d t}=}= & U \cos \theta \sin \psi+V(\sin \phi \sin \theta \sin \psi+\cos \phi \cos \psi) \\
& +W(\cos \phi \sin \theta \sin \psi-\sin \phi \cos \psi) \tag{5.3}
\end{align*}
$$

$$
\frac{d Z^{\prime}}{d t}=-U \sin \theta+V \sin \phi \cos \theta+W \cos \phi \cos \theta
$$

Integration of these equations gives the aircraft ground track.
A further relationship may be obtained between the rate of
change of the Euler angles $(\psi, \theta, \phi)$ and the components of the angular velocity in the moving axes system, viz,

$$
\begin{align*}
& \dot{\psi}=(R \cos \phi+Q \sin \phi) \sec \theta \\
& \dot{\theta}=Q \cos \phi-R \sin \phi  \tag{5.4}\\
& \dot{\phi}=P+\dot{\psi} \sin \theta
\end{align*}
$$

### 5.3 FORCE EQUATION

The total external force, $\underline{F}$, acting at the aircraft center of mass is given by

$$
\begin{equation*}
F=\frac{\mathrm{d}}{\mathrm{dt}}(\mathrm{~m} \underline{V})=m\left[\frac{\delta \underline{V}}{\delta t}+\underline{\Omega} \times \underline{V}\right] \tag{5.5}
\end{equation*}
$$

where $m$ is the mass of the aircraft and $\frac{\delta V}{\delta t}$ is the rate of change of $\underline{V}$ with respect to the moving reference frame OXYZ i.e.

$$
\begin{equation*}
\frac{\delta \underline{V}}{\delta t}=U \underline{\hat{i}}+V \hat{j}+W \hat{k} \tag{5.6}
\end{equation*}
$$

If $E$ has components $F_{X}, F_{Y}$ and $F_{z}$ along the respective axes then

$$
\underline{F}=F_{x} \hat{\underline{i}}+F_{Y \hat{j}}+F_{z} \underline{\hat{k}}=m\left\{U \underline{\hat{i}}+V \hat{j}+W \hat{k}+\left|\begin{array}{lll}
\hat{i} & \hat{j} & \hat{k} \\
P & Q & R \\
U & V & W
\end{array}\right|\right\}
$$

thus

$$
\begin{align*}
& F_{x}=m(\dot{U}+Q W-R V) \\
& F_{Y}=m(\dot{V}+R U-P W)  \tag{5.7}\\
& F_{z}=m(\dot{W}+P V-Q U)
\end{align*}
$$

The forces $F_{X}, F_{Y}$ and $F_{z}$ are given by

$$
\begin{align*}
& \mathbf{F}_{\mathbf{x}}=\mathrm{X}_{\mathrm{AERO}}-\mathrm{mg} \sin \theta \\
& \mathrm{~F}_{\mathbf{y}}=\mathrm{Y}_{\mathrm{AERO}}+\mathrm{mg} \sin \phi \cos \theta  \tag{5,8}\\
& \mathrm{~F}_{\mathrm{z}}=\mathrm{z}_{\mathrm{AERO}}+\mathrm{mg} \cos \phi \cos \theta
\end{align*}
$$

where $X_{\text {AERO }}$, etc., are the components of the total aerodynamic force acting at the aircraft center of mass.

Substituting equations (5.8) in equations (5.7), the following equations are obtained for the aircraft accelerations,

$$
\begin{align*}
& \dot{U}=\frac{X_{\text {AERO }}}{m}-g \sin \theta-Q W+R V \\
& \dot{\mathrm{~V}}=\frac{\mathrm{Y}_{\text {AERO }}}{\mathrm{m}}+\mathrm{g} \cos \theta \sin \phi-\mathrm{RU}+\mathrm{PW}  \tag{5.9}\\
& \dot{W}=\frac{\mathrm{z}_{\text {AERO }}}{m}+g \cos \theta \cos \phi+Q U-P V
\end{align*}
$$

### 5.4 MOMENT EQUATION

The derivation of the equations for the total moment acting about the aircraft center of mass is complicated by the fact that the center of mass changes position due to the tilting nacelles. Thus the centers of gravity of the principal aircraft component masses of the wings $\left(m_{w}\right)$, fuselage (including tails) $\left(m_{f}\right)$, and nacelles $\left(m_{N}\right)$, move with respect to the reference axes OXYZ placed at the instantaneous overall center of gravity of the aircraft. The equation of motion for such a mass element will first be obtained and the total moment found by adding the contributions of all the elements.

### 5.5 EQUATION OF MOTION FOR A MASS ELEMENT

With reference to Figure (5.1) O'xyz is a right-handed set of axes placed at the center of gravity of the representative mass. The axes are parallel to the set oxyz. The mass, m,
rotates about its own center of gravity with angular velocity $\underline{\omega}$ which, in general, differs from $\underline{\Omega}$ the angular velocity of the aircraft. If $\underline{r}$ is the radius vector from $O$ to $O^{\prime}$ then the velocity of the center of mass of the element is

$$
\begin{equation*}
V=\frac{\delta \underline{r}}{\delta t}+\underline{\Omega} \times \underline{r} \tag{5.10}
\end{equation*}
$$

The angular momentum of this mass about $O$ is

$$
\begin{equation*}
\underline{h}=m(\underline{r} \times \underline{V})+\underline{h o} \tag{5.11}
\end{equation*}
$$

where ho is the angular momentum of $m$ about its own center of mass and is given by

$$
\begin{equation*}
\underline{h o}=\overline{\mathrm{I}} \underline{\omega} \tag{5.12}
\end{equation*}
$$

where $_{\bar{I}}$

$$
\bar{I}=\left[\begin{array}{ccc}
I_{x x} & -I_{x y} & -I_{x z}  \tag{5.13}\\
-I_{y x} & I_{y y} & -I_{y z} \\
-I_{z x} & -I_{z y} & I_{z z}
\end{array}\right]
$$

and $I_{x x}$, etc., are the moments and products of inertia of the mass about o'xyz.

The total moment, $G$, about the aircraft center of mass is given by

$$
\begin{equation*}
G=\frac{\mathrm{d} \underline{h}}{d t}=\frac{\delta \underline{h}}{\delta t}+\underline{\Omega} \times \underline{h} \tag{5.14}
\end{equation*}
$$

Using equations (5.10), (5.11) and (5.12) in (5.14) the
moment becomes

$$
\begin{align*}
\underline{G} & =m\left[\frac{\delta \underline{r}}{\delta t} \times\left(\frac{\delta \underline{\underline{r}}}{\delta t}+\underline{\Omega} \times \underline{r}\right)+\underline{r} \times \frac{\delta}{\delta t}\left(\frac{\delta \underline{r}}{\delta t}+\underline{\Omega} \times \underline{r}\right)\right]+\frac{\delta}{\delta t}(\bar{I} \underline{\omega}) \\
& +m \underline{\Omega} \times\left[\underline{r} \times\left(\frac{\delta \underline{r}}{\delta t}+\underline{\Omega} \times \underline{r}\right)\right]+\underline{\Omega} \times\left(\bar{I}_{\underline{\omega}}\right) \tag{5.15}
\end{align*}
$$

which reduces to

$$
\begin{align*}
\underline{G}= & 2 m \underline{\Omega}\left(\underline{\underline{r}} \cdot \frac{\delta \underline{r}}{\delta t}\right)+m \underline{r} \times \frac{\delta^{2} \underline{r}}{\delta t^{2}}+m \frac{\delta \underline{\Omega}}{\delta t}(\underline{r} \cdot \underline{r})-m \underline{r}\left(\underline{r} \cdot \frac{\delta \underline{\Omega}}{\delta t}\right)  \tag{5.16}\\
& -2 m \frac{\delta \underline{\underline{r}}}{\delta t}(\underline{\Omega} \cdot \underline{r})-m(\underline{r} \cdot \underline{\Omega})(\underline{\Omega} \times \underline{r})+I \frac{\delta \underline{\omega}}{\delta t}+\underline{\Omega} \times(\bar{I} \underline{\omega})
\end{align*}
$$

The only masses that possess angular velocities different from that of the aircraft are the nacelles, which are free to pitch about $O^{\prime \prime}$ with angular rate $i=\frac{d i_{N}}{d t}$. Thus $\underline{\omega}$ may be written generally as

$$
\begin{equation*}
\underline{\omega}=P \underline{i}+\left(Q+i_{N}\right) \hat{j}+R \underline{\hat{k}} \tag{5.17}
\end{equation*}
$$

Now, with $\underline{r}=X \underline{\hat{i}}+Y \hat{i}+Z \hat{\underline{k}}$, where $X, Y$, and $Z$ are the instantaneous coordinates of the individual mass center relative to the aircraft mass center, the various terms of equation (5.16) are, in component form,

$$
\begin{aligned}
& \underline{r} \cdot \frac{\delta \underline{r}}{\delta t}=X X+Y Y+Z Z \\
& \underline{r} x \frac{\delta^{2} \underline{r}}{\delta t^{2}}=(Y Z-Z Y) \underline{i}-(X Z-Z X) \hat{i}+(X Y-Z X) \underline{\hat{k}} \\
& \frac{\delta \underline{\Omega}}{\delta t}(\underline{r} \cdot \underline{r})=\left(X^{2}+Y^{2}+Z^{2}\right)(\dot{P} \underline{\hat{i}}+\dot{Q} \hat{j}+\dot{\mathrm{R}} \hat{\hat{k}}) \\
& \text { r. } \frac{\delta \Omega}{\delta t}=X \dot{P}+Y \dot{Q}+Z \dot{R} \\
& \underline{\Omega} \cdot \underline{\underline{r}}=X P+Y Q+Z R \\
& (\underline{r} \cdot \underline{\Omega})(\underline{\Omega} X \underline{r})=(X P+Y Q+X R)[(Q Z-R Y) \underline{\hat{i}}-(P Z-R X) \hat{j}+(P Y-X Q) \hat{\hat{k}}] \\
& \overline{\mathrm{I}} \frac{\delta \omega}{\delta t}=\left(I_{x x} \dot{P}-I_{x z} R\right) \hat{i}+I_{Y Y}\left(\dot{Q}+\dot{I}_{N}\right) \hat{\dot{j}}+\left(I_{z z} \dot{R}-I_{x z} \dot{P}\right) \underline{\hat{k}} \\
& \underline{\Omega} \times\left(\bar{I}_{\underline{\omega}}\right)=\left(Q R I_{Z Z}-Q P I_{X Z}-R Q I_{Y Y}-R i_{N} I_{Y Y}\right) \underline{\hat{i}} \\
& -\left(P R I_{z z}-P^{2} I_{x z}-P R I_{x x}+R^{2} I_{x z}\right) \hat{j} \\
& +\left(Q R I_{X Z}+P Q I_{Y Y}+P i_{N} I_{Y Y}-P Q I_{X X}\right) \underline{\hat{k}}
\end{aligned}
$$

where, in the last two terms, the products of inertia $I_{x y}$ and $I_{y z}$ are zero from symmetry considerations.

$$
5.0-7
$$

Substituting the above relations into equation (5.16) and noting that $\dot{Y}$ and $\ddot{Y}$ are always zero (no lateral motion of the individual masses) the following expressions are obtained for the components of the moment $\underline{G}=\Delta L \underline{\hat{i}}+\Delta M \hat{j}+\Delta N \hat{k}$ :

$$
\Delta L=\dot{P}\left[I_{x x}+m\left(Y^{2}+Z^{2}\right)\right]-(\dot{R}+P Q)\left[I_{x z}+m x Z\right]
$$

$$
+R Q\left[I_{Z Z}-I_{Y Y}+m\left(Y^{2}-Z^{2}\right)\right]+m Y Z\left(R^{2}-Q^{2}\right)-I_{Y Y} R i_{N}
$$

$$
+m(Y \ddot{z}-2 \dot{X} Y R-2 \dot{X} Z R+2 Z \dot{Z} P-X Y(\dot{Q}-P R))
$$

$$
\Delta M=\dot{Q}\left[I_{Y Y}+m\left(X^{2}+Z^{2}\right)\right]-\left(R^{2}-P^{2}\right)\left[I_{X Z}+m X Z\right]
$$

$$
\begin{equation*}
+\operatorname{PR}\left[I_{x x}-I_{z z}+m\left(z^{2}-x^{2}\right)\right]+I_{y y^{\prime}} i_{N} \tag{5.19}
\end{equation*}
$$

$$
+m[\ddot{X} Z-X \ddot{Z}+2 Q(Z \dot{Z}+X \dot{X})-X Y(\dot{P}+R Q)+Y Z(P Q-\dot{R})]
$$

$$
\begin{equation*}
\Delta N=\dot{R}\left[I_{z z}+m\left(X^{2}+Y^{2}\right)\right]-(\dot{P}-R Q)\left[I_{x z}+m x Z\right] \tag{5.20}
\end{equation*}
$$

$$
+P Q\left[I_{Y Y}-I_{X X}+m\left(X^{2}-Y^{2}\right)\right]+I_{Y Y} P i_{N}
$$

$$
+m\left[2 X \dot{X} R-Y \ddot{X}-2 X Z P-2 Y \dot{Z} Q-Y Z(\dot{Q}+P R)+X Y\left(Q^{2}-P^{2}\right)\right]
$$

Summing the rolling moment equation:

$$
\begin{align*}
L= & I_{X X} \dot{P}-I_{X Z}(\dot{R}+P Q)+\left(I_{Z Z}-I_{Y Y}\right) R Q  \tag{5.21}\\
& +m_{N}\left(R^{2}-Q^{2}\right)\left(z_{N R}-Z_{N L}\right) Y_{N}+m_{N}\left\{Y_{N}\left(\ddot{z}_{N R}-\ddot{z}_{N L}\right)\right. \\
- & 2 Q\left(\dot{X}_{N R}-\dot{X}_{N L}\right) Y_{N A}-2 R\left(\dot{X}_{N R} z_{N R}+\dot{X}_{N L} z_{N L}\right)+2 P\left(\dot{z}_{N R} z_{N R}+\right. \\
& \left.\left.\dot{z}_{N L} z_{N L}\right)-(\dot{Q}-P R)\left(X_{N R}-X_{N L}\right) Y_{N}\right\}+2 m_{f} Z_{f}\left(P \dot{z}_{f}-\right. \\
& \left.R \dot{X}_{f}\right)+2 m_{W} z_{W}\left(P \dot{z}_{W}-R \dot{X}_{W}\right)-R I_{Y Y}^{N}\left(i_{N L}+i_{N R}\right)
\end{align*}
$$

where $I_{X X}, I_{X Z}, I_{Z Z}$ and $I_{Y Y}$ are the inertias of the aircraft about its center of gravity, and the subscripts $f, w, N L$ and $N R$ stand for fuselage, wing, left nacelle and right nacelle. The remaining symbols are defined in the List of Symbols. Similar expressions are obtained for the pitching moment and yawing
moment. In the interests of brevity the remainder of the discussion will be limited to equation (5.21).

Evaluation of the terms of the rolling moment equation indicate that this equation may be simplified considerably without a significant change in accuracy. For example, terms containing ( $\dot{X}_{N R}-\dot{X}_{N L}$ ) may be dropped because $\dot{X}_{N R}$ is normally identical to $\dot{X}_{N L}$, i.e. the nacelles are raised or lowered together at the same rate. Equation (5.21) may thus be written

$$
\begin{equation*}
L=I_{X X} \dot{P}-I_{X Z}(\dot{R}+P Q)+\left(I_{Z Z}-I_{Y Y}\right) R Q+m_{N} Y_{N}\left(\ddot{X}_{N R}-\ddot{Z}_{N L}\right) \tag{5.22}
\end{equation*}
$$

where the last term has been retained in consideration of the high differential nacelle accelerations encountered during hover maneuvers.

From the relationships presented in Appendix $C$ the last term of equation (5.22) may be rewritten as

$$
\begin{gather*}
-\ell m_{N} y_{N}\left[i_{N R} \cos \left(i_{N R}-\lambda\right)+i_{N L}^{2} \sin \left(i_{N L}-\lambda\right)\right. \\
\left.-i_{N R}^{2} \sin \left(i_{N R}-\lambda\right)-i_{N L} \cos \left(i_{N L}-\lambda\right)\right] \tag{5.23}
\end{gather*}
$$

which may be approximated to

$$
\begin{equation*}
-\ell m_{N} Y_{N}\left[\ddot{i}_{N R} \cos \left(i_{N R}-\lambda\right)-\ddot{i}_{N L} \cos \left(i_{N L}-\lambda\right)\right] \tag{5.24}
\end{equation*}
$$

since the nacelle rates appear as squared terms.

Similar treatment of the pitching moment and yawing moment equations results in the following final form of the moment equations.

$$
\begin{aligned}
L_{A E R O} & =I_{X X} \dot{P}-I_{X Z}(\dot{R}+P Q)+\left(I_{Z Z}-I_{Y Y}\right) R Q \\
& -\ell m_{N} Y_{N}\left[\dot{i}_{N R} \cos \left(i_{N R}-\lambda\right)-\ddot{i}_{N L} \cos \left(i_{N L}-\lambda\right)\right] \\
M_{A E R O} & =I_{Y Y} \dot{Q}-I_{X Z}\left(R^{2}-P^{2}\right)+\left(I_{X X}-I_{Z Z}\right) P R \\
& +\dot{i}_{N R}\left\{I_{Y Y O}^{N}+\ell m_{N}\left[X_{R} \cos \left(i_{N R}-\lambda\right)-z_{R} \sin \left(i_{N R}-\lambda\right)\right]\right\}(5.25) \\
& +i_{N L}\left\{I_{Y Y_{O}}+\ell m_{N}\left[X_{L} \cos \left(i_{N L}-\lambda\right)-Z_{L} \sin \left(i_{N L}-\lambda\right)\right]\right\} \\
N_{A E R O} & =I_{Z Z} \dot{R}-I_{X Z}(\dot{P}-R Q)+\left(I_{Y Y}-I_{X X}\right) P Q \\
& +\ell m_{N} Y_{N}\left[i_{N R} \sin \left(i_{N R}-\lambda\right)-i_{N L} \sin \left(i_{N L}-\lambda\right)\right]
\end{aligned}
$$

where the moments $L_{\text {AERO }}, M_{\text {AERO }}$ and $N_{\text {AERO }}$ represent the sum of the aerodynamic moments and rotor/engine gyroscopic moments about the aircraft center of mass. $I_{Y Y_{O}}^{N}$ is the nacelle pitch inertia referred to the nacelle-fixed axes system described in Appendix C. Equations for the aircraft inertias are also presented in that Appendix.

### 5.6 EQUATIONS OF MOTION FOR NACELLES

The equation of motion for a nacelle is required in order to obtain the moment exerted by the nacelle on the wing tip at the pivot. This moment is then used in the equations for wing twist.

The angular momentum of a nacelle about its pivot point is given by

$$
\begin{align*}
\underline{h}_{p} & =\left(\underline{r}-\underline{r}_{p}\right) \times m_{N} \underline{V}+\underline{h}_{N} \\
& =m_{n}(\underline{r} \times \underline{V})+\underline{h}_{0}-m_{n} \underline{r}_{p} \times \underline{V} \tag{5.26}
\end{align*}
$$

where $\underline{r}$ is the radius vector from aircraft c.g. to nacelle $\underline{v}$ is the velocity of the nacelle c.g.

$\mathrm{m}_{\mathrm{N}}$ is the nacelle mass
and $r_{-p}$ is the radius vector from aircraft cig. to
The term $m_{n}(\underline{r} \times \underline{V})+\underline{h}_{o_{N}}$ is the angular momentum of the nacelle about the aircraft cog. $\left(=\underline{h}_{C G}^{N}\right)$

$$
\text { i.e. } \quad \underline{h}_{p}=\underline{h}_{C G}^{N}-m_{N}\left(\underline{r}_{p} \times \underline{v}\right)
$$

The moment about the pivot is

$$
\begin{equation*}
G_{p}=\frac{d h_{p}}{d t}=\frac{d h_{N}}{d t}-m_{n} \frac{d}{d t}\left(\underline{r}_{p} \times \underline{V}\right)=\underline{G}_{C G}^{N}-\Delta \underline{G} \tag{5.27}
\end{equation*}
$$

Since the quantity $\underline{G}_{c g}^{N}$ has already been obtained (equations (5.18), (5.19) and (5.20)), only the remaining term needs to be evaluated.

$$
\begin{align*}
& \Delta \underline{G}=m_{N} \frac{d}{d t}\left(\underline{r}_{p} \times \underline{V}\right)=m_{N}\left\{\frac{\delta \underline{r}_{p}}{\delta t} \times \underline{V}+\underline{r}_{p} \times \frac{\delta \underline{V}}{\delta t}+\underline{\Omega}\left(\underline{r}_{p} \times \underline{V}\right)\right\} \\
&=m_{N}\left\{\frac{\delta \underline{r}_{p}}{\delta t} \times\left(\frac{\delta \underline{r}}{\delta t}+\underline{\Omega} \times \underline{r}\right)+\underline{r}_{p} \times \frac{\delta}{\delta t}\left(\frac{\delta \underline{r}}{\delta t}+\underline{\Omega} \times \underline{r}\right)\right.  \tag{5.28}\\
&\left.+\underline{\Omega} \times\left[\underline{r}_{p} \times\left(\frac{\delta \underline{r}}{\delta t}+\underline{\Omega} \times \underline{r}\right)\right]\right\}
\end{align*}
$$

Expansion of these terms results in the following expression

$$
\begin{aligned}
\Delta \underline{G} & =m_{\mathbb{N}}\left\{\frac{\delta \underline{r} p}{\delta t} \times \frac{\delta \underline{r}}{\delta t}+\underline{\Omega}\left(\underline{r} \cdot \frac{\delta \underline{r}}{\delta t}\right)-\underline{r}\left(\frac{\delta \underline{r} p}{\delta t} \cdot \underline{\Omega}\right)+\underline{r}_{p} \times \frac{\delta^{2} \underline{r}}{\delta t^{2}}+\frac{\delta \underline{\Omega}}{\delta t} \quad\left(\underline{r} \cdot \underline{r}_{p}\right)\right. \\
& -\underline{r}\left(\underline{r}_{p} \cdot \frac{\delta \underline{\Omega}}{\delta t}\right)+\underline{\Omega}\left(\frac{\delta \underline{r}}{\delta t} \cdot \underline{r} p\right)-2 \frac{\delta \underline{r}}{\delta t}(\underline{r}-\underline{\Omega}) \\
& \left.+\underline{r_{p}}\left(\frac{\delta \underline{r}}{\delta t} \cdot \underline{\Omega}\right)-\left(\underline{r_{p}} \cdot \underline{\Omega}\right)(\underline{\Omega} \times \underline{r})\right\}
\end{aligned}
$$

We require only the $\hat{\hat{j}}$ component of this vector in order to obtain the nacelle pivot pitching moment.

The components of the vectors $\underline{r}_{p}, \underline{r}$ and $\underline{\Omega}$ are

$$
\begin{aligned}
& \underline{r}_{p}=X_{p} \hat{i}+Y_{N} \hat{j}+z_{p} \hat{\underline{k}}=-X_{C G} \hat{\hat{i}}+Y_{N} \hat{j}-z_{C G} \underline{\hat{k}} \\
& \underline{r}=X_{N} \underline{\hat{i}}+Y_{N} \underline{\hat{j}}+z_{N} \hat{k} \\
& \underline{\Omega}=P \underline{\hat{i}}+Q \hat{j}+R \underline{\hat{j}}
\end{aligned}
$$

Noting that the $\underline{\hat{j}}$ components of $\frac{\delta \underline{\underline{p}}}{\delta t}, \frac{\delta \underline{r}}{\delta t}$ are zero (since $Y_{N}$ is a constant), the above expression yields

$$
\begin{align*}
\Delta M=m_{N}\{ & \ddot{x}_{N} z_{C G}-\ddot{z}_{N} x_{C G}+\dot{z}_{C G} \dot{x}_{N}+\dot{z}_{N} \dot{x}_{C G}+P Q X_{N} z_{N}  \tag{5.30}\\
& \left.-R Q x_{N} y_{N}\right\}
\end{align*}
$$

Combining this equation with Equation (5.19) and using the transformations given in Appendix $C$, the final equation for the right-hand nacelle pivot actuator pitching moment becomes, after some simplification,

$$
\begin{align*}
A_{N R} & =-\dot{i}_{N R}\left[I_{Y Y_{O}}^{N}+\ell^{2} m_{N}\left(1-\frac{m_{N}}{m}\right)\right]-\ell^{2} m_{N}\left(1-\frac{m_{N}}{m}\right)\left[\dot{Q}-P R \cos 2\left(i_{N R}-\lambda\right)\right. \\
& \left.+\left(R^{2}-P^{2}\right) \sin \left(i_{N R}-\lambda\right) \cos \left(i_{N R}-\lambda\right)\right]-\left(R^{2}-P^{2}\right) I_{z Z_{O}}^{N} \sin i_{N R} \cos i_{N R} \\
& -I_{Y Y_{O}} \dot{Q}+\ell \frac{m_{N}}{m}\left[X_{A E R O} \sin \left(i_{N R}-\lambda\right)+z_{A E R O} \cos \left(i_{N R}-\lambda\right)\right] \\
& -\ell m_{N} Y_{N}\left\{(\dot{R}-P Q) \sin \left(i_{N R}-\lambda\right)-(\dot{P}+R Q) \cos \left(i_{N R}-\lambda\right)\right\} \\
& +M_{N R_{A E R O}} \tag{5.31}
\end{align*}
$$

where $M_{N R R_{\text {AERO }}}$ includes the moment resulting from nacelle aerodynamic loads and the rotor gyroscopic moments. The terms $X_{A E R O}$ and $Z_{A E R O}$ are, respectively, the total aircraft aerodynamic $X$ and $Z$ forces.

The corresponding equation for the left nacelle actuator moment is outained by substituting $-Y_{N}=Y_{N}$ and changing the $R$ subscript to L .

### 5.7 DETERMINATION OF ROTOR GYROSCOPIC MOMENTS

The gyroscopic moments are most readily obtained as follows. A set of axes $O^{\prime \prime} x^{\prime} y^{\prime} z^{\prime}$ is taken at the rotor hub (rotor c.g.) parallel to the nacelle-fixed set of axes $0 x_{0} y_{0} z_{0}$. Associated with each axis are the corresponding unit vectors $\hat{i}^{\prime} \hat{i}^{\prime}$ and $\hat{k}^{\prime}$. The angular velocity of the rotor with respect to these axes is the vector

$$
\underline{\omega}=\Omega_{R} \hat{\hat{i}}^{\prime}
$$

where $\Omega_{\mathrm{K}}$ is the rotor rotational speed.

The angular momentum of the rotor with respect to its c.g. is

$$
\begin{equation*}
\underline{h}_{0}=\bar{I}_{R} \underline{\omega} \tag{5.33}
\end{equation*}
$$

where $\bar{I}_{R}$ is the inertia matrix

$$
\left[\begin{array}{llll}
I_{R_{x}} & & & \\
& & & \\
& I_{R_{y}^{\prime}} & \\
& & & I_{R_{z}}
\end{array}\right]
$$

the off-diagonal terms being zero since the axes $U^{\prime \prime} x^{\prime} y^{\prime} z^{\prime}$ are principal axes of inertia of the rotor and hub.

In component form the angular momentum of the rotor is

$$
\begin{equation*}
\underline{h}_{0}=I_{R_{Y}}, \Omega_{R} \hat{\underline{i}}^{\prime}=I_{R} \Omega_{R} \hat{\underline{i}}^{\prime} \tag{5.34}
\end{equation*}
$$

Witn respect to the inertial axes $O X Y Z$, the components of $h_{0}$ are

$$
\begin{equation*}
\underline{n}_{0}=I_{R^{\Omega} \Omega_{R}} \cos i_{N} \hat{\hat{i}}-I_{R} \Omega_{R} \sin i_{N} \underline{\hat{k}} \tag{5.35}
\end{equation*}
$$

The huid moment is therefore given by

$$
\begin{align*}
\underline{G}_{H U B} & =\frac{d \underline{h_{0}}}{\delta t}=\frac{\delta \underline{h}_{0}}{\delta t}+\underline{\Omega} \times \underline{h_{0}}  \tag{5.36}\\
\text { where } \quad \underline{\Omega} & =P \underline{i}+Q \hat{j}+R \hat{k} \tag{5.37}
\end{align*}
$$

Suvstitution of equations (5.35) and (5.37) into equation (5.36) results in the following equations for the rotor gyroscopic moments.

$$
\begin{align*}
& L_{\text {gyro }}=I_{R} \dot{\Omega}_{R} \cos i_{N}-I_{R} \Omega_{R}\left(i_{N}+Q\right) \sin i_{N} \\
& =I_{R} P \Omega_{R} \sin i_{N}+I_{R} R_{R} \cos i_{N}  \tag{5.39}\\
& \text { ingro }_{\text {gyro }}=-I_{R} \dot{\Omega}_{R} \sin i_{N}-I_{R} \Omega_{R}\left(i_{N}+Q\right) \cos i_{N} \tag{5.40}
\end{align*}
$$

The above terms appear in the Computer Representation (Appendix E) as additions to the rotor aerodynamic forces and moments.

### 6.0 AIRFRAME AERODYNAMICS

This section presents the mathematical equations and representations of the aerodynamic data for the aircraft without rotors. The contribution of the rotors is described in Section 7. The overall airframe aerodynamics are obtained from the following components:
(a) Fuselage
(b) Wings
(c) Horizontal Tail
(d) Vertical Tail
(e) Nacelles

The data and equations for each of the aerodynamic components are discussed below, together with the substantiating methods. The aerodynamic data are presented in local wind axes. Resolution to aircraft body axes is accomplished as described in the mathematical model (Appendix $E$ ). Where required, the equations have been written so as to be applicable over the entire range of angle of attack $\pm 180$ degrees.

### 6.1 FUSELAGE

The aerodynamic lift, drag and pitching moment coefficients of the fuselage were estimated using the methods of Reference $\mathbf{I}$. The forces and moments are referred to the point on the fuselage corresponding to the wing quarter chord position. This reference point was selected in order to minimize the number of force and moment transfer equations in the mathematical
model. Wing-to-body carryover effects have been included in fuselage loads.

The equations for the fuselage forces and moments are:
Lift:

$$
\begin{aligned}
C_{L_{F}}= & K_{42}+K_{3} \operatorname{Sin} \alpha_{F} \operatorname{Cos} \alpha_{F}+K_{4} \operatorname{Sin} \alpha_{F} \operatorname{Cos} \alpha_{F} \\
& \operatorname{Sin} \alpha_{F} \operatorname{Cos} \alpha_{F}
\end{aligned}
$$

Drag:

$$
\begin{aligned}
C_{D F}= & C_{D_{O}}\left(1+K_{O}\left|\beta_{F}\right|^{3}\right)+K_{2}\left(\sin \alpha_{F} \operatorname{Cos} \alpha_{F}\right)^{2}+K_{1} \\
& \left|\operatorname{Sin} \alpha_{F} \operatorname{Cos} \alpha_{F}\right|+\Delta C_{D_{L G}}
\end{aligned}
$$

Side Force:

$$
C_{Y_{F}}=K_{7} \operatorname{Sin} \beta_{F} \operatorname{Cos} \beta_{F}+K_{8} \operatorname{Sin} \beta_{F} \operatorname{Cos} \beta_{F}\left|\operatorname{Sin} \beta_{F} \operatorname{Cos} \beta_{F}\right|
$$

Pitching Moment: $\quad C_{M_{F}}=C_{M_{O_{F}}}+K_{5} \sin \alpha_{F} \operatorname{Cos} \alpha_{F}+K_{6} \sin \alpha_{F} \operatorname{Cos} \alpha_{F} \mid$

$$
\operatorname{Sin} \alpha_{F} \operatorname{Cos} \alpha_{F} \mid+\Delta C_{M_{L G}}
$$

Yawing Moment: $\quad C_{\mathrm{T}_{\mathrm{F}}}=\mathrm{C}_{\mathrm{N}_{\mathrm{OF}}}+\mathrm{K}_{\mathrm{g}} \operatorname{Sin} \beta_{\mathrm{F}} \operatorname{Cos} \beta_{\mathrm{F}}+\mathrm{K}_{10} \operatorname{Sin} \beta_{\mathrm{F}} \operatorname{Cos} \beta_{\mathrm{F}}\left|\operatorname{Sin} \beta_{\mathrm{F}} \operatorname{Cos} \beta_{\mathrm{F}}\right|$
Rolling Moment: $\quad C_{\boldsymbol{R}_{F}}=0$

$$
\text { where } \begin{aligned}
\alpha_{F} & =\operatorname{Tan}^{-1}\left(\frac{W}{U}\right), C_{L_{F}}=\frac{L_{F}}{\frac{1}{2} \rho V_{F U S}^{2} S_{W}} \text { etc. } \\
\beta_{F} & =\operatorname{Tan}^{-1}\left[\frac{V}{\sqrt{U^{2}+W^{2}}}\right], C_{M_{F}}=\frac{M_{F}}{\frac{1}{2} \rho V_{F U S}^{2} S_{W} C_{W}}
\end{aligned} \text { etc. }
$$

and $\Delta \mathrm{C}_{\mathrm{D}_{\mathrm{LG}}}$, $\Delta \mathrm{C}_{\mathrm{M}_{\mathrm{LG}}}$, are the landing gear contributions to fuselage drag and pitching moment coefficients, when the landing gear is extended.

The fuselage forces and moments are then resolved into body axes at the aurcraft C.G.

### 6.2 NACELLES

The forces and moments acting on the nacelles were estimated using the cross-flow methods of Reference 12 . For convenience the resulting forces and moments are referred to the rotor hub, so that they may be added directly to the rotor forces and moments. The following equations are for the forces and moments on two nacelles:

$$
\begin{aligned}
& C_{L_{N}}=K_{32} \sin \alpha_{N} \cos \alpha_{N} \\
& C_{D_{N}}=C_{D_{O_{N}}}+K_{30}\left|\alpha_{N}\right|+K_{31} \alpha_{N}{ }^{2} \\
& C_{M_{N}}=C_{M_{O_{N}}}+K_{34} \sin \alpha_{N} \cos \alpha_{N}+K_{35} \sin \alpha_{N} \cos \alpha_{N}\left|\sin \alpha_{N} \cos \alpha_{N}\right| \\
& C_{Y_{N}}=K_{36} \sin \beta_{N} \cos \beta_{N}+K_{37} \sin \beta_{N} \cos \beta_{N}\left|\sin \beta_{N} \cos \beta_{N}\right| \\
& C_{N_{N}}=C_{N_{O_{N}}}+K_{38} \sin \beta_{N} \cos \beta_{N}+K_{39} \sin \beta_{N} \cos \beta_{N}\left|\sin \beta_{N} \cos \beta_{N}\right| \\
& C_{X_{N}}=O
\end{aligned}
$$

The nacelle forces and moments in nacelle axes are:

$$
\begin{aligned}
\Delta X_{N}^{\prime} & =q_{N} S_{W}\left[-C_{D_{N}} \cos \alpha_{N}+C_{L_{N}} \sin \alpha_{N}-C_{Y_{N}} \sin \beta_{N} \cos \alpha_{N}\right] \frac{1}{2} \\
\Delta Y_{N}^{\prime} & =q_{N} S_{W}\left[C_{Y_{N}} \cos \beta_{N}-C_{D_{N}} \sin \beta_{N}\right] \frac{1}{2} \\
\Delta Z_{N}^{\prime} & =q_{N} S_{W}\left[-C_{L_{N}} \cos \alpha_{N}-C_{D_{N}} \cos \beta_{N} \sin \alpha_{N}-C_{Y_{N}} \sin \beta_{N} \sin \alpha_{N}\right] \frac{1}{2} \\
\Delta \mathcal{L}_{N}^{\prime} & =q_{N} S_{W} b_{W}\left[-\left(\frac{D_{W}}{b_{W}}\right] C_{M_{N}} \sin \beta_{N} \cos \alpha_{N}-C_{N_{N}} \sin \alpha_{N}\right] \frac{1}{2} \\
\Delta M M_{N}^{\prime} & =q_{N} S_{W} C_{W}\left[C_{M_{N}} \cos \beta_{N}\right] \frac{1}{2} \\
\Delta N_{N}^{\prime} & =q_{N} S_{W} b_{W}\left[C_{N_{N}} \cos \alpha_{N}-\left(\frac{C_{W}}{b_{W}}\right) C_{M_{N}} \sin \beta_{N} \cos \alpha_{N}\right] \frac{1}{2}
\end{aligned}
$$

### 6.3 HORIZONTAL TAIL

Aerodynamics of the horizontal tail were obtained using the methods of Reference 1 in combination with test data. The horizontal tail includes a plain elevator.

The angle of attack of the horizontal tail, including interference effects, for zero elevator deflection, is

$$
\alpha_{H T}=\operatorname{Tan}^{-1}\left[\frac{w_{H T}}{u_{H T}}\right]-\varepsilon+i_{H T}
$$

where $\varepsilon$ is the total downwash at the tail due to wing, rotor and ground effects and $i_{H T}$ is the tail incidence angle.

The effect of elevator deflection on the effective tail angle of attack is introduced through the elevator effectiveness parameter, $\tau_{H T}$, which is a function of the elevator and horizontal tail areas. Thus the effective horizontal tail angle of attack is

$$
\alpha_{e_{\mathrm{HT}}}=\alpha_{\mathrm{HT}}+\tau_{\mathrm{HT}} \delta_{\mathrm{e}}
$$

where $\delta_{e}$ is the elevator deflection.
The tail downwash angle, $\varepsilon$, depends on wing angle of attack and on rotor slipstream deflection. At a given rotor angle of attack, the slipstream deflection is a function of rotor thrust coefficient, $C_{T_{S}}$, where the coefficient is based on the slipstream dynamic pressure. Figure 6.1 presents data on downwash angles measured during tests on a tilt rotor wind tunnel model (Reference 7). As can be seen, the downwash at low values of thrust coefficient is the same as the value of the power-off wing

$$
6.0-4
$$



Figure 6.1. Variation of Horizontal Tail Downwash Angle with Thrust Coefficient
downwash $\left(C_{T_{S}}=0\right)$. Above values of $C_{T_{S}}$ in the neighborhood of $C_{T_{S}}=.5$ the downwash increases with increasing thrust coefficient. The values in the increasing portion of $\varepsilon v_{s} C_{T}$ were found to correspond approximately to the slipstream deflection angle $\varepsilon_{p}$. Therefore, the approach adopted in the mathematical
model was to test if the rotor slipstream downwash ( $\bar{\varepsilon}_{p}$ ) exceeded the wing downwash and, if so, to use the computed slipstream downwash value as the tail downwash angle. Otherwise the wing downwash value was used. Thus if

$$
\begin{aligned}
& \bar{\varepsilon}_{p} \geq \varepsilon_{0}+\frac{d \varepsilon}{d \alpha}\left(\bar{\alpha}_{w}-\ell_{A C} \frac{\dot{W}_{W}}{\bar{U}^{2}}\right) \\
& \text { then } \quad \varepsilon=\frac{\bar{\varepsilon}_{p}(1-G E F)}{\sqrt{1-M^{2}}}
\end{aligned}
$$

otherwise

$$
\varepsilon=\left[\varepsilon_{0}+\frac{d \varepsilon}{d \alpha}\left(\bar{\alpha}_{W}-\ell_{A C} \frac{\dot{W}}{U^{2}}\right)\right] \frac{(1-G E F)}{\sqrt{1-M^{2}}}
$$

In these expressions $\varepsilon_{0}$ is the wing downwash angle at zero wing angle-of-attack, $\frac{d_{\varepsilon}}{d \alpha}$ is the downwash derivative, $\ell_{A C}$ is the distance from the wing to the tail aerodynamic centers, and $\ell_{A C} \frac{\dot{W}}{U^{2}}$ is the familiar downwash lag term. In general, the quantities $\varepsilon_{O}$ and $\frac{d \varepsilon}{d \alpha}$ depend on the average of the left and right flaperon deflections. The effect of differential deflection of aileron/spoiler in producing an asymmetrical downwash field at the horizontal tail was not included because of the small contribution this makes to total aircraft rolling moment. The term (l-GEF) in the above equations is the ground effect factor. This quantity was obtained from Reference 10 and is a function of the wing span and height of the horizontal tail above the ground. This factor, when multiplied by the downwash which would be found out of ground effect, yields the downwash in ground effect. Ground effect is discussed in more detail in Section 10 .

The lift and drag forces acting on the horizontal tail are required over the complete range of angle of attack -180 to $+180^{\circ}$, since the tilt rotor can fly backwards. The following sketch shows the schematic variation of lift and drag coefficients over this range plotted as a function of the effective horizontal tail angle of attack, $\alpha_{e_{H T}}$.



The angle of attack for $C_{L_{H T}}$ is denoted by $\hat{\alpha}_{H T}$ and is the value of the effective angle of attack at the stall less 2 degrees i.e.

$$
\hat{\alpha}_{\mathrm{HT}_{+}}=\left(\alpha_{\mathrm{HT}_{\mathrm{STALL}}}-2^{\circ}\right)+{ }^{\tau} \mathrm{HT}^{\delta}{ }_{e}
$$

Similarly the angle of attack for stall at negative angles of attack is

$$
\hat{\alpha}_{\mathrm{HT}_{-}}=-\left(\alpha_{H T_{S T A L L}}-2^{\circ}\right)+\tau_{H T} \delta_{e}
$$

The slope of the lift curve within this range of positive and negative angles of attack is given by

$$
C_{L_{\alpha}}=\frac{C_{L_{\alpha H T}}\left(\frac{a_{\alpha}}{a}\right)}{\sqrt{1-M^{2}}}
$$

where $a_{g} / a$ is the ratio of tail lift-curve slopes in and out of ground effect, and $\sqrt{1-M^{2}}$ is the Prandtl-Glauert correction factor for the effect of Mach number on lift-curve slope.

Within this region on the lift curve the value of lift coefficient is given by $C_{L_{H T}}=C_{L_{\alpha}} \alpha_{e H T}$ and the corresponding drag coefficient by

$$
C_{D_{H T}}=C_{D_{O_{H T}}}+\frac{2 C_{L_{H T}}^{2}}{\pi A R_{H T}}
$$

After stall angle of attack is passed the lift is assumed to fall linearly to zero at $\alpha_{e}= \pm 90^{\circ}$.

In these regions the lift is given by

$$
c_{I_{\alpha}}=c_{L_{\alpha}} \hat{\alpha} \pm \frac{\left( \pm 90-\alpha_{e_{H T}}\right)}{\left( \pm 90-\hat{\alpha}_{H T}{ }^{2}\right)}
$$

where the appropriate signs are taken depending on the sign of aeht.

The corresponding drag is obtained by assuming a linear variation of drag from the value at $C_{L_{\text {MAX }}}$ to a value of $C_{D}=1.1$ (flat plate normal to stream) at $\alpha_{e_{H T}}=90^{\circ}$. Thus

$$
6.0-8
$$

$$
c_{L_{H T}}=c_{L_{\alpha T A L L}} \hat{\alpha}_{H T_{ \pm}}
$$


and

$$
\mathrm{C}_{\mathrm{D}_{\mathrm{HT}}}=\mathrm{C}_{\mathrm{D}_{\mathrm{HT}}}+\frac{\left(\alpha_{\mathrm{STALL}}-\hat{\alpha}_{\mathrm{HT}}\right)\left(1.1-\mathrm{C}_{\mathrm{D}_{\mathrm{HT}}}\right)}{\left( \pm 90-\hat{\alpha}_{\mathrm{HT} \mathrm{H}_{ \pm}}\right)}
$$

If the effective angle of attack of the horizontal tail exceeds $\pm 90^{\circ}$ the tail will point trailing-edge first into the relative wind. Under this condition early stalling is precipitated because of the sharp "leading edge" and blunt "trailing edge". In order to represent this, it was assumed that the attainable $\mathrm{C}_{\text {MAX }}$ of the tail under these conditions is half that occurring in normal flight.

Thus if $\quad 90^{\circ}<\alpha_{\mathrm{e}_{\mathrm{HT}}} \leq\left(180-\frac{1}{2} \hat{\alpha}_{\mathrm{HT}_{-}}\right)$

$$
\text { or }\left(-180+\frac{1}{2} \hat{\alpha}_{\mathrm{HT}_{+}}\right) \leq \alpha_{\mathrm{e}_{\mathrm{HT}}}<-90^{\circ}
$$

then

$$
\begin{aligned}
C_{L_{H T}} & =.5 C_{L_{\alpha}} \hat{\alpha}_{H T} \frac{\left(\alpha_{e_{H T}}-90^{\circ}\right)}{\left(90^{\circ}-\frac{1}{2} \hat{\alpha}_{H T_{-}}\right)} \\
\text {or } \quad C_{L_{H T}} & =.5 C_{L_{\alpha}} \hat{\alpha}_{H T_{+}} \frac{\left(\alpha_{e_{H T}}+90^{\circ}\right)}{\left(-90+\frac{1}{2} \hat{\alpha}_{H T_{+}}\right)}
\end{aligned}
$$

The corresponding drag coefficients are:

$$
\begin{aligned}
& \text { for } 90^{\circ}<\alpha_{\mathrm{e}_{\mathrm{HT}}} \leq\left(180-\frac{1}{2} \hat{\alpha}_{\mathrm{HT}_{-}}\right) \text {; } \\
& C_{L_{H T}^{S T A L L}}=0.5 C_{L_{\alpha}} \hat{\alpha}_{H T_{-}} \\
& C_{D_{H T}}=\frac{2 C_{L_{H T A L L}}^{2}}{\pi A R_{H T}}+C_{D_{O_{H T}}} \\
& \text { 6.0-9 }
\end{aligned}
$$

which gives $\left.C_{D_{H T}}=c_{D_{H T S T A L L}}+\frac{\left(\alpha_{e_{H T}}+0.5 \hat{\alpha}_{H T-}-180^{\circ}\right)\left(1.1-c_{D_{H T S T A L L}}\right)}{\left(0.5 \hat{\alpha}_{H T-}-90^{\circ}\right.}\right)$ and for $\left(-180+\frac{1}{2} \hat{\alpha}_{\mathrm{HT}_{+}}\right) \leq \alpha_{\mathrm{e}_{\mathrm{HT}}}<-9,0^{\circ}$;

$$
\begin{aligned}
& \mathrm{CL}_{\mathrm{HT}_{\text {STALL }}}=0.5 \mathrm{C}_{\mathrm{L}_{\alpha}}{ }^{\hat{\alpha} \mathrm{HT}_{+}} \\
& C_{D_{H T}}=\frac{2 C_{\mathbb{L}_{H T A L L}}}{\pi A R_{H T A L L}}+C_{D_{O_{H T}}}
\end{aligned}
$$

which gives $\left.C_{D_{H T}}=C_{D_{H T}}-\frac{\left(\alpha_{e_{H T A L L}}+180^{\circ}-.5 \hat{\alpha}_{H T}\right.}{\left(.5 \hat{\alpha}_{H_{T}}-90^{\circ}\right)}\right)\left(1.1-C_{D_{H T}}\right)$

In the range $\left(180-.5 \hat{\alpha}_{H T_{-}}\right) \leq \alpha_{e_{H T}} \leq 180^{\circ}$ when the tail has unstalled

$$
\begin{aligned}
& C_{L_{H T}}=C_{L_{\alpha}}\left(\alpha_{e_{H T}}-180^{\circ}\right) \\
& C_{D_{H T}}=C_{D_{O_{H T}}}+\frac{2 C_{L_{H T}^{2}}^{2}}{\pi A R_{H T}}
\end{aligned}
$$

and similarly for the range $-180^{\circ} \leq \alpha_{e_{\mathrm{HT}}}<\left(-180+.5 \hat{\alpha}_{\mathrm{HT}}^{+},()\right.$

$$
\begin{aligned}
& c_{L_{H T}}=c_{L_{\alpha}}\left(\alpha_{e_{H T}}+180^{\circ}\right) \\
& c_{D_{H T}}=c_{D_{O_{H T}}}+\frac{2 C_{L_{H T}}}{\pi A R_{H T}}
\end{aligned}
$$

The above equations define the variation of tail lift and drag over the entire range of angle of attack. The tail pitching moment is not computed since it makes only a small contribution to the total aircraft pitching moment.

### 6.4 VERTICAL TAIL

The aerodynamic forces and moments acting on the vertical tail were estimated using the methods of Reference 1 . The angle of attack of the vertical tail is given by 6.0-10

$$
\alpha_{V T}=-\operatorname{Tan}^{-1}\left[\frac{v_{V T}}{\sqrt{\bar{u}^{2} V T T^{+W^{2}}}}\right]+\beta_{\mathrm{VT}}\left(\frac{d \sigma}{d \beta}\right)
$$

where $u_{V T}, v_{V T}$ and $w_{V T}$ are the components of velocity at the vertical tail aerodynamic center as given in Appendix C. The term $\beta_{F}\left(\frac{d \sigma}{d \beta}\right)$ is the sidewash correction for the presence of the fuselage.

As in the treatment of the horizontal tail, the effect of rudder deflection is obtained using a rudder effectiveness parameter ${ }^{\tau}{ }^{\mathrm{VT}}$. Thus the effective angle of attack of the vertical tail when the rudder is deflected is

$$
{ }^{\alpha} \mathrm{e}_{\mathrm{VT}}={ }^{\alpha}{ }_{\mathrm{VT}}+{ }^{\tau}{ }_{\mathrm{VT}}{ }^{\delta}{ }_{\mathrm{RUD}}
$$

The treatment of the vertical tail aerodynamics through the complete angle of attack range $-180^{\circ}$ to $+180^{\circ}$ then follows the same lines as that for the horizontal tail aerodynamics previously described.

The vertical tail forces and moments in body axes are then obtained from:

$$
\begin{aligned}
& x_{A E R O}^{V T T}=\bar{q} S_{V T}{ }^{\eta}{ }_{V T}\left[-C_{D_{V_{T}}} \cos \left(\beta_{V T}-\sigma\right) \cos \left(\alpha_{H T}{ }^{-i_{H T}}\right)\right. \\
& \left.-C_{Y_{V T}} \sin \left(\beta_{V T}-\sigma\right) \cos \left(\alpha_{H T}-i_{H T}\right)\right] \\
& Y_{\mathrm{AERO}}^{\mathrm{VT}}=\bar{q} \mathrm{~S}_{\mathrm{VT}} \eta_{\mathrm{VT}}\left[\mathrm{C}_{\mathrm{Y}_{\mathrm{VT}}} \cos \left(\beta_{\mathrm{VT}}-\sigma\right)-\mathrm{C}_{\mathrm{D}_{\mathrm{VT}}} \sin \left(\beta_{\mathrm{VT}}-\sigma\right)\right] \\
& \mathrm{z}_{\mathrm{AERO}}^{\mathrm{VT}}=\overline{\mathrm{q}} S_{\mathrm{VT}} n_{V T}\left[-\mathrm{C}_{\mathrm{D}_{\mathrm{VT}}} \cos \left(\beta_{\mathrm{VT}}-\sigma\right) \sin \left(\alpha_{\mathrm{HT}}-\mathrm{i}_{\mathrm{HT}}\right)-\mathrm{C}_{\mathrm{Y}_{\mathrm{VT}}} \sin \left(\beta_{\mathrm{VT}}-\sigma\right)\right. \\
& \left.\sin \left(\alpha_{H T}-i_{H T}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
& M_{A E R O}^{V T}=Z_{\text {AERO }}^{V T}\left(X_{C G}-X_{V T}\right)+X_{A E R O}^{V T}\left(z_{V T}-Z_{C G}\right) \\
& \underset{\mathrm{N}_{\mathrm{AERO}}}{\mathrm{VT}}=-\mathrm{Y}_{\mathrm{AERO}}^{\mathrm{VT}} \quad\left(\mathrm{X}_{\mathrm{CG}}-\mathrm{X}_{\mathrm{VT}}\right)
\end{aligned}
$$

### 6.5 WING AERODYNAMICS

The treatment of the wing aerodynamics is the most complex of all the components. Because wing flexibility must be represented, each wing panel required a separate treatment. The approach adopted for simulation purposes was first to obtain the aerodynamic forces and moments on the complete wing considered as rigid and uninfluenced by slipstream interference effects. With this data as a basis the effects of elastic deflection were introduced as an increment in the effective angle of attack of each wing panel and the rotor slipstream interference was then calculated. This approach is described in detail below.

### 6.5.1 BASIC WING AERODYNAMICS

The basic wing lift, drag and pitching moment coefficients for the wing in the presence of the fuselage rotors-off, were calculated using the methods of Reference 1 . This data is applicable to low speed flight. Corrections for Mach number effects are introduced through the Prandtl-Glauert factor $\sqrt{1-M^{2}}$. Beyond stall angle of attack, the lift, drag and pitching moment curves are extended linearly to $\pm 90^{\circ}$
angle of attack in order to provide a representation of wing behavior at low transition speeds when wing angles of attack approach $90^{\circ}$. The data was calculated for the complete range of flaperon settings.

The complete wing basic lift, drag and pitching moment data also applies to each individual wing panel provided the data is obtained at the appropriate panel angle of attack. This approximation is acceptable if the angles of attack of each wing panel are not substantially different. This condition is normally fulfilled.

In addition to the above data, the effects of spoiler deflection on panel lift, drag and pitching moment are required. These were estimated using the data of Reference 1 . As can be seen from the equations presented in Appendix $E$ the spoiler effectiveness is strongly dependent upon flaperon deflection, a result of the spoilers being slot-lip spoilers.

### 6.5.2 ROTOR SLIPSTREAM INTERFERENCE

Before the basic wing aerodynamic data can be utilized in the calculation of the wing forces, the effects of the rotor slipstream must be calculated. The calculation procedure presented here has been developed and used at Boeing for some years, and gives acceptable agreement with wind tunnel test data on a wide variety of both tilt rotor and tilt wing configurations.

The method uses momentum theory to obtain the direction and
speed of the rotor slipstream in the neighborhood of the wing. From this the effective angle of attack of that part of the wing that is immersed in the slipstream is calculated. The lift, drag and pitching moment on the wing are then calculated for this angle of attack as if the entire wing were immersed. The area of the wing immersed in the slipstream is now computed and, using the ratio of the immersed to total wing area, the forces acting on the immersed portion are approximated.

At the angle of attack of the wing outside the slipstream, the wing forces and moments are obtained from the basic wing data as if no slipstream effects were present. These forces are then scaled by the ratio of unimmersed to total wing areas to obtain approximately the forces acting on the unimmersed wing. The sum of the approximations to immersed and unimmersed wing forces is now formed. This sum is then multiplied by a correction factor to obtain the final forces.

This correction factor is obtained from a consideration of the mass flows associated with the rotor-wing combination. In the following outline of the method only one rotor is considered.

From the following sketch, which shows the forces acting on the rotor, the inclination of

the resultant force on the rotor to the freestream direction is given by

$$
\tau_{R}=\alpha_{R}+\operatorname{Tan}^{-1}\left(\frac{N F}{T}\right)
$$

The resultant force on the rotor is

$$
R=\sqrt{T^{2}+N F^{2}+S F^{2}}
$$

where $T, N F$ and $S F$ are the thrust, normal force and sideforce, respectively.

The mass flow through the disc is

$$
\mathrm{m}=\rho \mathrm{A} \mathrm{~V}^{\prime}
$$

where $A$ is the disc area and $V^{\prime}$ is obtained from the induced velocity triangle at the disc plane.

$$
V^{\prime}=\sqrt{\left(V_{0}+V_{i} \cos \tau\right)^{2}+\left(v_{i} \sin \tau\right)^{2}}
$$

The resultant force on the rotor is related to the mass flow by (Glauert's assumption)

$$
R=2 m v_{i}=2 \rho A V^{\prime} v_{i}
$$

From these equations the following quartic equation is obtained for the induced velocity at the disc.

$$
v_{*}^{4}+2 V_{*} v_{*}^{3} \cos \tau+v_{*}^{2} v_{*}^{2}=1
$$

where the nondimensional notations

$$
v_{*}=\frac{v_{i}}{\sqrt{\frac{R}{2 \rho A}}} \quad V_{m}=\frac{V_{0}}{\sqrt{\frac{R}{2 \rho A}}}
$$

have been introduced.

This equation is then solved for $V *$ and the direction of the slipstream just behind the rotor disc is calculated from

$$
\varepsilon_{p}=\operatorname{Tan}^{-1}\left[\frac{V_{*} \sin \tau}{V_{*} \cos \tau+V_{*}}\right]
$$

The rotor thrust coefficient $\mathrm{CT}_{\mathbf{S}}$ is defined as

$$
C_{T_{S}}=\frac{T}{\left(q+\frac{T}{A}\right)} A
$$

NOTE: Because the rotor diameter to wing chord is large the slipstream is considered
with $T=R \cos \left(\tau-\alpha_{R}\right)$ to be uncontracted in the vicinity of the wing.
and $q=\frac{1}{2} \rho V^{2}=\frac{1}{4} V_{*}^{2} R$
then $C_{T_{S}}=\frac{\cos \left(\tau-\alpha_{R}\right)}{\cos \left(\tau-\alpha_{R}\right)+\frac{V_{*}^{2}}{4}}$
The aspect ratio of the slipstream-immersed wing area is given by

$$
A R_{i}=\frac{S_{i}}{c^{2}}
$$

where $S_{i}$ is the immersed area calculated by the method described in Appendix $D$, and $c$ is the wing chord.

The lift on the wing, if the slipstream were absent, is obtained by calculating the effective angle of attack of the wing outside the slipstream from

$$
\alpha_{0}=\sin ^{-1}\left[\frac{w W}{\sqrt{u_{w}^{2}+w_{W}^{2}}}\right]+\theta_{t}
$$

where $w_{W}, u_{w}$ are the velocites at the wing aerodynamic center and $\theta_{t}$ is the elastic twist at the point. The lift coefficient ( $C_{L}^{*}$ ) for this angle of attack is obtained from the aerodynamic data for the appropriate flaperon/spoiler deflection.

Similarly the lift $\left(C_{L}^{\prime \prime}\right)$ and drag ( $C_{D}^{\prime \prime}$ ) coefficients of the wing in the slipstream (assuming wing is completely immersed) are obtained from the aerodynamic data at the angle of attack

$$
\alpha_{s}=\alpha_{0}-\varepsilon
$$

The total lift coefficient of the wing with slipstream is therefore

$$
C_{L_{S}}=K_{A}^{\prime}\left[\frac{S_{i}}{S}\left(C_{L}^{\prime \prime} \cos \varepsilon-C_{D}^{\prime \prime} \sin \varepsilon\right)+C_{L}^{*}\left(1-C_{T_{s}}\right)\left(1-\frac{S_{i}}{s}\right)\right]
$$

where

$$
C_{L_{s}}=\frac{L}{q_{s} S_{w}}
$$

in which $q_{s}$ is the nominal slipstream dynamic pressure, defined by $q_{S}=q+\frac{T}{A}$

The factor $K_{A}^{\prime}$ is a correction factor to account for the fact that the lift-sharing between the immersed and unimmersed portions
of the wing is not simply proportional to the respective areas.

From considerations of the mass flows associated with the wing-rotor combination the factor $K_{A}^{\prime}$ was obtained in the form

$$
K_{A}^{\prime}=\frac{V_{*}+\frac{C_{L_{\alpha i}}}{C_{L_{\alpha}}} V_{*}}{V_{*}+V_{*}}
$$

where, from wing theory,

$$
\frac{C_{L_{\alpha i}}}{C_{L_{\alpha}}}=\frac{1}{1+\frac{C_{L_{\alpha}}}{\pi}\left[\frac{1}{A R}-\frac{1}{A R}\right]}
$$

The drag and pitching moments for the wing with slipstream are obtained similarly and are given by:
$C_{D_{S}}=K_{A}^{\prime}\left\{\frac{S_{i}}{S}\left(C_{L}^{\prime \prime} \sin \varepsilon+C_{D}^{\prime \prime} \cos \varepsilon\right)+C_{D}^{*}\left(1-C_{T_{S}}\right)\left(1-\frac{S_{i}}{S}\right)\right\}$
$C_{\mathrm{NI}_{S}}=K_{A}^{\prime}\left\{\frac{S_{i}}{S} \quad C_{M}^{\prime \prime}+C_{M}\left(1-C_{T_{S}}\right)\left(1-\frac{S_{i}}{S}\right)\right\}$
The rolling moment and yawing moment coefficients for the

$$
\begin{aligned}
& \text { wing are given by } \\
& \text { wing are given by } \\
& C_{\mathscr{L} S}=\left(K_{20}+K_{21} \bar{C}_{L}\right)\left(1-\bar{C}_{T_{S}}\right) \beta_{F}+\overline{\mathrm{Y}}_{A C}\left(\frac{1-C_{T_{S}}}{2 b_{W}}\right)\left(C_{L_{L W}^{*}}^{*}-C_{L_{R W}}^{*}\right) \\
& +\Delta C_{\mathscr{L}_{S_{\text {POWER }}}} \\
& C_{n_{S}}=K_{22} \bar{c}_{L}{ }^{2}\left(1-C_{T_{S}}\right) \beta_{F}+\bar{Y}_{A C} \frac{\left(1-C_{T_{S}}\right)}{2 b_{W}}\left(C_{D_{R W}}^{*}-C_{D_{L W}}^{*}\right) \\
& +\Delta C^{\eta_{S_{\text {POWER }}}}
\end{aligned}
$$

where the increment in rolling moment due to power is ${ }^{\Delta C_{\mathscr{L}}}{ }_{\text {POWER }}=\frac{1}{4}\left\{\left[C_{L_{S_{L W}}}-\left(1-\bar{C}_{T_{S}}\right) C_{E_{L W}}\right]\left[1-\frac{1}{2}\left(\frac{S_{i}}{S}\right)_{L W}\right]\right.$

$$
-\left[c_{L_{S_{R W}}}-\left(1-\bar{C}_{T_{S}}\right) c_{ \pm_{R W}}\left[-\frac{1}{2}\left(\frac{S_{i}}{S}\right)_{R W}\right]\right\}
$$

and the increment in yawing moment is
$\Delta C_{n_{S_{\text {POWER }}}}=\frac{1}{4}\left\{\left[C_{D_{S_{R W}}}-\left(1-\bar{C}_{T_{S}}\right) C_{D_{R W}^{*}}^{*}\right]\left[1-\frac{1}{2}\left(\frac{S_{i}}{S}\right)_{R W}\right]\right.$

$$
-\left[C_{D_{S W}}-\left(1-\bar{C}_{T_{S}}\right) C_{D_{L W}^{*}}^{*}\right]\left[1-\frac{1}{2}\left(\frac{S_{i}}{S}\right)_{L W}\right]
$$

Figure 6.2 shows a correlation between the wing-in-slipstream method described above and experimental results for the Boeing inodel 160 tilt rotor aircraft. As may be seen the simple treatment gives acceptable predictions of wing forces and moments.


Figure 6.2. Correlation of Theory with Test for Predictions of Slipstream Forces and Moments
6.0-20

### 7.0 ROTOR AERODYNAMICS

The rotor aerodynamics as used in the mathematical model are described in this section. Also presented are the methods used to compute the rotor aerodynamics, a discussion on wing upwash as it effects the rotor, and a description of the technique used to account for rotor on rotor interference in skewed flight. In addition, correlation of the methods described in tnis section with test data for soft-in-plane hingeless rotors are presented. Calculation of the Model 222 rotor forces and moments was not practicable because of the complexity and size of the programs required to represent the lag-flap coupling effects of the rotor. In this mathematical model, the rotor forces and moments are input from a series of curve plot fit equations. These equations were generated by computing rotor data using the computer programs discussed in Section 7.2, and then a least squares curve fit program was used to obtain the curve fit equations. The rotor forces and moments used in the mathematical model include the six basic forces and moments (thrust, power, normal force, side force, pitching moment, yawing moment), hub pitcining and yawing moments due to aircraft pitch and yaw rate, and changes to the six basic forces and moments due to cyclic pitch application.

### 7.1 FORMAT AND RANGE OF DATA

kotor forces and moments are functions of thirteen variables. In order to reduce the size of the data bank, these variables were combined and non-dimensionalized. Each rotor force and
moment can be written as:

$$
\begin{aligned}
& F=f\left(V, V_{t}, \theta_{0 .}\right. 7 \\
&\text { where or } \left.T, \alpha, B, P, Q, R, A_{l}, B_{1}, P_{N}^{R}, Q_{N}^{R}, R_{N}^{R}\right) \\
& V=\text { Forward speed } \\
& V_{t}=\text { Rotor tip speed } \\
& \theta_{0.75}=\text { Collective pitch at the } .75 \text { radius } \\
& T=\text { Rotor thrust } \\
& \alpha=\text { Rotor angle of attack } \\
& \beta=\text { Rotor sideslip angle } \\
& P=\text { Body axis roll rate } \\
& Q=\text { Body axis pitch rate } \\
& R=\text { Body axis yaw rate } \\
& R_{1}=\text { Longitudinal cyclic pitch } \\
& B_{1}=\text { Lateral cyclic pitch } \\
& P_{N}^{R}=\text { Rotor wind axis roll rate } \\
& Q_{N}^{R}=\text { Rotor wind axis pitch rate } \\
& R_{N}^{R}=\text { Rotor wind axis yaw rate }
\end{aligned}
$$

Forward speed and tip speed were combined to form rotor advance ratio and collective pitch or thrust were retained. Rotor angles of attack and sideslip and body axis roll, pitch and yaw rates were combined into a resultant angle of attack. Longitudinal and lateral cyclic pitch angles are retained. By combining the thirteen variables in this manner, Equation 7-1 can be expressed as:

$$
\begin{equation*}
F=f\left(\mu, \theta .75 \text { or } C_{T}, \alpha_{R}\right)+\left[\Delta F=f\left(A_{1}, B_{1}\right)\right]+\left[\Delta F=f\left(P_{N}^{R}, Q_{N}^{R}, R_{N}^{R}\right)\right] \tag{7-2}
\end{equation*}
$$

where $\mu=$ rotor advance ratio
$\alpha_{R}=$ rotor resultant angle of attack

By using this functional relationship, basic rotor forces and moments can be written as functions of three variables plus increments due to cyclic pitch control application and wind axis pitch roll and yaw rates at the rotor hub. This is the format used in the mathematical model. In addition, the rotor forces and moments are non-dimensionalized by dividing forces by ( $\rho \pi R^{2} V_{t}{ }^{2}$ ), moments by ( $\left.\rho \pi R^{2} V_{t}{ }^{2} R\right)$, and power by ( $\rho \pi R^{2} V_{t}{ }^{3}$ ).

$$
\alpha_{R}=\operatorname{TAN}^{-\prime} \frac{\sqrt{v_{R}^{2}+\omega_{R}^{2}}}{u_{R}}
$$

NOTE: $U_{R}, w_{R}, \omega_{R}$ ARE ROTOR HUB Velocities in shaft axes


The above sketch shows a rotor under condition of combined angle of attack ( $\alpha_{\mathrm{T} . \mathrm{L} .}$ ) and sideslip ( $\beta$ ). The resultant angle of attack $\left(\alpha_{R}\right)$ is the angle between the " $u_{R}$ " component of velocity at the rotor hub and the total velocity $\left(V_{R}\right)$ at the hub. The velocity components that define this resultant angle are the rotor hub velocities resolved to shaft axes and
derived in Appendix C. They include body axes pitch, roll and yaw rates. Other functional relationships that define the rotor resultant angle of attack are shown in Appendix $D$. Also shown on the sketch is the rotor sideslip angle ( $\zeta$ ). This angle represents the inclination of the plane containing the resultant velocity. Rotor wind axis forces and moments are defined relative to this plane. Since the resultant angle is defined from 0 through $180^{\circ}$ the inclination of the rotor sideslip angle ( $\zeta$ ) determines the signs of the rotor forces and moments when they are resolved back to body axes.

After the functional format for the rotor data was established, the ranges of the variables were established. Discrete speeds and rotor rpm conditions were selected. A range of rotor resolved angles of attack and thrust levels were selected at each combination. These conditions were carefully selected to cover the total operating envelope of the Model 222. The ranges of the rotor data are shown in Table 7.1.

### 7.2 PROGRAMS USED TO COMPUTE ROTOR DATA

Rotor data used in the mathematical model were predicted from Boeing-developed computer programs. Hover and cruise performance (thrust-power) were obtained from a propeller performance analysis computer program (B-92). This analysis establishes a radial distribution of induced velocity based on a prescribed wake contraction schedule to calculate rotor induced and total power coefficients at specified thrust or

TABLE 7.1 RANGE OF ROTOR DATA

| Total Velocity <br> (V) ~ KTS | Rotor Speed (rpm) | Resultant Angle of Attack Range $\left(\alpha_{R}\right) \sim \operatorname{deg}$ | Rotor Thrust $(T) \sim L b$ |
| :---: | :---: | :---: | :---: |
| 0 | 551 | $0 \rightarrow 180^{\circ}$ | $500 \rightarrow 7000$ |
| 45 | 551 | $0 \rightarrow 180^{\circ}$ | $500 \rightarrow 7000$ |
| 60 | 551 | $0 \rightarrow 180^{\circ}$ | $2000 \rightarrow 6500$ |
| 90 | 551 | $0 \rightarrow 180^{\circ}$ | $2000 \rightarrow 6500$ |
| 120 | 400 | $0 \rightarrow 45^{\circ}$ | $500 \rightarrow 2600$ |
| 142 | 386 | $0 \rightarrow 20^{\circ}$ | $-700 \rightarrow 3500$ |
| 160 | 386 | $0 \rightarrow 20^{\circ}$ | $-500+4750$ |
| 200 | 386 | $0 \rightarrow 20^{\circ}$ | $-500 \rightarrow 6000$ |
| 240 | 386 | $0 \rightarrow 20^{\circ}$ | $0 \rightarrow 3700$ |
| 280 | 386 | $0 \rightarrow 20^{\circ}$ | $0 \rightarrow 3800$ |
| 320 | 386 | $0 \rightarrow 20^{\circ}$ | $0 \rightarrow 4800$ |
| 360 | 386 | $0 \rightarrow 20^{\circ}$ | $0 \rightarrow 3500$ |

thrust coefficients. The radial airload distribution is also defined. A detailed description of this program is given in Reference 4.

Transition performance data, in-plane forces and moments and cyclic pitch effectiveness throughout the flight envelope were estimated using computer program D88 (Reference 5).

The D-88 computer program is an aeroelastic analysis for the study of aerodynamic, dynamic, and structural characteristics of current and advanced rotor and prop/rotor concepts. Airloads are calculated considering the effects of section geometry, compressibility and non-uniform inflow. An iterative process between the airloads and coupled flap-pitch dynamic response establishes blade accelerations which in turn are used to compute hub loads and rotor aerodynamic performance.

The rotor analysis is based on the idealization of a continuous, elastic, non-uniform beam into one composed of lumped discrete masses connected by weightless elastic sections. Associated with each mass is a flat rigid airfoil segment, with the mass center located at the midpoint. The aerodynamic loads generated by each segment are assumed to act.at the mass center.

The effects of non-uniform inflow are included by considering a discontinuous constant circulation along part of the rotor blade, of sufficient strength to maintain the desired thrust. A vortex is assumed to trail from the inboard and outboard
circulation discontinuities, of equal and opposite strength. By summing the effects of all the vortices on a given blade around the azimuth the non-uniform induced flow for each blade at every dynamic bay is determined.: Total velocity at each point in the blade is computed by vector addition of the velocity components.

The local angle of attack of each blade element is then computed at every blade station for specified azimuth angles and the aerodynamic coefficients $\left(C_{L}, C_{D}, C_{M}\right)$ are looked up from tables of coefficients as a function of Mach number. From these coefficients the airloads are computed. The vertical, tangential and pitching aerodynamic loads are then harmonically analyzed into 10 harmonics and act as the forcing functions for each blade section.

To obtain a thrust match, an iteration process is performed on the airloads until a steady collective pitch angle is obtained which corresponds to the desired thrust. To perform the dynamic analysis, the lumped mass and elastic bay elements of the idealized rotor blade are transformed into a sequence of transfer matrix products, by means of the Associated Matrix Method. This method replaces each blade element by an equivalent "transfer matrix" that transfers the dynamic system variables, shear, moment, deflection and slope, inboard across the element. Therefore, multiplying the system variables outboard of the element by the transfer matrix gives the variables
inboard of the element. The whole mass, elastic blade idealization is then reduced to a sequence of transfer matrix products.

In-plane elastic rotor derivatives (both static and rate) in axial flow were calculaced using computer program C-4l (Reference 2).

Dynamic derivatives for a rotor system are defined taking account of the modal behavior of the blades in two general flap-lag modes. These derivatives are given as matrix arrays of the partial derivatives of rotor forces with respect to unit amounts of elementary linear and angular motions of the hub and unit displacements in the blade modes. These effects are separated into inertial, damping and gyroscopic, and stiffness effects. Thus an element $m_{i j}$ in the inertia derivative matrix is $\partial F_{i} / \partial g_{j}$, i.e., the force in the $i$ direction due to unit acceleration in the $j$ direction, all other quantities being held constant.

Similarly element $d_{i j}$ of the damping derivative matrix will represent $\partial F_{i} / \partial g_{j}$ which might for appropriate (ij) be the aggregate gyroscopic and aerodynamic pitching moment due to unit velocity of yaw.

Similarly the elements of the stiffness derivative matrix represent such quantities as the normal force due to unit amount of shaft angle of attack, and generalized forces in the blade freedoms due to unit displacements in each of the other freedoms.

The matrices are of order $15 \times 15$ maximum. The first 6 rows and columns refer to forces in the vertical, lateral and axial directions and moments in the yaw pitch and roll directions due to unit acceleration, rates and displacements in each of the directions. These are the only numbers present if the rotor blades are assumed rigid. Three additional rows and columns are added for each blade mode considered. A limit of two blade modes is currently applied. The final three rows are for cyclic and collective pitch.

These derivative matrices provide a ready means for evaluating the contribution of the rotor to the coefficients of the aircraft dynamic equations. This program also provides the inplane elastic rotor derivatives.

Elastic rotor rate derivatives in transition were estimated using computer program C-49 (Reference 3 ). This program evaluates hub force and moment derivatives for shaft angles varying from cruise to hover conditions. Dynamic derivatives suitable for transient analysis are computed. The dynamic derivatives are the partial differentials of hub forces and moments with respect to hub positions rates and accelerations and include inertial and gyroscopic effects as well as aerodynamic effects. For the static derivatives a constant shaft angle to the relative wind is assumed and the resulting blade motion computed. The effects of blade aerodynamic and inertia and gyroscopic forces are combined to give the hub derivatives
due to constant shaft angle and constant rate of change of shaft angle.

The output rotor forces and moments of these programs are in rotor wind axis.

### 7.3 ROTOR SIGN CONVENTION

The rotor sign conventions as used in this mathematical model are shown in Figure 7.1. Positive directions of all rotor forces, moments and cyclic pitch angles are noted.

### 7.4 CURVE FIT FORMAT

The rotor data generated for the Model 222 mathematical model was curve fit at each advance ratio. A curve fit which is third order in angle of attack and second order in thrust coefficient or collective pitch was found to yield the most accurate results. The curve fits have the following general form.

$$
C_{F}=\sum_{v=0}^{2} \sum_{u=0}^{3}\left[A(u+4 v)^{u} C^{v}\right]
$$

The double summation is expanded starting with the inner quantity i.e. set $v$ and expand $u$ from 0 to 3. Repeat until the summations are satisfied. The expansion of the generalized form is

$$
\begin{aligned}
C_{F} & =A_{0}+A_{1} \alpha+A_{2} \alpha^{2}+A_{3} \alpha^{3} \\
& +\left(A_{4}+A_{5} \alpha+A_{6} \alpha^{2}+A_{7} \alpha^{3}\right) C_{T} \\
& +\left(A_{8}+A_{9} \alpha+A_{10} \alpha^{2}+A_{11} \alpha^{3}\right) C_{T}{ }^{2}
\end{aligned}
$$


7.0-11

All of the rotor forces and moments are curve fit in this format. The coefficients of the equation were obtained from a least squares fit of the computed rotor data. The criteria used to determine the final coefficients was to have not more than a 5\% difference between the curve fit equations and the computed rotor data at the nominal aircraft trim condition. In general this criteria was met.

### 7.5 EFFECT OF WING UPWASH ON ROTOR PERFORMANCE

The rotor operates in the upwash field associated with the lifting wing. Thus, the rotor behaves as if it were operating at an increased angle of attack. The effective upwash angles were calculated using lifting line theory. In the mathematical model the upwash angles are input in the form of a table of upwash angles as a function of wing lift coefficient, and nacelle incidence angle.

### 7.6 ROTOR/ROTOR INTERFERENCE

In order to obtain the correct lateral stick gradient when flying sidewards or at large sideslip angles, a calculation for rotor-on-rotor interference is included in the mathematical model. In Reference ll, the wake skew angle is defined as in

7.0-12

Also presented in this reference are contour charts of the normal component of induced velocity near a rotor with a triangular disc loading for six different skew angles in the range from $0^{\circ}$ to $90^{\circ}$. For the Model 222 geometry, a curve of normal induced velocity/average induced velocity as a function of skew angle was obtained. For the case of the Model 222 flying sidewards, the downwind rotor is assumed to be operating at a lower angle of attack than the upwind rotor, and will therefore generate different forces and moments. The downwash angle is calculated from the normal component of induced velocity. The rotor/rotor interference is washed out as a function of nacelle angle and sideslip angle such that there is no interference in the high transition speed and cruise modes. The equations derived are shown in Appendix E, under the rotor/rotor interference section.

### 7.7 ISOLATED ROTOR AERODYNAMICS

The equations utilized to represent the isolated rotor aerodynamics are presented below. These equations are then resolved into body axis forces and moments to be used in the equation of motion.
7.7.1 Thrust (1)

$$
C_{T_{R}}=\left[C_{T_{O R R}} \cos A_{l C_{R}} \cos B_{1 C_{R}}\right]
$$

where $\quad C_{T_{O R R}}=\sum_{v=0}^{2} \sum_{u=0}^{3}\left[A_{T}(u+4 v) \quad{ }_{R R}^{u}{ }_{0.95}^{\theta^{v}}\right]_{0.9}$
(I) In the equations that follow, subscript RR denotes right rotor. The left rotor is identical provided due regard is paid to sign convention and azimuth reference.

$$
\begin{aligned}
& A_{(u+4 v)}=\text { function of } \mu\left(\mu=\frac{V}{V_{t}}\right) \text { and is obtained } \\
& \text { from Appendix } F \\
& { }^{A_{1}} C_{R}=\text { Lateral cyclic pitch } \\
& B_{1 C_{R}}=\text { Longitudinal cyclic pitch } \\
& { }^{\theta} 0.75^{=} \text {Blade pitch angle at } 75 \% \text { blade radius } \\
& \alpha_{R R}=\operatorname{Tan}^{-1}\left\{\frac{\sqrt{v_{R R}^{2}+\left(w_{R R}+u_{R R} \varepsilon_{w R R}\right)^{2}}}{u_{R R}}\right\}+\varepsilon_{i_{L R}} \\
& u_{R R}, V_{R R}, W_{R R}=\text { rotor shaft axis velocity components } \\
& \varepsilon_{w_{R R}}=\text { Wing upwash angle } \\
& \varepsilon_{i_{L R}}=\text { Rotor/rotor interference angle }
\end{aligned}
$$

The effect of close proximity to the ground is accounted for by use of the following relationships

$$
C_{T_{R R}}=c_{T_{R R}}^{\prime}\left(\frac{T_{I G E}}{T_{O G E}}\right)_{R R}
$$

where $\left(\frac{T_{\text {AGE }}}{\mathrm{T}_{\text {GE }}}\right)$ is defined in Section 10 under the discussion of ground effect.
7.7.2 Power

$$
C_{P_{R R}}=C_{P_{O_{R R}}}=\sum_{v=0}^{2} \sum_{u=0}^{3}\left[\begin{array}{ll}
A_{p}(u+4 v) \alpha_{R R} u & C_{T T_{R R}}^{\prime V}
\end{array}\right]
$$

where: $A_{p}(u+4 v)$ may be obtained from Appendix $F$ as a function of $\mu_{R R}$
7.7.3 Normal Force

where: $\quad C_{N F} O_{O_{R R}}=\sum_{v=0}^{2} \sum_{u=0}^{3}\left[A_{N F}(u+4 v){ }_{\alpha}^{u} C_{R R} \dot{T}_{R R}.\right]$
$\bar{A}_{N F}(u+4 v)=$ Function of $\mu_{R R}$ and may be obtained from Appendix $F$.

$$
\begin{aligned}
& \frac{d C_{N F_{R R}}}{d A_{l} C_{R}}=D_{N F_{1}} C_{T_{R R}}+D_{N F_{2}} \mu_{R R}^{2}+D_{N F_{3}} \mu_{R R}+D_{N F_{4}} \\
& \frac{d C_{N F_{R R}}}{d B_{1 C_{R}}}=E_{N F_{1}} C_{T_{R R}}+E_{N F_{2}} \mu_{R R}^{2}+E_{N F_{3}}{ }^{\mu_{R R}}+E_{N F_{4}}
\end{aligned}
$$

The coefficients in the above 2 equations may be obtained from Appendix $F$.
7.7.4 Side Force
$c_{S F_{R R}}=C_{S F_{O R R}}+\frac{d C_{S F_{R R}}}{d A_{l C_{R}}} A_{1 C_{R}}+\frac{d C_{S F_{R R}}}{d B_{l C_{R}}} B_{1 C_{R}}$
where: $\quad C_{S F_{o_{R R}}}=\sum_{v=0}^{2} \sum_{u=0}^{3}\left[A_{S F}(u+4 v) \alpha_{R R}^{u} C_{T_{R R}^{\prime}}^{v}\right]$
$A_{S F}(u+4 v)=$ function of $\mu_{R R}$ and may be obtained from Appendix $F$.

$$
\begin{aligned}
& \frac{d C_{S F_{R R}}}{d A_{1} C_{R}}=D_{S F_{1}} C_{T_{R R}}+D_{S F_{2}}{ }^{\mu}{ }^{2}{ }_{R R}+D_{S F_{3}}{ }^{\mu}{ }_{R R}+D_{S_{5}} \\
& \frac{d C_{S F_{R R}}}{d^{B}{ }_{l C_{R}}}=E_{S F_{1}} C_{T_{R R}}+E_{S F_{2}} \mu_{R R}^{2}+E_{S F_{3}}{ }^{\mu_{R R}}+E_{S F_{4}}
\end{aligned}
$$

The coefficients in the above 2 equations may be obtained from Appendix F .
7.7.5 Hub Pitching Moment

$$
C_{P M_{R R}}=C_{P M_{O_{R R}}}+\frac{d C_{P M_{R R}}}{d A_{1 C_{R}}} A_{1 C_{R}}+\frac{d C_{P M_{R R}}}{d B_{1 C_{R}}}{ }_{1 C_{R}}+\frac{d C_{P M_{R R}}}{d Q} Q_{N R}^{R}
$$

where: $\quad C_{P M_{O_{R R}}}=\sum_{v=0}^{2} \sum_{u=0}^{3}\left[A_{P M}(u+4 v) \alpha_{R R}^{u} C_{T_{R R}}^{v}\right]$
$A_{P M}(u+4 v)=$ function of $\mu_{R R}$ and may be obtained from Appendix $F$.

$$
\frac{d C_{P i{ }_{R R}}}{d Q}=\sum_{v=0}^{2} \sum_{u=0}^{3}\left[H_{P M}(u+4 v) \alpha_{R R}^{u} C_{T_{R R}}^{v}\right]
$$

$$
H_{P M}(u+4 v)=\text { function of } \mu_{R R} \text { and may be obtained from Appendix } F
$$

$$
\begin{aligned}
& Q_{N R}^{R}=Q_{N R}^{N} \cos \zeta_{H R}+R_{N R}^{N} \sin \zeta_{H R} \\
& Q_{N R}^{N}=Q+i_{N_{R}}
\end{aligned}
$$

$$
R_{N R}^{N}=-R \cos i_{N_{R}}-P \sin i_{N_{R}}
$$

$$
\zeta_{\mathrm{HR}}=\text { right rotor sideslip angle }
$$

$$
i_{N R}=\text { right nacelle velocity }
$$

$$
i_{N R}=\text { right nacelle angle }
$$

$$
\frac{\mathrm{dC}_{\mathrm{PM}_{\mathrm{RR}}}}{\mathrm{~dB}_{1 C_{R}}}=\mathrm{E}_{\mathrm{PM}_{1}} C_{\mathrm{T}_{\mathrm{RR}}}+\mathrm{E}_{\mathrm{PM}_{2}}{ }^{2} \mathrm{RR}+\mathrm{E}_{\left.\mathrm{PM}_{3}{ }^{\mu} \mathrm{RR}+\mathrm{E}_{\mathrm{PM}_{4}}\left(\mu_{\mathrm{RR}} \leq \cdot 35\right), ~\right)}
$$

$$
=\mathrm{E}_{\mathrm{PM}_{1}} C_{\mathrm{T}_{\mathrm{RR}}}+\mathrm{E}_{\mathrm{PM}_{5} \mu_{\mathrm{RR}}}+\mathrm{E}_{\mathrm{PM}_{6} \mu_{\mathrm{RR}}}+\mathrm{E}_{\mathrm{PM}_{7}}\left(\mu_{\mathrm{RR}}>.35\right)
$$

Values for the coefficients in the above 2 sets of equations may be found in Appendix $F$.
7.7.6 Hub Yawing Moment
$C_{Y M_{R R}}=C_{Y M_{O_{R R}}}+\frac{d C_{Y M_{R R}}}{d A_{l C_{R}}} A_{1 C_{R}}+\frac{d C_{Y M_{R R}}}{d B_{l C_{R}}} B_{l C_{R}}+\frac{d C_{Y M_{R R}}}{d R} R_{N R}^{R}$
where: $\quad C_{Y_{M_{O_{R R}}}}=\sum_{v=0}^{2} \sum_{u=0}^{3}\left[A_{Y M}^{(u+4 v)} \quad \alpha_{R R}^{u} C_{T_{R R}}^{\prime v}\right]$
$A_{Y M}(u+4 v)$ is a function of $\mu_{R R}$ and may be obtained from Appendix
F.
7.0-16

$$
\frac{d C_{Y_{M M_{R R}}}}{d K}=\sum_{v=0}^{2} \sum_{u=0}^{3}\left[J_{Y_{N I}}(u+4 v)^{\alpha}{ }_{R R}^{u} C_{T R}^{\prime} v\right]
$$

$J_{Y M}(u+4 v)$ is a function of $\mu_{R R}$ and may be obtained from Appendix F.

$$
\begin{aligned}
\mathrm{R}_{\mathrm{NK}}^{\mathrm{K}} & =\mathrm{R}_{\mathrm{NR}}^{\mathrm{N}} \cos \zeta_{\mathrm{HR}}-Q_{\mathrm{NR}}^{N} \sin \zeta_{\mathrm{HR}} \\
\mathrm{~K}_{\mathrm{iNR}}^{N} & =-\mathrm{R} \cos i_{\mathrm{NR}}-P \sin i_{\mathrm{NR}} \\
\mathbb{Q}_{\mathrm{NR}}^{N} & =Q+\dot{i}_{\mathrm{NR}} \\
\zeta_{\mathrm{HR}} & =\text { Right rotor sideslip angle } \\
i_{\mathrm{NR}} & =\text { Right nacelle velocity } \\
i_{N R} & =\text { Right nacelle angle }
\end{aligned}
$$

$$
\begin{aligned}
& \frac{d C_{Y M_{R R}}}{d A_{1} C_{R}}=D_{Y M 1} C_{T_{R R}}+D_{Y M 2} \mu^{2} R R \\
&=D_{Y M 1} C_{T M 3}{ }^{\mu}+D_{Y R}+D_{Y M 4}\left(\mu_{R R} \leq .35\right) \\
&{ }^{2} R
\end{aligned}
$$

$$
\frac{\mathrm{dC}_{\mathrm{YM}_{\mathrm{RR}}}}{\mathrm{~dB} \mathrm{IC}_{\mathrm{R}}}=\mathrm{E}_{\mathrm{YM1}} C_{T}+\mathrm{E}_{\mathrm{YM} 2}{ }^{\mu_{R R}}+\mathrm{E}_{\mathrm{YM} 3} \mu_{\mathrm{RR}}+\mathrm{E}_{\mathrm{YM} 4}\left(\mu_{\mathrm{RR}} \leq .35\right)
$$

$$
=E_{Y M 1} C_{T_{R R}}+E_{Y M 5} \mu_{R R}^{2}+E_{Y M 6}{ }^{\mu} R R=E_{E M 7}\left(\mu_{R R}>.35\right)
$$

Values for the coefficients in the above 2 sets of equations may be found in Appendix $F$.

Notes: (1) Application of rotor equations for left rotor follow similar format with subscript "RR" changed to "LR".
(2) When solving equations with double summations for values of $\mu$ not given in tables, solve equations for the two values of $\mu$ closest to the value desired and then interpolate linearly for exact value of $\mu$.

This section presents the results of correlation studies that were conducted to verify the adequacy of the rotor prediction methods used for the Model 222 tilt rotor. In general, prediction of trends is excellent with quite good agreement in absolute magnitudes.
7.8.1 Model 213 Four Blade Hingeless Rotor Correlation Figure 7.2 presents correlation with rotor derivatives measured on a $1 / 9$ scale dynamically similar model of a tilt/ stowed rotor conversion model. In this test the rotor hub forces and moments were carefully measured over a range of RPM in which the lead-lag modal frequency progressed from less than 1 per rev at 900 RPM to values significantly greater than 1 per rev as the rotor was feathered. The measured values confirm the predicted behavior trend and the quantitative correlation is also excellent.

### 7.8.2 Correlation with Model 222 26-Foot Diameter Rotor

 Test in NASA-Ames $40 \times 80$-Foot TunnelFigure 7.3 shows the schematic of the windmilling test stand and its instrumentation. Test data were obtained from strain gages mounted on the outer portion of the wing as shown, and calibrated to measure normal force, pitching moment and yawing moment. Comparison with test data was made by calculating the moments about the wing strain gage locations using forces and moments predicted by the $C-41$ program. The results of this


Figure 7.2. Model 213 1/9 Scale Conversion Model - 85 Ft/Sec Derivative Variation with RPM


Figure 7.3. 26 Ft. Rotor Test Stand in NASA's $40^{\prime} \times 80^{\prime}$ Tunnel
comparison for alpha derivatives are given in Figure 7.4 and for cyclic pitch derivatives in Figures 7.5 and 7.6 .

The analysis did not attempt to account for force and moment contributions from nacelle and wing aerodynamic interference. Nevertheless, quite good correlation is observed. These plots also show the values of derivatives predicted by several other programs. These include D-88 program which accounts for compressible non-linear downwash and L-22 which uses linear airfoil theory and uncoupled flap-lag freedoms. C-49 accounts for unsteady aerodynamics while C-41 uses a linear representation. C-4l and C-49 use a modal representation of blade freedoms (2 coupled flap-lag modes) while D-88 and L-22 make use of a finite element discrete mass representation.

The rotor derivative data was also compared with C-41 using a total unresolved moment approach. Total moments about the center of the wing tip gages and the reference azimuth position (orientation of the moment vector in the rotor disc plane) were calculated from the $C-41$ hub forces and moments and compared with test results (Figure 7.7). The interesting conclusion which is not apparent from the resolved forces and moments is that the total moment is predicted well but there are slight differences in the reference azimuth position.

### 7.8.3 Correlation witn Model 222 1/4.622 Scale Model vata

The subject model is a dynamically similar version of the M222. The test data presented in Figures 7.8 and 7.9 were taken


Figure 7.4. Correlation of 26 Ft Rotor Test Data with Various Rotor Derivative Programs



Figure 7.5. Correlation of 26 Ft Rotor Test Data with Various Rotor Derivative Programs Cyclic Moment Derivatives


Figure 7.6. Correlation of 26 Ft Rotor Test Data with Various Rotor Derivative Programs Cyclic Force Derivatives


Figure 7.7. Rotor Moment and Azimuth Angle Due to Angle of Attack - Correlation with 26 Ft Rotor Data
with the model mounted on a pedestal in the tunnel. The rotors were given angles of attack to the free stream by pitching the complete model with zero sideslip angle and yawing the model at zero angle of attack. The yawing data contains minimal wing induced flow effects and comparison with the pitch data indicates the importance of induced flow on the rotor forces and moments. Forces and moments were computed for the isolated rotor and it is seen from Figure 7.8 that correlation with test data is excellent when wing induced effects are small; in Figure 7.9 wing effects introduce perceptible shifts which increase with dynamic pressure.


Figure 7.8. Comparison of Calculated and Test Rotor Hub Force and Moment Derivatives for M222




7.) -28
Figure 7.9. Comparison of Calculated and Test Rotor Hub Force and Moment Derivatives for M222 1/4.622 Scale Model (Pitch Sweep) $\Omega=386$ RPM

### 8.0 CONTROL SYSTEM DESCRIPTION

This section describes the control system, stability augmentation systems, load alleviation system and thrust management system utilized in the mathematical model. A more complete description is given in Reference 8. .

### 8.1 CONTROL AERODYNAMIC CONFIGURATION

Control of the Model 222 aircraft is accomplished by utilization of longitudinal cyclic, differential longitudinal cyclic, collective and differential collective pitch, and differential nacelle tilt control in conjunction with the airplane control surfaces. The airplane control surfaces consist of conventional elevator and rudder and a flaperon and spoiler arrangement. The primary controls in each axis for each regime of flight are shown in Table 8.1.

The rotor controls provide a major portion of the control capability from hover through the low transition speed range, but airplane surface controls are operative in all regimes of flight, including hover. The rotor controls are phased out during transition as nacelle incidence decreases, speed increases, and the surface controls become more effective.

### 8.2 LONGITUDINAL CONTROL

Longitudinal control in hover is provided by longitudinal cyclic pitch. This is phased out through transition as the elevator becomes more effective. The elevator provides longitudinal control in the cruise mode.

TABLE 8.1 FLIGHT CONTROL MIXING

| FLIGHT MODE | PRIMARY CONTROLS |
| :---: | :---: |
| Helicopter (Hover) <br> Pitch <br> Roll <br> Yaw <br> Height Control | Longitudinal Cyclic <br> Differential Collective <br> Differential Longitudinal <br> Cyclic and Differential <br> Nacelle Tilt <br> Collective/Engine Power |
| Transition <br> Pitch <br> Roll <br> Yaw | Longitudinal Cyclic and Elevator <br> Differential Collective, Differential Longitudincal Cyclic, Differential Nacelle Tilt, Aileron and Spoiler <br> Differential Longitudinal Cyclic, Differential Nacelle Tilt, and Rudder |
| Airplane <br> Pitch <br> Roll <br> Yaw | Elevator <br> Aileron and Spoiler <br> Rudder |

### 8.3 LATERAL CONTROL

Lateral control in hover is provided by differential collective pitch, together with differential engine fuel flow (power). The differential engine power is provided to ensure maintaining roll control in the event of a cross shaft failure. It also serves to minimize the cross shaft torque. In transition, differential collective and differential cyclic are scheduled as a function of nacelle tilt.

When differential cyclic pitch is commanded the nacelles are also actuated to tilt differentially, thereby increasing the thrust vectoring effect of the cyclic pitch. Differential deflection of the nacelles is $\pm 1.55$ degrees per degree of cyclic plus approximately $\pm 0.20$ degrees of differential nacelle tilt due to elasticity of wing and nacelles. This results in a large increase in control power as compared to the control power available from cyclic alone The control for requirements may, therefore, be met with modest amounts of cyclic control resulting in low blade stresses and long rotor fatigue life. Collective pitch is also scheduled with nacelle tilt so that when the nacelles are tilted differentially, pitch is increased on the rotor whose disc is tilted down, and decreased on the rotor which is tilted up. This maintains the thrust approximately equal on the two rotors, ensuring that thrust vectoring rather than differential thrust is achieved by the differential cyclic pitch and differential nacelle tilt.

The wing has full span flaps and spoilers mounted on the trailing edge. The flaps are single slotted of 30 percent chord with a fixed hinge point 14.6 percent below the wing chord line. The flaps act as flaperons for roll control and deflect downward only by a maximum of 20 degrees from the nominal flap setting. Maximum incremental lift from the flaps is attained at approximately 35 degrees deflection and the maximum rolling moment occurs at the same time, so the flaperon deflection for roll control is limited to a maximum total flap deflection of 35 degrees. If, for example, the flaps are symmetrically deflected 30 degrees, only 5 degrees additional deflection is utilized for roll control. Full span spoilers of 12.7 percent chord are located forward of the flaps and hinged to the rear spar. The spoilers are "slot-lipped", i.e., they open up the slot forward of the flap with the flaps extended resulting in a large increase in roll control as compared to the control power with flaps closed. Maximum deflection of the spoilers for roll control is 45 degrees from the closed position.

Maximum spoiler rolling moment coefficient is also attained with flaps deflected approximately 35 degrees. Spoiler effectiveness with the flaps retracted is approximately onethird that attainable with the flaps extended. Spoiler rolling moment is further reduced at high speed by limiting the spoiler actuator force capability, thereby restricting the spoiler extension at speeds above 175 knots.

The spoilers and flaps are also used in conjunction with download alleviation devices referred to as umbrellas mounted on the leading edge of the wing for download relief in the hoves and low-speed ranco on the upper and lover firg surfaces: Maximum deflections of the surfaces for downozt Ileviation are: flaps 70 degrees, spoilers 110 degrees form asod, and mbrellar afterge-of... the -upper surface up th ? 0 derreas from verticel and aft edge. of-lower-surface down to 10 degrees from verticial, The umbrellas and spoilers retract at 50 knots automatically.

### 8.4 DIRECTIONAL CONTROL

Directional control in hover is provided by differential longitudinal cyclic pitch, which, as discussed above under lateral. control, also actuates differential nacelle tilt to amplify the thrust vectoring effect of the cyclic pitch.

In transition, the differential cyclic and its associated nacelle tilt are phased out as the rudder becomes more effective. This results in near zero initial roll acceleration in response to a yaw input.

### 8.5 THRUST/COLLECTIVE CONTROL

In hover, forward motion of the thrust/collective lever mechanically commands both increased sollective pitch and increased power. The governor provides a fine adjustment to the collective pitch to maintain rpm. Over travel of the pilot's lever, beyond the normal max power position, provides a collective
pitch landing flare capability. The over travel is entered by going through a "gate", which shuts down the rotor governor and leaves the pilot's lever directly connected to collective pitch, just like a helicopter collective pitch lever.

The collective pitch is also scheduled through transition as a function of nacelle incidence, minimizing the adjustment needed from the governor and also providing the pitch variation with differential nacelle tilt required for roll and yaw control.

In cruise the mechanical interconnection of the thrust/collective lever with collective pitch is phased out completely so that a pure power demand system with governed pitch, like a conventional fixed wing airplane, is provided. The control system block diagrams are shown in Appendix E.

### 8.6 CONTROL FEEL

Control force gradient variation with dynamic pressure prevents excessive sensitivity of control at high speed. In the model 222, the force gradients of the primary controls (longitudinal and lateral stick, and pedals) are varied linearly with dynamic pressure. The rudder and elevator deflections vary linearly with pilot's rudder pedal and longitudinal stick travel. Aileron deflection is programmed linearly and spoiler deflection nonlinearly with lateral stick deflection, to provide near-linear rolling moment effectiveness to near cruise speed. As mentioned earlier, spoiler deflection is limited at high speed by limiting the actuator capacity. The control force breakout forces and gradients are shown in Appendix $F$.

### 8.7 STABILITY AUGMENTATION SYSTEMS

Stability augmentation systems are provided to enhance aircraft flying qualities. The system consists of longitudinal, lateral and direction SAS. The longitudinal stability augmentation system incorporates a pitch rate feedback and a longitudinal stick pickoff. In addition, a lagged pitch rate signal is incorporated to provide some degree of attitude stabilization without the autopilot. (An autopilot is not represented in this simulation.) These signals are shaped and put through an authority limit. The longitudinal SAS commands longitudinal cyclic ptich to provide the required damping in hover and transition. It is not required in the cruise mode and is phased out at 175 knots. The block diagram of the longitudinal SAS is given in Appendix E.

The lateral stability augmentation system is operative in all flight modes. It consists of roll rate feedback for increased damping in roll, lagged roll rate feedback to provide roll attitude stability, and a lateral stick pickoff. In addition a sideslip feedback is incorporated to decrease the strong dihedral effect. These feedback loops are shaped and phased to yield good aircraft dynamic characteristics. A lateral SAS authority limit is incorporated in the circuit. The output of the lateral stability augmentation system is input to the control system in terms of equivalent lateral stick, since the drive actuator is in series with, and commands the same control as, the pilots lateral stick control linkage. The
lateral SAS never opposes the pilots' command. The block diagram of this system is shown in Appendix E.

A directional stability augmentation system is provided and operates in all flight regimes. The yaw channel consists of yaw rate feedback for increased directional damping in hover and low speed flight modes, lagged yaw rate feedback to provide yaw attitude stability, and a rudder pedal pickoff for quickening. Directional damping provided by the rotors is quite high in the higher transition and cruise speed ranges. No additional yaw rate damping is therefore needed in cruise. A feedback is provided to modify the effective yawing moment due to roll rate which exists in the basic unaugmented aircraft configuration in the cruise speed range. The feedback gains, and the relative phasing of these gains have been optimized to provide good directional dynamic response. A directional SAS authority limit is incorporated. The SAS command is input to the control system in terms of equivalent inches of rudder pedal. The block diagram for the directional stability augmentation system is shown in Appendix $E$.

The stability augmentation systems used for the simulation are not set up to investigate individual component failures. Modifications are required in order to do malfunction type studies with this simulation.

### 8.8 LOAD ALLEVIATION SYSTEM (LAS)

Propeller type aircraft experience significant blade loads during exposure to skewed flow due to steady state or transient conditions (climb, sideslip, gusts, etc). The tilt rotor configuration can have similar problems. However, since cyclic pitch is a basic part of the tilt rotor control system it provides the means to significantly reduce the sensitivities to these effects. It also can be used to reduce the destabilizing moments which come from the rotors and thus improve static stability.

An automatic load alleviation system is provided and operates via the swashplate to reduce both transient and steady state hub forces and moments and the destabilizing moments at the nacelle pivot. It is not a required system for the Model 222, but will significantly enhance the static stability and the fatigue margins of the aircraft.

The overall objectives to be achieved through the use of cyclic feedback control are:

- Reduce rotor hub forces and moments for both steady state operation and gust encounter
- Improve flying qualities of the aircraft by using the cyclic control system to reduce pilot workload and improve short period response by reducing destabilizing forces and moments of the rotors
- Reduce aircraft structural loads resulting from gust turbulence
- Improve ride qualities by damping the response to gust turbulence

The load alleviation system, as mechanized in this simulation consists of angle of attack, angle of sideslip, and dynamic pressure sensors which drive through appropriate gains and filters to reduce the longitudinal and lateral moments at the nacelle pivot. The lateral cyclic pitch used for load alleviation is authority limited and drives separate actuators in each hub. The longitudinal cyclic pitch is summed in with the longitudinal SAS. The block diagram for this system as mechanized is shown in Appendix . This system is operative from low transition speed (approx. 50 knots) through dive speed and reduces the pivot moments from $50 \%$ in the 150 to 200 knot range to $100 \%$ in all other modes of flight.

### 8.9 THRUST MANAGEMENT SYSTEM

The thrust and power management system for a tilt rotor aircraft must be compatible with both the helicopter and airplane configurations. Thrust control for the hover task, rpm control, gust response (especially in the cruise flight regime), and effect on aircraft flying qualities must all be considered. Classically, helicopters have used collective pitch demand to control thrust and fuel governing to control rpm while fixedwing aircraft have used fuel flow demand to control thrust and
collective pitch governing to control rpm. Each system has its advantages. For a tilt rotor aircraft it is desirable from a practical viewpoint to have one type of governing for both the helicopter and fixed-wing flight regimes. Collective pitch governing was chosen for Model 222 for several reasons:

- It is more readily adapted to the hover flight regime than the fuel governor is to cruise
- It has better gust response characteristics
- It is fast acting and has high accuracy
- Thrust response to pilot control can be easily shaped with feed forward loops
- It has been demonstrated successfully in hover, transition and cruise in the CL-84 aircraft

With collective pitch governing there are two areas in the thrust management system to be considered: (1) style of the collective pitch governor; and (2) the feed forward loops for shaping pilot thrust control. The block diagram for this system as mechanized in this simulation is shown in Appendix E.

Several different governor configurations were considered for The M222 in order to determine the governor system best suited to meet the following objectives: (1) 0.3 percent steady state error in 2.5 to 3 seconds; (2) 2 percent rpm overshoot; and (3), satisfactory effect on aircraft flying qualities in the $8.0-11$
all-operational mode (i.e., all aircraft components operational and performing as designed) and various failure modes. A single governor reference which used the rpm signal from each rotor and averaged them was chosen as the configuration that best satisfied the design criteria. To achieve the required accuracy and transient response goals, integral as well as proportional feedback of rpm was necessary in both the hover and cruise regimes. Governor gain is scheduled with nacelle incidence to maintain a near optimum level of governing throughout the flight envelope. Gains are varied linearly as the rotor rpm is changed from 551 in hover to 386 in cruise. The second requirement of the governor system is shaping the rotor thrust output for a pilot throttle input. Considerations in determining the proper shaping include:
(1) throttle sensitivity;
(2) time constant to reach 63\% of steady-state thrust; and
(3) allowable thrust overshoot

Variable pilot's control sensitivity is employed to give the optimum sensitivity in the hover power range yet maintain full power control within a reasonable throttle throw (8 inches). Shaping of the pilot command with collective quickening is done to improve the thrust time constant and thrust response transient shaping so that the pilot may perform the precision hover task with a minimum of difficulty. In the cruise regime, shaping of the thrust output is unnecessary and is phased out during transition.

The thrust/collective pitch control system is designed in such a manner that, during hover, when the pilot moves his control, he commands both a change in engine fuel setting and, mechanically, a change in collective setting. The governor then operates with a time lag to trim the collective to the value required to maintain rpm. The mechanical collective change feature is washed out as a function of nacelle incidence so that when nacelle incidence is decreased to zero, the pilot commands only engine fuel. In addition, the reference setting schedule for collective has been established to maintain equal thrust output from both rotors during application of differential nacelle tilt.

As was mentioned previously, additional details on the Model 222 control system may be obtained from Reference 8.

### 9.0 ENGINE REPRESENTATION

This section describes the engine performance and dynamic model representation that is used in the mathematical model. The basic engine cycle performance data consists of tabulated values of four variables: power, fuel flow, gas generator shaft rpm, and power turbine shaft rpm. These parameters are a function of Mach number and turbine inlet temperature. All data are in referred, normalized format as shown in Table 9.1. Because of the normalized, referred format, all data are valid for any ambient conditions. The effects on engine performance of operating at non-optimum power turbine speed are included in this model. The referred format also facilitates including engine thermodynamic and mechanical limits. Limitations on engine cycle operation may be input on any combination of the following: fuel flow, torque, gas generator speed, gas generator referred rpm or output shaft speed. A detailed description of this routine is in Reference 9. The flow cinarts which describe this routine mathematically are shown in Appendix $E$.

A simplified dynamic model of the Lycoming T53-L-13 engine was formulated for use in the tilt rotor mathematical model. This model was coupled to the output of the engine performance program described above. The model consists basically of 2 first order lags in series with variable time constants and gains. The output of the model is rate limited to reflect actual engine performance. This simplified model gives satisfactory

$$
9.0-1
$$

results for both large and small power transients. The block diagram for this system is shown as part of the thrust management system block diagram shown in Appendix E.

TABLE 9.1 ENGINE CYCLE DATA FORMAT

| VARIABLE | SYMBOL | REFERRED, NORMALIZED FORM |
| :---: | :---: | :---: |
| Thrust | $\mathrm{F}_{\mathrm{N}}$ | $\mathrm{F}_{\mathrm{N}} / \delta \mathrm{F}_{\mathrm{N}}^{*}$ |
| Power | SHP | SHP / $\delta \sqrt{\theta}$ SHP* |
| Gas Generator rpm | $\mathrm{N}_{\mathrm{I}}$ | $\mathrm{N}_{\mathrm{I}} / \sqrt{\theta} \mathrm{N}_{\mathrm{I}}^{*}$ |
| Power Turbine rpm | $\mathrm{N}_{\text {II }}$ |  |
| Fuel Flow | $W_{\text {E }}$ |  |
| Turbine Inlet Temperature | T | $\begin{aligned} & \mathrm{W}_{\mathrm{f}} / \delta \sqrt{\theta} \mathrm{SHP} * \\ & \mathrm{~T} / \theta \end{aligned}$ |
| Where: | ```* = Max. Power Setting, Static, Sea Level, Standard Day 0 = Ambient Temperature ( }\mp@subsup{}{}{\circ}\mathrm{ R) Divided by }518.69\mp@subsup{}{}{\circ}\textrm{R``` |  |
|  |  |  |
|  | $\delta=A$ | (psia) Divided |

The effects of operating near the ground on the rotors and airframe are included in this model. The presence of the ground on the airframe imposes a boundary condition which inhibits the downward flow of air normally associated with the lifting action of the wing and tail. The reduced downwash has three main effects;

- A reduction in the downwash angle at the tail
- An increase in the wing lift curve slope
- An increase in the tail lift curve slope

These have been accounted for by the methods given in Reference 10 Appendix $B-7$. The data given in the reference for the change in wing and tail lift curve slope has been used directly. The equation specified for the change in downwash angle at the tail due to ground proximity was modified for convenience. The equation as stated is:

$$
\frac{(\Delta \varepsilon)}{\varepsilon} g=\frac{b_{1}^{2}+4(h-H)^{2}}{b_{1}^{2}+4(h+H)^{2}}
$$

where $(\Delta E)_{g}=$ the change in tail dọwnash angle due to ground proximity
$\varepsilon \quad=$ the downwash remote from ground
$h \quad=$ the height of the tail root quarter chord point above the ground
$H \quad=$ the height of the wing root quarter chord point above the ground
$b_{1} \quad=a$ function of wing lift and wing flap geometry

For this mathematical model, the $b_{1}$ in the above equation was taken to be equal to the wing span, $b_{w}$. This results in a small error in the change in horizontal tail downwash. It is, however, sufficiently accurate for this simulation.

Ground effects on the rotor are difficult to predict analytically, especially in forward flight. Wind tunnel test data for the Model 160 powered model, Reference 6 was plotted as a thrust ratio versus effective rotor height/diameter ratio, for two rotor advance ratios. This data, shown in Figure 10.1 was curve fit and linearly interpolated for advance ratio. The resulting equation is as follows: - (for the right rotor. The left rotor is identical except for subscripts)

$$
\begin{aligned}
& \left|\frac{T_{I G E}}{T_{O G E}}\right|_{R R}=\left[\left(\frac{h}{\bar{D}}\right)_{\underset{R R}{E F F}}^{2}\left(.1741-.6216 \mu_{R R}\right)+\left(\left.\frac{h}{\bar{D}}\right|_{\underset{R R}{E F F}} ^{(1.4779} \mu_{R R}-.4143\right)\right. \\
& \left.\begin{array}{l}
\mathrm{EFF} \\
\mathrm{RR} \\
+1.2479-.8806 \mu_{R R}
\end{array}\right] \\
& \text { where }\left|\frac{h}{D}\right|_{E F F}=\frac{h_{R R}}{2 R\left[\left|\sin \left(\theta+i_{N_{R}}\right) \cos \phi\right|+.0174\right]} \\
& \text { RR } \\
& h_{R R}=-z_{\text {DOWN }}+\left(L_{S} \cos i_{N_{R}}-x_{C G}\right) \sin \theta \\
& +\left[\left(L_{S} \sin i_{N_{R}}+z_{C G}\right) \cos \phi-Y_{N} \sin \phi\right] \cos \theta \\
& =\text { Rotor hub height above the ground } \\
& L_{S} \quad=\text { Distance from the nacelle pivot to the rotor hub } \\
& X_{C G}=\text { Longitudinal distance from the pivot to the CG } \\
& Z_{C G}=\text { Vertical distance from the pivot to the CG } \\
& \theta \quad=\text { Aircraft pitch attitude } \\
& \phi \quad=\text { Aircraft roll attitude } \\
& i_{N_{R}}=\text { Right rotor nacelle angle } \\
& \mathrm{Y}_{\mathrm{N}} \quad=\text { Wing semispan }
\end{aligned}
$$



Figure 10.1. Effect of Rotor Height on Thrust Augmentation Ratio
10.0-3

The equation for the effective rotor height to diameter ratio (h/D) EFF was derived by dividing the rotor hub height by $\left[\sin \left(\theta+i_{N}\right) \cos \phi\right]$. This yields the rotor height along the shaft. For the cruise condition the hub height is infinite, (h/D) EFF is infinite and the augmentation ratio due to ground effect is unity. Some special conditions which must be observed when using these equations are noted in Figure 10.1 .

### 11.0 AIRFRAME REPRESENTATION (PREPROCESSOR)

An airframe representation/preprocessor calculation is included in the mathematical model that enables the user to input the location of major structural elements of the aircraft in terms of water line, butt line and station line location. All lengths, center of gravity distances and inertias used in the equations are then calculated. This feature enables the user to quickly change the location of major structural elements to assess their impact on vehicle response.

In the derivation of the basic equations of motion, the aircraft was divided into three principal mass elements. The fuselage mass element $\left(m_{f}\right)$, the wing mass element $\left(m_{W}\right)$ and the tilting nacelle mass element $\left(m_{N}\right)$. The components of the three mass elements are shown below and are available from a standard mass properties buildup of the Model 222 .

- fuselage mass $\quad\left\{\begin{array}{l}\text { Fuselage and contents } \\ \text { Horizontal tail and contents } \\ \text { Vertical tail and contents } \\ \text { Crew and trapped liquids } \\ \text { Cargo }\end{array}\right.$
- wing mass $\quad$ element $\left(m_{W}\right) \quad\left\{\begin{array}{l}\text { Wing and contents } \\ \text { Fuel carried in wing } \\ \text { Fixed nacelles and/or engines }\end{array}\right.$
- tilting nacelle $\begin{gathered}\text { mass element }\end{gathered}$ Tilting nacelle (including rotors) $\left(m_{N}\right)$

These three mass elements along with their respective distances from the nacelle pivot to the center of each mass element are used to compute the aircraft center of gravity distances with

$$
11.0-1
$$

respect to the nacelle pivot. The equations for these center of gravity distances, derived in Appendix $C$, and including the effects of nacelle tilt are:

$$
\begin{aligned}
& x_{C G}=\frac{m_{f} \ell_{f}+m_{W} \ell W}{m}+\ell\left(\frac{m_{N}}{m}\right)\left[\cos \left(i_{N L}-\lambda\right)+\cos \left(i_{N R}-\lambda\right)\right] \\
& z_{C G}=\frac{m_{f} h_{f}+m_{w} h_{W}}{m}-\ell\left(\frac{m_{N}}{m}\right)\left[\sin \left(i_{N L}-\lambda\right)+\sin \left(i_{N R}-\lambda\right)\right]
\end{aligned}
$$

The masses and distances used in these equations are defined on the sketch below.

11.0-2

The quantities required to compute mf, $\ell_{f}, m_{w}, \ell_{w}, m, \ell, m_{N}$, $\lambda, h_{f}, h_{W}$ are available from an aircraft three-view drawing and a standard mass properties buildup. The quantities $\ell$ and $\lambda$ (defined in the sketch) are easily obtainable from a drawing. The mass quantities $\left(m, m_{N}, m_{f}, m_{W}\right)$ are computed from a mass properties buildup by adding up the components of each mass element as described in the previous paragraph. The lengths $\ell_{f}, \ell_{W}, h_{f}$ and $h_{W}$ are computed by summing the weight moments of tne components of each mass element about the nacelle pivot. The equations for these operations have been derived and are presented in Appendix $E$ under the preprocessor equations. The input data to these equations include the weight of each component, and its location in terms of water line, fuselage station line, and butt line.

When the center of gravity distance of each mass element has been determined, the component and total aircraft mass moments of inertia can be computed. The equations for the total aircraft mass moments of inertia are presented in Appendix $C$. The moments of inertia of each mass element are computed by application of the parallel axis theorem. The moments of inertia of each component about its own center of gravity must be known. The parallel axis theorem states:

$$
I_{x x}=\sum_{i=1}^{N}\left[I_{x x_{O_{i}}}+m_{i}\left(y_{i}^{2}+z_{i}^{2}\right)\right]
$$

$$
\begin{aligned}
& I_{y y}=\sum_{i=1}^{N}\left[I_{y y_{O_{i}}}+m_{i}\left(z_{i}^{2}+x_{i}^{2}\right)\right] \\
& I_{z z}=\sum_{i=1}^{N}\left[I_{z z_{O_{i}}}+m_{i}\left(x_{i}^{2}+y_{i}^{2}\right)\right] \\
& I_{x z}=\sum_{i=1}^{N}\left[I_{x z_{O_{i}}}+m_{i}\left(x_{i} z_{i}\right)\right]
\end{aligned}
$$

where $N$ represents the number of component masses.

These equations have been expanded to compute the moments of inertia of each mass element and are shown in Appendix $E$ under the preprocessor section. The only additional input data required are the inertias of each component about their own centers of gravity. These are readily available from the mass properties buildup of the Model 222.

Other lengths required for the mathematical model are computed in this section. The input data for these computations are in terms of the water line, butt line and fuselage station line locations of the elements in question.

Two aero-elastic degrees of freedom are included in the tilt rotor mathematical model. These are first mode wing vertical bending and first mode wing torsion. The stability and control characteristics of flexible airplanes may be significantly influenced by distortions of the structure under transient loading conditions. When the separation in frequency between the elastic degrees of freedom and the rigid body motions is not large, then significant aerodynamic and inertial coupling can occur between the two. Many of the important effects of elastic distortion, however, can be accounted for simply by modifying the aerodynamic equations. The assumption is made that the changes in aerodynamic loading take place so slowly that the structure is at all times in static equilibrium. This is equivalent to assuming that the natural frequencies of vibration of the structure are much higher than the frequencies of the rigid body motions. Thus a change in load produces a proportional change in the shape of the airplane, which in turn influences the load. This is known as the method of "quasistatic" deflections where all the coupling occurs in the aerodynamic equations.

The wing uncoupled natural frequencies were investigated to determine which method would be used. Table 12.1 shows the 12.0-1

```
TABLE 12.1 WING UNCOUPLED FREQUENCIES (BLADES OFF)
    CRUISE CONFIGURATION
```

| Symmetric Mode | Frequency |
| :--- | :---: |
| Vertical Bending | 3.6 cps |
| Chordwise Bending | 5.4 cps |
| Torsion | 6.1 cps |
|  |  |
| Antisymmetric Mode | Frequency |
| Vertical Bending | 11.2 cps |
| Chordwise Bending | 9.1 cps |
| Torsion . | 5.7 cps |

Model 222 wing uncoupled frequencies for the cruise condition for both the symmetric and anti-symmetric modes. As can be noted in the table, the lowest vertical bending frequency is 3.6 cps and the lowest wing torsional frequency is 5.7 cps . The rigid body short period mode varies from approximately 0.40 cps to 1.35 cps. Since the rigid body short period modes are separated from the elastic modes by a substantial margin, the method of "quasistatic" deflection is used to represent the wing bending and torsion modes, with the only coupling in the aerodynamic terms (through angle of attack). The wing twists and bends instantaneously when subjected to an applied load. The assumptions made in deriving the wing bending and torsion relationships are as follows:

- No coupling between bending and torsion modes
- Wings are cantilevered from the fuselage
- Elliptical loading assumed for the rigid untwisted wing
- Aerodynamic loads act at the wing quarter chord
- Wing elastic axis coincident with cross shaft
- Wing center of mass assumed to lie on the elastic axis
- First wing torsional mode assumed linear from tip to root

In the mathematical model, wing twist at the tip is calculated using the following equation:
12.0-3

$$
\begin{aligned}
& K_{\theta_{t}}^{\theta} t=M_{A C T}-I_{E} \Omega_{E} R+q \frac{c_{W}^{2} b_{W}}{2} c_{m_{0}} \\
& \quad+q c_{W}^{2}\left(\frac{d C_{m_{c / 4}}}{d C_{\ell}}+\frac{\mathbf{x}_{W A C}}{c_{W}}\right)\left(\frac{C_{I_{\alpha}} b_{W}}{6 \pi}\right)\left({ }^{4 \theta_{t}}+3 \pi \alpha_{R I G I D}\right)
\end{aligned}
$$

where: $K_{\theta_{t}}=$ Wing torsional spring constant
$\theta_{t} \quad=$ wing twist angle in degrees
MACT $=$ Nacelle actuator pitching moment
IE $\quad=$ Engine inertia
$\Omega_{\mathrm{E}} \quad=$ Engine speed
R $\quad$ Body yaw rate
q $\quad=$ Dynamic pressure
$c_{w} \quad=$ Wing reference chord
$b_{w} \quad=$ Wing reference span
$c_{\text {tho }}=$ Wing zero lift pitching moment coefficient $\frac{\mathrm{dCm}_{\mathrm{C} / 4}}{\mathrm{dC}_{\ell}}=$ Wing pitching moment slope with lift coefficient
$C_{L_{\alpha}} \quad=$ Wing lift curve slope
$\alpha_{\text {RIGID }}=$ wing angle of attack without twist

Assuming a linear mode shape from the wing tip to the root and a cantilevered wing (zero twist at root), the wing twist at the aerodynamic center location of the wing is obtained by linear interpolation. The wing twist represents the change in angle of attack of the wing tip and aerodynamic center and are used in the aerodynamic equations.

Wing vertical bending deflection is also treated on a "quasistatic" basis. The form of the equation used in the mathematical model for the wing tip deflection is as follows:

$$
h_{1}=K_{W_{1}} z_{A E R O}^{N}+k_{W_{2}} z_{A E R O}^{W}-K_{W_{3}} L_{A E R O}^{N}-K_{W_{4}} \overline{\mathbf{a}}_{T}-K_{W_{5}} \overline{\mathbf{a}}_{\text {WAC }}
$$

```
where: \(h_{1} \quad=\) wing tip deflection
    \(z_{\text {AERO }}^{W}=\) Wing lift
    \(\mathrm{Z}_{\text {AERO }}^{\mathrm{N}}=\) Total wing lift
    \(\stackrel{N}{N} \quad=\) Nacelle rolling moment
    \(\bar{a}_{T} \quad=\) Vertical acceleration of the nacelle
    \(\bar{a}_{\text {WAC }} \quad=\) Vertical acceleration of the wing aero-
                        dynamic center
    \(\mathrm{K}_{\mathrm{W}_{1}} \rightarrow \mathrm{~K}_{\mathrm{W}_{5}}=\) Constants for Model 222 wing
```

The form of the equation for the wing deflection at the aerodynamic center is written similarly:

$$
h_{l_{W A C}}=K_{W_{6}} z_{A E R O}^{N}+K_{W_{7}} z_{A E R O}^{W}-K_{W_{8}} L_{A E R O}^{N}-K_{W_{9}} \bar{a}_{T}-K_{W_{10}} \bar{a}_{W A C}
$$

The symbols represent the same quantities as the tip deflections except the quantities $\mathrm{K}_{\mathrm{w}_{6}}$ to $\mathrm{K}_{\mathrm{w}_{10}}$ are different from $K_{1}$ to $K_{5}$.

These equations are derived in Appendix A. Since the wings are assumed cantilevered, these equations may be written for
the left and right sides. The equations as used in the mathematical model are written in Appendix E.

The wing tip and aerodynamic center vertical bending velocities are computed by dividing the change in vertical bending deflection by the simulation time frame. The vertical bending deflections and velocities are then added to the velocity components at the wing tip and aerodynamic center. These velocity components are then used in the calculation of the aerodynamic angle of attack.

In addition to the aerodynamic coupling via angle of attack, as discussed above, the wing tip vertical forces and moments act as the driving functions to a set of second order equations that are forced at the wing vertical bending frequency. This results in giving the pilot a "seat of the pants" feel for the vibratory aspects of the wing vertical bending mode. The equations were written in this manner to see if the pilot could induce a P.I.O. (pilot induced oscillation) during the piloted simulations due to wing vertical bending.

### 13.0 CONCLUSIONS AND RECOMMENDATIONS

1. Formulation of an eleven degree of freedom tilt rotor mathematical model and setting up an in-house hybrid simulation program using this model have been successfully completed.
2. The simulation model has been successfully checked out and validated at the Ames Research Center.
3. The in-house simulation model is "real time" and executes in 40 milliseconds. The Ames simulation is also real time with a 50 millisecond time frame. This increased time is due to the all digital nature of the Ames simulation.
4. It is desirable to shorten the frame time of the simulation. This may be accomplished by streamlining the following elements of the mathematical model:

- Slipstream aerodynamics
- Input aerodynamic data in body axes rather than wind axes to eliminate axes transforms

5. The simulation could be improved by incorporating advances in methodology in such areas as:

- Rotor Representation - Formulate a simplified analytical model to adequately represent the dynamics and aerodynamics of soft-in-plane hingeless
rotors for all flight regimes. This would avoid the necessity for complex time-consuming table look ups of rotor data.
- Slipstream Aerodynamics - Simplify the analytical representation based on wind tunnel test data.
- Interference Effects - Improve the prediction of the tail downwash environment at low transition speeds.


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As described in Section 12 the large separation which exists between the natural frequencies of vibration of the wing structure and the aircraft rigid body motions, enables the elastic deformations of the wing structure to be calculated on a quasistatic basis.

In the simple treatment presented below, the bending and torsion modes are considered to be uncoupled. The wing is treated as a cantilever with a built-in root end. The wing is free to twist about the elastic axis which is assumed to coincide with the nacelle pivot line. The center of mass of each chordwise strip is also taken to lie on the pivot line. The unloaded wing has neither geometric nor aerodynamic twist.

## WING TWIST

Spanwise twisting of the wing takes place under the action of the nacelle aerodynamic and inertial moments, the wing lift distribution, and the spanwise distribution of aerodynamic pitching moment. The nacelle aerodynamic moments consist of rotor hub loads, transferred to the pivot, together with the aerodynamic loads on the nacelle itself. Nacelle inertial moments include the gyroscopic effects of the rotor drive system.

With reference to Figure $A .1, M_{N}$ is the moment supplied or absorbed by the nacelle tilt actuator. If $\mathrm{K}_{\theta}$ is the wing stiffness as seen by the wing tip, then

$$
\begin{equation*}
M_{N}=K_{\theta}{ }^{\theta} T \tag{A-1}
\end{equation*}
$$

The total moment about the elastic axis due to wing aerodynamics, nacelle loads and engine gyroscopic torque is

$$
\begin{equation*}
T=\int_{0}^{b / 2} m d y+M_{N}+M_{g y r o} \tag{A-2}
\end{equation*}
$$

The aerodynamic moment about the elastic axis at any station $y$ is given by

$$
\begin{equation*}
m=m_{c / 4}+\ell x \tag{A-3}
\end{equation*}
$$

where $\ell$ is the section lift and $x$ is the distance from the quarter chord to the elastic axis. In terms of the section aerodynamic coefficients,

$$
\begin{equation*}
m(y)=\frac{1}{2} \rho v^{2} c^{2} \quad c_{m_{c / 4}}+\frac{1}{2} \rho v^{2} c^{2} C_{\ell} \frac{x}{c} \tag{A-4}
\end{equation*}
$$

The section lift coefficient, $C_{\ell}$, is given by

$$
\begin{align*}
c_{\ell} & =k \frac{d c_{\ell}}{d \alpha}\left(\alpha-\alpha_{0}\right) \sqrt{1-\left(\frac{2 y}{b}\right)^{2}} \\
& =k a_{0}\left(\alpha_{R}-\varepsilon_{p}-\alpha_{0}+\theta_{t}(y)\right) \sqrt{1-\left(\frac{2 y}{b}\right)^{2}} \tag{A-5}
\end{align*}
$$

where $\quad \alpha_{R}$ is the wing root section angle of attack
${ }^{E} p$ is the rotor induced downwash, assumed constant
$\alpha_{0}$ is the section zero-lift angle
$\theta_{t}$ is the structural twist at station $y$


Figure A.1. Wing Geometry for Derivation of Flexibility

The factor $k \sqrt{1-\left(\frac{2 y}{b}\right)^{2}}$ is introduced so that, for the untwisted wing, the lift distribution is elliptical. The value of $k$ is obtained from the rigid wing elliptical loading as

$$
\begin{equation*}
\mathrm{k}=\frac{4}{\pi} \frac{\mathrm{C}_{\mathrm{L}_{\alpha}}}{\mathrm{a}_{\mathrm{o}}} \tag{A-6}
\end{equation*}
$$

Thus the equation for $C_{\ell}$ becomes, with $\alpha_{\text {RIGID }}=\alpha_{R}{ }^{-\varepsilon} \bar{p}^{-\alpha} o^{\prime}$

$$
\begin{equation*}
C_{\ell}=\frac{4}{\pi} c_{L_{\alpha}}\left[\alpha_{R I G I D} \sqrt{1-\left(\frac{2 y}{b}\right)^{2}}+\theta_{t} \sqrt{1-\left(\frac{2 y}{b}\right)^{2}}\right] \tag{A-7}
\end{equation*}
$$

In equation ( $\mathrm{A}-4$ ) we can write, for low angles of attack,

$$
\begin{equation*}
c_{m_{c / 4}}=c_{m_{0}}+\frac{d c_{m_{c} / 4}}{d C_{\ell}} c_{\ell} \tag{A-8}
\end{equation*}
$$

and therefore

$$
\begin{equation*}
m(y)=\frac{1}{2} \rho V^{2} c^{2}\left\{c_{m_{0}}+\left(\frac{d c_{m_{c / 4}}}{d c_{\ell}}+\frac{x}{c}\right) \quad c_{\ell}\right\} \tag{A-9}
\end{equation*}
$$

The equation for the total wing twisting moment, equation (A-2), can now be written as,

$$
T=M_{\text {actuator }}+M_{G Y R O}+\frac{1}{4} \rho V^{2} c^{2} C_{m_{O}} b+\frac{1}{2} \rho V^{2} c^{2}
$$

$$
\begin{equation*}
\left(\frac{d c_{m_{c / 4}}}{d c_{\ell}}+\frac{x}{c}\right)^{b / 2} \int_{0}^{2} c_{\ell} d y \tag{A-10}
\end{equation*}
$$

Using equation ( $\mathrm{A}-7$ ), assuming a linear structural twist from root to tip and performing the indicated integrations, the equation for total wing twisting moment becomes

$$
\begin{align*}
T=K_{\theta} \theta_{T}= & M_{\text {actuator }}{ }^{+M}{ }_{g Y r o}+\frac{1}{4} \rho V^{2} b c^{2} C_{m_{o}}+\frac{1}{2} \rho V^{2} c^{2}\left(\frac{d C_{m_{C} / 4}}{d C_{l}}+\frac{x}{c}\right) \\
& \times \frac{C_{L_{\alpha b}}}{6 \pi}\left(3 \pi \alpha_{R I G I D}+4 \theta_{T}\right) \tag{A-11}
\end{align*}
$$

The equation for the actuator moment is given in the equations of motion, Section 5.0.

Rearranging, and writing $q=q_{s}\left(1-C_{T_{s}}\right)=\frac{1}{2} \rho V^{2}$

$$
\begin{equation*}
{ }^{\theta} T=\frac{M_{N}+M_{g y r o}+\frac{1}{2} q_{S}\left(1-c_{T_{S}}\right) c_{W}^{2}\left[6 \pi \alpha_{\text {rigid }}\left(\frac{d_{C_{m}}}{d C_{L}}+\frac{x}{c}\right)+b_{W} c_{m_{0}}\right]}{K_{\theta}-\frac{2}{3 \pi} q_{S} b_{W} c_{W}^{2} C_{L_{\alpha}}\left(1-C_{T_{S}}\right)\left(\frac{d C_{m}}{d C_{I}}+\frac{x}{c}\right)} \tag{A-12}
\end{equation*}
$$

where $C_{m_{0}}$, the zero-lift wing section pitching moment coefficient, is a function of flap deflection:

$$
\begin{equation*}
c_{m_{0}}=c_{1}+c_{2} \delta_{f}+c_{3} \delta_{f}^{2} \tag{A-13}
\end{equation*}
$$

Knowing the tip value of twist, the twist at any other spanwise station is obtained by assuming a linear variation of twist from zero at the root to the tip value.

## WING VERTICAL BENDING

The spanwise bending moment at any spanwise station $Y$, on the wing is the sum of the bending moments due to wing aerodynamic lift, wing weight, nacelle lift, nacelle weight and net torque on the nacelle. The expressions for each contribution to the bending moments are derived below.

- Bending moment due to wing loading.

Assuming an elliptical distribution of lift the bending moment is given by

$$
\begin{align*}
M^{a}\left(y_{1}\right) & =\int_{y_{1}}^{b / 2} \ell(y)\left(y-y_{1}\right) d y  \tag{A-14}\\
& =\frac{\ell_{0} b^{2}}{4} \quad \int_{Y_{1}}^{b / 2} \sqrt{1-\left(\frac{2 y}{b}\right)^{2}}\left(\frac{2 y}{b}-\frac{2 y_{1}}{b}\right) d\left(\frac{2 y}{b}\right)
\end{align*}
$$

Where $\ell_{0}$ is the lift per unit length at the wing root. Introducing the spanwise variable $\theta=\cos ^{-1}\left(\frac{2 y}{b}\right)$ making the required substitutions and integrating, the bending moment at any point y is:

$$
\begin{equation*}
M^{a}(y)=\frac{\ell_{0} b^{2}}{4}\left[\frac{1}{2}(\sin \theta-\theta \cos \theta)-\frac{1}{6} \sin ^{3} \theta\right] \tag{A-15}
\end{equation*}
$$

- Bending due to nacelle net yerticall load.

The net vertical force on nacelle is

$$
F=F^{a}-n W_{N}
$$

where $F^{a}$ is the aerodynamic force and $n W_{N}$ is the inertial load on the nacelle. The bending moment due to nacelle
force is

$$
\begin{equation*}
M^{N}(y)=\frac{F b}{2} \quad(1-\cos \theta) \tag{A-16}
\end{equation*}
$$

- Bending due to wing weight.

Assuming a uniform distribution of wing weight

$$
M^{w}\left(y_{1}\right)=-n \int_{Y_{1}}^{b / 2} w(y)\left(y-y_{1}\right) d y
$$

and $w(y)=2 W / b$ where $W$ is the weight of one wing panel
$\therefore M^{W}\left(y_{1}\right)=\frac{2 n W}{b} \int_{y_{1}}^{b / 2}\left(y-y_{1}\right) d y$
i.e. $M^{W}(y)=-\frac{n W b}{2}\left(1-\cos \theta-\frac{1}{2} \sin ^{2} \theta\right)$

- Bending due to nacelle torque (rolling moment)

$$
\begin{equation*}
T(y)=\text { constant }=T \tag{A-18}
\end{equation*}
$$

A-6

Total bending moment at station $y$ is therefore

$$
\begin{equation*}
M(y)=M^{a}(y)+M^{N}(y)+M^{W}(y)+T \tag{A-19}
\end{equation*}
$$

Assuming a linear variation of $E I$ from root to tip given by

$$
\begin{equation*}
E I(y)=E I_{0}\left[1-a\left(\frac{2 y}{b}\right)\right]=E I_{0}(1-a \cos \theta) \tag{A-20}
\end{equation*}
$$

the curvature of the wing due to bending is

$$
\begin{align*}
\frac{M(y)}{E I(y)} & =\frac{d^{2} z}{d y^{2}}=\frac{\ell_{0} b^{2}}{8 E I_{O}}\left[\frac{(\sin \theta-\theta \cos \theta)-1 / 13 \sin ^{3} \theta}{1-a \cos \theta}\right]+\frac{F_{a} b}{2 E I_{0}}\left[\frac{1-\cos \theta}{1-a \cos \theta}\right] \\
& -\frac{n W_{N} b}{2 E I_{0}}\left[\frac{1-\cos \theta}{1-a \cos \theta}\right]-\frac{n W_{W} b}{2 E I_{0}}\left[\frac{1-\cos \theta-\frac{1}{2} \sin ^{2} \theta}{1-a \cos \theta}\right] \\
& +\frac{T}{E I_{0}}\left[\frac{1}{(1-a \cos \theta)}\right] \tag{A-21}
\end{align*}
$$

Double integration of this equation yields the following expression for the bending deflection of the wing at any point $y$ on the span:-

$$
\begin{align*}
z(y) & =\frac{L b^{3}}{8 \pi E I_{0}} \phi_{1}+\frac{b^{3} F^{a}}{8 E I_{0}} \phi_{2}-\frac{n W_{N} b^{3}}{8 E I_{0}} \phi_{3} \\
& -\frac{n W_{w} b^{3}}{8 E I_{0}} \phi_{4}+\frac{T b^{3}}{4 E I_{0}} \phi_{5} \tag{A-22}
\end{align*}
$$

where $\phi_{I}=\int_{0}^{y}\left\{\int_{0}^{y} \frac{(\sin \theta-\theta \cos \theta)-\frac{1}{3} \sin ^{3} \theta}{1-\operatorname{acos} \theta} d y\right\} d y$

$$
\begin{aligned}
\phi_{2}=\phi_{3} & =\int_{0}^{y}\left\{\int_{0}^{y} \frac{1-\cos \theta}{1-a \cos \theta} d y\right\} d y \\
\phi_{4} & =\int_{0}^{y}\left\{\int_{0}^{y} \frac{1-\cos \theta-\frac{1}{2} \sin ^{2} \theta}{1-\operatorname{acos} \theta} d y\right\} d y
\end{aligned}
$$

$$
\phi_{5}=\int_{0}^{y}\left\{\int_{0}^{y} \frac{d y}{1-a \cos \theta}\right\} d y
$$

and where the wing lift ( 2 wing panels) $L=\frac{\pi}{4} \ell_{0} b$. The function $\phi_{1}$ through $\phi_{5}$ were obtained numerically and are presented in Figure A. 2 .

Since $\quad L=-2 z_{\text {AERO }}^{W}$

$$
\begin{aligned}
& F^{a}=-z_{A E R O}^{N} \\
& T=-L_{A E R O}^{N} \\
& n W_{W}=\frac{1}{2} m_{W} \frac{z_{A E R O}}{m}=\frac{1}{2} m_{W} \bar{a}_{W A C} \\
& n W_{N}=m_{N} \bar{a}_{T}
\end{aligned}
$$

where $m_{w}$ is the mass of two wing panels $m$ is the total aircraft mass $\bar{a}_{\text {WAC }}$ is the acceleration of the wing aerodynamic center $\bar{a}_{T}$ is the acceleration of the wing tip
and since the values of $\phi_{1}$ through $\phi_{5}$ are constant for any given station $y$ on the wing we can write the final equation for wing bending in the form

$$
\begin{aligned}
\mathrm{h}_{1}= & \mathrm{K}_{W_{1}} z_{A E R O}^{N}+\mathrm{K}_{W_{2}} z_{\mathrm{AERO}}^{\mathrm{W}}-\mathrm{K}_{W_{3}} \mathrm{~L}_{\text {AERO }}^{N}-K_{W_{4}} \overline{\mathrm{a}}_{T} \\
& -\mathrm{K}_{W_{5}} \bar{a}_{W A C}
\end{aligned}
$$

where

$$
\begin{aligned}
& h_{1}=-z \\
& K_{W_{1}}=\frac{b^{3} \phi 2}{8 E I_{0}}
\end{aligned}
$$

$$
\begin{aligned}
& K_{W_{2}}=\frac{b^{3} \phi_{1}}{4 \pi E I_{o}} \\
& K_{W_{3}}=\frac{b^{3} \phi_{5}}{4 E I_{o}} \\
& K_{W_{4}}=\frac{m_{N} b^{3} \phi_{2}}{8 E I_{o}} \\
& K_{W_{5}}=\frac{m_{W^{3}} b^{3} \phi_{4}}{8 E I_{o}}
\end{aligned}
$$

This is the form given in the computer representation. The bending deflection at the aerodynamic center and at the wing tip are obtained using the values of $\phi_{1} \rightarrow \phi_{5}$ appropriate to these stations.


Figure A.2. Wing Bending Functions

## APPENDIX B - DERIVATION OF LANDING GEAR EQUATIONS

Presented below are the equations for landing gear forces and moments arising from ground contact. The derivation accounts for brake and friction forces together with a simplified representation of the oleo dynamics. Nose wheel steering is not included.

With reference to Figure $B-1$ the distance from the center of gravity to the bottom of the right main wheel following a positive pitch rotation is

$$
\begin{equation*}
h_{\theta}=x \sin \theta-z \cos \theta-r \tag{B-1}
\end{equation*}
$$

where $X$ and $Z$ are the coordinates of the hub of the wheel relative to the C.G. and $r$ is the tire radius. If the aircraft is now rolled right, through the angle $\phi$, the bottom of the right gear moves through a distance

$$
\begin{equation*}
h_{\phi}=[Y \sin \phi+(Z+r)(\cos \phi-1)] \cos \theta \tag{B-2}
\end{equation*}
$$

The height of the bottom of the wheel above the ground is therefore

$$
\begin{equation*}
h=H_{C G}+h_{\theta}-h_{\phi} \tag{B-3}
\end{equation*}
$$

and the oleo deflection during ground contact is given by

$$
\begin{equation*}
h_{\mathbf{T}}=\frac{\mathrm{H}_{\mathrm{CG}}+\mathrm{h}_{\theta}-\mathrm{h}_{\phi}}{\cos \phi \cos \theta} \tag{B-4}
\end{equation*}
$$

By differentiation of equation $B-4$ and making small angle assumptions regarding the aircraft pitch and roll angles during touchdown, the rate of change of oleo strut deflection is

$$
B-1
$$



Figure B.1. Geometry of Landing Gear
obtained as

$$
\begin{equation*}
\dot{\mathrm{h}}_{\mathrm{T}}={\frac{\dot{\mathrm{H}}_{\mathrm{CG}}}{\cos \phi \cos \theta}+\mathrm{XQ}-\mathrm{YP},{ }_{Q} .} \tag{B-5}
\end{equation*}
$$

Assuming that the oleo response is that of a second order system, the equation of motion for the landing gear is

$$
\begin{equation*}
\mathrm{F}_{\mathrm{G}}=\mathrm{K}_{\mathrm{ST}} \mathrm{~h}_{\mathrm{T}}+\mathrm{D}_{S T}{\dot{h_{T}}}_{\mathrm{T}} \tag{B-6}
\end{equation*}
$$

where $K_{S T}$ and $D_{S T}$ are the equivalent spring rates and damping for the oleo, and $F_{G}$ is the force on the landing gear strut.

## Tire Friction and Side Force

The friction force acting on each tire during ground contact is resolved into a force $F_{\mu}$ along the line of intersection of the plane of the wheel and the ground plane, positive forward, and a side force $F_{\mu}$ at right angles to $F_{\mu}$ lying in the ground plane and positive to starboard. The friction force $F_{s}$ is assumed to be proportional to oleo force and the amount of braking exerted by the pilot. The side force is proportional to the oleo force.

The components of tire friction are:

$$
\begin{align*}
& F_{\mu}=\left(\mu_{0}+\mu_{1} B_{G}\right) F_{G Z} \frac{u}{|u|}  \tag{B-7}\\
& F_{S}=\mu_{S} F_{G Z} \frac{v}{|v|} \tag{B-8}
\end{align*}
$$

where $\mu_{s}, \mu_{l}$ and $\mu_{s}$ are the coefficients for rolling friction, brake friction and sliding friction. $B_{G}$ is expressed as a percentage of full brake pedal deflection. The signs of the forward and sidewards velocity are introduced to properly orient the tire forces.

The force and moment contributions of each landing gear to the aircraft total forces and moments are, assuming small angles;

$$
\begin{align*}
& \Delta X_{n}=F_{\mu_{n}}-F_{G Z_{n}} \theta \\
& \Delta Y_{n}=F_{s_{n}}+F_{G Z_{n}} \phi \\
& \Delta Z_{n}=F_{\mu_{n}} \theta-F_{S_{n}} \phi+F_{G Z_{n}} \\
& \Delta M_{n}=-\Delta Z_{n} X_{n}+\Delta X_{n}\left(Z_{n}+r_{n}+h_{T_{n}}\right) \\
& \Delta L_{n}=\Delta Z_{n} Y_{n}-\Delta Y_{n}\left(Z_{n}+r_{n}+h_{T}\right) \\
& \Delta N_{n}=-\Delta X_{n} Y_{n}+X_{n} \Delta Y_{n}
\end{align*}
$$

where $n=1,2$ and 3 denote the left main gear, right main gear and nose gear, respectively.

The total contribution of the landing gear forces to the forces and moments at the center of gravity of the aircraft are:

$$
\begin{aligned}
\Delta X_{L G} & =\sum_{n=1}^{3} \Delta X_{n} \\
\Delta Y_{L G} & =\sum_{n=1}^{3} \Delta Y_{n} \\
\Delta Z_{L G} & =\sum_{n=1}^{3} \Delta Z_{n} \\
\Delta L_{L G} & =\sum_{n=1}^{3} \Delta L_{n} \\
\Delta M_{L G} & =\sum_{n=1}^{3} \Delta M_{n} \\
\Delta N_{L G} & =\sum_{n=1}^{3} \Delta N_{n}
\end{aligned}
$$

CENTER OF GRAVITY/INERTIA EQUATIONS

## C.l Velocity Transformations

The calculation of aerodynamic forces on wings, fuselage, nacelles and tail surfaces requires that the angle of attack and relative wind velocity at these surfaces be known. These velocities are obtained most conveniently in terms of the velocity of the pivot reference point.

With reference to Figure C.l , the velocity of a general point in the aircraft relative to the airplane center of gravity is

$$
\begin{equation*}
V=\frac{\delta r}{\delta t}+\underline{\Omega} \times \underline{r} \tag{C-1}
\end{equation*}
$$

where $\underline{r}$ is the radius vector from the c.g. to the point and $\underline{\Omega}$ is the angular velocity of the aircraft. Thus, expanding equation $C-1$, the velocity of the pivot relative to the c.g. is

$$
\begin{align*}
u_{p}^{\prime} & =\dot{X}_{p}+Q Z_{P}-Y_{P} R \\
v_{p}^{\prime} & =\dot{Y}_{p}-P Z_{P}+X_{P} R  \tag{C-2}\\
w_{p}^{\prime} & =\dot{Z}_{p}+P Y_{P}-Q X_{P}
\end{align*}
$$

where $X_{p}, Y_{p}$ and $Z_{p}$ are the distances of the pivot from the c.g., measured positively forward, to the right and downwards, respectively. If we measure all distances from the pivot location then $X_{p}=-X_{C G}$, $Y_{P}=-Y_{C G}=0, Z_{P}=-Z_{C G}$ and the velocity of the pivot relative to inertial space can be written,

$$
\begin{gather*}
u_{\mathrm{p}}=\mathrm{U}+\mathrm{u}_{\mathrm{p}}^{\prime}=\mathrm{U}-\dot{x}_{\mathrm{CG}}-\mathrm{QZ}_{C G} \\
\mathrm{v}_{\mathrm{p}}=\mathrm{V}+\mathrm{v}_{\mathrm{p}}^{\prime}=\mathrm{V}+\mathrm{PZ}_{C G}-\mathrm{X}_{\mathrm{CG}} \mathrm{R}  \tag{c-3}\\
\mathrm{w}_{\mathrm{p}}=\mathrm{W}+\mathrm{w}_{\mathrm{p}}^{\prime}=\mathrm{w}+\mathrm{QX}_{C G}-\dot{z}_{\mathrm{CG}} \\
\mathrm{C}-1
\end{gather*}
$$

where $U, V$ and $W$ are the components of the velocity of the airplane center of gravity.

The velocity of a point in the aircraft relative to the pivot is

$$
\begin{align*}
& \mathrm{u}=\dot{\mathrm{X}}+\mathrm{QZ}-\mathrm{YR} \\
& \mathrm{v}=\dot{\mathrm{Y}}+\mathrm{RX}-\mathrm{PZ}  \tag{C-4}\\
& \mathrm{w}=\dot{\mathrm{Z}}+\mathrm{PY}-\mathrm{QX}
\end{align*}
$$

where $X, Y$, and $Z$ are measured from the pivot to the point. By adding equations $(C-3)$ and ( $C-4$ ) the velocities of the following components are obtained relative to inertial space. The indicated distances are measured relative to the pivot. Velocity of Horizontal Tail Aerodynamic Center

$$
\begin{align*}
& u_{H T}=u_{P}+z_{H T} Q \\
& v_{H T}=v_{P}+x_{H T} R-z_{H T}{ }^{P}  \tag{C-5}\\
& w_{H T}=w_{P}-x_{H T} Q
\end{align*}
$$

Velocity of Vertical Tail Aerodynamic Center

$$
\begin{align*}
& u_{V T}=u_{\mathrm{P}}+z_{V T^{Q}} \\
& \mathrm{v}_{\mathrm{VT}}=u_{\mathrm{P}}+\mathrm{x}_{\mathrm{VT}^{\mathrm{R}}}-z_{\mathrm{VT}^{P}}  \tag{C-6}\\
& \mathrm{w}_{\mathrm{VT}}=\mathrm{w}_{\mathrm{P}}+\mathrm{x}_{\mathrm{VT}} \mathrm{Q}
\end{align*}
$$

Velocity of Left Wing Aerodynamic Center - Body Axes

$$
\begin{gather*}
u_{L W}^{\prime}=u_{P}+Q\left(z_{W A C}+h_{l_{L W A C}}\right)+Y_{W A C} R \\
v_{L W}^{\prime}=u_{P}+x_{W A C} R-P\left(z_{W A C}+h_{1_{L_{W A C}}}\right)  \tag{C-7}\\
w_{L W}^{\prime}=w_{P}-Y_{W A C} P-x_{W A C} Q+\dot{h}_{1_{L_{W A C}}} \\
C-3
\end{gather*}
$$

where ${ }^{h_{1}} L_{\text {WAC }}$ is the elastic deflection of the left wing aerodynamic center. The equations for the right wing are obtained by substituting

$$
\begin{array}{rlrl} 
& \mathrm{Y}_{\mathrm{R}_{\mathrm{WAC}}} & =-\mathrm{Y}_{\mathrm{L}_{\mathrm{WAC}}} \\
\text { and } \quad \mathrm{h}_{1_{\mathrm{R}_{\mathrm{WAC}}}} & =\mathrm{h}_{1_{\mathrm{L}_{\mathrm{WAC}}}}
\end{array}
$$

Velocity of Left Wing Aerodynamic Center-Chord Axes
In order to compute wing angle-of-attack the velocity components are required relative to the wing chord line. If the wing chord makes an angle $i_{w}$ with the body centerline then

$$
\begin{align*}
& u_{L W}=u_{L W}^{\prime} \cos i_{w}-w_{L W}^{\prime} \sin i_{w} \\
& v_{L W}=v_{L W}^{\prime}  \tag{C-8}\\
& w_{L W}=w_{L W}^{\prime} \cos i_{w}+w_{L W}^{\prime} \sin i_{w}
\end{align*}
$$

The equations for the right wing are obtained by changing the subscript. Velocity of Left Rotor Hub - Body Axes

$$
\begin{align*}
& u_{R L}^{\prime}=u_{P}+R Y_{N}-L_{s}\left(\dot{i}_{N L}+Q\right) \sin i_{N L}+Q h_{1_{L}} \\
& v_{R L}^{\prime}=v_{P}+L_{s}\left(R \cos i_{N L}+P \sin i_{N L}\right)-P h_{l_{L}}  \tag{C-9}\\
& w_{R L}=w_{P}-P Y_{N}-L_{S}\left(i_{N L}+Q\right) \cos i_{N L}+\dot{h}_{l_{L}}
\end{align*}
$$

where $L_{S}$ is the distance from the rotor pivot point to the rotor hub and $h_{l_{L}}$ is the deflection of the wing tip. The equations for the right hub are obtained by changing subscripts and substituting $Y_{N}=-Y_{N}$.
-
Velocity of Left Rotor Hub - Shaft Axes

Since the rotor aerodynamic forces and moments are functions of the shaft angle of attack and sideslip, the velocity components are required relative to shaft axes.

$$
\begin{align*}
& u_{R L}=u_{R L}^{\prime} \cos i_{N L}-w_{R L}^{\prime} \sin i_{N L} \\
& v_{R L}=v_{R L}^{\prime}  \tag{c-10}\\
& w_{R L}=w_{R L}^{\prime} \sin i_{N L}+w_{R L}^{\prime} \cos i_{N L}
\end{align*}
$$

The corresponding equations for the right hub are obtained by changing the subscript.

## C. 2 Center of Gravity and Inertia Equations

Equations are required that express the overall aircraft center of gravity position and inertias in terms of the centers of gravity and inertias of the individual mass components. In order to do this a fixed reference point is chosen in the aircraft defined by the intersection of the line joining the nacelle pivots and the vertical plane of symmetry of the aircraft, see Figure C.I. A set of axes PXPYPZP is taken at this pivot reference point, parallel to the axes oXYz at the aircraft center of gravity. If the location of the aircraft center of gravity with respect to the pivot reference axes is ( $X_{C G}^{\prime}, Y_{C G}^{\prime}, Z_{C G}^{\prime}$ ) and if $\left(\ell_{f}, h_{f}\right)$ and ( $\left.\ell_{w}, h_{W}\right)$ are the $x$ and $z$ coordinates of the fuselage and wing masses measured from the pivot, then the following relationships are obtained between the centers of mass of the components and the aircraft center of gravity.

$$
C-5
$$

Fuselage CG Relative to Aircraft CG

$$
\begin{align*}
& x_{f}=\ell_{f}-x_{C G}^{\prime}  \tag{C-11}\\
& x_{f}=h_{f}-z_{C G}
\end{align*}
$$

Wing CG Relative to Aircraft CG

$$
\begin{align*}
& \mathrm{x}_{\mathrm{w}}=\ell_{\mathrm{w}}-\mathrm{x}_{\mathrm{CG}}^{\prime}  \tag{C-12}\\
& \mathrm{z}_{\mathrm{w}}=\mathrm{h}_{\mathrm{w}}-\mathrm{z}_{\mathrm{CG}}^{\prime}
\end{align*}
$$

Nacelle CG Relative to Aircraft CG

$$
\begin{align*}
& x_{N R}=\ell \cos \left(i_{N R}-\lambda\right)-x_{\mathrm{CG}}^{\prime} \\
& \mathrm{x}_{\mathrm{NL}}=\ell \cos \left(i_{\mathrm{NL}}-\lambda\right)-\mathrm{x}_{\mathrm{CG}}^{\prime}  \tag{C-13}\\
& \mathrm{z}_{\mathrm{NR}}=\ell \sin \left(i_{\mathrm{NR}}-\lambda\right)-\mathrm{z}_{\mathrm{CG}}^{\prime} \\
& \mathbf{z}_{\mathrm{NL}}=\ell \sin \left(i_{\mathrm{NL}}-\lambda\right)-\mathrm{z}_{\mathrm{CG}}^{\prime}
\end{align*}
$$

where $\ell$ is the distance from the nacelle pivot point to the nacelle c.g., and $\lambda$ is the angular depression of the nacelle center of mass below the nacelle pivot, when the nacelle is in the down position, see Figure C.l.

Aircraft Center of Gravity Position
By taking moments about the pivot, the aircraft center of gravity is given by

$$
\mathrm{C}-6
$$

$$
\begin{align*}
& x_{C G}^{\prime}=\frac{m_{f} \ell_{f}+m_{W} \ell_{W}}{m}+\ell\left(\frac{m_{N}}{m}\right)\left[\cos \left(i_{N L^{-\lambda}}\right)+\cos \left(i_{N R^{-\lambda}}\right)\right] \\
& z_{C G}^{\prime}=\frac{m_{f} h_{f}+m_{W} h_{W}}{m}-\ell\left(\frac{m_{N}}{m}\right)\left[\sin \left(i_{N L^{-\lambda}}\right)+\sin \left(i_{N R^{-\lambda}}\right)\right] \tag{C-14}
\end{align*}
$$

The equations of motion (Section 5) require the first and second time derivatives of the center of gravity position. They are as follows: Center of Gravity Velocity Relative to Pivot Point

$$
\left.\dot{\mathrm{x}}_{\mathrm{CG}}^{\prime}=-\ell\left(\frac{m_{\mathrm{N}}}{m}\right)\left[i_{\mathrm{NR}^{\sin }} \sin i_{\mathrm{NR}^{-\lambda}}\right)+i_{\mathrm{NL}} \sin \left(i_{N L^{-\lambda}}\right)\right]
$$

$$
\begin{equation*}
\dot{z}_{\mathrm{CG}}^{1}=-\ell\left(\frac{m_{N}}{m}\right)\left[i_{N R} \operatorname{Cos}\left(i_{N_{R}}-\lambda\right)+i_{N L} \cos \left(i_{N L}-\lambda\right)\right] \tag{c-15}
\end{equation*}
$$

Center of Gravity Acceleration Relative to Pivot Point

$$
\begin{aligned}
& \ddot{x}_{C G}^{\prime}=-\ell\left(\frac{m_{N}}{m}\right)\left[{\stackrel{\infty}{i_{N R}}}^{\sin }\left(i_{N R}-\lambda\right)+\stackrel{\infty}{i}_{N L} \sin \left(i_{N L}-\lambda\right)+\stackrel{\circ}{i}_{N L}^{2} \cos \right. \\
& \left.\left(i_{N L}-\lambda\right)+i_{N_{R}}^{2} \cos \left(i_{N R}-\lambda\right)\right] \\
& \ddot{z}_{C G}^{\prime}=-\ell\left(\frac{m_{N}}{m}\right)\left[\stackrel{\infty}{i}_{N R} \operatorname{Cos}\left(i_{N R}-\lambda\right)+\stackrel{\infty}{i}_{N L} \operatorname{Cos}\left(i_{N L}-\lambda\right)-\stackrel{\circ}{i}_{N L}^{2} \sin \right. \\
& \left.\left(i_{N L}-\lambda\right)-\stackrel{\circ}{i}_{N R^{2}}^{2} \sin \left(i_{N R}-\lambda\right)\right]
\end{aligned}
$$

## Pilot Station Velocities - Body Axes

The velocities at the pilot's station are required in order to drive the visual display. From equations ( $C-3$ ) and (C-4) the components of velocity of the pilot's station in body axes are:

$$
C-7
$$

$$
\begin{aligned}
& u_{P A}=u_{P}+Q Z_{P A}-R Y_{P A} \\
& v_{P A}=v_{P}+R \ell_{P A}-P Z_{P A} \\
& w_{P A}=w_{P}+P Y_{P A}-Q \ell_{P A}
\end{aligned}
$$

## C-3 Pilot Station Acceleration - Body Axes

The pilot station acceleration is also required to drive the visual display. These accelerations are derived here.

The velocity at the pilot's station is

$$
\underline{V}_{P A}=\underline{V}_{C G}+\underline{\Omega}^{x} \underline{r}_{P A}+\frac{\delta \underline{r}_{P A}}{\delta t}
$$

where $r_{P A}$ is the vector from the aircraft CG to the pilot's station and $\frac{{ }^{\delta} \underline{r}_{P A}}{\delta t}$ is the rate of change of the pilot's station with respect to the aircraft CG.

The pilot's station acceleration is

$$
\begin{aligned}
& \underline{a}_{P A}=\frac{d \underline{V}_{P A}}{d t}=\frac{d \underline{V}_{C G}}{d t}+\frac{d}{d t}\left(\underline{\Omega} \times \underline{r}_{P A}\right)+\frac{d}{d t}\left(\frac{\delta \underline{r}_{P A}}{\delta t}\right) \\
= & \underline{a}_{C G}+\frac{\delta}{\delta t}\left(\underline{\Omega} \times \underline{r}_{P A}\right)+\underline{\Omega} \times\left(\underline{\Omega} \times \underline{r}_{P A}\right)+\frac{\delta^{2} \underline{r}_{P A}}{\delta t^{2}}+\underline{\Omega} \times \frac{\delta \underline{r_{P A}}}{\delta t} \\
= & \underline{-}_{C G}+\frac{\delta \underline{\Omega}}{\delta t} \times \underline{r}_{P A}+2 \Omega \times \frac{\delta \underline{r}_{P A}}{\delta t}+\underline{\Omega}\left(\underline{r}_{P A} \cdot \underline{\Omega}\right)-\Omega^{2} \underline{r}_{P A}+\frac{\delta^{2} \underline{r}_{P A}}{\delta t^{2}}
\end{aligned}
$$

with $\underline{\Omega}=P \underline{i}+Q \underline{\hat{j}}+R \underline{k}$

$$
\begin{aligned}
& \frac{\delta \Omega}{\delta t}=\dot{P} \hat{i}+\dot{Q} \hat{j}+\dot{R} \hat{k} \\
& \underline{r}_{P A}=\left(X_{P A}-X_{C G}\right) \underline{i}+\left(Y_{P A}-Y_{C G}\right) \underline{\hat{j}}+\left(Z_{P A}-z_{C G}\right) \underline{\hat{k}} \\
& \frac{\delta r_{P A}}{\delta t}=\left(\dot{X}_{P A}-\dot{X}_{C G}\right) \underline{i}+\left(\dot{Y}_{P A}-\dot{Y}_{C G}\right) \hat{j}+\left(\dot{Z}_{P A}-\dot{z}_{C G}\right) \underline{\hat{k}} \\
& C-8
\end{aligned}
$$

and noting that $Y_{C G}$ and the time derivatives of $X_{P A}, Y_{P A}, Z_{P A}$ are always zero, the above equation yields the pilot's station accelerations as: -

$$
\begin{aligned}
a_{x_{P A}}= & \frac{x_{A E R O}}{m}+(\dot{Q}+P R)\left(Z_{P A}-z_{C G}\right)+\left(Q^{2}+R^{2}\right)\left(X_{C G}-\ell_{P A}\right) \\
& +Y_{P A}(P Q-\dot{R})-8 Q \dot{Z}_{C G}-\ddot{x} C G \\
a_{Y_{P A}}= & \frac{Y_{A E R O}}{m}(\dot{P}-Q R)\left(z_{C G}-z_{P A}\right)+(\dot{R}+P Q)\left(\ell_{P A}-X_{C G}\right) \\
& -Y_{P A}\left(R^{2}+P^{2}\right)+2\left(P \dot{Z}_{C G}-R \dot{x}_{C G}\right) \\
a_{z_{P A}}= & \frac{z_{A E R O}}{m}(\dot{Q}-P R)\left(X_{C G}-\ell_{P A}\right)+\left(P^{2}+Q^{2}\right)\left(Z_{C G}-Z_{P A}\right) \\
& +Y_{P A}(\dot{P}+Q R)+2 Q \dot{X}_{C G}-\ddot{z}_{C G}
\end{aligned}
$$

where $a_{x_{C G}}=\frac{z_{\text {AERO }}}{m}$ etc.

$$
X_{P A}=\ell_{P A}, \text { the distance from the pivot to the pilot's station }
$$

## C. 4 Aircraft Inertias

The aircraft roll inertia about the aircraft center of gravity is, from the parallel axis theorem,

$$
\begin{equation*}
I_{x x}=I_{x x}^{f}+I_{x x}^{W}+I_{x x}^{N L}+I_{x X}^{N R}+m_{f} Z_{f}^{2}+m_{W} z_{w}^{2}+2 m_{N} Y_{N}^{2}+m_{N} z_{N L}^{2}+m_{N} z_{N R}^{2} \tag{C-17}
\end{equation*}
$$

where $I_{x x}^{f}$, etc., are the inertias of the various components about their individual centers of gravity.
C-9

In the case of the nacelles the inertias $I_{x x}^{N L}, I_{x x}^{N R}$ are dependent on the nacelle tilt angle, $i_{N}$. These inertias are related to the inertias of the nacelle with respect to a set of nacelle-fixed axes O"xyz placed as shown in Figure 5.1. The relationships are

$$
\begin{align*}
& I_{x x}^{N}=I_{x x_{O}}^{N}+\left(I_{z z_{o}}^{N}-I_{x x_{O}}^{N}\right) \sin ^{2} i_{N}-I_{x z_{O}} \sin 2 i_{N} \\
& I_{y y}^{N}=I_{Y y_{O}}^{N}  \tag{C-18}\\
& I_{z z}^{N}=I_{z z_{o}}^{N}+\left(I_{x x_{o}}^{N}-I_{z z_{o}}^{N}\right) \sin ^{2} i_{N}+I_{x z_{o}} \sin 2 i_{N} \\
& I_{x z}^{N}=I_{x z_{o}}^{N} \cos 2 i_{N}+\frac{1}{2}\left(I_{x x_{O}}-I_{z z_{o}}\right) \sin 2 i_{N}
\end{align*}
$$

Using equations ( $\mathrm{C}-18$ ) together with $(\mathrm{C}-13),(\mathrm{C}-11)$ and ( $\mathrm{C}-12$ ), in equation ( $\mathrm{C}-17$ ), the roll inertia becomes

$$
\begin{aligned}
I_{x x}= & I_{X X}^{f}+I_{X X}^{W}+2 I_{x x_{O}}^{N}+\left(I_{z Z_{O}}^{N}-I_{X x_{O}}^{N}\right)\left(\sin ^{2} i_{N L}+\sin ^{2} i_{N R}\right) \\
& -I_{x z_{O}}^{N}\left(\sin 2 i_{N L}+\sin 2 i_{N R}\right)+2 m_{N} Y_{N}^{2}+m_{f} h_{f} Z_{f} \\
& +m_{W} h_{W} z_{W}-m_{f} Z_{f} z_{C G}^{\prime}-m_{W} z_{w} z_{C G}^{\prime} \\
& -m_{N} z_{N L} Z_{C G}^{\prime}-m_{N} z_{N R} Z_{C G}^{\prime} \\
& -\ell m_{N}\left[z_{N R} \sin \left(i_{N R}-\lambda\right)+z_{N L} \sin \left(i_{N L}-\lambda\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
= & I_{x x}^{f}+I_{x X}^{W}+2 I_{x x_{0}}^{N}+\left(I_{z z_{0}}^{N}-I_{x x_{0}}^{N}\right)\left(\sin ^{2} i_{N L}+\sin ^{2} i_{N R}\right) \\
& -I_{x z_{0}}^{N}\left(\sin 2 i_{N L}+\sin 2 i_{N R}\right)+2 m_{N} Y_{N}^{2}+m_{f} h_{f} Z_{f} \\
& +m_{W} h_{w} z_{w}-\ell m_{N}\left[Z_{N R} \sin \left(i_{N R}-\lambda\right)+z_{N L} \sin \left(i_{N L}-\lambda\right)\right]
\end{aligned}
$$

since the terms containing $\mathrm{Z}_{\mathrm{CG}}^{\prime}$ sum to zero.

Similarly

$$
\begin{aligned}
& I_{x z}=I_{x z}^{f}+I_{x z}^{W}+I_{x z}^{N}\left(\cos 2 i_{N L}+\cos 2 i_{N R}\right) \\
& +\frac{1}{2}\left(I_{X x_{0}}^{N}-I_{z Z_{O}}^{N}\right)\left(\sin 2 i_{N L}+\sin 2 i_{N R}\right)+m_{f}^{\ell} f_{f}^{Z} \\
& +m_{w} z_{w}{ }_{w}+\ell m_{N}\left[z_{N R} \cos \left(i_{N R}-\lambda\right)+z_{N L} \cos \left(i_{N L}-\lambda\right)\right] \\
& \left(I_{Z Z}-I_{Y Y}\right)=I_{Z Z}^{f}-I_{Y Y}^{f}+I_{Z Z}^{W}-I_{Y Y}^{W}+2\left(I_{Z Z_{O}}^{N}-I_{Y Y_{O}}^{N}\right) \\
& +\left(I_{x x_{0}}^{N}-I_{z Z_{0}}^{N}\right)\left(\sin ^{2} i_{N L}+\sin ^{2} i_{N R}\right)+I_{x Z_{0}}^{N}\left(\sin 2 i_{N L}+\sin 2 i_{N R}\right) \\
& -\left(m_{f} h_{f} Z_{f}+m_{w} h_{w} Z_{w}\right)+m_{N} \ell\left[z_{N L} \sin \left(i_{N L}-\lambda\right)\right. \\
& \left.+z_{N R} \sin \left(i_{N R}-\lambda\right)\right]+2 \mathrm{~m}_{\mathrm{N}} \mathrm{y}_{\mathrm{N}}^{2}
\end{aligned}
$$

Similar expressions are obtained for $I_{y y}$ and $I_{z z}$ and these are presented in Appendix E.

## APPENDIX D - CALCULATION OF SLIPSTREAM-IMMERSED WING AREAS

The wing areas washed by the rotor slipstreams are required in the calculation of wing lift and drag. These immersed areas depend on rotor shaft inclination, wing angle of attack and sideslip, and rotor thrust. The equations presented in Appendix $E$ for the immersed areas $S_{i_{L}}$ and $S_{i_{R}}$ were obtained as follows.


The above sketch shows a rotor under conditions of combined angle of attack ( $\alpha_{\text {T.L. }}$ ) and sideslip ( $\beta$ ). The resultant angle of attack of the shaft is given by

$$
\begin{equation*}
\alpha_{R}=\cos ^{-1}\left(\cos \alpha_{T . L .} \cos \beta\right) \tag{D-1}
\end{equation*}
$$

If the rotor shaft is inclined to the fuselage centerline at angle $i_{N}$ and the fuselage is at angle of attack $\alpha_{f}$ then

$$
\begin{equation*}
\alpha_{T \cdot L}=\alpha_{f}+i_{N} \tag{D-2}
\end{equation*}
$$

$$
D-1
$$

The rotor "sideslip" angle, $\zeta$, is defined by

$$
\begin{equation*}
\zeta=\operatorname{Tan}^{-1}\left[\frac{\operatorname{Tan} \beta}{\operatorname{Sin} \alpha_{T . L}}\right] \tag{D-3}
\end{equation*}
$$

and is the angle shown in the sketch.

Figure D. 1 presents four views of the geometry of rotor slipstream/wing planform interaction.

Figure D.l[a] is a view of the plane taken through the rotor shaft parallel to the aircraft vertical plane of symmetry. The line PT is the wing chord, the distances $P C$ and $h_{p}$ are the horizontal and vertical coordinates of the pivot measured from the wing leading edge, and $\ell$ is the spinner-to-pivot shaft length.

Figure D.l[b] is a view taken normal to the rotor disc plane. In this view, the traces of the slipstream on planes taken through the wing leading and trailing edges parallel to the disc plane appear as circles. This assumes that the slipstream is a sheared circular cylinder.

Figure D.l[c] is a section taken in the plane containing the rotor shaft and the freestream velocity vector $\mathrm{V}_{\infty}$. The angle $\varepsilon$ is the deflection of the slipstream relative to the freestream direction. Planes are taken through the wing leading and trailing edges parallel to the rotor disc. These intersect the rotor shaftline at the points $O$ and $T$, and intersect the slipstream centerline at the points $O^{\prime}$ and $O^{\prime \prime}$. These points enable the slipstream traces shown in (b) to be constructed.

Figure (D, l[d]) is a view taken perpendicular to the wing surface showing the areas washed by the slipstream. For convenience this view combines the immersed areas of both left and right wings. In general, the imprint of the slipstream on the wing will be bounded in the chordwise direction by curved lines; however, the approximation is made that these lines are straight.

The immersed area of the right wing panel is (assuming that the tip is immersed),

$$
\begin{align*}
S_{i_{R}} & =\frac{1}{2}(P M+T N) C \\
& =\frac{1}{2}(P R+R M+T S+S N) C \tag{D-4}
\end{align*}
$$

From Figure D. l[b] $\quad P R=00^{\prime} \sin \zeta$
From Figure D, l[c] $00^{\prime}=(\ell-O D) \operatorname{Tan}\left(\alpha_{R}-\varepsilon\right)$
From Figure D.l[a] $O D=P C \cos \left(i_{N}-i_{W}\right)-h_{p} \sin \left(i_{N}-i_{W}\right)(D-7)$
From Figure $D .1[b] \quad R M=R^{\prime} M^{\prime}=\sqrt{\frac{D_{S}^{2}}{4}-O^{\prime} R^{\prime 2}}$
From Figure D.l[b] $O^{\prime} R^{\prime}=0 O^{\prime} \cos \zeta+O P$ (D-9)
From Figure $D .1[a] \quad O P=P C \sin \left(i_{N^{-}} i_{W}\right)+h_{p} \cos \left(i_{N}{ }^{-} i_{W}\right)(D-10)$

These equations define the leading edge intersection PM. If RM is zero or negative, the slipstream does not intersect the leading edge and the wing is considered to be unaffected by the slipstream.

For the trailing edge intersection, $T N:$

$$
\begin{equation*}
\mathrm{TS}=00^{\prime \prime} \sin \zeta \tag{D-1l}
\end{equation*}
$$



$$
\begin{array}{ll}
O O^{\prime}=\left(\ell+c \cos \left(i_{N^{\prime}} i_{W}\right)-O D\right) \operatorname{Tan}\left(\alpha_{R}-\varepsilon\right) & (D-12) \\
S N=S^{\prime} N^{\prime}=\frac{D_{S}^{2}}{4}-O " S^{\prime 2} & (D-13) \\
O S^{\prime}=O O " \cos \zeta+T T^{\prime} & (D-14) \\
T T^{\prime}=O P-c \sin \left(i_{N}-i_{W}\right) & (D-15) \tag{D-15}
\end{array}
$$

If we write

$$
\xi_{1}=\mathrm{PR}, \xi_{2}=\mathrm{RM}, \xi_{3}=\mathrm{TS}, \text { and } \xi_{4}=\mathrm{SN}
$$

then, using the above equations,

$$
\xi_{1}=\left[\ell-P C \cos \left(i_{N}-i_{W}\right)+h_{p} \sin \left(i_{N}-i_{W}\right)\right] \operatorname{Tan}\left(\alpha_{R}-\varepsilon\right) \sin \xi(D-16)
$$

and

$$
\xi_{2}=\sqrt{\frac{D_{S}^{2}}{4} \frac{-\left\{\left[\ell-P C \cos \left(i_{N}-i_{W}\right)+h_{P} \sin \left(i_{N}-i_{W}\right)\right] \operatorname{Tan}\left(\alpha_{R}-\varepsilon\right) \cos \zeta\right.}{\left.+P C \sin \left(i_{N}-i_{W}\right)+h_{p} \cos \left(i_{N}-i_{W}\right)\right\}}}
$$

The corresponding equations for $\xi_{3}$ and $\xi_{4}$ are obtained by replacing $P C$ in ( $D-16$ ) and ( $D-17$ ) with ( $P C-C$ )

Thus the immersed area of the right wing panel is given

$$
\begin{equation*}
\text { by } \quad s_{i_{R}}=\frac{1}{2} c\left(\xi_{1}+\xi_{2}+\xi_{3}+\xi_{4}\right) \tag{D-18}
\end{equation*}
$$

From the symmetry of Figure $D .1(d), S N=B S$ and $R M=A R$. The total immersed area of both wing panels is

$$
S_{i_{T}}=\frac{1}{2} \quad c(A M+B N)=\frac{1}{2} c\left(2 \xi_{2}+2 \xi_{4}\right)=C\left(\xi_{2}+\xi_{4}\right) \quad(D-19)
$$

and therefore the $1 m m e r s e d$ area of the lett wing $1 s$ obtained from

$$
\begin{equation*}
s_{i_{L}}=s_{i_{T}}-s_{i_{R}} \tag{D-20}
\end{equation*}
$$

The above equations correspond to those presented in Appendix E for calculating immersed wing area.

$$
D-5
$$

## APPENDIX E COMPUTER REPRESENTATION

The equations derived in previous sections of this report have been collected and written in a format to facilitate computer programming. The complete set of equations which define the Model 222 simulation mathematical model are contained in this section. The computer block diagram for the simulation is also included. Each element of this block diagram contains an index number. Figure $E .1$ lists the index number, the name of the element, and its page number in this appendix. In addition the input and output, where appropriate, to each element are identified by their index numbers.

| INDEX NUMBE' | BLOCK DIAGKAIM ELEIMENT NAMES | PAGE |
| :---: | :---: | :---: |
| 1. | Control Mixing, Load Alleviation System | E-4 |
|  | and Actuator Dynamics | E-7 |
| 2. | Stability Augmentation System | E-7 |
| 3. | Density Calculation | E-9 |
| 4. | Engines and Thrust Management System | E-10 |
| 5. | Rotor Control Coordinate Axis Transforms | E-14 |
| 6. | Center of Gravity Calculation | E-15 |
| 7. | Aerodynamic Coordinate Transforms | E-16 |
| 8. | Wing Equations (Including Interference) | E-19 |
| 9. | Wing A.C. to Elastic Axis Transform | E-33 |
| 10. | Wing Force and Moment Resolution to Center of Gravity | E-34 |
| 11. | Horizontal and Vertical Tail Aerodynamics (Including Interference) | E-35 |
| 12. | Tail Force and Moment Resolution to Center of Gravity | E-43 |
| 13. | Nacelle Aerodynamics | E-44 |
| 14. | Landing Gear Equations | E-46 |
| 15. | Fuselage Aerodynamics | E-49 |
| 16. | Fuselage Force and Moment Resolution to Center of Gravity (Includes Landing Gear) | E-50 |
| 17. | Wing/Rotor Interference | E-51 |
| 18. | Rotor/Kotor Interference | E-52 |
| 19. | Rotor Aero Input Equations | E-53 |
| 20. | Rotor Equations | E-54 |
| 21. | Rotor Force and Moment Resolution | E-61 |
| 22. | Wing Vertical Bending | E-64 |
| 23. | Wing Torsion Total Force and Moment Summation About | E-66 |
| 24. | Center of Gravity | E-68 |
| 25. 26. | Basic Equations of Motion | E-77 |
| 27. | Aircraft Condition Calculation and Ground Track | E-78 |
| 28. | Gust Model | E-80 |
| 29. | Preliminary Calculation (Preprocess) | E-81 |
| 30. | Trim Loops | E-87 |

Figure E.1. Block Diagram Element Index Numbers

$$
E-2
$$




E-4
Model 222 Load alleviation system (las.)


Mode 222
Nacelle, flap, flaperon, \& spoiler controls


LONGITUDINAL SAS


outpur: $\delta, \theta, \sigma, a, M, p$




ENG 1 SUBROUTNE, FLOW CHART


Rotor Control Coordinate $A_{\text {xis }}$ Transform

LEFT

$$
\begin{aligned}
& A_{1 C L}^{\prime \prime}=A_{1 C i}^{\prime} \cos \phi_{p}+B_{1 C L}^{\prime} \sin \phi_{p} \\
& B_{1 C L}^{\prime \prime}=-A_{1 C L}^{\prime} \sin \phi_{p}+B_{1 C L}^{\prime} \cos \phi_{p} \\
& A_{1 C L}=A_{1 C L}^{\prime \prime} \cos \xi_{H L}-B_{1 C L}^{\prime \prime} \sin \xi_{H L} \\
& B_{1 C L}=A_{1 C L}^{\prime \prime} \sin \xi_{H L}+B_{1 C L}^{\prime \prime} \cos \xi_{H L}
\end{aligned}
$$

RIGHT

$$
\begin{aligned}
& A_{1 C R}^{\prime \prime}=A_{1 C R}^{\prime} \cos \phi_{P}+B_{1 C R}^{\prime} \sin \phi_{P} \\
& B_{1 C R}^{\prime \prime}=-A_{1 C R}^{\prime} \sin \phi_{P}+B_{1 C R}^{\prime} \cos \phi_{P} \\
& A_{1 C R}=+A_{1 C R}^{\prime \prime} \cos \xi_{H R}+B_{1 C R}^{\prime \prime} \sin \zeta_{H R} \\
& B_{1 C R}=-A_{1 C R}^{\prime \prime} \sin \xi_{H R}+B_{1 C R}^{\prime \prime} \cos \zeta_{H R}
\end{aligned}
$$

Rotor Side slip Angle

$$
\begin{aligned}
& \zeta_{H R}=\tan ^{-1} \frac{v_{R R}}{w_{R R}+\epsilon_{w R R} U_{R R}} \\
& \zeta_{H L}=\tan ^{-1} \frac{v_{R L}}{\omega_{R L}+\epsilon_{W R L} U_{R L}}
\end{aligned}
$$

FORM THE $\sin t \cos$ of $\xi_{H R} \not \xi_{H L}$

Note: $\phi_{p}$ is the control phase angle. $\phi_{p}$ is positive FOR THE CONTROL AXIS MOVED OPpOSITE TO ROTOR ROTATION.

CENTER OF Gadity Calculation
C.G. Location w, r. t. Pivot

$$
\begin{aligned}
& x_{c G}=\frac{m_{f} l_{f}+m_{w} h_{w}}{m}+\rho\left(\frac{m_{N}}{m}\right)\left[\cos \left(i_{N L}-\lambda\right)+\cos \left(i_{N R}-\lambda\right)\right] \\
& Z_{c G}=\frac{m p h_{f}+m_{\omega} h_{w}}{m}-\rho\left(\frac{m_{N}}{m}\right)\left[\sin \left(i_{N L}-\lambda\right)+\sin \left(i_{N R}-\lambda\right)\right]
\end{aligned}
$$

CG. Velocitr w.r.t Pruot

$$
\begin{aligned}
& \dot{x}_{C G}=-\rho \frac{m_{N}}{m}\left[i_{N C} \sin \left(i_{N L}-\lambda\right)+i_{N R} \sin \left(i_{N R}-\lambda\right)\right] \\
& \dot{z}_{C G}=-\rho \frac{m_{N}}{m}\left[i_{N L} \cos \left(i_{N L}-\lambda\right)+i_{N R} \cos \left(i_{N R}-\lambda\right)\right]
\end{aligned}
$$

C. G. Acceleration w.r.t. Pivot

$$
\begin{aligned}
\ddot{x}_{C G}=-\rho \frac{m_{N}}{m} & {\left[\ddot{i}_{N L} \sin \left(i_{N L}-\lambda\right)+i_{N L}^{2} \cos \left(i_{N L}-\lambda\right)\right.} \\
& \left.+\ddot{i}_{N R} \sin \left(i_{N R}-\lambda\right)+i_{N R}^{2} \cos \left(i_{N R}-\lambda\right)\right] \\
\ddot{z}_{C G}=-\Omega \frac{m_{N}}{m} & {\left[\ddot{i}_{N L} \cos \left(i_{N L}-\lambda\right)-\ddot{i}_{N L}^{2} \sin \left(i_{N L}-\lambda\right)\right.} \\
& \left.+\ddot{i}_{N R} \cos \left(i_{N R}-\lambda\right)-\dot{i}_{N R}^{2} \sin \left(i_{N R}-\lambda\right)\right]
\end{aligned}
$$

Fuselage Pivet Velocity

$$
\begin{aligned}
& u_{p}=u-z_{c G} g-\dot{x}_{c G} \\
& v_{p}=v+z_{c G} p-x_{c G} r \\
& w_{p}=w+x_{c G} g-\dot{z}_{c G}
\end{aligned}
$$

AERODYNAMIC COORDINATE TRANSFORM

LEFT WING A.C. VELOCITY - BODY AXES

$$
\begin{aligned}
& U_{L W}^{\prime}=u_{p}+z_{W A C} g+y_{W A C} r+q h_{I L W A C} \\
& V_{L W}^{\prime}=v_{p}+x_{W A C} r-z_{W A C} p-p h_{I C W A C} \\
& W_{L W}^{\prime}=W_{p}-Y_{W A C} p-x_{W A C} q+\dot{h}_{I L W A C}
\end{aligned}
$$

Right Wing A.C. Velocity - Body Axes

$$
\begin{aligned}
& u_{R W}^{\prime}=u_{p}+z_{W A C} g-Y_{W A C} r+q h_{\text {IRWAC }} \\
& v_{R W}^{\prime}=v_{p}+x_{W A C} r-z_{W A C} p-p h_{\text {IRWAC }} \\
& w_{R \omega}^{\prime}=w_{p}+Y_{W A C} p-x_{W A C} q+h_{\text {IRWAC }}
\end{aligned}
$$

LEFT ROTOR HUB VELOCITY - BODY AXES

$$
\begin{aligned}
& u_{R L}^{\prime}=u_{p}+r Y_{N}-L_{S} \sin i_{N L}\left(i_{N L}+q\right)+g h_{1 L} \\
& V_{R L}^{\prime}=v_{P}+L_{S}\left(r \cos i_{N C}+p \sin i_{N C}\right)-p h_{1 L} \\
& W_{R L}^{\prime}=w_{P}-p Y_{N}-L_{S}\left(i_{N L}+g\right) \cos i_{N L}+\dot{h}_{1 L}
\end{aligned}
$$

Right Rotor Hub Velocity - Body Axes

$$
\begin{aligned}
& u_{R R}^{\prime}=u_{p}-r Y_{N}-L_{S} \sin i_{N R}\left(i_{N R}+q\right)+q h_{1 R} \\
& v_{R R}^{\prime}=v_{p}+L_{S}\left(r \cos i_{N R}+p \sin i_{N R}\right)-p h_{1 R} \\
& w_{R R}^{\prime}=w_{p}+p Y_{N}-L_{S}\left(i_{N R}+q\right) \cos i_{N R}+i_{1 R}^{0}
\end{aligned}
$$

Aerodynamic Coordinate Transform (Conto)
left Rotor Hub Velocity - Shaft axes

$$
\begin{aligned}
& u_{R L}=u_{R L}^{\prime} \cos i_{N L}-\omega_{R L}^{\prime} \sin i_{N L} \\
& v_{R L}=v_{R L}^{\prime} \\
& \omega_{R L}=u_{R L}^{\prime} \sin i_{N L}+\omega_{R L}^{\prime} \cos i_{N L}
\end{aligned}
$$

Right Rotor Hub Velocitik - shaft axes

$$
\begin{aligned}
u_{R R} & =U_{R R}^{\prime} \cos i_{N R}-\omega_{R R}^{\prime} \sin i_{N R} \\
V_{R R} & =\nu_{R R}^{\prime} \\
\omega_{R R} & =U_{R R}^{\prime} \sin i_{N R}+\omega_{R R}^{\prime} \cos i_{N R}
\end{aligned}
$$

LEFT Wing A.C. VELOCITY - Chord AXES

$$
\begin{aligned}
& u_{L w}=u_{L w}^{\prime} \cos i w-w_{L w}^{\prime} \sin i_{w} \\
& v_{L w}=v_{L w}^{\prime} \\
& w_{L w}=u_{L w}^{\prime} \sin i w+w_{L w}^{\prime} \cos i w
\end{aligned}
$$

Right Wing A.C. Velocity - Chord Axes

$$
\begin{aligned}
& u_{R W}=u_{R W}^{\prime} \cos i_{\omega}-\omega_{R \omega}^{\prime} \sin i_{w} \\
& \nu_{R \omega}=\nu_{R W}^{\prime} \\
& \omega_{R W}=u_{R \omega}^{\prime} \sin i_{w}+\omega_{R \omega}^{\prime} \cos i_{w}
\end{aligned}
$$

Aeroornamic Cooroinate Transform (Cont'd.)

Horizontal Stagmizer A.C. Velocity

$$
\begin{aligned}
& u_{H T}=u_{\rho}+z_{H T} q \\
& v_{H T}=v_{P}+x_{H T} r-z_{H T} \rho \\
& w_{H T}=w_{P}-x_{H T} g
\end{aligned}
$$

VERTICAL FIN A.C. VELOCITY

$$
\begin{aligned}
& u_{V T}=u_{p}+z_{V T} g \\
& w_{V T}=v_{\rho}+x_{V T r}-z_{V T} \rho \\
& w_{V T}=w_{p}-x_{V T} q
\end{aligned}
$$

Wing Equations

$$
\begin{aligned}
& \tau_{R R}=\alpha_{R R}+\operatorname{Tan}^{-1}\left(\frac{N F_{R}}{T_{R}}\right) \\
& R_{R R}=\sqrt{T_{R}^{2}+N F_{R}^{2}+S F_{R}^{2}} \\
& V_{A_{R}}=\frac{V_{R R}}{\sqrt{\frac{1 R_{R R} \mid+10}{2 \rho A}}} \\
& v_{\#_{R}}^{4}+2 v_{\#_{R}} v_{*_{R}}^{3} \cos \tau_{R R}+v_{A_{R}}^{2} V_{\#_{R}}^{2}=1 \quad \text { (solve for } v_{k} \text { ) } \\
& \epsilon_{P_{R R}}=\operatorname{Tan}^{-1} \quad \frac{v_{x_{R}} \sin \tau_{R R}}{V_{A_{R}}+v_{\#_{R}} \cos \tau_{R R}} \\
& C_{T S R R}=\frac{\cos \left(\tau_{R R}-\alpha_{R R}\right)}{\cos \left(\tau_{R R}-\alpha_{R R}\right)+\frac{V_{R R}}{4}} \\
& \tau_{L R}=\alpha_{L R}+\operatorname{Tan}^{-1}\left(\frac{N F_{L}}{T_{L}}\right) \\
& R_{L R}=\sqrt{T_{L}^{2}+N F_{L}^{2}+S \cdot F_{L}^{2}} \\
& V_{*_{L}}=\frac{V_{L R}}{\sqrt{\frac{\left|R_{L R}\right|+10}{2 \rho A}}} \\
& v_{*_{L}}^{4}+2 V_{* L} v_{*_{L L}}^{3} \cos \tau_{L R}+v_{* L}^{2} V_{A L}^{2}=1 \text { (solve for } v_{* L} \text { ) } \\
& \epsilon_{P_{L R}}=\operatorname{Tan}^{-1} \frac{\tau_{N_{L}} \sin \tau_{L R}}{V_{A_{L}}+v_{L_{L}} \cos \tau_{L R}} \\
& C_{T S R}=\frac{\cos \left(\tau_{L R}-\alpha_{L R}\right)}{\cos \left(\tau_{L R}-\alpha_{L R}\right)+\frac{V_{A L}^{2}}{4}} \\
& \text { E-19 }
\end{aligned}
$$

Wing Equations (Cont'd.)

$$
\begin{aligned}
& \bar{\xi}=\left(\xi_{H R}+\zeta_{H E}\right) .5 \\
& \bar{\alpha}_{R}=\left(\alpha_{R R}+\alpha_{L R}\right) .5 \\
& \left.\begin{array}{l}
\bar{\epsilon}_{P}=\left(\epsilon_{P R R}+\epsilon_{P_{L R}}\right) .5 \\
\bar{C}_{T S}=\left(C_{T S R R}+C_{T S L R}\right) .5
\end{array}\right\} \text { USed in Tail Aerodymemics } \\
& \left.\begin{array}{l}
\bar{i}_{N}=\left(i_{N L}+i_{N R}\right) \cdot 5 \\
C_{L W}=\frac{\left(C_{L \text { RN }}+C_{L S W W}\right)}{\left(1-C_{T S}\right)} \cdot 5
\end{array}\right\} U_{\text {sem }} \text { in } W_{\text {ing }} / \text { Rotor Interfere. } \\
& \xi_{R 1}=\left[L_{S}-P C \cos \left(\bar{i}_{N}-i_{\omega}\right)+h_{P} \sin \left(\bar{i}_{N}-i_{\omega}\right)\right] \tan \left(\bar{\alpha}_{R}-\bar{\epsilon}_{P}\right) \sin \bar{\xi} \\
& \xi_{R 2}=\sqrt{\frac{D^{2}}{4}-\left\{\left[L_{S}-P C \cos \left(i_{N}-i_{\omega}\right)+h_{p} \sin \left(\bar{i}_{N}-i_{\omega}\right)\right] \tan \left(\bar{\alpha}_{R}-\bar{\epsilon}_{p}\right)_{\cos \bar{\xi}}\right.} \\
& \left.+P C \sin \left(\bar{i}_{N}-i_{\omega}\right)+h_{P} \cos \left(i_{N}-i_{\omega}\right)\right\}^{2} \\
& I F: \xi_{R z}=0 \text { or } I_{\text {imaginary, }} S_{\text {yew }}=0 \text { and } S_{\text {Sig }}=0 \text {, } \\
& \text { also }\left(\frac{C_{\text {wat }}}{C_{\alpha \alpha}}\right)_{R w}=0 \text { and }\left(\frac{C_{k+i}}{c_{t \alpha}}\right)_{L w}=0.0
\end{aligned}
$$

Form $\xi_{R 3} B y$ Replacing $P_{N}$ in $\xi_{R 1}$ Equation WITH $\left(P C-C_{W}\right){ }^{S_{R 1}}$

Form $\xi_{R 4}$ By replacing $\begin{gathered}P C \\ \text { with }\left(P C-C_{w}\right)\end{gathered} \xi_{\text {Rr }}$ equation
IF: $\xi_{R 4}=0$ or Imaginary; $S_{i R w}=0$ and $S_{i z w}=0$, also $\left(\frac{C_{L w i}}{C_{L \alpha}}\right)_{R w}=C_{0} 0$ and $\left(\frac{C_{L \alpha i}}{\left.C_{L \alpha}\right)_{L W}}=0.0\right.$

IF: Unsreclas open; SET $C_{L \omega}=0.0$
UMBRELLA LOGIC:

SET UMBRELLAS ${ }^{\prime N}$ CLOSED ${ }^{2}$ (Hysteresis Fin $\pm, 0$; gr $\pm .1 \frac{L B}{F T}$ )

Wing Equations (Cont'd)

$$
\begin{aligned}
& s_{i_{2 j}}=\frac{C_{w}}{2}\left[\xi_{R 1}+\xi_{R 2}+\xi_{R 3}+\xi_{R 4}\right] \\
& \left(\frac{S_{i}}{S}\right)_{R w}=2\left(\frac{S i R}{S w}\right) \\
& S_{i_{r}}=c_{w}\left[\xi_{R 2}+\xi_{R 4}\right] \\
& S_{i_{\omega}} \text { " } S_{i_{T}}-S_{i_{R}} \\
& \left(\frac{S_{i}}{S}\right)_{\omega w}=2\left(\frac{S_{i L}}{S_{w}}\right) \\
& \left(R_{i}\right)_{L \omega}=\left(\frac{S_{i_{L}}}{C_{\omega}{ }^{2}}\right) \\
& \left(R_{i}\right)_{R \omega}=\left(\frac{S_{i_{R}}}{C_{\omega}^{2}}\right) \\
& R_{\omega}=\frac{S_{w}}{C_{\omega}^{2}} \text { (PRORIHNDRV) } \\
& \left(\frac{C_{L \alpha i}}{C_{L \alpha}}\right)_{L \omega}=\frac{1}{1+\frac{C_{L \alpha_{\omega}}}{\pi}\left[\frac{1}{\left(\mathcal{R}_{i}\right)_{L \omega}}-\frac{1}{R_{w}}\right]} \\
& \left(\frac{C_{L \alpha i}}{C_{L \alpha}}\right)_{R \omega}=\frac{1}{1+\frac{C_{L \alpha}}{\pi}\left[\frac{1}{\left(R_{i}\right)_{R \omega}}-\frac{1}{R_{\omega}}\right]}
\end{aligned}
$$

Wing Equations (continuod)

$$
\begin{aligned}
& \bar{q}_{S}=\left[\frac{1}{2} \rho\left(u^{2}+v^{2}+w^{2}\right)+\frac{\left(T_{L}+T_{R}\right) \cdot 5}{A}\right] \\
& q_{s_{R N}}=\left[\frac{1}{2} \rho\left(u_{R \omega}^{2}+v_{R \Delta}^{2}+w_{R \omega}^{2}\right)+\frac{T_{R}}{A}\right]
\end{aligned}
$$

$$
q_{s L w}=\left[\frac{1}{2} \rho\left(U_{w}^{2}+1+w_{L w}^{2}\right)+\frac{T_{c}}{A}\right]
$$

Wing Equations (Cont'd)

Wing Angle of Attack ano sidesuip

$$
\begin{aligned}
& \alpha_{L \omega_{0}}=\sin ^{-1}\left[\frac{\omega_{L \omega}}{\sqrt{{U_{L \omega}}^{2}+\omega_{L \omega}^{2}}}\right]+\theta_{\text {tUWAC }} \\
& \alpha_{R W 0}=\sin ^{-1}\left[\frac{\omega_{R \omega}}{\sqrt{U_{R \omega}^{2}+\omega_{R \omega}^{2}}}\right]+\theta_{\text {tewac }} \\
& \beta_{L \omega 0}=\sin ^{-1}\left[\frac{v_{L \omega}}{\sqrt{u_{L \omega}^{2}+v_{L \omega}^{2}+\omega_{L \omega}^{2}}}\right] \\
& \beta_{R w_{0}}=\sin ^{-1}\left[\frac{v_{k w}}{\sqrt{U_{R \omega}^{2}+v_{R \omega}^{2}+w_{k w}^{2}}}\right] \\
& \alpha_{L W S S_{0}}=\alpha_{L W O}-\epsilon_{\text {PLR }} \\
& \alpha_{\text {RWSSO }}=\alpha_{\text {RWO }}-\epsilon_{\text {PRR }} \\
& \bar{\alpha}_{\omega}=\left(\alpha_{L w_{0}}+\alpha_{R w_{0}}\right) .5 \\
& \alpha_{L \omega \text { rigid }}=\sin ^{-1}\left[\frac{\omega_{L \omega}}{\sqrt{U_{L \omega}^{2}+\omega_{L \omega}^{2}}}\right]-\epsilon_{\text {PLR }} \\
& \alpha_{R W \text { rigid }}=\sin ^{-1}\left[\frac{W_{R W}}{\sqrt{U_{R W}^{2}+W_{R W}^{2}}}\right]-\epsilon_{P R R} \\
& \alpha_{L w o}^{\prime}=\alpha_{L w o}-i_{w}-\theta_{\text {tiwac }} \\
& \alpha_{R W_{0}}^{\prime}=\alpha_{R W_{0}}-i_{w}-\theta_{\text {tRWAC }}
\end{aligned}
$$

notr: if $\alpha_{L \omega_{S S_{0}}}$ or $\alpha_{\text {RUSSO }} \geq \alpha_{\text {Max }}$; Print out stall warning

Wing Equations (contio)
CALLULATION OF INCRETHANTAL LIFT, DRAG AND MOMANT COEFFICIANTS
Calculate:

$$
\begin{aligned}
& C_{L L W_{O}}=C_{L} @ \alpha=\alpha_{L \omega_{S S_{d}}}, \quad \delta=\delta_{a_{L \omega}}+\delta_{f}, \quad \delta_{S P}=\delta_{S P_{L}} \\
& C_{L R W_{0}}=C_{L} @ \alpha=\alpha_{R w_{S S O}}, \delta=\delta_{a_{R w}}+\delta_{f}, \delta_{S P}=\delta_{S P R} \\
& C_{L L w_{0}}^{ \pm}=C_{L} @ \alpha=\alpha_{L w_{0}}, \delta=\delta_{a_{L W}}+\delta_{f}, \quad \delta_{S P}=\delta_{S P_{L}} \\
& C_{L R \omega_{0}}=C_{L} @ \alpha=\alpha_{R \omega_{0}}, \quad \delta=\delta_{a_{R O}} \delta \delta_{\rho}, \delta_{S P}=\delta_{S P_{R}} \\
& C_{L}=C_{L} \text { e } \alpha_{f}+i_{\omega}, \quad \delta=\delta_{f}
\end{aligned}
$$

Calculats:

$$
\begin{array}{rlrl}
\alpha_{\nu, L}^{+} & =14.6^{\circ}-0.12 & \delta & \left(0^{\circ} \leq \delta \leq 40^{\circ}\right) \\
& =9.8^{\circ} & \left(\delta>40^{\circ}\right) \\
\alpha_{\nu . c .}^{-} & =-16.7^{\circ}-.1138 \delta & \left(0^{\circ} \leq \delta \leq 40^{\circ}\right) \\
& =-21.25^{\circ} & \left(\delta>40^{\circ}\right)
\end{array}
$$

If: $\quad \alpha_{0, L}^{-} \leq \alpha \leq \alpha_{0, L}^{+}$

$$
C_{L}=0.134+C_{U_{W}} \alpha+\Delta C_{L \delta}+F \Delta C_{L S P}
$$

WHARE:

$$
\begin{array}{rlrl}
\Delta C_{L \delta} & =.0269 \delta & & \left(0^{0} \leq \delta \leq 22.22900^{\circ}\right) \\
& =-2.437137+.20607 \delta-.003128 \delta^{2} & \left(22.22906^{\circ}<\delta \leq 29.706^{\circ}\right) \\
& =.442180+.0263 \delta-.000338 \delta^{2} & \left(\delta>29.786^{\circ}\right) \\
\Delta C_{L S P} & =-0.01132 \delta_{S P} & \left(\delta^{\circ} \leq \delta_{S P} \leq 30^{\circ}\right) \\
& =.076-.018666 \delta_{S P}+.00016 \delta_{S P}^{2} & & \left(\delta_{S P}>300\right) \\
F & =1.003412+.011163 \delta+.002168 \delta^{2} & \left(0^{\circ} \leq \delta \leq 20.1665^{\circ}\right) \\
F=-.756323+.185684 \delta-.002159 \delta^{2} & \left(\delta>20.1665^{\circ}\right)
\end{array}
$$

WING EQUATIONS (CONTO)
IF: $\quad \alpha_{N . L}^{+}<\alpha \leq \alpha_{N . L}^{+}+8.534^{\circ}$
CALCULATE: $\quad C_{L N . L}^{\prime}=0.134+C_{L / W} \alpha_{N . L}^{+}+\Delta C_{L \delta}+F \Delta C_{L S P}$

$$
\begin{aligned}
& \alpha_{\text {DOH }}=\alpha-\alpha_{N, L}^{+}+14.6^{\circ} \\
& \Delta C_{L_{1 / L}}=-.00547619 \alpha_{\text {sur }}^{2}+.18254762 \alpha_{\text {Dung }}-1.421571513 \\
& C_{L}=C_{L N, L}^{\prime}+\Delta C_{L N, L} \text { AND PRINT STALL WaRNING }
\end{aligned}
$$

If: $-\left(\alpha_{N . L}^{+}+8.534^{\circ}\right)<\alpha<90^{\circ}$
CALCULATE! : $\quad C_{L N . L}^{\prime}=0.154+C_{L_{40}} \alpha_{\text {NV }}^{+}+\Delta C_{L \delta}+F \Delta C_{L S P}$
$C_{L}=\frac{C_{\text {Lea }}^{\prime}\left(90^{\circ}-\alpha\right)}{90^{\circ}-\left(\alpha^{+} \omega . L+8.534^{\circ}\right)}$ and PRiNt Stall Waranva.

IF: $\left(\alpha_{0.1}^{-}-8.534^{\circ}\right) \leq \alpha<\alpha_{0 . L}^{-}$
Calculate:

$$
\begin{aligned}
& C_{L N . L}^{\prime}=0.134+C_{L L_{W}} \alpha_{N, L}+\Delta C_{L}+F \Delta C_{L S P} \\
& \alpha_{D U N T}=\alpha_{N . L}^{-}-\alpha+14.60 \\
& \Delta C_{L N, L}=-.00547619 \alpha_{\text {DUI }}^{2}+0.18254762 \alpha_{\text {DUN }}-1.421571513 \\
& C_{L}=C_{L N, L}^{\prime}-\Delta C_{L D, L} \quad \text { AND PRINT STALL WARNING }
\end{aligned}
$$

If: $\quad-90^{\circ} \leq \alpha<\left(\alpha_{N .6}^{-}-8.534^{\circ}\right)$
Calculate:-

$$
\begin{aligned}
& C_{L N . L}^{\prime}=0.134+C_{L N_{N}} \alpha_{N, L}^{-}+\Delta C_{L \delta}+F \Delta C_{L S P} \\
& C_{L}=\frac{C_{L N . L}^{\prime}\left(90^{\circ}+\alpha\right)}{90^{\circ}+\alpha_{N . L}^{-}-8.534^{\circ}} \quad \text { ANO PRINT STOL U UsperNG }
\end{aligned}
$$



Wing Equations (Contio)

Calculate:-

$$
\begin{aligned}
& C_{D_{L W}}=C_{D} e \alpha=\alpha_{L \omega S S O}, \delta=\delta_{f}+\delta_{a_{L W}}, \delta_{S P}=\delta_{S P_{L}} \\
& C_{D_{R W}}=C_{D} \text { e } \alpha=\alpha_{\text {RwSSo }}, \delta=\delta_{f}+\delta_{a p w}, \delta_{S P}=\delta_{S P_{R}} \\
& C_{D_{L W_{0}}}^{+1}=C_{D} \text { @ } \alpha=\alpha_{L w_{0}}, \delta=\delta_{1}+\delta_{a_{L W}}, \delta_{S P}=\delta_{S P_{L}} \\
& C_{D N_{0}}^{+\prime}=C_{D} \text { e } \alpha=\alpha_{R \omega_{0}}, \delta \cdot \delta_{p}+\delta_{a p \nu}, \delta_{S p}=\delta_{S P_{p}}
\end{aligned}
$$

AS FOLLOWS:
IF: $-20^{\circ} \leq \alpha \leq 20^{\circ}$
CALCULATE: $C_{D}=.00059250 \alpha^{2}+.002109993 \alpha+.01765$

$$
+\sum_{v=0}^{4} \sum_{u=0}^{4}\left[A_{D(u+5 v)} \delta^{u} \alpha^{v}\right]+\Delta C_{D_{S P}}
$$

UHERE: $\quad \Delta C_{D_{3 p}}=-000098784 \delta_{s p}+.000009622 \delta_{s p}^{2}$
If: $20^{\circ}<\alpha \leqslant 90^{\circ}, \quad \alpha_{\text {DUTr }}=20^{\circ}$
Caculata:

$$
\begin{aligned}
& C_{D}^{+}=0.31105749+\sum_{w=0}^{4} \sum_{u=0}^{4}\left[A_{D(u+5 w)} \delta^{u} \alpha_{D 0 \sim}^{w}\right]+\Delta C_{D S P} \\
& C_{D}=C_{D}^{+}+\frac{\left(1-C_{D}^{+}\right)}{70^{\circ}}\left(\alpha-20^{\circ}\right)
\end{aligned}
$$

IF: $-90^{\circ} \leq \alpha<-20^{\circ}, \alpha_{00 n}=-20^{\circ}$
C ALCUATE $: C_{D}^{-}=0.19124389+\sum_{w=0}^{4} \sum_{u=0}^{4}\left[A_{D(u+5 w)} \delta^{u} \alpha_{D U N}^{w}\right]+\Delta C_{D_{S N}}$

$$
C_{D}=C_{0}^{-}-\frac{\left(1-C_{D}^{-}\right)}{70^{\circ}}\left(\alpha+20^{\circ}\right)
$$



WING EqUATIONS (CONTD)

Calculate:

$$
\begin{aligned}
& C_{M L \omega}=C_{M} \& \alpha=\alpha_{L \omega_{s s_{0}}}, \quad \delta=\delta_{\rho}+\delta_{a_{c \omega}} \\
& C_{\text {Haw }}=C_{M} e \alpha \cdot \alpha_{\text {Russo }}, \delta=\delta_{f}+\delta_{\text {Re }} \\
& C_{M L \omega_{0}}^{\prime}=C_{M} \otimes \alpha=\alpha_{L \omega_{0}}, \quad \delta=\delta_{f}+\delta_{a L \omega} \\
& C_{M_{R W} W_{j}}^{\prime}=C_{M} Q \alpha=\alpha_{R \omega_{0}}, \quad \delta=\delta_{f}+\delta_{a p \omega}
\end{aligned}
$$

As fours:
If:- $-20^{\circ} \leqslant \alpha \leqslant 20^{\circ}$
Calculate:- $\quad C M^{\prime}=-030117-.0003162 \alpha$

$$
\begin{aligned}
& \Delta C_{M_{\delta}}=.0000778 \delta^{2}-.010033 \delta \quad\left(0^{0} \leq \delta \leq 45^{\circ}\right) \\
& \Delta C_{M_{\delta}}=.0000322 \delta^{2}-.0049045 \delta-.1384272 \quad\left(\delta 745^{\circ}\right) \\
& C_{M}=C_{M}^{\prime}+\Delta C_{H \delta}
\end{aligned}
$$

IF: $\quad \alpha>20^{\circ}$
Calculate: $\quad C_{M}^{\prime}=-036441+\Delta C_{M 8}$

$$
C_{M}=C_{n}^{\prime} \frac{\left(90^{\circ}-\alpha\right)}{70^{\circ}}
$$

IF $\alpha<-20^{\circ}$
Calculate $C_{n}^{\prime}=-.023793+\Delta C_{M_{\delta}}$

$$
C_{m}=C_{m}^{\prime} \frac{\left(90^{\circ}+\alpha\right)}{70^{\circ}}
$$

NOTE: $\alpha, \alpha_{\text {DUI }}, \delta, \delta_{a_{R W}}, \delta_{a_{\text {LI }}}, \delta_{\text {SP }}, \delta_{S R}$ INDEGREAS

WING EQUATIONS (CONT'D)

Calculate: -

$$
\begin{aligned}
& C_{L \omega}^{\prime \prime \prime}=C_{L L W} \quad ; \quad C_{D_{L \omega}}^{\prime \prime \prime}=C_{D_{L W}} ; C_{\text {HLN }}^{\prime \prime}=C_{\text {HLN }} \\
& C_{L \text { RW }}^{\prime \prime \prime}=C_{L_{R W O}} ; C_{D_{R W}^{\prime \prime}}^{\prime \prime}=C_{D_{R W}} ; C_{\text {HRN }}^{\prime \prime}=C_{\text {MRW }} \\
& C_{L \text { WWMAX }}^{\prime \prime}=C_{L \text { max }}+\Delta C_{L_{\delta}}+\Delta C_{L \text { sP }} \\
& C_{L \text { RW MAX }}^{\prime \prime}=C_{L_{\text {nAX }}}+\Delta C_{L_{\delta}}+\Delta C_{L_{S P}} \\
& C_{L R N}^{\prime \prime}=C_{L_{R W_{0}}^{*}}^{*} \quad ; \quad C_{D R V}^{* \prime}=C_{D_{R N O}}^{\prime \prime} ; C_{M A W}^{*}=C_{M A N_{O}}^{\prime} \\
& C_{L L}^{*}=C_{L L \omega_{0}}^{*} \quad ; C_{D L N}^{*}=C_{D L \omega_{0}}^{*_{L N}^{\prime}} ; C_{M L N}^{*}=C_{M_{L N}}^{*^{\prime}} \\
& C_{L L_{\text {max }}}^{*}=C_{L_{\text {wimax }}^{\prime \prime}}^{\prime \prime} \\
& C_{L_{\text {awnax }}^{+}}^{+}=C_{L_{\text {andax }}^{\prime \prime}}^{\prime \prime} \\
& \bar{C}_{L}=\frac{C_{c_{0}}\left(\frac{a_{a t}}{a}\right)_{w}}{\sqrt{1-M^{2}}} \\
& C_{L L W}^{\prime \prime \prime \sigma \sigma}=C_{L L \omega}^{\prime \prime \prime} \frac{\left(\frac{a_{3}}{a}\right)_{\omega}}{\sqrt{1-M^{2}}} ; \quad C_{L R \omega}^{\prime \prime \prime \sigma \sigma}=C_{L}^{\prime \prime \prime} \quad \frac{\left(\frac{a_{3}}{a}\right)_{\omega}}{\sqrt{1-M^{2}}} ; \\
& C_{L_{L \omega}}^{\nabla^{I G G}}=C_{L_{L W}^{*}}^{*^{\prime}} \frac{\left(\frac{a_{t}}{a}\right)_{\omega}}{\sqrt{1-M^{2}}} ; \quad C_{L_{R \omega}}^{*^{I G E}}=C_{L_{R \omega}}^{\phi^{\prime}} \frac{\left(\frac{a_{9}}{a}\right)}{\sqrt{1-M^{2}}}
\end{aligned}
$$

Wing Equations (Cont'd)

$$
\begin{aligned}
& \text { IF: } C_{L R W}^{\text {IGE }} \geq C_{\text {CRWMAX }}^{\prime \prime} ; \operatorname{set} \Delta C_{\text {DRW }}^{\text {IGE }}=0.0 \nleftarrow C_{L \text { REW }}^{\text {IGE }}=C_{\text {LEWMax }}^{\prime \prime}
\end{aligned}
$$

$$
\begin{aligned}
& \text { IF: } C_{L R W}^{\prime \text { GE }} \geq C_{L \text { EWMAX }}^{\star} ; \operatorname{ser} \Delta C_{D R W}^{\text {TGF }}=0.0 \& C_{L R W}^{\text {TGE }}=C_{L R W M A X}^{\star} \\
& \text { IF: }(\mathrm{ag} / \mathrm{a})>1.0 \text {; sET } K g=-1.0 \\
& \left(a_{9} / a\right) \leqslant 1.0 ; \operatorname{set} K_{99}=+1.0
\end{aligned}
$$

Calcurate

$$
\begin{aligned}
& C_{L L W}^{\prime \prime}=C_{L L W}^{\text {TGF }} \\
& C_{D L W}^{\prime \prime}=C_{D I W}^{\prime \prime \prime}+\Delta C_{D L W}^{I G E} \\
& C_{L \text { ew }}^{\prime \prime}=C_{\text {Rew }}^{\text {IGE }} \\
& C_{D R W}^{\prime \prime}=C_{\text {DRW }}^{\prime \prime \prime}+\Delta C_{\text {DRW }}^{\text {IGE }} \\
& C_{\text {LIW }}^{\star}=C_{\text {LiW }}^{\text {IGG }} \\
& C_{D W}^{\star}=C_{D L W}^{\star 1}+\Delta C_{D W}^{T S F} \\
& C_{\text {Lew }}^{*}=C_{\text {Lew }}^{\text {TGTE }} \\
& C_{\text {DRW }}^{\star}=C_{\text {DRW }}^{\nrightarrow \prime}+\Delta C_{D R W}^{\stackrel{T G F}{J}}
\end{aligned}
$$

Wing Equations (cont'd.)

$$
\begin{aligned}
& C_{L S L W}=K_{A L}^{\prime}\left\{\left(\frac{S_{i}}{S}\right)_{L W}\left(C_{L L W}^{\prime \prime} \cos \epsilon_{P L R}-C_{D L W}^{\prime \prime} \sin \epsilon_{P L R}\right)+C_{L L W}^{*}\left(1-C_{T S L R}\right)\left[1-\left(\frac{S_{i}}{S}\right)_{L W}\right]\right\} \\
& C_{L S R W}=K_{A_{R}}^{\prime}\left\{\left(\frac{S_{i}}{S}\right)_{R W}\left(C_{L R W}^{\prime \prime} \cos E_{P R R}-C_{D R W}^{\prime \prime} \sin E_{P R R}\right)+C_{L R W}^{*}\left(1-C_{T S R R}\right)\left[1-\left(\frac{S_{i}}{S}\right)_{R W}\right]\right\} \\
& C_{D S L W}=K_{A_{L}}^{\prime}\left\{\left(\frac{S_{i}}{S}\right)_{L W}\left(C_{L L W}^{\prime \prime} \sin \epsilon_{P_{L R}}+C_{D L W}^{\prime \prime} \cos \epsilon_{P L R}\right)+C_{D L W}^{*}\left(1-C_{T S L R}\right)\left[1-\left(\frac{S_{i}}{S}\right)_{W W}\right]\right\} \\
& C_{D S R N}=K_{A R}^{\prime}\left\{\left(\frac{S_{i}}{S}\right)_{R W}\left(C_{L R I N}^{\prime \prime} \sin \epsilon_{P R R}+C_{D R W}^{\prime \prime} \cos \epsilon_{P R R R}\right)+C_{D R W}^{*}\left(1-C_{\text {TSRR }}\right)\left[1-\left(\frac{S_{i}}{S}\right)_{R W}\right]\right\} \\
& C_{M S L W}=K_{A L}^{\prime}\left\{\left(\frac{S_{i}}{S}\right)_{L W}\left(C_{M L W}^{\prime \prime}\right)+C_{M L W}^{*}\left(1-C_{T S L R}\right)\left[1-\left(\frac{S_{i}}{S}\right)_{L W}\right]\right\} \\
& C_{\text {MSRW }}=K_{A R}^{\prime}\left\{\left(\frac{S_{i}}{S}\right)_{R W}\left(C_{\text {MRW }}^{\prime \prime}\right)+C_{\text {MRW }}^{*}\left(1-C_{\text {TSRR }}\right)\left[1-\left(\frac{S_{i}}{S}\right)_{R W}\right]\right\}
\end{aligned}
$$

Wing Equations (cont'd.)

$$
\begin{aligned}
& \Delta C_{d s \text { powse }}=\frac{1}{4}\left\{\left[C_{L s t w}-\left(1-\bar{C}_{T S}\right) C_{L L w}^{ \pm}\right]\left[1-\frac{1}{2}\left(\frac{S_{i}}{5}\right)_{w w}\right]\right. \\
& \left.-\left[C_{\text {LsRW }}-\left(1-\bar{C}_{T s}\right) C_{\text {Lew }}^{*}\right]\left[1-\frac{1}{2}\left(\frac{s_{i}}{s_{2}}\right)_{\text {Rw }}\right]\right\} \\
& \Delta C_{V_{\text {S Powere }}}=\frac{1}{4}\left\{\left[C_{\text {DSRW }}-\left(1-\bar{C}_{T S}\right) C_{\text {DRW }}^{*}\right]\left[1-\frac{1}{2}\left(\frac{S_{i}}{S}\right)_{R W}\right]\right. \\
& \left.-\left[C_{D S L W}-\left(1-\bar{C}_{T s}\right) C_{D L W}^{*}\right]\left[1-\frac{1}{2}\left(\frac{S_{S}}{S}\right)_{L W}\right]\right\} \\
& C_{Z s w}=\left(K_{20}+K_{2}, \bar{C}_{L}\right)\left(1-\bar{C}_{T s}\right) \beta_{f}+\left(\frac{1-\bar{C}_{T s}}{2 b_{w}}\right)\left(K_{2}\right)\left(c_{L L W}^{\star}-C_{\text {ew }}^{*}\right) \bar{Y}_{y c} \\
& +\Delta C_{\text {ysponge }}
\end{aligned}
$$

$$
\begin{aligned}
& \left.-\left[C_{L R W}^{*} \operatorname{Sin}\left(\alpha_{R \omega_{O}}-i_{w}\right)+C_{L L}^{*} \operatorname{SiN}\left(i_{\omega}-\alpha_{L \omega_{O}}\right)\right]\right\} \bar{Y}_{A C} \\
& +\Delta C_{\text {YSPOWER }}
\end{aligned}
$$

Wing EqUATIONS

SPECIAL CONDITIONS (FOR UMBRELLAS OPEN)
IF: Umargllas CLosfo; Go through wing equations
IF: Umbrigas open ; Calculate the wing forces and moments as follows:

$$
\begin{aligned}
& x_{A E E O}^{\angle W}=\quad f e_{u} q_{S \angle W}\left(1-C_{T S \angle R}\right)\left[\frac{-U_{L W}}{\left|u_{L W}\right|+.1}\right] \\
& X_{\text {zERO }}^{\text {oW }}=f e_{u} q_{\operatorname{sew}}\left(1-C_{\text {ToR }}\right)\left[\frac{-u_{R W}}{\left|u_{R W}\right|+.1}\right] \\
& Y_{\text {AERO }}^{L W}=0.0 \\
& Y_{\text {AGR }}^{R W}=0.0 \\
& \left.\begin{array}{l}
Z_{\text {AERO }}^{\text {tWI }}=T_{L}\left(\frac{D}{T}\right)_{L} \\
Z_{\text {AERO }}^{\text {tWI }}=T_{R}\left(\frac{D}{T}\right)_{R}
\end{array}\right\} \text { GO TO WING BENDING }
\end{aligned}
$$

$$
\begin{aligned}
& M_{\text {AERO }}^{\omega}=0.0
\end{aligned}
$$

$$
\begin{aligned}
& \phi\left(\frac{M}{T}\right)_{R}=K_{\frac{M}{T}}\left[\frac{h}{D}\right]_{E R F}^{2}+K_{\frac{M^{2}}{T}}\left[\frac{h}{D}\right]_{E R F}+K_{M_{T}}{ }^{\frac{T}{T}} \\
& \text { IF: }\left[\frac{h}{D}\right]_{E_{R} F_{R}}>1.3 ;\left(\frac{D}{T}\right)_{L}=K_{\frac{D^{4}}{T}} ; \notin\left(\frac{M}{T}\right)_{L}=K_{M_{T}^{4}} \\
& \text { IF: }\left[\frac{h}{D}\right]_{R R}^{E F F} \leq 1.3 ;\left(\frac{D}{T}\right)_{R}=K_{\frac{D_{1}}{T}}\left[\frac{h}{D}\right]_{R R}^{2}+K_{\frac{D 2}{T}}\left[\frac{h}{D}\right]_{E F F}+K_{\frac{D^{3}}{T}} \\
& \left.\phi\left(\frac{M}{T}\right)_{R}=K_{\frac{M 1}{T}}\left[\frac{h}{D}\right]_{R R}^{2}+K_{M_{T}^{2}}\left[\frac{h}{D}\right]_{R R}\right]_{R F}+K_{M^{3}}{ }^{3}
\end{aligned}
$$

Wing A.C. to Elastic $A_{x i s}$ Transform

PITCHING MOMENT

$$
\begin{aligned}
M_{A R R O}^{R W}= & C_{\text {MSRW }} q_{\text {SEW }} \frac{S_{W}}{2} C_{W}-X_{W A C} Z_{A E R O}^{R W} \\
& +Z_{W A C} X_{\text {AERO }}^{R W} \\
M_{A E R O}^{L W}= & C_{\text {MSLW }} q_{S L W} \frac{S_{W}}{2} C_{W}-X_{W A C} Z_{\text {AERO }}^{L W} \\
& +Z_{W A C} X_{\text {AERO }}^{\angle W}
\end{aligned}
$$

Vertical Forces

$$
\begin{aligned}
& Z_{\text {AERO }}^{\text {Rn }}=\left[-C_{\text {LEW }}-C_{\text {SEW }} \alpha_{\text {eco }}^{\prime}\right] q_{\text {sew }} \frac{S_{w}}{2} \\
& Z_{\text {AFRO }}^{\angle W /}=\left[-C_{L S L W}-C_{\text {DSLW }} \alpha_{L W O}^{\prime}\right] q_{\text {LW }} \frac{S_{W}}{z}
\end{aligned}
$$

NOTE: $Z_{\text {AFRO }}^{R W '}$ \& $Z_{\text {AERO }}^{\text {LW' }}$ ARE USED IN VERTICAL BENDING EQ.'S

Wing Force $\ddagger$ Moment Resolution - Body Axes @C.G.

$$
\begin{aligned}
& X_{A E R_{0}}^{L W}=\left[-C_{D S L W}+C_{L S L W} \alpha_{L W O}^{\prime}\right] q_{S L W} \frac{S_{W}}{2} \\
& X_{\text {AERO }}^{\text {oW }}=\left[-C_{\text {SEW }}+C_{\text {Lsein }} \alpha_{\text {iwo }}^{\prime}\right] q_{\text {sen }} \frac{S_{w}}{2} \\
& Y_{A E R O}^{L W}=\left[-C_{\text {PSLW }} \beta_{L W 0}\right] q_{s \angle W} \frac{S_{w}}{2} \\
& Y_{\text {AREs }}^{E W}=\left[-C_{\text {SEW }} \beta_{\text {pto }}\right] q_{\text {sen }} \frac{S_{w}}{2} \\
& \left.\begin{array}{l}
Z_{\text {AERO }}^{L W} \\
Z_{\text {AERO }}^{R W}
\end{array}\right\} \text { FROM VERTICAL BENDING } \\
& \mathcal{L}_{\text {AERO }}^{\omega}=C_{z S W} \bar{q}_{s} s_{\omega} b_{\omega} \\
& M_{\text {AFRO }}^{W}=M_{A E R O}^{L W}+M_{\text {AERO }}^{R W}+X_{C G}\left(Z_{\text {AFRO }}^{L W}+Z_{\text {AERO }}^{R W}\right) \\
& -Z_{C G}\left(X_{A F R O}^{2 W}+X_{A E R_{0}}^{R W}\right) \\
& \eta_{\text {AERO }}^{\omega}=C_{\text {SW }} \bar{q}_{s} S_{\omega} b_{\omega}
\end{aligned}
$$

Note: Observe Wing Equation Special Conditions

HORIZONTAL AND VERTICAL TAIL AERODYNAMICS

WING AND TAIL ALTITUDE - GROUND EFFECT

$$
\begin{aligned}
& h_{w C / 4}=-z_{D a N}+\left(x_{\omega M}-x_{C G}\right) \sin \theta+\left(z_{C G}-z_{\omega M C}\right) \cos \theta \\
& h_{T_{C / 4}}=-z_{D o w}+\left(x_{H T}-x_{C G}\right) \sin \theta+\left(z_{C G}-z_{W T}\right) \cos \theta
\end{aligned}
$$

Horizontal Tail Angle Of Attack

$$
\begin{aligned}
& \ell_{A C}=x_{\omega_{B C}}-x_{H T} \quad\left(\rho_{R E U H I I N A R Y}\right) \\
& G E F=\left[b_{\omega}^{2}+4\left(h_{T C / 4}-h_{\omega_{C / /}}\right)^{2}\right] /\left[b_{\omega}^{2}+4\left(h_{T C / 4}+h_{\omega_{C / /}}\right)^{2}\right]
\end{aligned}
$$

IF: $\quad \bar{\epsilon}_{p} \frac{(1-G E F)}{\sqrt{1-M^{2}}}>\left[\epsilon_{0}+\frac{d \epsilon}{d \alpha}\left(\bar{\alpha}_{\omega}-\operatorname{loc} \frac{\dot{\omega}}{u^{2}}\right)\right] \frac{(1-G \epsilon F)}{\sqrt{1-M^{2}}}$

$$
\epsilon=\frac{\bar{\epsilon}_{P}\left(1-\sigma_{F F}\right)}{\sqrt{1-M^{2}}}
$$

IF:- $\frac{\bar{\epsilon}_{p}(1-G E F)}{\sqrt{1-M^{2}}}<\left[\epsilon_{0}+\frac{d \epsilon}{d \alpha}\left(\bar{Q}_{\omega}-\operatorname{lnc} \frac{\dot{\omega}}{u^{2}}\right)\right] \frac{(1-G E P)}{\sqrt{1-M^{2}}}$

$$
\epsilon=\left[\epsilon_{0}+\frac{d \epsilon}{d \alpha}\left(\bar{Q}_{w}-\operatorname{lnc} \frac{\dot{\omega}}{u^{2}}\right)\right] \frac{(1-G \angle F)}{\sqrt{1-M^{2}}}
$$


WHORE $\epsilon_{0}=f\left(\delta_{L} \delta_{\alpha}\right) .5$ AND $\frac{d \epsilon}{d \alpha}=f\left(\delta_{L}+\delta_{R}\right) .5$
IF: $\left|\alpha_{H T}\right|>180^{\circ} ; \underbrace{\alpha_{H T}=-\left(\operatorname{sinN} \alpha_{H T}\right) 360^{\circ}+\alpha_{H T}}_{\text {USE ONLY IN EQ. FOR FORE E AND }}$ Monowr conf.

HORIZONTAL AND UARTICAL TAIL ALRODUNAMICS (CONTO)

HORIzONTAL TAFIL LIET ANO DRAG.

$$
\begin{aligned}
& \alpha_{e_{H T}}=\alpha_{H T}+\tau_{H T} \delta_{e} \\
& \hat{\alpha}_{\omega_{+}}=\left(\alpha_{H T_{\text {smi }}}-2^{\circ}\right)+\tau_{N T} \delta_{e} \\
& \hat{\alpha}_{\text {ir }}^{-}=-\left(\alpha_{H T_{\text {stmi }}}-2^{\circ}\right)+\tau_{\text {mr }} \delta e \\
& C_{L \alpha}=C_{L \alpha \mu T}\left(\frac{a_{q}}{a}\right)_{M_{T}} / \sqrt{1-M^{2}} \\
& \text { WHARE }\left(\frac{a_{I}}{a}\right)_{H T}=f\left(h_{T / / 4}\right) ; \quad L_{H T}=f\left(\mathrm{se}_{\mathrm{S}} / \mathrm{s}_{T}\right)
\end{aligned}
$$

IF:- $\quad \hat{\alpha}_{N I_{-}} \leq \alpha_{e n T} \leq \hat{\alpha}_{H r_{+}}$

$$
\begin{aligned}
& C_{L_{H T}}=C_{L \alpha} \alpha_{e_{W T}} \\
& C_{D_{W T}}=C_{D_{O_{H T}}}+\frac{2 C_{L N T}^{2}}{\pi / R_{H T}}
\end{aligned}
$$

IF:- $\quad \hat{\alpha}_{N r_{+}}<\alpha_{e_{H T}} \leq 90^{\circ}$

$$
\begin{aligned}
& C_{L H T}=C_{L \alpha} \hat{\alpha}_{H T_{+}}\left[\frac{90^{\circ}-\alpha_{\text {OHT }}}{90^{\circ}-\hat{\alpha}_{H T_{+}}}\right] \\
& C_{L_{H r_{\text {GTHLL }}}}=C_{L \alpha} \hat{\alpha}_{H T_{+}}
\end{aligned}
$$

Horizontal and vertical tail aerodinatics (CONT'D)

HORIZONTML TAIL LIPT AND DRAC (CONTID)

If:-

$$
\begin{aligned}
& 90^{\circ}<\alpha_{e_{H T}} \leqslant\left(180^{\circ}-.5 \hat{\alpha}_{W T_{-}}\right) \\
& C_{L \mu T}=.5 C_{L \alpha} \hat{\alpha}_{\mu T}-\frac{\left(\alpha_{e_{\mu T}}-90^{\circ}\right)}{\left(90^{\circ}-. \hat{\alpha}_{\omega T_{-}}\right)} \\
& C_{L \text { HITML }}=.5 C_{L \alpha} \hat{Q}_{H_{T_{-}}} \\
& C_{D_{H T} T_{\text {STML }}}=\frac{2 C_{L A T \text { STML }}^{2}}{\pi R_{\text {ITT }}}+C D_{D_{H T}}
\end{aligned}
$$

IF:- $\left(180^{\circ}-.5 \hat{\alpha}_{H T_{-}}\right) \leqslant \alpha_{e_{N T}} \leqslant 180^{\circ}$

$$
\begin{aligned}
& C_{L_{H T}}=C_{L \alpha}\left(\alpha_{\left.e_{H T}-180^{\circ}\right)}\right. \\
& C_{D_{H T}}=C_{D_{O H T}}+\frac{2 C_{H T T}^{2}}{\pi A_{H T}}
\end{aligned}
$$

If:-

$$
\begin{aligned}
& -90^{\circ} \leq \alpha_{e_{H T}}<\hat{\alpha}_{H T_{-}} \\
& C_{L H T}=C_{L \alpha} \hat{\alpha}_{N T_{-}} \frac{\left(-90^{\circ}-\alpha_{N_{N H}}\right)}{\left(-90^{\circ}-\hat{\alpha}_{N T_{-}}\right)} \\
& C_{L_{\text {MTITALL }}}=C_{L_{\psi}} \hat{\alpha}_{\text {HT_ }} \\
& C_{D_{\text {WTITOLL }}}=C_{D_{\text {OHT }}}+\frac{2 C_{L_{\text {WTSTOL }}}^{2}}{\pi R_{\text {HT }}} \\
& C_{D_{H T}}=C_{D_{H T \text { STMLL }}}+\frac{\left(\alpha_{e_{H T}}-\hat{\alpha}_{W T}\right)\left(1.1-C_{D_{N T S T A M}}\right)}{\left(-90^{\circ}-\hat{\alpha}_{W T_{-}}\right)}
\end{aligned}
$$

HORIZONTAL AND VERTICAL TAIL AERODYNAMICS (CONTD)

HORIZONTAL TAIL LIFT AND DRAG: (CONTD)

IF:-

$$
\begin{aligned}
& -\left(-180^{\circ}+5 \hat{\alpha}_{m_{H}}\right)<\alpha_{e_{H T}}<-90^{\circ} \\
& C_{L H T}=.5 C_{L \alpha} \alpha \hat{\alpha}_{H T_{H}} \frac{\left(\alpha_{q+}+90^{\circ}\right)}{\left(-90^{\circ}+.5 \hat{\alpha}_{N T_{+}}\right)}
\end{aligned}
$$

$$
C_{L H T_{S} T H L}=.5 C_{L+} \hat{\alpha}_{H T_{+}}
$$

$$
C_{D_{\text {HTSTMCL }}}=C_{D_{O H T}}+\frac{2 C_{L W T S T M L}^{2}}{\pi R_{H T}}
$$

$$
C_{D_{H T}}=C_{D_{H T S T B U}}-\frac{\left(\alpha_{e_{H T}+180^{\circ}-.5 \hat{\alpha}_{H T_{+}}}\left(1.1-C_{Q_{N T S T O L}}\right)\right.}{\left(.5 \hat{\alpha}_{H T_{+}}-90^{\circ}\right)}
$$

IF:- $\quad-180^{\circ} \leqslant \alpha_{e_{H T}}<\left(-180^{\circ}+.5 \hat{\psi}_{H T_{+}}\right)$

$$
\begin{aligned}
& C_{L H T}=C_{L H}\left(\alpha_{e_{H T}+180^{\circ}}\right) \\
& C_{D_{H T}}=C_{D_{O H T}}+\frac{2 C_{L H T}^{2}}{\pi R_{H T}}
\end{aligned}
$$

HORIZONTAL AND VERTICAL TAIL ALREDOYNAMICS (CONTD)
VERTICAL TAIL AERODYNAMICS
VERTICAL TAIL ANGLE OF ATTREM AND SIDESLIP

$$
\begin{aligned}
\beta_{V T}= & \operatorname{TAN}^{-1} \frac{v_{V T}}{\sqrt{U_{U T}^{2}+w_{L}^{2}}} \\
\alpha_{V T} & =-\beta_{U T}+\beta_{f}\left(\frac{d \sigma}{d B}\right) \quad\left\{\begin{array}{c}
\text { NOTE: THIS VALUE OF NUT IS USED IN RESOLUTION } \\
\text { OF FORCES AND MOMENTS }
\end{array}\right\}
\end{aligned}
$$


$\hat{\alpha}_{V T_{+}}=\left(\alpha_{V T_{\text {STOL }}}-2^{\circ}\right)+Z_{V T} \delta_{R U D}$
$\hat{\alpha}_{V T_{-}}=-\left(\alpha_{V T_{\text {STAR }}} 2^{\circ}\right)+Z_{U T} \delta_{R V O}$

$$
C_{y_{\alpha}}=C_{y_{\alpha u T}} / \sqrt{1-M^{2}}
$$

TAIL DYNAMIC PRESSURE AND SIDAWMSH

$$
\begin{aligned}
& \bar{q}=\frac{\rho}{2}\left(u^{2}+v^{2}+w^{2}\right) \\
& \sigma=\frac{d \sigma}{d \beta} \beta_{F}
\end{aligned}
$$

VERTICAL TAIL LIFT AND DRAG
IF:- $\hat{\alpha}_{v T_{-}} \leq \alpha_{e v T} \leq \hat{\alpha}_{v T_{+}}$

$$
\begin{aligned}
& C_{Y_{U T}}=C_{Y_{\alpha}} \alpha_{Q U T} \\
& C_{D_{U T}}=C_{D_{O U T}}+\frac{2 C_{Y U T}^{2}}{\pi / R_{U T}}
\end{aligned}
$$

HORIZONTAL AND VERTICAL TAIL ALVRDYNAMICS (CONT'O)
VERTICAL TAIL LIPT AND DRAG (CONTID)

IF:-

$$
\begin{aligned}
& \hat{\alpha}_{u T_{t}}<\alpha_{e u r} \leqslant 90^{\circ} \\
& C_{Y_{U T}}=C_{Y \alpha} \quad \hat{X}_{\nu T_{r}}\left[\frac{90^{\circ}-\alpha_{Q U T}}{90^{\circ}-\hat{\alpha}_{U T H}}\right] \\
& C_{\text {yutstanc }}=C_{Y_{\alpha}} \hat{\alpha}_{\text {ur }_{+}} \\
& C_{D_{\text {ut smal }}}=C_{D_{0 u r}}+\frac{2 C_{y_{\text {urstronl }}^{2}}^{2}}{\pi R u t} \\
& C_{D_{V T}}=C_{D_{u T} \text { mau }}+\frac{\left(\alpha_{\text {Qur }}-\hat{\alpha}_{v 7+} \chi\left(1.1-C_{D_{u r} \text { moun }}\right)\right.}{90^{\circ}-\hat{\alpha}_{u r_{+}}}
\end{aligned}
$$

If:- $\quad 90^{\circ}<\alpha_{\text {evt }} \leqslant\left(180^{\circ} \div 5 \hat{\alpha}_{\nu T-}\right)$

$$
\begin{aligned}
& c_{y \nu T}=.5 c_{\gamma_{\alpha}} \hat{\alpha}_{\nu T_{-}}\left(\frac{\alpha_{e \nu r}-90^{\circ}}{90^{\circ}-5 \hat{\alpha}_{\nu r_{-}}}\right) \\
& C_{\text {Yutstou }}=.5 C_{\text {Yo }} \hat{\alpha}_{\text {ur_ }} \\
& C_{D_{\text {utstion }}}=C_{D_{\text {our }}}+\frac{2 C_{Y_{v T \text { Trac }}}^{2}}{\pi R_{\nu T}} \\
& C_{D_{u T}}=C_{\text {PuTstrou }}+\frac{\left(\alpha_{\text {eur }}+.5 \hat{\alpha}_{u T_{-}}-180^{\circ}\right)\left(1.1-C_{\text {Dut Smok }}\right)}{\left(.5 \hat{\alpha}_{u T_{-}}-90^{\circ}\right)}
\end{aligned}
$$

If: - $\left(180^{\circ}-.5 \hat{\alpha}_{\nu r_{1}}\right) \leq \alpha_{e_{\nu T}} \leq 180^{\circ}$

$$
\begin{aligned}
& C_{Y U T}=C_{Y /}\left(\alpha_{Q U T}-180^{\circ}\right) \\
& C_{D_{U T}}=C_{D_{0 U T}}+\frac{2 C_{Y U T}^{2}}{\pi R_{U T}}
\end{aligned}
$$

HORIZONTAL AND UERTICAL TAIL AETODNNAMIGS (COUT'D)

VERTICAL THIL LIFT AND DRAG (COWT'D)

IF:- $-90^{\circ} \leq \alpha_{e_{u r}}<\hat{\alpha}_{\nu r_{-}}$

$$
\begin{aligned}
& C_{y \nu T}=C_{y \alpha} \hat{\alpha}_{\nu T_{-}} \frac{\left(-90^{\circ}-\alpha_{\ell \nu T}\right)}{\left(-90^{\circ}-\hat{\alpha}_{\nu T-}\right)} \\
& C_{\text {YuTsmou }}=C_{\text {Y̌ }_{\alpha}} \hat{\chi}_{\text {uT_ }}
\end{aligned}
$$

$$
\begin{aligned}
& C_{D_{V T}}=C_{D_{V T \text { TTAL }}}+\frac{\left(\alpha_{\text {evr }}-\hat{\alpha}_{\nu r-}\right)\left(1.1-C_{\text {Dutsmac }}\right)}{\left(-90^{\circ}-\hat{\alpha}_{\nu T-}\right)}
\end{aligned}
$$

If:- $\quad\left(-180^{\circ}+.5 \hat{\chi}_{\nu r_{r}}\right)<\alpha_{\text {eur }}<-90^{\circ}$

$$
\begin{aligned}
& C_{\text {YOT }}=.5 C_{Y \alpha} \hat{\alpha}_{U T_{t}} \frac{\left(\alpha_{u n t}+90^{\circ}\right)}{\left(-90^{\circ}+.5 \hat{\alpha}_{\nu T_{+}}\right)} \\
& C_{\text {yut stan }}=.5 C_{y \alpha} \hat{\alpha}_{\nu i_{+}}
\end{aligned}
$$

$$
\begin{aligned}
& C_{D_{V T}}=C_{D_{\text {VT STOLL }}}-\frac{\left(\alpha_{e_{V T}}+180^{\circ}-5 \hat{\alpha}_{\nu T_{+}} X_{1.1}-C_{D V T \text { STIOL }}\right)}{\left(.5 \hat{\alpha}_{\nu T_{+}}-90^{\circ}\right)}
\end{aligned}
$$

If:- $\quad-1800 \leqslant \alpha_{\nu r}<\left(-180^{\circ}+.5 \vec{\alpha}_{u r_{+}}\right)$

$$
\begin{aligned}
& C_{y_{U r}}=C_{Y \alpha}\left(\alpha_{e \nu r}+180^{\circ}\right) \\
& C_{D_{\nu r}}=C_{D_{0 V r}}+\frac{2 C_{Y U T}^{2}}{\pi Q_{\nu T}}
\end{aligned}
$$

HORIZONTAL AND UERTICAL TANL AAEXODYNAMICS (CONTID)

TAIL ÉQUATIONS LOGIC

HORIZONTAL TAIL

1. IF $h \omega_{C / 4}>100$ FT., SET GEF $=0.0$
2. IF THE UMGRURLAS OPAN ; SET $\epsilon=\frac{\bar{G}_{P}(1-G L P)}{\sqrt{1-M^{2}}}$
3. If $\alpha_{e_{A T}}>\widehat{\alpha}_{\text {HTt }}$ PRINT stall warntag.
4. If $\mathcal{X e n t ~}^{4}<\hat{\mathcal{Q}}_{\mathrm{HI}}$ PRINT stoge warning

VERTICAL Thil

1. If Xeut $\rightarrow \vec{x}_{u r}$ + Priwt Stoul Mprininca
2. If $\alpha$ eut $<\vec{\alpha}_{u r}$ - PRint Stoll Wprovina

Tail Force and Moment Resolution to C.G.
Horizontal Tail nots:-If Unarcuas open and $M>0$; SET $\eta_{\text {MT }}=5 \eta_{n T}$

$$
\begin{aligned}
& X_{A E R O}^{H T}=\left[-C_{D H T} \cos \left(\alpha_{H T}-i_{H T}\right) \cos \left(\beta_{V T}-\sigma\right)+C_{L H T} \sin \left(\alpha_{H T}-i_{H T}\right)\right] \bar{q} S_{H T} \eta_{H T} \\
& Y_{A E R O}^{H T}=\left[-C_{D H T} \sin \left(\beta_{V T}-\sigma\right)\right] \bar{q} S_{H T} \eta_{H T} \\
& Z_{A E R O}^{H T}=\left[-C_{L H T} \cos \left(\alpha_{H T}-i_{H T}\right)-C_{D H T} \cos \left(\beta_{V T}-\sigma\right)_{\sin }\left(\alpha_{H T}-i_{H T}\right)\right] \bar{q} S_{H T} \eta_{H T} \\
& \mathcal{L}_{A E R O}^{H T}=-Y_{A E R O}^{H T}\left(Z_{H T}-Z_{C G}\right) \\
& M_{A E R O}^{N T}=Z_{A E R O}^{H T}\left(X_{C G}-X_{H T}\right)+X_{A E R O}^{H T}\left(Z_{H T}-Z_{C G}\right) \\
& \sum_{A E R O}^{H T}=-Y_{A E R O}^{H T}\left(X_{C G}-X_{H T}\right)
\end{aligned}
$$

VERTICAL TAIL

$$
\begin{aligned}
& X_{A E R O}^{V T}=\left[-C_{D V T} \cos \left(\beta_{V T}-\sigma\right) \cos \left(\alpha_{H T}-i_{N T}\right)-C_{V V T} \sin \left(\beta_{V T}-\sigma\right) \cos \left(\alpha_{H T}-\alpha_{N T}\right)\right] \times \\
& \bar{q} S_{V r} \#_{V T} \\
& Y_{A G R}^{V T}=\left[C_{y V T} \cos \left(\beta_{V T}-\sigma\right)-C_{D V T} \sin \left(\beta_{V T}-\sigma\right)\right] \bar{q} S_{V T} \eta_{V T} \\
& Z_{A E R O}^{v r}=\left[-C_{D V T} \cos \left(\beta_{V T}-\sigma\right) \sin \left(\alpha_{H T}-i \omega T\right)-C_{y V T} \sin \left(\beta_{V T}-\sigma\right) \sin \left(\alpha_{H T}-i{ }_{H r}\right)\right] \bar{q}_{V T} V_{V T} \\
& \mathcal{Z}_{A F R O}^{V T}=-Y_{A \in R 0}^{V T}\left(Z_{V T}-Z_{C G}\right) \\
& m_{\text {AGR }}^{V T}=Z_{A G R O}^{V T}\left(x_{C G}-x_{v T}\right)+x_{A E C_{0}}^{V T}\left(z_{V T}-Z_{C G}\right) \\
& n_{A G R 0}^{V T}=-Y_{A T R_{0}}^{V T}\left(X_{C G}-X_{V T}\right)
\end{aligned}
$$

TOTAL TAIL CONTRIBUTION

Nacelle Aerodynamics
Nacelle Angle of Attack and Sideslip

$$
\begin{array}{ll}
\alpha_{R N}=\tan ^{-1} \frac{w_{R R}}{u_{R R}} & ; q_{R N}=\frac{1}{2} \rho V_{R R}^{2} \\
\alpha_{L N}=\tan ^{-1} \frac{w_{R L}}{u_{R L}} & ; q_{L N}=\frac{1}{2} \rho V_{L R}^{2} \\
\beta_{R N}=\tan ^{-1} \frac{v_{R R}}{\sqrt{u_{R R}^{2}+w_{R R}^{2}}} \\
\beta_{L N}=\tan ^{-1} \frac{v_{R L}}{\sqrt{u_{R L}^{2}+w_{R L}^{2}}}
\end{array}
$$

Nacelle Wind Axis Force \& Moment Coff.'s

$$
\text { WHERE: } \quad C_{\text {DARN }}=C_{\text {DON }}
$$

$$
C_{\text {DOLT }}=C_{\text {DON }}
$$

$$
\begin{aligned}
& C_{L R N}=K_{32} \sin \alpha_{R N} \cos \alpha_{R N} \\
& C_{L L N}=K_{32} \sin \alpha_{L N} \cos \alpha_{L N} \\
& C_{\text {MAN }}=C_{\text {MON }}+K_{34} \sin \alpha_{n N} \cos \alpha_{N N}+K_{25}\left(\sin \alpha_{0 N} \cos \alpha_{N}\right)\left|\sin \alpha_{N N} \cos \alpha_{R N}\right| \\
& C_{M L N}=C_{M O N}+K_{34} \sin \alpha_{2 N} \cos \alpha_{L N}+K_{N O}\left(\sin \alpha_{L N} \cos \alpha_{L N}\right)\left|\sin \alpha_{L N} \cos \alpha_{2 N}\right|
\end{aligned}
$$

SPECIAL CONDITIONS

1. IF: $V_{R R}^{2} \leq 1(f t / \mathrm{sec})^{2}$; RIGHT NaCELLE AGRO $\equiv 0.0$
2. IF: $V_{R}^{2} \leq 1(f+/ \mathrm{sec})^{2} ;$ HEFT NACELLE AERO $\equiv 0.0$ /

Hoco value of $\alpha_{L N}$ / $\beta_{1 N}$

Nacelle Aerodynamics (Contd.)

Nacelle Forces a Moments - nacelle axes

$$
\begin{aligned}
& \Delta X_{R N}^{\prime}=q_{R N} S_{W}\left[-C_{D R N} \cos \alpha_{R N}+C_{L R N} \sin \alpha_{R N}-C_{Y R N} \sin \beta_{R N} \cos \alpha_{R N}\right] \frac{1}{2} \\
& \Delta Y_{R N}^{\prime}=q_{R N} S_{W}\left[C_{Y R N} \cos \beta_{R N}-C_{D R N} \sin \beta_{R N}\right] \frac{1}{2} \\
& \Delta Z_{R N}^{\prime}=q_{R N} S_{W}\left[-C_{R R N} \cos \alpha_{R N}-C_{D R N} \cos \beta_{R N} \sin \alpha_{R N}-C_{Y R N} \sin \beta_{R N} \sin \alpha_{R N}\right] \frac{1}{2}
\end{aligned}
$$

RIGHT

$$
\Delta \mathscr{L}_{R N}^{\prime}=q_{R N} s_{w} b_{w}\left[-\left(\frac{C_{W}}{b_{N}}\right) C_{M R N} \sin \beta_{R N} \cos \alpha_{R N}-C_{N R N} \sin \alpha_{R N}\right] \frac{1}{2}
$$

$$
\Delta m_{R N}^{\prime}=f_{R N} S_{w} c_{w}\left[c_{M R N} \cos \beta_{R N}\right] \frac{1}{2}
$$

$$
\Delta \eta_{R N}^{\prime}=q_{R N} s_{w} b_{w}\left[c_{N R N} \cos \alpha_{R N}-\left(\frac{c_{w}}{b_{w}}\right) C_{M R N} \sin \beta_{R N} \cos \alpha_{R N}\right] \frac{1}{2}
$$

$$
\Delta X_{L N}^{\prime}=q_{L N} S_{W}\left[-C_{P L N} \cos \alpha_{L N}+C_{L I N} \sin \alpha_{L N}-C_{Y L N} \sin \beta_{L N} \cos \alpha_{L N}\right] \frac{1}{2}
$$

$$
\Delta Y_{L N}^{\prime}=q_{L N} S_{W}\left[C_{Y L N} \cos \beta_{L N}-C_{D C N} \sin \beta_{L N}\right] \frac{1}{2}
$$

LEFT

$$
\Delta Z_{L N}^{\prime}=q_{2 N} S_{W}\left[-C_{L L N} \cos \alpha_{L N}-C_{D L N} \cos \beta_{L N} \sin \alpha_{L N}-C_{Y L N} \sin \beta_{L N} \sin \alpha_{L N}\right] \frac{1}{2}
$$

$$
\begin{aligned}
& \left.\Delta \mathscr{z}_{L N}^{\prime}=q_{L N} s_{W} b-\left(\frac{c_{W}}{b_{W}}\right) c_{M L N} \sin \beta_{L N} \cos \alpha_{L N}-c_{N L N} \sin \alpha_{L N}\right] \frac{1}{2} \\
& \Delta m_{L N}^{\prime}=q_{L N} s_{W} c_{W}\left[c_{M L N} \cos \beta_{L N}\right] \frac{1}{2} \\
& \Delta n_{L N}^{\prime}=q_{L N} s_{W} b_{W}\left[c_{N L N} \cos \alpha_{L N}-\left(\frac{c_{W}}{b_{W}}\right) c_{M L N} \sin \beta_{L N} \cos \alpha_{L N}\right] \frac{1}{2}
\end{aligned}
$$

$$
\begin{aligned}
& C_{Y R N}=K_{36} \operatorname{SN} \beta_{R N} \operatorname{Cos} \beta_{N N}+K_{3 N}\left(\operatorname{Sin} \beta_{m \omega} \operatorname{Cos} \beta_{R N}\right)\left|\operatorname{Sin} \beta_{N N} \operatorname{Cos} \beta_{R N}\right| \\
& C_{Y L N}=K_{36}^{\prime} \operatorname{Sin} \beta_{\omega} \operatorname{Cos} \beta_{\omega \omega}+K_{37}^{\prime}\left(S_{N} \beta_{\omega} \operatorname{Cos} \beta_{\omega}\right)\left|\operatorname{Sin} \beta_{\omega N} \operatorname{Cos} \beta_{\omega}\right| \\
& C_{N E N}=C_{N O R N}+K_{S B} \operatorname{Sin} B_{N D} \cos \theta_{m \nu}+K_{N}\left(\operatorname{Sin} \beta_{R N} \cos \beta_{R N}\right)\left|\sin \beta_{R D} \cos \beta_{R v}\right| \\
& C_{N L N}=C_{N o i N}+K_{10} \operatorname{SiN}_{1 N} \beta_{L N} \operatorname{Cos} \beta_{L N}+K_{41}\left(\operatorname{Sin} \beta_{L D} \cos \beta_{L N}\right)\left|\sin \beta_{L N} \operatorname{Cos} \beta_{L N}\right| \\
& C_{X R N}=C_{X L N} \equiv 0.0
\end{aligned}
$$

LANDING GEAR EQUATIONS

PERFORM THE FOLLOWING CALCULATIONS FOR EACH OF THREF LANDING GEAR ie. $n=1,2,43$.

Landing GEar -A/C Location

$$
\begin{aligned}
& X_{n}=-X_{c \sigma}+X_{6 n} \\
& Y_{n}=Y_{6 n} \\
& Z_{n}=-Z_{c \sigma}+Z_{6 n}
\end{aligned}
$$

n= 1 LEFT MAIN GEAR
$n=2$ RIGHT MAIN GEAR
$n=3$ NOSE GEAR
strut Deflection

$$
\begin{aligned}
& h_{G \theta n}=X_{n} \sin \theta-Z_{n} \cos \theta-r_{n} \\
& h_{\text {Gp }}=\left[Y_{n} \sin \phi+\left(Z_{n}+r_{n}\right)(\cos \phi-1)\right] \cos \theta \\
& h_{\text {rn }}=\left(-Z_{\text {Down }}+h_{\operatorname{con}}-h_{\operatorname{son}}\right) /(\cos \phi \cos \theta)
\end{aligned}
$$

Rate of Strut Deflection

$$
\dot{h}_{T n}=-\dot{Z}_{D \omega \omega}\left(\frac{1}{\cos \phi \cos \theta}\right)+X_{n g}-Y_{n} \rho
$$

Vertical force

$$
F_{G Z n}=K_{s i n} h_{r n}+g_{s i n} \dot{h}_{T n}
$$

NOTE: COMPUTE $F_{G Z n}$ ont IF $h_{\text {Th }}<0$;

$$
\text { If } h_{r n}>0 ; F_{G z n}=0.0
$$

REMAINING CALCULATIONS MAT BE SET TO ZERO.

Landing Gear Equations (cont'd.)

Longitudinal force:

$$
F_{\mu n}=+\left(\mu_{0}+\mu_{1} B_{\Delta n}\right) F_{G z n} \frac{+u}{|u|}
$$

ie. if $u>0$ Fum is negative

$$
\begin{array}{ll}
\text { if } u<0 & F_{\text {mu }} \text { is positive } \\
\text { if } u=0 & F_{\text {un }} \equiv 0.0
\end{array}
$$

NOTE: $B_{6 n}$ is percent brake pedal deflection

Side force:

$$
\begin{aligned}
& F_{\text {sm }}=\mu_{s} F_{6 z n} \frac{+V}{|v|} \\
& \text { ie. IF } V>0 \quad F_{s n} \text { is negative } \\
& \text { if } V<0 \quad F_{\text {sn }} \text { is positive } \\
& \text { if } v=0 \quad F_{\text {sn }}=0.0
\end{aligned}
$$

Force $\&$ Moment Contribution of each gear

$$
\begin{aligned}
& \Delta X_{n}=F_{\mu n}-F_{G Z n} \theta \\
& \Delta Y_{n}=F_{s_{n}}+F_{G Z_{n} \phi} \\
& \Delta Z_{n}=F_{\mu n} \theta-F_{s n} \phi+F_{G Z n} \\
& \Delta M_{n}=-\Delta Z_{n} X_{n}+\Delta X_{n}\left(Z_{n}+r_{n}+h_{T n}\right) \\
& \Delta Z_{n m}=\Delta Z_{n} Y_{n}-\Delta Y_{n}\left(Z_{n}+r_{n}+h_{T n}\right) \\
& \Delta \eta_{n}=-\Delta X_{n} Y_{n}+X_{n} \Delta Y_{n}
\end{aligned}
$$

Landing Gear Equations (Cont'd.)

$$
\begin{aligned}
& \Delta X_{L G}=\sum_{1}^{3} \Delta X_{n} \\
& \Delta Y_{L G}=\sum_{1}^{3} \Delta Y_{n} \\
& \Delta Z_{L G}=\sum_{1}^{3} \Delta Z_{n} \\
& \Delta Z_{L G}=\sum_{1}^{3} \Delta Z_{n} \\
& \Delta M_{L G}=\sum_{1}^{3} \Delta m_{n} \\
& \Delta U_{L G}=\sum_{1}^{3} \Delta U_{a}
\end{aligned}
$$

Fuselage Aerodynamics

Fuselage Input Equations

$$
\begin{array}{ll}
\alpha_{F}=\tan ^{-1} \frac{\omega}{u} & \beta_{F}=\tan ^{-1} \frac{v}{\sqrt{u^{2}+\omega^{2}}} \\
\alpha_{p}^{\prime}=\sin \alpha_{F} \cos \alpha_{F} & \beta_{F}^{\prime}=\sin \beta_{F} \cos \beta_{F} \\
V_{F}=\sqrt{u^{2}+2^{2}+\omega^{2}} & \\
q_{F}=\frac{1}{2} \rho V_{F}^{2} & \text { (q) same as tain dYnamic pressure } \\
V_{\text {PUS }}=V_{F} \sqrt{\sigma_{h}} &
\end{array}
$$

Fuselage Wind $A \times i s$ Coff's.

$$
\begin{aligned}
& C_{D F}=C_{D O F}\left(1+K_{0}\left|\beta_{F}\right|^{\prime}\right)+K_{2} \alpha_{F}^{2}+K_{1}\left|\alpha_{F}\right|+\Delta C_{D L G} \\
& C_{L F}=K_{3} \alpha_{F}^{\prime}+K_{F} \alpha_{F}^{\prime}\left|\alpha_{F}^{\prime}\right|+K_{42} \\
& C_{Y F}=K_{7} \beta_{F}^{\prime}+K_{B} \beta_{F}^{\prime}\left|\beta_{F}^{\prime}\right| \\
& C_{\text {MF }}=C_{\text {MOF }}+K_{5} \alpha_{F}^{\prime}+K_{G} \alpha_{F}^{\prime}\left|\alpha_{F}^{\prime}\right|+\Delta C_{\text {ALG }} \\
& C_{N F}=C_{\text {NOF }}+K_{G} \beta_{F}^{\prime}+K_{10} \beta_{F}^{\prime}\left|\beta_{F}^{\prime}\right|
\end{aligned}
$$

NOTE: If $G_{E A R}$ is up; $\Delta C_{D L G} \& \Delta C_{M K G} \equiv 0.0$
SPECIAL CONDITIONS

1. IF $V_{F}^{2} \leqslant 1(f+/ \mathrm{sec})^{2}$ fuselage AERO $=0.0$ HOLD VALUE OF $\alpha_{f} \neq \beta_{F}$

Fuselage Forces \& Moment about a/c c.g.

$$
\begin{aligned}
& x_{A F R_{0}}^{F \prime}=\left[-C_{D F} \cos \alpha_{F}+C_{L F} \sin \alpha_{F}-C_{Y F} \sin \beta_{F} \cos \alpha_{F}\right] q_{F} S_{W} \\
& Y_{A E R_{0}}^{F \prime}=\left[C_{Y F} \cos \beta_{F}-C_{D F} \sin \beta_{F}\right]_{q_{F}} S_{w} \\
& Z_{A E R O}^{F I}=\left[-C_{L F} \cos \alpha_{F}-C_{D F} \cos \beta_{F} \sin \alpha_{F}-C_{Y F} \sin \beta_{F} \sin \alpha_{F}\right] q_{f} S_{w} \\
& \mathscr{L}_{A F R 0}^{f \prime}=\left[-\left(\frac{c_{w}}{b_{w}}\right) C_{M F} \sin \beta_{f} \cos \alpha_{F}-c_{N E} \sin \alpha_{f}\right] q_{f} S_{w} b_{w}+ \\
& +Y_{A E F O}^{F \prime}\left[Z_{C G}-Z_{F A C}\right] \\
& M_{A E R O}^{F \prime}=\left[C_{M F} \cos \beta_{F}\right] q_{F} S_{w} C_{w}+Z_{A E R O}^{F \prime}\left[X_{C G}-X_{F A C}\right] \\
& -x_{\text {AGRO }}^{F \prime}\left[Z_{C G}-Z_{F A C}\right]+ \\
& n_{A E R O}^{F 1}=\left[C_{N F} \cos \alpha_{F}-\left(\frac{C_{w}}{b_{w}}\right) C_{M F} \sin \beta_{F} \sin \alpha_{F}\right] q_{F} s_{w} b_{w} \\
& -Y_{A E R O}^{F \prime}\left[X_{C G}-X_{F A C}\right]+ \\
& X_{A E R_{0}}^{F}=X_{A E R_{0}}^{F \prime}+\Delta X_{L G} \\
& Y_{A E R_{0}}^{F}=Y_{A \in R_{0}}^{F \prime}+\Delta Y_{L G} \\
& Z_{A G R O}^{F}=Z_{A G R O}^{F \prime}+\Delta Z_{L G} \\
& \mathscr{L}_{A E R O}^{F}=\mathcal{Z}_{\text {AERO }}^{F}+\Delta \mathcal{Z}_{L G} \\
& m_{\text {aGRO }}^{f}=m_{\text {MERO }}^{\prime \prime}+\Delta M_{\text {LG }} \\
& n_{\text {AGEO }}^{F}=n_{\text {AER }}^{F \prime}+\Delta N_{L G}
\end{aligned}
$$

Wing on Rotor Interference

Average Nacelle Incidence

$$
\bar{i}_{N}=0.5\left(i_{N L}+i_{N R}\right)
$$

Average Lift Coff.

$$
C_{L W}=0.5 \frac{\left(C_{L \text { SW }}+C_{L S L W}\right)}{\left(1-\bar{C}_{T S}\right)}
$$

$$
\text { Look-up: } \epsilon_{\text {WRR }} \& \epsilon_{W R L} @ \bar{i}_{N} \& C_{W}
$$

Wing Interference Logic

1. IF:Unoperuas open, set $C_{L W}=0.0 / \epsilon=\frac{\epsilon_{p}(1-G E P)}{\sqrt{1-M^{2}}}$

Rotor/ROTOR Interference
Positive sideslip ie $V>0.0$ (Logic require)

$$
\begin{aligned}
& x=1.5708-\epsilon_{P R R} \\
& \left(\frac{\delta v_{R L}^{*}}{v_{R R}^{*}}\right)=T_{1}+T_{R}(\chi)+T_{3}(x)^{2} \\
& \delta v_{R L}=\left(\frac{\delta v_{R L}^{*}}{V_{R R}^{*}}\right) \tau_{R R} \sqrt{\frac{R_{R R}}{2 \rho \pi R^{2}}} \\
& \epsilon_{i R L}^{\prime}=-\tan ^{-1}\left[\frac{\delta v_{R L}}{V_{L R}+1.0}\right] \\
& \epsilon_{i R L}=\left(\left|\beta_{R}\right|\right)\left(.40528 i_{N L}\right) \epsilon_{i R L}^{\prime} \\
& \epsilon_{i, R}=0.0
\end{aligned}
$$

Negative Sideslip ie. $v<0.0$

$$
\begin{aligned}
& x=1.5708-\epsilon_{P L R} \\
& \left(\frac{\delta V_{L R}^{*}}{V_{L R}^{*}}\right)=T_{1}+T_{2}(\chi)+T_{3}(X)^{2} \\
& \delta V_{L R}=\left(\frac{\delta V_{L R}^{*}}{V_{R R}^{*}}\right) v_{N L} \sqrt{\frac{R_{L R}}{2 \rho \pi R^{2}}} \\
& \epsilon_{i L R}^{\prime}=-\tan ^{-1}\left[\frac{\delta v_{L R}}{V_{R R}+1.0}\right] \\
& \epsilon_{i L R}=\left(\left|\beta_{F}\right|\right)\left(.40528 i_{N R}\right) \epsilon_{i L R}^{\prime} \\
& \epsilon_{i R L}=0.0
\end{aligned}
$$

NOTE: $V_{X R} \nmid V_{n L}$ FROM wiNG EQUATIONS.

Rotor Aero Input Equations

Right Rotor

$$
\begin{aligned}
& \text { RotoR } \alpha_{R R}=\tan ^{-1}\left\{\frac{\sqrt{v_{R R}^{2}+\left(w_{R R}+u_{R R} \epsilon_{W R R}\right)^{2}}}{u_{R R}}\right\}+\epsilon_{i<R} \\
& V_{R R}=\sqrt{u_{R R}^{2}+v_{R R}^{2}+w_{R R}^{2}} ; \mu_{R R}=\frac{V_{R R}}{\Omega_{R} R}
\end{aligned}
$$

LEFT ROTOR

$$
\begin{aligned}
& \frac{T O R}{\alpha_{L R}}=\tan ^{-1}\left\{\frac{\sqrt{V_{R L}^{2}+\left(W_{R L}+u_{R L} \epsilon_{W R}\right)^{2}}}{u_{R L}}\right\}+\epsilon_{i R L} \\
& V_{L R}=\sqrt{u_{R L}^{2}+V_{R L}^{2}+W_{R L}^{2}} ; \mu_{L R}=\frac{V_{L R}}{\Omega_{L} R}
\end{aligned}
$$

Rotor Angular Rate Transforms

$$
\begin{aligned}
& R_{I G H T}-N_{A C E L L E} A \times E S \\
& P_{N R}^{N}=-p \operatorname{cosi} i_{M R}+r \sin i_{N R} \\
& Q_{N R}^{N}=q+i_{N R} \\
& R_{N R}^{N}=-r \cos i_{N R}-p \sin i_{N R}
\end{aligned}
$$

Right Wind AXES

$$
\begin{aligned}
& P_{N R}^{R}=P_{N R}^{N} \\
& Q_{N R}^{R}=Q_{N R}^{N} \cos \xi_{H R}+R_{N R}^{N} \sin \xi_{N R} \\
& R_{N R}^{R}=R_{N R}^{N} \cos \xi_{N R}-Q_{N R}^{N} \sin \xi_{H R}
\end{aligned}
$$

Left -Nacelle Axes

$$
\begin{aligned}
& P_{N L}^{N}=p \cos i_{N L}-r \sin i_{N L} \\
& Q_{N L}^{N}=q+i_{N L} \\
& R_{N L}^{N}=r \cos i_{N L}+\rho \sin i_{N L}
\end{aligned}
$$

LEFT WIND AXES

$$
\begin{aligned}
& P_{N L}^{R}=P_{N L}^{N} \\
& Q_{N L}^{R}=Q_{N L}^{N} \cos \zeta_{N L}-R_{N L}^{N} \sin \xi_{H L} \\
& R_{N L}^{R}=R_{N L}^{N} \cos \zeta_{N L}+Q_{N L}^{N} \sin \zeta_{N L}
\end{aligned}
$$

Note: Use Wind Axis Rates in Rotor Routine

ROTOR EQUATIONS
RIGHT ROTOR
THRUST

$$
C_{T R R}^{\prime}=\left[\frac{I_{1} S+1}{\tau_{2} s+1}\right]\left[C_{T_{O R R}} \operatorname{Cos} A_{1 C_{R}} \operatorname{Cos} B_{1 C_{R}}\right]
$$

WHERE:-

GROUND EFFECT

$$
\begin{aligned}
& h_{R R}=-z_{D o w n}+\left(L_{S} \operatorname{Cos} i_{N R}-x_{C Q}\right) \operatorname{Sin} \theta \\
& +\left[\left(L_{s} \sin i_{\nu R}+Z_{C a}\right) \cos \phi-Y_{N} \sin \phi\right] \cos \theta \\
& \left(\frac{h}{D}\right)_{\substack{E R F \\
R R}}=\frac{h_{R R}}{2 R\left[\left|\sin \left(\theta+i_{N R}\right) \cos \phi\right|+.0174\right]} \\
& \left(\frac{T 1 G E}{T O Q E}\right)_{R R}=\left[\left(\frac{h}{D}\right)_{R R}^{2} \underset{R F F}{ }\left(.1741-.6216 \mu_{R R}\right)\right. \\
& +\left(\frac{h}{D}\right)_{\substack{R R}}\left(1.4779 \mu_{R R}-.4143\right) \\
& \left.+1.2479-.8806 \mu_{R R}\right] \\
& C_{T_{R R}}=C_{T_{R R}}^{\prime}\left(\frac{T_{I G F}}{T_{\text {TOFF }}}\right)_{R R}
\end{aligned}
$$

SPACIRL Conditions: If $\mu_{R R} \geq 0.283 ;\left(\frac{T_{\text {IN E }}}{T_{\text {AGA }}}\right)_{R R}=1.0$
$O R \quad$ IF $\left(\frac{h}{D}\right)_{R R} \geq 1.3 ;\left(\frac{T / G \sigma}{T O G \sigma}\right)_{R R}=1.0$

$$
E-54
$$

Rotor Equations (Continuer)

Power

$$
C_{P_{R R}}=C_{P_{P R R}}
$$

WHERE:-

NORMAL FORCE

$$
C_{U F_{R R}}=C_{N F_{R R}}+\frac{d C_{N F R R}}{d A_{I C_{R}}} A_{I C R}+\frac{d C_{N E R}}{d B_{C_{R}}} B_{1 C_{R}}
$$

WHERE :-

$$
\begin{aligned}
& \frac{d C_{N F_{R R}}}{d A_{I C R}}=D_{N F_{1}} C_{T R R}+D_{N F_{2}} \mu_{R R}^{2}+D_{N F_{3}} \mu_{R R}+D_{N F_{4}} \\
& \frac{d C_{N F R R}}{d B_{I C R}}=E_{N F_{1}} C_{T R R}+E_{N F_{2}} \mu_{R R}^{2}+E_{N F_{3}} \mu_{R R}+E_{N F_{4}}
\end{aligned}
$$

Rotor Equationes (Continuato)

Side Forct

$$
C_{S F_{R R}}=C_{S F O R R}+\frac{d C_{S F R R}}{d A_{I C R}} A_{I C R}+\frac{d C_{S F R R}}{d B_{I C R}} B I C R
$$

WHIRE:-

$$
\begin{aligned}
& \frac{d C_{S F R R}}{d A_{I C R}}=D_{S F_{1}} C_{T R R}+D_{S F_{2}} \mu_{R R}^{2}+D_{S F_{3}} \mu_{R R}+D_{S F_{4}} \\
& \frac{d C_{S R R R}}{d B_{1} C_{R}}=E_{S F_{1}} C_{T R R}+E_{S F_{2}} \mu_{R R}^{2}+E_{S F_{3}} \mu_{R R}+E_{S F_{4}}
\end{aligned}
$$

Rotor Equations (Continuma)
hub Pitching Monowt

$$
C_{P H_{R R}}=C_{P M_{O R R}}+\frac{d C_{P M_{P R}} A_{I C R}}{d A_{I C R}}+\frac{d C_{P M_{R R}}}{d B_{I C R}} B_{I C R}+\frac{d C_{P M R R}}{d Q} Q_{N R}^{R}
$$

$W_{\text {MURE }}$ : -

$$
\begin{aligned}
& C_{P H_{O R R}}=\sum_{N=0}^{2} \sum_{u=0}^{3}\left[A_{P H}(u+4 \sigma) \alpha_{R R}^{u} C_{T R R}^{\prime \sigma}\right] \\
& A_{P H}(u+4 \sigma)=f\left(\mu_{R R}\right) \\
& \frac{d C_{P M_{R R}}}{d Q}=\sum_{N=0}^{2} \sum_{u=0}^{3}\left[H_{P r(u+4 \sigma)} \alpha_{R R}^{u} C_{T R R}^{1 N}\right] \\
& H_{P n(u+4 v)}=f\left(\mu_{R R}\right)
\end{aligned}
$$

$$
\begin{aligned}
& \text { INTARPDLATE } \\
& \text { CPMORR AND } \\
& \frac{d C_{\text {PMRO }}}{d Q} \\
& \text { LINLTARLY } \\
& \text { BETWLEN } \mu^{\prime} \text { 's }
\end{aligned}
$$

vore:-
For $\mu_{R D} \leq 0.35$

$$
\frac{d C_{P M_{R R}}}{d A_{K R}}=D_{P r_{1}} C_{T_{R R}}+D_{P H_{2}} \mu_{R R}^{2}+D_{P r_{3}} \mu_{R R}+D_{P H_{4}}
$$

For $\mu_{R A}=0.35^{\circ}$, THE Cosfers. Above CHAnc| AS Fowaws

$$
\begin{aligned}
& D_{P H_{2}} \equiv D_{P H_{5}} \\
& D_{P H_{3}}=D_{P H_{6}} \\
& D_{P H_{4}} \equiv D_{P H_{7}}
\end{aligned}
$$

ROTOR Equations (COntinued)
hub Pitching Moment (Continued)

$$
\frac{d C_{P M R R}}{d B_{M_{C R}}}=E_{P H_{1}} C_{T R R}+E_{P H_{2}} \mu_{R R}^{2}+E_{P r_{3}} \mu+E_{P r_{4}}
$$

NOTE:- USE THE ABOVE EQUATION FOR $\mu_{R R} \leq 0.35$

$$
\text { For } \begin{aligned}
\mu_{R R}>0.35: \quad E_{P H_{2}} & =E_{P H_{1}} \\
E_{P M_{3}} & \equiv E_{P H_{6}} \\
E_{P M_{4}} & \equiv E_{P H_{7}}
\end{aligned}
$$

Hub Yawing Moment

$$
C_{Y M_{R R}}=C_{Y M_{O R R}}+\frac{d C_{Y M R R}}{d A_{1 C R}} A_{I C R}+\frac{d C_{Y M R R}}{d B_{I C R}} B_{I C R}+\frac{d C_{Y M_{R R}}}{d R} R_{U R}^{R}
$$

$W_{\text {HIRE :- }}$

Rotor Equations (Continued)
hus Yawing Moment (Continuo)

FOR $\mu_{R R} \leq 0.35$

$$
\frac{d C_{y M_{R R}}}{d A_{1 C R}}=D_{y M_{1}} C_{T_{R R}}+D_{y \Pi_{2}} \mu_{R R}^{2}+D_{y \Pi_{3}} \mu_{R R}+D_{y M_{4}}
$$

FOR $\mu_{\text {AR }}>0.35$

$$
\begin{aligned}
& D_{y 1_{2}} \equiv D_{y+15} \\
& D_{y H_{3}} \equiv D_{y m 4} \\
& D_{y H_{4}} \equiv D_{y m 7}
\end{aligned}
$$

$$
\begin{aligned}
& \text { For } \mu_{R R} \leq 0.35 \\
& \qquad \frac{d C_{Y M R R}}{d B_{M R R}}=E_{Y M_{1}} C_{T R R}+E_{Y m_{2}} \mu_{R R}^{2}+E_{Y M_{3}} \mu_{R R}+E_{Y \Pi_{4}}
\end{aligned}
$$

FOR $\mu_{R R}=0.35$

$$
\begin{aligned}
& E_{y M_{2}} \equiv E_{y m 5} \\
& E_{4 m_{3}} \equiv E_{y m 4} \\
& E_{4 M_{4}} \equiv E_{y m 7}
\end{aligned}
$$

Rotor Equations (Contd)

Rotor Force $\&$ Moment Calculation

$$
\begin{aligned}
& T_{R}=f_{T R} C_{T R R} \rho \pi R^{4} \Omega_{R}^{2} \\
& N . F_{R}=f_{\text {ir }} C_{N F R R} \rho \pi R^{4} \Omega_{R}^{2} \\
& \text { S. FR }=f_{\text {SF }} C_{\text {STR }} \rho \pi R^{4} \Omega_{R}^{2} \\
& M_{R}=f_{P T_{R}} C_{P M R R} \rho \pi R^{5} \Omega_{R}^{2} \\
& M_{R}=f_{Y M_{R}} C_{Y M R R} \rho \pi R^{5} \Omega_{R}^{2} \\
& Q_{R R E Q}=f_{Q_{R}} C_{P R R} \rho \pi R^{5} \Omega_{R}^{2} \\
& R H P_{R R}=f_{P_{R}} C_{P R R} \rho \pi R^{5} \Omega_{R}^{3} / 550 \text { or } R H P_{R R}=Q_{R R C A}\left(\frac{\Omega_{R}}{550}\right)
\end{aligned}
$$

LEFT ROTOR FOLLOWS sIMILAR FORMAT
WITH SUBSCRIPTS CHANGED.
THE LEFT ROTOR ALTITUOE FQUATION is AS FOLLOWS:

$$
\begin{aligned}
h_{C e}= & -Z_{0 o w N}+\left(L_{s} \cos i_{N L}-x_{C \sigma}\right) \sin \theta \\
& +\left[\left(L_{s} \sin i_{N L}+Z_{C \sigma}\right) \cos \phi+V_{N} \sin \phi\right] \cos \theta
\end{aligned}
$$

OR;

$$
h_{L R}=h_{R R}+2 Y_{N} \sin \phi \cos \theta
$$

Rotor Force \& Moment Resolution

Hub Moments - Nacelle Axes
LEFT

$$
\begin{aligned}
\mathscr{L}_{L R H}= & -Q_{L R E Q}-I_{P} \dot{\Omega}_{L} \\
M_{L R H}= & M_{L} \cos \xi_{H L}-\eta_{L} \sin \xi_{H L} \\
& -I_{P} \Omega_{L}\left(p \sin i_{N L}+r \cos i_{N L}\right) \\
M_{L R H}= & -M_{L} \cos \zeta_{H L}-M_{L} \sin \xi_{H L}+I_{P} \Omega_{L}\left(\dot{i}_{N L}+g\right)
\end{aligned}
$$

RIGHT

$$
\begin{aligned}
\mathscr{L}_{R R H}= & Q_{R R E Q}+I_{P} \dot{\Omega}_{R} \\
M_{R R H}= & M_{R} \cos \xi_{H R}+M_{R} \sin \xi_{H R} \\
& +I_{P} \Omega_{R}\left(p \sin i_{N R}+r \cos i_{N R}\right) \\
M_{R R H}= & M_{R} \cos \xi_{H R}-M_{R} \sin \xi_{H R}-I_{P} \Omega_{R}\left(i_{N R}+g\right)
\end{aligned}
$$

Note: Nacelle Axes are right hanged systems

Rotor Forces \& Moment Resolution (Cont'd.)

Left Tip Pivot - Body Axes@ tip ( $\omega /$ NACELLE AERO)

$$
\begin{aligned}
& X_{A R R O}^{N L}=\left(T_{L}+\Delta X_{L N}^{\prime}\right) \cos i_{N L}-\sin i_{N L}\left(N \cdot F_{1} \cos \zeta_{H L}+S_{I} F_{I L} \sin \xi_{H L}\right. \\
& \left.-\Delta Z_{L N}^{\prime}\right) \\
& Y_{A E R O}^{N L}=5 \cdot F_{L L} \cos \zeta_{H L}-N \cdot F_{L} \sin \zeta_{H L}+\Delta Y_{L N}^{\prime} \\
& Z_{A \in e_{0}}^{N L \prime}=-\left(T_{L}+\Delta X_{L N}^{\prime}\right) \sin i_{N L}-\cos i_{N L}\left(N \cdot F_{L} \cos \zeta_{H_{L}}+S F_{L} \sin \zeta_{H_{L}}\right. \\
& \left.-\Delta Z_{L N}^{\prime}\right) \\
& \mathcal{Z}_{A E R O}^{N L \prime}=\left(Z_{L R H}+\Delta Z_{L N}^{\prime}\right) \cos i_{N L}+\sin i_{N L}\left(\Pi_{L R N}+\Delta \Pi_{L N}^{\prime}+L_{S} Y_{A E R O}^{N L}\right) \\
& M_{\text {AERO }}^{N L}=M_{L R H}+\Delta M_{L N}^{\prime}+N \cdot F_{L} L_{S} \cos \xi_{H L}+S . F_{L} L_{S} \sin \xi_{H L} \\
& -L_{s} \Delta Z_{L N}^{\prime}-I_{E} \Omega_{E L} r \\
& \eta_{A E R O}^{N L}=\cos i_{N L}\left(\eta_{L R H}+\Delta n_{L N}^{\prime}+L_{S} Y_{A E R_{0}}^{N L}\right)-\sin i_{N L}\left(\mathscr{Z}_{L R H}+\Delta Z_{L N}^{\prime}\right) \\
& +I_{E} \Omega_{E L} g
\end{aligned}
$$

Nacelle Equation Input - Left

$$
M_{N L A E R S}=M_{L R H}+\Delta M_{I N}^{\prime}+\left(N \cdot F_{L} \cos F_{H L}+5 \cdot F_{L} \sin S_{H L}-\Delta Z_{L N}^{\prime}\right) L_{S}
$$

GLAS INPUTS - LEFT

$$
\begin{aligned}
& m_{\substack{\text { LARS } \\
\text { GARS }}}=m_{L R H}+L_{S}\left(N \cdot F_{L} \cos 3_{N L}+S \cdot F_{L} \sin S_{M L}\right) \\
& n_{N L A R O D}=\eta_{L R H}+L_{S}\left(S F_{L} \cos \zeta_{N L}-N F_{L} \sin S_{M L}\right)
\end{aligned}
$$

Rotor Force \& Moment Resolution (cont'd.)
Right Tip Pivot-Body Axes @tip (w/nacelle aero)

$$
\begin{aligned}
& X_{A E R O}^{N R}=\left(T_{R}+\Delta X_{R N}^{\prime}\right) \cos i_{N R}+\sin i_{N R}\left(-N \cdot F_{R} \cos \zeta_{H R}\right. \\
& \left.+5 . F_{R} \sin \xi_{H R}+\Delta Z_{R N}^{\prime}\right) \\
& Y_{A \in R_{0}}^{N R}=-S \cdot F_{R} \cos \xi_{H R}-N \cdot F_{R} \sin \xi_{N R}+\Delta Y_{R N}^{\prime} \\
& Z_{A E R 0}^{N E P}=-\left(T_{R}+\Delta X_{R N}^{\prime}\right) \sin i_{N R}+\cos i_{N R}\left(-N \cdot F_{R} \cos \xi_{H R}\right. \\
& \left.+5 \cdot F_{R} \sin J_{H R}+\Delta Z_{R N}^{\prime}\right) \\
& \mathscr{L}_{R E R O}^{N R}=\left(\mathscr{Z}_{R R H}+\Delta Z_{R N}^{\prime}\right) \cos i_{N R}+\sin \operatorname{SinR}_{N R}\left(\eta_{R R H}+L_{S} Y_{A B R_{0}}^{N R}+\Delta \eta_{R N}^{\prime}\right) \\
& M_{\text {AERD }}^{N R}=M_{\text {RRN }}+\Delta M_{R N}^{\prime}+N \cdot F_{R} L_{S} \cos \xi_{H R}-S \cdot F_{R} L_{S} \sin \zeta_{H R} \\
& -L_{s} \Delta Z_{R N}^{\prime}-I_{E} \Omega_{R R} r \\
& \eta_{\text {AGRO }}^{N R}=\cos i_{N R}\left(M_{\text {PRN }}+\Delta \Pi_{P N}^{\prime}+\angle_{S} Y_{\text {NERO }}^{N R}\right)-\sin i_{N R}\left(Z_{R P N}+\Delta Z_{R N}^{\prime}\right) \\
& +I_{E} \Omega_{E R} g
\end{aligned}
$$

Nacelie Equation Input - Right

$$
M_{N E A E R O}=M_{R P H}+\Delta M_{R N}^{\prime}+\left(N \cdot F_{R} \cos \xi_{N R}-S \cdot F_{R} \sin \zeta_{N R}-\Delta Z_{N R}^{\prime}\right) Z_{S}
$$

GLAS inputs - RIGNT

Wing Vertical Bending

Right Wing Tip Deflection

$$
\begin{aligned}
& \bar{a}_{R T}=\frac{Z_{A F R O}}{m}+Y_{N P} \dot{P} \\
& \bar{a}_{R W A C}=\frac{Z_{A E R O}}{m}+Y_{W A C} \dot{P} \\
& h_{I R}=K_{W I} Z_{A F R O}^{N R}+K_{W Z} Z_{A G R O}^{R W \prime}+K_{W B} Z_{A F R O}^{N R I}-K_{W T} \bar{a}_{R T}-K_{W S} \bar{a}_{R W A C} \\
& \dot{h}_{I R}=\Delta h_{I R} / \Delta t
\end{aligned}
$$

 AND At is Tho TIME FRATV

RIGHT WING AC. DEFLECTION

Force and Moment Effects

$$
\begin{aligned}
& \ddot{Z}_{A E R O}^{N R}=-2 \xi_{w_{1}} \omega_{\omega 1} \dot{Z}_{A G R O}^{N R} \mid-\omega_{\omega,}^{2} Z_{A E R O}^{N R}+\omega_{\omega,}^{2} Z_{\text {AERO }}^{N R}
\end{aligned}
$$

Wing Vertical Bending (Comt'd.)

LEFT Wing. Tip Deflection

$$
\begin{aligned}
& \bar{a}_{C T}=\frac{Z_{\text {aces }}}{m}-Y_{N} \dot{\rho} \\
& \bar{a}_{\text {LAC }}=\frac{Z_{\text {AEC }}}{\bar{n}}-Y_{\text {WAC }} \bar{P}
\end{aligned}
$$

$$
\begin{aligned}
& \ddot{h}_{L_{L}}=\Delta h_{L_{L}} / \Delta t
\end{aligned}
$$

 AND $\triangle t$ is tate tine frame

Left Wing A.C. DefLECTION

$$
\begin{aligned}
& \dot{h}_{1_{1} \text { WM }}=\Delta h_{I_{\text {WaC }}} / \Delta t
\end{aligned}
$$

 FRiDGES AND $\triangle t$ is The Time FRAME

Force and Moment Effects

$$
\begin{aligned}
& \ddot{Z}_{A E R_{0}}^{N L}=-2 \xi_{w_{1}} \omega_{\omega_{1}} \dot{Z}_{A G R_{0}}^{N L}-w_{\omega_{1}}^{2} Z_{A E e_{0}}^{N L}+w_{\omega_{1}}^{2} Z_{A G R O}^{N L /}
\end{aligned}
$$

$$
\begin{aligned}
& \ddot{\mathscr{L}}_{A E R O}^{N L}=-2 \xi_{W 3} \omega_{W 3} \dot{\mathcal{L}}_{A E R O}^{N L}-\omega_{W_{3}}^{2} \mathcal{L}_{A E R_{0}}^{N L}+\omega_{W S}^{2} \mathcal{L}_{A K R O}^{N L N}
\end{aligned}
$$

FORM $Z_{\text {AERO }}^{N L}, Z_{A E R O}^{L W}, \not \mathscr{L}_{\text {AERO }}^{N L}$

Wing Torsion
LEFT WING TWIST@ TIP

$$
\begin{aligned}
& K_{o t} \theta_{t L N}=M_{N L K T}-I_{E} \Omega_{E L} r \\
& +q_{s L W} \frac{c_{\omega}^{2} b_{\omega}}{2} C_{m 0}\left(1-C_{\text {TsaR }}\right) \\
& +\left(1-C_{T s L e}\right) q_{5<\omega} C_{w}^{2}\left(\frac{d C_{M \omega c / 4}}{d C_{L}}+\frac{X_{w a c}}{C_{w}}\right)\left(\frac{C_{L \alpha} b_{w}}{6 \pi}\right)\left(4 \theta+3 \pi_{t \alpha \alpha}\right)
\end{aligned}
$$

Right Wing Twist e Tip

$$
\begin{aligned}
K_{\theta_{t}} \theta_{t R W} & =M_{N R A C T}-I_{E} \Omega_{E R} r \\
& +q_{S R W} \frac{C_{\omega}^{2} b_{\omega}}{2} C_{M 0}\left(1-C_{T S R R}\right) \\
& +\left(1-C_{\text {TERR }}\right) q_{S R W} C^{2}\left(\frac{d C_{M W C / 4}}{d C_{L}}+\frac{X_{W A C}}{C_{\omega}}\right)\left(\frac{C_{L \alpha} b_{W}}{6 \pi}\right)\left(4 \theta_{t A \omega}+3 \pi \alpha_{R W R I S I R}\right)
\end{aligned}
$$

WHERE: $\quad C_{M O}=C_{1}+C_{2} \delta_{F}+C_{3} \delta_{f}^{2}$

$$
\begin{aligned}
& \theta_{t W A C}=\frac{Y_{W A C}}{Y_{N}} \theta_{t C W} \\
& \theta_{t R W A C}=\frac{Y_{W A C}}{Y_{N}} \theta_{t R W}
\end{aligned}
$$

NOTE: If umbrellas are open ; set terms containing

$$
q_{5}\left(1-C_{T S}\right) \text { EQUAL To zERO }
$$

TOTAL FOMCE AND MOMENT SUMMATION ABOUT C.G.

$$
\begin{aligned}
& X_{A E R O}=X_{A E R O}^{N L}+X_{A E R O}^{N R}+X_{A E R O}^{F}+X_{A E R O}^{L W}+X_{A F R O}^{R W}+X_{A E R O}^{\top} \\
& Y_{A E R 0}=Y_{A E R 0}^{N L}+Y_{A F R 0}^{N R}+Y_{A E R}^{F}+Y_{A E R 0}^{L W}+Y_{A E R O}^{R W}+Y_{A E R O}^{T} \\
& Z_{A E R O}=Z_{A E R O}^{N L}+Z_{A R R O}^{N R}+Z_{A E R O}^{F}+Z_{A E R O}^{L W}+Z_{A E R O}^{N W}+Z_{A E R O}^{T} \\
& \mathcal{L}_{A E R 0}=\mathcal{L}_{A E R O}^{N L}+\mathcal{L}_{A E R O}^{N R}+\mathcal{L}_{A E R 0}^{F}+\mathcal{L}_{A E R O}^{W}+\mathcal{L}_{A E R O}^{T} \\
& +Y_{N}\left(Z_{A E R O}^{N R}-Z_{A E R O}^{N L}\right)+Z_{C G}\left(Y_{A E R O}^{N L}+Y_{A E R O}^{N R}\right) \\
& M_{\text {AFRO }}=M_{\text {AFRS }}^{N L}+M_{\text {ARE }}^{N E}+M_{\text {AFRO }}^{F}+M_{\text {AERO }}^{W}+M_{\text {ACRC }}^{T} \\
& +X_{C G}\left(Z_{A E R O}^{N L}+Z_{A E R O}^{N R}\right)-Z_{C G}\left(X_{A E R_{0}}^{N L}+X_{A E R O}^{N R}\right) \\
& M_{A E R O}=M_{\text {AER }}^{N L}+M_{\text {AER }}^{N R}+M_{\text {AER }}^{F}+M_{\text {AER }} \omega+M_{\text {AER }}^{T} \\
& +Y_{N}\left(X_{A E R R_{0}}^{N L}-X_{A G R O}^{N R}\right)-X_{C G}\left(Y_{A G R a}^{N L}+Y_{A E R_{0}}^{N R}\right)
\end{aligned}
$$

Nomenclature
$\rho_{f}, h_{f} \sim$ FUSELAGE MASS GENTER w.r.t. PIVOT FUSE. CENTER, AXES.
lw, Kw $\sim$ WING " " $"$ " $"$
~ NACELEE PIVOT TO NACELE CG DISTANCE
$\lambda$ ~ ANGLE BETWEEN NACELLE SHAFT AXIS ANO ITS C.G. to pivot axis.
$m_{f} \sim$ MASS OF FUSELAGE
mw ~ MASS OF BOTH WINGS
$m_{N} \sim$ MASS OF ONE NACELLE
IN ~ NACELLE SHAFT TO FUSELAGE X-AXIS ANGLE
$I_{x x}^{(f)}, I_{y y}^{(f)}, I_{z z}^{(f)}, I_{x z}^{(f)}$-FUSFLAGE INERTIAS ABOUT its CG.
$I_{x x}^{(\omega)} I_{Y y}^{(\omega)}, I_{t z 1}^{(\omega)} I_{x Z}^{(\omega)}$ - WING INERTIAS ABOUT THEIR CG.
$I_{x x}^{\prime}, I_{y}^{\prime}, I_{z z}^{\prime}, I_{x z}^{\prime} \sim$ MOMENTS of InERTIA of one nacelle ABOUT ITS CG.
pig, r - FusELAGE BODY Axis ANGULAR RATES
u, $V, w \sim$ FUSELAGE BODY AXIS LINEAR RATES
Ip ~ ROTOR POLAR MOMENT OF INERTIA
$\Omega \sim$ ROTOR SPEED, ANGULAR


SUBSCRIPTS
R ~ RIGHT
$\angle \sim$ LEFT
$\omega$ ~ WING
$f$ - fuselage

Basic Equations of Motion

Preliminary Calculations

Euselage C.G.w.r.t. $A / C$ C.G.

$$
\begin{aligned}
& x_{f}=l_{f}-x_{c G} \\
& z_{f}=h_{f}-z_{c c}
\end{aligned}
$$

Wing C.G writ. A/C C.G.

$$
\begin{aligned}
& x_{\omega}=f_{\omega}-x_{c \sigma} \\
& z_{\omega}=h_{\omega}-z_{c \sigma}
\end{aligned}
$$

NACELLE C.G.'s w.r.t. A/C C.G.

$$
\begin{aligned}
& x_{R}=\rho \cos \left(i_{N R}-\lambda\right)-x_{C G} \\
& z_{R}=-l \sin \left(i_{N R}-\lambda\right)-z_{C G} \\
& x_{L}=\rho \cos \left(i_{N L}-\lambda\right)-x_{C G} \\
& Z_{L}=-\rho \sin \left(i_{N L}-\lambda\right)-z_{C G}
\end{aligned}
$$

Preliminary Calculations
Inertia TERMS

$$
\begin{aligned}
& \sum_{i} I_{j}^{(l)}= I_{i j}^{(f)}+I_{i j}^{(W)}+2 I_{i j}^{\prime} \\
& I_{x x}=\sum_{k}^{\prime} I_{x x}^{(k)}+\left(I_{Z Z}^{\prime}-I_{x x}^{\prime}\right)\left(\sin ^{2} i_{N R}+\sin ^{2} i_{N L}\right) \\
&-I_{x Z}^{\prime}\left(\sin Z i_{N R}+\sin Z i_{N L}\right)+2 m_{N} Y_{N}^{2} \\
&+m_{f} h_{f} Z_{f}+m_{N} h_{N} Z_{W}+ \\
&-\operatorname{lm}_{N}\left[Z_{R} \sin \left(i_{N R}-\lambda\right)+Z_{L} \sin \left(i_{N L}-\lambda\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
J_{x x}=\sum_{k} & \left(I_{z z}{ }^{(k)}-I_{y y}{ }^{(k)}\right)+\left(I_{x x}^{\prime}-I_{z Z}^{\prime}\right)\left(\sin ^{2} i_{N R}+\sin ^{2} i_{N K}\right) \\
& +I_{x z}^{\prime}\left(\sin 2 i_{N R}+\sin 2 i_{N L}\right)+Z m_{N} Y_{N}^{2} \\
& -\left(m_{f} h_{f} z_{f}+m_{W} h_{W} z_{W}\right) \\
& +\operatorname{lm}_{N}\left[z_{R} \sin \left(i_{N R}-\lambda\right)+Z_{L} \sin \left(i_{N L}-\lambda\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
I_{x z}^{(P)}= & I_{x Z}^{(f)}+I_{x Z}^{(\omega)}+\frac{1}{2}\left(I_{x x}^{\prime}-I_{Z z}^{\prime}\right)\left(\sin 2 i_{N R}+\sin 2 i_{N L}\right) \\
& +I_{x Z}^{\prime}\left(\cos 2 i_{N R}^{\prime}+\cos Z_{i N L}^{\prime}\right)+\left(m_{f} l_{f} Z_{f}+m_{\omega} l_{W} Z_{\omega}\right) \\
& +m_{N} l\left[Z_{R} \cos \left(i_{N R}-\lambda\right)+Z_{L} \cos \left(i_{N L}-\lambda\right)\right]
\end{aligned}
$$

INERTIA TERMS

$$
\begin{aligned}
& I_{y y}=\sum_{k} I_{y y}^{(h)}+m_{f}\left(\rho_{f} x_{f}+h_{f} Z_{f}\right)+m_{w}\left(\rho_{w} x_{w}+h_{w} Z_{w}\right) \\
& +m_{N} l\left[x_{R} \cos \left(i_{N R} \lambda\right)-z_{R} \sin \left(i_{N R}-\lambda\right)\right] \\
& +m_{N} l\left[x_{L} \cos \left(i_{N L}-\lambda\right)-z_{L} \sin \left(i_{N L}-\lambda\right)\right] \\
& J_{y y}=\left(I_{x x}^{(f)}-I_{z z}^{(f)}\right)+\left(I_{x x}^{(\omega)}-I_{z z}^{(\omega)}\right)+ \\
& +\left(I_{x x}^{\prime}-I_{Z Z^{\prime}}^{\prime}\right)\left(\cos Z_{c_{N R}}+\cos Z_{i_{N L}}\right) \\
& -2 I_{X Z}^{\prime}\left(\sin 2 i_{N R}+\sin 2 i_{N L}\right)+m_{f}\left(-\rho_{f} x_{f}+h_{f} z_{f}\right) \\
& +m_{w}\left(-\rho_{w} X_{w}+h_{w} z_{w}\right) \\
& -m_{N} l\left[X_{R} \cos \left(i_{N R}-\lambda\right)+Z_{R} \sin \left(i_{N R}-\lambda\right)+X_{L} \cos \left(i_{N L}-\lambda\right)\right. \\
& \left.+Z_{L} \sin \left(i_{N L}-\lambda\right)\right] \\
& I_{x Z}{ }^{(g)}=I_{x Z}{ }^{(f)}+I_{x Z}{ }^{(\omega)}+\frac{1}{2}\left(I_{x x}^{\prime}-I_{z Z}^{\prime}\right)\left(\sin Z_{i N R}+\sin Z_{i{ }_{i L}}\right) \\
& +I_{x z}^{\prime}\left(\cos Z_{i N R}+\cos 2 i_{N K}\right) \\
& -m_{N} l\left[X_{R} \sin \left(i_{N R}-\lambda\right)+X_{L} \sin \left(i_{N L}-\lambda\right)\right] \\
& +m_{f} h_{f} x_{f}+m_{\omega} h_{\omega} X_{\omega}
\end{aligned}
$$

InERtiA TERMS

$$
\begin{aligned}
I_{z Z}=\sum_{k} & I_{z z}^{(\rho)}+\left(I_{x x}^{\prime}-I_{Z Z}^{\prime}\right)\left(\sin ^{2} i_{N R}+\sin ^{2} i_{N L}\right) \\
& +I_{x Z}^{\prime}\left(\sin 2 i_{N R}^{\prime}+\sin 2 i_{N L}^{\prime}\right)+2 m_{N} Y_{N}^{2} \\
& +m_{f} l_{f} X_{f}+m_{W} l_{W} X_{W} \\
& +m_{N} l\left[X_{R} \cos \left(i_{N R}-\lambda\right)+X_{L} \cos \left(i_{N L}-\lambda\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
J_{Z Z}=\sum_{K} & \left(I_{Y Y}^{(k)} I_{X x}^{(l)}\right)+\left(I_{x x}^{\prime}-I_{Z Z}^{\prime}\right)\left(\sin ^{2} i_{N R}+\sin ^{2} i_{N L}^{\prime}\right) \\
& +I_{x Z}^{\prime}\left(\sin 2 i_{N R}+\sin 2 i_{N L}\right)-2 m_{N} Y_{N}^{2} \\
& +m_{f} \rho_{f} X_{f}+m_{W} l_{\omega} x_{W} \\
& +m_{N} l\left[X_{R} \cos \left(i_{N R}-\lambda\right)+x_{L} \cos \left(i_{N L}-\lambda\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
I_{x z}^{(r)}= & I_{x z}^{(f)}+I_{x z}^{(\omega)}+\frac{1}{2}\left(I_{x x}^{\prime}-I_{z z}^{\prime}\right)\left(\sin Z_{N R}+\sin 2 i_{N L}\right) \\
& +I_{x z}^{\prime}\left(\cos 2 i_{N R}+\cos Z i_{N L}\right)+m_{f} h_{f} X_{f}+m_{\omega} h_{\omega} X_{\omega} \\
& -\rho m_{N}\left[x_{R} \sin \left(i_{N R}-\lambda\right)+X_{L} \sin \left(i_{N L}-\lambda\right)\right]
\end{aligned}
$$

BASIC Equations - FINAL SIMPlification

Roll Equation

$$
\begin{aligned}
I_{x x} \dot{p} & =-J_{x x} r_{q}+I_{x Z}^{(p)}(\dot{r}+p q) \\
& +\rho_{m_{N}} Y_{N}\left\{\ddot{i}_{N R} \cos \left(i_{N R}-\lambda\right)-i_{N C} \cos \left(i_{N C}-\lambda\right)\right\} \\
& +\mathcal{L}_{A E R D}
\end{aligned}
$$

Pitch equation

$$
\begin{aligned}
I_{y Y g} \dot{g}= & -J_{Y Y} p r-I_{X Z}^{(g)}\left(p^{2}-r^{2}\right) \\
& -\ddot{i}_{N R}\left\{I_{Y Y}^{\prime}+l m_{N}\left[-Z_{R} \sin \left(i_{N R}-\lambda\right)+X_{R} \cos \left(i_{N R}-\lambda\right)\right]\right\} \\
& -\ddot{i}_{N L}\left\{I_{Y Y}^{\prime}+\operatorname{lm}_{N}\left[-Z_{L} \sin \left(i_{N L}-\lambda\right)+X_{L} \cos \left(i_{N L}-\lambda\right)\right]\right\} \\
& +M_{A E R O}
\end{aligned}
$$

YAW EQUATION

$$
\begin{aligned}
I_{z Z} \dot{r}= & -J_{z Z} p q-(r q-\dot{p}) I_{X Z}^{(r)} \\
& -\operatorname{lm}_{N} V_{N}\left\{\ddot{i}_{N R} \sin \left(i_{N R}-\lambda\right)-\dot{i}_{N L} \sin \left(i_{N L}-\lambda\right)\right\} \\
& +n_{A E R O}
\end{aligned}
$$

Basic Equations
Right Nacelle Actuator Pitching Moment Equation

$$
\begin{aligned}
M_{N R A C T}= & -\ddot{i}_{N R}\left[I_{Y Y}^{\prime}+\rho^{2} m_{N}\left(1-\frac{m_{N}}{m}\right)\right] \\
& -\rho^{2} m_{N}\left(1-\frac{m_{N}}{m}\right)\left[-p r \cos 2\left(i_{N R}-\lambda\right)+\dot{g}\right. \\
& \left.+\left(r^{2}-P^{2}\right) \sin \left(i_{N R}-\lambda\right) \cos \left(i_{N R}-\lambda\right)\right] \\
& -\left(r^{2}-\rho^{2}\right)\left[I_{Z Z}^{\prime} \sin i_{N R} \cos i_{N R}\right]-I_{Y Y}^{\prime} \dot{q} \\
& +\rho \frac{m_{N}}{m}\left[X_{A E R O} \sin \left(i_{N R}-\lambda\right)+Z_{A E R O} \cos \left(i_{N R}-\lambda\right)\right] \\
& -\rho_{m_{N}} Y_{N}\left\{(\dot{r}-p g)\left[\sin \left(i_{N R}-\lambda\right)\right]\right. \\
& \left.-(\dot{p}+r q)\left[\cos \left(i_{N R}-\lambda\right)\right]\right\} \\
& +m_{N R A E R O}
\end{aligned}
$$

Note: The above equation must be calculated for wing torsion calculation only

Basic Equations
Left Nacelle Actuator Pitching Moment Equation

$$
\begin{aligned}
& M_{\text {NLACT }}=-\ddot{i}_{N L}\left[I_{Y Y}^{\prime}+l^{2} m_{N}\left(1-\frac{m_{N}}{m}\right)\right] \\
& -\Omega^{2} m_{N}\left(1-\frac{m_{N}}{m}\right)\left[-\operatorname{pr} \cos 2\left(i_{N L}-\lambda\right)+\dot{g}\right. \\
& \left.+\left(r^{2}-\rho^{2}\right) \sin \left(i_{N_{C}}-\lambda\right) \cos \left(i_{N_{L}}-\lambda\right)\right] \\
& -\left(r^{2}-P^{2}\right)\left[I_{Z Z}^{\prime} \sin i_{N C} \cos i_{N L}\right]-I_{Y Y}^{\prime} \dot{g}^{\circ} \\
& +\rho \frac{m_{N}}{m}\left[x_{A_{\text {Nan }}} \sin \left(i_{N_{L}}-\lambda\right)+Z_{A_{A K O}} \cos \left(i_{N_{4}}-\lambda\right)\right] \\
& +\Omega m_{N} Y_{N}\left\{(\dot{r}-\rho g)\left[\sin \left(i_{N L}-\lambda\right)\right]\right. \\
& \left.-\left(\dot{p}+r_{q}\right)\left[\cos \left(i_{N L}-\lambda\right)\right]\right\} \\
& +M_{\text {NLAERO }}
\end{aligned}
$$

Note: The above equation must be calealated for wing torsion calculation ouch.

Mikes Equations of Motion ( $\mu, \theta, \phi$-EuLER SYstem)

$$
\begin{aligned}
& \dot{\nu}=\frac{Y A E R_{0}}{m}+g \cos \theta \sin \phi-r u+p \omega \\
& \dot{\omega}=\frac{Z_{A E R O}}{m}+g \cos \theta \cos \phi+q u-p^{2}
\end{aligned}
$$

Euler Angle Calculation - $\psi, \theta, \phi$ system

$$
\begin{aligned}
& \dot{\psi}=(r \cos \phi+g \sin \phi) / \cos \theta \\
& \dot{\theta}=g \cos \phi-r \sin \phi \\
& \dot{\phi}=p+\dot{\psi}_{\sin } \theta
\end{aligned}
$$

Aircraft Condition Calculations

Ground Track
NORTH WARD VELOCity

$$
\begin{aligned}
\dot{x}_{\text {NoRA }}= & u \cos \theta \cos \psi+2(\sin \phi \sin \theta \cos \psi \\
& -\cos \phi \sin \psi) \\
& +\omega(\cos \phi \sin \theta \cos \psi+\sin \phi \sin \psi)
\end{aligned}
$$

Eastward Velocity

$$
\begin{aligned}
\dot{\varphi}_{\text {EAst }}= & U \cos \theta \sin \psi+V(\sin \phi \sin \theta \sin \psi+\cos \phi \cos \psi) \\
& +w(\cos \phi \sin \theta \sin \psi-\sin \phi \cos \psi)
\end{aligned}
$$

Down ward Velocity

$$
\dot{z}_{D o w n}=-U \sin \theta+V \sin \phi \cos \theta+W \cos \phi \cos \theta
$$

AIRCRAFT CONDITION CALCULATIONS (CONTINUeD)
PILOT STATION ACCELERATIONS (BODY AXES)

$$
\begin{aligned}
a_{X P A}= & \frac{x_{P F R O}}{m}+(\dot{q}+p r)\left(z_{P A}-z_{C G}\right) \\
& +\left(q^{2}+r^{2}\right)\left(x_{C G}-\rho_{P A}\right)+Y_{P A}(p q-\dot{r}) \\
& -2 g \dot{z}_{C G}-\ddot{x}_{C G} \\
a_{Y P A}= & \frac{Y_{A F R O}}{m}+(\dot{p}-q r)\left(z_{C G}-Z_{P A}\right)+(\dot{r}+p q)\left(\rho_{P A}-x_{C G}\right) \\
& -Y_{P A}\left(r^{2}+p^{2}\right)+2\left(p \dot{z}_{C G}-r \dot{X}_{C G}\right) \\
a_{Z P A}= & \frac{Z_{A E R O}}{m}+(\dot{q}-p r)\left(x_{C G}-\rho_{P A}\right)+\left(p^{2}+q^{2}\right)\left(z_{C G}-Z_{P A}\right) \\
& +Y_{P A}(\dot{p}+q r)+2 q \dot{X}_{C G}-\ddot{z}_{C G}
\end{aligned}
$$

Pilot Station Velocities (Boor Axes)

$$
\begin{aligned}
& u_{P A}=u_{P}+q z_{P A}-r Y_{P A} \\
& v_{P A}=v_{P}+r l_{P A}-p Z_{P A} \\
& w_{P A}=w_{P}+p Y_{P A}-q l_{P A}
\end{aligned}
$$

GUST MODEL

The Gust Model to be used with the simulation will consist of:

NASA-ANES program number NAPS-BO. The out put of this program is in the form of gust components that will be added to the inertial components of the basic equations of motion. In proaction, the following equations will be used in formulating the input to the aerodyummic coordinate transforms etc.,.

$$
\begin{array}{ll}
u=u^{\prime}+u_{g} & P=P^{\prime}+P_{g} \\
v=v^{\prime}+v_{g} & Q=Q^{\prime}+Q_{g} \\
w=w^{\prime}+w_{g} & R=R^{\prime}+R_{g}
\end{array}
$$

The primed terms abow are decried from the basic equations.
Alterations to nomenclature in the Vertol equations has been nosiotad at this time for: the sate of simplicity.
$\qquad$
CENTER OF GRAVITY CALCULATIONS

$$
\begin{aligned}
& m=\frac{1}{32.174}\left[w_{f}^{\prime}+w_{N T}^{\prime}+w_{A T}^{\prime}+w_{\nu T}^{\prime}+w_{\psi}^{\prime}+w_{N F}^{\prime}+w_{C R}^{\prime}+w_{\text {EUR }}^{\prime}+w_{c}^{\prime}\right] \\
& m_{N}=\frac{1}{32.174}\left(\frac{\omega^{\prime} N r}{2}\right) \\
& m_{f}=\frac{1}{32.174}\left[w_{f}^{\prime}+w_{M T}^{\prime}+w_{\nu T}^{\prime}+w_{C R}^{\prime}+w_{c}^{\prime}\right] \\
& m_{\omega}=\frac{1}{32.174}\left[w_{\omega}^{\prime}+w_{\text {FUn }}^{\prime}+w_{\nu F}^{\prime}\right] \\
& \ell_{f}^{\prime}=\left[(F S)_{p}-(F S)_{f^{\prime}}\right] \frac{1}{12} \\
& \ell_{H 7}^{\prime}=\left[(F S)_{P}-(F S)_{M T C E}\right] \frac{1}{12} \\
& l_{U T}^{\prime}=\left[(F S)_{P}-(F S)_{U T C G}\right] \frac{1}{12} \\
& \ell_{P A}^{\prime}=\left[(F S)_{P}-(F S)_{P A}\right] \frac{1}{12} \\
& l_{c}^{\prime}=\left[(F S)_{p}-(F S)_{c}\right] \frac{1}{12} \\
& l_{f}=\frac{1}{m_{f}(32.174)}\left[w_{f}^{\prime} l_{f}^{\prime}+w_{M T}^{\prime} l_{M T}^{\prime}+w_{\nu T}^{\prime} l_{\nu T}^{\prime}+w_{C R}^{\prime} l_{P A}+w_{c}^{\prime} l_{c}^{\prime}\right] \\
& \ell_{\omega}^{\prime}=\left[(F S)_{p}-(F S)_{\omega}\right] \frac{1}{12} \\
& \ell_{F U E L}^{\prime}=\left[(F S)_{P}-(F S)_{F U E Z}\right] \frac{1}{12} \\
& \ell_{N F}^{\prime}=\left[(F S)_{P}-(F S)_{N F}\right] \frac{1}{12} \\
& \ell_{\omega}=\frac{1}{m_{\omega}(32.174)}\left[w_{\omega}^{\prime} l_{\omega}^{\prime}+w_{\text {FoR }}^{\prime} l_{F U R}^{\prime}+w_{N F}^{\prime} l_{N F}^{\prime}\right]
\end{aligned}
$$

PRELIMINARY CALCULATIONS (CONT'D.)

$$
\begin{aligned}
& z_{f}^{\prime}=\left[(w L)_{p}-(w L)_{f^{\prime}}\right] \frac{1}{12} \\
& z_{i r r}^{\prime}=\left[(W L)_{P}-(W L)_{\text {HTCC }}\right] \frac{1}{12} \\
& z_{\nu T}^{\prime}=\left[(\omega L)_{P}-(\omega L)_{\nu T C G}\right] \frac{1}{12} \\
& Z_{P A}=\left[(\omega L)_{P}-(\omega L)_{P A}\right] \frac{1}{12} \\
& z_{c}^{\prime}=\left[(W L)_{p}-(W L)_{c}\right] \frac{1}{12} \\
& h_{P}=\frac{1}{(32.174)\left(m_{f}\right)}\left[w_{f}^{\prime} z_{f}^{\prime}+w_{H T}^{\prime} z_{H T}^{\prime}+w_{\nu T}^{\prime} z_{V T}^{\prime}+w_{C R}^{\prime} Z_{P A}+w_{c}^{\prime} z_{C}^{\prime}\right] \\
& z_{w}^{\prime}=\left[(w L)_{p}-(w L)_{\omega}\right] \frac{1}{12} \\
& z^{\prime} \text { FUUL }=\left[(w L)_{p}-(w L)_{\text {FUUR }}\right] \frac{1}{12} \\
& z^{\prime} N F=\left[(W L)_{P}-(W L)_{N F}\right] \frac{1}{12} \\
& h_{\omega}=\frac{1}{(32.174) m_{\omega}}\left[w_{\omega}^{\prime} z_{\omega}^{\prime}+w_{\text {FUR }}^{\prime} z_{\text {FVER }}^{\prime}+w_{\nu F}^{\prime} z^{\prime} \omega F\right] \\
& x_{W A C}=\left[(F S)_{P}-(F S)_{W R C}\right] \frac{1}{12} \\
& Y_{\omega_{A C}}=\left[(B L)_{\omega_{A C}}\right] \frac{1}{12} \\
& Z_{W_{P C}}=\left[(W L)_{P}-(W L)_{W_{A C}}\right] \frac{1}{12} \\
& Y_{N}=\left[(B L)_{N}\right] \frac{1}{12}
\end{aligned}
$$

PRELIMINARY CALCULATIONS (CONTD)

$$
\begin{aligned}
& X_{H T}=\left[(F S)_{P}-(F S)_{H T}\right] \frac{1}{12} \\
& Z_{H T}=\left[(W L)_{P}-(W L)_{H T}\right] \frac{1}{12} \\
& X_{\nu T}=\left[(F S)_{P}-(F S)_{\nu T}\right] \frac{1}{12} \\
& Z_{u T}=\left[(w L)_{p}-(w L)_{v T}\right] \frac{1}{12} \\
& A=3.14159 R^{2} \\
& \vec{Y}_{\omega_{B C}}=\left[(\overline{B L})_{w B C}\right] \frac{1}{12} \\
& X_{G 2}=X_{G 1}=\left[(F S)_{P}-(F S)_{G 2}\right] \frac{1}{12} \\
& Z_{G_{2}}=Z_{G_{1}}=\left[(w L)_{P}-(w L)_{G_{2}}\right] \frac{1}{12} \\
& Y_{G 2}=\left[(B L)_{G 2}\right] \frac{1}{12} \\
& Y_{G 1}=-Y_{G 2} \\
& Y_{G 3}=0 . \\
& Y_{P A}=\left[(B L)_{P A}\right] \frac{1}{12} ; \text { POSITIVE FOR PILOT IN RIGHT SEAT } \\
& X_{f x c}=\left[(F S)_{p}-(F S)_{f x c}\right] \frac{1}{12} \\
& Z_{f A C}=\left[(W L)_{P}-(W L)_{f A C}\right] \frac{1}{12}
\end{aligned}
$$

PRELIMINARY CALCULATIONS (CONTD)

$$
\begin{aligned}
& X_{C / 2}=\left[(F S)_{P}-(F S)_{C / 2}\right] \frac{1}{12} \\
& Z_{G 3}=\left[(W L)_{P}-(W L)_{G 3}\right] \frac{1}{12} \\
& X_{G 3}=\left[(F S)_{P}-(F S)_{G 3}\right] \frac{1}{12} \\
& Y_{N F}=\left[(B L)_{N F_{C G}}\right] \frac{1}{12} \\
& Y_{H T}=\left[(B L)_{H T_{C G}}\right] \frac{1}{12} \\
& Y_{W}=\left[(B L)_{W_{C G}}\right] \frac{1}{12} \\
& Y_{F U E 2}=\left[(B L)_{F U U C C G}\right] \frac{1}{12}
\end{aligned}
$$

INERTIA CALCULATIONS

$$
\begin{aligned}
& \eta_{f^{\prime}}^{\prime}=l_{f}-l_{P}^{\prime} \\
& \delta_{f^{\prime}}^{\prime}=h_{f}-z_{f}^{\prime} \\
& \eta_{H T}^{\prime}=l_{f}-l_{H T}^{\prime} \\
& \delta_{H T}^{\prime}=h_{f}-z_{M T}^{\prime} \\
& \eta_{V T}^{\prime}=l_{f}-l_{V T}^{\prime} \\
& \delta_{V T}^{\prime}=h_{P}-z_{U T}^{\prime} \\
& \eta_{C R}^{\prime}=l_{f}-l_{P A} \\
& \delta_{C R}^{\prime}=h_{f}-Z_{P A}
\end{aligned}
$$

PRELIMINARY CALCULATIONS (CONT'D)

$$
\begin{aligned}
& \eta_{c}^{\prime}=l_{f}-l_{c}^{\prime} \\
& \delta_{c}^{\prime}=h_{f}-z_{o}^{\prime} \\
& I_{y y}^{(\rho)}=I_{y y_{0}}^{\left(\omega_{\prime^{\prime}}\right)}+I_{y y_{0}}^{(\mu r)}+I_{y y_{0}}^{(\nu T)}+I_{y y_{0}}^{(\underline{q} \Omega)}+I_{y y_{0}}^{(\epsilon)}+\frac{\omega_{f}^{\prime}}{32.174}\left(\eta_{f^{\prime}}^{\prime 2}+\delta_{f^{\prime}}^{\prime 2}\right) \\
& +\frac{w_{i / 7}^{\prime}}{32.174}\left(\eta_{\mu 7}^{\prime 2}+\delta_{M T}^{\prime 2}\right)+\frac{w_{\nu T}^{\prime}}{32.174}\left(\eta_{V T}^{\prime 2}+\delta_{\nu T}^{\prime 2}\right) \\
& +\frac{W_{c R}^{\prime}}{32.174}\left(\eta_{C R}^{\prime 2}+\delta_{c R}^{\prime 2}\right)+\frac{W_{c}^{\prime}}{32.174}\left(\eta_{c}^{\prime 2}+\delta_{c}^{\prime 2}\right) \\
& I_{x x}^{(\rho)}=I_{x x_{0}}^{\left(\omega_{0}\right)}+I_{x x_{0}}^{(\mu \tau)}+I_{x x_{0}}^{(\nu T)}+I_{x x_{0}}^{(C A)}+I_{x x_{0}}^{(\alpha)}+\frac{w_{f}^{\prime}}{32.174} \delta_{f}^{\prime 2}+\frac{w^{\prime} \mu T}{32.174}\left(\delta_{\mu T}^{\prime 2}+Y_{\mu T}^{2}\right) \\
& +\frac{w_{\nu T}^{\prime}}{32.174} \delta_{\nu T}^{\prime 2}+\frac{w_{c R}^{\prime}}{32.174} \delta_{c R}^{2}+\frac{w_{c}^{\prime}}{32.174} \delta_{c}^{\prime 2} \\
& I_{z z}^{(f)}=I_{z z_{0}}^{\left(\omega_{\rho_{0}}\right)}+I_{z z_{0}}^{(H T)}+I_{z z_{0}}^{(\nu T)}+I_{z z_{0}}^{(R R)}+I_{z z_{0}}^{(e)}+\frac{W_{f}^{\prime}}{32.174} \eta_{f^{\prime}}^{\prime 2}+\frac{W_{H T}^{\prime}}{32.174}\left(\eta_{H T}^{\prime 2}+Y_{H T}^{2}\right) \\
& +\frac{w_{Y T}^{\prime}}{32.174} \eta_{V r}^{\prime 2}+\frac{w_{C R}^{\prime}}{32.174} \eta_{C R}^{\prime 2}+\frac{w_{C}^{\prime}}{32.174} \eta_{c}^{\prime 2} \\
& I_{x z}^{(f)}=I_{x z_{0}}^{\left(w_{0}\right)}+I_{x z_{0}}^{(1 / 7)}+I_{x z_{0}}^{(\nu 7)}+I_{x z_{0}}^{(c \beta)}+I_{x z_{0}}^{(c)}+\frac{W_{f}^{\prime}}{32.174} \eta_{f}^{\prime \prime} \delta_{f^{\prime}}^{\prime} \\
& +\frac{W_{H T}^{\prime}}{32.174} \eta_{H T}^{\prime} \delta_{H T}^{\prime}+\frac{W_{V T}^{\prime}}{32.174} \eta_{V T}^{\prime} \delta_{V T}^{\prime}+\frac{w_{C R}^{\prime}}{32.174} \eta_{C R}^{\prime} \delta_{C R}^{\prime} \\
& +\frac{W^{\prime} c}{32,174} \eta_{c}^{\prime} \delta_{c} \\
& H_{w^{\prime}}^{\prime}=l_{w}-l_{\omega}^{\prime} \\
& \Delta_{w_{w}^{\prime}}^{\prime}=h_{\omega}{ }^{-} z_{w}^{\prime} \\
& H_{w_{\text {FUNR }}^{\prime}}^{\prime}=l_{\omega}-l_{\text {sun }}^{\prime} \\
& \Delta^{\prime} w^{\prime} \text { fuer }=h_{w}-z_{\text {fute }}^{\prime}
\end{aligned}
$$

$$
\begin{aligned}
& H_{\text {w'UF }}^{\prime}=l_{w}-l_{\text {NF }}^{\prime} \\
& \Delta_{\omega_{N F}}^{\prime}=h_{\omega}-z_{N F}^{\prime} \\
& I_{y y}^{(\omega)}=I_{y y_{0}}^{\left(\omega^{\prime} \psi\right)}+I_{y y_{0}}^{\left(\omega^{\prime} \mathrm{Fwn}\right)}+I_{y y_{0}}^{\left(W_{\nu \rho}^{\prime}\right)}+\frac{\omega_{\omega}^{\prime}}{32.174}\left(H_{\omega_{\psi}^{\prime}}^{\prime 2}+\Delta_{\omega_{\omega}^{\prime}}^{\prime 2}\right) \\
& +\frac{W^{\prime} \text { FUEl }}{32.174}\left(H_{W^{\prime} \text { FUN }}^{\prime 2}+\Delta_{W^{\prime} \text { GeEZ }}^{\prime 2}\right)+\frac{W_{N F}^{\prime}}{32.174}\left(H_{W_{N E}^{\prime}}^{\prime 2}+\Delta_{W^{\prime} N \sigma}^{\prime 2}\right)
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{W_{\text {FUn }}}{32.174}\left(\Delta_{W \prime \text { FUR }}^{\prime 2}+Y_{\text {FUN }}^{2}\right)+\frac{W_{N F}^{\prime}}{32.174}\left(\Delta_{W_{\text {NF }}}^{2}+Y_{N F}^{2}\right)
\end{aligned}
$$

$$
\begin{aligned}
& I_{x z}^{(w)}=I_{x z_{0}}^{\left(w_{\psi}^{\prime}\right)}+I_{x z_{0}}^{\left(w^{\prime} F \nu \Omega\right)}+I_{x z_{0}}^{\left(\omega^{\prime} \mu \mu\right)}+\frac{w^{\prime} \omega}{32.174} H_{w_{\omega}^{\prime}}^{\prime} \Delta_{w_{w}^{\prime}}
\end{aligned}
$$

Math Model Trim Loops - Steady Flight
$\frac{\text { Initialize }}{\text { FLIGHT CONDItion. }} U, V, h, \Omega, P, q, r$ at desired
Close the following trim feedback loops TO TRIM MATH MODEL FOR FLIGHT.

$$
\begin{aligned}
& i_{\text {REF }}=K_{T 1} \int \dot{u} d t+K_{T 2} \dot{U} \\
& \phi=-K_{T 3} \int \dot{v} d t-K_{T+} i \\
& \omega=k_{T 5} \int \dot{w} d t+k_{T 6} \dot{w} \\
& \delta_{S}=-K_{T 7} \int \dot{p} d t-K_{r o} \dot{p} \\
& \delta_{R}=-K_{T 9} \int \dot{r} d t-K_{T 10} \dot{r} \\
& \delta_{B}=-K_{T 11} \int \dot{q} d t-K_{T 12} \dot{q} \\
& \dot{c}_{N R}=K_{T / 3} \iint \ddot{i}_{N R} d t d t+K_{T / 4} \int \ddot{i}_{N R} d t \\
& i_{N L}=K_{T / 3} \iint \ddot{i}_{N L} d t d t+K_{T / 4} \int \ddot{i}_{N R} d t
\end{aligned}
$$

NOTE: 1.) HOLD integrated value when going to flight.
2.) START SECOND TRIM FROM FIRST TRIM VALUES.
3.) Determine K's Experimentally to minimize trim time.
4.) TRIM WTHH ALL ACTUNTOR OVNAMKS, SAS, AND GOVERNOR IN OPERATING CONDITION TO INSURE PROPER COMMIT CONTRA AND COLLECTING POSITIONS.


Math MODE TRIM LOOP OPTIONS
1.) When specifying indef, form:

$$
\theta=K_{T n} \int \dot{u} d t+K_{T 1 Q} \dot{u}
$$

Note: This option will be commonly used in cruise flight when the nacelles ane down and Locked.
presented in this section is the input data for the mathematical model. A general description of the Model 222 tilt rotor was given in Section 4.0. Model 222 dimensional data and control surface deflections and travels are given on the following pages. Weight, balance and moment of inertia data for five nominal design operating conditions are defined in Figure F.l. Center of gravity envelopes for the condition of nacelle incidence zero (cruise configuration) and nacelle incidence 90 degrees (hover configuration) are illustrated in Figure F.2. The mathematical model input data are given in Section F.l to F. 5 and are referenced by page number to the equations presented in Appendix E.

MODEL 222 DIMENSIONAL DATA

## WING

| AREA (THEO.) | 200 FT 2 |
| :---: | :---: |
| ASPECT RATIO | 5.61 |
| SPAN (BETWEEN ROTOR ¢) | 33.42 FT |
| TAPER RATIO | 1.00 |
| CHORDS: |  |
| ROOT | 71.8 IN |
| TIP | 71.8 IN |
| MEAN AERODYNAMIC | 71.8 IN |
| SWEEPBACK | 0 DEGREES |
| DIHEDRAL | 0 DEGREES |
| INCIDENCE |  |
| ROO' | 2.0 DEGREES |
| TIP | 2.0 DEGREES |
| AIRFOIL SECTION |  |
| ROOT | NACA 63 ${ }^{221}$ (MODIFIED) |
| TIP | NACA 634221 (MODIFIED) |

FUSELAGE

LENGTH
DEPTH (NOT INCLUDING SPONSONS) 5.45 FT
WIDTH (NOT INCLUDING SPONSONS) 5.45 FT
WETTED AREA (INCLUDING SPONSONS) $582 \mathrm{FT}^{2}$

MODEL 222 DIMENSIONAL DATA (Continued)

## NACELLES

ENGINE

LENGTH

DEPTH

WIDTH
WETTED AREA (PER NACELLE)

TILTING
5.58 FT
2.37 FT
2.37 FT
$21 \mathrm{FT}^{2}$
3.70 FT
3.35 FT
2.37 FT
$22 \mathrm{FT}^{2}$

HORIZONTAL TAIL

AREA (EXPOSED)
AREA (THEO)

SPAN
ASPECT RATIO

TAPER RATIO
DISTANCE $(\bar{c} / 4)_{W}$ to $(\bar{c} / 4)_{\mathrm{HT}}$ CHORDS

ROOT

TIP

MEAN AERODYNAMIC

SWEEPBACK AT 0 PERCENT CHORD

DIHEDRAL
$46.3 \mathrm{FT}^{2}$
$58.3 \mathrm{FT}^{2}$
15.75 FT
4.26
.379
20.29 FT
66.0 IN
25.0 IN
48.0 IN
$14^{\circ} 51^{\prime}$

0 DEGREES

MODEL 222 DIMENSIONAL DATA (Continued)
INCIDENCE
ROOT 0 DEGREES
TIP 0 DEGREES
AIRFOIL SECTION
ROOT
TIP
NACA 64AO10 (MODIFIED
NACA 64A010 (MODIFIED)
VERTICAL TAIL
AREA (EXPOSED, EXCLUDES DORSAL) $35.5 \mathrm{FT}^{2}$
AREA (REFERENCE)
$43.3 \mathrm{FT}^{2}$
SPAN (REFERENCE)
8.14 FT

ASPECT RATIO
1.53

TAPER RATIO
.303
DISTANCE $(\bar{c} / 4)$ W to $(\bar{c} / 4)_{V T}$
18.88 FT CHORDS (REFERENCE)

ROOT
8.17 FT

TIP
2.48 FT

MEAN AERODYNAMIC
5.83 FT

SWEEPBACK AT 0 PERCENT CHORD
$46^{\circ} 28^{\prime}$
AIRFOIL SECTION
NACA 64A008
CONTROL SURFACES
FLAPERON

| AREA (AFT OF HINGE) | 52.5 FT |
| :--- | :---: |
| SPAN (LENGTH EACH SIDE) | 151.56 IN |
| CHORD ( $\%$ OF WING CHORD) | 30 |
| SWEEPBACK OF HINGE LINE | $0^{\circ}$ |
| OILERS |  |
| AREA | $19.15 \mathrm{FT}^{2}$ |

Control Surface Deflections (Positive deflection is trailing edge down unless indicated otherwise)


## Rotor Control Authorities

Longitudinal Cyclic

Differential Longitudinal Cyclic Maximum Longitudinal Cyclic for Combined Inputs $\quad \pm 7^{\circ}$ Collective Pitch
Differential Collective

Lateral Cyclic
$\pm 2.5^{\circ}$ for Pitch Trim Plus
Maneuver
$+2.7^{\circ}$ for Combined SAS + Lbad
Alleviation
$\pm 4.5^{\circ}$ Maximum for Roll Command
(Function of Nacelle Incidence)
$0^{\circ}$ to $56.5^{\circ}$ (at .75R)
$+3.0^{\circ}$ Maximum for Yaw Command $\pm 4.8^{\circ}$ for Maximum Roll Command (Function of Nacelle incldence) $\pm 2.7^{\circ}$ for Rotor Load Alleviation

## Nacelle Deflection Authorities

Nacelle Tilt $0^{\circ}$ to $105^{\circ}$
Differential Nacelle $1.55^{\circ}$ Per Degree Differential Tilt

Longitudinal Cyclic
Pilot Control Movements
Stick - Longitudinal

- Lateral

Rudder Pedals
$\pm 6$ Inches
$\pm 5$ Inches
$\pm 2.5$ Inches
MASS PROPERTIES

|  |  | NACELLE |  |  |  |  |  |  | NACELLE VERTICAL |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  | BALANCE |  |  | INERTIA - SLUG FTT |  |  |  |
| SUB-groups | weight | $\begin{array}{\|c} \hline x \\ \text { F.S.) } \\ \hline \end{array}$ | $\begin{gathered} \text { (w.s.) } \end{gathered}$ | $\underset{(w, L .)}{2}$ | $\underset{(\text { ROLL })}{I_{X X}}$ |  | $\begin{aligned} & I_{2 z}{ }_{2} \\ & (\text { YAWW } \end{aligned}$ | ${ }^{1} \times 2$ | $\begin{gathered} x \\ (\text { F.S. }) \end{gathered}$ | $\begin{gathered} Y \\ \text { (w.s.) } \end{gathered}$ | $\stackrel{w_{\text {w.L. }}}{ }$ | $\begin{gathered} I_{X X} \\ (R O L L) \end{gathered}$ | $\underset{(\mathrm{PITCH})}{\mathrm{I} \mathrm{I}_{2}}$ | $\begin{gathered} \mathrm{I}_{\mathrm{I} X} \\ \text { (YAW } \end{gathered}$ | ${ }^{1} \mathrm{xz}$ |
| - fuselage ${ }^{*}$ CONTENTS | 3696 | 149.7 |  | -9.0 | 903 | 6058 | 6152 |  | 200.7 54.3 |  |  | 389 | 388 | 45 |  |
| - horizomtal | 130 | 419.4 | 27.7 | 13.4 | 23 | 15 | 38 |  |  |  |  |  |  |  |  |
| $-\underset{\text { tail }}{\text { vertical }}$ | 96 | 406.7 |  | 63.8 | 15 | 22 | 7 |  |  |  |  |  |  |  |  |
| - wing contents | 1695 | 182.2 | 107.4 | 40.3 | 1266 | 200 | 1346 |  |  |  |  |  |  |  |  |
| - nacelle contents | (4267) | 160.0) | (210.1) | (42.4) | (224) | (1393) | (1507) |  |  |  |  |  |  |  |  |
| $\begin{aligned} & \text { FIXED } \\ & \text { TILTING } \end{aligned}$ | $\begin{aligned} & 1475 \\ & 2792 \end{aligned}$ | $202.7$ | $\begin{aligned} & 227.8 \\ & 200.7 \end{aligned}$ | $\begin{aligned} & 43.1 \\ & 42.0 \end{aligned}$ | $\begin{aligned} & 43 \\ & 45 \end{aligned}$ | $\begin{aligned} & 115 \\ & 368 \end{aligned}$ | $\begin{gathered} \varepsilon 1 \\ 3 \varepsilon_{6} \end{gathered}$ |  |  |  |  |  |  |  |  |
| OPER. MEIGHT EMPTY | 9884 | 165.8 |  | 22.6 | 48674 | 12255 | 57263 | +90 | 176.8 |  | 34.6 | 50751 | 13308 | 56238 | 1141 |
| - observer | 200 | 150.0 |  | 1.0 | 6 | \% | ) |  |  |  |  |  |  |  |  |
| - FUEL | 1280 | 178.6 | 109.0 | 37.0 | 380 |  | 46 |  |  |  |  |  |  |  |  |
| - instru. \& RES | 1157 | 178.6 <br> 171.5 |  | .5 .5 | 110 110 | 275 250 | 325 |  |  |  |  |  |  |  |  |
| basic design |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| gross weight | 23541 | 168.3 |  | 20.1 | 52843 | 13191 | 6157\% | 474 | 1:6.3 |  | 28.6 | 55176 | 14482 | 50533 | 1197 |
| - observer | 200 | 150.0 |  | 1.0 |  |  |  |  |  |  |  |  |  |  |  |
| - fuel | 1375 | 178.6 | 109.0 | 37.0 | 380 | 32 | -15 |  |  |  |  |  |  |  |  |
|  | 1157 | 178.6 |  | .5 <br> .5 | 110 | 275 250 | 300 225 |  |  |  |  |  |  |  |  |
| - other | 1864 | 171.5 |  | . 5 | 120 | 350 | 375 |  |  |  |  |  |  |  |  |
| alternate |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| gross weikht | 15500 | 168.8 |  | 17.9 | 53308 | 13697 | 62254 | 412 | 173.7 |  | 25.5 | 55781 | 15120 | 61204 | 1182 |
| : FUEL | 127 | (178.6 |  | ${ }^{37.0}$ | 110 | 275 | 400 |  |  |  |  |  |  |  |  |
| TYPICAL FLIGHT |  |  |  | 22.0 | 52573 | 12787 | 61359 | 453 | 177.1 |  | 31.6 | 54826 | 13971 | 60290 | 1104 |
| $\frac{\text { TAKE-OFF WEIGHT }}{\text { FUEL }}$ | 12322 | 1 168.3 | 109.0 | 32.0 | ${ }_{18}{ }^{18}$ | 12 | 195 |  |  |  |  |  |  |  |  |
| - instru. 4 RES | 1157 | 178.6 |  | . 5 | 110 | 275 | 300 |  |  |  |  |  |  |  |  |
| MEAN OPERATING gross weight | 11800 | 167.9 |  | 21.1 | 51053 | 12717 | 59743 | 425 | 177.1 |  | 31.1 | 53291 | 13856 | 58673 | 1101 |

- rotor blade weight (complete as removed from hub), balance and inertias (io) are as follows:
 Figure F.1. Mass Properties
F-7/-8

F. 1 CONTROL SYSTEIA INPUT DATA

The input data for the control system, SAS, thrust management, and load alleviation system are in this section, and are referenced by page number to the equations presented in Appendix E. Figures F. 3 to F. 14 present the scheduled function.
F.1.1 Control System Input Data
(1)

Control Mixing (Page E.4)
$K_{\delta R U D}=-8 \mathrm{deg} /$ inch
$K_{\delta_{R}}=1.0$
$K_{\delta_{S}}=1.0$
$K_{\delta_{B}}=1.0$
$K_{\delta e}=-3.33 \mathrm{deg} /$ inch
$K_{\delta}{ }_{s}=0$
Actuator Dynamics
$\omega_{N}=20 \mathrm{rad} / \mathrm{sec}$
$\zeta=1.0$

Lead-lag Dynamics
$\omega_{\mathrm{L}-\mathrm{L}}=35.5 \mathrm{rad} / \mathrm{sec}$
$\zeta=.18$

Scheduled Functions - Refer to Graphs
a) Scheduled Longitudinal Cyclic vs Nacelle Incidence
(1) Gains and time constants not shown on these pages are noted on the block diagrams.
b) Cyclic Gain vs Nacelle Incidence (Pedals)
c) Differential Collective Gain vs Nacelle Incidence (Pedals)
d) Differential Collective Gain vs Nacelle'Incidence (Lateral Stick)
e) Longitudinal Cyclic Gain vs Nacelle Incidence (Long. Stick)
f) Lateral Cyclic Gain vs Nacelle Incidence
g) Elevator Deflection vs Nacelle Incidence
h) Scheduled Lateral Cyclic vs Nacelle Incidence

Load Alleviation System (LAS) (Page E.5)
${ }^{\top}$ LAS $=0.2 \mathrm{sec}$.

## LAS Functions

$G_{B_{1_{\alpha}}}, G_{A_{1_{\beta}}},{ }^{G_{A_{1}}}$ vs Dynamic Pressure

Nacelle and Airplane Controls (Page E.6)
Nacelle Actuator Dynamics
$\omega_{N R}=\omega_{N L}=10 \mathrm{rad} / \mathrm{sec}$
$\zeta_{\mathrm{NL}}=\zeta_{\mathrm{NR}}=1.0$

## Scheduled Functions

a) Scheduled Flap Angle vs Nacelle Incidence
b) Flaperon vs Lateral Stick
c) Spoiler Deflection vs Lateral Stick
d) Spoiler Actuator Limit

Stability Augmentation System (Page E. 7 and E.8)
Gains, time constants and scheduled functions noted on block diagrams.
F-12

Roll SAS Authoritiy Limit $= \pm 2$ inches
Yaw SAS Authority Iimit $= \pm 1$ inch
Pitch SAS Authority Limit $= \pm 2.7^{\circ}$

Thrust Management System (Page E.13)

$$
\begin{aligned}
& \left(N_{I I} / N_{I I_{M A X}}\right)_{\mathrm{REF}}=.8865 \\
& \Omega_{\mathrm{REF}}=57.6923 \mathrm{rad} / \mathrm{sec} \\
& { }_{\eta_{\mathrm{TR}}}=1.0 \\
& \mathrm{I}_{\mathrm{P}}=564 \mathrm{slug}-\mathrm{ft} \\
& \mathrm{G}_{\mathrm{G} 1}=2.5 \mathrm{deg} / \mathrm{sec} / \mathrm{rad} / \mathrm{sec} \\
& \mathrm{G}_{\mathrm{G} 2}=2.66 \mathrm{deg} / \mathrm{rad} / \mathrm{sec} \\
& \mathrm{G}_{\mathrm{G} 3}=.05 \mathrm{deg} / \mathrm{sec} / \mathrm{deg}
\end{aligned}
$$

## Scheduled Functions

a) Turbine Inlet Temperature vs Throttle Position
b) ${ }^{\tau} \mathrm{D} V \mathrm{~V}(\triangle H P)$
c) $T^{T} e^{\delta / \sqrt{ } \theta} \mathrm{VS}$ SHP
d) Output Gain Ratio vs Power Output
e) Governor Gain Schedule
E) RPM Select Schedule
g) Throttle Collective Gain Schedule
h) Incremental Collective Schedule
i) Variable Authority Iimit

Rotor Control Coordinate Axis Transforms (Page E.14)
$\phi_{P}=-12$ degrees



Figure F.3. Differential Long. Cyclic for Yaw Control vs Nacelle Incidence

$$
F-14
$$



Figure F.5. Longitudinal Cyclic for Pitch Control Gain vs Nacelle Incidence


Figure F.6. Load Alleviation System Gain Schedule.

$$
F-17
$$



Figure F.7. Programmed Cyclic, Elevator, and Flap Deflection vs Nacelle Incidence


Figure F.8. Roll Control Deflection vs Stick Deflection


Figure F.9. Spoiler Actuator Limit vs Airspeed
F-20
-

Figure F.10. Assumed Throttle Travel Model 222 Simulation Both Engines


Figure F.12. Engine Characteristics Lycoming T53-L13 Engine


Figure F.13. Thrust Management System-Scheduled Parameters


Figure F.14. Thrust Management System-Scheduled Parameters

## F. 2 ENGINE INPUT DATA

The input data for the engine performance subroutine is given in this section, and are referenced by page number to the equation presented in Appendix E. Plotted data are shown in Figures F.l5 to F.l8.
F.2.1 Turbine Engine Performance Input Data

Engine Performance Data (Pages E.10, E. 11 and E.12)
SHP* $=1550$
WDTIND $=1.0$
NIIND $=1.0$
NLEIND $=0$
QIND $=1.0$
$\dot{\mathrm{w}}_{\mathrm{MAX}} / \mathrm{w}^{\star}{ }^{*}=1.11$
$N_{I_{\text {MAX }}} / N_{\text {I }}=1.04$
$\left(N_{I} / \sqrt{ } \theta_{I} / N_{I}\right)_{M A X}=0$
$Q_{i V A X} / Q^{*}=1.446$
$N_{\mathrm{II}_{\mathrm{MAXX}}} / \mathrm{N}_{\mathrm{I} \mathrm{I}}^{\mathrm{t}}=1.128$
$\mathrm{N}_{\mathrm{I}}^{\mathrm{I}}=25425 \mathrm{RPM}$
$\left(N_{I I} / N_{I I_{M A X}}\right)_{R E F}=.8865$
$\Omega_{\text {REF }}=57.6923$

Tabular Engine Cycle Input Data
a) Values of Referred Horsepower
b) Values of Referred Fuel Flow
c) Values of Referred Gas Generator Speed
d) Values of Referred Power Turbine Speed
VALUES OF REFERRED HORSEPOWER SHP/ $\delta \sqrt{\theta} /$ SHP*

| MACH NO. $T / \theta=1600$ | $T / \theta=1800$ | $T / \theta=2000$ | $T / \theta=2200$ | $T / \theta=2400$ | $T / \theta=2600$ | $T / \theta=2800$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | .035 | .330 | .630 | .920 | 1.200 | 1.340 | 1.400 |
| .2 | .075 | .375 | .670 | .960 | 1.245 | 1.390 | 1.450 |
| .4 | .125 | .425 | .720 | 1.010 | 1.295 | 1.440 | 1.500 |
| .6 | .180 | .480 | .775 | 1.065 | 1.350 | 1.495 | 1.550 |
| .8 | .240 | .534 | .835 | 1.125 | 1.410 | 1.550 | 1.600 |

2. VALUES OF REEERRED FUEL FLOW W $/ \delta \sqrt{\theta} /$ SHP*

3. VALUES OF REFERRED GAS GENERATOR SPEED $N_{I} / \sqrt{\theta} / N_{\mathrm{I}}$

4. VALUES OF REFERRED POWER TURBINE SPEED $N_{I I} / \sqrt{\theta} / N_{I I}^{*}$



Figure F.15. Turbine Engine Performance - Engine Cycle 1.78


Figure F.16. Turbine Engine Performance-Engine Cycle 1.78


Figure F.17. Turbine Engine Performance - Engine Cycle 1.78


Figure F.18. Turbine Engine Performance - Engine Cycle 1.78
F. 3 ROTOR AERODYNAMIC INPUT DATA

The input data for the rotor aerodynamics are given in this section, and are referenced by page number to the equations presented in Appendix E. Tabulated coefficients of the curve fit equations are shown in Figures F. 19 to F. 27 .
F.3.1 Rotor Aerodynamic Input Data

Rotor Thrust (Page E.54)
${ }^{\tau_{1}}=.10$
$\tau_{2}=.10$
$R=13 \mathrm{Ft}$.

Rotor Force and Moment Calculations (Page E. 60)
$\mathbf{f}_{\mathrm{TR}}=\mathrm{f}_{\mathrm{TL}}=1.0$
$\mathrm{f}_{\mathrm{NFR}}=\mathrm{f}_{\mathrm{NF}_{\mathrm{L}}}=1.0$
$\mathrm{f}_{\mathrm{SFR}}=\mathrm{f}_{\mathrm{SF}}^{\mathrm{L}}$ $=1.0$
$\mathrm{f}_{\mathrm{PMR}}=\mathrm{f}_{\mathrm{PM}}^{\mathrm{I}} \mathrm{I}=1.0$
$\mathrm{f}_{\mathrm{YMR}}=\mathrm{f}_{\mathrm{YM}_{\mathrm{L}}}=1.0$
$\mathrm{f}_{\mathrm{QR}}=\mathrm{E}_{\mathrm{Q}_{\mathrm{L}}}=1.0$
$\mathrm{f}_{\mathrm{PR}}=\mathrm{f}_{\mathrm{P}_{\mathrm{L}}}=1.0$

| $\mu$ | 0 | . 1014 | . 1351 | . 2027 | . 3723 | . 4565 | . 6432 | . 8038 | 1.125 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{A}_{\mathrm{T} 0}$ | . $131167 \times 10^{-2}$ | -. $539532 \times 10^{-2}$ | -. $203368 \times 10^{-1}$ | -. $427861 \times 10^{-1}$ | $-.470950 \times 10^{-1}$ | -. $439534 \times 10^{-1}$ | -. $522445 \times 10^{-1}$ | $-.708703 \times 10^{-1}$ | $-.878713 \times 10^{-1}$ |
| ${ }^{\text {A }}$ T1 | 0 | $-.119838 \times 10^{-3}$ | . $841758 \times 10^{-4}$ | . $481855 \times 10^{-3}$ | . $266417 \times 10^{-3}$ | 0 | 0 | 0 | 0 |
| Ar 2 | 0 | . $26831 \times-10^{-5}$ | . $260200 \times 10^{-5}$ | . $399507 \times 10^{-6}$ | $-.596957 \times 10^{-5}$ | 0 | 0 | 0 | 0 |
| $\mathrm{A}_{\text {T3 }}$ | 0 | $-.904463 \times 10^{-8}$ | -. $127543 \times 10^{-7}$ | $-.681927 \times 10^{-8}$ | . $36307 \times 10^{-7}$ | 0 | 0 | 0 | 0 |
| $\mathrm{A}_{\text {T4 }}$ | . $747733 \times 10^{-3}$ | . $784148 \times 10^{-3}$ | . $254258 \times 10^{-2}$ | . $250025 \times 10^{-2}$ | . $186317 \times 10^{-2}$ | . $108914 \times 10^{-2}$ | . $408704 \times 10^{-3}$ | . $766317 \times 10^{-3}$ | . $704819 \times 10^{-3}$ |
| $\mathrm{A}_{\mathrm{T} 5}$ | 0 | . $8905 \times 10$ | -. $387201 \times 10$ | -. $220100 \times 10$ | -. $270408 \times 10^{-4}$ | 0 | 0 | 0 | 0 |
| $A_{\text {T } 6}$ | 0 | -. $699401 \times 10^{-7}$ | . $413387 \times 10^{-6}$ | $.151901 \times 10^{-6}$ | $-.510959 \times 10^{-6}$ | 0 | 0 | 0 | 0 |
| ${ }^{A_{T} 7}$ | 0 | . $156844 \times 10^{-9}$ | -. $151355 \times 10^{-8}$ | -. $407612 \times 10^{-9}$ | . $306610 \times 10^{-7}$ | 0 | 0 | 0 | 0 |
| ${ }^{\text {A }}$ T8 | . $120357 \times 10^{-4}$ | . $996174 \times 10^{-5}$ | -. $925255 \times 10^{-4}$ | $-.825240 \times 10^{-5}$ | $-.307987 \times 10^{-5}$ | $.100012 \times 10^{-4}$ | . $214303 \times 10^{-4}$ | $.163128 \times 10^{-4}$ | . $161866 \times 10^{-4}$ |
| AT9 | 0 | $-.592706 \times 10^{-8}$ | . $371573 \times 10^{-5}$ | . $215847 \times 10^{-6}$ | $-.111356 \times 10^{-6}$ | 0 | 0 | 0 | 0 |
| AT10 | 0 | -. $10436 \times 10^{-8}$ | -. $449157 \times 10^{-7}$ | $-.725582 \times 10^{-9}$ | . $769226 \times 10^{-7}$ | 0 | 0 | 0 | 0 |
| ${ }^{\text {a }}$ T11 | 0 | . $388825 \times 10^{-11}$ | . $168184 \times 10^{-9}$ | -. $196520 \times 10^{-11}$ | $1-.167159 \times 10^{-8}$ | 0 | 0 | 0 | 0 |

Figure F.19. Coefficients of Curve Fit Equations for Thrust Coefficient

| $\rightarrow$ | 0 | . 1014 | . 1351 | . 2027 | .3723 | . 4565 | . 6432 | . 8038 | 1.125 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\frac{\mu}{\text { APO }}$ | .118188×10-3 | $\frac{.117752 \times 10^{-3}}{}$ | . $575597 \times 10^{-3}$ | $.204675 \times 10^{-3}$ | $.283014 \times 10^{-3}$ | $.362823 \times 10^{-3}$ | $.511711 \times 10^{-3}$ | . $539983 \times 10^{-3}$ | $.12581 \times 10^{-2}$ |
| ${ }^{\text {A P }} 1$ | 0 | $.262107 \times 10^{-5}$ | $-.835529 \times 10^{-5}$ | $-.551959 \times 10^{-5}$ | $-.952757 \times 10^{-5}$ | 0 | 0 | 0 | 0 |
| $A^{\text {P }} 2$ | 0 | $.379347 \times 10^{-7}$ | . $973647 \times 10^{-7}$ | $.145048 \times 10^{-6}$ | -. $615163 \times 10^{-7}$ | 0 | 0 | 0 | 0 |
| $\mathrm{A}_{\mathrm{P}} 3$ | 0 | $-.147777 \times 10^{-9}$ | $-.180303 \times 10^{-9}$ | $-.391787 \times 10^{-9}$ | $.131638 \times 10^{-8}$ | 0 | 0 | 0 | 0 |
| $\mathrm{A}_{\text {P } 4}$ | $.223227 \times 10^{-1}$ | $.143376 \times 10^{0}$ | . $143289 \times 10^{0}$ | $.304517 \times 10^{0}$ | $.383466 \times 10^{0}$ | $.466013 \times 10^{0}$ | $.662959 \times 10^{0}$ | $.854989 \times 10^{0}$ | $.114909 \times 10^{-1}$ |
| $\mathrm{A}_{\text {P5 }}$ | 0 | $-.263395 \times 10^{-2}$ | $-.391803 \times 10^{-2}$ | -. $56844 \times 10^{-2}$ | -. $278572 \times 10^{-2}$ | 0 | 0 | 0 | 0 |
| $A_{P 6}$ | 0 | $.115613 \times 10^{-4}$ | $.421318 \times 10^{-4}$ | $.326549 \times 10^{-4}$ | $.129237 \times 10^{-4}$ | 0 | 0 | 0 | 0 |
| $A_{P 7}$ | 0 | -. $239424 \times 10^{-7}$ | $-.174886 \times 10^{-6}$ | $-.109523 \times 10^{-6}$ | -. $595181 \times 10^{-6}$ | 0 | 0 | 0 | 0 |
| AP8 | $.601436 \times 10^{1}$ | . $153503 \times 10^{1}$ | -. $759331 \times 10^{0}$ | $-.718612 \times 10^{1}$ | $-.297074 \times 10^{-1}$ | . $186185 \times 10^{-1}$ | $-.810072 \times 10^{-1}$ | . $960492 \times 10^{-3}$ | . $631924 \times 10^{0}$ |
| $A^{\text {P }} 9$ | 0 | . $212258 \times 10^{0}$ | $.443411 \times 10^{0}$ | . $50642 \times 100$ | $-.477549 \times 10^{-2}$ | 0 | 0 | 0 | 0 |
| $A^{\text {P }} 10$ | 0 | -. $297147 \times 10^{-2}$ | -. $703113 \times 10^{-2}$ | $-.624918 \times 10^{-2}$ | $.327815 \times 10^{-3}$ | 0 | 0 | 0 | 0 |
| $A^{\text {P }} 11$ | 0 | . $985059 \times 10^{-5}$ | . $27991 \times 10^{-4}$ | . $221158 \times 10^{-4}$ | $-.543824 \times 10^{-5}$ | 0 | 0 | 0 | 0 |

Figure F.20. Coefficients of Curve Fit Equations for Power Coefficient
$\mu=0$

|  | $\mu=0$ | $\mu=.1014$ | $\mu=.2351$ | $\mu=.2027$ | $\mu=.3723$ | $\mu=.4565$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{A}_{\mathrm{NF}} 0$ | 0 | $-.154857 \times 10^{-5}$ | $.204691 \times 10^{-5}$ | $.450018 \times 10^{-4}$ | $-.982094 \times 10^{-4}$ | . $179585 \times 10^{-6}$ |
| A | 0 | . $467546 \times 10^{-5}$ | . $942028 \times 10^{-5}$ | . $211604 \times 10^{-4}$ | . $19641 \times 10^{-3}$ | . $323623 \times 10^{-3}$ |
| ${ }_{\text {A }} \mathrm{NF}$ | 0 | -. $285003 \times 10^{-7}$ | -. $100977 \times 10^{-6}$ | $-.251209 \times 10^{-6}$ | $-.639116 \times 10^{-5}$ | . $684873 \times 10^{-6}$ |
| ${ }_{\text {A }} \mathrm{NF} 2$ | 0 | $128888 \times 10^{-10}$ | $.270048 \times 10^{-9}$ | . $746916 \times 10^{-9}$ | . $657985 \times 10^{-7}$ | -. $249423 \times 10^{-7}$ |
| ${ }^{\text {A }}$ NF3 | 0 | $-.128888 \times 10^{-10}$ | . $224724 \times 10^{-3}$ | . $929207 \times 10^{-3}$ | -. $157595 \times 10^{-2}$ | $-.809587 \times 10^{-4}$ |
| ${ }^{\text {A }}$ NF4 | 0 | -. $102821 \times 10^{-3}$ | . $224724 \times 10^{-3}$ | . $929207 \times 10^{-2}$ | -. $24356 \times 10^{-2}$ |  |
| ${ }^{A}{ }_{N F 5}$ | 0 | $.749984 \times 10^{-3}$ | .125801x10-2 | $.178593 \times 10^{-2}$ | . $24356 \times 10^{-2}$ | . $572817 \times 102$ |
| ${ }_{\text {A }}{ }_{\text {NF } 5}$ | 0 | $-.945697 \times 10^{-5}$ | -. $132674 \times 10^{-4}$ | $-.217638 \times 10^{-4}$ | . $234598 \times 10^{-3}$ | $.109167 \times 10^{-2}$ |
|  | 0 | . $293263 \times 10^{-7}$ | . $348545 \times 10^{-7}$ | . $65423 \times 10^{-7}$ | -. $470641 \times 10^{-5}$ | -. $963463 \times 10^{-4}$ |
| NF7 |  |  | . $275072 \times 10^{-1}$ | $-.543699 \times 10^{-1}$ | . $443939 \times 10^{-1}$ | . $53212 \times 10^{-2}$ |
| ${ }^{\text {A }}{ }_{\mathrm{NF} 8}$ | 0 | . $109342 \times 10$ |  | -. $543699 \times 10$ | . $44393910{ }^{-1}$ | 159926×10 0 |
| ${ }^{\text {A }} \mathrm{NF} 9$ | 0 | . $943903 \times 10^{-1}$ | . $696523 \times 10^{-1}$ | . $18035 \times 10$ | -. $138981 \times 10^{-1}$ | . $159926 \times 10$ |
| ${ }_{\text {A }}^{\text {NF10 }}$ | 0 | $-.103604 \times 10^{-2}$ | $-.923809 \times 10^{-3}$ | -. $232838 \times 10^{-2}$ | . $632501 \times 10^{-3}$ | -. $115516 \times 10$ |
|  | 0 | $.317682 \times 10^{-5}$ | . $29846 \times 10^{-5}$ | $.738683 \times 10^{-5}$ | -. $814596 \times 10^{-5}$ | . $994553 \times 10^{-2}$ |
|  | $u=.5147$ | $\mu=.6432$ | $4=.772$ | $\mu=.9008$ | $\mu=1.03$ | $\mu=1.158$ |
|  | . $137987 \times 10^{-5}$ | . $161374 \times 10^{-5}$ | . $245107 \times 10^{-5}$ | -. $132502 \times 10^{-6}$ | $-.173383 \times 10^{-5}$ | $.585875 \times 10^{-5}$ |
|  | $.439095 \times 10^{-3}$ | $.760961 \times 10^{-3}$ | . $109347 \times 10^{-2}$ | . $140037 \times 10^{-2}$ | . $170687 \times 10^{-2}$ | . $199778 \times 10^{-2}$ |
| ${ }_{\text {A }}^{\text {NF }}$ [ ${ }^{\text {a }}$ | . $228487 \times 10^{-5}$ | . $440349 \times 10^{-6}$ | $.823616 \times 10^{-6}$ | -. $191532 \times 10^{-6}$ | $-.139327 \times 10^{-5}$ | $-.209278 \times 10^{-5}$ |
| ${ }_{\text {A }}^{\text {NF } 2}$ | $-.762918 \times 10^{-7}$ | $-.152524 \times 10^{-7}$ | $-.545001 \times 10^{-7}$ | . $257742 \times 10^{-7}$ | . $512513 \times 10^{-7}$ | . $76607 \times 10^{-7}$ |
| ${ }_{\text {A }}{ }^{\text {a }}$ 3 | $-.792358 \times 10^{-3}$ | -. $58501 \times 10^{-3}$ | -. $578165 \times 10^{-4}$ | -. $185878 \times 10^{-3}$ | . $212865 \times 10^{-2}$ | -. $228619 \times 10^{-2}$ |
| ${ }_{\text {A }}{ }^{\text {a }} 4$ |  |  |  |  |  | . $123733 \times 10^{-1}$ |
| ${ }_{\text {A }}^{\text {NF } 5}$ | . $665352 \times 10^{-2}$ | . $282596 \times 10^{-2}$ | $-.625689 \times 10^{-2}$ | $-.258884 \times 10^{-2}$ | $-.513321 \times 10^{-2}$ $.878786 \times 10^{-3}$ | $-.583625 \times 10^{-3}$ |
| ${ }^{\text {A }}$ NF6 | $-.208908 \times 10^{-3}$ | . $266667 \times 10^{-3}$ | . $122656 \times 10^{-2}$ | . $658596 \times 10^{-3}$ |  |  |
| ${ }_{\text {A }}^{\text {NF } 7}$ | . $776787 \times 10^{-5}$ | -. $517496 \times 10^{-5}$ | $-.361453 \times 10^{-4}$ | $-.249126 \times 10^{-4}$ | $-.316126 \times 10^{-4}$ | $.101001 \times 10^{-4}$ |
| ${ }^{\text {A }}$ NF8 | . $324054 \times 10^{-1}$ | . $487661 \times 10^{-1}$ | -. $60177 \times 10^{-1}$ | . $804597 \times 10^{-2}$ | -.162981x10 0 | $.466581 \times 10^{-1}$ |
| ${ }^{\text {A }}$ NF9 9 | $-.688518 \times 10^{-1}$ | . $113439 \times 10^{0}$ | . $642961 \times 10^{0}$ | . $208291 \times 10^{-1}$ | . $515108 \times 1)^{0}$ | -. $898786 \times 10^{0}$ |
| ${ }_{\text {A }}^{\text {NFiO }}$ | . $875982 \times 10^{-2}$ | $-.243064 \times 10^{-1}$ | -. $967895 \times 10^{-1}$ | -. $520537 \times 10^{-2}$ | $-.665886 \times 10^{-1}$ | $.324115 \times 10^{-1}$ |
|  | -. $327294 \times 10^{-3}$ | . $563958 \times 10^{-3}$ | . $300992 \times 10^{-2}$ | . $261473 \times 10^{-3}$ | . $221409 \times 1 \mathrm{~J}^{-2}$ | . $439838 \times 10^{-3}$ |

Figure F.21. Coefficients of Curve Fit Equations for Normal Force Coefficient


|  | $\mu=0$ | $\mu=.1014$ | $\mu=.1351$ | $\mu=.2027$ | $\mu=.3723$ | $\mu=.4565$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | $.377218 \times 10^{-5}$ | $.688095 \times 10^{-5}$ | $-.79344 \times 10^{-5}$ | $-.310285 \times 10^{-6}$ | $-.884622 \times 10^{-8}$ |
| ${ }_{\text {A }}^{\text {pmo }}$ | 0 | . $539482 \times 10^{-5}$ | . $125905 \times 10^{-4}$ | . $316671 \times 10^{-4}$ | . $820375 \times 10^{-4}$ | . $141415 \times 10^{-4}$ |
|  | 0 | -. $12652 \times 10^{-6}$ | -. $252414 \times 10^{-6}$ | $-.581828 \times 10^{-6}$ | $-.27116 \times 10^{-6}$ | $.345339 \times 10^{-7}$ |
| ${ }^{\text {P }}$ PM2 | 0 |  |  | . $226169 \times 10^{-8}$ | -. $151223 \times 10^{-7}$ | $-.105759 \times 10^{-8}$ |
| ${ }^{\text {A }}$ PM3 | 0 | . $537895 \times 10^{-9}$ | $101607 \times 10^{-8}$ |  |  | $-.738879 \times 10^{-5}$ |
| ${ }^{\text {A PM }} 4$ | 0 | . $172117 \times 10^{-3}$ | . $562525 \times 10^{-3}$ | . $201324 \times 10^{-2}$ | $.193688 \times 10^{-3}$ | $-.738879 \times 10^{-5}$ <br> $159412 \times 10^{-2}$ |
| ${ }^{\text {A }}$ PM5 | 0 | . $146772 \times 10^{-2}$ | . $1652 \times 10^{-2}$ | $206142 \times 10$ | . $464123 \times 10^{-2}$ |  |
|  | 0 | -. $11507 \times 10^{-4}$ | -. $121292 \times 10^{-4}$ | -. $131115 \times 10^{-4}$ | -. $229179 \times 10^{-4}$ | . $508798 \times 10^{-5}$ |
|  | 0 | . $185794 \times 10^{-7}$ | . $164608 \times 10^{-7}$ | . $913233 \times 10^{-8}$ | -. $119169 \times 10^{-6}$ | -. $10134 \times 10^{-6}$ |
|  | 0 | $-.165488 \times 10^{-1}$ | $-.189205 \times 10^{-1}$ | . $766096 \times 10^{-1}$ | $-.133686 \times 10^{-1}$ | $55878 \times 10^{-3}$ |
| ${ }^{\text {A PM8 }}$ | 0 | . $181983 \times 10^{-2}$ |  | $-.692308 \times 10^{-2}$ | -. $487350 \times 10^{-2}$ | . $216812 \times 10^{-1}$ |
| ${ }^{\text {A }}$ PM9 | 0 | . $181983 \times 10^{-2}$ | . $224173 \times 10^{-6}$ | $-.692303 \times 10^{-2}$ |  |  |
| $A_{\text {PM10 }}$ | 0 | -. $236138 \times 10^{-4}$ | . $16657 \times 10^{-6}$ | $175 \times 10^{-4}$ | $10^{-3}$ |  |
| ${ }_{\text {APM11 }}$ | 0 | $.796661 \times 10^{-7}$ | $-.470343 \times 10^{-8}$ | $-.176422 \times 10^{-6}$ | $-.598035 \times 10$ | $472 \times 1$ |
|  | $4=.5147$ | $\underline{=.} 6432$ | $\nu=.772$ | $=.9008$ | $\mu=1.03$ | $\mu=1.158$ |
|  | $-.163818 \times 10^{-6}$ | -. $771885 \times 10^{-6}$ | $-.248968 \times 10^{-5}$ | . $145349 \times 10^{-6}$ | . $168996 \times 10^{-5}$ | . $274231 \times 10^{-6}$ |
|  | $-.177418 \times 10^{-5}$ | -. $922672 \times 10^{-4}$ | -. $202383 \times 10^{-3}$ | -. $308712 \times 10^{-3}$ | -. $383796 \times 10^{-3}$ | $-.470485 \times 10^{-3}$ |
| ${ }_{\text {A }}{ }^{\text {PM }}$ |  |  |  | . $536055 \times 10^{-7}$ | $-.912361 \times 10^{-7}$ | . $25489 \times 10^{-6}$ |
| ${ }^{\text {A PM2 }}$ | $.356703 \times 10^{-7}$ | . $111958 \times 10^{-6}$ | . $704986 \times 10^{-6}$ | . $536055 \times 10^{-9}$ | -. $27077 \times 10^{-9}$ | $-.640161 \times 10^{-8}$ |
| ${ }_{\text {A }}^{\text {PM }}$ 3 | $-.137756 \times 10^{-8}$ | -. $216285 \times 10^{-9}$ | $-.328566 \times 10^{-7}$ | . $53226 \times 10^{-9}$ | -. $27077 \times 10^{-9}$ | $-.640161 \times 10^{-8}$ $-.321648 \times 10^{-3}$ |
| ${ }_{\text {A }}^{\text {PM4 }}$ | . $401368 \times 10^{-4}$ | . $225413 \times 10^{-4}$ | $.778302 \times 10^{-3}$ | $-.483831 \times 10^{-3}$ | . $497667 \times 10^{-3}$ |  |
|  | -. $240092 \times 10^{-2}$ | -. $399324 \times 10^{-2}$ | -. $641948 \times 10^{-2}$ | -. $70195 \times 10^{-2}$ | -. $100188 \times 10^{-1}$ | -. $687551 \times 10^{-2}$ |
|  | $-.120839 \times 10^{-4}$ | $-.665199 \times 10^{-4}$ | $-.128724 \times 10^{-3}$ | $-.119008 \times 10^{-3}$ | $-.416504 \times 10^{-4}$ | -. $344122 \times 10^{-3}$ |
| ${ }_{\text {A PM6 }}$ | . $387065 \times 10^{-6}$ | . $204615 \times 10^{-5}$ | . $638786 \times 10^{-5}$ | . $396756 \times 10^{-5}$ | . $168316 \times 10^{-5}$ | . $12192 \times 10^{-4}$ |
| ${ }_{\text {A }}^{\text {A }}$ A 7 | - $.227747 \times 10$ | . $261322 \times 10^{-1}$ | -. $424199 \times 10^{-1}$ | . $428319 \times 10^{-1}$ | . $392666 \times 10^{-1}$ | . $256492 \times 10^{-1}$ |
| ${ }_{\text {A }}^{\text {PM8 }}$ | $-.227747 \times 10^{-1}$ $-.751961 \times 10^{-1}$ | -. $183726 \times 100$ | . $359543 \times 10^{-1}$ | . $143102 \times 10^{-1}$ | $.949637 \times 10^{-1}$ | $-.135728 \times 10$ |
| ${ }^{\text {A }}{ }_{\text {PM9 }}$ | $-.751961 \times 10^{-1}$ | -. $183726 \times 10$ | $.359543 \times 10^{-1}$ $-.33401 \times 10^{-3}$ | . $.856042 \times 10^{-2}$ | $.12569 \times 10^{-1}$ | $.336543 \times 10^{-1}$ |
| ${ }_{\text {PM }}{ }_{\text {PM10 }}$ | . $10527 \times 10^{-2}$ | . $210223 \times 10^{-1}$ | $-.33401 \times 10^{-3}$ | . $856042 \times 10^{-2}$ |  | $\begin{array}{r} .336543 \times 10^{-1} \\ -.115452 \times 10^{-2} \end{array}$ |
| ${ }^{\text {PMM11 }}$ | $-.393332 \times 10^{-4}$ | $-.726143 \times 10^{-3}$ | $-.78287 \times 10^{-4}$ | -.33099x10 | -.435573×10 |  |
| NOTE | a must be pitching | degrees w nt equati | hen these on. | efficient | are used | in the |

Figure F.23. Coefficients of Curve Fit Equations for Pitching Moment Coefficient

|  | $\underline{i}=0$ | $\therefore=.1014$ | $\underline{\mu}=.1351$ | $\mu=.2027$ | $\mu=.3723$ | $\nu=.4565$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{\text {A }}$ YMO | 0 | . $109092 \times 10^{-5}$ | . $286933 \times 10^{-7}$ | $-.110543 \times 10^{-4}$ | $-.115949 \times 10^{-4}$ | $\begin{aligned} & .75428 \times 10^{-6} \\ & .199993 \times 10^{-3} \end{aligned}$ |
| ${ }_{\text {A }}^{\text {YM }}$ I | 0 | $. \therefore 26647 \times 10^{-4}$ | $.191392 \times 10^{-4}$ | . $250948 \times 10^{-4}$ | $\begin{aligned} & .442163 \times 10^{-4} \\ & 079871 \times 10^{-6} \end{aligned}$ | $672974 \times 10^{-6}$ |
| ${ }_{\text {A }}^{\text {YM1 }}$ | 0 | $-.248631 \times 10^{-6}$ | -. $229016 \times 10^{-6}$ | $-.220845 \times 10^{-6}$ | .979871×10-6 |  |
| ${ }_{\text {A }}{ }_{\text {YM }}$ | 0 | . $435436 \times 10^{-9}$ | . $68199 \times 10^{-9}$ | $.154233 \times 10^{-9}$ | -. $231738 \times 10^{-7}$ | $989016 \times 10^{-4}$ |
| ${ }_{\text {A }}^{\text {YM4 }}$ | 0 | . $216295 \times 10^{-3}$ | $61904 \times 10$ | $.303865 \times 10^{-4}$ | $.76278 \times 10$ | $\begin{aligned} & .989016 \times 10^{2} \\ & .890334 \times 10^{-2} \end{aligned}$ |
| A YM5 | 0 | $.636091 \times 10^{-4}$ | . $533431 \times 10^{-4}$ | $\begin{array}{r}.880677 \times 10^{-3} \\ \hline .4\end{array}$ | $.600766 \times 10$ | . $890334 \times 10^{-4}$ |
| ${ }^{\text {A }}$ YM6 ${ }^{\text {YMS }}$ | 0 | $.290233 \times 10^{-5}$ | $.265736 \times 10^{-5}$ | $-.16908 \times 10$ | $-.24055 \times 10^{-3}$ | $\begin{array}{r} .101251 \times 10^{-4} \\ -.239883 \times 10^{-6} \end{array}$ |
| ${ }^{\text {A. }} \text { YM7 }$ | 0 | -. $182158 \times 10^{-7}$ | $-.164399 \times 10^{-7}$ | $.10332 \times 10^{-6}$ $-.171322 \times 10^{-1}$ | $.289645 \times 10^{-1}$ $-.27995 \times 10^{-1}$ | $.646468 \times 10^{-2}$ |
| ${ }^{\text {A Y Y } 18}$ | 0 | -. $158447 \times 10^{-1}$ | $-.2112 \times 10^{-1}$ | $\cdots .171322 \times 10^{-1}$ | $116529 \times 10^{-1}$ | $486429 \times 10^{-1}$ |
| ${ }^{\text {A }} \mathrm{Y}$ M9 9 | 0 | $147256 \times 10^{-2}$ | . $323912 \times 10^{-3}$ | -. $426356 \times 10^{-2}$ | . $116529 \times 10^{-1}$ | $-.277348 \times 10^{-2}$ |
| ${ }_{\text {A }}^{\text {A }}$ M10 | 0 0 | $-.278633 \times 10^{-4}$ $.579483 \times 10^{-7}$ | $\begin{array}{r} -.158877 \times 10^{-5} \\ .374405 \times 10^{-8} \end{array}$ | $\begin{array}{r} .873694 \times 10^{-6} \\ -.533888 \times 10^{-6} \end{array}$ | $.133698 \times 10^{-4}$ | $.407947 \times 10^{-4}$ |
|  | $\mu=.5147$ | - $=.6432$ | $\mu=.772$ | ${ }^{-}=.9008$ | $\mu=1.03$ | $\mu=1.158$ |
|  | . $811278 \times 10^{-6}$ | -. $589829 \times 10^{-6}$ | -. $140611 \times 10^{-5}$ | $-.959155 \times 10^{-6}$ | . $232776 \times 10^{-5}$ |  |
| YMO | . $2549 \times 10$ | $.358: 53 \times 10^{-3}$ | $.40651 \times 10^{-3}$ | $.423457 \times 10^{-3}$ | . $404494 \times 10^{-3}$ | $.400109 \times 10^{-3}$ |
| ${ }_{\text {A }}^{\text {A }} \mathrm{YML}$ | . $298133 \times 10^{-5}$ | $-.483638 \times 10^{-6}$ | $.711836 \times 10^{-6}$ | $.366298 \times 10^{-6}$ | $-.978679 \times 10^{-6}$ | $.483519 \times 10^{-6}$ |
| ${ }_{\text {A }}^{\text {A }}$ Y2 | $.298133 \times 10$ $-.7069 \times 10^{-8}$ | . $165027 \times 10^{-7}$ | -. $242848 \times 10^{-7}$ | -. $201739 \times 10^{-7}$ | $.384779 \times 10^{-7}$ | -. $178582 \times 10^{-7}$ |
| A YM3 $\mathrm{A}_{\text {YM }}$ | $-.138358 \times 10^{-3}$ | $-.376316 \times 10^{-3}$ | $-.111605 \times 10^{-3}$ | $-.498142 \times 10^{-3}$ $.700776 \times 10^{-2}$ | $\begin{array}{r} -.859507 \times 10^{-3} \\ .495335 \times 10^{-2} \end{array}$ | $\begin{array}{r} -.170495 \times 10^{-3} \\ .937562 \times 10^{-3} \end{array}$ |
| ${ }_{\text {A }}^{\text {YM4 }}$ | . $84517 \times 10^{-2}$ | . $731418 \times 10^{-2}$ | . $111654 \times 10^{-2}$ | $.700776 \times 10^{-2}$ $.179799 \times 10^{-3}$ | $.495335 \times 10^{-2}$ <br> $475445 \times 10^{-3}$ | $\begin{aligned} & .937562 \times 10^{-3} \\ & .428652 \times 10^{-3} \end{aligned}$ |
| ${ }^{\text {A }}$ YM 6 | $.511468 \times 10^{-4}$ | . $25651 \times 10^{-3}$ | $-.397326 \times 10^{-3}$ | -. $179799 \times 10^{-3}$ | $.475445 \times 10^{-3}$ $-.172489 \times 10^{-4}$ | $\begin{array}{r} .428652 \times 10^{-3} \\ -.224772 \times 10^{-4} \end{array}$ |
| ${ }^{\text {A }}$ YM 7 | $-.113402 \times 10^{-5}$ | $-.882347 \times 10^{-5}$ | . $127859 \times 10^{-4}$ | $.100219 \times 10^{-4}$ $.392748 \times 10^{-1}$ | $-.172489 \times 10^{-4}$ $.62677 \times 10^{-1}$ | $\begin{array}{r} -.224772 \times 10^{-4} \\ .242592 \times 10^{0} \end{array}$ |
| ${ }^{\text {A }}$ YM8 | . $235303 \times 10^{-3}$ | . $322547 \times 10^{-1}$ | $.931734 \times 10^{-2}$ | $.392748 \times 10^{-}$ | . $62677 \times 10$ | $\begin{aligned} & .242592 \times 10 \\ & .629868 \times 10 \end{aligned}$ |
| ${ }^{\text {A }}$ YM9 | . $664771 \times 10^{-1}$ | . $729796 \times 10^{-1}$ | -. $283071 \times 10^{-1}$ | -. $697861 \times 10^{-1}$ | $-.284635 \times 10^{-1}$ | $-.600989 \times 10^{-1}$ |
| $\mathrm{A}_{\text {YM10 }}$ | $-.647118 \times 10^{-2}$ | $-.170565 \times 10^{-1}$ | $.292923 \times 10^{-1}$ | $.166632 \times 10$ |  | $.245749 \times 10^{-2}$ |
| ${ }^{\text {A YMII }}$ | $.240142 \times 10^{-3}$ | . $575827 \times 10^{-3}$ | -. $960904 \times 10^{-}$ | -.821801x10 |  |  |
| NO | $\begin{gathered} : \quad \text { must } \\ \text { in th } \end{gathered}$ | n degrees <br> wing momen | hen these equation. | efficient | are used |  |

Figure F.24. Coefficients of Curve Fit Equations for Yawing Moment Coefficient

$\mathrm{H}_{\mathrm{PMO}}$
$\mathrm{H}_{\mathrm{PM}}$
$\mathrm{H}_{\mathrm{PM}}$
$\mathrm{H}_{\mathrm{PM}}$
$\mathrm{H}_{\mathrm{PM}} 4$
$\mathrm{H}_{\mathrm{PM}}$
$\mathrm{H}_{\mathrm{PM}}$
$\mathrm{H}_{\mathrm{PM}}$
$\mathrm{H}_{\mathrm{PM}}$
$\mathrm{H}_{\mathrm{PM}}$
$\mathrm{H}_{\mathrm{PM}}$
$\mathrm{H}_{\mathrm{PM}} 0$


*

$\left.\begin{array}{l}\mathrm{D}_{\mathrm{SF}_{1}}=. .025 \\ \mathrm{D}_{\mathrm{SF}_{2}}=-.0025356 \\ \mathrm{D}_{\mathrm{SF}_{3}}=-.0002264 \\ \mathrm{D}_{\mathrm{SF}_{4}}=-.000044\end{array}\right\} \begin{aligned} & \text { Coeff. of } \\ & \mathrm{dC}_{\mathrm{SF}} \\ & \frac{\mathrm{dA}_{1 \mathrm{C}}}{} \sim \frac{1}{\mathrm{DEG}}\end{aligned}$


## F. 4 AIRFRAME AEROUYNAMIC INPUT DATA

The input data for the airframe aerodynamic data are given in this section and are referenced by page number to the equations presented in Appendix E. Plotted aerodynamic data are presented in Figures F. 28 to F. 30 .

## F.4.1 Input Data

## PAGE

E. $21 \quad C_{L_{\alpha W}}=3.941 / \mathrm{rad}$
E. $28 \quad C_{L_{\text {iLAX }}}=1.232$
E. $31 \quad K_{20}=-.0975 / R A D$
$\mathrm{K}_{21}=-.0916 / \mathrm{RAD}$
$K_{22}=.015$
$K_{\mathscr{L}}=1.0$
$K_{n}=1.0$
E. $32 \mathrm{f}_{\mathrm{eu}}=60 \mathrm{ft}^{2}$
$D / T=.05$
$\mathrm{KDI} / T=0.0$
$\mathrm{K}_{\mathrm{D} 2 / \mathrm{T}}=0.0$
$K_{D 3 / T}=.05$
$\mathrm{KD}_{\mathrm{L}} / \mathrm{T}=.05$
$\mathrm{KMl} / \mathrm{T}=0.0$
$\mathrm{K}_{\mathrm{M} 2} / \mathrm{T}=0.0$
$\mathrm{KM3} / \mathrm{T}=0.0$
$K_{114 / T}=0.0$
E. 36
${ }^{\top} \mathrm{HT}=.52$
$\alpha_{H T}$ STALL $=16 \mathrm{DEG}$
$C_{L_{\alpha H T}}=.0611 /$ DEG
$C_{D_{\text {OHT }}}=.0084202$
E. 39

$$
\begin{array}{ll}
\frac{\mathrm{d} \sigma}{\mathrm{~d} \beta}= & -.025 \\
{ }^{\tau} \mathrm{VT} & =.55 \\
\alpha_{\mathrm{VT}}^{\mathrm{STALL}} & =20.0 \mathrm{DEG} \\
\mathrm{C}_{\mathrm{X}_{\alpha \mathrm{VT}}} & =.05461 / \mathrm{DEG} \\
\mathrm{C}_{\mathrm{D}_{\mathrm{OVT}}} & =.0078915
\end{array}
$$

PAGE
E. $43 \quad \eta_{H T}=1.0$
$n_{V T}=1.0$
$\alpha_{N} \leq .5236 R A D />.5236 R A D$
E. $44 \quad \mathrm{C}_{\text {DON }}=.001821 /-.016179$
$\mathrm{K}_{30}=.04773 /-.2034$
$K_{31}=.16086 /-.071138$
$\mathrm{K}_{32}=.1087$
$C_{\text {MON }}=0$
$K_{34}=0$
$K_{35}=0$
$K_{36}=-.1087$
$K_{37}=0$
$K_{36}=-.1087$
$K_{37}=0$
$C_{\text {NON }}=0$
$K_{38}=0$
$K_{39}=0$
$K_{40}=0$
$K_{41}=0$
E. $49 \quad \mathrm{C}_{\text {DOF }}=.0075705$
$\begin{array}{ll}\mathrm{K}_{\mathrm{O}} & =18 \\ \mathrm{~K}_{1} & =-.03 \\ \mathrm{~K}_{2} & =.2561 \\ \Delta \mathrm{C}_{\text {DIG }} & =.05\end{array}$
$\begin{array}{ll}\mathrm{K}_{3} & =.922 / \mathrm{RAD} \\ \mathrm{K}_{4} & =0 \\ \mathrm{~K}_{5} & =.67709 \\ \mathrm{~K}_{6} & =0 \\ \mathrm{~K}_{7} & =-.478\end{array}$

```
E. \(49 \quad \mathrm{~K}_{8}=0\)
\(\begin{array}{ll}K_{8} & =0 \\ K_{9} & =-.131 / \mathrm{RAD} \\ \mathrm{K}_{10} & =0\end{array}\)
\(\mathrm{K}_{10}=0\)
\(\mathrm{C}_{\text {MOF }}=.0001883\)
\(\mathrm{C}_{\text {NOF }}=0\)
\(\Delta \mathrm{C}_{\mathrm{MLG}}=0\)
\(K_{42}=.0537\)
\(\begin{array}{lll}\mathrm{E} .52 & \mathrm{~T}_{1}=0 \\ & \mathrm{~T}_{2}=-.04808 \\ \mathrm{~T}_{3}==.3795\end{array}\)
E. \(54 \begin{array}{ll}{ }^{\tau} 1 & =.1 \\ & { }^{\tau} 2\end{array}=.1\)
E. \(60 \mathrm{f}_{\mathrm{TR}}=\mathrm{f}_{\mathrm{NFR}}=\mathrm{f}_{\mathrm{SFR}}=\mathrm{f}_{\mathrm{PMR}}=\mathrm{f}_{\mathrm{YMR}}=\mathrm{f}_{\mathrm{QR}}=\mathrm{f}_{\mathrm{PR}}=1.0\)
E. \(66 \quad \frac{\mathrm{dC}_{\text {MWC }} / 4}{\mathrm{dC}_{\mathrm{I}}}=-.03215\)
\(\mathrm{Cl}_{1}=-.065\)
\(\mathrm{C}_{2}=-.00251 / \mathrm{DEG}\)
\(\mathrm{C}_{3}=0.0 \quad 1 / \mathrm{DEG}^{2}\)
\(C_{L_{\alpha}}=3.94 / \mathrm{RAD}\)
```

$$
\begin{aligned}
& \begin{array}{l}
\text { Wing Aerodynamic Input Data } \\
\text { Coefficients of } \sum_{v=0}^{4} \sum_{u=0}^{4}\left[A_{D}(u+5 v) \delta_{\alpha}^{u}\right]^{n} \quad \text { (Page E. 26) }
\end{array} \\
& A_{\text {DO }}=.582990 \times 10^{-3} \\
& A_{D 1}=.126170 \times 10^{-2} \\
& A_{D_{2}}=.391649 \times 10^{-4} \\
& { }^{A_{D_{3}}}=.110058 \times 10^{-5} \\
& A_{D_{4}}=-.159415 \times 10^{-7} \\
& A_{D_{5}}=.245484 \times 10^{-3} \\
& A_{D_{6}}=.265950 \times 10^{-3} \\
& A_{D_{7}}=.404673 \times 10^{-5} \\
& A_{D_{8}}=-.152693 \times 10^{-6} \\
& A_{D_{9}}=.102320 \times 10^{-8} \\
& A_{D_{10}}=-.313543 \times 10^{-5} \\
& \begin{array}{l}
A_{D_{11}}=.624554 \times 10^{-6} \\
{ }^{A_{D_{12}}}=.141804 \times 10^{-6}
\end{array} \\
& A_{D_{13}}=-.821732 \times 10-8 \\
& A_{D_{14}}=.119984 \times 10^{-9} \\
& A_{D_{15}}=-.474069 \times 10^{-5} \\
& A_{D_{16}}=.771740 \times 10^{-6} \\
& { }^{A_{D_{17}}}=-.800800 \times 10^{-7} \\
& A_{D_{18}}=.208761 \times 10^{-8} \\
& A_{D_{19}}=-.114899 \times 10^{-10} \\
& A_{D_{20}}=.238184 \times 10^{-6} \\
& A_{D 21}=.196213 \times 10^{-7} \\
& A_{D_{22}}=-.204613 \times 10^{-8} \\
& A_{D_{23}}=.133330 \times 10^{-10} \\
& A_{24}=.492127 \times 10^{-13}
\end{aligned}
$$



Figure F.28. Model 222 Downwash Functions @ $C_{T}=0, i_{w}=+2.0^{\circ}$

Figure F.29. Variation of Lift Curve Slope with Ground Height


ESTIMATED ROTOR/ROTOR INTERFERENCE PARAMETER


Figure F.30. Rotor/Rotor and Wing/Rotor Interference

## F. 5 GEOMETRIC, WEIGHTS AND BALANCE DATA

The input data for the Model 222 geometry, weights and balance are presented in this section, and are referenced in Appendix E. Input data for the preprocessor calculations are not presented, but are easily obtainable from an aircraft threeview drawing and the weights and balance data presented in this section. It should be emphasized that the lengths and inertias presented here were calculated using the preprocessor.
F.5.1 Input Data

## Page Page



Page

$$
\begin{aligned}
& \text { E. } 46 \\
& \begin{aligned}
\mathrm{X}_{\mathrm{G}_{1}} & =-3.7 \mathrm{ft} \\
\mathrm{X}_{\mathrm{G}_{2}} & =-3.7 \mathrm{ft} \\
\mathrm{X}_{\mathrm{G}_{3}} & =10.67 \\
\mathrm{r}_{1} & =1.065 \mathrm{ft} \\
\mathrm{r}_{2} & =1.065 \mathrm{ft} \\
\mathrm{r}_{3} & =.60 \mathrm{ft} \\
\mathrm{~K}_{\text {ST }_{1}} & =3840 \mathrm{lb} / \mathrm{ft} \\
\mathrm{~K}_{\mathrm{ST}_{2}} & =3840 \mathrm{lb} / \mathrm{ft} \\
\mathrm{~K}_{\mathrm{ST}_{3}} & =3840 \mathrm{lb} / \mathrm{ft} \\
\mathrm{D}_{S T_{1}} & =600 \mathrm{lb} / \mathrm{ft} / \mathrm{sec} \\
\mathrm{D}_{S T_{1}} & =600 \mathrm{lb} / \mathrm{ft} / \mathrm{sec} \\
\mathrm{D}_{S T_{2}} & =600 \mathrm{lb} / \mathrm{ft} / \mathrm{sec} \\
\mu_{0} & =.03 \\
\mu_{I} & =.005 \\
\mu_{S} & =.5
\end{aligned} \\
& \text { E. } 50 \quad{ }^{*} \mathrm{X}_{\mathrm{FAC}}=.84 \mathrm{ft} \\
& \mathrm{Z}_{\mathrm{FAC}}=3.66 \mathrm{ft} \\
& \text { E. } 61 \text { I } I_{P}=564 \mathrm{slu}^{\mathrm{slf}}{ }^{2} \\
& \mathrm{E} .62 \quad \mathrm{I}_{\mathrm{E}} \quad \because 1.47 \mathrm{slug}-\mathrm{ft}^{2} \\
& \text { E. } 64 \quad \mathrm{~K}_{\mathrm{W}_{1}}=.59678 \times 10^{-4} \\
& K_{W_{2}}=.1637 \times 10^{-4} \\
& \mathrm{~K}_{\mathrm{W}_{3}}=.58356 \times 10^{-5} \\
& \mathrm{~K}_{\mathrm{W}_{4}} \xlongequal{=} .2959 \times 10^{-2} \\
& \mathrm{~K}_{\mathrm{W}_{5}}=.1656 \times 10^{-2}
\end{aligned}
$$

Page
E. $64 \quad \zeta_{W_{1}}=.5$
$\zeta_{W_{2}}=0.5$
$\zeta_{W_{3}}=0.5$
${ }^{\zeta} W_{W_{4}}=.5$
${ }^{\zeta} W_{5}=.5$
$\mathrm{K}_{\mathrm{W}_{6}}=.1709 \times 10^{-4}$
$\mathrm{K}_{\mathrm{W}_{7}}=.05768 \times 10^{-4}$
$\mathrm{K}_{\mathrm{W}_{8}}=.1221 \times 10^{-5}$
$K_{W_{g}}=.0847 \times 10^{-2}$
$\mathrm{K}_{\mathrm{W}_{10}}=.0559 \times 10^{-2}$
$\omega_{W 1}=19.92 \mathrm{rad} / \mathrm{sec}$
$\omega_{\mathrm{W} 2}=19.92 \mathrm{rad} / \mathrm{sec}$
$\omega_{W}{ }_{W}=19.92 \mathrm{rad} / \mathrm{sec}$
$\omega_{\mathrm{W} 4}=19.92 \mathrm{rad} / \mathrm{sec}$
E. $66 \mathrm{~K}_{\theta t}=0.98 \times 10^{5} \mathrm{FT}-\mathrm{LB} / \mathrm{RAD}$
$\frac{\mathrm{X}_{\text {WAC }}}{\mathrm{C}_{\mathrm{W}}}=.275$
E. $79 \quad Y_{P A}=0$
$\ell_{P A}=6.75 \mathrm{ft}$
$Z_{P A}=4.75 \mathrm{ft}$
E. 68 to

## E. 78

Equations of motion input constant (Weight $=12321 \mathrm{lb}$, nominal CG)

$$
\mathrm{m}_{\mathrm{W}}=138.32 \text { slugs }
$$

$$
\mathrm{m}_{\mathrm{N}}=43.39 \text { slugs }
$$

$$
m_{f}=157.88
$$

$$
m=382.98 \text { (12321 LBS) }
$$

$$
I_{x x}^{(f)}=789.3 \text { slug-ft }{ }^{2}
$$

$$
I_{y y}^{x x}(f) 10845.6 \text { slug-ft }{ }^{2}
$$

$$
I_{z z}(f) 10707.4 \text { slug-ft } t^{2}
$$

$$
I_{x z}^{(f)}=399.9 \text { slug-ft } t^{2}
$$

$$
I_{x x}^{(W)} 23978.4 \text { slug-ft } t^{2}
$$

$$
I_{Y Y}^{(W)}=664.8 \text { slug-ft }{ }^{2}
$$

$$
I_{z Z}^{Y Y}(W)=24513.6 \text { slug-ft } t^{2}
$$

$$
I_{x z}^{(w)}=384.5 \text { slug-ft } t^{2}
$$

$$
I_{x x}^{\prime}=22.5 \text { slug-ft } t^{2}
$$

$$
I_{Y Y}^{\prime}=194.0 \text { slug-ft }{ }^{2}
$$

$$
I_{z z}^{\prime}=195.4 \text { slug-ft }{ }^{2}
$$

$$
I_{x z}^{\prime}=-20.0 \text { slug-ft } t^{2}
$$

$$
\ell_{\mathrm{f}}=.6917 \mathrm{ft}
$$

$$
\mathrm{h}_{\mathrm{f}}=4.075 \mathrm{ft}
$$

$$
\ell_{\mathrm{w}}=-.775 \mathrm{ft}
$$

$$
\mathrm{h}_{\mathrm{w}}=.30417
$$

$$
\mathrm{x}_{\mathrm{N}}=16.666 \mathrm{ft}
$$

$$
\ell=3.3624 \mathrm{ft}
$$

$$
\lambda \quad=2.841 \mathrm{DEG}
$$

$$
\mathrm{L}_{\mathrm{s}}=4.94 \mathrm{ft}
$$

$$
F-52
$$

F. 6 SIMULATION INPUT DATA

This section presents the input data required to drive the Flight Simulator for Advanced Aircraft (FSAA). Figure F. 31 shows the instrumentation requirements and Figure F. 32 shows the Model 222 control force gradients and breakout forces.

CAB INSTRUMENTATION:


Figure F.31. Model 222 Pilot Station Requirements


Figure F.32. Model 222 Control Force Gradients and Breakout Forces

The math model described in this report was mechanized in the Boeing Hybrid Simulation Laboratory for the purpose of developing and evaluating math model simplifications. This was accomplished in a parallel time frame to the NASA simulation, which also used the described math model.

The Hybrid Simulation Laboratory is a large scale hybrid computation complex. It is capable of providing simultaneous operation of several hybrid and analog simulations, depending on problem size. The complex is totally state of the art, with recent acquisition of two mini-computers for the purpose of multivariable function generation.

The Hybrid Simulation Laboratory complex is comprised of the following elements:

- Digital

IBM $360 / 44$ system
25600 byte core
32 priority interrupts
16 hi-speed floating point register
2 hi-speed, 1 low speed channels
2-800 B.P.I. tape transport
1-2311 disk system
2 - 2315 disk system
1 - hi-speed card read/punch
1 - hi-speed line printer

2 - alpha-meric scope/keyboard
1 - console typewriter
l - ball printer
Basic Computer Arts Function Generation System (BOA)
1 - Interdata processor with 24000 byte core
1 - Interdata processor with 16000 byte core
2 - 16 channels analog to digital
2 - 16 channels digital to analog
2 - read only memory software systems

- Analog

4-3/4 expanded Applied Dynamics (AD-4)
771 amplifiers (all solid state)
4 resolver expansions
2 display consoles
10 ufd integrator system in 6 decades
$1-1 / 8$ expanded AD4 maintenance console
128 channels 100 KC analog to digital converters
128 channels digital to analog converters
1 applied dynamics 256

- Analog Output

4-8 channel Brush strip chart recorders
4-8 channel Varian Statos III strip chart recorders
4 - XY plotters

## - Software System

Integrated disk resident state of the art system embracing
"real time" languages:
Assembly Language
Modified Fortran IV, Level G and non-real time languages

Non-procedural block modeling, DSL/44
Fortran IV, Level G
Full utility system
Other special hybrid oriented programs

## G. 1 SIiUULATION ARCHITECTURE

The tilt rotur simulation model utilized the entire hybrid facility. When tied to Boeing's Nudge Base Simulator, four Consoles of Applied Dynamics from (AD-4's) analog, the Applied Dynamics 256 ( $\mathrm{AD}-256$ ) and two Simulator Laboratory analog computers were in use. In addition the IBM 360/44 digital computer and two Basic Computing Acts (BCA) function generators were utilized. Figure G.l shows the utilization of the hybrid facility and also shows the location of the major elements of the tilt rotor mathematical model.

In programming the digital portion of the tilt rotor simulation, core size and execution time were of immediate concern. Along with the complex wing and rotor representations, there was a large number of functions which had to be handled in the digital computer with trade-offs considered on core used if functions were programmed as tables and execution time for digital table
look-up versus curve-fit equations. In most cases, curve-fit equations were used to p:rogram functions. A program was written to curve fit the various functions needed, and the equation programmed for the real time task.

The single largest difficulty was the rotor representation. To program the curve-fit equations for each of the eight functions, for both rotors, would take 30 milliseconds (timing estimates without rotor indicated only 10 milliseconds were available). To program as tables and look-up answers, would not only take too long, but use too much core. So the rotor data was put in the function generation mini-computers (BCA). To get the rotor data into format for the $B C A$, several steps had to be executed; 1) data points were input to the curve-fit program which punched out the coefficients of the curve-fit expansion, 2) these coefficients were input to a program that punched data in the correct format at the correct breakpoints to be input to 3) the BCA program which punched a deck of cards that are input to the function generator mini-computer.

Although the BCA enabled the programming of the rotor without using digital time or core, it did not have enough room to hold 8 functions $x 2$ rotors for the rotor 'maps' of the size required. To obviate this, the BCA was multiplexed, such that only one rotor's results would be calculated each BCA cycle, with the left and right rotor being alternated. In this case it took 8 milliseconds per BCA cycle, resulting in a total rotor update every 16 milliseconds.


Figure G.I. Utilization of the Hybrid Laboratory for the Model 222 Math Model

As programming progressed, timing estimates showed the time frame would be a problem. The objective was a 40. millisecond (ms) time frame which results in 7 updates/cycle for the 3.5 cycle per second first mode vertical bending calculations. Due to the large number of angles and trigonometric functions, the complexity of the model and the real time requirement, every effort was made to reduce the time frame.

A parallel real time task method, where a 40 ms . time frame could be achieved, was selected. This method had two real time tasks, a 'Fast' real time task that was calculated every frame and a 'slow' real time task that was executed every 3 frames. Thus, it was important to separate the equations to ensure only low frequency equations were placed in the slow loop. In order to minimize execution time, the system routines for taking the sine, cosine and square root, having unnecessary accuracy at the expense of time, were discarded and replaced by streamlined routines. Since there are a total of 21 sinecosine pairs and 20 square-roots, the saving was substantial. The need for the time savings is emphasized by the fact that at present, using the parallel real time task, the total execution time is 38 ms . leaving 2 ms . for the foreground options, shown in Figure G. 2 to be executed. The 40 ms . time frame objective has been achieved.

The digital portions of the simulation were programmed using the General Hybrid Program (GHP) structure, which utilizes a

Direct control of the analog computer state and the interval timer (initial condition, hold, operate)

Change aircraft trim conditions (airspeed, lateral speed, altitude, rate of climb, trim pitch attitude, trim nacelle angle)

Control of line printer real-time printout

Control of line printer trim printout

Ability to change values of simulation flags (landing gear on/off ground effect on/off, vertical bending on/off, wing twist on/off)

- Ability to change real time phases (dual phase, total phase, plot phase; used to plot any digital function)

Figure G.2. Foreground Options

G-8
phase overlay scheme. The basic phase overlay structure is shown in Figure G.3. This figure also summarizes what is contained in each phase. Of most interest to this discussion are three phases; 1) the Preprocess phase, 2) the Run phase and 3) the Dual phase. The Preprocess phase loads the simulation data and sets up the analog computer by setting the potentiometers to the correct values and by reading out a test condition to ensure that no components are statically bad. Once the analogs are set up and checked, control is transferred by GHP to the run phase. This phase is in control while actually 'running' hybrid and executing the real time task. It is in the run phase that various options are provided, while the simulation is being used. These foreground options have been described in Figure G.2. . The two line printer options, the line printer trim sheet and the line printer real time printout, are powerful tools allowing visibility into the simulation equations.

The Dual phase contains the two real time tasks, the fast (RTFAST) and the slow (RTSLOW). Figures G. 4 and G. 5 show what each real time task contains. The execution time of RTFAST is 32 ms . while that of RTSLOW is 18 ms . Since RTFAST is executed every 'frame' but RTSLOW only every 3 frames, the execution time is $32+18 / 3$ or 38 ms .

The digital listing for the simulation program is shown in Figure G.6. This listing contains the fast and slow real

Figure G.3. GHP Phase Overlay Structure (Digital Core Allocations)

1. ANALOG TO DIGITAL I/O

- Reads analog ADC lines - Converts to floating point

2. ENTRY FOR STATIC CHECK OF FAST REAL TIME TASK

- Used for test cases W/O I/O

3. ANGLE INITIALIZATION SECTION

- $\sin , \cos i_{\mathrm{NL}} \& i_{\mathrm{NR}}$
- $\mathbf{I}^{\prime}$
- Sin, cos $i_{N L^{-\lambda}}, i_{N R^{-\lambda}}$ using trig
- $\delta_{\text {FLAP }}{ }^{2}$

4. VELOCITY SECTION (VELOCITIES, VELOCITIES², FREESTREAM, DYN. PRESSURE, TRANSFORM.)

- Fuselage (also afus, $\boldsymbol{s}_{\text {fus }}$, sin, cos)
- Doors open/close logic-f(inAC, $q_{\text {FUSE }}$ )
- Rotor Hub - body axes, shaft axes, freestream
- Wing A/C - body axes, chord axes, freestream
- rail

5. ROTOR SECTION - LEFT AND RIGHT

- $a, \zeta, \sin , \cos$ of $a$ and $\zeta$
- Rotor angular rate transform. $p, q, R$
- $\Omega, v_{\text {rIP }}, \mu, \Omega^{2}, \mu^{2}$
- Rotor control axes transform $A_{1 C}{ }^{\prime} B_{1 C}$ WRT $\phi_{P}$, $\zeta$
- Rotor EQS for CNF, CSF, CP, CYM
- Forces and moments from coefficients

T, N.F., S.F., M,N,Q

- Hub moments - Nacelle axes
- Resolution of forces \& moments -
body axes at tip
- Summation with nacelle aero
- Gust load alleviation system

6. WING SECTION - LEFT AND RIGHT

- $q_{L W}, q_{R W}, q_{\text {WING }}$
- Doors open/close check
- If doors open
- Calc. $\mathrm{X}, \mathrm{Y}, \mathrm{Z}, \mathrm{L}, \mathrm{M}, \mathrm{N}_{\mathrm{AERO}}$
- Leave wing section \& Set q's=0
- If doors closed
- $a, \beta, a_{S S O}, a_{\mathrm{RIG}}, \bar{a}$
- Aileron, Spoiler, Flap contribution
to lift, drag, moment; Call AILSP
- Contribution due to totally washed wing; call CLCDCM
- Contribution due to totally - unwashed wing; call CLCDCM $\bar{C}_{\mathrm{L}}$; Call CCF2
- $a, \sin \& \cos \alpha, \sin a \cos \beta, \sin a \cos \varepsilon$
- a, в check for stall
- Aero Calc \& resolution
- Wing/rotor interference

7. TAIL SECTION

- Etail ${ }^{\text {co }}$ if necessary; logic for doors opentclosed
- $a_{\mathrm{HT}}, \sin , \cos \left(a_{\mathrm{HT}}{ }^{-1} \mathrm{HT}\right)$
- 7 region $\mathrm{C}_{\mathrm{L}_{\mathrm{HT}}}, \mathrm{C}_{\mathrm{D}_{\mathrm{HT}}}$ curve

- 7 Region $C_{\text {LVT }}, C_{\text {DVT }}$, curve
- If doors open; $1 / 2$ efficiency of horizontal tail
- Vertical tail Aero
- Total tail Aero

8. EQUATION OF MOTION SECTION

- Call gear aubroutine
- Total Fuse Aero
- Calculate total aircraft Aero $\mathrm{X}_{\text {AERO }}, \mathrm{y}_{\text {AERO, }} \mathbf{z}_{\text {AERO }}$, $L_{\text {AERO' }}{ }^{\prime}{ }^{\prime}{ }_{\text {AERO' }}{ }^{\prime} N_{\text {AERO }}$
- break $z_{\text {aERO }}$, LaERO, MaERO into vertical bending/nonvertical bending parts
- EOM coefficients
- Vertical bending equations with flag
- Torsion equation with flag
- Fill dac array (64)

9. DIGITAL TO ANALOG I/O

- Convert dacs to integer
- Write values

1. ANALOG TO DIGITAL I/O

- Reade discretes
- Assigns flags to discretes
- Check for trim sheet flag
- Read 3rd console ADCS if required

2. 

ENTRY FOR STATIC CHECK OF SLOW REAL TIME TASK

- Used for test cases w/O I/O

PRELIMINARY CALCULATIONS

- $\sin ^{2} i_{N L}, i_{N R} ; \sin , \cos 2 i_{N R}, 2 i_{N L} ; h^{2}, q_{F U S E}$
- XCG, 2CG
- $V_{\text {NORTH }}, V_{\text {EAST }}$, ground track

4. ALR ENGINE MODEL

- $6, \mathrm{~T}^{\circ} \mathrm{F}, \rho, \mathrm{a}, \mathrm{M}, \sqrt{1-\mathrm{M}^{2}}$
- TEA, preliminary engine routine calculations
- SHP, $\Omega E, \quad Q$ : call engine

5. FUSELAGE SECTION

- $C_{D F}, C_{L F}, C_{Y F}, C_{M F}, C_{N F}$
- Aero calculation

6 GROUND EFFECT SECTION (WING \& ROTOR)

- (ag/a)w, K99, ( $\left.T_{T G E} / T_{O G E}\right), D / T,(M / T)$

7. NACELLE SECTION LEFT \& RIGHT

- ${ }^{\alpha}{ }_{\text {NAC }}{ }^{\beta_{N A C}}$; $\sin$ cos $\alpha_{\text {NAC }}{ }^{\beta_{\text {NAC }}}$
- $C_{D}, C_{L}, C_{Y}, C_{M}, C_{N}$ nacelle
- Aero calculation

8. WING IMMERSED AREA SECTION - LEFT \& RIGHT

- $\quad T_{,} V_{*}$, Look-up $V_{*}$,

- $\zeta_{\mathrm{R} 1} \cdot \zeta_{\mathrm{R} 2} \cdot 5_{\mathrm{R} 3} \cdot \zeta_{\mathrm{R} 4}$
- $S_{i_{R}}, S_{i_{L}}, S_{i_{L}} / S, S_{i_{R}} / S, S_{i_{T}}, A R_{1}, C_{L_{i}}, K_{a}$

9. ROTOR/ROTOR INTERFERENCE
10. GROUND EFFECT - TAIL

- $\mathrm{H}_{\mathrm{w}_{\mathrm{C}} / 4}, \mathrm{H}_{\mathrm{T}_{\mathrm{C}} / 4},\left(\mathrm{a}_{\mathrm{g}} / \mathrm{a}\right)_{\mathrm{t}}$
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Figure G.6. (Continued)



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Figure G.6. (Continued)

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Figure G.6. (Continied)
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Figure G.6. (Continued)

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time tasks (RTFAST and RrSLOW), the aileron-spoiler subroutine (AILSP), the total lift, drag and moment subroutine (CLCDCM), the engine power subroutine (ENGINE), and the landing gear subroutine (GEAR) . The aileron spoiler subroutine calculates the lift, drag and pitching moment increments. The total wing lift,drag, and moment characteristics are computed in subroutine CLCDCM. Engine performance is computed in the engine subroutine and landing gear forces and moments in the gear subroutine. This listing contains an index of all the variables immediately following each subroutine. The index specifies by location number where the particular variable is defined and used in the subroutine. A master index is provided at the end which specifies the subroutine in which a particular variable is located.

In programming the analog portion of the simulation, size also was of prime concern, where in this case size implies an equipment limitation. From the beginning, equipment was allocated with maximum efficiency, but due to the complexity of the engine/governor, the phasing of the controls, the number of second order representations of actuators, and the number of functions needed to program these sections, the result was 1) three analogs used with a minimum of spare equipment and 2) 31 out of 32 BCA channels needed to program functions (includes
rotor). When the capability of using the nudge-base simulator is added, the simulation uses every piece of hardware available in the hybrid laboratory. Figure G. 7 shows the definition of the symbols used on the analog diagrams for the Model 222 simulation presented in Figure G.8.

The scale factors for any of the elements shown in the analog diagrams may be determined by referring to Figure G.9 , which is the subroutine used to static check the analog boards. The subroutine shows all the equations on the analogs and all of the scale factors. This program is used for static check only, in the operate mode the real time task continuously updates the analog. As an example, if the scale factor (and value) for potentiometer 240, which is used in the pitch equation of motion on board $1 E$ console 1 is required the following steps are taken. Refer to the potentiometer calculation section of Figure G.9. This lies between statement numbers 0416 and 0518. Look up the definition of pot 240 [Pl(240)]. This appears in statement number 0444 and is P1(240)=P1C/PMX. P1C is contained in common $|X 1 C|$, statement number 0008 and PMX is contained in common |XMAX|, statement number 0013. Substituting numerical values and dividing would yield the scale factor for pot 240 .

## G. 2 TRIM LOOPS

The Model 222 trim loops are on the analog. The aircraft accelerations are used in feedback loops to drive the aircraft into


Figure G.7. Analog Symbols
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TRIM LOOPS

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Figure G.8. Analog Diagrams for Model 222 Simulation


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Figure G.8. (Continued)

Figure G.8. (Continued)



ROTOR FUNCTION LOGIC

Figure G.8. (Continued)

Figure G.8. (Continued)



Figure G.8. (Continued)
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Figure G.8. (Continued)

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Figure G.8. (Continued)


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Figure G.8. (Continued)

Figure G．9．Analog Static Check Routine（Digital）

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Figure G.9. (Continued)


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SHPDU SHPNX






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[^13]equilibrium. The equations for these feedback loops are shown in Appendix E. Several trim options are available: for a given initial condition of altitude, $u$ and $v$ components of velocity, rotor RPM and initial rates ( $p, q, r$ ) the aircraft can be trimmed with attitude for specified nacelle angle or with nacelle angle for specified attitude. In addition, the aircraft can be trimmed in backwards or sidewards flight. The trim gains used vary with the flight condition. Trim is generally attained in 5-10 seconds for any flight condition using this technique.

## G. 3 SIMULATION PROGRAM OUTPUT

The primary output of the mathematical model are:

- Trim sheet information
- Dynamic time histories of aircraft response

Figure G. 10 shows a typical trim sheet with 180 aircraft parameters printed out and Figure G.ll contains the definitions of all the parameters. Four brush recorders with eight channels of output each are available for recording of the aircraft real time response. Figure G.l2 shows a typical example of the output from one recorder. These data are extremely useful in analyzing aircraft responses and in optimizing stability augmentation and control systems.

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\end{gathered}
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01
\end{gathered}
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\begin{gathered}
\text { CO WNG R } \\
0.23539 \mathrm{E}-\mathrm{OI}
\end{gathered}
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flight no. =
FLIGHT NO. = RJN NJ. =

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& 0 . \mathrm{F}^{\text {WIST LW }} \\
& 0 . \mathrm{D}^{\text {DELE SAS }} \\
& \text { DELSPL }
\end{aligned}
$$

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\begin{aligned}
& 2.95530 \mathrm{E}-91 \\
& \text { SHP RQR }
\end{aligned}
$$

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0.39021 E 03 \\
M \operatorname{ROT} L \\
-0.13081 E 03
\end{gathered}
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\begin{gathered}
M \operatorname{ROT} R \\
-0.35299 E^{R} J 3
\end{gathered}
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\begin{gathered}
x \mathrm{NAC}^{L} \\
-0.47215 \mathrm{~F} 02
\end{gathered}
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\begin{gathered}
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Y \mathrm{NAC} L^{2} \\
0.12310 \mathrm{~J}
\end{gathered}
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\text { DELS SAS } \\
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0 E L S R R \\
-3.9=000 \mathrm{E}-02
\end{gathered}
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\begin{gathered}
1 \times x \\
0.50275 E 05
\end{gathered}
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\begin{gathered}
x \text { NAC R } \\
-0.47178 E 02 \\
Y \text { NAC R }
\end{gathered}
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& \text { CD NAC L } \\
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& \frac{\ddot{x}}{2} \\
& x_{0} \\
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Figure G.11. Definition of Trim Sheet Parameters

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| co |  |  |  |  | Risnt nacreis - 0 compl ( $\mathrm{C}_{\mathrm{L}}$ | contr. (CPiR) | $\left.c_{T_{\text {LR }}}\right)$ | (CuFice) |
| cosef. (Cop ) |  |  |  | LEFT NACMLS $\left(C_{O_{L N}}\right)$ | $C_{D_{0}}$ | (Csfin) | LAPT ROTLR PITONAK MOMANT COAFF: (CPILR) | $\qquad$ |
| $\begin{aligned} & \left(c_{M P}\right) \\ & \left(c_{0}\right. \end{aligned}$ | lier Courf. (C |  |  |  |  |  | R/CNT pond mols. $\operatorname{Cos} F F\left(C_{7 A A}\right)$ | RIGAT porte ubint <br> Fance COwt. $\left(C_{\mu} \mu_{\mu}\right)$ |
| Fusdince ymumal Hompur contr. $\left(G_{N P}\right)$ | $\text { DORAG COFFF. }\left(C_{D r}\right)$ |  |  |  |  | $\left(\mathrm{C}_{2} \mathrm{MN}\right)$ | (CPMPOH) |  |
| $\left(6_{0}\right) \sim 0 \times \pi, \sim$ | $(\epsilon)=0 \sim 6 .$ | LEFT ROTOR <br>  |  |  |  |  |  |  |



Figure G.12. Typical Time History Response to A . 25 Inch Longitudinal Stick Pulse at 150 Knots

## AMES RESEARCH CENTER

This section presents the validation plan which was submitted to NASA prior to the checkout and validation period at the Ames Research Center, and the simulation acceptance and pilot operating instructions and limitations submitted after the checkout period.

## H. 1 VALIDATION PLAN AND CRITERIA

Validation of large scale hybrid math model is an extremely time consuming and difficult process. The validation plan and criteria to be used by Boeing Vertol personnel in checking out and validating the NASA Model 222 simulation is developed in the following section. The following items will be considered in this validation:

- Trim Checks
- Range of data: 25 kt increments from minimum to maximum speed. (including backward and sideward flight).
- Accuracy (when compared against Boeing Vertol check cases)

Trim Data Tolerance
Pilot control position
Stick and pedal position slope with speed
Thrust or wing lift Pitch, roll, yaw angles Collective pitch

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\begin{aligned}
& \pm 0.25 \mathrm{in} . \\
& \pm 10 \% \\
& \pm 21 / 2 \% \\
& \pm 1.0^{\circ} \\
& \pm .5^{\circ}
\end{aligned}
$$

These requirements are subject to change pending a detailed selection of the trim conditions.

- Dynamic Responses (Response to control pulses)
- Range of data: An axis by axis check with SAS and LAS systems on and off. Same speeds as for trim data.
- Accuracy (when compared against Boeing Vertol check cases)

Period $+10 \%$
Time to double (or half) amplitude $\pm 10 \%$

- Stability Derivative Checks
- Range of data: Selected stability and control derivatives will be obtained at no more than five conditions.
- Accuracy when compared against Boeing Vertol check cases) Selected major stability and control derivatives $\left(L_{v}, N_{v}, L_{p}, N_{p}, L_{r}, N_{r}, X_{u}, M_{w}, M_{w}, M_{q}, Z_{w}\right.$,

$$
\left.L_{\delta_{S}}, N_{\delta_{S}}, L_{\delta_{r}}, N_{\delta_{r}}, M_{\delta_{B}}, M_{i_{N}}, Z_{\delta_{T H}}\right) \quad \pm 10 \%
$$

- Validation of Time Frame

Run selected dynamic response checks at hover and cruise in real time and $1 / 10$ real time i.e. reduced interval of integration. Damping of predominant modes shall not change by more than $\pm 10 \%$.

- Transport Lag Checks

The transport lag i.e. aircraft response following control input shall not be greater than one to two time frames (average $11 / 2$ time frames)

- Pilot acceptance will be based on a subjective comparison between the results obtained in the Boeing nudge base simulator and those obtained on the FSAA.

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## H. 2 SIMULATION ACCEPTANCE

Following the checkout period at Ames, the following simulation acceptance document was submitted.

The math model, as programmed, is considered acceptable for initial evaluations.

The following differences exist between the math model and the aircraft described in Boeing's proposal for Phase II.

1. The data bank in the math model gives very conservative values of power around the autorotation region. The math model uses data from computer program D-88. Boeing's proposal uses data from wind tunnel model tests which were compared with D-88 predictions. The wind tunnel data showed consistently lower power required in and near autorotation. Revision of the rotor data bank to incorporate the wind tunnel data was not practicable within the time available. The extent of the difference is indicated by a minimum rate of descent at 80 knots from the math model of 2600 feet per minute compared to about 2000 feet per minute from the wind tunnel model data as reported in Volume I, Appendix G.
2. There is no autopilot in the math model. This was not required by the contract statement of Work.
3. The landing gear dynamics in the model are an existing CSC program and do not represent the Model 222. There is some H-3
indication that the CSC gear causes lateral instability on the ground at less than $25 \%$ power.
4. The wing bending and torsion modes were not checked rut due to lack of time. Data obtained at Vertol indicate that these have no measurable effect on performance or flying qualities.
5. The representation of the SAS gives proper dynamic characteristics in the SAS on and SAS off modes. Individual component failures are not represented because of mechanization differences between the aircraft and the math model.
6. The actuator dynamics, which were included in the math model used on Boeing's nudge base simulator, were removed from the FSAA simultion in order to keep the time frame to a minimum. Evaluation on the Sigma 8 Computer at NASA showed no measurable difference as a result of removing the actuator dynamics. (The actuators have time constants less than the 50 millisecond time frame of the FSAA simulation.)
7. Boeing's proposed aircraft provides a pilot override for flap position and for rpm selection. These are not included in the math model. The chekcout and validation of the tilt rotor math model was accomplished in two phases. These were the math model aceptance and the simulator acceptance.
A. Math Model Acceptance:
8. Trim checks were calculated for a range of speeds from hover to 250 knots in 25 knot intervals. These were checked against previously computed trim conditions.

The results agreed within the tolerances specified in the reference.
2. Due to the limited time available for validation, dynamic responses were not checked over the full range of condition noted in the reference. However, dynamic responses were computed for a representative sample of condition. These compared favorably with those generated at Boeing and were generally within the specified tolerances. Differences could be explained by the different methods which were used to mechanize the equations; e.g., ---in the Boeing hybrid mechanization, rotor data were interpolated parabolicly for angle of attack and linearly for advance ratio. In the NASA mechanization, curve fit equations were solved at each angle of attack and linearly interpolated for advance ratio. This tends to produce differences in areas where the data is highly non-linear such as in transition.
3. Stability derivative checks were made at four speeds; $0,75,150$ and 250 knots. These were generally within the $\pm 10 \%$ accuracy specified by Boeing. Differences between the results are explainable and primarily due to the differenct ways the equations were mechanized.
4. Time frame studies and transport lag checks were made. Neither proved to be probiem areas even though the NASA
simulation has a frame time in excess of 50 milliseconds. No lags were apparent in the simulation cab due to transport lag.

## B. Boeing Pilot Acceptance:

The simulation is considered acceptable for initial evaluation however, Boeing's evaluation pilot made the following comments:

1. Controls:

- Power lever location not optimum but acceptable for evaluation using seat arm rest for support.
. Nacelle tilt switch - spring gradient slightly weak, $5^{\circ}$ detents appear to be not in center of available travel. Switch occasionally sticks producing uncommanded nacelle actuation. Suitable for evaluation.
- Stick forces - breakout and damping poor- difficult to achieve positive trim detent. Occasional shift in stick trim from one run to the next. (Simulator equipment problem)


## 2. Motion:

- B V pilot considered motion cues unsatisfactory.
- Lateral motion washouts and/or recentering produced spurious jerks and pulses which were disorienting.

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- Roll angular acceleration cues weak.
. Pitch, yaw and heave satisfactory.
- Longitudinal acceleration cues - long period cab tilt ok, short period were jerky and disconcerting with recentering reversals apparent.
- Summary: There was enough spurious motion that overall the tilt/motion cues were detrimental and the pilot preferred fixed base.

3. Model Flying Qualities:

Generally similar to $B V$ in-house simulation except
for:

- Vertical response slightly overdamped.
- Unable to cut engine(s) until last day. As a result, not able to properly check out power lever governor override.
- Pedal fixed turns in prop mode not as well coordinated - $30^{\circ}$ banked turns show $1 / 2$ to $3 / 4$ ball slip to $T \& S$ indicator, with $S / S$ ind. reading $1^{\circ}-2^{\circ}$.

4. Boeing was not able to evaluate the Model 222's response to gusts, since the gust model has not been defined. Response to random turbulence was evaluated.

## C. General:

The original time alloted for the checkout and validation of this model was extremely short, particularly in view of the computer software problems and the difficulties encountered in establishing the gains for the FSAA motion drive equations. As a result, the checkout period had to be extended by NASA for two weeks.

## H. 3 OPERATING INSTRUCTIONS AND LIMITATIONS

As part of the simulation checkout, a set of operating instructions and limitations were prepared. For the most part these refer to the piloted simulation and the mathematical model and do not imply limitations on the Model 222 aircraft.

## General:

1. I.C. - Set power lever trim to "0". (suggest using left-hand arm rest)
2. Stick grip has both Mag. brake and vernier "beep" force retrim. If Mag. braking is desired, use only in hover -- advise beep retrimming for transition and prop mode. (In real A/C, Mag. brake will deactivate above 150 KIAS)
3. Flaps and RPM are programmed automatically as a function of nacelle angle. In the Model 222 there will be manual flap and RPM override controls.
4. Nacelle angle has "q" interlock. Nacelles cannot be programmed "up" above 160 KIAS. IF 160 KIAS is exceeded with nacelle angle greater than $0^{\circ}$, they will automatically program to $0^{\circ}$ at a rate of $2^{\circ} / \mathrm{sec}$.
5. Nacelle angle switch gives nacell"e rate proporfional to displacement. Switch is spring loaded to center off position and has a detent either side of center, corresponding to approximately $\pm 5^{\circ} / \mathrm{sec}$ rate. Full displacement will give approximately $\pm 10^{\circ} /$ sec. For smoothest nacelle operation, use proportional feature; avoid "flick" type beep inputs.
6. Wing leading edge umbrellas automatically open or close at 50 KIAS.
7. Normal power lever travel is $8^{\prime \prime}$. This range represents flight idle to maximum power. There is a soft detent at 8 inches of travel which, when exceeded, overrides the governor. In this condition, the power lever controls collective pitch directly. This is provided for use as desired in autorotation and single engine landings.
8. The Model 222 is designed to go through transition at speeds between zero and 160 knots. Typical trimmed nacelle incidences at various speeds are:

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In investigating handling characteristics in the transition mode, it is recommended that these values be used as initial conditions.

In performing normal transitions to and from hover, it is recommended that the nacelle tilt be used as the primary speed control rather than flying at fixed tilt and using the stick for speed control.

## Limitations:

1. Observe torque limits:

75\% twin engine
$100 \%$ single engine
2. Autorotation:

Engines must be failed from console to achieve zero torque. Transitions can be made from airplane to helo mode with power lever full back, but some residual torque remains ( $10 \%$ total) and $N_{R}$ trims out nominally at 70\%.

- Autorotative sink rate at $i_{N}=90^{\circ}$ approximately $3500 \mathrm{ft} / \mathrm{min}$. Minimum rate of sink is about 2600 $\mathrm{ft} / \mathrm{min}$ at 80 knots at $\mathrm{i}_{\mathrm{N}}=60^{\circ}(70 \% \mathrm{RPM})$. Model gives higher descent rates than airplane.
- Power lever has detent at approximately 8". Pushing through detent will override governor (single engine failure or auto collective) and give direct control of collective pitch.

Technique on engine cut in hover - advance power lever to detent, remaining engine will go to $100 \%$ torque. Use override as required, but once into direct C.P. control $N_{R}$ will bleed off in same manner as turbine helo with one engine over-pitched. At topping power, model gross weight is too high for single engine hover. At max single engine power, vertical speed is -300 FPM.
3. At speeds above the normal flight envelope with nacelles tilted, the math model data bank is extrapolated from a curve fit and is not representative of the full scale aircraft. Speed and nacelle incidence limits for valid simulation are shown in the following table:

$$
\begin{array}{crrrr}
i_{N}-\text { DEG } & 90 & 60 & 45 & 30 \\
\text { Speed - KEAS } & 100 & 125 & 125 & 140
\end{array}
$$

These speeds should not be exceeded.
4. Aircraft oscillates if power is reduced below approximately $25 \%$ on the ground.
5. Tho math model is not set up to readily perform SAS or governor hardover studies. These may be approximated by setting the appropriate authority limits.
6. The Model 222 autopilot has not been incorporated into the simulation.


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