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15TH ORDER RESONANCE ON THE DECAYING ORBIT OF TETR-3

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ABSTRACT

The orbit of TETR-3 (1971-83B), inclination: 33°, passed through resonance with 15th order geopotential terms in February 1972. The resonance caused the orbit inclination to increase by 0.015°. Analysis of 48 sets of mean Kepler elements for this satellite in 1971-1972 (across the resonance) has established the following strong constraint for high degree, 15th order gravitational terms (normalized):

$$
\begin{aligned}
10^9(C,S)_{15} &= (28.3 \pm 1.5,\ 7.4 \pm 1.5) = \\
&0.001(C,S)_{15,15} - 0.015(C,S)_{17,15} + 0.073(C,S)_{19,15} \\
&- 0.219(C,S)_{21,15} + 0.477(C,S)_{23,15} - 0.781(C,S)_{25,15} \\
&+ 1.000(C,S)_{27,15} - 0.963(C,S)_{29,15} + 0.622(C,S)_{31,15} \\
&+ 0.119(C,S)_{33,15} - 0.290(C,S)_{35,15} + 0.403(C,S)_{37,15} \\
&- 0.223(C,S)_{39,15} - 0.058(C,S)_{41,15} + \cdots
\end{aligned}
$$

This result combined with previous results on high inclination 15th order and other resonant orbits suggests that the coefficients of the gravity field beyond the 16th degree are significantly smaller than Kaula's rule $(10^{-5}/\ell^2)$.

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INTRODUCTION

In the last two years considerable progress has been made in the study of the high degree and order geopotential from decaying, low altitude resonant orbits.[1,2] These orbits are distinguished from the many near resonant ones, in actually passing through perfect commensurability. As a consequence the decaying, resonant orbits suffer much larger perturbations over longer times. Therefore; these commensurate orbits can, in principle, be studied with much simpler techniques. In addition the decaying, resonant orbits offer the novelty of having essentially unpredictable perturbations through commensurability.[3] This arises because the changes are critically dependent on the satellite's longitude. But due to high and uncertain drag, the longitude is rapidly lost for these satellites without continual tracking.

The decaying, resonant orbits receiving the most attention are, naturally, the most plentiful ones. These are the ones commensurable with 15th order geopotential terms. Orbits established near 15 revolutions a day (at about 500 km altitude) are common to begin with. In addition, those initially above the critical altitude will decay (from the effects of atmospheric drag) sufficiently fast to pass through perfect commensurability in a reasonable time (say from 5 to 10 years from launch).

In his 1973 paper,[1] King-Hele presents good results on ten such (15 rev./day) orbits of satellites launched since 1965. (Many more have passed through this resonance without tracking adequate to see the perturbations.) However, only six of these are sufficiently distinct to be useful in discriminating the odd degree geopotential terms to which they are sensitive. Furthermore, these six orbits are all at inclinations over 50°, making them particularly sensitive to the low (odd) degree terms. If this sensitivity were exclusive, it would be an advantage and enable a good determination of the first few such terms from the available data. But this is not the case, so that even with six orbits King-Hele has not demonstrated an acceptable solution for some of these low degree terms [i.e., (15,15), (17,15) and (19,15)].

It appears that good discrimination of 15th order resonances will only come when a better range of inclinations are available. This is essentially the same consideration as in the satellite determination of the general geopotential field. To further this end, we study the 15th order commensurability (in 1972) of the orbit of TETR-3 (1971-83B). It is the first low inclination orbit (33°) used for this purpose and as such, is very sensitive to the high degree terms rather poorly represented by the previously analyzed orbits. A strong constraint on 15th order terms from the TETR orbit is derived, compared and combined with the previous results to yield a reasonable set of terms through (39,15).

DATA

Table 1 presents the (analyzed) mean elements for TETR-3 as determined at Goddard Space Flight Center in 1971-72 from Minitrack radio interferometer tracking measurements. TETR-D (TETR-3 in orbit) is a magnetically stabilized octahedron (20 kg, 0.3 m on a side) launched September 28, 1971 and tracked till June 1972 when the satellite was "retired". A unified S-band transponder failed soon after orbit was achieved and the primary mission for TETR-D was lost. This was to calibrate the NASA's Manned Space Flight Network. Nevertheless excellent (and compatible) elements on TETR-3 were obtained from the tracking data by two methods.

The unstarred sets in Table 1 are the routine Goddard Brouwer mean elements for this orbit determined from independent data at (usually) one week intervals. These are essentially Brouwer single primed mean elements[4] (with the Brouwer long period zonal terms kept in). They are equivalent to conventional mean elements defined as osculating values less only short period terms which time-average to zero over the anomalistic orbit period. Two corrections to the Goddard reported (double primed) quantities have been made. The major one, converting to single primed elements used the following zonal coefficients (from the original orbit determinations): $10^3 J_2 = +1082.48$, $10^6 J_3 = -2.56$, $10^6 J_4 = -1.84$ and $10^6 J_5 = -0.06$. The second, very small, correction to the single primed elements is for long period terms implicit in them because the short period Brouwer terms only orbit average to zero with respect to the true anomaly, not the mean anomaly. This second correction is given on p. 371 of Kozai's 1959 Astronomical Journal paper.[5]

The starred sets in Table 1 are mean elements converted by an analytic-numeric filter[6] from precise osculating values. The osculating elements (input to the filter) were determined by least squares fitting to (usually) 4 day data arcs using a precise trajectory calculated by numerical integration (See Appendix). The filter determines the Kepler elements of a best fitting secularly precessing ellipse to a one day arc of osculating data from the precise trajectory. The osculating data is first smoothed analytically by the removal of short and intermediate period terms due to the geopotential. The quality of these specially filtered mean elements is significantly better than the routine Brouwer elements. For example, independent processing of the inclination data shows that, after removing long term geopotential, radiation pressure and luni-solar gravity effects, the Brouwer inclinations have "residuals" (observed minus computed values) about a mean value of $\pm 0.0008°$ (rms). The equivalent residual for the filtered elements is $\pm 0.0004°$ (rms).

The double starred sets in Table 1 were filtered mean elements received after the analysis reported here was completed. They were not used in the results. However, preliminary tests show that these results are not significantly altered (but somewhat sharpened) with this "new" data.

ANALYSIS

This analysis of the 15th order resonance pass for TETR-3 is essentially the same as that for the 11th order pass of the orbit of Vanguard 3 (1959-7A).[2] Independent sets of mean elements are treated as observables.

Their long term variations are analyzed by a rapidly integrating semi-numeric program (ROAD) which accounts for all significant geopotential, radiation, drag and luni-solar gravity effects. The ROAD program calculates (by a least squares, differential orbit correction process) a pair of geopotential coefficients which "absorbs" the resonance perturbations across the commensurability. Although all the data is used in this orbit-geopotential fitting process, the heaviest weight is put on the inclination data (following King-Hele[1]) which is the least corrupted by uncertainties in the drag. Using the analytic variation of the inclination due to all the resonance terms, constraints for these terms are developed and evaluated from the ROAD results.

The chief distinguishing feature of this resonance (contrasted with Vanguard 3) is that it is "seen" directly in the inclination "observations" which show an increase of about 0.015° across the time of exact commensurability (Figure 1). In the case of Vanguard 3, odd zonal geopotential gravity dominated the long term inclination variation and the resonant effects could only be seen in "residuals" with the other perturbations removed.

Figure 1 shows the inclination "observations" (from Table 1: unstarred and singly starred only) on TETR with their standard errors as given previously. The dashed curve shows the evolution of the inclination for the orbit computed (by ROAD) without resonant geopotential effects but with all other relevant perturbations. There is a small secular decrease due to atmospheric rotation and minor periodic changes due to odd zonal (geopotential), luni-solar gravity, and radiation pressure effects. The solid curve shows the same computation with two resonant geopotential terms added. The characteristic "step" in the inclination with preceding and succeeding building and dying oscillations[3] is unmistakable.

Plotted against the lower right hand axis (in Figure 1) is the characteristic longitude rate which dominates this resonance (commensurability occurring when $\dot{\psi} = 0$). The significance of this particular rate arises from the characteristic geopotential variation whose longitude argument (ψ) is stationary at resonance. (The fact that $\ddot{\psi}$ is nearly constant shows that the drag forces along track on the orbit predominates over the resonant ones, even in the vicinity of the commensurability.)

The variation of the inclination due to a particular gravitational harmonic term (ℓ, m, p, q) in Kaula's development of the potential[7] is [from the Lagrange Planetary Equations]:

$$\dot{I} = \frac{1}{na^2(1-e^2)^{1/2} \sin I} \left\{ \cos I \frac{\partial T}{\partial \omega} - \frac{\partial T}{\partial \Omega} \right\}, \tag{1}$$

where,

$$T = \frac{\mu \, a_e^\ell}{a^{\ell+1}} F_{\ell,m,p} (I) \, G_{\ell,p,q} (e) \, S_{\ell,m,p,q} ,$$

and:

$$S_{\ell,m,p,q} = \begin{bmatrix} C_{\ell,m} \\ -S_{\ell,m} \end{bmatrix}_{\ell-m \text{ odd}}^{\ell-m \text{ even}} \cos \psi_{\ell,m,p,q} + \begin{bmatrix} S_{\ell,m} \\ C_{\ell,m} \end{bmatrix}_{\ell-m \text{ odd}}^{\ell-m \text{ even}} \sin \psi_{\ell,m,p,q} ,$$

with the orbit longitude (ψ) defined as:

$$\psi_{\ell,m,p,q} = (\ell - 2p) \, \omega + (\ell - 2p + q) \, M + m \, (\Omega - \theta).$$

In the above expressions, μ is the Earth's Gaussian gravity constant, a_e is its mean equatorial radius, θ is the hour angle of Greenwich, a is the orbit's semimajor axis, n its mean motion, and I, e, ω, Ω, and M its inclination, eccentricity, argument of perigee, ascending node, and mean anomaly. The F functions are sinusoidal with frequency proportional to ℓ-m, and the G functions are generally monotonic of order $e^{|q|}$. For TETR ($I = 33°$, $e = 0.01$) this implies special sensitivity to those terms for which q is low and ℓ-m is high. The $C_{\ell,m}$ and $S_{\ell,m}$ are the usual gravitational harmonic coefficients (fully normalized).

Orbital resonance occurs when $\dot{\psi} = 0$ for any gravitational term since, at that time $\dot{I}$ is constant (to first order) and I can increase linearly with time. Note that in Figure 1 the inclination increase near resonance is roughly linear. Examination of just the resonance variation shows this precisely. There is a point of inflextion ($\dot{I}$ = const, $\ddot{I} = 0$) at exact commensurability.[3] For a near circular orbit the dominant commensurabilities are those for which q = 0 and $\ell - 2p = 1$, so that $\dot{\psi} \approx 0$ when $m = \dot{M}/\dot{\theta}$. For $\dot{M} \doteq 15$ revs/day, the resonant order is m = 15 and the degrees for the dominant series are odd since 2p is even. Other resonant series exist on TETR for m = 15 ($q \neq 0$) near the dominant one (ℓ, m, p, q = ℓ, 15, $(\ell-1)/2$, 0; $\ell \geqslant 15$, odd) but these have much less effect on the orbit because the G functions for them are small. In addition, commensurabilities for m = 30, 45, 60, etc. (q = 0) also exist at the dominant resonance. These have minor effect because their degrees ($\ell \geqslant 30$, 45, etc.) are large so that their potential effect at altitude are scaled down. In addition, the passage through these resonances are faster (permitting less buildup of perturbations) than the dominant one and the expected gravity terms (C, S) are smaller $(10^{-5}/\ell^2)$.

The dominant resonant orbit longitude is thus:

$$\psi_{m,q} = \psi_{15,0} = \omega + M + 15\,(\Omega - \theta),$$

and it is the rate of this longitude which is seen to go through zero at resonance in Figure 1. This argument is the same for all degrees of the series so that an evaluation of the actual inclination variation essentially can determine only the amplitudes of a sine and cosine of this argument. These (determinable) amplitudes are, in turn, weighted sums of the $S_{\ell,15}$ and $C_{\ell,15}$ gravity coefficients according to factors which [from Equation (1)] depend on the degree and the particular F and G functions of that degree.[1,2] Evaluating these factors from Equation (1) and normalizing with respect to the highest weight, these two determinable amplitudes (constraints) are found to be:

$$\begin{aligned}(C,S)_{15} = {} & 0.00133\,(C,S)_{15,15} - 0.015\,(C,S)_{17,15} + 0.073\,(C,S)_{19,15} \\ & - 0.219\,(C,S)_{21,15} + 0.477\,(C,S)_{23,15} - 0.781\,(C,S)_{25,15} + 1.000\,(C,S)_{27,15} \\ & - 0.963\,(C,S)_{29,15} + 0.622\,(C,S)_{31,15} + 0.119\,(C,S)_{33,15} - 0.290\,(C,S)_{35,15} \\ & + 0.403\,(C,S)_{37,15} - 0.223\,(C,S)_{39,15} - 0.058\,(C,S)_{41,15} + \ldots \end{aligned} \tag{2}$$

The (15,15) term, while negligible, is given to three significant figures because this was the (lumped) term actually solved for in the data reduction. The series is carried to the point where the "influence" (weight) factors are less than 20%.

Clearly this is a slowly converging sum even with the benefit of coefficients decreasing according to Kaula's rule[7] ($10^{-5}/\ell^2$).

DATA REDUCTION

The data in Table 1 was analyzed in the ROAD program for all relevant long period variations (zonal Earth gravity, radiation pressure, atmospheric drag, luni-solar gravity, motion of the pole) and a single pair of resonant coefficients $(C,S)_{15,15}$. The inclination data was most heavily weighted in this analysis which was in all essential aspects identical to that performed on Vanguard 3 to determine its resonant coefficients.[2] For example, the most critical aspect of the analysis was again the calculation of the satellite's mean anomaly to insure the proper phase for the resonance. This was accomplished in ROAD to within ±5° of the observations with the aid of an empirically determined 3rd degree secular term in the mean anomaly. As before, the secular term was only weakly correlated with the resonant coefficients because the mean anomaly data was not strongly weighted.

The coefficients determined by ROAD, producing the (solid curve) inclination evolution in Figure 1, were:

$$10^6\,(C,S)_{15,15} = (21.3 \pm 1.1,\ 5.6 \pm 1.1),$$

with a correlation of -0.84 between these parameters. Using this result in Equation (2) determines the cosine and sine constraint for TETR-3 as

$$10^9\,(C,S)_{15} = (28.3 \pm 1.5,\ 7.4 \pm 1.5).$$

Further analysis of the data for the nearby resonances of even degree with $q = \pm 1$ produced no significant change in this result nor did the analysis for the resonance with 30th order terms.

RESULTS AND DISCUSSION

The most convenient way to present the results of the TETR analysis is in a (C,S) diagram of the kind used by Kozai to compare individual gravitational terms[8] (see Figure 2). Here the determined constraint is represented as a rotated 1 σ ellipse with considerably different semi-major and minor axes due to the relatively high correlation. It should be remarked (happily) that TETR-3 yields the first 15th order constraint in the first quadrant. The previous ten analyzed orbits have all given these lumped terms in the 3rd quadrant.[1] But this fact alone strongly suggests that even with 11 orbits, we still have an overly biased sample to obtain a good separation of terms. The TETR result is also only the second occurrence of an inclination increase through the resonance; a nice accident too, but of no bearing on the problem of separation. According to Allan's analysis,[3] there is almost an equal chance of inclination increase or decrease in a strongly dragged resonance such as this.

But the occurrence of the high correlation between sine and cosine coefficient has not been remarked on. These are important and have only been reported once before (for Ariel 3) on 15th order resonances.[9] They are likely to be high because they arise from the (generally) unequal sampling of the sine and cosine potential functions during the passage. Heavy sampling occurs in that (local) portion of the potential closest to the commensurability; light sampling takes place elsewhere. There may be special conditions for the passage which produce zero or small correlation, but the likelihood of them appears to be small. For Ariel 3, the reported correlation was -0.82. For the 11th order Vanguard 3 resonance,[2] the correlation was -0.51. The highly correlated constraints should significantly alter least squares solutions for individual terms to force the "calculated" constraint to "line up" with the major axis of the "observed" constraint. But such adjusted solutions cannot be tried until more correlations are known.

Obvious tests of the observed TETR constraint are with calculated values from gravitational fields containing a significant number of gravitational terms. For the TETR case the fields should extend to at least (27,15). But since no such field has been calculated from other data, the best we can do is use tentative 15th order fields estimated from previously analyzed decaying, resonant orbits[1,10] (Figure 2). The calculated constraint from three such fields (listed in Table 2) are shown here. The first is a 4 orbit, 4 term solution[10] from 1972 which is complete to (21,15) only. This solution contains only one orbit, with inclination less than 60°, which is especially sensitive to high degree terms ($\ell > 21$). It gives a small (and poor) TETR calculation, mainly because the maximum degree is not high enough. The second is a 6 orbit, 6 term solution, complete through (25,15). It uses the "best" data from King-Hele's 1973 analysis[1] for the six distinct orbits of inclination 51°, 56°, 63°, 74°, 80° and 90°. The result on TETR is much too large. Even though higher degree terms (sensitive to TETR) are represented, they are not well determined. The terms of degree 23 and 25 are clearly absorbing the effects of terms of higher degree which are not solved for but which have significant effect on two of the orbits (56°, 63°).

These orbit sensitivities and constraints (including TETR) are shown in Table 3 to degree 39.

Calculations using Kaula's rule for the coefficients show that for the higher inclination orbits the influence of terms beyond 25 should be negligible. But for the orbits below 70° inclination, higher order terms should have significant effect compared to the precision of the constraint. Unfortunately there are, as yet, not quite enough orbits of high inclination to separate the low degree terms completely using the resonant data alone. The unreasonable high C values in the 6 term, 6 satellite solution show this. King-Hele has found[1] that (least squares) solutions for less than five terms do not recover the constraints for the six distinct orbits ($I > 50°$) satisfactorily. A 5 term (7 orbit) solution (listed in Table 2) does but also has an unreasonably distorted set of C values. However, it is a smoother (least squares) solution than the 6 orbit, 6 term one (with smaller values as a set). By chance (?) it comes the closest in calculating the TETR result ($\sim 8\,\sigma$).

The addition of the TETR result might be expected to help resolve the uncertainty and distortion in the lower degree C values by more accurately absorbing the higher degree effects to which it is sensitive. However (solutions show), this single orbit still cannot do the whole job because its sensitivity extends too far. It is tempting to turn this drawback into an asset by using outside information and seeking a solution (with TETR) as high as that sensitivity extends (i.e., $\ell \leqslant 39$).

One logical way to do this, in the context of weighted least squares, is to constrain the whole set of coefficients to zero with uncertainties (errors) given by a rule, such as Kaula's, for the expected size of the coefficients (rms). The fourth field in Table 2 is such a "Bayesian" least squares solution for (15,15) through (39,15), ℓ odd, using the results on the seven orbits in Table 3 as conditioning data. The residuals (observed constraints minus computed quantities, including a priori information) are all less than 1σ except for only two which lie between 1σ and 2σ. This is a very compatible solution for 40 condition equations. The striking feature of it is the significant adjustment of $C_{15,15}$ and $C_{23,15}$ from King-Hele's 5 term solution (in Table 2). This adjustment essentially eliminates the distortion (noted by King-Hele) and restores $C_{15,15}$ to previous values compatible with exact solutions for the orbit constraints.

The formal standard deviations of this "Bayesian" solution suggest that many terms are not significantly different from zero. It seemed meaningful to ask, therefore, whether a reasonable 7 term (exact) solution for the seven orbits existed with zeros for the least significant coefficients. After some experimentation field number 5 (Table 2) was found to satisfy these requirements.

It also seemed worthwhile to compare another complete solution through (39,15) computed on a different basis so as to have a range of possibilities for the coefficients. We chose to compute an exact minimum coefficient power solution; that is a coefficient set which both satisfied the condition equations (without error) and had a minimum sum of squares. Field 6 (Table 2) is this solution.

However it may be rather easily seen that this "minimum power" solution is actually equivalent to a special case of a "Bayesian" least squares fit. This is a "fit" without error on the orbit equations and with zero a priori estimates and equal error for the coefficients. These conditions are not terribly different from the realistically "errored" Bayesian solution. With regard to the exact 7 term solution, the large number of "insignificant" coefficients and the good Bayesian fit, make it seem probable that a reasonable set can be found which matches the orbit results perfectly. The real question is: what is a reasonably full latitude for these terms, given this limited data?

It is true that the formal standard deviations of the Bayesian solution "cover" the two other "varied" solutions with TETR for all but a few coefficients of high degree. But a fair judgement (aware of the limited expectations with the TETR data) would be that these statistics are only to be trusted to about the 25th degree. Nevertheless, these limited results, when compared with Kaula's rule, suggest that many of the terms beyond degree 15 may be significantly smaller (rms) than the rule (see Figure 3). However it should be said that no complete fields have been published beyond about degree 16. But these fields beyond degree 16 do contain four or five orders. Nevertheless more definitive results must wait on the analysis of further odd and even degree, 15th order resonant orbits and the extension of the complete fields. Meanwhile, the present orbits should considerably strengthen the more "complete" solutions for the gravitational fields from diverse data sources.[11]

CONCLUSIONS

The analysis of 15th order, odd degree, gravitational coefficients has been strengthened by the inclusion of resonance data on the decaying orbit of TETR-3 (1971-83B). A strong constraint on these terms, especially those above degree 21, has been developed. In combination with other resonant data and a priori information, the TETR data shows that the relevant 15th order terms (except for the 23rd degree) are significantly less than Kaula's rule ($10^{-5}/\ell^2$) at least as high as the 25th degree. Recent high order comprehensive gravity solutions appear to confirm this judgement.

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REFERENCES

1. D. G. King-Hele, "Resonance effects in decaying satellite orbits, and their use in studies of the geopotential," Royal Aircraft Establishment, Farnborough, Hants., England, May 1973. Presented to: The First International Symposium on the Use of Artificial Satellites for Geodesy and Geodynamics, Athens, Greece, May 1973.

2. C. A. Wagner, "11th order resonance terms in the geopotential from the orbit of Vanguard 3," Goddard Space Flight Center Document X-592-73-130, Greenbelt, Md., May 1973.

3. R. R. Allan, "Resonant effect on inclination for close satellites," Royal Aircraft Establishment Technical Report 71245, Farnborough, Hants., England, 1971.

4. D. Brouwer, "Solution of the problem of artificial satellite theory without drag," Astronomical Journal, 64, 378-397, 1959.

5. Y. Kozai, "The motion of a close earth satellite," Astronomical Journal, 64, 367-377, 1959.

6. B. C. Douglas, J. G. Marsh, and N. E. Mullins, "Mean elements of GEOS 1 and GEOS 2," Goddard Space Flight Center Document X-553-72-85, Greenbelt, Md., 1972.

7. W. M. Kaula, "Theory of satellite geodesy," Blaisdell Press, Waltham, Mass., 1966.

8. Y. Kozai, "The earth gravitational potential derived from satellite motion," Space Science Reviews, 5, 818-879, 1966.

9. R. H. Gooding, "Lumped geopotential coefficients $\overline{C}_{15,15}$ and $\overline{S}_{15,15}$ obtained from resonant variation in the orbit of Ariel 3," Royal Aircraft Establishment Technical Report, 71068, Farnborough, Hants., England, 1971.

10. D. G. King-Hele., "15th order harmonics in the geopotential from analysis of decaying satellite orbits," Royal Aircraft Establishment, Farnborough, Hants., England, 1972. Presented to: The 15th COSPAR Meeting, Paper a.2, Madrid, Spain, May 1972.

11. F. J. Lerch, C. A. Wagner, B. H. Putney, M. L. Sandson, J. E. Brownd, J. A. Richardson, and W. A. Taylor, "Gravitational field models GEM 3 and 4," Goddard Space Flight Center Document X-592-72-476, Greenbelt, Md., 1972.

Table 1

Mean Elements for TETR-3 (1971-83B)

TIME (MJD)	a(e.r.)	e	i°	ω°	Ω°	M°
41223.43402780	1.07682137	.0129728	33.0656	50.1704	310.3399	203.2011
~~41225.00000000~~	~~1.07682451~~	~~.0130227~~	~~33.0867~~	~~63.7723~~	~~300.2136~~	~~170.0824~~
41228.00000000	1.07682388	.0130055	33.0654	63.8772	300.2135	169.9763
~~41229.00000000~~	~~1.07682388~~	~~.0130158~~	~~33.0650~~	~~63.9105~~	~~300.2109~~	~~169.9449~~
41232.00000000	1.07679237	.0129280	33.0827	128.9158	254.9216	120.0564
~~41239.00000000~~	~~1.07676211~~	~~.0122275~~	~~33.0819~~	~~195.8435~~	~~209.6265~~	~~69.9196~~
41246.00000000	1.07673781	.0119118	33.0828	265.8229	164.3192	18.1575
~~41253.00000000~~	~~1.07670958~~	~~.0121681~~	~~33.0836~~	~~336.1554~~	~~119.0133~~	~~327.4897~~
41260.00000000	1.07667588	.0127857	33.0854	43.6429	73.7029	281.3267
~~41267.00000000~~	~~1.07663746~~	~~.0129116~~	~~33.0862~~	~~108.6592~~	~~28.3901~~	~~239.8736~~
41274.00000000	1.07660281	.0124083	33.0867	174.9488	343.0704	198.5537
~~41281.00000000~~	~~1.07655735~~	~~.0118359~~	~~33.0852~~	~~244.3447~~	~~297.7465~~	~~156.4820~~
*41282.50000000	1.07654520	.0118291	33.0860	259.4619	288.0326	121.8461
~~41288.00000000~~	~~1.07651141~~	~~.0119667~~	~~33.0839~~	~~315.1793~~	~~252.4217~~	~~115.5061~~
**41289.50000000	1.07649921	.0120736	33.0832	329.9796	242.7015	81.7247
~~41295.00000000~~	~~1.07646876~~	~~.0125000~~	~~33.0800~~	~~23.3576~~	~~207.1764~~	~~79.6115~~
**41296.50000000	1.07645605	.0126408	33.0816	37.5709	197.3583	46.9119
~~41302.00000000~~	~~1.07641167~~	~~.0128701~~	~~33.0797~~	~~88.7769~~	~~161.7256~~	~~49.0948~~
**41303.50000000	1.07639207	.0128114	33.0795	102.6743	152.0106	17.3405
~~41309.00000000~~	~~1.07633928~~	~~.0124443~~	~~33.0805~~	~~154.0862~~	~~116.3673~~	~~22.2281~~
**41310.50000000	1.07632283	.0123497	33.0825	168.5665	106.6477	350.7757
~~41316.00000000~~	~~1.07627466~~	~~.0118048~~	~~33.0835~~	~~222.5650~~	~~71.7439~~	~~355.9151~~
**41317.50000000	1.07625900	.0117333	33.0846	237.5079	61.2758	324.7568
~~41323.00000000~~	~~1.07622073~~	~~.0116187~~	~~33.0838~~	~~293.2910~~	~~25.6220~~	~~330.6088~~
**41324.50000000	1.07620792	.0117188	33.0650	308.5027	15.8954	299.7729
~~41330.00000000~~	~~1.07616899~~	~~.0120797~~	~~33.0847~~	~~3.0976~~	~~340.2404~~	~~308.9275~~
**41331.50000000	1.07615632	.0122710	33.0854	17.3375	330.5126	279.6673
~~41337.00000000~~	~~1.07611192~~	~~.0126246~~	~~33.0849~~	~~69.3722~~	~~294.8532~~	~~293.7288~~
41337.00000000	1.07611192	.0126210	33.0850	69.3540	294.8532	293.7478
~~**41338.50000000~~	~~1.07609815~~	~~.0126284~~	~~33.0860~~	~~83.2393~~	~~285.1229~~	~~265.5022~~
41344.00000000	1.07604544	.0124319	33.0845	134.4293	249.4554	283.0449
~~**41345.50000000~~	~~1.07602803~~	~~.0123428~~	~~33.0861~~	~~148.6760~~	~~239.7237~~	~~255.2167~~
41351.00000000	1.07598383	.0118063	33.0883	201.9487	204.0429	273.4837
~~**41352.57003889~~	~~1.07596889~~	~~.0116980~~	~~33.0891~~	~~217.4964~~	~~193.8568~~	~~271.0669~~
41358.00000000	1.07593209	.0114129	33.0906	272.7014	158.6251	263.7065
~~*41359.50000000~~	~~1.07591976~~	~~.0114804~~	~~33.0915~~	~~287.8110~~	~~148.8511~~	~~236.3026~~
41365.00000000	1.07587172	.0117639	33.0932	342.8484	113.2022	257.3512
~~41372.00000000~~	~~1.07579098~~	~~.0122960~~	~~33.0938~~	~~59.5821~~	~~67.7690~~	~~257.3816~~
*41375.50000000	1.07575199	.0124627	33.0945	82.9066	45.0473	80.5517
~~41379.00000000~~	~~1.07572168~~	~~.0124062~~	~~33.0954~~	~~115.3073~~	~~22.3255~~	~~264.5439~~
41386.00000000	1.07565552	.0118197	33.0981	181.9337	336.8738	273.3697
~~*41390.50000000~~	~~1.07561875~~	~~.0114508~~	~~33.0993~~	~~226.5291~~	~~307.6529~~	~~201.9129~~
41393.00000000	1.07559641	.0113223	33.0986	251.9490	291.4146	282.1022
~~41400.00000000~~	~~1.07552366~~	~~.0114362~~	~~33.0994~~	~~322.7620~~	~~245.9479~~	~~293.6286~~
41407.00000000	1.07545954	.0120878	33.0987	30.9567	200.4751	311.5559
~~41421.00000000~~	~~1.07535183~~	~~.0119055~~	~~33.0998~~	~~162.0676~~	~~109.4868~~	~~261.3858~~
*41433.50000000	1.07524750	.0112280	33.0999	287.0293	28.2241	74.7455
~~41435.00000000~~	~~1.07523941~~	~~.0112860~~	~~33.0995~~	~~302.1484~~	~~18.4706~~	~~55.2133~~
41442.00000000	1.07519551	.0117855	33.0969	11.2812	332.5542	87.0364
~~41449.00000000~~	~~1.07514314~~	~~.0121613~~	~~33.0944~~	~~77.7114~~	~~287.4273~~	~~123.9285~~
*41451.50000000	1.07511181	.0121557	33.0949	100.6591	271.1636	215.9072
~~*41454.50000000~~	~~1.07506818~~	~~.0120024~~	~~33.0939~~	~~128.7574~~	~~251.6530~~	~~182.5027~~
41461.00000000	1.07500251	.0114424	33.0951	191.1261	209.3518	351.4873
*41465.50000000	1.07497197	.0111024	33.0957	236.1756	180.0727	302.0997
*41472.50000000	1.07492571	.0110832	33.0981	307.5119	134.5111	346.0104
~~*41475.50000000~~	~~1.07485368~~	~~.0116324~~	~~33.0994~~	~~16.6991~~	~~88.9413~~	~~35.3304~~
*41482.50000000	1.07482292	.0118547	33.0995	45.1677	69.4099	7.4043
*41486.50000000	1.07477178	.0119711	33.0989	82.6678	43.3651	91.7436
STANDARD SIGMAS	0.00002000	0.0000200	0.000800	0.200000	0.250000	2.000000

NOTES: All unstarred sets are Goddard's conventional Brouwer mean elements. The data given are modified single primed values (see text). The starred sets are "filtered" mean elements determined from precise orbit determination (see text). The double starred sets were received after the results, reported here, were obtained. They are given for completeness (see text). The standard sigmas are the data weights used in the ROAD orbit-harmonic determinations. The standard sigmas for the inclination data in the starred sets are 0.0004°.

Table 2

Gravitational Coefficients of 15th Order from Decaying, Resonant Orbits (Coefficients Fully Normalized: Units of 10^{-9})

		Field					
					Contains TETR Constraint		
ℓ		1	2	3	4	5	6
15	C	-18.3	-21 ±4	-12.4	-20.4 ±1.5	-19	-21
	S	-8.9	-8 ±3	-9.5	-7.9 ±0.6	-8	-8
17	C	5.3	2 ±3	6.5	2.7 ±2.4	0	4
	S	10.1	9 ±3	8.5	9.4 ±0.8	10	10
19	C	-13.5	-1 ±5	-5.3	-10.7 ±3.2	-14	-11
	S	-21.8	-17 ±4	-15.8	-13.8 ±1.0	-14	-13
21	C	26.5	-25 ±14	6.2	-1.4 ±5.3	0	2
	S	10.2	14 ±11	6.6	6.7 ±1.8	8	6
23	C		73 ±15	56.8	36.4 ±8.4	40	27
	S		-8 ±15	-3.6	2.6 ±2.8	0	3
25	C		-55 ±24		-10.3 ±9.1	-4	-6
	S		13 ±20		-1.9 ±3.0	0	-3
27	C				-14.6 ±4.1	-12	-16
	S				6.9 ±1.3	6	7
29	C				-14.3 ±9.3	-23	-20
	S				0.7 ±3.1	0	2
31	C				5.5 ±9.8	-3	12
	S				1.2 ±3.2	8	2
33	C				2.9 ±9.1	0	8
	S				0.1 ±3.0	-6	0
35	C				-1.6 ±8.2	0	-5
	S				-0.1 ±2.7	0	0
37	C				-0.6 ±7.2	0	-3
	S				0.4 ±2.4	0	1
39	C				-0.5 ±6.6	0	-2
	S				-0.2 ±2.2	0	0

COMMENTS:

1 is a 4 satellite solution with orbits of 51°, 63°, 74° and 80° inclinations.[10]

2 is a 6 satellite solution with orbits of 51°, 56°, 63°, 74°, 80° and 90° inclinations. The standard deviations include an estimate of the influence of neglected higher degree terms.

3 is a 7 satellite "least squares" solution with orbits of 51°, 56°, 63°, 74°, 74°, 80° and 90° inclinations.[1]

4 is a 7 satellite "Bayesian least squares" solution with orbits of 33°, 51°, 56°, 63°, 74°, 80° and 90° inclinations. The standard deviations do not include estimates of the truncation error.

5 is a 7 satellite "exact" solution with the same orbits as solution 4.

6 is a "minimum power" solution which exactly satisfies the (unerrored) constraints for the orbits of solutions 4 and 5 <u>and</u> minimizes the sums of the squares of the coefficients included.

Table 3

Constraints for 15th Order Resonances

					Influence Factors for $(C,S)_{\ell,15}$												
Satellite	I°	a(e.r.)	e	$10^9(C,S)_{15}$	ℓ = 15	17	19	21	23	25	27	29	31	33	35	37	39
TETR-3 (1971-83B)	33.1°	1.076	0.012	28.3 ±1.5, 7.4 ±1.5	.001	-.015	.073	-.219	.477	-.781	1.000	-.963	.622	.119	-.290	.403	-.223
Explorer 44 Rocket (1971-58B)	51.1°	1.078	0.011	-19.8 ±3.2, -25.9 ±1.3	.091	-.517	1.000	-.771	-.110	.508	.000	-.086	.000	.045	.004	-.024	-.006
Cosmos 72 (1965-53B)	56.0°	1.079	0.003	-45.7 ±1.0, -22.8 ±2.2	.196	-.853	1.000	-.050	-.562	-.009	.351	.101	-.194	-.144	.070	.132	.014
Cosmos 373 (1970-87A)	62.9°	1.080	0.007	-1.0 ±1.4, -12.5 ±1.1	.40	-1.00	.20	.54	.13	-.24	-.25	-.03	.15	.14	.02	-.08	-.05
Cosmos 387 (1970-111A)	74.0°	1.084	0.001	-26.0 ±0.9, -5.0 ±0.5	1.00	-.33	-.59	-.45	-.20	.03	.16	.19	.15	.07	.01	-.05	-.07
Ariel 3 (1967-42A)	80.2°	1.085	0.007	-19.9 ±1.2, -7.7 ±0.8	1.000	.347	.059	-.097	-.172	-.195	-.184	-.153	-.102	-.062	-.027	.000	.019
Burner Rocket (1971-54A)	90.2°	1.087	0.002	-20.5 ±2.3, -5.1 ±2.2	1.000	+.511	+.323	+.217	+.151	+.106	+.081	+.062	+.036	+.040	.000	.000	.000

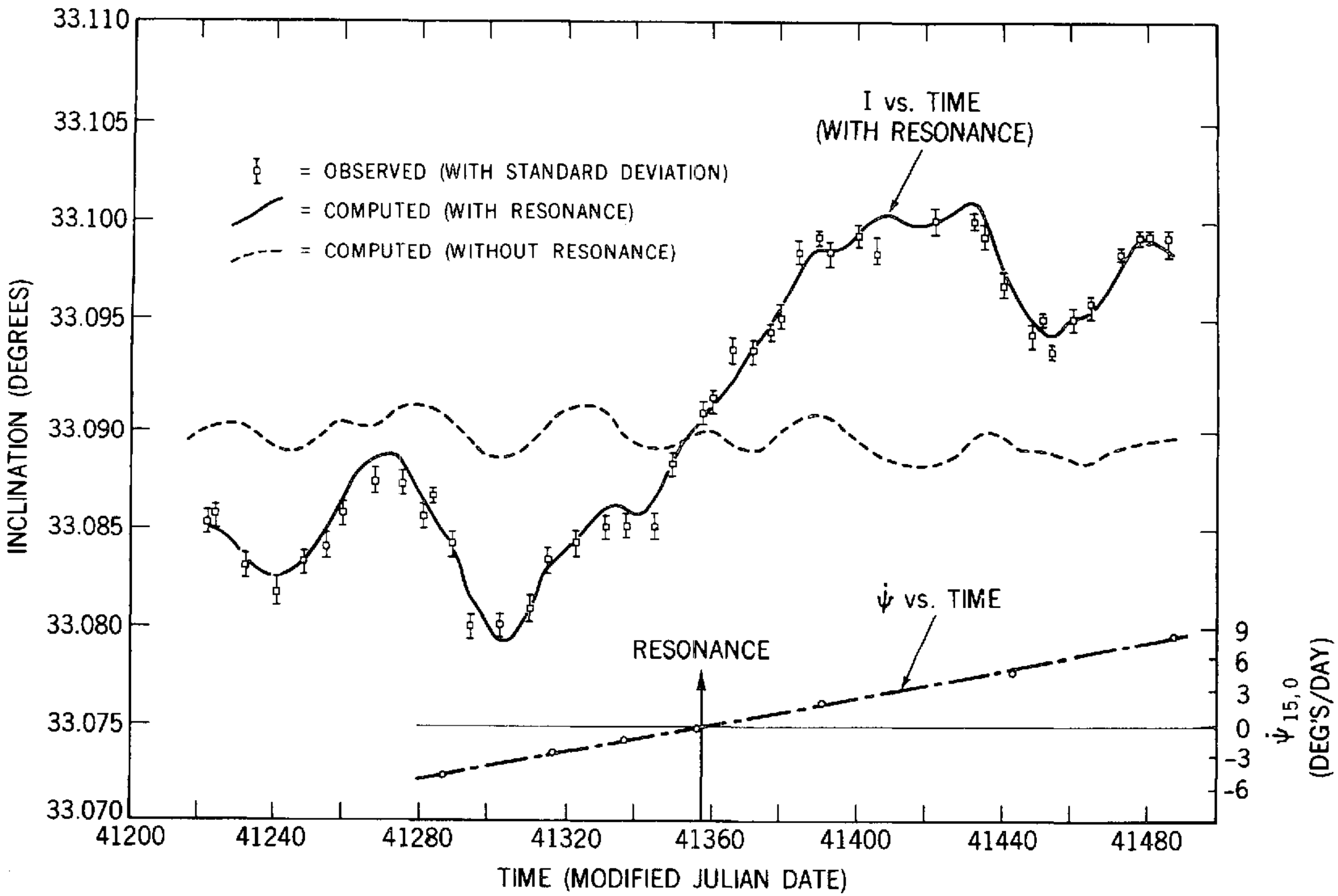

Figure 1. Variation of Inclination for TETR-3

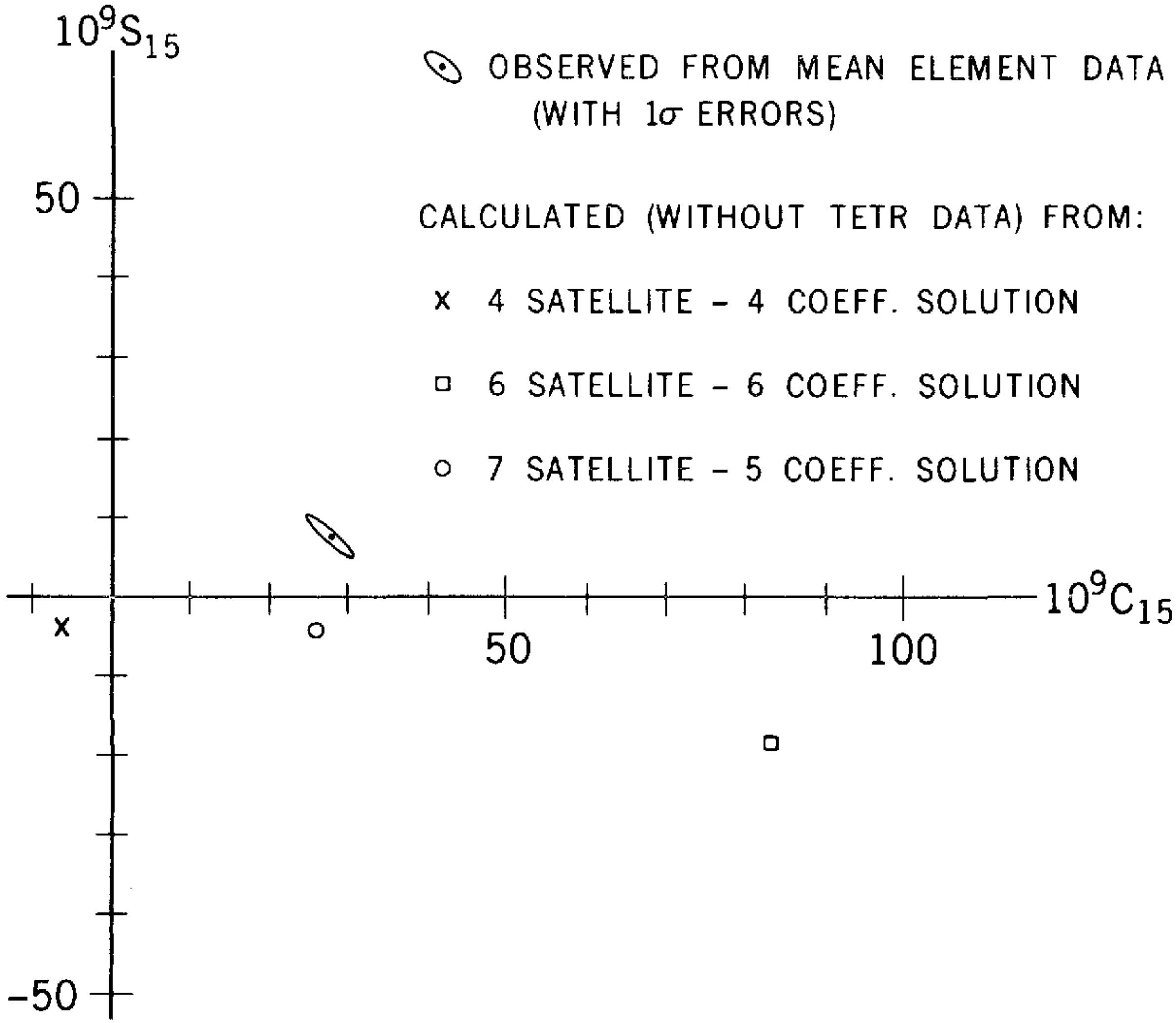

Figure 2. TETR-3 15th Order Constraint

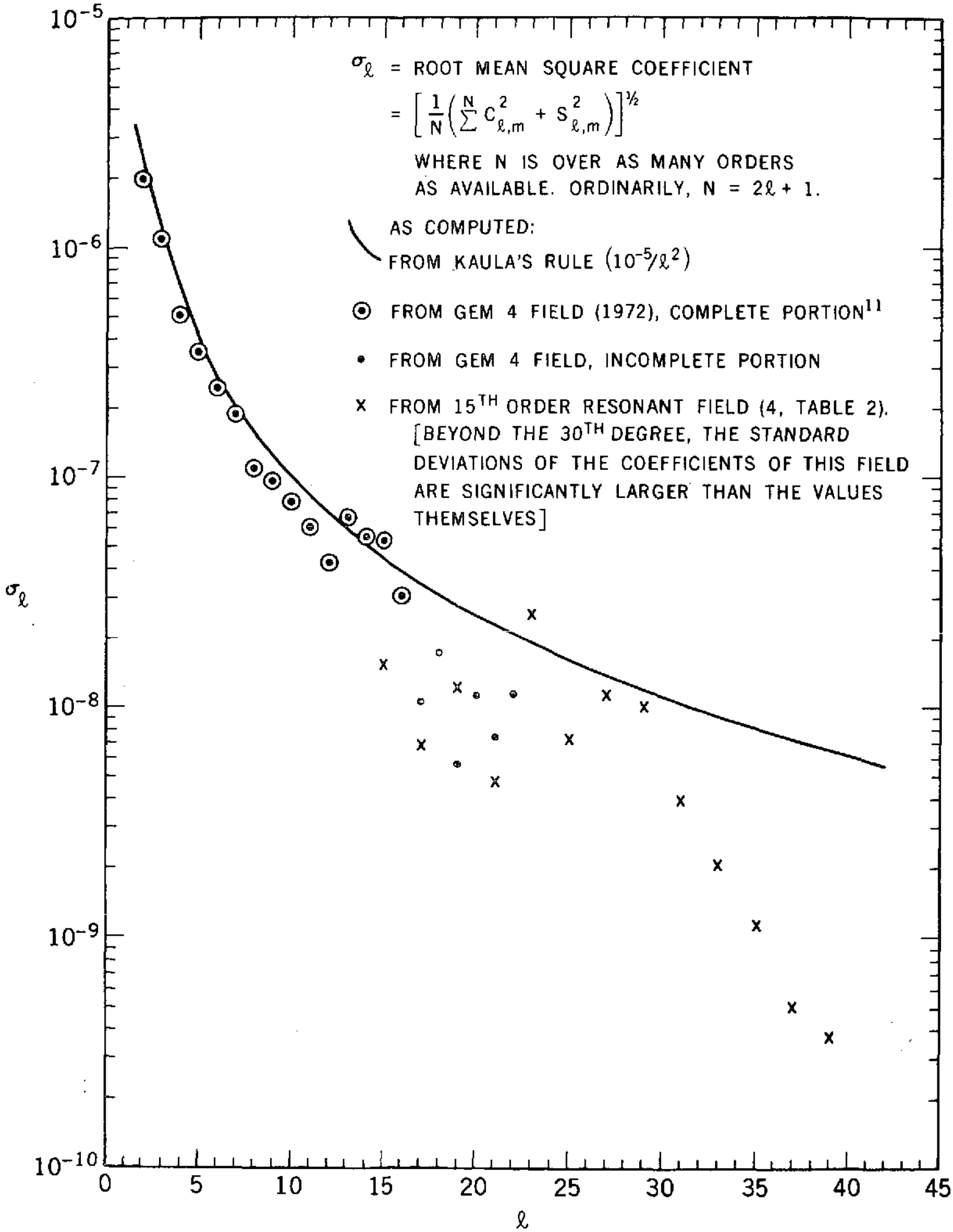

Figure 3. RMS Potential Coefficient by Degree

APPENDIX

PRECISE ORBIT DETERMINATION FOR TETR-3

INTRODUCTION

The NASA/GSFC developed GEODYN precision orbit determination program[A1] has been used in conjunction with certain ancillary routines to determine the orbit of the NASA Test and Training Satellite IV (TETR-3, 7108302) for 22 epochs. Data taken by the NASA Minitrack system was used in this recovery. The span of data reduced was from November, 1971 through June, 1972.

TETR-3 was only tracked by five of the NASA minitrack stations. Alaska and Winkfield were not used, having latitudes which did not permit visibility with this 33° inclined orbit. The data taken by Johannesburg, Tananarive, Orroral, Santiago and Quito was dense, providing excellent orbital recoverability.

The on-board S-band transponder on TETR-3 never functioned properly causing the complete cessation of all tracking, including minitrack, for this mission at the end of June, 1972. However, the orbit of TETR-3 entered deep resonance with the 15th order terms of the geopotential early in 1972 and the orbital evolution from minitrack provided excellent data for the study of these resonance terms in the geopotential.

The Keplerian osculating elements for TETR-3 at the epoch of May 17, 1972 were:

$$a = 6854.660\,\text{km}$$

$$e = 0.01238$$

$$I = 33.07920°$$

$$\text{Apogee ht.} = 565.9\,\text{km}$$

$$\text{Perigee ht.} = 396.2\,\text{km}$$

The Orbital Recovery

The minitrack data from TETR-3 was reduced in arcs of four days in length. The 1969 Standard Earth II gravity model[A2] was employed; complete to (16,16) with resonance coefficients as high as (22,14). The minitrack station coordinates were obtained from Marsh, Douglas and Klosko.[A3]

The GEODYN program employs full state-of-the-art force modeling including BIH polar motion and UT1 time corrections, full luni-solar and earth tide perturbations, and corrections for precession and nutation of the earth's polar axis. GEODYN uses a Cowell 11th order integrator. For TETR-3, a 75-second fixed integration step was employed. A Jacchia model atmosphere[A4] was used with a ballistic coefficient adjusted in each orbital arc.

The entire available set of minitrack data over the selected arcs was used. Routine Goddard orbit determination uses only a few "normal points" of smoothed data per pass. These data, about 30 points per station pass, contained timing corrections for delay times at the individual sites and used the airplane calibration corrections. Tropospheric refraction corrections were not applied to the data. Schmid has shown[A5] that the tropospheric refraction subtracts out to first order for minitrack measurements. The ionospheric refraction corrections were not applied due to the uncertainty in the available models incorporated into the GEODYN system for minitrack data. Dunn[A6] has shown that for arcs of a few days in length the ionospheric refraction effects largely cancel and therefore this is probably an insignificant error source.

The entire history of solar and magnetic flux values were modeled as daily values throughout the period of interest for this study. In this fashion, by using the full state of the art force models available and using all available minitrack data, a precise orbital computation in the given arc length of four days was achieved.

Table A1 presents a summary of these data reduction orbital solutions giving the number of passes, the number of observations, the recovered ballistic coefficient (C_D) and the rms of fit for each arc. The fits to the data are generally quite satisfactory, at the usual minitrack level of 0.3 mils accuracy. The determined orbits themselves (osculating elements) are given in Table A2.

Mean Element Determination for TETR-3

Mean elements for TETR-3 were recovered using a new technique which combines both analytic and numerical procedures.[A7] Briefly, the osculating elements in Table A1 were integrated by the GEODYN program with intermediate mean elements being produced every minute for one day. These intermediate mean elements were produced by analytically subtracting off the short period perturbations of the geopotential to degree and order (4,4). These mean elements were then numerically averaged by fitting to a precessing Kepler ellipse. This technique has been shown to result in little loss of accuracy in going from osculating to mean elements. These special mean elements of TETR-3 produced for this study are presented as the starred element sets in Table 1.

REFERENCES

A1. T. V. Martin, C. C. Goad, M. M. Chin, and N. E. Mullins, "GEODYN, Vol. 1, 2, 3 and 4," Wolf Research and Development Corporation, Riverdale, Md., 1972.

A2. E. M. Gaposchkin and K. Lambeck, "1969 Smithsonian Standard Earth 2," Smithsonian Astrophysical Observatory Special Report 315, Cambridge, Mass., 1970.

A3. J. G. Marsh, B. C. Douglas, and S. M. Klosko, "A unified set of station coordinates derived from geodetic satellite tracking data," Goddard Space Flight Center Document X-553-71-370, Greenbelt, Md., 1971.

A4. L. G. Jacchia, "Static diffusion models of the upper atmosphere with empirical temperature profiles," Smithsonian Contributions to Astrophysics, Vol. 8, 215-257, 1965.

A5. P. Schmid, "NASA minitrack interferometer refraction corrections," NASA TN D-5966, 1971.

A6. P. Dunn and J. Diamante, "Minitrack system calibration using USB data from ERTS," Wolf Research and Development Corporation, Riverdale, Md., 1973.

A7. B. C. Douglas, J. G. Marsh, and N. E. Mullins, "Mean elements of GEOS and GEOS 2," Goddard Space Flight Center Document X-553-72-85, Greenbelt, Md., 1972.

Table A1

The Orbital Solutions for TETR-3

Arc	Date Start	Date Stop	Number of Passes	Number of Observations	RMS of Fit (x .3 mils)	Recovered Drag Coefficient C_D (Area/mass = .0445 cm^2/gm)
1	71/11/27	71/11/31	28	1521	1.425	2.736
2	71/12/04	71/12/08	15	1122	0.868	2.894
3	71/12/11	71/12/15	27	1484	1.067	3.272
4	71/12/18	71/12/22	32	1658	1.214	3.290
5	71/12/25	71/12/29	30	1908	1.202	3.375
6	72/01/01	72/01/05	22	1245	1.111	3.660
7	72/01/08	72/01/12	26	1537	.994	3.556
8	72/01/15	72/01/19	26	1396	.945	2.806
9	72/01/22	72/01/26	23	1407	.816	2.860
10	72/01/29	72/02/02	18	1418	.921	3.493
11	72/02/05	72/02/09	12	1009	.832	3.066
12	72/02/12	72/02/15	15	914	1.078	2.675
13	72/02/28	72/03/04	18	1547	1.270	2.695
14	72/03/14	72/03/18	22	1430	2.134	3.206
15	72/04/26	72/04/30	25	2299	1.137	3.092
16	72/05/14	72/05/18	26	2650	1.401	3.071
17	72/05/17	72/05/20	13	1589	0.777	2.810
18	72/05/28	72/05/31	9	624	1.165	2.244
19	72/06/04	72/06/07	18	1471	1.076	2.756
20	72/06/11	72/06/14	24	2724	0.907	2.979
21	72/06/14	72/06/18	28	2196	1.527	2.180
22	72/06/18	72/06/21	22	1738	0.889	4.271

Table A2

Precise Osculating Elements for TETR-3

Arc	Date YYMMDD	hhmmss	a(km)	e	I°	ω°	Ω°	M°
1	71 11 27	00 00 00	6867.5320	0.011486801	33.091790	250.249716	291.239486	257.562037
2	71 12 04	00 00 00	6868.8252	0.011076075	33.101449	321.925797	245.937689	216.090905
3	71 12 11	00 00 00	6867.0146	0.011678642	33.091119	32.507824	200.627436	178.114120
4	71 12 18	00 00 00	6863.4262	0.012339996	33.065595	98.809203	155.273843	147.102969
5	71 12 25	00 00 00	6862.5517	0.012174602	33.066823	165.841122	109.867261	119.143247
6	72 01 01	00 00 00	6865.9139	0.011813023	33.095764	237.164494	64.484823	90.503873
7	72 01 08	00 00 00	6866.8202	0.012044779	33.100460	308.375059	19.150942	65.100429
8	72 01 15	00 00 00	6862.2126	0.012475018	33.073718	14.860169	333.776306	47.141732
9	72 01 22	00 00 00	6861.6261	0.013125726	33.074356	79.682299	288.338288	33.843197
10	72 01 29	00 00 00	6865.9910	0.013420358	33.103665	145.617101	242.954616	22.782131
11	72 02 05	00 00 00	6862.6698	0.012409840	33.085961	213.321821	197.588800	13.489006
12	72 02 12	00 00 00	6860.1674	0.011982187	33.076384	282.618934	152.113630	5.582155
13	72 02 28	00 00 00	6858.9505	0.011977390	33.077769	77.488670	48.274350	209.419407
14	72 03 14	00 00 00	6863.3635	0.012460344	33.117396	219.016478	310.914887	332.345352
15	72 04 26	00 00 00	6856.8002	0.010583139	33.092743	281.298305	31.444333	201.976449
16	72 05 14	00 00 00	6854.7168	0.012632566	33.076953	95.488455	274.431847	342.039158
17	72 05 17	00 00 00	6854.6609	0.012384475	33.079209	122.400240	254.928833	309.646498
18	72 05 28	00 00 00	6854.9117	0.011486745	33.087669	233.854968	183.296120	64.864925
19	72 06 04	00 00 00	6855.0024	0.010709449	33.090276	305.295784	137.792332	108.477867
20	72 06 11	00 00 00	6858.2457	0.010533324	33.116296	14.122878	92.184854	157.885672
21	72 06 14	00 00 00	6858.1190	0.011106346	33.117387	44.882174	72.655660	127.548320
22	72 06 18	00 00 00	6853.0662	0.011454693	33.086522	76.913336	46.600330	217.161852