Analysis of longitudinal, torsional and bending vibrations of drill string

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The paper considers longitudinal, torsional and bending vibrations of the drill string that occur during the bottom-hole deepening. The equations are presented that show the possibility of applying vibration velocity to evaluate the strength and reliability of the string pipes in case of vibrations.

Key words: drill string, oscillations, vibration velocity, tension.

Longitudinal vibrations of the drill string occur during RIH operations (SPO), works of roller-cutter bit during drilling, when pumping the drilling fluid and the rotation of the column. Torsional vibrations occur by interaction of the elements of (BHA) and the bit with the well walls and the work of roller-cutter bit at wave-type bottom [1].

Let's consider the influence of longitudinal oscillation of roller-cutter bit on the drill string during bottom deepening.

The perturbation strength in the axial direction resulted from the interaction of the bit face is transmitted to the drill string and the pipes. The axial bit displacement is accompanied by the increase of potential energy in the column pipe with the bit movement downward, the potential energy become kinetic one used for rock failure. The elastic waves associated with rolling cutters from tooth to tooth. The alternating half-waves of compression - tension cause the change of potential energy of heavy bottom, which leads to the increase of the axial load on the drill and bottom failure [2].

The efforts in cross section tubes differ from the efforts resulted from the static calculations. The elastic vibrations that occur in the drill string during the bit interaction with the bottom, both directly and indirectly, affect additionally the value of the internal force factors. For the mathematical description of the mechanical system – the drill string in the process of deepening the bottom to determine its internal security and reliability factors it is needed to record not only the equations of the model of oscillatory processes, but also to clarify the relationship of the model of wells deepening.

Having modelled the drill string in the form of a rod with distributed parameters and appropriate boundary conditions, we constructed the diagram represented in the figure. Thus, the weight drill pipes (WDP) are taken as concentrated mass, damping mass is ignored.

Given that the drill string is linear deformed system, the total weight factors can be defined in its cross-sections through the deformation under the known formulas of the elementary theory of materials strength:

$$N(x,t) = -E(x)F_k(x)\frac{\partial u(x,t)}{\partial x}; \qquad (1)$$

$$M_{кр}(x,t) = -G(x)I_p(x)\frac{\partial \varphi(x,t)}{\partial x},$$

where E – module of longitudinal elasticity, F_k – surface of cross section of column, G – rigidity modulus, I_p – polar moment of inertia of surface of cross section of column, $u(x,t)$ – longitudinal elastic displacement of cross-section, φ – angle of twist in the considered section.

Given the static modes drilling resulted in internal static load, general efforts are equal to the static and dynamic components:

$$
\begin{aligned}
N(x,t) &= N_{с}(x,t) + N_{д}(x,t);\\
M_{кр}(x,t) &= M_{кр.с}(x,t) + M_{кр.д}(x,t).
\end{aligned}
\qquad (2)
$$

Meanwhile, this dynamic components of these forces are determined through the inertial forces by means of formulas:

$$
\begin{aligned}
N_{д}(x,t) &= -\int_0^x m(x)\frac{\partial^2 u(x,t)}{\partial t^2}dx;\\
M_{кр.д}(x,t) &= -\int_0^x I_m(x)\frac{\partial^2 \varphi}{\partial t^2}dx,
\end{aligned}
\qquad (3)
$$

The equation of elastic vibrations of the drill string can be obtained from the equation (1) and by their differentiation them by x, find expressions that bind the components of beam deformation with external distributed loads, and then enter the appropriate distributed inertial forces

$$
\begin{aligned}
q(x,t) &= \frac{\partial}{\partial x}\left[E(x)F_k(x)\frac{\partial u(x,t)}{\partial x}\right];\\
q_{кр}(x,t) &= \frac{\partial}{\partial x}\left[G(x)I_p(x)\frac{\partial \varphi(x,t)}{\partial x}\right].
\end{aligned}
\qquad (4)
$$

Let’s describe the external load as:

$$
\begin{aligned}
q(x,t) &= q_a(x,t) - m(x)\frac{\partial^2 u(x,t)}{\partial t^2};\\
q_{кр}(x,t) &= q_a(x,t) - I_m(x)\frac{\partial^2 \varphi(x,t)}{\partial t^2},
\end{aligned}
\qquad (5)
$$

where $q_a(x,t)$ – external linear load (interaction of columns and walls of the well with drilling fluid).

Thus, we get the equation of longitudinal and torsional vibrations of the drill string:

$$
\begin{aligned}
\frac{\partial}{\partial x}\left[E(x)F_k(x)\frac{\partial u(x,t)}{\partial x}\right] - m(x)\frac{\partial^2 u(x,t)}{\partial t^2} &= q_x(x,t);\\
\frac{\partial}{\partial x}\left[G(x)I_p(x)\frac{\partial \varphi(x,t)}{\partial x}\right] - I_m(x)\frac{\partial^2 \varphi(x,t)}{\partial t^2} &= q_{кр}(x,t),
\end{aligned}
\qquad (6)
$$

where $q_x(x,t)$ i $q_{кр}(x,t)$ – linear function of the external load (weight, fluid friction).

Taking the first approximation that the linear functions of the external unit load approaches to zero, after transformations and provided designations

$$\chi^2 = \frac{m(x)}{E(x)F_k(x)},\ \lambda^2 = \frac{I_m(x)}{G(x)I_p(x)}$$

we have the differential equations of longitudinal and torsional free elastic vibrations of the drill string as the core system:

$$\frac{\partial^2 u(x,t)}{\partial t^2} = \chi^2 \frac{\partial^2 u(x,t)}{\partial x^2};$$
$$\frac{\partial^2 \varphi(x,t)}{\partial t^2} = \lambda^2 \frac{\partial^2 \varphi(x,t)}{\partial x^2}. \qquad (7)$$

As the drill string during drilling takes the form of a spatial spiral and the hole axis is curved, the stiffness axis of the column does not coincide with the axis of the centers of gravity of the cross sections. Bending vibrations are accompanied by torsional vibrations relatively to the stiffness axis and vice versa. The set of specified oscillations is explained by the presence of inertial oscillations among them, which are proportional to the distance between the stiffness axes and centers of gravity. The equations of bending oscillations of drill string are considered below.

The oscillations of the drill string caused by the work of the bit at the bottom, are forced [3]. The form of these oscillations varies depending on the BHA, as well as geological and technical conditions and drilling modes.

Let's analyze small longitudinal and torsional oscillations of the drill string described by the equations (7).

The first equation of the system (7) relates to longitudinal oscillations of the drill string and the limit conditions are as follows: at $X = 0$

$$\frac{\partial u(x,t)}{\partial x} = hu(x,t), \qquad (8)$$

where C_0 – stiffness of drilling line system;

$$h = \frac{C_0}{EF_k},$$

at $X = H + L$ (H – length of steel drill string (SDS), L – length of casing)

$$EF_k \frac{\partial u}{\partial x} = R_в \sin(\omega,t), \qquad (9)$$

where $R_в$ – bottom reaction.

Considering the process of bottom deepening is quasistatic one and on the basis of the above mentioned conclusions in the first approximation we take

$$u = \int_0^t \left(v_м, R_в, \frac{\partial \varphi}{\partial t}\right) dt, \qquad (10)$$

where v_M – instantaneous mechanical velocity of deepening the bottom during drilling.

Having differentiated by the time the first system equation (7) and designated ***∂u(x,t)/∂t=V*** we write it as follows

$$\frac{\partial^2 V}{\partial t^2} = \chi^2 \frac{\partial^2 V}{\partial x^2}. \qquad (11)$$

Appropriately, the expressions (8) and (9) are represented at $X = 0$

$$\frac{\partial V}{\partial x} = hV, \qquad (12)$$

at $X = H + L$

$$EF_k \frac{\partial V}{\partial t} = R_s \sin(\omega, t). \qquad (13)$$

The solution of equation (11) may be represented as the product of two functions, one of which depends on the time and the other - on the location of sections:

$$V = T(x)V(t) \qquad (14)$$

or

$$V = (A\sin\omega t + B\cos\omega t)\left(C\sin\frac{\omega x}{\chi} + D\cos\frac{\omega x}{\chi}\right). \qquad (15)$$

If we consider the longitudinal oscillations of the drill string as the thin rod at a specific moment of time $t = t_1$, the first multiplier T is constant:

$$V = T\left(C\sin\frac{\omega x}{\chi} + D\cos\frac{\omega x}{\chi}\right). \qquad (16)$$

As the number of proper oscillations of frequencies w_1, w_2, w_3 ... for such construction is unlimited, each value w has partial solution that is similar to the equation (15). In the first approximation we consider that during deepening the bottom there harmonic oscillations of drill string resulted from bit-rock interaction during roller-bit transfer along wave-type bottom:

$$V(x,t) = V(x)\sin\omega t. \qquad (17)$$

Adding this solution to the equation (11), we have

$$\frac{d^2 V}{dx^2} - \frac{\omega^2}{\chi^2} V = 0. \qquad (18)$$

The general solution of the equation (18) we write as follows:

$$V(x) = C\sin x\sqrt{\frac{\omega^2}{\chi^2}} + D\cos x\sqrt{\frac{\omega^2}{\chi^2}}. \qquad (19)$$

The resulting expression determines the shape of the elastic line of the drill string during its oscillations.

The Constants C and D are defined considering (12) and (13):

$$C=\frac{hV\chi}{\omega},\ D=\frac{hVEF_k\cos\frac{\omega}{\chi}l-R(t)}{EF_k\frac{\omega}{\chi}\sin\frac{\omega}{\chi}l}. \quad (20)$$

Having added the constants C and D to the equation (19) we have:

$$V(x)=\frac{hV\chi}{\omega}\sin\frac{\omega}{\chi}l+\frac{hVEF_k\cos\frac{\omega}{\chi}l-R(t)}{EF_k\frac{\omega}{\chi}}\operatorname{ctg}\frac{\omega}{\chi}l. \quad (21)$$

For practical application of this result it is important that it is possible to determine the stress state of the drill string in the form of the elastic line. These solutions are realized with the longitudinal oscillations under the influence of external disturbances. The frequencies of the resonant oscillations are determined by the natural oscillations that meet certain own forms.

The solutions (19) and (21) are valid for columns of considerable length with low bending rigidity, especially for the column stretched part. To calculate the oscillations of the drill string while drilling the inclined holes aimed a spatial trajectory it is necessary to use the analog as a curved rod with non-stretched axis, which is a system with variable parameters by length. The results of the use of such models give better initial parameters on calculations of reliability of the drill string.

The oscillations of the drill string and its elements can be considered random, and their parameters should be calculated using the apparatus of the theory of possible fluctuations. At this stage of research, we assume that the reaction of the drill string as a mechanical system at narrow-band random oscillation will be equivalent to the effect of harmonic oscillation, and the response system for broadband oscillation – reaction to the action of polyharmonic drilling. Thus, in the first approximation we assume that the drill string has harmonic oscillations.

The failure of drill string elements are associated with the action of many geological and technical factors, if stress or strain in its elements exceed the permissible value and the element is destroyed or has any partial deformation resulting in the rejection of element (e.g., bit) during the further work. In this case, the criterion is excessive tensions.

The majority of the fatigue damage in the drill string occurs when tension during oscillation is not very high, but the destruction is caused by a large number of cycles of stress. The criterion of failure in this case will be to reach the material endurance.

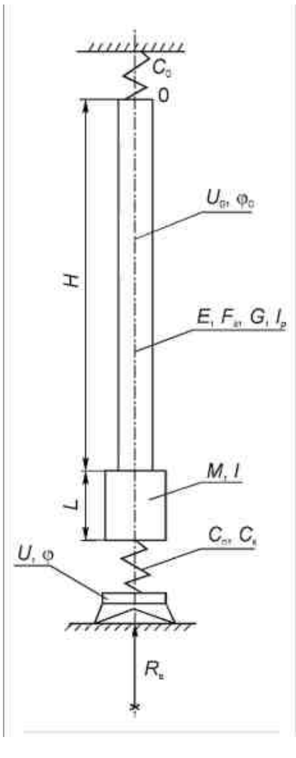

Fig. 1. Modeling scheme of drill string: C_0 – stiffness of drilling line system which are changed during the deepening the bottom; H – length of drill string; L – length of casing; E – module of elasticity; F_k – surface of cross section of pipes; G – module of displacement; I_p – polar moment of inertia of string; M – weight of casing; I – casing inertia moment; $C_п$, $C_к$ – respectively longitudinal and torsional stiffness of buffer; $U_0(x,t)$, $\varphi_0(x,t)$ – longitudinal and torsional displacement of sections of drill string; $U(t)$, $\varphi(t)$ – longitudinal and torsional displacement of bit; $R_в$ – bottom reaction

The presence of stress concentrators reduces the longevity of the drilling tool. In this regard, there are used various principles of reducing oscillation stresses. Recently, many countries use vibration velocity as indicator of reliability of structures in case of vibrations [4]. Mean square value of vibration velocity is linear related to the root-mean-square stress and does not depend on constructive features of drill string. Then, we use maximum stress for evaluation of stiffness

$$\sigma_{макс} = v_{макс} A^{*} \sqrt{E\rho}, \qquad (22)$$

where $v_{макс}$ – maximum amplitude of related vibration velocity of element, A^* - factor considering the distribution of stresses and related amplitudes of vibration speeds according to the volume of drill string.

Below, under [5], it is provided the value of factor A^* for rods:

Longitudinal oscillations of rods - 1,00;

torsional oscillations of circular rods - 0,75;

bending oscillations of freely overhanging beams - 2,00;

bending oscillations of thin wall rods fixed from both sides, under such tones: basic - 1,73, higher - 2,00.

The possibilities of the practical use of the obtained criterion are proved by its use in turbine construction and for calculation vibration stress of internal combustion engines [6].

Let's consider the simplified equations of elastic vibrations of the drill string as rod with variable cross section along the length, for which we use the equations (2) and (3). We differentiate by x the longitudinal force, the torsional moment will enable to find expressions related to deformation of system with external parameters and loads. It is necessary to repeat calculations for longitudinal and torsional oscillations. It will be resulted in differential equation of bending oscillations of drill string in surface x, $\boldsymbol{y}$:

$$\frac{\partial^2}{\partial x^2}\left[E(x)I_p(x)\frac{\partial^2 y(x,t)}{\partial x^2}\right] + m(x)\frac{\partial^2 y(x,t)}{\partial x^2} = q_a(x,t). \qquad (23)$$

The function may be represented as the sum $y(x,t)=y_1(x,t)+y_2(x,t)$, where the first element describes the free longitudinal oscillations of construction, the second element – forced oscillations. Respectively, the homogenous differential equation will be:

$$\frac{\partial^2}{\partial x^2}\left[E(x)I_p(x)\frac{\partial^2 y_1(x,t)}{\partial x^2}\right] = m(x)\frac{\partial^2 y_1(x,t)}{\partial x^2} = 0. \qquad (24)$$

Let's consider the oscillations of linear drill string in case of hinge support. If we do not take into consideration of displacement, the form of elastic line of string which we module by elastic rod is sinusoid:

$$y(x) = Y_0 \sin\frac{i\pi x}{L}, \qquad (25)$$

where Y_0 – amplitude of oscillations at any point of linear link, i – number of oscillations form, L – length of linear link of drill string.

Then, we use the works results [5, 6]. Bending moment:

$$M(x) = EI_y''(x) = -EI_p \frac{i^2\pi^2}{L^2} Y_0 \sin\frac{i\pi x}{L}. \qquad (26)$$

Maximum stress in the link of drill string:

$$\sigma_{макс}=EI_p\frac{i^2\pi^2}{WL^2}Y_0, \qquad (27)$$

where W – resistance moment.

Let’s calculate the frequency of free oscillations of this string link

$$\omega_i=\frac{i^2\pi^2}{L^2}\sqrt{\frac{EI_p}{\rho F_k}}. \qquad (28)$$

We write the expressions for the length of semi-wave

$$\left(\frac{L}{i}\right)^2=\frac{\pi^2}{\omega_i}\sqrt{\frac{EI_p}{\rho F_k}}. \qquad (29)$$

We re-write the expression (27) as follows:

$$\sigma_{макс}=v_{макс}\sqrt{3E\rho}. \qquad (30)$$

Thus, the maximum stress of bend is proportional to maximum vibration velocity of the elements of the drill string and for acceptable conditions of fixation does not depend on their constructions and dimensions.

The definition of vibration velocity by means of found equations (23) and (24) enables to define theoretically the maximum stress in string points avoiding complex experimental records.

Having introduced the notion of factors of dynamism η and form stress k, we define the safe vibration velocity for drill string generated by the bit (kinetic drilling) [5]:

$$v_{макс}=v_{дол}k\eta,$$

where

$$k=\int_0^1\bar{y}d\bar{x}/\int_0^1\bar{y}^2d\bar{x}, \quad \bar{y} \text{ i } \bar{x}$$

this is the formless form of oscillations and movable coordinate.

Due to this equation, we define the vertical vibration velocity of roller-bit resulting in any possible fatigue fracture of over-bit elements of BHA.