NONLINEAR ADAPTIVE CONTROLLER FOR OMNI-DIRECTIONAL WALKER: DYNAMIC MODEL IMPROVEMENT AND EXPERIMENT

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1. Introduction.

Walking rehabilitation has drawn a great attention in recent years. Walkers are commonly used to assist patient to improve stability and muscle strength. So far, walking rehabilitation has only allowed a few basic motions with the help of crutches, canes, and parallel bars. However, walking is a complex combination of motions, which includes not only forward and backward motions, but also right and left motions, oblique motions, and rotations. In this paper, an omni-directional walker (ODW) [1] is shown which can support people with walking impairments for both walking rehabilitation and walking support. This walker allows omni-directional movement. In walking rehabilitation, the user follows the movement of the ODW to perform walking training. In walking support, the ODW supports the user to reach the user’s intended goal. The training programs are stored in the walker so that rehabilitation can be carried out without the presence of physical therapists. Its effectiveness in walking rehabilitation was verified by clinical tests [2].

The accuracy of the path tracking of this walker needed to be improved to accurately follow the exercise programs prescribed by physical therapists. The COG shifts and load changes caused by user...
changes caused by the users during walking training are the main reason leading to the path tracking error [3]. In previous studies, a COG dynamic model was derived for the ODW that considered the COG shift and load change [4]. We supposed that the COG and the center of the ODW were at the same position. An adaptive controller was developed to ensure the COG of the ODW follows the reference path. However, the COG of the ODW is not the same with the center where the user is during walking training. Therefore, the control objective is better to the center than the COG in order that the position of the user follows the reference path. In the present paper, a new center-dynamic model of the ODW that describes the relationship between the input of the ODW system and the center position of the ODW is presented. A nonlinear adaptive control method is proposed to control the ODW following the reference path based on this center-dynamic model. A linear path, as one of the basic walking rehabilitation paths, is followed using the improved method in an experiment. The experiment results demonstrate the feasibility and effectiveness of the proposed method.

The paper is organized as follows: Section 2 introduces the structure of ODW and its dynamic analysis; Section 3 presents an adaptive control for the ODW; Section 4 shows experiment results. Conclusions are provided in Section 5.

2. Structure and Kinetics of the ODW. The structure of the ODW is shown in Figure 1. The most important feature of the walker is the omni-directional wheel. The arrangement of four omni-wheels at the bottom of the walker body enables the walker to move in any direction while maintaining its orientation. Two telescopic poles are designed to support both the upper part of the walker and the load from the user. The walker height is adjustable from 900 to 1200 mm to accommodate the different heights of the users. It also has four force sensors embedded in the armrest. The extra loads caused by the user can be measured by the force sensors.

![Omni-directional walker](image)

**Figure 1.** Omni-directional walker

The coordinate settings and structural model are shown in Figure 2. The parameters and coordinate system are as follows:

\[
\begin{align*}
\sum (x, y, O) & : \text{Absolute coordinate system.} \\
\sum (x', y', O) & : \text{Translation coordinate system of the ODW.} \\
v & : \text{Speed of the omni-directional walker.} \\
\alpha & : \text{Angle between the } x'\text{-axis and the direction of } v. \\
v_i & : \text{Speed of an omni-directional wheels } (i = 1, 2, 3, 4). \\
f_i & : \text{Force on omni-wheel } i. \\
L & : \text{Distance from the center of the ODW to each omni-wheel.} \\
l_i & : \text{Distance from the COG to the middle of omni-wheel } i.
\end{align*}
\]
\( \theta_i \): Angle between the \( x' \)-axis and the position of omni-wheel \( i \) as measured from the center.

\( \phi_i \): Angle between the \( x' \)-axis and the position of the first omni-directional wheel as measured from the COG.

\( r_0 \): Distance between the center of the ODW and the COG.

\( \beta \): Angle between the \( x' \)-axis and \( r_0 \).

Using the coordinate system shown in Figure 2, kinematic and kinetic analysis can be carried out considering COG shift and load change for the nonlinear system. Based on the kinematics on the center and kinematics on the COG, the relationship between the COG and the center of this walker is

\[
\dot{X}_G = K' \cdot \dot{X}
\]

where

\[
\dot{X}_G = \begin{bmatrix} \dot{x}_G \\ \dot{y}_G \\ \dot{\phi} \end{bmatrix}, \quad \dot{X} = \begin{bmatrix} \dot{x}_C \\ \dot{y}_C \\ \dot{\theta} \end{bmatrix}, \quad K' = (K'_G \cdot K_G)^{-1} \cdot K'_G \cdot K_C,
\]

\[
(K'_G \cdot K_G)^{-1} \cdot K'_G \cdot K_C = \begin{bmatrix}
1 & 0 & \frac{1}{2} \sin \theta \cdot (\lambda_1 - \lambda_3) + \cos \theta \cdot (\lambda_2 - \lambda_4) \\
0 & 1 & \frac{1}{2} \sin \theta \cdot (\lambda_2 - \lambda_4) - \cos \theta \cdot (\lambda_1 - \lambda_3) \\
0 & 0 & 1
\end{bmatrix}
\]

The dynamic equation of the ODW is:

\[
M_0 \ddot{X}_G = K'_G F
\]

where \( K'_G \) is the transpose of matrix \( K_G \). \( M_0 \), \( F \) and \( \ddot{X}_G \) are defined as:

\[
K_G = \begin{bmatrix}
-\sin \theta_1 & \cos \theta_1 & \lambda_1 \\
\sin \theta_2 & -\cos \theta_2 & -\lambda_2 \\
\sin \theta_3 & -\cos \theta_3 & -\lambda_3 \\
-\sin \theta_4 & \cos \theta_4 & \lambda_4
\end{bmatrix}, \quad M_0 = \begin{bmatrix}
M + m & 0 & 0 \\
0 & M + m & 0 \\
0 & 0 & I + mr_0^2
\end{bmatrix},
\]

\[
F = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}, \quad \ddot{X}_G = \begin{bmatrix} \ddot{x}_G \\ \ddot{y}_G \\ \ddot{\phi} \end{bmatrix}^T
\]

where \( M \) is the mass of the omni-directional walker; \( m \) is the equivalent mass that the user imposes on the omni-directional walker, which varies according to the user’s weight and walking disability; and \( I \) is the inertia of mass of the walker. \( mr_0^2 \) is the inertia of mass cause by \( m \). In (2), \( f_1, f_2, f_3, \) and \( f_4 \) are the inputs of the system; \( \ddot{x}_C, \ddot{y}_C \) and \( \ddot{\theta} \) are the output of the system. COG shift influences both the distance \( r_0 \) between the center \( C \) and the COG \( G \) and the angle \( \beta \) between \( x' \)-axis and \( r_0 \). Load change refers to the value of \( m \). So \( m \) and \( r_0 \) are variable parameters.
Substituting using (1), we can write the dynamic equations in matrix form:

\[ M_0 \cdot K' \cdot \ddot{X} + M_0 \cdot \dot{K}' \cdot \dot{X} = K_G^T \cdot F \]  \hspace{1cm} (3)

3. Adaptive Control. In this section, in order to deal with the COG shift and the load changes caused by the users, we develop a adaptive control method for the motion control of the omni-directional walker inspired by Slotine and Li [5].

Considering the nonlinear system (3), the controller (4) and adaptive law (5) are chosen as follows:

\[ F = K_G(K_G^T K_G)^{-1} \left( \hat{M}_0 K' (\ddot{X}_d + \lambda \dot{e}) + \hat{M}_0 K' \dot{X} + \hat{A} \dot{S} + KS \right) \]  \hspace{1cm} (4)

\[ \dot{\lambda} = \Gamma^{-1} HS \]  \hspace{1cm} (5)

where

\[ X = [x_C, y_C, \theta]^T, \quad X_d = [x_{Cd}, y_{Cd}, \theta_d]^T, \quad A = M_0 - M_0 K' \]

\[ S = \dot{e} + \lambda e, \quad e = X_d - X, \quad \dot{e} = \dot{X}_d - \dot{X}, \quad e = [e_x, e_y, e_\theta]^T, \]

\[ \hat{M}_0 = \begin{bmatrix} \hat{M} + \hat{m} & 0 & 0 \\ 0 & \hat{M} + \hat{m} & 0 \\ 0 & 0 & \hat{I} + \hat{m} \hat{r}_0^2 \end{bmatrix}, \]

\[ H = \begin{bmatrix} \ddot{\theta} + \dddot{X}_d + \lambda_1 \dot{e}_x + e(\ddot{\theta}_d + \lambda_3 \dot{e}_\theta) & 0 & 0 \\ 0 & \dddot{\theta}_d + \lambda_3 \dot{e}_\theta & 0 \\ 0 & 0 & \dddot{\theta}_d + \lambda_3 \dot{e}_\theta \end{bmatrix} \]

\[ \lambda = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}, \quad K = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} \Gamma_1 & 0 & 0 \\ 0 & \Gamma_2 & 0 \\ 0 & 0 & \Gamma_3 \end{bmatrix} \]

The path tracking accuracy can be improved by manually adjusting the design parameters of adaptive control (\( \lambda, K, \Gamma \)). The proof of the convergence is abbreviated. All the signals in the closed-loop system are bounded and \( e \) converges to zero as \( t \to \infty \), the designed system is stable.

4. Experiment Results. In this section, the proposed adaptive control algorithm is verified in a path tracking experiment. In the experiment, one camera hung up records the real-time data of the walker including \( x \) distance, \( y \) distance and orientation angle \( \theta \) to feed them back to the ODW. Reference path was stored in the ODW. In order to elucidate the performance of the adaptive controller, one bag was hanged on one side of the ODW to represent the load change and COG shifted of the ODW caused by trainer in this experiment. The use of a load rather than a subject is because that a subject imposes pressure on the armrest, which is difficult to measure accurately. The target path of experiment is an oblique line.

Experiment parameters of the adaptive controller are adjusted by trial and error under the conditions that \( m = 0 \) kg. We carried out two tests. One was without any load. The other was with an 8.8 kg load. The experiment results are shown in Figure 3 and Figure 4. In Figure 3, the maximum distance between reference position and response position is 11 cm. In Figure 4, the maximum distance is 9 cm. Position error with load is smaller than that with load. The two tests are carried out in the same experimental environment. The main reason is that even when there is no load, the COG is not the same with the center due to the asymmetric structure of ODW. Figure 4 demonstrated almost the same
path tracking result with Figure 3. These experiment results showed that the adaptive response could track the reference target rapidly no matter with or without load.

It had been proved that PI control could not adapt to the system whose parameters \((m, r_0)\) changed in the previous experiment [3]. Therefore, this control strategy can successfully deal with the nonlinearity of the ODW in real control system.

![Figure 3](image3.png)

**Figure 3.** Experiment results with \(m = 0\) kg, \(r_0 = 16\) cm

![Figure 4](image4.png)

**Figure 4.** Experiment results with \(m = 8.8\) kg; \(r_0 = 15\) cm

5. **Conclusions.** In this paper, to reduce the error in path tracking of the ODW due to the COG shift and load change, a center model and an adaptive control algorithm were derived. The experiment results have showed the effectiveness of the proposed algorithm in that the tracking responses are similar even when the COG position and load are changed. Future work will focus on improving the path tracking accuracy by searching the optimal parameters.

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