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Stationary configurations of the Standard Model Higgs potential: Electroweak stability and rising inflection point

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We study the gauge-independent observables associated with two interesting stationary configurations of the Standard Model Higgs potential (extrapolated to high energy according to the present state of the art, namely the next-to-next-to-leading order): i) the value of the top mass ensuring the stability of the SM electroweak minimum and ii) the value of the Higgs potential at a rising inflection point. We examine in detail and reappraise the experimental and theoretical uncertainties which plague their determination, finding that i) the stability of the SM is compatible with the present data at the 1.5σ level and ii) despite the large theoretical error plaguing the value of the Higgs potential at a rising inflection point, the application of such a configuration to models of primordial inflation displays a 3σ tension with the recent bounds on the tensor-to-scalar ratio of cosmological perturbations.

I. INTRODUCTION

The extrapolation of the Standard Model (SM) Higgs potential at high energy via the renormalization group equation (RGE) is an interesting task [1,2]. On one hand, the SM can be considered valid up to some energy scale only if the electroweak minimum is stable, or at least metastable. On the other hand, the shape of the Higgs potential at high energy could have some impact on the dynamics of the early Universe. Two stationary configurations of the SM Higgs potential turn out to be particularly relevant: i) the case of two degenerate vacua [3], that is the condition for electroweak vacuum stability, and ii) the case of a rising inflection point at high energy [4], close to the Planck scale.

Our goal is to perform a detailed and updated study of the gauge-independent observables associated with such stationary configurations, building on the recent progress made in both the theoretical and the experimental sides. We extrapolate the SM Higgs potential up to high energy according to the present state of the art, namely the next-to-next-to-leading order (NNLO), and study in particular i) the value of the top mass ensuring stability of the SM electroweak minimum and ii) the value of the Higgs potential at the rising inflection point. We examine in detail and reappraise in a critical way the experimental and theoretical uncertainties which plague their determination.

The inputs necessary to carry out the extrapolation are the low energy values of the three gauge couplings, the top quark and the Higgs masses. Since the discovery of the Higgs boson [5,6], a lot of work has been done in the direction of refining the calculation of the RGE-improved Higgs effective potential at high energy. As for the matching with the experimental inputs at low energy, progress was made in Refs. [7–10]. A better understanding of how to extract gauge-invariant observables [11,12] from the (gauge-dependent) effective potential is worth mentioning. Particularly relevant for the sake of the present analysis are the insights about the effective potential expansion in the case that the Higgs quartic coupling is small [13,14], this progress justifies the method used in previous analysis [7–9] to carry out the calculation of the effective potential at NNLO.

On the experimental side, the situation has also changed a bit in the last years. It is worth mentioning the better knowledge of the Higgs mass, now measured with significant precision by ATLAS and CMS [15]: $m_H = 125.09 \pm 0.21(\text{stat}) \pm 0.11(\text{syst})$ GeV. On the other hand, it was recognized that the error in the determination of the strong coupling constant has been previously underestimated by a factor of 2, so that the present world average is $\alpha_s = 0.1181 \pm 0.0013$ [16]. Another open issue much debated in the literature is the determination of the top pole mass from the Monte Carlo (MC) mass [17–19]; the combined measurements of the Tevatron and LHC give, for the latter, $m_t^{\text{MC}} = 173.34 \pm 0.76$ GeV [20].

Taking into account all these developments, we update the analysis of Refs. [7–10,21–23], reappraising in a critical way the experimental and theoretical uncertainties which plague the determination of the top mass value ensuring stability of the SM electroweak minimum. We find that stability of the SM is consistent with the present data at the 1.5σ level. Previous claims of a stronger tension at the 2σ level [7–9] are, in our opinion, due to the previous underestimation of the experimental error on α_s.

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We also examine in a systematic way the theoretical and experimental uncertainties plaguing the calculation of the value of the Higgs potential at a rising inflection point. It turns out that, despite the largeness of these uncertainties, applications of such configuration to models of primordial inflation are disfavored at 3σ by the recent bounds on the tensor-to-scalar ratio of cosmological perturbations. Claims of an even stronger tension [24,25] are, in our opinion, due to an underestimation of the theoretical errors involved in the calculation.

The paper is organized as follows. In Sec. II, we present the details of the NNLO calculation, as the matching, running and effective potential expansion. We introduce the two stationary configurations in Sec. III, discussing the gauge invariance of the observables related to them. Section IV is devoted to the study of two degenerate vacua, while Sec. V is devoted to the inflection point configuration. Comments about the effect of higher-dimensional operators from an unknown gravitational sector are made in Sec. VI. The conclusions are drawn in Sec. VII.

II. CALCULATION'S STATE OF THE ART

The normalization of the Higgs quartic coupling λ is chosen in this paper so that the potential for the physical Higgs ϕ contained in the Higgs doublet $\mathcal{H} = (0 \quad (\phi + v)/\sqrt{2})^T$ is given, at tree level, by

$$V(\phi) = \frac{\lambda}{6}\left(|\mathcal{H}|^2 - \frac{v^2}{2}\right)^2 \approx \frac{\lambda}{24}\phi^4, \qquad (1)$$

where $v = 1/(\sqrt{2}G_\mu)^{1/2} = 246.221$ GeV and G_μ is the Fermi constant from muon decay [26]. The approximation in Eq. (1) holds when considering large field values. According to our normalization, the physical Higgs mass satisfies the tree-level relation $m_H^2 = \lambda v^2/3$. In addition, the mass of the fermion f reads, at tree level, $m_f = h_f v/\sqrt{2}$, where h_f denotes the associated Yukawa coupling.

In order to extrapolate the behavior of the Higgs potential at very high energies, we adopt the $\overline{\text{MS}}$ scheme and consider the matching and RGE evolution for the relevant couplings which, in addition to the Higgs quartic coupling λ, are the gauge couplings g, g', g_3; the top Yukawa coupling, h_t; and the anomalous dimension of the Higgs field, γ. We then compute the RGE-improved Higgs effective potential.

As anticipated, we perform this calculation according to the present state of the art, namely the NNLO. Before discussing in detail the procedure associated to matching, running, and effective potential expansion, we review the basic ideas of the RGE to introduce our notation.

In applications where the effective potential $V_{\text{eff}}(\phi)$ at large ϕ is needed, as is the case for our analysis, potentially large logarithms appear, of the type $\log(\phi/\mu)$ where μ is the renormalization scale, which may spoil the applicability of

perturbation theory. The standard way to treat such logarithms is by means of the RGE. The fact that, for fixed values of the bare parameters, the effective potential must be independent of the renormalization scale μ means that [27]

$$\left(\mu\frac{\partial}{\partial\mu} + \beta_i\frac{\partial}{\partial\lambda_i} - \gamma\frac{\partial}{\partial\phi}\right)V_{\text{eff}} = 0, \qquad (2)$$

where

$$\beta_i = \mu\frac{d\lambda_i}{d\mu}, \qquad \gamma = -\frac{\mu}{\phi}\frac{d\phi}{d\mu} \qquad (3)$$

are the β-functions corresponding to each of the SM couplings λ_i and the anomalous dimension of the background field, respectively.

The formal solution of the RGE is

$$V_{\text{eff}}(\mu, \lambda_i, \phi) = V_{\text{eff}}(\mu(t), \lambda_i(t), \phi(t)), \qquad (4)$$

where

$$\mu(t) = e^t\mu, \qquad \phi(t) = e^{\Gamma(t)}\phi,$$
$$\Gamma(t) = -\int_0^t \gamma(\lambda(t'))dt' \qquad (5)$$

and $\lambda_i(t)$ are the SM running couplings, determined by the equation

$$\frac{d\lambda_i(t)}{dt} = \beta_i(\lambda_i(t)) \qquad (6)$$

and subject to the boundary conditions $\lambda_i(0) = \lambda_i$. The usefulness of the RGE is that t can be chosen in such a way that the convergence of perturbation theory is improved, which is the case for instance when $\phi(t)/\mu(t) = \mathcal{O}(1)$.

Since in our calculation the boundary conditions are given at the top quark mass, m_t, we will take $\mu = m_t$ in Eq. (5) from now on.

A. Matching

In order to derive the values of the relevant parameters $(g, g', g_3, h_t, \lambda)$ at the top pole mass, m_t, we exploit the results of the most recent analysis about the matching procedure, performed by Bednyakov et al. [10]. This paper uses as input parameters at m_Z those from the 2014 release of the Particle Data Group (PDG) [26]. Such parameters are evolved in the context of the SM as an effective theory with five flavors and then matched to the six-flavors theory at m_t. The procedure is carried out by including corrections up to $\mathcal{O}(\alpha_{\text{em}}^2)$, $\mathcal{O}(\alpha_{\text{em}}\alpha_s)$, and $\mathcal{O}(\alpha_s^4)$, where α_{em} and α_s are the fine structure constant and the strong gauge coupling, respectively. The theoretical uncertainties in the results due to unknown higher-order corrections are estimated

TABLE I. Coefficients for Eq. (13) in the Landau gauge.

p	t	W	Z	ϕ	χ
N_p	-12	6	3	1	3
C_p	$3/2$	$5/6$	$5/6$	$3/2$	$3/2$
κ_p	$h^2/2$	$g^2/4$	$(g^2 + g'^2)/4$	3λ	λ

considering both scale variations and truncation errors. The matching is thus carried out at the NNLO (actually even slightly beyond for the strong gauge coupling contribution).

Although in our analysis we use the complete results of Ref. [10] [see in particular their Eq. (6) and Table I], we provide here some simplified expressions which capture the dominant dependences and sources of uncertainty. It is well known that, for the sake of the present calculation, the most significant uncertainties are those associated with the determination of $g_3(m_t)$, $h_t(m_t)$, $\lambda(m_t)$, while uncertainties in the matching of $g(m_t)$, $g'(m_t)$ are negligible. Let us consider the former three couplings in turn, following closely the procedure of Bednyakov *et al.* [10] but updating it when necessary by means of the latest (September 2015) release of the PDG [16]:

(i) The uncertainty in the value of $g_3(m_t)$ is essentially dominated by the experimental error on the value of $\alpha_s^{(5)}$, the strong coupling constant at m_Z in the SM with five flavors,

$$g_3(m_t) \simeq 1.1636 + 5.8 \times 10^{-3} \frac{\alpha_s^{(5)} - \alpha_s^{(5,\mathrm{exp})}}{\Delta\alpha_s^{(5,\mathrm{exp})}}, \quad (7)$$

where the present (PDG revised version of September 2015) [16] world average experimental value, $\alpha_s^{(5,\mathrm{exp})} = 0.1181$, and its associated 1σ error, $\Delta\alpha_s^{(5,\mathrm{exp})} = 0.0013$, have been used as reference values. Notice that Ref. [10] used instead as a reference value $\alpha_s^{(5,\mathrm{exp})} = 0.1185$, with 1σ error given by $\Delta\alpha_s^{(5,\mathrm{exp})} = 0.0006$ [26]; previous analyses like e.g. Refs. [7–9] used $\alpha_s^{(5,\mathrm{exp})} = 0.1184$, with 1σ error given by $\Delta\alpha_s^{(5,\mathrm{exp})} = 0.0007$ [28]. The experimental error is approximately doubled at present because of a previous underestimation of the uncertainty in the lattice results.

(ii) The uncertainty on $\lambda(m_t)$ is dominated by the experimental uncertainty on the Higgs mass m_H and by the theoretical uncertainty associated to the matching procedure (scale variation and truncation, here added in quadrature)

$$\lambda(m_t) \simeq 0.7554 + 2.9 \times 10^{-3} \frac{m_H - m_H^{\mathrm{exp}}}{\Delta m_H^{\mathrm{exp}}}$$
$$\pm 4.8 \times 10^{-3}, \quad (8)$$

where we used as reference values the most recent ATLAS and CMS combination,[1] $m_H^{\mathrm{exp}} = 125.09$ GeV, with 1σ error given by $\Delta m_H^{\mathrm{exp}} = 0.24$ GeV [15]. Notice that in Eq. (8) the theoretical error is pretty large, being equivalent to a 1.6σ variation in m_H. In the previous literature, there is some difference about the size of the theoretical error: for instance, the theoretical error of the recent analysis by Bednyakov *et al.* [10] is about the half of the one quoted in the well-known analysis by Degrassi *et al.* [7], due to the inclusion [10] of all corrections up to $\mathcal{O}(\alpha_{\mathrm{em}}^2)$, $\mathcal{O}(\alpha_{\mathrm{em}}\alpha_s)$, and $\mathcal{O}(\alpha_s^4)$. On the other hand, Buttazzo *et al.* [9] quote an error which is five times smaller than the one of Ref. [7]; according to our previous analysis [22], the upper error is as small as the one of Ref. [9], but the lower error is indeed consistent with Ref. [7]. Clearly, it would be worth it to assess and further refine the present error in the matching of λ.

(iii) The uncertainty on $h_t(m_t)$ is mainly affected by the experimental error on the top pole mass m_t itself, but also the theoretical uncertainty associated to the matching procedure (scale variation and truncation, here added in quadrature) is sizable,

$$h_t(m_t) \simeq 0.9359 + 4.4 \times 10^{-3} \frac{m_t - m_t^{\mathrm{exp}}}{\Delta m_t^{\mathrm{exp}}}$$
$$\pm 1.4 \times 10^{-3}, \quad (9)$$

where we used as reference values those of the first joint Tevatron and LHC analysis, $m_t^{\mathrm{exp}} = 173.34$ GeV and $\Delta m_t^{\mathrm{exp}} = 0.76$ GeV [20]. The theoretical uncertainty in Eq. (9) is consistent with the one of Refs. [7,9].[2] Since the top mass is extracted by fitting MC computed distributions to experimental data, what is really measured is a MC parameter, m_t^{MC}.

According to some authors [29], although it is common to identify the top quark pole mass m_t with m_t^{MC}, the uncertainty in the translation from the MC mass definition to a theoretically well-defined short distance mass definition at a low scale should currently be estimated to be of the order of 1 GeV [30]; if this were the case [10,21,22], the customary

[1]Again, we update the result of Ref. [10] by using the most recent LHC data, instead of $m_H^{\mathrm{exp}} = 125.7$ GeV and $\Delta m_H^{\mathrm{exp}} = 0.4$ GeV, quoted in the 2014 version of the PDG [26]. Notice also that within our convention the value of λ is six times the one of Ref. [10].

[2]The latter quotes an error on the matching of h_t which is three times smaller than the one in Eq. (9) but includes an error of $\mathcal{O}(\Lambda_{\mathrm{QCD}})$, i.e. about 0.3 GeV, in the definition of m_t, to account for nonperturbative uncertainties associated with the relation between the measured value of the top mass and the actual definition of the top pole mass.

confidence ellipses with respect to m_t^{MC} and m_H (more on this in Sec. IV) should be taken with a grain of salt.

Other authors [18,19] argued that measurements relying on the reconstruction of top-decay products yield results which are actually close to the top quark pole mass, although there are theoretical uncertainties due to the approximations contained in the MC shower models, namely missing loop and width corrections and color-reconnection effects. The discrepancy between MC and pole masses was estimated in Ref. [31], by identifying the MC mass as a Soft-Collinear Effective Theory (SCET) jet mass, evaluated at a scale given by the shower infrared cutoff, i.e. $\mathcal{O}(1 \text{ GeV})$, in $e^+ e^- \rightarrow t\bar{t}$ collisions. As discussed in Ref. [19], such a discrepancy amounts to about 200 MeV. In addition, the renormalon ambiguity affecting the pole mass was recently estimated [18] as the size of the last converging term in the $\overline{\text{MS}}$/pole relation, obtained extrapolating to higher orders the four-loop computation in Ref. [32] and amounting to less than 100 MeV. The four-loop correction was also obtained in semianalytical form in Ref. [33], finding agreement with Ref. [32].

B. Running

The β-functions can be organized as a sum of contributions with increasing number of loops,

$$\frac{d}{dt}\lambda_i(t) = \kappa \beta_{\lambda_i}^{(1)} + \kappa^2 \beta_{\lambda_i}^{(2)} + \kappa^3 \beta_{\lambda_i}^{(3)} + \cdots, \quad (10)$$

where $\kappa = 1/(16\pi^2)$ and the apex on the β-functions represents the loop order.

Here, we are interested in the RGE dependence of the couplings $(g, g', g_3, h_t, \lambda, \gamma)$. The one-loop and two-loop expressions for the β-functions in the SM are well known and can be found e.g. in Ford *et al.* [34]. The complete three-loop β-functions for the SM have been computed more recently: as for the SM gauge couplings, they have been presented by Mihaila *et al.* in Refs. [35,36]; as for λ, h_t, and the Higgs anomalous dimension, they have been presented by Chetyrkin and Zoller in Refs. [37,38] and by Bednyakov *et al.* in Refs. [39–42]. The dominant four-loop contribution to the running of the strong gauge coupling has been also computed recently; see Refs. [43,44]. In our NNLO analysis, we include all these contributions.[3]

C. RGE-improved effective potential

As already stressed, t can be chosen in such a way that the convergence of perturbation theory is improved.

Without sticking, for the time being, to any specific choice of scale, the RGE-improved effective potential at high field values can be rewritten as

$$V_{\text{eff}}(\phi, t) \approx \frac{\lambda_{\text{eff}}(\phi, t)}{24}\phi^4, \quad (11)$$

where $\lambda_{\text{eff}}(\phi, t)$ takes into account wave function normalization and can be expanded as sum of tree level plus increasing loop contributions:

$$\lambda_{\text{eff}}(\phi, t) = e^{4\Gamma(t)}[\lambda(t) + \lambda^{(1)}(\phi, t) + \lambda^{(2)}(\phi, t) + \cdots]. \quad (12)$$

In particular, the one-loop Coleman-Weinberg contribution [46] is

$$\lambda^{(1)}(\phi, t) = 6\frac{1}{(4\pi)^2}\sum_p N_p \kappa_p^2(t)\left(\log\frac{\kappa_p(t)e^{2\Gamma(t)}\phi^2}{\mu(t)^2} - C_p\right), \quad (13)$$

where, generically, p runs over the top quark, W, Z, Higgs, and Goldstone bosons contributions. The coefficients N_p, C_p, κ_p are listed in the table below for the Landau gauge (see e.g. Table 2 of Ref. [11] for a general R_ξ gauge).

The two-loop contribution $\lambda^{(2)}(\phi, t)$ was derived by Ford *et al.* in Ref. [34] and, in the limit $\lambda \rightarrow 0$, was cast in a more compact form in Refs. [7,9]. We verified, consistently with these works, that the error committed in this approximation is less than 10% and can thus be neglected.

It is clear that when $\lambda(t)$ becomes negative, the Higgs and Goldstone contributions in Eq. (13) are small but complex, and this represents a problem in the numerical analysis of the stability of the electroweak vacuum. Indeed, in Refs. [7,9], the potential was calculated at the two-loop level, but by setting to zero the Higgs and Goldstone contributions in Eq. (13). In Ref. [22], we rather decided to calculate the potential only at the tree level because, for the sake of the analysis of the electroweak vacuum stability, the numerical difference with respect to the previous method is negligible.

Some authors [14,47] recently showed that the procedure of Refs. [7,9] is actually theoretically justified when λ is small (say $\lambda \sim \hbar$); in this case, the sum over p does not have to include the Higgs and Goldstone's contributions, which rather have to be accounted for in the two-loop effective potential, which practically coincides with the expression derived in Refs. [7,9].

For the stationary configurations we are interested in—two degenerate minima and a rising point of inflection—it happens that λ is small (and could be negative); we thus adopt the procedure outlined in Ref. [14].

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[3]We do not include the four-loop contribution to the Higgs self-coupling, anomalous dimension of the Higgs field and top Yukawa β-functions calculated very recently by Chetyrkin and Zoller [45]. A numerical estimate of these terms by the same authors leads to the conclusion that they are negligible with respect to the other sources of uncertainty.

III. ON THE GAUGE (IN)DEPENDENCE

Another aspect of the present calculation is represented by the well-known fact that the RGE-improved effective potential is gauge dependent, although the value of the top mass corresponding to the stability bound is not [9]. To introduce the issue, let us consider the argument presented by Di Luzio *et al.* in Ref. [11] for the case of two degenerate vacua, generalizing it to any stationary configuration (here, we use m_t instead of m_H).

Let us assume that all the parameters of the SM are exactly determined, except for the top mass. After choosing the renormalization scale t, the RGE-improved effective potential, $V_{\text{eff}}(\phi, m_t; \xi)$, is a function of ϕ, the top mass m_t, and the gauge-fixing parameters, collectively denoted by ξ. One can think of m_t as a free parameter, the variation of which modifies the shape of the effective potential, as sketched in Fig. 1 for the Landau gauge. Starting from top to bottom, the shape of the Higgs potential changes, going from stability to instability by increasing the top mass.

The absolute stability bound on the top mass can be obtained by defining a critical mass, m_t^c, for which the value of the effective potential at the electroweak minimum, ϕ_{ew}, and at a second minimum, $\phi_c > \phi_{\text{ew}}$, are the same:

$$\frac{\partial V_{\text{eff}}}{\partial \phi}\bigg|_{\phi_{\text{ew}}, m_t^c} = \frac{\partial V_{\text{eff}}}{\partial \phi}\bigg|_{\phi_c, m_t^c} = 0,$$

$$V_{\text{eff}}(\phi_{\text{ew}}, m_t^c; \xi) - V_{\text{eff}}(\phi_c, m_t^c; \xi) = 0. \quad (14)$$

Slightly reducing m_t, one finds another particular value of the top mass, m_t^i, such that the Higgs potential displays a rising point of inflection at $\phi_i > \phi_{\text{ew}}$:

$$\frac{\partial V_{\text{eff}}}{\partial \phi}\bigg|_{\phi_{\text{ew}}, m_t^i} = \frac{\partial V_{\text{eff}}}{\partial \phi}\bigg|_{\phi_i, m_t^i} = 0, \qquad \frac{\partial^2 V_{\text{eff}}}{\partial \phi^2}\bigg|_{\phi_i, m_t^i} = 0. \quad (15)$$
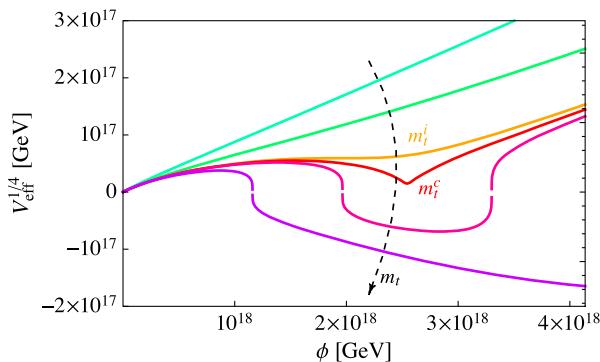

FIG. 1. Sketch of the shape of SM effective potential at high energy in the Landau gauge, for increasing values of the top mass from top to bottom. Two peculiar stationary configurations can be identified: two degenerate vacua and a rising point of inflection, associated, respectively, to m_t^c and m_t^i.

Due to the explicit presence of ξ in the vacuum stability and/or inflection point conditions, it is not obvious *a priori* which are the physical (gauge-independent) observables entering the vacuum stability and/or inflection point analysis.

The basic tool, in order to capture the gauge-invariant content of the effective potential, is given by the Nielsen identity [48]

$$\left(\xi \frac{\partial}{\partial \xi} + C(\phi, \xi) \frac{\partial}{\partial \phi} \right) V_{\text{eff}}(\phi, \xi) = 0, \quad (16)$$

where $C(\phi, \xi)$ is a correlator of which the explicit expression will not be needed for our argument. The equation means that $V_{\text{eff}}(\phi, \xi)$ is constant along the characteristics of the equation, which are the curves in the (ϕ, ξ) plane for which $d\xi = \xi / C(\phi, \xi) d\phi$.

In particular, the identity says that the effective potential is gauge independent where it is stationary. Due to the fact that the value of the effective potential at any stationary point ϕ_s (as is the case for ϕ_{ew}, ϕ_c, ϕ_i) is gauge invariant,

$$\frac{\partial V_{\text{eff}}(\phi, \xi)}{\partial \phi}\bigg|_{\phi_s, m_t} = 0 \rightarrow \frac{\partial V_{\text{eff}}(\phi, \xi)}{\partial \xi}\bigg|_{\phi_s, m_t} = 0, \quad (17)$$

its value at the extremum can be calculated working in any particular gauge, say the Landau gauge,

$$V_{\text{eff}}(\phi_s, m_t; \xi) = V_{\text{eff}}(\phi_s^L, m_t; 0), \quad (18)$$

where ϕ_s^L is the field evaluated in a stationary point in Landau gauge.

We want to check explicitly that, contrary to ϕ_s, the particular values of m_t ensuring criticality, m_t^c, and an inflection point, m_t^i, are gauge independent. Let us call them collectively m_t^s ($s = c$, i) and denote by $\bar{V}_s$ the associated value of the effective potential in the Landau gauge:

$$V_{\text{eff}}(\phi_s, m_t^s; \xi) = V_{\text{eff}}(\phi_s^L, m_t^s; 0) \equiv \bar{V}_s. \quad (19)$$

Inverting Eq. (19) (together with the stationary condition) would yield gauge-dependent field and top mass values: $\phi_s = \phi_s(\xi)$ and $m_t^s = m_t^s(\xi)$. We apply a total derivative with respect to ξ to Eq. (18) and obtain

$$\frac{\partial V_{\text{eff}}}{\partial \xi}\bigg|_{\phi_s, m_t^s} + \frac{\partial V_{\text{eff}}}{\partial m_t}\bigg|_{\phi_s, m_t^s} \frac{\partial m_t^s}{\partial \xi} + \frac{\partial V_{\text{eff}}}{\partial \phi}\bigg|_{\phi_s, m_t^s} \frac{\partial \phi_s}{\partial \xi} = 0, \quad (20)$$

where the third and first terms in the lhs vanish because of the stationary condition and the Nielsen identity, respectively. Since in general $\frac{\partial V_{\text{eff}}}{\partial m_t}\big|_{\phi_s, m_t^s} \neq 0$, we obtain that $\frac{\partial m_t^s}{\partial \xi} = 0$. We can conclude that the peculiar values of m_t

ensuring stationary configurations, like two degenerate vacua or an inflection point, are gauge independent.

Notice that the above argument can be easily generalized to the case in which we treat as free parameters not only the top mass but all other input parameters entering in the calculation of the effective potential, as for instance the Higgs mass and α_s. Let us call them $\vec{f} = (m_t, m_H, \alpha_s, \ldots)$, so that $V_{\text{eff}}(\phi, \vec{f}; \xi)$. In this case, the generalization of Eq. (20) is simply

$$\left. \frac{\partial V_{\text{eff}}}{\partial \xi} \right|_{\phi_s, \vec{f}^s} + \sum_i \left. \frac{\partial V_{\text{eff}}}{\partial f_i} \right|_{\phi_s, \vec{f}^s} \frac{\partial f_i^s}{\partial \xi} + \left. \frac{\partial V_{\text{eff}}}{\partial \phi} \right|_{\phi_s, \vec{f}^s} \frac{\partial \phi_s}{\partial \xi} = 0.$$

(21)

As before, the last and the first terms in the lhs vanish because of the stationary condition and the Nielsen identity, respectively. Since in general $\left. \frac{\partial V_{\text{eff}}}{\partial f_i} \right|_{\phi_s, \vec{f}^s} \neq 0$, we obtain $\frac{\partial f_i^s}{\partial \xi} = 0$. The peculiar values of the input parameters ensuring stationary configurations, like two degenerate vacua or an inflection point, are thus gauge independent.

Working in the Landau gauge is thus perfectly consistent in order to calculate the value of the effective potential at a stationary point, $\bar{V}_s$, or the value of the top mass providing it,[4] m_t^s. Nevertheless, one has to be aware that the truncation of the effective potential loop expansion at some loop order introduces an unavoidable theoretical error both in $\bar{V}_s$ and in m_t^s. For this sake, it is useful to define the parameter α via

$$\mu(t) = \alpha \phi$$

(22)

and study the dependence of $\bar{V}_s$ and m_t^s on α. The higher the order of the loop expansion to be considered, the less the dependence on α. This will be shown explicitly in the next two sections, where we study in detail the case of two degenerate vacua and the case of a rising inflection point at high energies, respectively.

IV. TWO DEGENERATE VACUA

As discussed in the previous section, once m_H and $\alpha_s^{(5)}$ have been fixed, the value of the top mass for which the SM displays two degenerate vacua, m_t^c, is a gauge-invariant quantity. This value is, however, plagued by experimental and theoretical errors, which we now discuss in turn.

The experimental error is the one associated to the precision with which we know the input parameters at the matching scale m_t. The uncertainty on m_t^c due to varying $\alpha_s^{(5)}$ in its 1σ range is ± 0.37 GeV, while the uncertainty due to varying m_H in its 1σ range is ± 0.12 GeV. This can be graphically seen in Fig. 2, where

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[4]Of course, this applies also to the other input parameters, but here we focus on m_t as it is the one associated to the largest uncertainty.
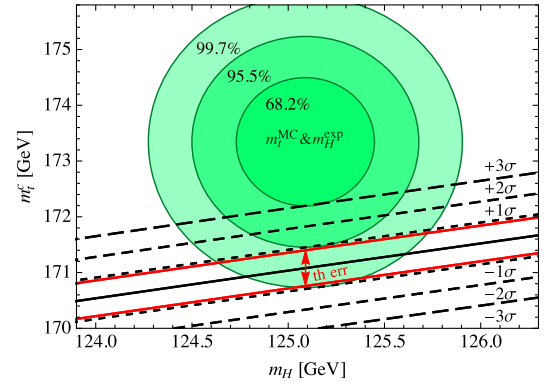

FIG. 2. Lines for which the Higgs potential develops a second degenerate minimum at high energy. The solid line corresponds to the central value of $\alpha_s^{(5)}$; the dashed lines are obtained by varying $\alpha_s^{(5)}$ in its experimental range, up to 3σ. The (red) arrow represents the theoretical error in the position of the lines. The (green) shaded regions are the covariance ellipses obtained combining $m_t^{\text{MC}} = 173.34 \pm 0.76$ GeV and $m_H^{\text{exp}} = 125.09 \pm 0.24$ GeV; the probability of finding m_t^{MC} and m_H inside the inner (central, outer) ellipse is equal to 68.2% (95.5%, 99.7%).

m_t^c is displayed as a function of m_H for selected values of $\alpha_s^{(5)}$; in particular, the solid line refers to its central value, while the dotted, short, and long dashed lines refer to the 1σ, 2σ, and 3σ deviations, respectively. In the region below (above) the line, the potential is stable (metastable).

Theoretical errors comes from the approximations done in the three steps of the calculation, matching at m_t, running from m_t up to high scales, and effective potential expansion:

(i) As already seen in Sec. II A, the theoretical error in the NNLO matching (scale variation and truncation) of λ is equivalent to a variation in m_H by about 1.6σ (± 0.38 GeV), which in turn means an error on m_t^c of about ± 0.19 GeV. The theoretical error in the NNLO matching (scale variation and truncation) of h_t is equivalent to a variation in m_t by about 0.3σ, which translates into an uncertainty of about ± 0.25 GeV on m_t^c. Combining in quadrature, the theoretical uncertainty on m_t^c due to the NNLO matching turns out to be about ± 0.32 GeV: this means that the position of the straight lines in Fig. 2 can be shifted up or down by about 0.32 GeV, as represented by the (red) arrow for the central value of $\alpha_s^{(5)}$.

(ii) The theoretical uncertainty associated to the order of the β-functions in the RGE can be estimated by adding the order of magnitude of the unknown subsequent correction. For the NNLO, it turns out that the impact of the subsequent four-loop correction on m_t^c is of order 10^{-5} GeV and is thus negligible.
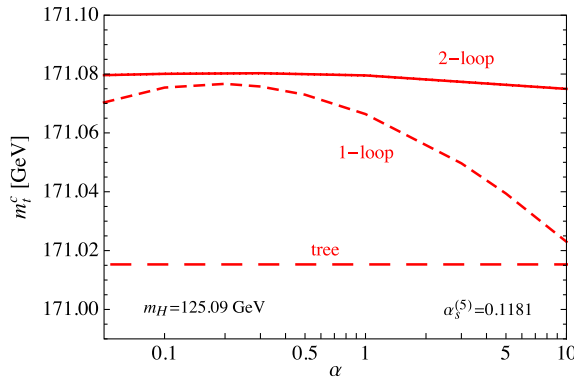
FIG. 3.   Dependence of m_t^c on α, for the increasing level of the effective potential expansion. We choose the central values for m_H and $\alpha_s^{(5)}$.

(iii) A way to estimate the theoretical uncertainty on m_t^c associated to the order of the expansion of the effective potential is to study its dependence on α. In Fig. 3, we show this dependence by varying α in the interval 0.1–10, while keeping the other parameters fixed at their central values. For comparison, we display the dependence obtained by performing the calculation of the effective potential at the tree (long-dashed), one-loop (short-dashed), and two-loop (solid) levels. A few comments are in order here. The renormalization scale $\mu(t)$ depends on α via Eq. (22) so that, at the tree level, the Higgs potential does not depend explicitly on $\mu(t)$ but only implicitly via the dependence of the running couplings; for this reason, in Fig. 3, the dependence on α for tree level is negligible, and we find $m_t^c \approx 171.02$ GeV. At one loop, the dependence on α is explicit, but the variation is anyway small, being about 0.05 GeV. The two-loop order further improves the independence on α; we can conclude that the error on m_t^c at the NNLO is very small, ± 0.005 GeV.

Summarizing, the theoretical error that mostly affects the NNLO calculation of m_t^c is the one in the matching, while those in the truncation of the loop expansion in the β-functions and in the effective potential expansions are negligible.

We can conclude that the result of the NNLO calculation is

$$m_t^c = 171.08 \pm 0.37_{\alpha_s} \pm 0.12_{m_H} \pm 0.32_{th} \text{ GeV}, \quad (23)$$

where the first two errors are the 1σ variations of $\alpha_s^{(5)}$ and m_H. Our results for the value of m_t^c update and improve[5]

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[5]The calculation of the theoretical error associated to the truncation at the two-loop order has (to our knowledge) not been shown so far.

but, modulo the doubling of the experimental error in $\alpha_s^{(5)}$, are essentially consistent with those of the most recent literature [7–10,21,22].

The above value of m_t^c has to be compared with the experimental determination of the top pole mass, m_t. The present value of the MC top mass is $m_t^{MC} = 173.34 \pm 0.76$ GeV [20] and would imply a difference with m_t^c at the level of about 1.7σ. This can be graphically seen in Fig. 2, where the (shaded) ellipses are the covariance ellipses for a two-valued Gaussian density, obtained by combining the present experimental values of the MC top mass and Higgs mass, so that the probability of being inside the smaller, central, and larger ellipses is, respectively, 68.2%, 95.5%, and 99.7%. However, as discussed in Sec. II A, the uncertainty in the identification between the pole and MC top mass is currently estimated to be of order 200 MeV [18,19] (or even 1 GeV for the most conservative groups [29]); this would further reduce the difference, say at the 1.5σ level. Statistically speaking, a tension at the 1.5σ level supports the stability and metastability at the 14% and 86% C.L., respectively. Physically speaking, in our opinion, it is even too strong to use the term "tension" when speaking of a 1.5σ deviation.

As a result, the updated calculation of the experimental and theoretical uncertainties on m_t^c in addition to the uncertainty in the identification of the MC and pole top masses lead us to conclude that the configuration with two degenerate vacua is at present compatible with the experimental data.

Our conclusions agree with those of Ref. [10]. Previous claims that stability is disfavored at more than the 95% level [7,9] are due, in our opinion, to the previous underestimation of two uncertainties—the experimental one in the determination of $\alpha_s^{(5)}$ and the theoretical one in the identification of the MC and pole top masses—together with a less conservative interpretation (with respect to ours) of the significance of the results.

It is clear that, in order to discriminate in a robust way between stability and metastability, it would be crucial to reduce the experimental uncertainties in both m_t and $\alpha_s^{(5)}$. A reduction of the theoretical error in the matching would also be welcome.

As a final remark, notice that a recent measurement of the top pole mass by the CMS Collaboration is $m_t^{CMS} = 172.38 \pm 0.66$ GeV [49]. The shaded ellipses in this case would change as shown in Fig. 4, and the discrepancy with m_t^c would thus further decrease, at less than 1σ. It will be very interesting to see if such a low value will be confirmed by future LHC data.

V. INFLECTION POINT

We now turn to the inflection point configuration assuming, as usual, that the potential at the electroweak minimum is zero. Such a configuration could be relevant
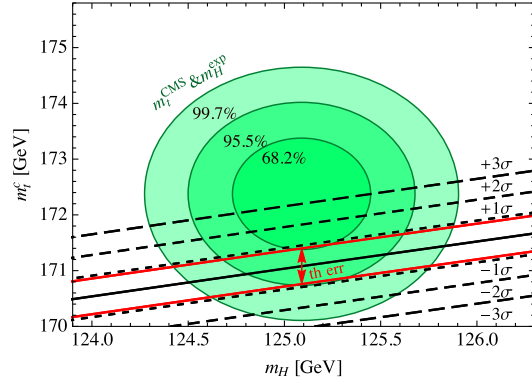
FIG. 4. Lines for which the Higgs potential develops a second degenerate minimum at high energy. The solid line corresponds to the central value of $\alpha_s^{(5)}$; the dashed lines are obtained by varying $\alpha_s^{(5)}$ in its experimental range, up to 3σ. The (red) arrow represents the theoretical error in the position of the lines. The (green) shaded regions are the covariance ellipses obtained combining $m_t^{CMS} = 172.38 \pm 0.66$ GeV [49] and $m_H^{exp} = 125.09 \pm 0.24$ GeV.
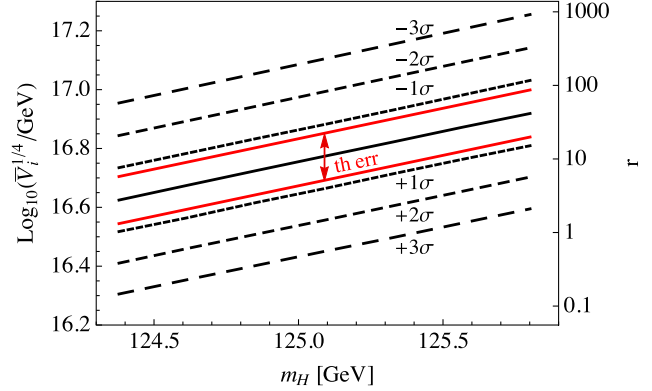

FIG. 5. Dependence of $\bar{V}_i^{1/4}$ on m_H for fixed values of $\alpha_s^{(5)}$. The (red) arrow and solid lines show the theoretical error due to the matching of λ (positive contribution to λ for the upper line, negative for the lower one). The right vertical axis displays the associated value of the tensor-to-scalar ratio r, according to Eq. (24).

for the class of models of primordial inflation based on a shallow false minimum [4,50–52]. In particular, the highness of the effective potential at an inflection point, let us call it $\bar{V}_i$, could be directly linked to the ratio of the scalar-to-tensor modes of primordial perturbations, r, via the relation

$$\bar{V}_i = \frac{3\pi^2}{2} r A_s, \tag{24}$$

where the amplitude of scalar perturbations is $A_s = 2.2 \times 10^{-9}$ [53]. It is thus relevant for models of primordial inflation to assess the size of the experimental and theoretical errors in the calculation of $\bar{V}_i$.

Experimental uncertainties on $\bar{V}_i$ can be estimated as follows. We let m_H vary in its 3σ experimental range, and for fixed values of $\alpha_s^{(5)}$, we determine m_t^i, the value of the top mass needed to have an inflection point (which is so close to m_t^c that one can read it from the stability line of Fig. 2). We then evaluate the effective potential at this point, $\bar{V}_i$. The result is displayed in Fig. 5: one can see that $\bar{V}_i^{1/4}$ spans 1 order of magnitude, as it varies from 2×10^{16} GeV up to 2×10^{17} GeV, for decreasing values of $\alpha_s^{(5)}$; the dependence on m_H is less dramatic but still relevant, being about 50%.

We divide the theoretical errors in three categories, as done before:

(i) Theoretical errors associated to matching of λ at NNLO have an impact on m_t^c of about ± 0.19 GeV (see Fig. 2 in the previous section) and an impact on $\log_{10}(\bar{V}_i^{1/4}/\text{GeV})$ of about ± 0.08, namely a 20% variation of $\bar{V}_i^{1/4}$. As for the NNLO matching of h_t,

the impact on m_t^c is of about ± 0.25 GeV, but there is no significant impact on $\bar{V}_i$. As a consequence of the theoretical error in the matching, the lines in Fig. 5 could be shifted up or down by about 0.08; for the central value of $\alpha_s^{(5)}$, this is represented in Fig. 5 by the (red) arrow and solid lines. The theoretical error due to the NNLO matching is thus slightly smaller than the experimental error due to the 1σ variation of $\alpha^{(5)}$.

(ii) The order of magnitude of the theoretical errors associated to the β-functions at NNLO can be estimated by studying the impact of the subsequent correction; it turns out that $\bar{V}_i^{1/4}$ changes at the per mille level. Such an error is thus negligible.

(iii) The theoretical uncertainty associated to the fact that we truncate the effective potential at two loops can be estimated by studying the dependence of $\bar{V}_i$ on α. We fix $\alpha_s^{(5)}$ and m_H at their central values and display in Fig. 6 the resulting value of $\bar{V}_i^{1/4}$ by means of the solid line; for comparison, we display also the dependences obtained at tree[6] (long-dashed) and one-loop (short-dashed) levels. Notice that the dependence of $\bar{V}_i$ on α at the tree-level is implicit,[7] but significant: the tree-level calculation of $\bar{V}_i^{1/4}$ is uncertain by more than 1 order of magnitude. The one-loop corrections flatten the dependence on α so that the uncertainty on $\bar{V}_i^{1/4}$ gets reduced down to

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[6]We checked that our results for the tree level with $\alpha = 1$ agree with those obtained in Refs. [22,52]. In these works, the potential was indeed calculated only at the tree level, without providing any estimate for the theoretical uncertainty because of neglecting higher loops.

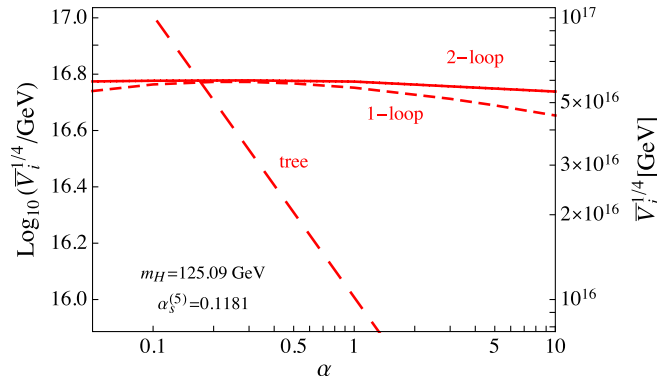[7]Indeed, $\bar{V}_{eff} \propto \lambda(\ln(\alpha\phi/m_t))$.

FIG. 6.   Dependence of $\bar{V}_i^{1/4}$ on α. For definiteness, $\alpha_s^{(5)}$ and m_H are assigned to their central values.
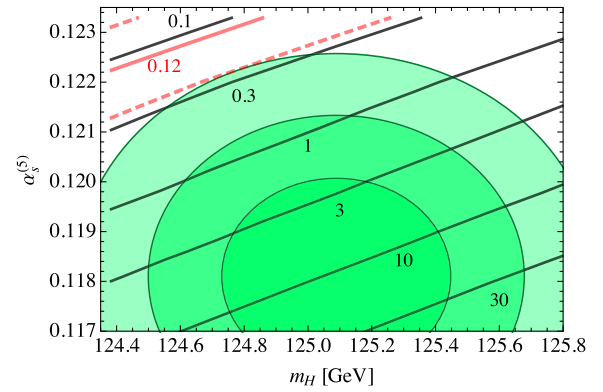

FIG. 7.   Contour levels of r in the plane $(m_H, \alpha_s^{(5)})$. The theoretical uncertainty corresponding to $r = 0.12$ is shown by means of the (red) dashed lines. The shaded regions are the covariance ellipses indicating that the probability of finding the experimental values of m_H and $\alpha_s^{(5)}$ inside the ellipses are, respectively, 68.2%, 95.4%, and 99.7%.

about 20%; this uncertainty is comparable to the theoretical one due to the matching. The two-loop correction further flattens the dependence on α and allows one to estimate $\bar{V}_i^{1/4}$ with a 5% precision.

Summarizing, the theoretical error that mostly affects the calculation of $\bar{V}_i$ at the NNLO is the one associated to the matching. The theoretical error in the truncation of the effective potential is smaller that the theoretical error in the matching only including the two-loop correction to the effective potential.

We can conclude that the result of the NNLO calculation is

$$\log_{10} \bar{V}_i^{1/4} = 16.77 \pm 0.11_{\alpha_s} \pm 0.05_{m_H} \pm 0.08_{th}, \quad (25)$$

where the first two errors refer to the 1σ variations of $\alpha_s^{(5)}$ and m_H, respectively, while the theoretical error is essentially dominated by the one in the matching.

As anticipated, a precise determination of $\bar{V}_i$ is important for models of inflation based on the idea of a shallow false minimum [4,50–52] as, in these models, $\bar{V}_i$ and r are linked via Eq. (24). In view of such an application, the right axis of Fig. 5 reports the corresponding value of r. Notice that the dependence of r on $\alpha_s^{(5)}$ and m_H is extremely strong; when the latter are varied in their 3σ range, r spans about 4 orders of magnitude, from 0.1 to 1000. In addition, the theoretical error in the matching implies an uncertainty on r by a factor of about 2.

According to the 2015 analysis of the Planck Collaboration, the present upper bound on the tensor-to-scalar ratio is $r < 0.12$ at 95% C.L. [54], as also confirmed by the recent joint analysis with the BICEP2 Collaboration [55]. Due to Eq. (24), this would translate into to the bound $\log_{10} \bar{V}_i^{1/4} < 16.28$ at 95% C.L.; this implies a tension with Eq. (25) at about 3σ.

This can be graphically seen in Fig. 7, where the contour levels of r in the plane $(m_H, \alpha_s^{(5)})$ are shown. Even invoking the uncertainty due to the matching (red-dashed lines), a value for r as small as 0.12 (red-solid line) could be

obtained only with $\alpha_s^{(5)}$ in its upper 3σ range, while the values of m_H, and also m_t (see Fig. 2), could stay inside their 1σ interval.

A value for r close to 0.2, as claimed by BICEP2 in 2013 [56], would have been compatible at about 2σ with a model based on a shallow false minimum. With the inclusion of the theoretical error and the slight changes in the central values of the input parameters, the present calculation supersedes the results obtained in the previous paper [52], where the calculation of the effective potential was done only at tree level (due to the yet unsolved problem of the inclusion of the one-loop correction when λ turns negative [14]).

The present results have to be compared with those of the most recent detailed analysis on the inflection point configuration, performed by Ballesteros *et al.* [25]. The latter work includes the two-loop correction to the Higgs effective potential but, at our understanding, does not include the theoretical error in the matching (which is actually the dominant one according to us). This may justify their stronger exclusion of the inflection point configuration. Apart from this detail, the results of the two analyses are in substantial agreement. The even stronger exclusion of the inflection point configuration found in Ref. [24] might be also due to an underestimation of the theoretical errors, together with the previous underestimation of the experimental uncertainty on $\alpha_s^{(5)}$.

VI. GRAVITATIONAL CONTRIBUTION: EFFECTS OF PHYSICS AT M_P

Through this work, we assumed the SM to be valid all the way up to the Planck scale. Near the cutoff of the theory, large Planckian effects are possible, but without a satisfactory comprehension of quantum gravity effects (with a reliable UV completion of the theory), there is no hope to

calculate them. The usual approach in this sense is to consider a tower of a nonrenormalizable operators suppressed by the cutoff in an effective theory scenario below M_P [12,57–61], leading to a modification of the SM Higgs potential:

$$V(\phi) = \frac{\lambda}{24}\phi^4 + \frac{\lambda_6}{6}\frac{\phi^6}{M_P^2} + \frac{\lambda_8}{8}\frac{\phi^8}{M_P^4} + \mathcal{O}\left(\frac{\phi^{10}}{M_P^6}\right). \quad (26)$$

Without any protecting symmetry, these corrections have to be taken into account. In this way, assuming order 1 values for the new unknown couplings λ_6 and λ_8, treated as free parameters, it is in principle possible to estimate the impact of gravitational physics. The effects of these higher-order operators turn out to be heavily dependent on the choice of the free couplings toward both stability and instability; it is not clear why gravitational physics should make the potential more unstable or vice versa. The approach of Eq. (26) has, however, raised some concerns [62], as the method is based on an effective theory expansion that breaks down when $\phi \sim M_P$. The use of an effective theory close to its cutoff might not be fully reliable.

VII. CONCLUSIONS

We studied in detail gauge-independent observables associated with two interesting stationary configurations of the SM Higgs potential extrapolated at the NNLO: i) the value of the top mass ensuring the stability of the SM electroweak minimum and ii) the value of the Higgs potential at a rising inflection point. We reappraised in a critical way the experimental and theoretical uncertainties plaguing their determination.

Considering the updated value of the experimental error on $\alpha_s^{(5)}$, the issue of the determination of the top pole mass from the MC one, and the theoretical uncertainty

associated to the matching, we find that the stability of the SM is compatible with the present data at the level of 1.5σ. Stability of the SM Higgs potential is thus, in our opinion, a viable possibility. In order to robustly discriminate between stability and metastability, higher precision measurements of the top quark pole mass and of $\alpha_s^{(5)}$ would be needed. For the time being, we can wonder if the fact that the Higgs potential is so close to a configuration with two degenerate vacua is telling us something deep [12].

As for the configuration of a rising inflection point, we find that, despite the large theoretical error plaguing the value of the Higgs potential at the inflection point, the application of such a configuration to models of primordial inflation based on a shallow false minimum displays a 3σ tension with the recent bounds on the tensor-to-scalar ratio of cosmological perturbations. The tension is essentially due to the value of $\alpha_s^{(5)}$, instead of the top or Higgs masses. Hence, if $\alpha_s^{(5)}$ will turn out to be in its present 3σ range, such models will be rescued; otherwise, modifications due to new physics will be necessarily introduced [25,63].

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