

How well does the chiral expansion converge in nuclear and neutron matter?

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The equations of state of nuclear and neutron matter (and, more generally, neutron-rich matter) play a fundamental role towards the understanding of a broad spectrum of systems, ranging from the skins of neutron-rich nuclei to the structure of compact stars.

After a brief review of different approaches to predict the properties of nuclear and neutron matter, we will focus on error quantification in effective field theory. The various sources of uncertainty and their impact in infinite matter will be explored.

We will then report on recent calculations of the nuclear and neutron matter equations of state at different orders of the chiral expansion as well as changing resolution scale. We will discuss the significance of such predictions as a foundation for future studies of convergence of the chiral perturbation series.

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1. Introduction and outline

Microscopic nuclear physics seeks to describe the structure of nuclear systems in terms of the underlying few-nucleon forces, namely, *ab initio*. Ideally, one wishes to derive nuclear forces from quantum chromodynamics (QCD), the fundamental theory of strong interactions. However, the well-known problem with QCD is that it is nonperturbative in the low-energy regime characteristic for nuclear physics, a fact which was perceived for a long time as an obstacle to a derivation of nuclear forces from QCD. The effective field theory (EFT) concept, chiral EFT in particular, has shown a way out of this problem and is presently a popular approach which starts from a low-energy realization of QCD [1, 2]. In chiral EFT, one retains the basic degrees of freedom typical of low-energy nuclear physics, pions and nucleons, while fitting unresolved nuclear dynamics at short distances to the properties of two- and few-nucleon systems alone.

Although finite nuclei are a natural testing ground for many-body theories, infinite matter is an alternative and convenient system for testing such theories. Being translationally invariant and consisting of nucleons subjected only to their mutual strong interactions, it is theoretically simpler than a finite nucleus and yet, through its saturation properties, provides strong constraints for theoretical models of nuclear forces in the medium.

The chiral EFT approach, central to this paper, has been shown to produce realistic equations of state (EoS) for nuclear and neutron matter with the inclusion of repulsive three-nucleon forces (3NF) arising at order $N^2\text{LO}$ (next-to-next-to-leading order) in the chiral power counting [3, 4, 5, 6]. In chiral EFT, the dominant two-pion-exchange component of this 3NF is constructed consistently with the $N^2\text{LO}$ two-body force.

The main purpose of this paper is to discuss error quantification in predictions of nucleonic matter obtained with chiral two- and three-body forces. The discussion will articulate through the following main points:

- We will start with a broad introduction and further underline the importance of studying infinite matter (symmetric or neutron-rich) as a testing ground for many-body theories.
- We will highlight two popular philosophies currently used to obtain predictions of nuclear and neutron matter properties, and comment on their respective advantages and disadvantages.
- We will then move to our focal point, namely, error quantification in chiral EFT, and address the main sources of uncertainty.
- We will present and discuss recent calculations of the energy per particle at different orders of chiral EFT. Prior to that, we will take the opportunity to review the development of the chiral two- and three-nucleon forces which we use.
- We will summarize our conclusions, work in progress, and future plans.

2. Infinite matter: an important theoretical “laboratory”

The typical outcome of nuclear matter investigations consists of the energy per particle and the pressure as functions of density (and additional variables, such as temperature, if appropriate).

Adopting what is known as the “local density approximation”, one can relate the EoS of infinite matter to finite systems. *Vice versa*, empirical information on nuclei have been extrapolated to an infinite system to provide constraints for theoretical models of nuclear matter.

When proton and neutron densities are different (that is, in the presence of isospin asymmetry), the energy per particle becomes a function of both the total density and the relative concentrations of neutrons and protons. The EoS of isospin-asymmetric matter naturally introduces the symmetry energy, similarly to the appearance of the symmetry term in the well-known Bethe-Weizsäcker formula. As will be discussed later in more details, the symmetry energy is typically approximated as the difference between the energy per particle of symmetric nuclear matter and pure neutron matter as a function of density.

Of particular contemporary interest is the EoS of *highly* neutron-rich matter, all the way to nearly pure neutron matter. This quantity is important for understanding wide ranging questions in modern nuclear structure physics, from the properties of rare isotopes to those of neutron stars. On the one hand, the density dependence of the symmetry energy is known to correlate strongly with the neutron skin thickness of a heavy nucleus. On the other hand, the pressure in neutron-rich matter is the main input for the structure equations of compact stars.

Therefore, microscopic predictions together with empirical constraints from observables that are sensitive to the EoS are an ideal combination to learn about the in-medium behavior of the nuclear force, particularly in isospin-asymmetric medium. We recall that the EoS is an important part of the input of transport models describing heavy-ion collisions and thus can be constrained through analyses of carefully selected observables in ion-ion scattering.

Concerning non-terrestrial observations, partnership between nuclear physics and astrophysics is increasingly important as better constraints on the high-density part of the EoS become available through more accurate measurements of neutron star masses.

In summary, studies of nucleonic matter are especially timely and important, as they support rich on-going and future experimental effort, both astrophysically and in terrestrial laboratories.

3. Meson theory *vs.* chiral approaches: general considerations

As mentioned earlier, the goal of microscopic nuclear physics is to derive the properties of nuclear systems from the basic few-nucleon interactions. Therefore, the first question to raise is how to derive those fundamental forces.

Meson-theoretic interactions, such as those obtained with one-boson-exchange models, have been, and still are, used extensively in nuclear structure calculations. At this point, it may be interesting to make some additional observations with regard to the structure of the nucleon-nucleon potential in chiral EFT as compared to the traditional meson-exchange model. In the latter, the pion, the intermediate-mass meson (typically, the fictitious σ), and the heavier bosons (ρ and ω), provide the long-, the intermediate-, and the short-range behaviors, respectively, required by the phenomenology of the nuclear force. In chiral EFT, multi-pion exchange diagrams describe the longer and intermediate range parts of the force, whereas “contact terms” encode the short-range components. Both philosophies have in common the outstanding role of the pion.

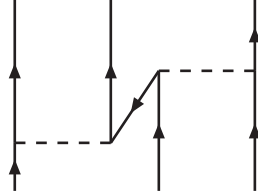


Figure 1: Three-nucleon force due to virtual excitation of a nucleon-antinucleon pair.

However, meson-exchange models (see, for instance, Ref. [7]) typically employ three-nucleon forces with little connection to the associated nucleon-nucleon (NN) force. The main problem is how to select the contributions to be retained in a systematic and model-independent way.

Relativistic approaches to nuclear matter have focused on the Dirac-Brueckner-Hartree-Fock (DBHF) method together with relativistic one-boson-exchange potentials [8], and are suitable to address a broad range of momenta and densities. This is desirable, since, in astrophysical contexts, the EoS of neutron-rich matter is required over many orders of magnitude in the nuclear density, potentially up to ten times that of saturated nuclear matter. The main strength of the DBHF framework is its ability to account in an efficient way for an important class of 3NF, namely virtual nucleon-antinucleon excitations, shown in Fig. 1, that lead to nuclear matter saturation properties close to the empirical density and energy [8].

In spite of the strong features offered by some meson-theoretic models, the problem shared with all non-EFT based approaches is that it is essentially impossible to estimate reliably the uncertainty associated with a particular prediction. On the other hand, EFT provides a well-defined framework to calculate observables where the truncation error decreases systematically as higher orders are included. More specifically, EFT is a theory in which the properties governed by low-energy physics are specified by the choice of degrees of freedom and symmetries, and can be computed systematically. Short-range physics is included through the processes of regularization and renormalization. Together with an organizational scheme to rank-order the various contributions, known as power counting, two- and few-nucleon forces emerge on equal footing in a controlled hierarchy, as shown in Fig. 2.

4. EFT uncertainty quantification

Estimates of theoretical uncertainties [9] for calculations of the EoS have mostly focused on varying the low-energy constants and resolution scale at which nuclear dynamics are probed [3, 4, 5, 10, 11, 12]. In recent work [13] we laid the foundation for order-by-order calculations of nuclear

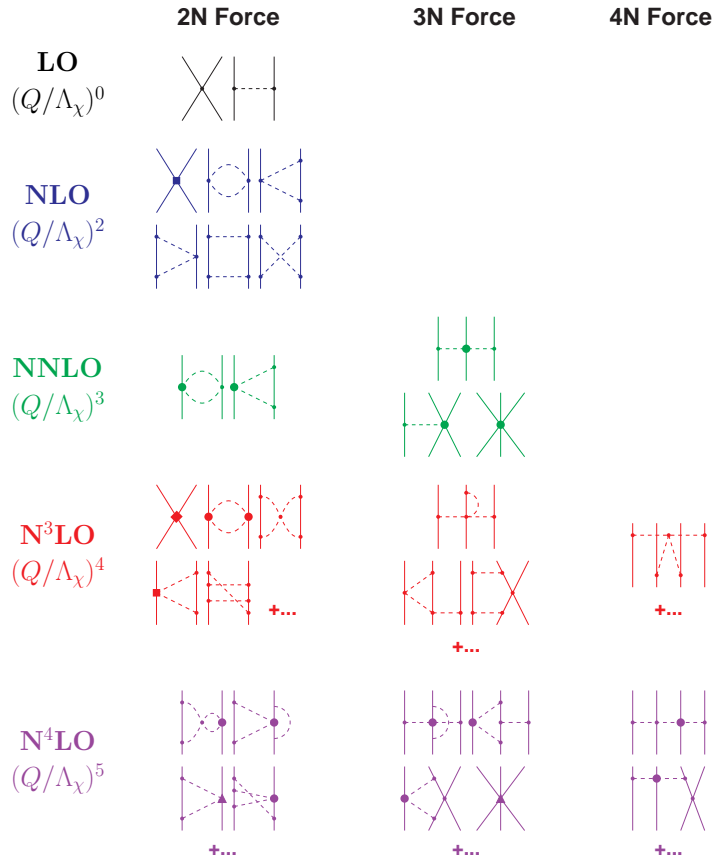


Figure 2: Chiral two- and few-nucleon forces up to 5th order of EFT.

many-body systems by presenting consistent NLO and N²LO chiral nuclear forces whose relevant short-range three-body forces are fit to $A = 3$ binding energies and the lifetime of the triton. We then assessed the accuracy with which infinite nuclear matter properties and the isospin asymmetry energy can be predicted from order-by-order calculations in chiral EFT. Identifying the dominant sources of uncertainty in nuclear many-body calculations is an important open problem, especially as more stringent constraints on the EoS of neutron-rich matter and its density dependence are becoming available [14].

Uncertainty originates from:

- The choice of the many-body method (a source of error not inherent to EFT).
- Error in the determination of the low-energy constants (LECs). Short-range LECs (NN) and long-range LECs (π N) must be considered separately.
- Regulator dependence.
- Truncation error.

In the following sections, we will address them systematically.

4.1 Uncertainty from the many-body method

A variety of many-body methods are available and have been used extensively in nuclear matter. They include: the coupled-cluster method, many-body perturbation, variational Monte Carlo or Green's function Monte Carlo methods.

In computing the EoS, we employ the nonperturbative particle-particle (pp) ladder approximation. In the traditional hole-line expansion, it represents the leading-order contribution. We use the so-called continuous choice for the single particle potential, which is determined selfconsistently applying the effective mass approximation. To quantify the uncertainty carried by these choices, it is insightful to compare with Refs. [15, 16]. In Ref. [15], the authors report on coupled-cluster calculations in symmetric nuclear matter including pp and hole-hole (hh) diagrams (as well as an exact treatment of the Pauli operator). The overall effect, as seen from comparing the first and last entries in Table II of Ref. [15], is very small around saturation density, consistent with Table II in Ref. [5], and grows to 1.5 MeV at the highest Fermi momentum included in the study. These calculations adopt the N³LO potential (with $\Lambda=500$ MeV) and only two-nucleon forces. On the other hand, in Ref. [16] coupled-cluster calculations in nucleonic matter were performed at N²LO with two- and three-body forces and with the inclusion of selected triples clusters, namely correlations beyond pp and hh ladders. Note that triples signify what is known, in the more traditional language, as three-hole line contributions. The effects of these triples is found to be negligible in neutron matter and about 1 MeV per nucleon in symmetric matter in the density range under consideration [16].

In the light of the above considerations, we conclude that a realistic estimate of the impact of using a nonperturbative approach beyond pp correlations is about ± 1 MeV in nuclear matter around saturation density and much smaller in neutron matter. As we show below, such uncertainties are significantly smaller than those associated with variations in the cutoff scale.

4.2 Uncertainty in the low-energy constants

In order to quantify the error associated with possible variations of the (short-range) NN LECs, we refer to recent findings from the Granada group [17]. They applied 205 samples of smooth local potentials, all with χ^2/datum of approximately 1, and found a variation of ± 15 keV in the triton binding energy.

From our part, we performed Brueckner-Hartree-Fock calculations in nuclear matter using local high-precision potentials from the Nijmegen group [18] and observed an uncertainty of ± 0.6 MeV in the energy per particle at normal density.

In summary, we conclude that the uncertainty arising from the error in the NN data has negligible impact on the many-body system.

Concerning the (long-range) π N LECs, they are likely to impact mostly peripheral partial waves (namely, those high partial waves where no contact terms are present). At NLO, that means D -waves and higher, whereas at N³LO no contacts are available in F -waves and higher. Therefore, one might anticipate that variations of the π N LECs (within the range allowed by π N scattering data) will have only minor impact in nuclear matter, since the sensitivity is limited to peripheral partial waves. Nevertheless, we stress that a systematic investigation with consideration of π N LECs uncertainty *consistently* in the 2NF and the 3NF, has not yet been done and is part of our future plans.

4.3 Regulator dependence and truncation error in the NN system

Chiral NN potentials are multiplied by a regulating function, which typically has the form:

$$f(p', p) = \exp[-(p'/\Lambda)^{2n} - (p/\Lambda)^{2n}], \quad (4.1)$$

where $\Lambda \lesssim 500$ MeV has been found to yield favorable perturbative properties. Potentials with different cutoff values constitute a family with similar long-range properties.

Cutoff dependence and truncation error are not to be identified with each other, although, generally, cutoff dependence is expected to decrease with increasing order. Figure 3 shows to which extent this is the case for the two-body system. There, we demonstrate the dependence of NN scattering phase shifts on the chiral order and on the choice of the cutoff scale in the regulating function, Eq. (4.1). The yellow, red and blue bands represent the NLO, N²LO, and N³LO results, respectively, obtained from varying the cutoff between 450 and 600 MeV. Although N²LO calculations can achieve sufficient accuracy in selected partial waves up to $E_{\text{lab}} = 200$ MeV, only the N³LO interactions achieve the level of high-precision potentials, characterized by a $\chi^2/\text{datum} \sim 1$.

At the two-body level, each time the chiral order is increased, the NN contact terms and/or the two-pion-exchange contributions proportional to the low-energy constants $c_{1,3,4}$ are refitted. We recall that at N²LO no new NN contact terms are generated, and therefore improved cutoff independence in the NN phase shifts (compare the yellow and red bands in Fig. 3) is due to changes in the two-pion-exchange contributions. At N²LO, subleading $\pi\pi NN$ vertices enter into the chiral NN potential. These terms encode the important physics of correlated two-pion-exchange and the excitation of intermediate $\Delta(1232)$ isobar states. Thus, only at this order is it possible to obtain a realistic description of the NN interaction at intermediate-range, traditionally generated through the exchange of a fictitious σ meson of intermediate mass. At N³LO in the chiral power counting, 15 additional NN contact terms (bringing the total to 24 free parameters at N³LO) result in a much improved description of NN scattering phase shifts.

We observe that, with increasing chiral order, the dependence on the cutoff scale is reduced. In the present work, we employ standard Weinberg power counting in which contributions to the NN potential are computed perturbatively with loop integrals renormalized through counterterms. Weinberg's scheme implicitly assumes that the counterterms introduced to renormalize the perturbative potential are sufficient to also renormalize its nonperturbative resummation, e.g., in the Lippmann-Schwinger equation for computing phase shifts. Kaplan *et al.* [19], however, pointed out the presence of problems with this assumption, which stimulated intense discussion in the literature [20]. In particular, Nogga *et al.* [21] performed a systematic investigation of Weinberg's power counting at lowest order and proposed a modified scheme in which contact terms from NLO are promoted to LO to take care of the cutoff dependence in 3P_0 and 3P_2 , and from N³LO to LO to address the same problem in 3D_2 . On the other hand, the consistency problem of Weinberg's power counting appears to be minimal when implemented together with finite-cutoff regularization below the high-energy scale of the EFT. It has been shown [22] that, at a given order, cutoff ranges can be identified where the χ^2 of the fit to the NN data is essentially flat (that is, cutoff independent). In other words, order-by-order renormalization can be accomplished successfully with finite cutoffs in the Weinberg power counting scheme.

In summary, in the NN system we see both excellent order-by-order convergence and reduced cutoff dependence with increasing order.

5. Description of the few-nucleon forces

Before moving to nucleonic matter, in this section we describe the few-nucleon forces as we will apply them in Section 6.

5.1 The chiral two-body force

In the present investigation we consider NN potentials at order $(q/\Lambda_\chi)^2$, $(q/\Lambda_\chi)^3$ and $(q/\Lambda_\chi)^4$ in the chiral power counting, where q denotes the small scale set by external nucleon momenta or the pion mass and Λ_χ is the chiral symmetry breaking scale. Chiral NN potentials at NLO and N²LO, corresponding to $(q/\Lambda_\chi)^2$ and $(q/\Lambda_\chi)^3$, have been constructed previously in Ref. [22] for cutoffs ranging from $\Lambda = 450$ MeV to about 800 MeV. With varying chiral order and cutoff scale, the low-energy constants in the two-nucleon sector are refitted to elastic NN scattering phase shifts and properties of the deuteron. The low-energy constants $c_{1,3,4}$ associated with the $\pi\pi NN$ contact couplings of the $\mathcal{L}_{\pi N}^{(2)}$ chiral Lagrangian are given in Table 1. We note that the c_i can be extracted from πN or NN scattering data. The potentials we use here [23, 24] follow the second path. At N²LO, taking the range determined in πN analyses as a starting point, values were chosen to best reproduce NN data at that order, see Table 2 of Ref. [24]. At N³LO, high-precision required a stronger adjustment of c_4 depending on the regulator function and cutoff. The fitting procedure is discussed in Ref. [24], where it is noted that the larger value for c_4 has, overall, a very small impact but lowers the 3F_2 phase shift for a better agreement with the phase shift analysis.

In Ref. [22], it was found that the two-body scattering phase shifts can be described well at NLO up to a laboratory energy of about 100 MeV, while the N²LO potential fits the data up to 200 MeV. Interestingly, in the latter case the χ^2/datum was found to be essentially cutoff independent for variations of Λ between 450 and approximately 800 MeV. Finally, we also use NN potentials constructed at next-to-next-to-next-to-leading order (N³LO) [24, 23], with low-energy constants $c_{1,3,4}$ as displayed in Table 1. However, at N³LO, NN potentials with cutoffs up to 800 MeV are not available. Therefore, in the present study, we limit the cutoff range to 450-600 MeV.

5.2 The chiral three-nucleon force

Three-nucleon forces make their appearance at third order in the chiral power counting. They are expressed as the sum of three contributions: the long-range two-pion-exchange part with $\pi\pi NN$ vertex proportional to the low-energy constants c_1, c_3, c_4 , the medium-range one-pion exchange diagram proportional to the low-energy constant c_D , and finally the short-range contact term proportional to c_E . The corresponding diagrams are shown in Fig. 4, labeled as (a), (b), (c), respectively.

The natural framework to include 3NFs in the particle-particle ladder approximation to the energy per particle in homogeneous nuclear matter would be the Bethe-Faddeev equation. To facilitate this inclusion, we employ the density-dependent NN interaction derived in Refs. [25, 26] from the N²LO chiral three-body force. This effective interaction is obtained by summing one particle line over the occupied states in the Fermi sea. Neglecting small contributions [27] from terms depending on the center-of-mass momentum, the resulting NN interaction can be expressed in analytical form with operator structures identical to those of free-space NN interactions. For symmetric nuclear matter all three-body forces contribute, while for pure neutron matter only terms proportional to the low-energy constants c_1 and c_3 are nonvanishing [26, 27]. Previous studies

NLO	Λ (MeV)	n	c_1	c_3	c_4
	450	2			
	500	2			
	600	2			
N ² LO	Λ (MeV)	n	c_1	c_3	c_4
	450	3	-0.81	-3.40	3.40
	500	3	-0.81	-3.40	3.40
	600	3	-0.81	-3.40	3.40
N ³ LO	Λ (MeV)	n	c_1	c_3	c_4
	450	3	-0.81	-3.40	3.40
	500	2	-0.81	-3.20	5.40
	600	2	-0.81	-3.20	5.40

Table 1: Values of n and low-energy constants of the dimension-two πN Lagrangian, $c_{1,3,4}$, at each order and for each type of cutoff in the regulator function given in Eq. (4.1). None of the c_i 's appear at NLO. The low-energy constants are given in units of GeV^{-1} .

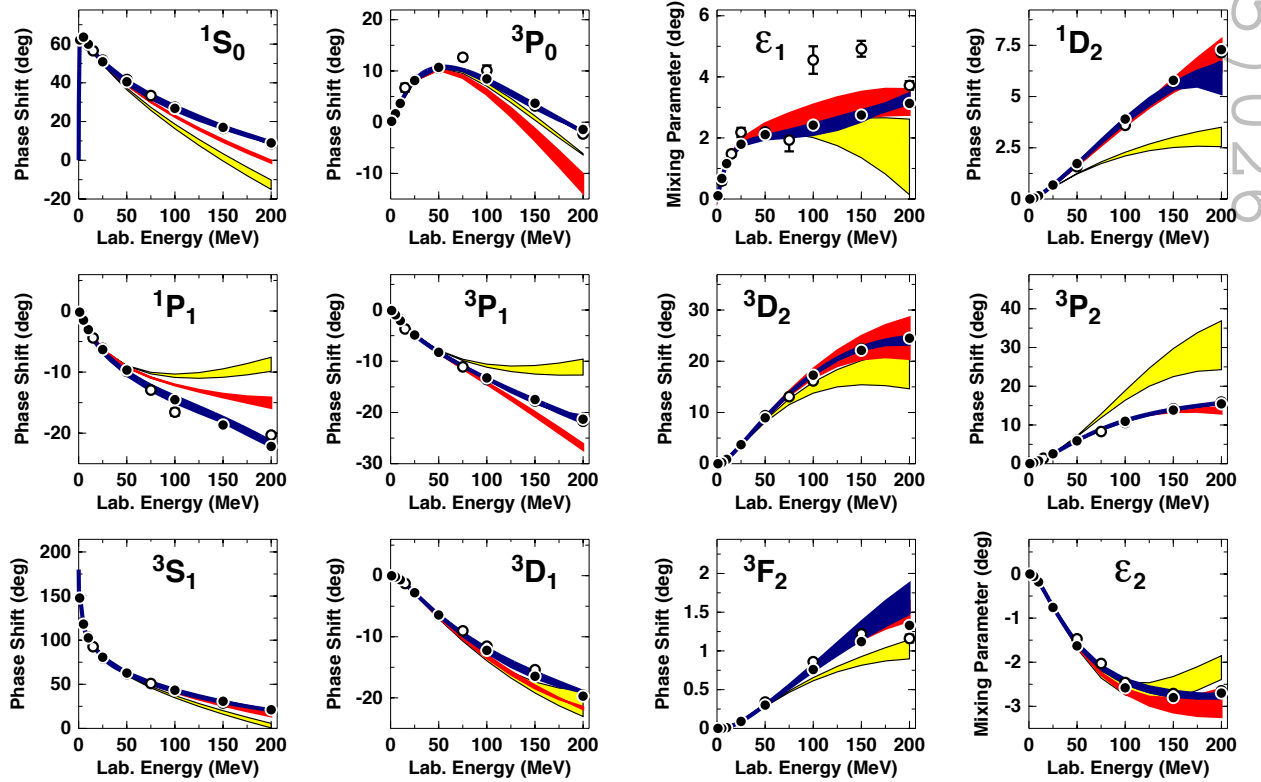


Figure 3: (Color online) Phase shifts for some selected NN partial waves. The yellow, red, and blue bands show the variations of the predictions with changing cutoffs between 450 and 600 at NLO, N²LO, and N³LO, respectively.

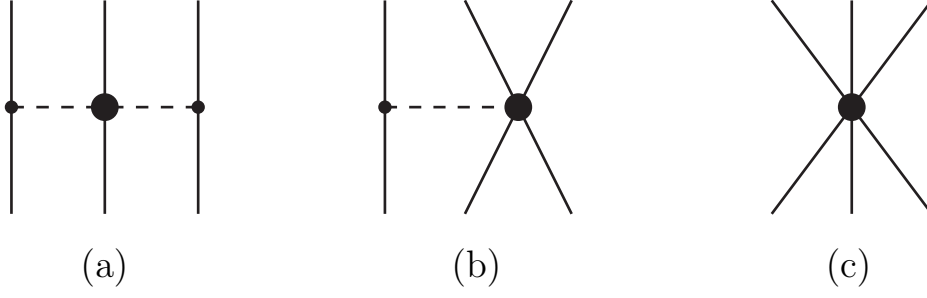


Figure 4: Diagrams of the 3NF at N^2LO . See text for more details.

(see e.g., Ref. [5]) have found that 3NF contributions to the energy per particle are dominant at the Hartree-Fock level. When three-body forces are approximated with a density-dependent NN interaction, certain topologies are missing at and beyond second order in perturbation theory. In the case of a pure contact interaction the additional topologies reduce the second-order contribution to the energy per particle by about 50% at saturation density [28], which corresponds in the present calculation to an uncertainty of $\Delta E/A \lesssim 1$ MeV at saturation density.

We fix the low-energy constants c_D and c_E that appear in the N^2LO 3NF within the three-nucleon sector. Specifically, we constrain them to reproduce binding energies of $A = 3$ nuclei together with the Gamow-Teller matrix element in tritium β -decay, following a well established procedure [29, 30, 31, 32, 33, 34, 35]. The values of c_D and c_E are given in Table 2 for the different chiral orders and cutoff scales. We note that the values at N^3LO in Table 2 are extracted from Refs. [29, 5], while those at N^2LO have been computed in Ref. [13]. Although efforts are in progress to incorporate potentially important N^3LO 3NF contributions [36, 37, 38], both in the fitting procedure and in the neutron and nuclear matter EoS presented in the following section, the current “ N^3LO ” study is limited to the inclusion of the N^2LO three-body force together with the N^3LO two-body force, an approximation that is commonly used in the literature but whose associated uncertainties have not been carefully analyzed. In Refs. [4, 39], calculations of the neutron matter energy per particle at N^3LO show a small effect (of about -0.5 MeV) at saturation density for the potentials of our purview [24]. The Hartree-Fock contributions to the energy per particle of symmetric nuclear matter from the N^3LO 3NF are attractive and on the order of 7 MeV at saturation density [4]. The inclusion of 3NFs at N^3LO , however, necessitates a refitting of the c_D and c_E low-energy constants, which has not yet been performed and would likely result in a smaller change to the total energy per particle at saturation density. Most recently, evidence has been reported [40] that sub-leading terms in the 3NF may provide important contributions to the triton binding energy, as well as indications that similar conclusions apply in symmetric nuclear matter.

In the case of the N^2LO NN interaction with $\Lambda = 600$ MeV, small charge-symmetry-breaking effects have emerged in the fitting procedure. This is visible in Fig. 5, where the c_D - c_E trajectories which reproduce the experimental 3H and 3He binding energies are displayed. Allowing for charge-symmetry breaking, the values for c_E are -0.833 and -0.885 for 3H and 3He , respectively. The value shown in Table 2 is the average of these two. The error in the $A = 3$ binding energies, when the average value of c_E is used, is ~ 40 keV. We do not know at present the origin of this (small)

N ² LO	Λ (MeV)	c_D	c_E
	450	-0.326	-0.149
	500	-0.165	-0.169
	600	0.456	-0.859
N ³ LO	Λ (MeV)	c_D	c_E
	450	-0.24	-0.11
	500	0.0	-0.18
	600	-0.19	-0.83

Table 2: Values of the c_D and c_E low-energy constants obtained using the N²LO 3NF in conjunction with NN interactions of different orders, for several values of the cutoff Λ . These constants do not appear at NLO.

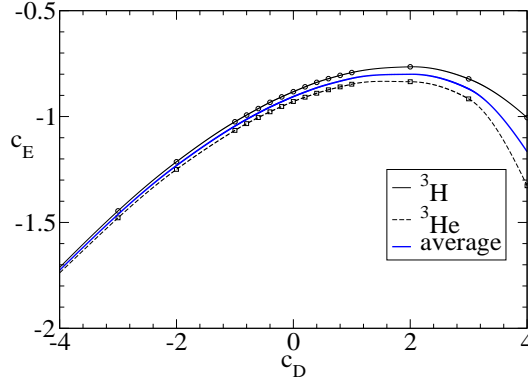


Figure 5: (Color online) c_E - c_D trajectories fitted to reproduce the experimental ${}^3\text{H}$ and ${}^3\text{He}$ binding energies in the case of the N²LO NN interaction plus 3NF with $\Lambda = 600$ MeV. The average curve (in blue) is also displayed.

charge-symmetry breaking effect.

6. Results and discussion

With the input as described in the previous sections, and in the light of the uncertainty considerations made above, we now move to nuclear and neutron matter.

Our results for the energy per particle as a function of the nuclear density are shown in Fig. 6 for symmetric nuclear matter. We note that the particle-particle ladder approximation employed in the present work is in good agreement with the perturbative results available at N³LO from Ref. [5] including up to third-order pp diagrams. In the left panel of Fig. 6, the shaded bands in yellow and red represent the spread of our complete calculations conducted at NLO and N²LO, respectively. The blue band is the result of a calculation that employs N³LO NN potentials together with N²LO 3NFs. In all cases shown, the cutoff is varied over the range 450-600 MeV. As noted before, the N³LO 3NFs and 4NFs are at present omitted, and the resulting convergence pattern gives an estimate on the theoretical uncertainty of the calculation (and not necessarily of the chiral EFT expansion per se). On the right-hand side of the figure, the individual curves corresponding to each

order and cutoff are displayed. We observe that at NLO the potentials constructed at lower cutoff scales do not exhibit saturation until very high densities. On the other hand, for the 600 MeV cutoff potential the 1S_0 partial wave (together with the 3S_1 partial wave) is sufficiently repulsive to enable saturation at a relatively smaller density. We observe that the convergence pattern for the low-cutoff ($\Lambda = 450 - 500$ MeV) potentials is significantly better than for the 600 MeV potential. Overall there is a large spread from cutoff variations both at NLO and N²LO beyond nuclear matter saturation density. Moreover, the bands at these two orders do not overlap, suggesting that their width is not a suitable representation of the uncertainty. Although the (incomplete) N³LO calculation reveals a strong reduction of the cutoff dependence, it is important to notice that an uncertainty of about 8 MeV remains at saturation density. While we do not expect much of a change in nuclear matter predictions from 4NFs [41, 42, 4], it is quite possible that the inclusion of N³LO 3NFs might reduce either the cutoff dependence or improve the convergence pattern. This will be an interesting subject for future investigations.

The results for neutron matter are presented in Fig. 7, where the left and right panels have the same meaning as in Fig. 6. Note that the range of densities under consideration is smaller for neutron matter in order to keep the Fermi momentum below the cutoff in all cases. We see a large spread at NLO for the largest densities considered, whereas the band has only moderate size at the next order and remains small for our N³LO calculation. Similar to what was observed in symmetric nuclear matter, the bands at NLO and N²LO do not overlap in neutron matter. In addition the N³LO band does not generally overlap with the N²LO band. Therefore, the variation obtained by changing the cutoff does not seem to provide a reliable representation of the uncertainty at the given order. A better way to estimate such uncertainty is to consider the difference between the predictions at two consecutive orders.

In Fig. 8 we present the results for the symmetry energy E_{sym} , which is defined as the strength of the quadratic term in an expansion of the energy per particle in asymmetric matter with respect to the asymmetry parameter α :

$$\bar{E}(\rho, \alpha) \approx \bar{E}(\rho, \alpha = 0) + E_{\text{sym}}\alpha^2 + \mathcal{O}(\alpha^4), \quad (6.1)$$

where $\bar{E} = E/A$ is the energy per particle and $\alpha = (\rho_n - \rho_p)/(\rho_n + \rho_p)$. The nearly linear behavior of $\bar{E}(\rho, \alpha)$ with α^2 has been confirmed by many microscopic calculations (see for instance Ref. [43] and more recently Refs. [44, 45]). It justifies the common approximation of neglecting powers beyond α^2 in the expansion above and thus defining the symmetry energy as the difference between the energy per particle in neutron matter and symmetric nuclear matter.

As mentioned in Sec. 2, systematic efforts are ongoing to set better empirical constraints on the symmetry energy, through both laboratory and astrophysical measurements. It is therefore important to have an understanding of the theoretical uncertainty affecting calculations of this quantity. The spread due to the change of the cutoff values in our NLO, N²LO, and N³LO calculations is represented by the three bands as before. As observed previously for symmetric matter, the spread due to cutoff variations remains large at N²LO, with some minimal overlap with the NLO band. The N³LO band reflects the large cutoff sensitivity previously observed in symmetric matter. Again, we conclude that the spread generated by changing the cutoff does not in general provide a reliable estimate of the theoretical uncertainty.

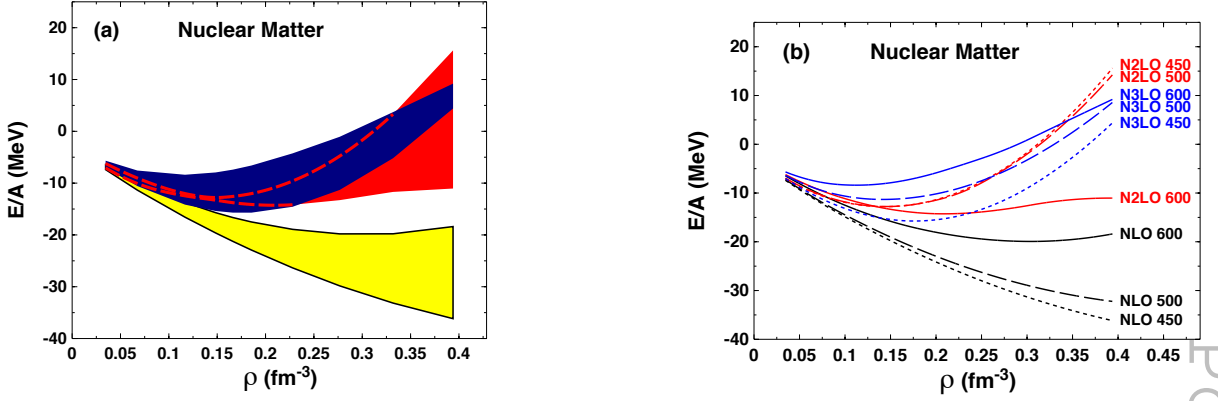


Figure 6: (Color online) Energy/nucleon (E/A) in symmetric nuclear matter as a function of density, ρ . Left frame: The yellow and red bands represent the uncertainties in the predictions due to cutoff variations as obtained in complete calculations at NLO and N^2 LO, respectively. The blue band is the result of a calculation employing N^3 LO NN potentials together with N^2 LO 3NFs. The dashed lines show the upper or lower limits of hidden bands. Right frame: predictions at the specified order and cutoff value.

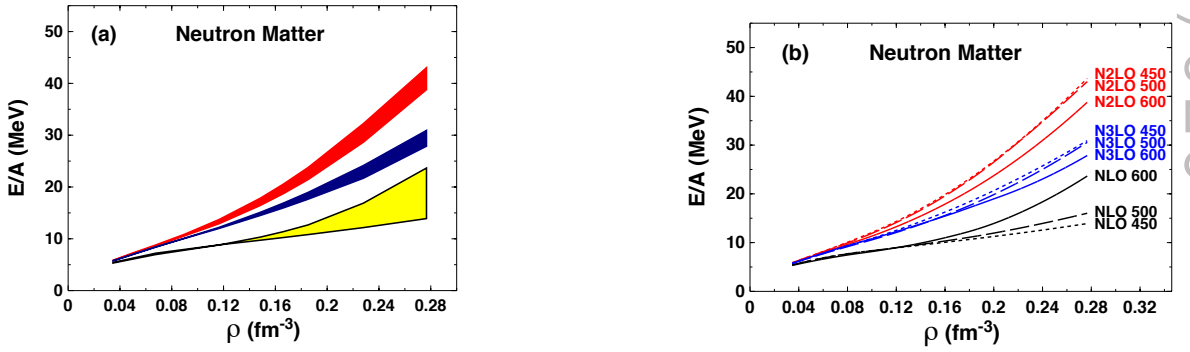


Figure 7: (Color online) As in Fig. 6 for pure neutron matter.

7. Conclusions and future plans

We have reported predictions for the energy per particle in symmetric nuclear matter and pure neutron matter, focusing on uncertainties related to order-by-order convergence. Compared to the consistent NLO and N^2 LO results for the EoS, which themselves exhibit relatively little overlap even for a large spread of momentum-space cutoffs $\Lambda = 450 - 600$ MeV, the results from employing N^3 LO two-nucleon and N^2 LO three-nucleon forces imply non-negligible uncertainties associated with missing higher-order terms in the chiral expansion. We find that the uncertainty associated with the cutoff variation is generally larger in symmetric nuclear matter than in pure neutron matter, but in the latter case we find that the results from one chiral order to the next have little overlap. This suggests that further systematic studies of the order-by-order convergence should be performed, together with variations in the resolution scale and low-energy constants,

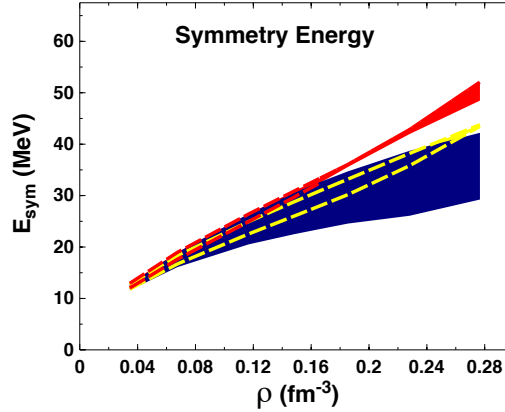


Figure 8: (Color online) The symmetry energy, E_{sym} , as a function of density, ρ . Meaning of bands and dashed lines as in the left panel of Fig. 6.

to accurately estimate the complete theoretical uncertainties in chiral EFT predictions of nuclear many-body systems. The two- and three-body potentials considered in the present work can serve as a basis for such future uncertainty estimates.

In the near future, we wish to make the analysis broader and more systematic. In particular, we plan to extend the present study by considering variations of the πN LECs within the uncertainty allowed by the πN analyses. For each set of LECs, we will construct a corresponding NN potential and consistently adjust the 3NF by refitting its parameters to the three-nucleon system.

Furthermore, it may be of interest to look at the 5th order ($N^4\text{LO}$) of chiral EFT. Based upon Ref. [46], a high-precision NN potential at $N^4\text{LO}$ will soon become available. Concerning three-nucleon forces at $N^4\text{LO}$, there are sizeable pion-exchange 3NFs [47] and, furthermore, a new set of ten 3NF contact interactions appear [48]. Thus, the fifth order may still play an important role in the chiral expansion.

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