

Electroweak structure of light nuclei

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Abstract. We review the most recent studies performed within the chiral effective field theory framework of electroweak processes, among which, in particular, the electromagnetic structure of light nuclei, the weak muon capture on deuteron and ${}^3\text{He}$, and the weak proton-proton capture reaction at energies of astrophysical interest. The presented results will be compared with the available experimental data and with those obtained within the conventional phenomenological approach.

1. Introduction

The traditional picture of a nucleus consisting of nucleons interacting among themselves and with external electroweak (EW) probes can be extensively tested by comparing theoretical and experimental results for many observables as, for instance, form factors, magnetic moments, and capture cross sections. Here we focus on the $A = 2, 3$ charge and magnetic form factors and static properties, the muon capture on deuteron and ${}^3\text{He}$, and the proton-proton (pp) weak capture reaction ($p + p \rightarrow d + e^+ + \nu_e$). The conventional approach, where realistic models for the nuclear interactions and currents are constructed phenomenologically (for a review see Ref. [1]), has proven to be extremely successful in reproducing the many experimental data, both in the electromagnetic and weak sector (for the observables of interest here see Refs. [2, 3, 4, 5]). However, it presents an “original sin”, i.e., no connection with the underlying theory of strong interaction, quantum chromodynamics (QCD), is clearly visible. An alternative approach now available is the so-called chiral effective field theory (χEFT) one. In a very schematic view, χEFT can be seen as a formulation of QCD in terms of effective degrees of freedom suitable for low-energy nuclear physics: pions and nucleons. The symmetries of QCD, in particular its (spontaneously broken) chiral symmetry, severely restrict the form of the interactions of nucleons and pions among themselves and with external EW fields, and make it possible to expand the Lagrangian describing these interactions in powers of Q/Λ_χ , where Q is pion momentum and $\Lambda_\chi \sim 700$ MeV is the chiral-symmetry-breaking scale. As a consequence, classes of Lagrangians emerge, each characterized by a given power of Q/Λ_χ and each involving a certain number of unknown coefficients, the so-called low-energy constants (LECs). While these LECs could in principle be determined by theory (for instance, in lattice QCD calculations), they are in practice constrained by fits to experimental data. The potentials and currents derived within this framework have power-law behavior for large momenta, and need to be regularized. This is accomplished in practice by introducing a momentum-cutoff function. Among the great advantages of the χEFT framework, the two following ones are here of interest: (i) the possibility of deriving nuclear EW currents consistently with the nuclear interaction, and (ii) the possibility of setting a hierarchy among the different contributions, both for the interactions and the



currents. In fact, it is well known that χ EFT can justify *a priori* the empirical observation that the contribution of three-nucleon interactions (TNIs) to nuclear structure is far less significant than that of the two-nucleon (NN) force. Furthermore, the χ EFT power counting allows to estimate and in principle improve the theoretical uncertainty of the calculation.

The idea of using χ EFT to derive the nuclear EW transition operators was first implemented by Park *et al* [6] in the nineties for the electromagnetic (EM) current and charge operators and few years later [7] for the weak axial current and charge operators. The so-called heavy-baryon chiral perturbation theory (HB χ PT) approach was used, where the baryons are treated as heavy static sources, and the perturbative expansion is performed in terms of the involved momenta over the baryon mass. Since at that time the χ EFT nuclear potentials were not yet as accurate as the conventional phenomenological ones, the first χ EFT calculations of EW observables were performed within a “hybrid” approach, in which nuclear wave functions were obtained from phenomenological potentials [5, 7]. Only very recently [8, 9, 10], these χ EFT weak operators have been used to study weak processes which involve few-nucleon systems in conjunction with modern accurate χ EFT potentials, in particular the NN potential derived at next-to-next-to-next-to leading order (N3LO) by Entem and Machleidt [11], augmented, when needed, by the TNI obtained at next-to-next-to leading order (N2LO), in the version of Ref. [12]. The cutoff Λ is varied between 500 and 600 MeV.

Few years ago, the problem of deriving the EM current and charge operators in χ EFT has been revisited by Pastore *et al* [13] and, in parallel, by Kölling *et al* [14]. Pastore *et al* have used time-ordered perturbation theory (TOPT) to calculate the EM transition amplitudes, which allows for an easier treatment of the so-called reducible diagrams than the HB χ PT approach. On the other hand, Kölling *et al* have used the method of unitary transformation, the same one used to derive the chiral potentials mentioned above. We will focus here only on the work of Pastore *et al*, but we would like to remark that the results obtained by these two groups, although with different methods, are in good agreement with each other. To be noted that TOPT has not yet been applied to derive the weak axial current and charge operators at N3LO, but work along this line is currently underway. Therefore, the results presented here have been obtained within the model of Pastore *et al* for the EM observables and that of Park *et al* for the weak sector. A summary description of the two models can be found in Ref. [15]. Here we only notice that (i) the leading-order (LO) contribution is that due to the one-body current operator, obtained from a non-relativistic reduction of the covariant single-nucleon current. (ii) The NN EM currents derived by Pastore *et al* have been found significantly different from those of Park *et al*. The largest differences arise at N3LO for the box diagrams and the contact terms. In particular, the contact terms of Park *et al* are much simpler than those of Pastore *et al* and can be written as sum of two terms, one isoscalar and one isovector, with two different LECs in front, usually fitted to reproduce the isoscalar and isovector combination of the $A = 3$ magnetic moments [8, 9]. We refer to Ref. [10] for the LECs fitting procedure of Pastore *et al*. (iii) At N2LO, the weak axial current operator presents a contact three-nucleon contribution whose LEC can be related with the LEC c_D entering one of the two contact terms present in the TNI at N2LO, via the relation $d_R = \frac{m}{\Lambda_\chi g_A} c_D + \frac{1}{3} m (c_3 + 2c_4) + \frac{1}{6}$, where g_A is the single-nucleon axial coupling constant, c_3 and c_4 are LECs of the πN Lagrangian, already part of the chiral NN potential at NLO, and Λ_χ MeV is the the chiral-symmetry-breaking scale mentioned above. Therefore, it has become common practice to fit c_D (and c_E — the other LEC entering the N2LO TNI) to the triton binding energy and the Gamow-Teller matrix element in tritium β -decay. The values obtained in this way for c_D and c_E are listed in Refs. [8, 16], and they have been used in Ref. [8] to study the muon capture on deuteron and ^3He , in Ref. [9] to investigate the pp weak capture, in Ref. [17] to study $A = 3$ and 4 elastic scattering observables, as cross sections and analyzing powers, and in Ref. [16] to calculate the nuclear matter equation of state in many-body perturbation theory.

2. Results

In this section we review some of the results on the electromagnetic structure of $A = 2, 3$ nuclei, extensively studied in Ref. [10]. Then we report the results for the muon capture on deuteron and ${}^3\text{He}$ [8], and for the pp reaction [9].

2.1. The electromagnetic structure of $A = 2, 3$ nuclei

The static properties of $A = 2, 3$ nuclei are summarized in table 1, where we present the χEFT results for the deuteron r.m.s. radius and quadrupole moment, and the charge and magnetic radii for the $A = 3$ nuclei. The experimental data are also reported. To be noticed that the deuteron and $A = 3$ magnetic moments results are not listed, since these observables are used to fit the LECs entering the EM current operators at N³LO [10]. The theoretical uncertainties in table 1 are due to the cutoff dependence and the fitting procedure. By inspection of the table, we can conclude that the static properties of the $A = 2, 3$ nuclei are nicely reproduced. It should be noticed that within the phenomenological approach, based on the Argonne v_{18} potential [18] (AV18), the quadrupole moment, calculated to be 0.275 fm^2 , is found in significant disagreement with the experimental value.

Table 1. Deuteron r.m.s. radius (r_d) and quadrupole moment (Q_d), and ${}^3\text{H}$ and ${}^3\text{He}$ charge (r_c) and magnetic (r_m) radii. The corresponding experimental values are also reported.

	Theory	Experiment
r_d [fm]	1.972 ± 0.004	1.9733 ± 0.0044
Q_d [fm ²]	0.2836 ± 0.0016	0.2859 ± 0.0003
$r_c({}^3\text{He})$ [fm]	1.962 ± 0.004	1.959 ± 0.030
$r_c({}^3\text{H})$ [fm]	1.756 ± 0.006	1.755 ± 0.086
$r_m({}^3\text{He})$ [fm]	1.905 ± 0.022	1.965 ± 0.153
$r_m({}^3\text{H})$ [fm]	1.791 ± 0.018	1.840 ± 0.181

The deuteron $B(q)$ structure function and magnetic form factor, together with the $A = 3$ magnetic form factors, and their isoscalar and isovector combinations, are reported in figure 1. Besides the fully consistent χEFT results, we present also the results obtained within the “hybrid” χEFT approach, where the nuclear wave functions are obtained from the AV18 potential for the deuteron and the AV18 augmented with the Urbana IX (UIX) TNI [19] for the $A = 3$ nuclei. From the inspection of the figure, we can conclude that the χEFT calculation is in agreement with the experimental data in a range of momentum transfer q much larger than one would naively expect (up to 1-2 times the pion mass). On the other hand, this χEFT calculation is unable to reproduce the first diffraction region of the $A = 3$ magnetic form factors. This problem is also present in the hybrid calculation, as well as in the phenomenological one (see for instance Ref. [2, 3]).

2.2. Muon capture on $A = 2, 3$ nuclei

The muon capture reactions here under consideration are $\mu^- + d \rightarrow n + n + \nu_\mu$ and $\mu^- + {}^3\text{He} \rightarrow {}^3\text{H} + \nu_\mu$, for which we are interested in the capture rate in the doublet hyperfine initial state (Γ^D) and in the total capture rate (Γ_0), respectively. The χEFT results of Ref. [8] are summarized in table 2, where the value for Γ^D obtained retaining in the final nn scattering state only the 1S_0 partial wave is also shown. The results of the table can be summarized as $\Gamma^D = (399 \pm 3) \text{ sec}^{-1}$ and $\Gamma_0 = (1494 \pm 21) \text{ sec}^{-1}$. The errors are due to cutoff dependence, the uncertainty

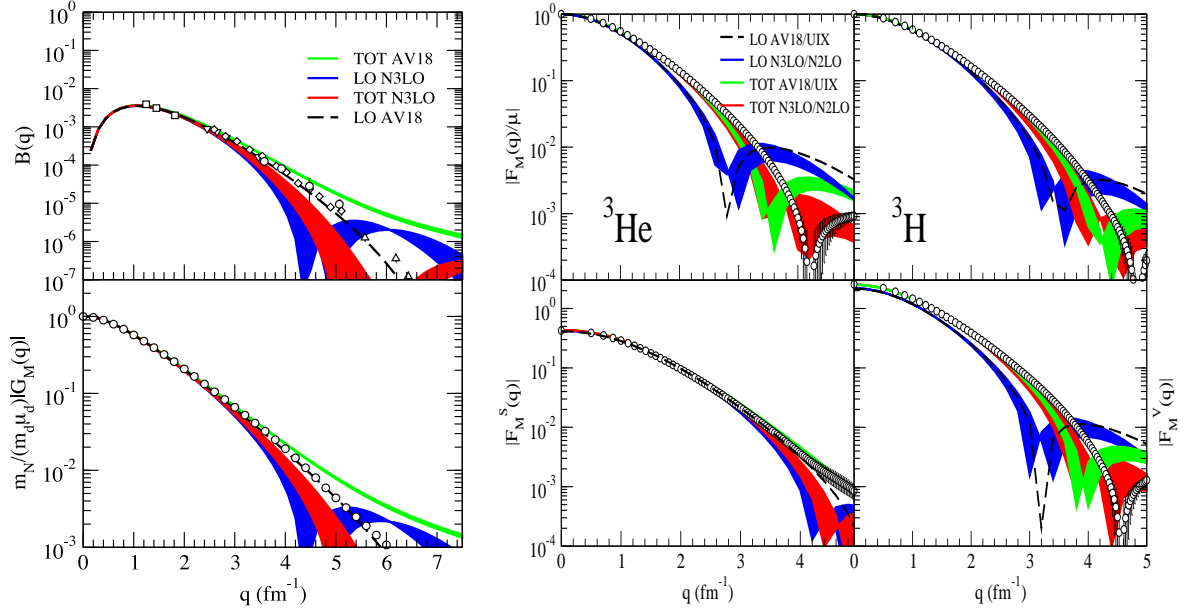


Figure 1. (Color online) The deuteron $B(q)$ structure function and magnetic form factor (on the left), and the $A = 3$ magnetic form factors, together with their isoscalar and isovector combinations (on the right), obtained at leading order (LO) and with inclusion of current operators up to N3LO (TOT), compared with the experimental data. The curves labelled “AV18” (or “AV18/UIX”) have been obtained within the hybrid χ EFT approach using the AV18 (and Urbana IX [19] TNI for $A = 3$) phenomenological potentials. The curves labelled “N3LO” (or “N3LO/N2LO”) have been obtained instead using the N3LO NN potential [11] (and N2LO [12] TNI for $A = 3$).

inherent the fitting procedure of the LEC d_R , and radiative corrections [8]. The experimental data for Γ^D are affected at present by quite large uncertainties and no significant comparison between theory and experiment can be made. However, the ongoing experiment conducted by the MuSun collaboration at the Paul Scherrer Institute (PSI) is expected to measure Γ^D with a 1% precision. The theoretical prediction for Γ_0 , instead, is in very nice agreement with the remarkably precise experimental determination of Ref. [20], $(1496 \pm 4) \text{ sec}^{-1}$. To be noticed that muon capture on ${}^3\text{He}$ can proceed also through the two- and three-body breakup channels, i.e. $\mu^- + {}^3\text{He} \rightarrow n + d + \nu_\mu$ and $\mu^- + {}^3\text{He} \rightarrow n + n + p + \nu_\mu$, with a 20% and 10% of branching ratio, respectively. No χ EFT calculation for these processes has ever been performed, but the first study within the conventional phenomenological approach has been performed in Ref. [21], although using only the one-body current operator. Work on these reactions with a model for the weak axial current which includes also NN contributions is currently underway.

2.3. Weak proton-proton capture

The astrophysical S -factor for pp weak capture is defined as $S(E) = E \exp(2\pi\eta) \sigma(E)$, where $\eta = \alpha/v_{\text{rel}}$, α being the fine structure constant and v_{rel} the pp relative velocity, and $\sigma(E)$ is the pp cross section at the center-of-mass energy E . Typically, $S(E)$ is given as a Taylor expansion around $E = 0$, and the coefficients of the expansion, $S(0)$, $S'(0)$ and $S''(0)$, are the quantities of interest [22]. Alternatively, the energy dependence of $S(E)$ can be made explicit by calculating

Table 2. Doublet capture rate for muon capture on deuteron and total capture rate for muon capture on ${}^3\text{He}$ obtained at leading order (LO) and with inclusion of current operators up to N2LO (TOT). The results calculated with the different values of two cutoff Λ are reported. The theoretical uncertainties are due to the fitting procedure of the LEC d_R .

	$\Gamma^D({}^1S_0)$ [sec^{-1}]	Γ^D [sec^{-1}]	Γ_0 [sec^{-1}]
LO - $\Lambda = 500$ MeV	238.8	381.7	1362
LO - $\Lambda = 600$ MeV	238.7	380.8	1360
TOT - $\Lambda = 500$ MeV	254.4 ± 9	399.2 ± 9	1488 ± 9
TOT - $\Lambda = 600$ MeV	255 ± 1	399 ± 1	1499 ± 9

it directly. Note that the Gamow peak for the pp reaction is at $E = 6$ keV in the Sun, and it becomes of about 15 keV in larger mass stars. Therefore, we have studied the pp reaction in the energy range $E = 3 - 100$ keV. Two ingredients are essential in the calculation: (i) the initial pp scattering state is calculated using the χEFT N3LO NN potential [11], augmented not only by the Coulomb interaction, but also by the higher order electromagnetic terms, due to two-photon exchange and vacuum polarization. The additional distortion of the pp wave function, induced primarily by vacuum polarization, reduces $S(0)$ by $\sim 1\%$ [22, 9]. (ii) To have the correct energy-dependence of the S -factor up to $E = 100$ keV, we have included, in addition to the S -wave (the 1S_0 partial wave), all the P -wave channels (3P_0 , 3P_1 , and 3P_2), and we have retained the explicit dependence on the momentum transfer via a standard multipole expansion. More details can be found in Ref. [9]. Finally, we recall that the model for the weak current is the same used in the successful studies of the muon capture reactions presented above. The S -factor at zero energy is found to be $S(0) = (4.030 \pm 0.006) \times 10^{-23}$ MeV fm 2 , with a P -wave contribution of 0.020×10^{-23} MeV fm 2 . The theoretical uncertainty is due again to the fitting procedure of the LECs and to the cutoff dependence. This value is $\simeq 1\%$ larger than the value reported in the literature [22], exactly due to the P -wave contributions. Note that within the potential model approach, using the AV18 potential, we have found $S(0) = (4.033 \pm 0.003) \times 10^{-23}$ MeV fm 2 , with a P -wave contribution of 0.033×10^{-23} MeV fm 2 , in nice agreement with the χEFT predictions, and with the value presented in Ref. [22] when the P -wave contributions are removed.

The energy dependence of $S(E)$ is shown in figure 2. The S - and $(S + P)$ -wave contributions are displayed separately, and the theoretical uncertainty is included—the curves are in fact very narrow bands. As expected, the P -wave contributions become significant at higher values of E . The study of the effects of these P -wave contributions, as well as the new determination of $S(0)$ on the stellar structure and evolution is currently underway.

3. Conclusions and outlook

We have presented the most recent studies of EW observables involving light nuclei within the χEFT framework. The considered observables are the $A = 2, 3$ electromagnetic form factors and static properties, the muon capture on deuteron and ${}^3\text{He}$ and the proton weak capture by proton. The theoretical predictions have been found in a good agreement with the available experimental data, except for the $A = 3$ magnetic form factors at high values of momentum transfer, in a region, though, well beyond the applicability range of χEFT . The calculation of the pp astrophysical S -factor has been performed, for the first time, retaining the P -wave contributions, which have been found of the order of $\sim 1\%$. The theoretical uncertainty estimated within the χEFT framework has been found smaller than 1% . To be noticed that the radiative proton-deuteron and the weak proton- ${}^3\text{He}$ capture reactions, also relevant in astrophysics, are at reach within the present framework, and work along these lines is currently underway.

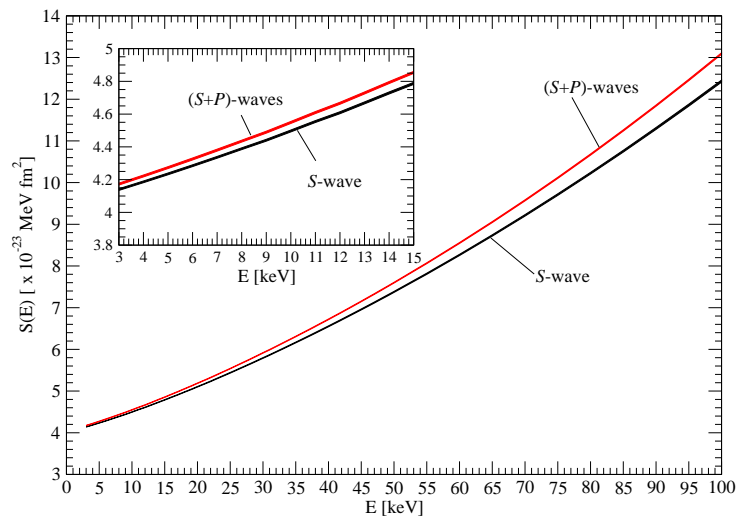


Figure 2. (Color online) The astrophysical S -factor for $E = 3 - 100$ keV. The S - and $(S + P)$ -wave contributions are displayed separately. In the inset, $S(E)$ is shown in the range 3–15 keV.

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