

Geometrical scale-factor stabilization of square cavity ring laser gyroscopes

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Abstract—We present the experimental activity on the GP2 ring laser gyro. GP2 was built for developing advanced stabilization techniques of the ring laser square resonator shape, i.e. the ring laser geometrical scale-factor. A method based on optical interferometry has been developed for canceling the deformations of the resonator. The method is based on the measurement and stabilization of the absolute length of the cavity perimeter and of the resonators formed by the opposite cavity mirrors. The optical frequency reference in the experiment is an iodine-stabilized He-Ne laser, with a relative frequency stability of 10^{-11} . The measurement of the absolute length of the two resonators has been demonstrated up to now on a test bench. We discuss the present performances of GP2 as a gyroscope as well as its next geometrical stabilization by means of the developed method.

I. INTRODUCTION

Ring lasers gyroscopes (RL) are inertial sensors able to measure absolute rotations [1]. If they are placed at rest in a ground-located laboratory, the measured rotation is that of our planet $\vec{\Omega}_{\oplus}$. Ultra sensitive, large frame RLs are promising detectors for ground based tests of General Relativity (frame-dragging). Earth's rotation frame-dragging or Lense-Thirring effect consists in a dragging of the local inertial frame of reference caused by the perturbation of the local metrics in the proximity of a spinning massive body like Earth [2]. This implies that the Earth rotation rate measured against the "fixed-stars" inertial frame differs from the rotation rate measured in the laboratory frame. On the Earth surface, relativistic effects are of the order of 1 part in 10^9 of $\vec{\Omega}_{\oplus}$. Frame dragging effects have been experimentally observed up to now only in space-experiments using orbiting satellites [3] [4].

The GINGER (Gyroscopes IN General Relativity) experiment [5] [6] aims at measuring the Lense-Thirring effect for the first time in a ground-based laboratory, by using an array of large RLs made solid with the ground. The requirements can be clearly understood by examining the sensor response. The basic setup of a RL is made up of a stable ring optical cavity along which an active medium, typically a He-Ne mixture, is present (Fig. 1). Two laser beams are generated and propagate in opposite directions along the loop. By mixing on a photodiode the beams exiting the cavity in the opposite

directions, a Sagnac beat frequency is measured [7]:

$$f_S = \frac{4\vec{A} \cdot \vec{\Omega}}{\lambda P}, \quad (1)$$

where $\vec{\Omega} = \vec{\Omega}_{\oplus} + \vec{\Omega}'$ is the rotation relative to the local Lorentz inertial frame (being $\vec{\Omega}'$ any correction term), \vec{A} is the area vector enclosed by the ring optical path P and λ is the wavelength of the laser. The sensitivity limit of a RL is given by the shot-noise:

$$\Omega_{sn} = \frac{vP}{4AQ} \sqrt{\frac{hf}{P_{out}T}}, \quad (2)$$

where v is the velocity of the laser beam along the cavity, Q is the quality factor of the resonator, h the Planck constant, P_{out} the detected optical power and T the measuring time. From equation 1 two important features follow: the dependence of Sagnac frequency f_S on the laser path geometry via the scale factor $k_S = 4A/\lambda P$, and the scalar nature of the sensor, being measured only the projection of the velocity vector $\vec{\Omega}$ along the area vector \vec{A} . The development of a high sensitivity RL requires:

- a large frame structure. To increase the size of the ring cavity, in fact, implies to increase the signal to noise ratio (SNR), being the signal proportional to the ratio A/P via the scale factor, and the shot noise of the sensor proportional to P via the quality factor Q .
- a high-Q resonator ($> 10^{11}$ and higher). Low-loss 'five-9s quality' super/mirrors must be used.
- a multi-axial system of RL, in order to reconstruct the modulus of the Earth rotational vector and compare the Earth rotation rate measured locally with the one provided by IERS (International Earth Rotation and Reference Systems Service). Otherwise, an absolute calibration of the RL orientation at a level of 0.1 nrad would be needed.
- to reduce the instrumental drift in the measurement of rotation rate to less than $\Omega' / \Omega_{\oplus}$. This needs a long-term strict control on the fluctuation of laser active medium, cavity geometry and, in a RLs array, of relative dihedral angles.
- to reduce all the sources of Earth-surface and environmental noise, installing the detector in a very stable geological environment, well coupled to the

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solid rock, in a low environmental noise laboratory, possibly located underground.

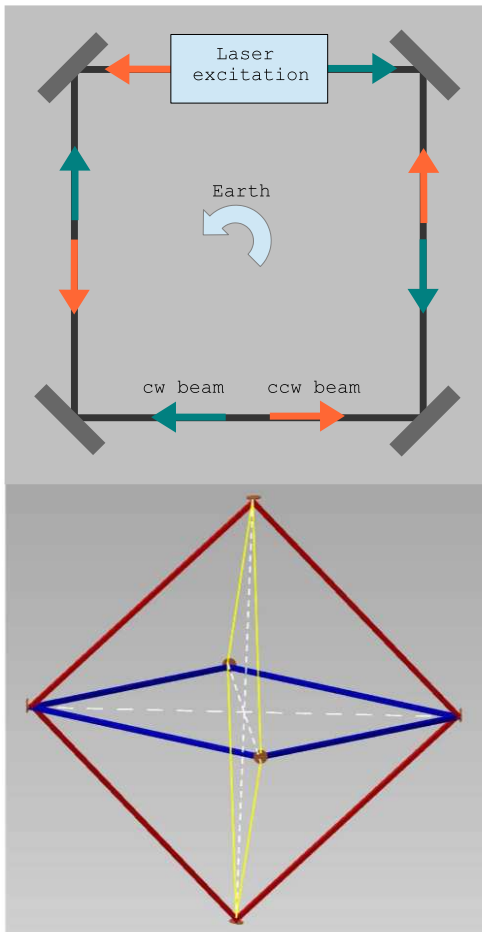


Fig. 1. Ring laser gyroscope. Above: setup of a square RL. Below: triaxial sensor made of 6 mirrors. Each couple of opposite mirrors is shared between two rings.

To this day, the best RL is the Grössring G, located at the Geodätisches Observatorium in Wettzell, Bavaria [8] [9]. It has achieved a resolution better than 5×10^{-13} rad/s with an integration time of few hours, becoming of geodetic interest for measuring short-term fluctuation, with periods of hours to days, in Earth rotation. This stability record is mainly due to the strong passive stabilization of the optical cavity. G, in fact, is a semi-monolithic device made in Zerodur, a glass ceramic with an especially small thermal expansion, high mechanical stability and consistency of shape and length. Four bars are rigidly connected to a base plate forming the edges of a square 4 m length in side; spherical supermirrors are attached to the face sides of the bars by molecular adhesion, ensuring a stable vacuum seal. It is kept in an underground controlled room and equipped with an active control of the cavity perimeter that stabilizes the circulating laser beams frequency against an optical frequency reference. Albeit G resolution is very close to that required to detect the relativistic effects on rotation, its design can't be upscaled. The next-generation RL able to detect tiny effects, as Lense-Thirring effect, must have a heterolithic multi-axial design equipped with a precise diagnostic system of laser beam path deformation, in order to stabilize the scale

factor k_S better than 10^{-10} . Recent studies and experimental activities made in this context on the middle-size heterolithic RL G-Pisa [10], have motivated the of the GINGER proposal. It will consist in a triaxial system of large square heterolithic RLs arranged in a octahedral structure (figure 1). A suitable location for this device could be the underground facility of LNGS (Laboratori Nazionali del Gran Sasso - Italy). To accomplish GINGER's goal needs a control of the systematic errors related to the fluctuation of the cavity geometry and the laser active medium parameters. We developed a set of spectroscopic diagnostic of the active medium and an off-line denoising method based on the Kalman filter approach. The denoising method subtracts the systematic effects induced by the non-linearity of the RL dynamics from the raw Sagnac data [11] [12].

II. RING CAVITY GEOMETRY CONTROL

The measure of Lense-Thirring effect in a ground-based laboratory requires a stabilization of the scale factor k_S better than 10^{-10} . This implies an accuracy on the mirror positioning better than 1 nm. It can be shown [13] that for a single square RL, the accuracy request on the mirror positions can be reduced if the absolute length of the diagonal cavities is stabilized in addition to the perimeter one. We showed that if the lengths of the two diagonals are locked to the same value, the perturbations to the mirror positions affect only quadratically the ring laser scale factor. This constraint reduces the mirror position fluctuation at a level of 1 part in 10^{10} , even if the two lengths are stabilized to values that differs at a micrometric scale. These results motivated the design of GP2 [6], an intermediate prototype of GINGER specifically devoted to test the active control strategies and, in particular, to implement the length stabilization of the diagonal resonators by means of optical interferometry.

III. GP2 RING LASER

GP2 is the seed device for the next generation heterolithic active-stabilized RLs. The optical setup consists in four supermirrors each one contained in a steel holder placed at the corner of a granite support fixed on a concrete base. Steel pipes connected by the mirror holders define the vacuum chamber that encloses the optical path of the circulating beams along a square loop. High finesse of the two diagonal resonators is guaranteed by a special mirror coating ensuring a reflectivity of about 99.9% at normal incidence, in addition to a reflectivity $> 99.999\%$ at 45 deg angle of incidence.

The vacuum chamber, that includes also the crossed diagonals optical path, and is filled with He-Ne. The active medium for the ring laser operation is excited by means of a capacitive radiofrequency discharge, applied through a pyrex capillary located in the middle of one side of the ring.

Fig. 2 shows a drawing of GP2 (above) and its installation in the clean room at INFN Pisa laboratories. The granite slab supporting the laser cavity is oriented with its normal axis parallel to the Earth's rotation axis in order minimize the orientation errors. The four mirrors holders (Fig. 5) are placed at the corner of a square granite slab and the vacuum chamber encloses the beam optical path along a square loop 1.60 m length in side. Fig. 3 shows a preliminary Sagnac spectrum; a Sagnac frequency of 184 Hz has been observed, as expected.

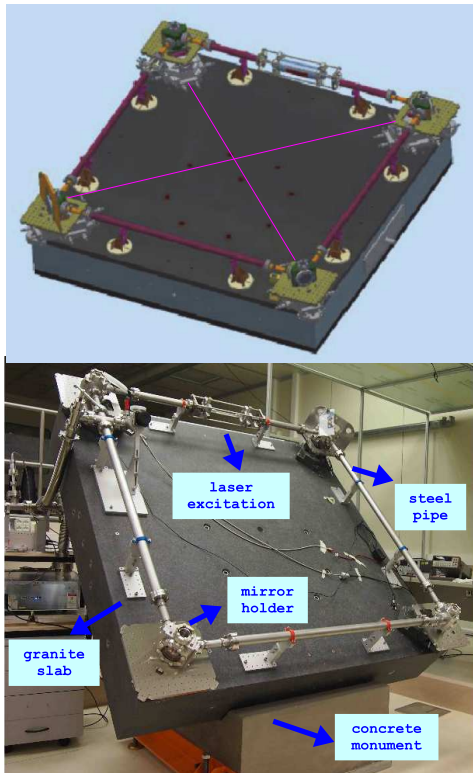


Fig. 2. GP2 ring laser gyroscope. In the lower picture is sketched the sensor installed inside the clean room of INFN Pisa section.

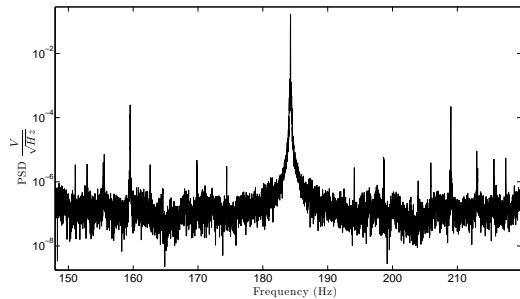


Fig. 3. Power spectral density of GP2 data. The Sagnac response peak at 184 Hz is observed.

To check the quality factor $Q = 2\pi f\tau$ of the laser cavity we made a ring-down time τ measurement of the laser by short-cutting the discharge capacitor. In Fig. 4 we report the laser intensity decay trace acquired by the oscilloscope. Fitting to data the exponential function $I = I_0 e^{-t/\tau} + C$, where I_0 is the initial intensity and C is a constant, we have obtained a measure for the ring-down time $\tau = 154.4 \pm 0.5 \mu\text{s}$. This corresponds to a quality cavity factor $Q = 4.6 \times 10^{11}$. The expected shot noise level is $\Omega_{sn} \sim 1(\text{nrad/s})/\sqrt{\text{Hz}}$.

The slab whereon the holders are mounted is made of precise black granite, a rock well suited for metrology application for his long term thermal and dimensional stability, high flatness accuracy, high bending strength and insensitivity to mechanical overloading. It has been machined with a precision better than $10 \mu\text{m}$ to guarantee a preliminary well positioning of the corner mirrors. The mirrors are accessible through big optical transparent windows installed parallel to them on the

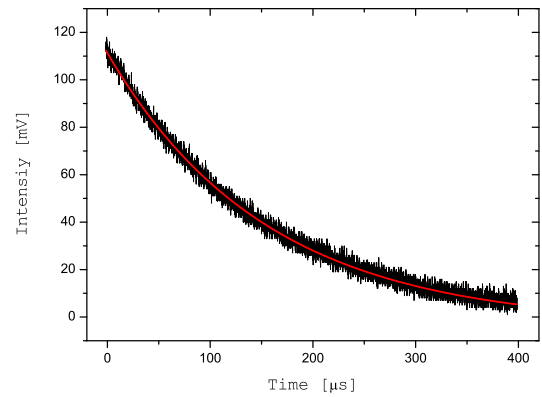


Fig. 4. Ring-down time measurement of GP2 laser cavity. Black line: trace of laser intensity decay acquired by the oscilloscope. Red line: exponential fit of data points. Fit results: $\tau = 154.4 \pm 0.5 \mu\text{s}$; $I_0 = -3.35 \pm 0.10 \text{ mV}$; $C = 114.63 \pm 0.14 \text{ mV}$.

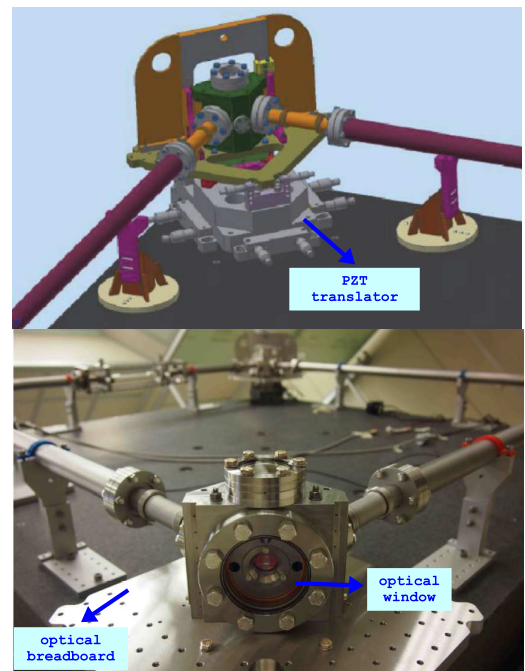


Fig. 5. GP2 Mirror holders. Above: drawing of the holder design; the piezo system is shown. Middle: picture of the holders installed on the granite slab. The big optical transparent window allowing the optical access to corner mirror is shown. Below: detail of holder. The optical setup needed to combine the circulating beams is also shown. The combiner is mounted on a small breadboard and consists in two high-reflection dielectric mirrors 0.5 inch in size. For the detection of the beat frequency, a photodiode integrated with an interference filter is used. The three allowed directions (monobeams exiting the cavity and external laser source entering in the diagonal resonator) are pointed out.

holders. The window allows the circulating beams to exit the cavity and to be monitored; an external optical setup detects the beat frequency. As showed in the lower part of Fig.5, it is possible to inject an external laser probe into the diagonal resonators.

In [13] we defined the eigenvectors basis of the cavity deformations, identified the rigid body motion of the cavity and then classified the residual 6 cavity deformation modes. These correspond to the minimum number of degrees of freedom to

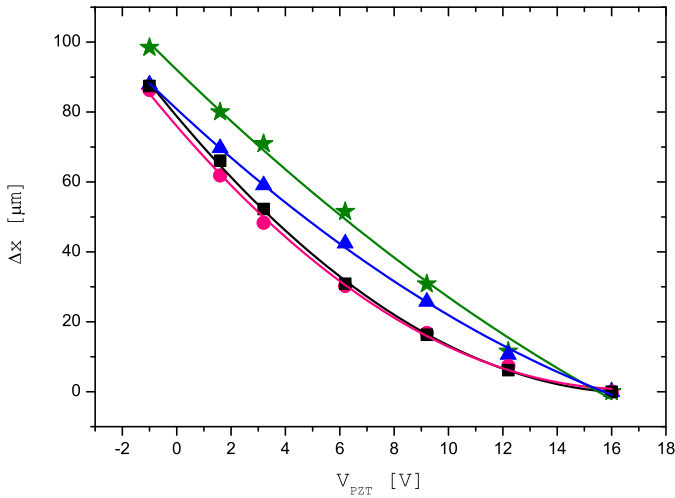


Fig. 6. PZT calibration data. The displacement data induced by an applied voltage are plotted for each 1-axial piezo moving the mirror holder along the ring cavity diagonal. Positive variation means displacement toward the center of the ring. A second order polynomial $\Delta x = A V_{PZT} + B V_{PZT}^2$ is fitted to data. From top to bottom: corner 4 (green line), $A_1 = (-9.3 \pm 0.2) \mu\text{mV}^{-1}$, $B_1 = (0.271 \pm 0.013) \mu\text{mV}^{-2}$; corner 3 (blue line), $A_2 = (-9.0 \pm 0.3) \mu\text{mV}^{-1}$, $B_2 = (0.269 \pm 0.018) \mu\text{mV}^{-2}$; corner 1 (black line), $A_3 = (-7.2 \pm 0.3) \mu\text{mV}^{-1}$, $B_3 = (0.132 \pm 0.017) \mu\text{mV}^{-2}$; corner 2 (magenta line), $A_4 = (-7.5 \pm 0.5) \mu\text{mV}^{-1}$, $B_4 = (0.10 \pm 0.04) \mu\text{mV}^{-2}$.

control for implementing a rigid constraint. GP2 has 6 piezoelectric transducers moving the mirrors holders: one 3-axial, and three 1-axial along the diagonals. In Fig. 7 is shown the measured frequency response of a 1-axial actuator. The response is flat up to about 80 Hz, where a resonance appears. This sets the limit of the control bandwidth to about few tens of Hz.

Each piezo stage has a dynamic range of $80 \mu\text{m}$. To have an estimate of the displacement response, we made a calibration of the 1-axial translators by measuring the displacement induced by an applied voltage. The PZT calibration data are plotted in Fig. 6. A second order polynomial fit provides a displacement constant mean value of $(7.4 \pm 0.6) \mu\text{mV}^{-1}$ for the mirrors mounted on the north side of the RL, and $(9.2 \pm 0.4) \mu\text{mV}^{-1}$ for the mirrors mounted on the south side.

A. Measurement principle

To stabilize the absolute length of a square RL diagonal resonators with respect to an interrogating high-stability laser we worked out an interferometric metrology technique and we tested it on two Fabry-Perot cavities simulating the ring diagonals on an optical bench. The technique we used is based on an accurate frequency measurement of the resonant longitudinal mode and an univocal determination of the interference order [14].

The laser source is a 10 mW diode laser emitting at 633 nm. A high spectral purity is gained referring it to an optical reference frequency provided by a 100 μW He-Ne laser frequency stabilized on the saturated absorption line R-127 11-5 of Iodine. The relative Allan deviation of the laser frequency has a minimum at 100 s at the level of 10^{-11} .

This is achieved implementing a light amplifier based on injection-locking. Fig. 8 shows the optical scheme that will be

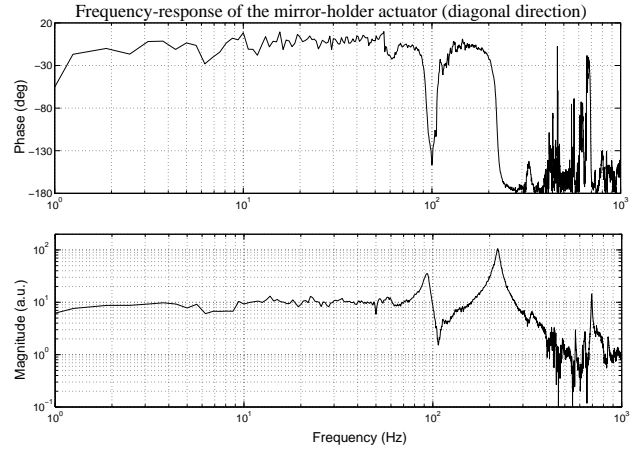


Fig. 7. Measured frequency response for the mirror translator shown in Fig. 5. The piezo transducers has been excited with a random noise and the mirror displacement has been acquired by means of laser based proximity sensor (resolution $\sim 100 \text{ nm}$)

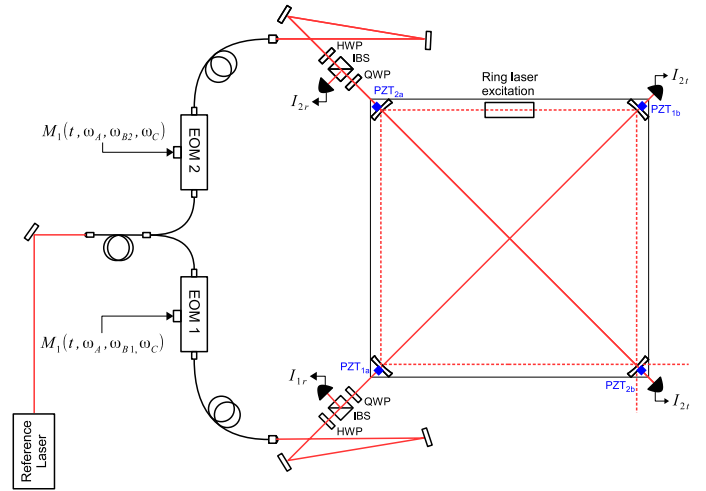


Fig. 8. The laser source block represents the injection locking setup. EOM: Electro-Optic Modulator. PBS: Polarizing Beam Splitter. HWP: Half Wave Plate. QWP: Quarter Wave Plate. PZT: Piezoelectric Transducer. VCO: Voltage Controlled Oscillator.

used on GP2. The frequency stabilized laser is split in two parts and modulated by two electro-optic modulators driven by a modulation signal $M_{i=1,2}(t)$

$$M_i(t) = \alpha \sin \omega_A t + \beta \sin (\omega_{B_i} + \Delta \sin \omega_C t) \quad (3)$$

where ω_A is the modulation providing the Pound-Drever-Hall signal for carrier lock; ω_{B_i} , is the modulation allowing us the detection of the i -th dynamical cavity resonance; ω_C is a frequency dithering applied for shifting the FSR detection in the kHz frequency range.

The electronic apparatus for the GP2 diagonals length stabilization is shown in Fig. 9. It is based on the multi-frequency digital lock-in amplifier (Zürich Instruments UHF2). It allows us to digitally synthesize the phase modulations and to process the two beams intensities I_{1r} and I_{2r} reflected from the two cavities. For each resonator, two phase error signals

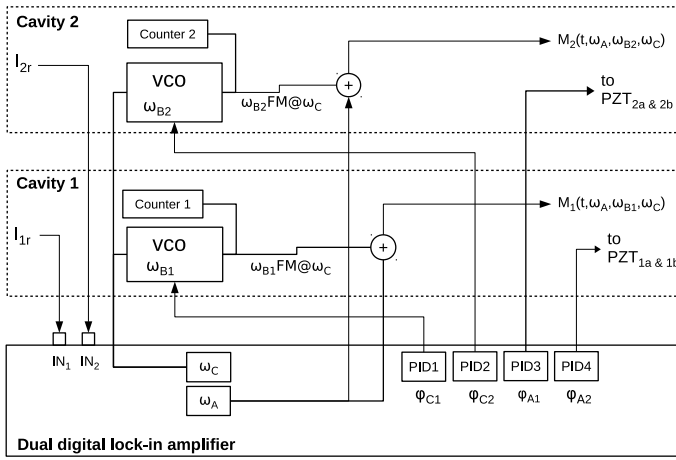


Fig. 9. Block scheme of the signal processing and absolute length control. The dual digital lock-in amplifier provides the phase error signals ϕ_{A1} , ϕ_{A2} , ϕ_{C1} , ϕ_{C2} that are processed by 4 digital PIDs. The 4 PIDs lock respectively the cavity lengths and the radio-frequencies ω_{B1} and ω_{B2} .

ϕ_A and ϕ_C at the modulation frequencies ω_A and ω_C are calculated and two digital PIDs are generated. Four PIDs are implemented. Two PIDs drive the cavities PZTs, and the other two PIDs drive the VCOs synthesizing the modulations at ω_{B_i} (see details in the figure caption). Once the four loops are closed, cavities optical frequency resonances are locked to the reference laser. The cavity dynamic resonances (in the range of few hundreds MHz) are measured with two RF frequency counters.

IV. CONCLUSION

We described the heterolithic, active-stabilized GP2 gyroscope. The expedients adopted to achieve a stabilization of the scale factor better than 10^{-10} are reported. An optical interferometry method already verified in [14] is going to be implemented by means of digital controls. The expected outcome provided by the implementation of the control scheme described in this paper is to make GP2 run as a gyroscope with a geometrical stability only limited by the stability of the laser frequency reference. The achievement of these results will be of key importance toward an heterolithic large frame detector for fundamental physics research.

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