

---

Alina Sirbu · Vittorio Loreto ·  
Vito D P Servedio · Francesca Tria

## Opinion dynamics with disagreement and modulated information

**Abstract** Opinion dynamics concerns social processes through which populations or groups of individuals agree or disagree on specific issues. As such, modelling opinion dynamics represents an important research area that has been progressively acquiring relevance in many different domains. Existing approaches have mostly represented opinions through *discrete* binary or *continuous* variables by exploring a whole panoply of cases: e.g. independence, noise, external effects, multiple issues. In most of these cases the crucial ingredient is an attractive dynamics through which similar or similar enough agents get closer. Only rarely the possibility of explicit disagreement has been taken into account (i.e., the possibility for a repulsive interaction among individuals' opinions), and mostly for discrete or 1-dimensional opinions, through the introduction of additional model parameters. Here we introduce a new model of opinion formation, which focuses on the interplay between the possibility of explicit disagreement, modulated in a self-consistent way by the existing opinions' overlaps between the interacting individuals, and the effect of external information on the system. Opinions are modelled as a *vector of continuous* variables related to *multiple possible choices* for an issue. Information can be modulated to account for promoting multiple possible choices.

---

A. Sirbu  
ISI Foundation, Via Alassio 11c, 10126 Turin, Italy  
E-mail: alina.sirbu@isi.it

V.Loreto  
Sapienza University of Rome, Physics Dept., P.le A. Moro 2, 00185 Rome, Italy  
ISI Foundation, Via Alassio 11c, 10126 Turin, Italy  
E-mail: vittorio.loreto@roma1.infn.it

V.D.P. Servedio  
Sapienza University of Rome, Physics Dept., P.le A. Moro 2, 00185 Rome, Italy  
E-mail: vito.servedio@roma1.infn.it

F. Tria  
ISI Foundation, Via Alassio 11c, 10126 Turin, Italy  
E-mail: tria@isi.it

Numerical results show that extreme information results in segregation and has a limited effect on the population, while milder messages have better success and a cohesion effect. Additionally, the initial condition plays an important role, with the population forming one or multiple clusters based on the initial average similarity between individuals, with a transition point depending on the number of opinion choices.

**Keywords** Opinion dynamics · interaction · disagreement · external information · numerical simulations

## 1 Introduction

Opinion formation is an important property of social systems. People are driven by their choices in many moments of their lives, based on opinions formed in time under the influence of many factors, such as their own personality, the culture they belong to, peer interaction, mass-media effects, etc. These choices range from selecting a lifestyle, a specific behaviour, a phone company, or a supermarket, to the city to leave in or whom to vote for. The way opinions form and choices are made is of interest to many classes of researchers, with implications in many fields from politics to economics, to marketing. Probably the most immediate example is represented by political opinions: here the question is how people synthesize the different sources of information and biases, to come up with a specific position to be eventually expressed as a vote whenever required. Another important example concerns marketing strategies. These strategies are typically devised using social research, under the assumption that in order to attract clients, the products have to be presented in an appealing way. An example is described in [9], where a supermarket chain, after careful analysis of the purchases of their clients, send customised promotional leaflets. By sending early advertising about products people need, they attract them before other chains, and increase sales also in departments other than that advertised. For a different client type, the advertising looks different, even though the supermarket is the same. This is one classical example of shaping external information and advertising in such a way as to attract as many individuals as possible. Yet another example is the decision to buy a specific product in a market segment, e.g. a mobile phone, a car, a book, etc. It is again a matter of opinion formation and evolution, based on an evaluation of how the product matches the needs, attitude and possibilities of an individual. For instance, while until a few years ago, the smart phone market was very narrow, nowadays it has extended significantly [8]. It would be interesting to know what changes people's opinion, and leads to a large propagation of a new technology.

Traditionally studied by sociologists, opinion formation has become an important topic for physicists and different classes of models have been investigated in the past years [5]. Models with opinions or options modelled through discrete variables, such as the voter, local majority rule [11], Sznaid [34], Axelrod [2], social impact [23, 29, 36] and their various extensions, have been applied to explain aspects of elections, strikes, dynamics of mobile markets, changes in the number of privately owned companies, financial

crises and culture formation [12, 13, 24, 28]. Although discrete variables are very suitable for modelling choices, the internal state of individuals, based on which discrete decisions are taken, may be continuous. Also, the opinion itself could take continuous values. The Deffuant-Weisbuch [7, 38] and Hegselmann-Krause [16] models analyse one-dimensional continuous opinions, while multiple dimensions are analysed in [25], where the opinion space is the unit simplex. The Continuous Opinions and Discrete Actions approach (CODA) [27] analyses internal probabilities for two or three discrete choices. Most of the original models deal with attractive dynamics, where individuals follow their neighbours. However, in reality, both attractive and repulsive interactions can be observed, and it has been argued that disagreement is a very important feature of a democratic society [19]. This element has been introduced in some of the existing models, by considering additional model parameters to control disagreement (e.g. [33, 37, 20, 35, 30, 21, 18, 1]). Additionally, the effect of external information is crucial when modelling real social systems. Again, some approaches consider this effect [3, 14, 32, 37, 6, 17]. However, an analysis for multi-dimensional continuous opinions is missing, both for external information and disagreement.

Here, the interplay between disagreement and external information in opinion dynamics is analysed, by the introduction of a new modelling approach. The model considers the probabilities that an individual will make a specific choice out of multiple possibilities (such as voting or choosing a market product). It includes both attractive and repulsive interactions, in a self consistent way, without the addition of a further parameter to the model. Modulated information is also possible, which promotes multiple choices at the same time. Additionally, an analysis of the effect of the initial condition is performed.

## 2 Methods

A fully connected social network of  $N$  individuals is considered, where each agent has to make a choice between  $K$  possible opinions on a given subject. Each individual maintains a set of probabilities for the  $K$  possibilities:  $\mathbf{x} = [p_1, p_2, \dots, p_K]$  with  $\sum_{k=1}^K p_k = 1$  i.e., an element in the simplex in  $K - 1$  dimensions. We define the similarity between two individuals  $i$  and  $j$  as the cosine overlap between the two opinions' vectors:

$$o^{ij} = \frac{\mathbf{x}^i \cdot \mathbf{x}^j}{|\mathbf{x}^i| |\mathbf{x}^j|} = \frac{\sum_{k=1}^K p_k^i p_k^j}{\sqrt{\sum_{k=1}^K (p_k^i)^2 \sum_{k=1}^K (p_k^j)^2}}. \quad (1)$$

At each time step a randomly selected pair of individuals,  $(i, j)$ , interacts, either by agreeing or disagreeing, based on their instantaneous overlap:

$$p_{\text{agree}}^{ij} = \min(1, \max(0, o^{ij} \pm \epsilon)), \quad (2)$$

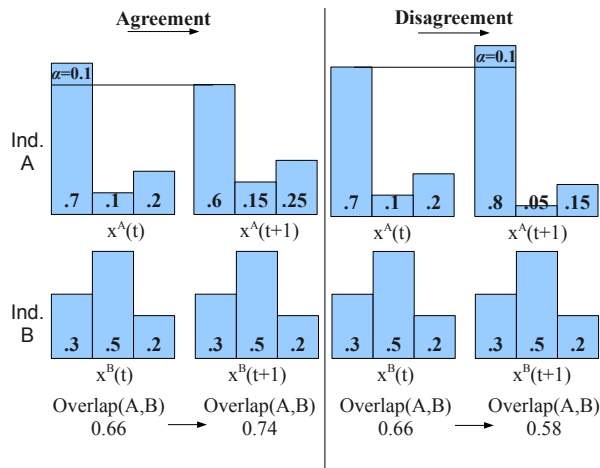
$$p_{\text{disagree}}^{ij} = 1 - p_{\text{agree}}^{ij}. \quad (3)$$

where  $\epsilon$  is a noise term which avoids lack of interaction due to null overlap, with the choice between the plus and minus signs (Equation 2) made randomly at each time step. The rule 2 (and 3) corresponds to imagine that, during the interaction, each individual perceives how close (or distant) he/she is from another individual and consistently agrees or disagrees. It is important to note here that our model does not impose “bounded confidence”: individuals with low overlap will tend to disagree, with an effect on their state, and it is however possible that two individuals agree even though their overlap is null (with low probability). Furthermore, the model introduces the possibility of both agreeing and disagreeing in a self consistent way. Interaction causes one of the individuals in the pair to change a random element  $l$  in the opinion vector:

$$p_l^i(t+1) = \begin{cases} p_l^i(t) \pm \alpha \operatorname{sign}(p_l^j - p_l^i) & \text{if } |p_l^j - p_l^i| > \alpha \\ p_l^i(t) \pm \frac{1}{2}(p_l^j - p_l^i) & \text{otherwise.} \end{cases} \quad (4)$$

where the plus sign occurs when the interaction results in agreement and the minus sign when the interaction results in disagreement. Hence, the position is changed by a small fixed step  $\alpha > 0$ , unless differences to the other individual are too small, in which case the change is half the difference. This allows complete agreement between individuals. The parameter  $\alpha$  determines the flexibility of agents since it fixes the time scale for local agreement or disagreement. The larger  $\alpha$  is, the faster the two individuals will agree or get separated. The rest of the elements in  $\mathbf{x}^i$  are adjusted to preserve the unit sum, by uniformly redistributing the amount the element  $l$  was changed by. Since 0 and 1 are absorbing values, this is performed iteratively. For instance, if we consider that position  $l$  has been increased by  $\alpha$ , the other positions would have to change by  $-\frac{\alpha}{K-1}$ . In attempting to do this, some positions may become negative. In this case, the negative positions will be set to 0, after being summed to obtain a new amount  $\alpha'$  for redistribution. This new amount will be redistributed to all non-null positions (except for  $l$ ), with the procedure repeated until no negative position is obtained. This method allows for the absolute value of the change on position  $l$  to be the same for agreement and disagreement. Figure 1 demonstrates graphically the update rule employed.

External information, e.g. mass-media, is introduced as a static agent  $\mathbf{I} = [I_1, I_2, \dots, I_K]$  with:  $\sum_{k=1}^K I_k = 1$ . After interacting with a peer, an individual interacts also with the information with probability  $p_I$ , following the same interaction rules. Hence, interacting with the external information does not imply less peer communication. In previous models, external information biased individuals towards one choice out of all possibilities (e.g. Axelrod, Sznajd). Here, this means setting one position in  $\mathbf{I}$  to 1 and others to 0. In reality, however, sources of information are so wide that only one possibility is never promoted. Our approach has the ability to model such complex influence, by choosing non zero values on more positions of  $\mathbf{I}$ , i.e., a *modulated information*. This was also true for the Deffuant model: if we consider that the continuous opinion is actually a probability to make a choice between two discrete options, then milder external information can be introduced by



**Fig. 1 Example of pairwise interaction.** We consider a generic interaction between an individual A and an individual B, in a dynamics where the number of possible opinions is fixed to  $K = 3$ . In particular, the individual A changes its opinion according to its interaction with the individual B. We depict both the cases of agreement (left) and disagreement (right). Left: The individual A changes its status according to an agreement with individual B. In this case, the opinion they discuss is the opinion  $l = 1$ , and its probability is decreased by the value  $\alpha = 0.1$  in the individual A to become more similar to the individual B (and, as such, the mutual overlap increases). The probabilities of the rest of the opinions are increased by equal amounts (in this case,  $\frac{\alpha}{2}$ ) to conserve the unity sum. Right: the same dynamics in case of disagreement. In this case, the individual A increases the probability of opinion 1 by the amount  $\alpha = 0.1$ , decreasing its overlap with the individual B. Equivalently, the probabilities of the rest of the opinions are decreased by equal amounts to conserve the unity sum.

using information values far from the extremes of the opinion interval. In fact, similarities between information effects in the Deffuant model and the one introduced here will be discussed in a later section.

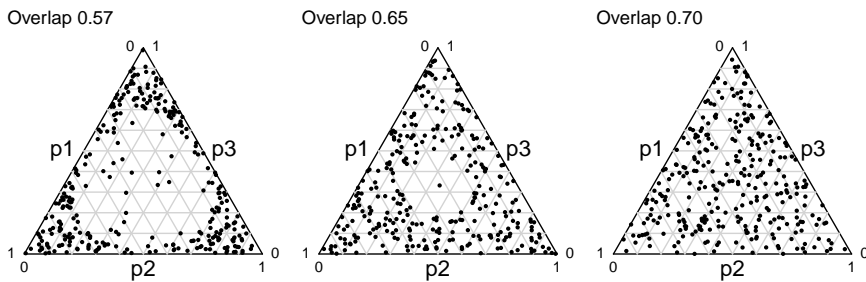
### 3 Results

#### 3.1 Role of the initial condition

An important parameter to take into account is the initial average overlap of the population, defined as :

$$\bar{o} = \frac{2 \sum_{i,j} o^{ij}}{N(N-1)}. \quad (5)$$

This value represents the probability that a randomly chosen pair of individuals will follow agreement dynamics, so it may have a large influence on the final state of the population. Thus, it is interesting to see how the dynamics depends on this feature of the initial condition, and we perform this analysis when no external information is present ( $p_I = 0$ ). A random sampling of



**Fig. 2** Initial conditions for  $K = 3$  - distribution of opinion values for a population of 300 individuals, with different average population overlap. The entropy threshold values used to generate these populations are, from left to right, 1.2, 1.5 and 1.6.

the simplex in  $K - 1$  dimensions yields a population with a relatively large average overlap. In order to generate populations with different initial  $\bar{o}$ , we consider the entropy  $S$  associated to the opinions probability, defined as:

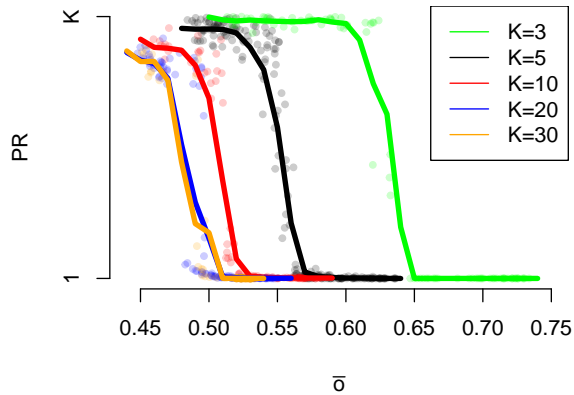
$$S = - \sum_{i=1}^K p_i \log_2(p_i) \quad (6)$$

and we remove from the random sampling some individuals that have  $S$  larger than a specific threshold. Specifically, we construct a population of  $N$  individual by random sampling each individual from a simplex in  $K - 1$  dimensions. In order to decrease the population average overlap, we compute for each sampled individual its entropy  $S$  and remove it from the population with probability 0.9 if  $S$  is above the chosen threshold. We continue to sample points in the  $K - 1$  simplex and to apply the above procedure till we reach a population of the desired size  $N$ . Decreasing the threshold, populations with decreasing  $\bar{o}$  can be obtained. Figure 2 shows the distribution of populations with different  $\bar{o}$  for  $K = 3$ . Throughout the rest of this paper, more fragmented or compact population will be generated in this manner, with the most compact initial condition corresponding to the random sampling of the simplex.

In order to study the effect of the initial condition, we have performed numerical simulations ( $N = 300$ ,  $\epsilon = 0.1$ ,  $p_I = 0$ ) for different  $K \in \{3, 5, 10, 20, 30\}$ , with corresponding  $\alpha \in \{0.0167, 0.01, 0.005, 0.0025, 0.00167\}$ . Hierarchical clustering of the final population has been performed, using complete linkage clustering [26], and cutting the tree at 0.8 similarity level. This ensures that if two agents  $i$  and  $j$  are in the same cluster then  $o_{ij} > 0.8$ . The *effective* number of clusters has been computed as the cluster participation ratio (PR):

$$PR = \frac{\left(\sum_{i=1}^C c_i\right)^2}{\sum_{i=1}^C c_i^2} \quad (7)$$

with  $C$  the number of clusters and  $c_i$  the size of cluster  $i$ . This measure is more significant than the number of clusters itself, as it also considers cluster



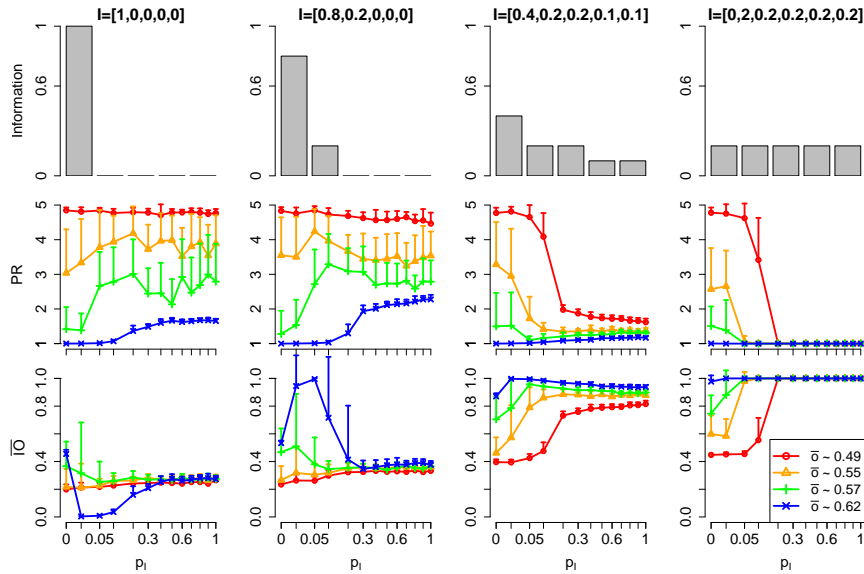
**Fig. 3** Effect of initial condition on the effective number of clusters in the final population. Dots represent individual instances, while lines are averages. Simulations have been performed with  $N = 300$ ,  $\epsilon = 0.1$ ,  $p_I = 0$  for different  $K \in \{3, 5, 10, 20, 30\}$ , with corresponding  $\alpha \in \{0.0167, 0.01, 0.005, 0.0025, 0.00167\}$ .

sizes. For instance, if the population consists of two clusters, PR would be 2 only if the clusters are equal in size, and very close to 1 if one of the clusters is extremely small compared to the other.

Figure 3 displays PR values for different realizations of the model, depending on the initial  $\bar{o}$ . This shows that the initial condition has a large effect on the final population, with a transition in the number of clusters obtained. Specifically, above a certain value of the initial overlap, the population forms one cluster, while below this value the population divides into  $K$  clusters, each of the form  $(1, 0, \dots)$ . The value of  $\bar{o}$  marking the transition decreases with  $K$ , showing that agreement in the population is facilitated by the existence of more opinion choices. This has also been observed for the model in [25], although without performing an analysis of the initial condition effect. However, it is important to note that in the case of agreement (and in the absence of external information), agents maintain a probability different from zero for (almost) all options, which means a generalised state of indecision. On the other hand, when clusters form, these adopt a more decided option with  $p_l = 0$  for many values of  $l$ . More details about the structure of the opinions in the different cases will be discussed in Section 3.4.

### 3.2 Effect of information

We now analyse the effect of external information on the final state of the population. Numerical simulations have been performed for  $N = 300$ ,  $K = 5$ ,  $\alpha = 0.01$ ,  $\epsilon = 0.1$  and  $p_I$  ranging from 0 to 1. Four types of information have been



**Fig. 4** Numerical results for a population of 300 individuals,  $K = 5$ ,  $\epsilon = 0.1$  and  $\alpha = 0.01$  are presented (100000 population updates (refer to figure 15 for the time needed for a population of size  $N$  to reach a stationary state), 20 instances for each parameter value). Four different information values are used, corresponding to each column. The top graphs show the histogram of the information, the middle the average number of clusters as a function of  $p_I$  while the bottom the average information overlap again as a function of  $p_I$ . The individual points correspond to  $p_I \in \{0, 0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$ . Error bars are displayed as well, showing standard deviation for each point. Where error bars are not visible, the standard deviation is within graphic point limits or null.

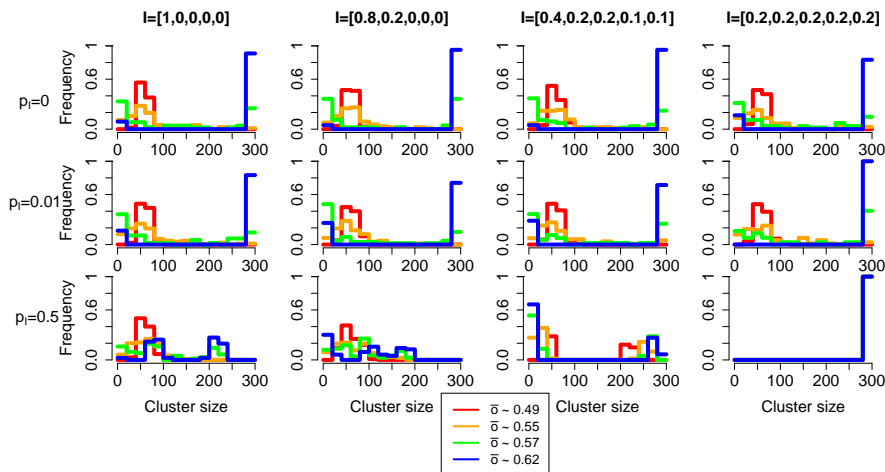
investigated, in order to study the effect of extreme and mild external messages on the population (specifically  $\mathbf{I} \in \{[1, 0, 0, 0, 0], [0.8, 0.2, 0, 0, 0], [0.4, 0.2, 0.2, 0.1, 0.1], [0.2, 0.2, 0.2, 0.2, 0.2]\}$ ). Since, as shown in section 3.1, the initial condition plays a very important role in the number of clusters obtained, four different initial conditions have been used, with  $\bar{o} \in \{0.49, 0.55, 0.57, 0.62\}$ . These values cover situations before, around and after the transition occurs (Figure 3), with  $\bar{o} = 0.62$  corresponding to the random sampling of the simplex.

The effective number of clusters has been determined as in the previous section, using the PR measure. The effect of information has been quantified by computing the average information overlap ( $\overline{IO}$ ) once the population has reached a stationary state:

$$\overline{IO} = \frac{1}{N} \sum_{i=1}^N o^{I_i} \quad (8)$$

where  $o^{I_i}$  represents the cosine overlap between agent  $\mathbf{x}^i$  and the external information  $\mathbf{I}$ . This average measure is an indicator of the percentage of individuals in the population adhering to the information.





**Fig. 5** Histograms of cluster sizes obtained over 20 simulations runs for  $K = 5$ ,  $N = 300$ ,  $\epsilon = 0.1$  and  $\alpha = 0.01$ . Four information types and four initial conditions are displayed, for  $p_I \in \{0, 0.01, 0.5\}$ .

Figure 4 displays  $\overline{IO}$  and the effective cluster number PR for the different parameter configurations, each point being an average over 20 realizations of the process. Several patterns can be observed. In general, extreme information is less successful in the population compared to milder messages, as  $\overline{IO}$  values indicate. Additionally, mild information favours cohesion in the population, i.e., a decreased number of clusters compared to the  $p_I = 0$  situation, while extreme information induces segregation (increased PR). These effects increase with  $p_I$ . Of course, the extent of the two effects observed depends on the initial condition. That is, the segregation effect of extreme information is small when the population starts from a tight community, i.e., large  $\bar{o}$ , with the number of clusters increasing when  $\bar{o}$  decreases. Similarly, the success of mild information is smaller when the initial population is not very compact and larger in the opposite case.

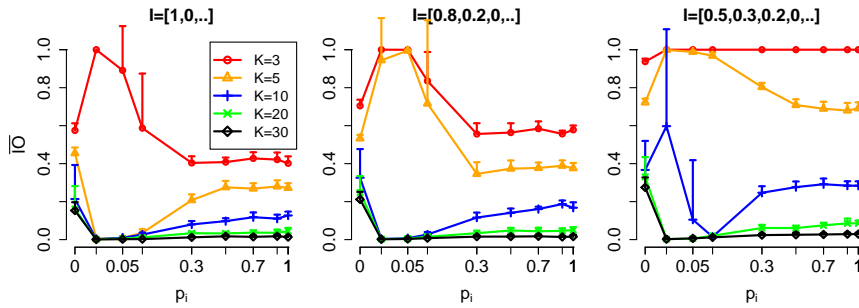
Several other interesting details can be observed. In general, even when the frequency of interaction with the external information ( $p_I$ ) is very large, the success in the population ( $\overline{IO}$ ) is bounded. This bound depends both on the initial condition and on the type of information, with a small value for extreme information and low initial  $\bar{o}$ , and (nearly) complete success for mild information and large  $\bar{o}$ . This is due to the disagreement dynamics based on the overlap between individuals and the external information, and is observable also in real life, where no matter how much propaganda there is, if the individuals do not agree enough with an idea, this is not adhered to.

To analyse in more detail the clustering patterns obtained, Figure 5 displays histograms of cluster sizes obtained in 20 runs, for  $p_I \in \{0, 0.01, 0.5\}$ . This shows that, in general, as expected, PR values of 5 correspond to clusters of similar size (around 60 individuals), while PR values close to 1 are gener-

ated by one very large cluster and one or several extremely small groups. The figure shows very clearly how the fraction of very large cluster sizes increases as the initial condition becomes more compact (red to blue lines) and as the information becomes milder (left to right columns).

Another very interesting phenomenon can be observed for compact initial conditions, i.e., random sampling of the simplex (Figures 4 and 5, blue lines). When the information is not too extreme  $\mathbf{I} = [0.8, 0.2, 0, 0, 0]$ , the entire population adheres to it provided  $p_I$  is very small, while as  $p_I$  increases, the media success decreases. On the other hand, when the information is very peaked ( $\mathbf{I} = [1, 0, 0, 0, 0]$ ), very small  $p_I$  leads to complete disagreement to the population, while larger  $p_I$  increases agreement. Cluster sizes, however, do not show a large change when increasing  $p_I$  from 0 to 0.01, showing that group composition is basically the same, even though  $\overline{IO}$  values change significantly. This indicates that a very low  $p_I$  allows for the dynamics to proceed in a similar manner as without information, and after groupings are formed, these are slowly swayed towards or away from the information. In the specific case here, a compact initial population forms first one cluster, which moves close to the information if this is mild, or far from it if peaked. This shows that, when facing a compact group, an external message is more efficient if presented gradually, provided it is not extremely different from the current convictions of the population. This suggests that peer influence is more effective than that from an external static source. This can be explained by the fact that peers are flexible, and move towards others freely, while external information is too rigid. An agent that is facing an external message which she does not agree with, will move away from it, while when the direct exposure to the external message is small, the interaction with other peers can sway her towards accepting the message. However, for this to happen, the message has to be close enough to the initial state of the population, i.e., acceptable by a large number of individuals. When this is not true, as is the case with an extreme external information, the entire population will disagree with the message, and the media campaign will have no effect.

It is important to note that the overlap of the external information with the population, which, as our results show, is one of the most important determinant for the success of a campaign, depends also on  $K$ . For instance, an extreme message  $[1, 0, \dots]$  has an average overlap with a random population of 0.447 when  $K = 5$  and 0.577 for  $K = 3$ . Figure 6 shows average overlap with the information obtained for different  $K$  values. Each initial population has been obtained by random sampling of the simplex, being thus a compact population. Three information types have been used. It is obvious that the three types of information have a very different effect depending on  $K$ . While for  $K = 3$ , information  $[0.5, 0.3, 0.2]$  is very mild, having complete success, for  $K = 30$   $[0.5, 0.3, 0.2, 0, \dots, 0]$  would be quite extreme, since only three out of thirty options are promoted, so the information overlap obtained decreases drastically. We can conclude that the success of all three information types in the population decreases with  $K$ . This indicates that it is easier to convince the public about a specific option when there are few choices, compared to when the number of choices is large. When  $p_I$  is small ( $\neq 0$ ), the phenomenon explained in the previous paragraph for compact populations

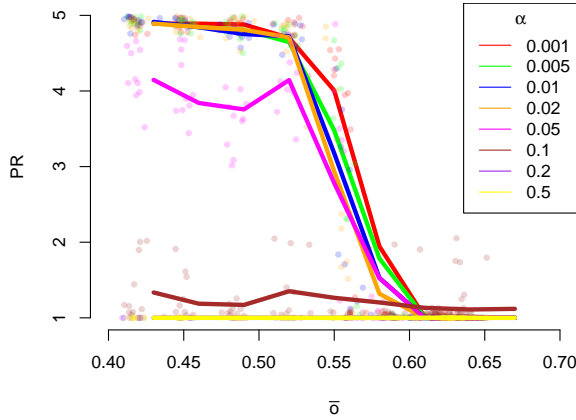


**Fig. 6** Average information overlap for compact populations of 300 individuals,  $K \in \{3, 5, 10, 20, 30\}$ ,  $\epsilon = 0.1$  and corresponding  $\alpha \in \{0.0167, 0.01, 0.005, 0.0025, 0.00167\}$  (values for  $\alpha$  have been scaled to maintain the scale, compared to the average value of  $p_i$ , when  $K$  increases). The graph includes 10 instances for each parameter value. Error bars show one standard deviation from the plotted mean. The individual points correspond to  $p_I \in \{0, 0.01, 0.05, 0.1, 0.3, 0.5, 0.7, 0.9, 1\}$ .

can be observed, for all  $K$ . Specifically, when information is mild enough, complete agreement in the population is obtained ( $\overline{IO} = 1$ ), while extreme information fails completely in attracting individuals ( $\overline{IO} = 0$ ). Again, mildness of information depends on  $K$ . For example, for low  $p_I \neq 0$ , information  $[0.8, 0.2, \dots]$  obtains full agreement for  $K \in \{3, 5\}$  and full disagreement for  $K \in \{10, 20, 30\}$ .

### 3.3 Robustness with respect to the choice of $\alpha$

The role of the value of the parameter  $\alpha$  is analysed to identify the effect of changing agent flexibility. In figure 7 we report the effective number of clusters PR resulting from simulations with different values of the flexibility  $\alpha$ , for  $K = 5$ . The same dependence of the results on the initial condition observed in Section 3.1 is conserved as long as  $\alpha$  is small enough ( $\alpha < 0.05$  in simulations with  $K = 5$ ), whereas when  $\alpha$  is too large ( $\alpha > 0.1$ ), the population converges to one cluster, regardless of the initial condition chosen. These results show that the model is robust with respect to  $\alpha$ , as long as the change in opinion is not forced to be very large, which is what is expected. A very large  $\alpha$  favours agreement in the population, since individuals that are very different can become very close on a single interaction, while for similar individuals which disagree, the change is not as drastic, since the difference between their opinions is very likely to be smaller than  $\alpha$  (and hence use the second rule in equation 4). It is important to note that decreasing  $\alpha$  further below a certain value (thus increasing simulation times) is not necessary since results are very similar.



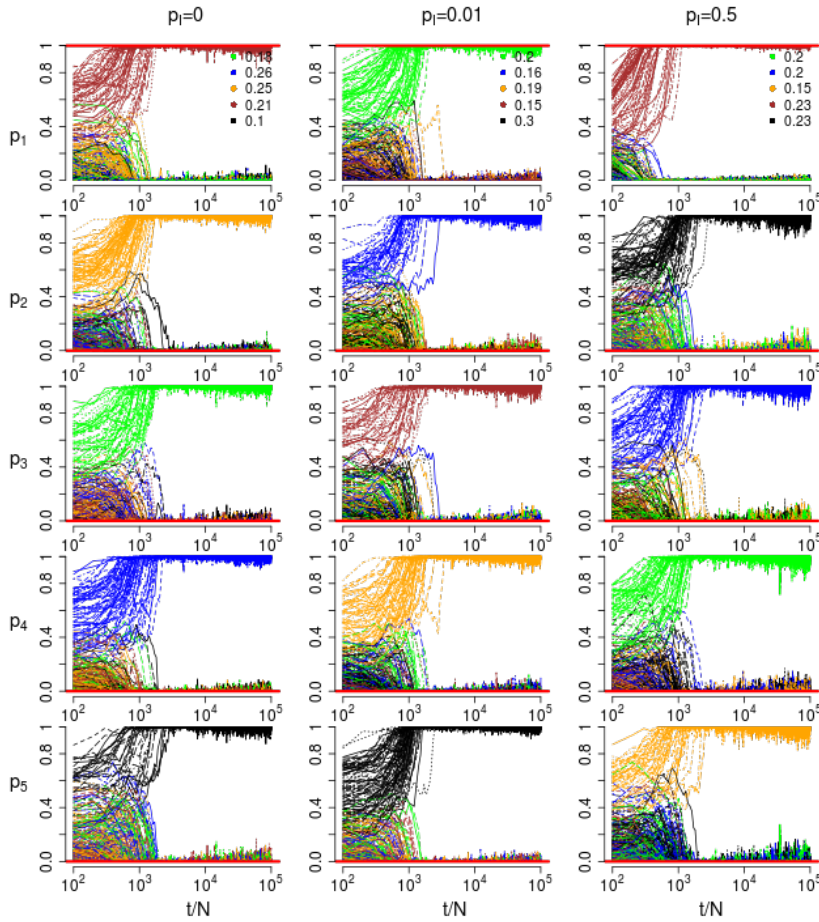
**Fig. 7** Effective number of clusters obtained for different initial conditions and  $\alpha$  values for  $K = 5$ .

### 3.4 Temporal patterns

In order to get a further insight on the dynamics of the system, this section discusses temporal patterns for two initial conditions ( $\bar{o} = 0.49$  and  $\bar{o} = 0.62$ ) and two information types ( $\mathbf{I} = [1, 0, 0, 0, 0]$  and  $\mathbf{I} = [0.4, 0.2, 0.2, 0.1, 0.1]$ ), each for  $p_I \in \{0, 0.01, 0.5\}$  and  $K = 5$ . Figures 8, 9, 10 and 11 show single simulation instances for each  $p_I$ . Plots display the evolution in time of each of the five elements of the opinion, for every individual in a population of 300. Each row corresponds to one position  $p_i$  in the opinion vector, while each column represents a different value of  $p_I$ . The value of the information is shown in red. Individual opinions are displayed in colours corresponding to the cluster they belong to at the end of the simulation (e.g. all green lines correspond to individuals which cluster together). The relative cluster sizes ( $\frac{c_i}{N}$ ) are also shown at the top, as a legend.

As figures show, opinions start in random position spanning the interval  $[0, 1]$ , and stabilise around a particular value. The system never reaches a frozen state, with small fluctuations preserved even after the clusters are formed (see Fig. 4 for the effective number of clusters in the different regimes). This is due to the parameter  $\epsilon$ , which allows for agreement even when the overlap between two individuals is zero, or disagreement even when the overlap is one.

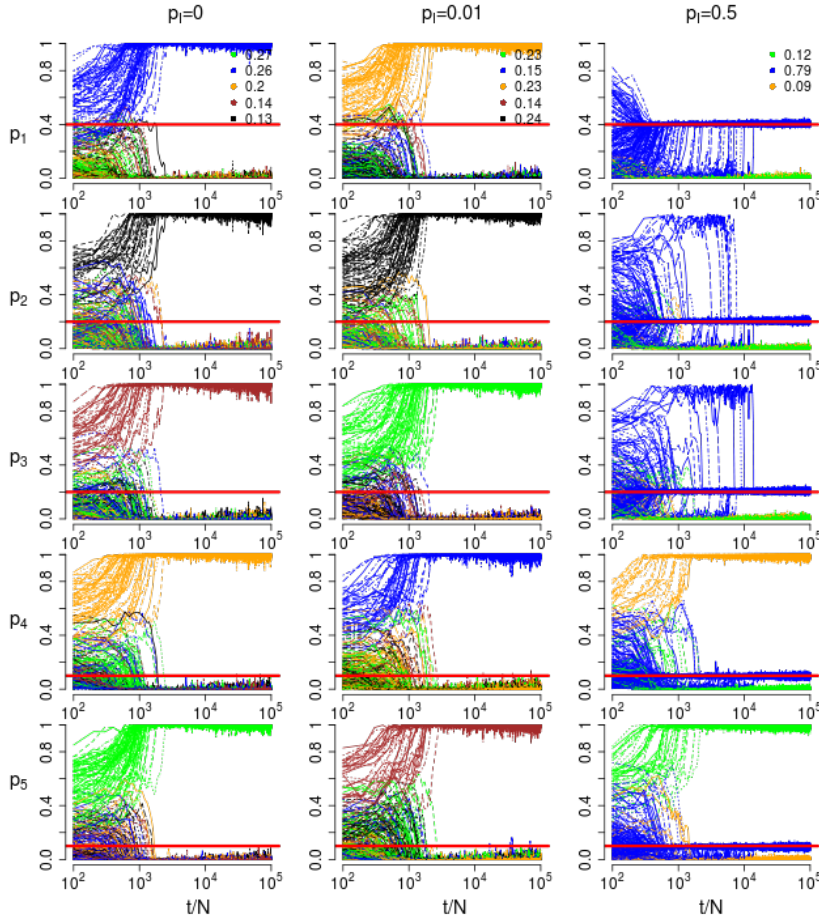
For the case of segregated initial populations, Figures 8 and 9 show the formation of five clusters within the population, for  $p_I = 0$ . These five clusters are maintained for extreme information, regardless of the value of  $p_I$ , due to the segregation effect of such information. For milder external message, on the other hand, the five clusters are only maintained when the frequency of exposure is small. In this situation, however, although the average overlap



**Fig. 8** Opinion values for  $K = 5$ ,  $N = 300$ ,  $\bar{d} = 0.49$ ,  $\mathbf{I} = [1, 0, 0, 0, 0]$ . Each row corresponds to the positions in the opinion vector  $\mathbf{x} = [p_1, p_2, p_3, p_4, p_5]$ , while each column represents a different  $p_I$ . Individual opinions are coloured based on the cluster membership, with cluster sizes included in the legend. The information value is represented in red. In this case five clusters are formed.

with the information is quite large (Figure 4), no individuals agree completely with the information. Larger  $p_I$  causes a large cluster to form around the external information value, showing the cohesion effect of a mild message (which appears only when the frequency of exposure is large enough).

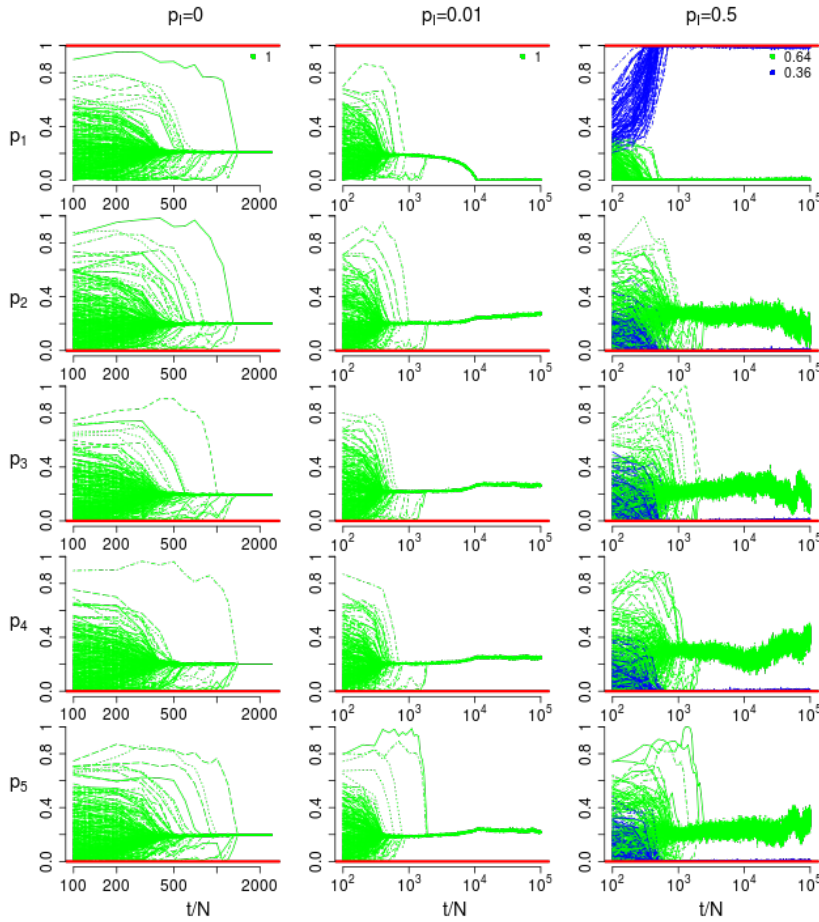
Figures 10 and 11 show similar graphs for a compact initial population. When no information is present, all individuals form one cluster. Plots for  $p_I = 0.01$  validate the explanation of the total agreement/disagreement observed in Figures 4 and 5. Specifically, low exposure to the information allows the population to initially form one cluster, similar to no exposure, which is afterwards slowly affected by the external message. When this is extreme (Figure 10), the cluster shifts away from the external information, since it



**Fig. 9** Opinion values for  $K = 5$ ,  $N = 300$ ,  $\bar{\sigma} = 0.49$ ,  $\mathbf{I} = [0.4, 0.2, 0.2, 0.1, 0.1]$ . Each row corresponds to the positions in the opinion vector  $\mathbf{x} = [p_1, p_2, p_3, p_4, p_5]$ , while each column represents a different  $p_I$ . Individual opinions are coloured based on the cluster membership, with cluster sizes included in the legend. The information value is represented in red. In this case five clusters are formed for  $p_I = 0$  and  $p_I = 0.01$  and three for  $p_I = 0.5$ .

is too dissimilar. On the contrary, when the information is mild, the cluster moves towards it resulting in complete agreement (Figure 11). For  $p_I = 0.5$ , the group, determined previously by the initial condition, does not form, and part of the population adheres to the information directly. For extreme information, the cluster overlapping with the information is small, while for the mild message, this dominates the population.

All in all, we observe that a small  $p_I$  allows for dynamics to be determined by the initial condition at the beginning of the system's evolution. Clusters are influenced by information after they are formed: these move away or close to the information value, depending on the cluster overlap with the external

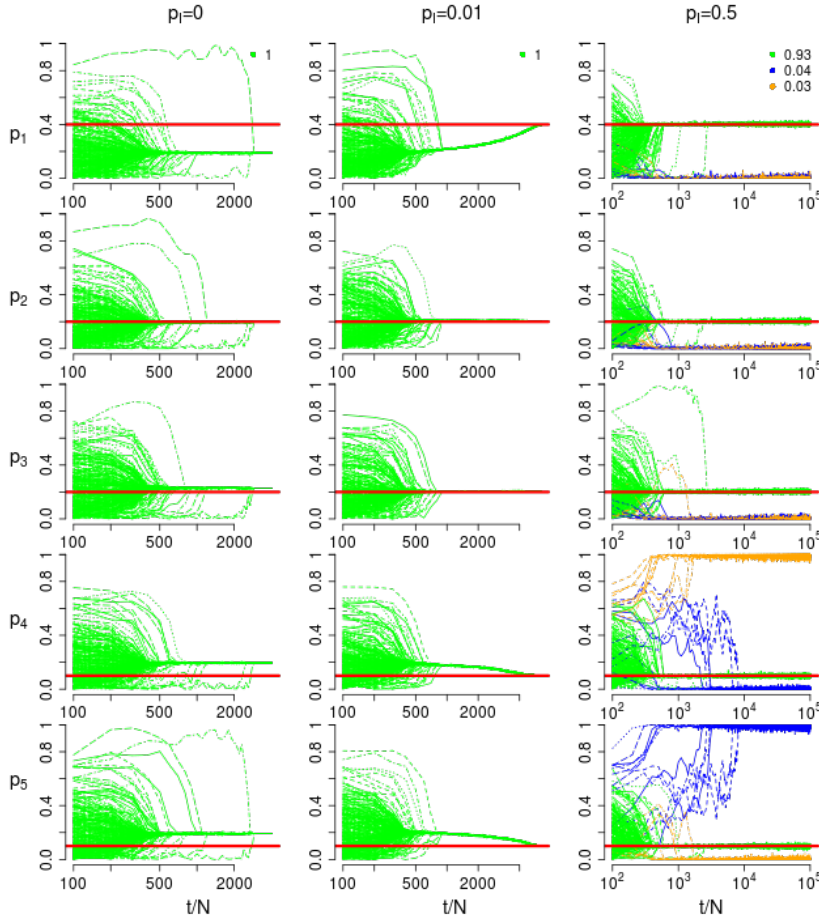


**Fig. 10** Opinion values for  $K = 5$ ,  $N = 300$ ,  $\bar{o} = 0.62$ ,  $\mathbf{I} = [1, 0, 0, 0, 0]$ . Each row corresponds to the positions in the opinion vector  $\mathbf{x} = [p_1, p_2, p_3, p_4, p_5]$ , while each column represents a different  $p_I$ . Individual opinions are coloured based on the cluster membership, with cluster sizes included in the legend. The information value is represented in red. In this case one cluster is formed for  $p_I = 0$  and  $p_I = 0.01$  and two for  $p_I = 0.5$ .

input. For larger  $p_I$ , the initial overlap of each individual with the external information is important: individuals who are far away form additional clusters that are then too distant from the rest of the population and from the information to be attracted back.

### 3.5 Biased population

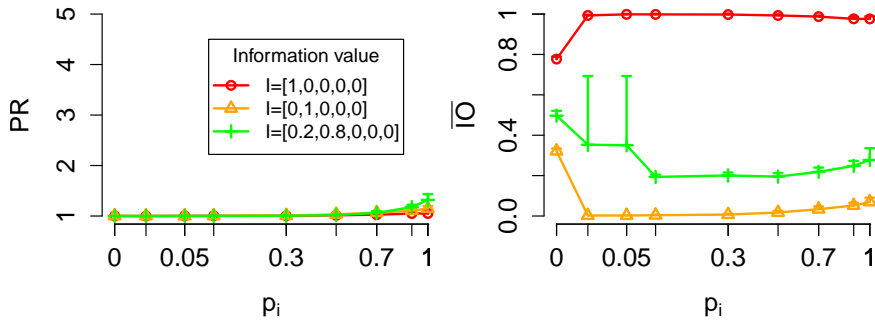
In the simulations presented until now, the population was random, i.e., not biased, on average, towards any opinion. Information, on the other hand, was considered extreme if it promoted strongly one of the possible choices.



**Fig. 11** Opinion values for  $K = 5$ ,  $N = 300$ ,  $\bar{\sigma} = 0.62$ ,  $\mathbf{I} = [0.4, 0.2, 0.2, 0.1, 0.1]$ . Each row corresponds to the positions in the opinion vector  $\mathbf{x} = [p_1, p_2, p_3, p_4, p_5]$ , while each column represents a different  $p_I$ . Individual opinions are coloured based on the cluster membership, with cluster sizes included in the legend. The information value is represented in red. In this case one cluster is formed for  $p_I = 0$  and  $p_I = 0.01$  and three for  $p_I = 0.5$ .

Extreme information was shown to have limited effect on such populations. However, there can be situations where the initial population is already biased towards one of the possibilities, in which case, extreme information can still have a large effect. Figure 12 shows the effect of different information types for such a biased population. It is obvious that if the bias in the population coincides with the opinion that the information promotes, then even an extreme information can induce complete (for low  $p_I$ ) or wide (for larger  $p_I$ ) agreement. However, if the choice promoted by extreme information is different, then all individuals disagree. Agreement with information, in the latter situation, increases when information becomes milder. This shows that it is





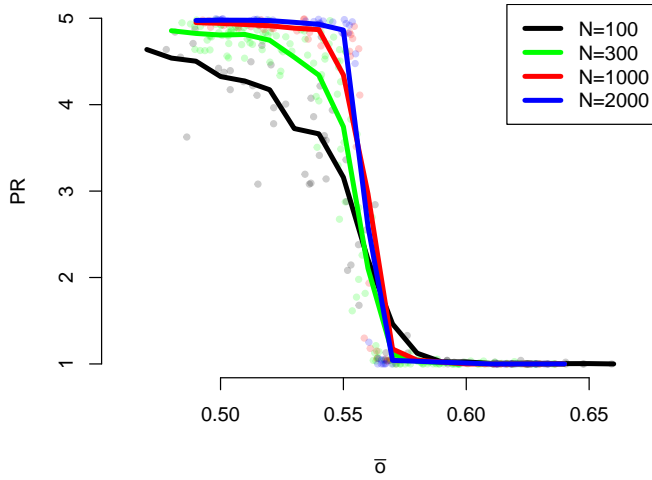
**Fig. 12** Effect of extreme information on a biased population, with a positive bias for the first opinion choice (average opinion value for the population around  $b = [0.4, 0.15, 0.15, 0.15, 0.15]$ ). The individual points correspond to  $p_I \in \{0, 0.01, 0.05, 0.1, 0.3, 0.5, 0.7, 0.9, 1\}$ .

the similarity of the information to the original population that determines its success, and not the information structure in itself. In these conditions, the previous observations for extreme information can be extended to any situation, if we consider information “extremism” as a feature dependent on the state of the population, i.e., information with low overlap to the population.

### 3.6 Population size

All previous simulations presented have been performed using a limited population size, i.e.,  $N = 300$ . Some effects seen in the results can be artifacts of the small population size, hence in this section we are analysing some of the previous simulation settings for increasing population sizes i.e.,  $N \in \{100, 300, 1000, 2000\}$ . Due to increased running times for larger populations, only the situation  $K = 5$  has been studied, with ten instances for each parameter set.

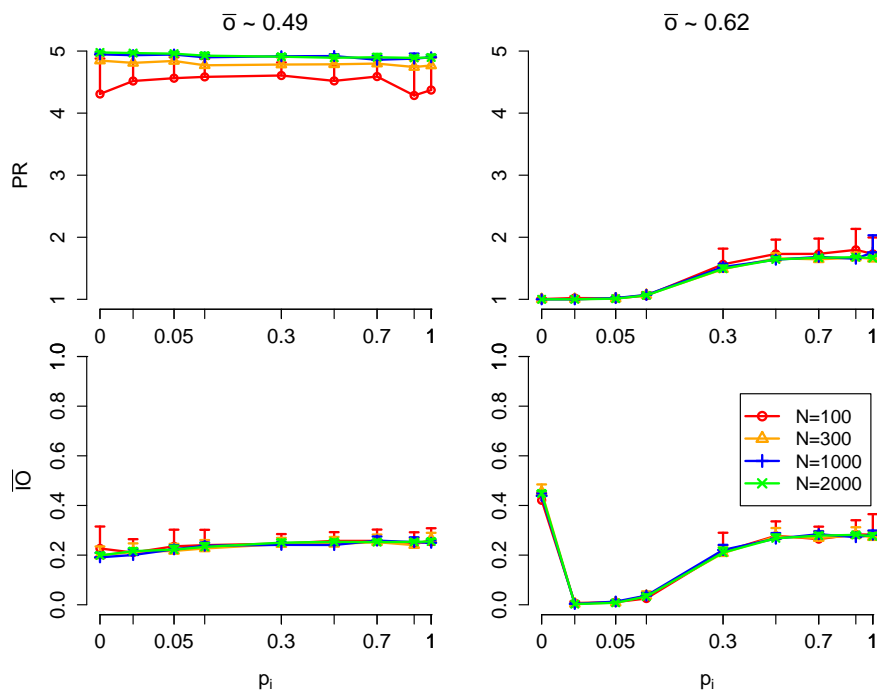
A first such analysis involves the effect of the initial condition, i.e., the average initial overlap in the population. Figure 13 shows the number of clusters for different instances with various initial  $\bar{o}$ . This shows that the transition between  $K$  clusters obtained for low  $\bar{o}$  and one cluster for large  $\bar{o}$ , observed in Section 3.1, is conserved for different populations sizes. Further this transition becomes steeper as  $N$  increases, with less frequent intermediate values at the transition point. This indicates that the intermediate number of clusters obtained for  $N \in \{100, 300\}$  could be an artifact of the small population size; however, the curves overlap very well, especially for larger  $N$ . These observations indicate a low dependence between the effect of the initial condition and the population size. As long as the population is not extremely small, (as seen for  $N = 100$ , where the curve seems more different than the others), results do not change much, at least qualitatively, by increasing the population size.



**Fig. 13** Effect of the initial condition for different population sizes and  $K = 5$ . Dots represent values for individual runs, while the lines show average number of clusters (computed by binning).

In a similar fashion, the external information effect has been studied for different population sizes, for  $K = 5$ . Figure 14 displays the average number of clusters and the final average  $\bar{IO}$  for four population sizes,  $I = [1, 0, 0, 0, 0]$  and two initial conditions (compact and dispersed initial population). This shows an extremely good overlap between the curves corresponding to each population size, indicating, for external information as well, that results are not influenced by  $N$ . Cluster sizes, in the case of segregated initial population ( $\bar{o} \sim 0.49$ ), become more uniform as the population grows, i.e., PR value is closer to 5. This suggests that, for an infinite such population, extreme external information would produce equally sized clusters in the population.

Further, we study relaxation times depending on  $N$  and information type. Since opinions do not converge to one value during simulations, relaxation time is defined as the number of updates (total number of interactions) required to obtain stable clusters and information agreement, even though opinions will still fluctuate. Figure 15 displays the scaling, averaged over 100 simulation runs, of the relaxation times vs.  $N$  for two information types (with  $p_I = 0.5$ ) and two initial conditions. This shows that, in general, clusters form faster when the initial population is more compact (right vs. left plot). The scaling of the relaxation time with  $N$  features a linear behaviour in all cases. An exception is a segregated population exposed to mild information, where the scaling is slightly more than linear with a fitted exponent  $\sim 1.11$ . Finite-size effects are visible for small population sizes, which features also larger



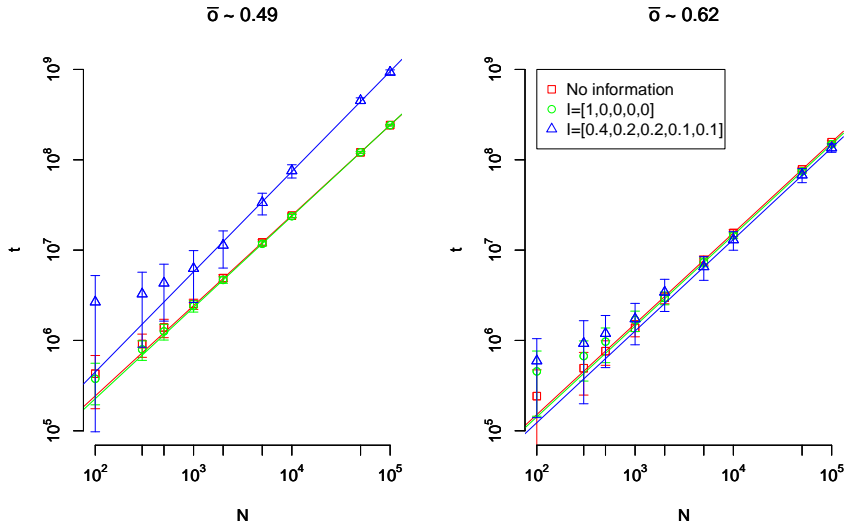
**Fig. 14** Effect of extreme information ( $I = [1, 0, 0, 0, 0]$ ) on  $K=5$  and two different initial conditions, for different population sizes. The individual points correspond to  $p_I \in \{0, 0.01, 0.05, 0.1, 0.3, 0.5, 0.7, 0.9, 1\}$ .

error bars. These finite-size effects can be explained by the sparser sampling of the opinion space during initialisation, for small populations.

The effect of external information on the relaxation time is also displayed in Figure 15. The two initial conditions show different trends. For a segregated starting point (left panel) the two types of information have opposite effects on the relaxation time. Specifically, exposure to mild information (which, as we saw before, has a cohesive effect) increases significantly the relaxation time, due to the contrasting effects of the initial condition and information type (i.e., segregation vs cohesion). For a compact initial condition (right panel) both information types have the same effect.

## 4 Conclusions

A new model for opinion dynamics was introduced, considering the internal probability of individuals to choose between several discrete options, i.e., with opinions represented as a vector of continuous values with unity sum. Both attractive and repulsive dynamics are considered, through a standalone mechanism, i.e., based on pairwise similarity and not introducing further model parameters. An important feature of the model is the ability to expose



**Fig. 15** Relaxation times for two initial conditions (left and right plots, respectively) and two information types, vs.  $N$ . Times shown represent the number of pairwise interactions. Points represent averages over 100 simulation runs, with error bars corresponding to one standard deviation from the mean. The continuous lines show distributions of  $t$  for large population sizes, fitted to the simulation times for  $N \geq 5000$ . The distributions are of the form  $t = \alpha N^\beta$ , with  $\beta = 1$  (i.e., linear dependency) for all situations except  $\bar{o} \sim 0.49$  and  $\mathbf{I} = [0.4, 0.2, 0.2, 0.1, 0.1]$ , when  $\beta = 1.11$  (left panel, blue line).

the system to modulated external information, so that more possibilities can be promoted (e.g. by mass media).

Numerical results showed that extreme information causes fragmentation and has limited success, while moderate information causes cohesion and has a better success in attracting individuals. This coincides to the success of marketing or election campaigns, where coalitions and milder messages are more effective. Additionally, information success is maximised when individuals do not interact too much with the information, showing the importance of the social effect in information spreading. An important factor driving the capacity of information to influence the population appears to be the initial similarity to the agents. This is a known fact in devising marketing strategies, for instance, where information is displayed in a way appealing to the target audience.

Similar effects of external information have been observed previously for other models (either discrete or non-vectorial opinions). For instance, [14] observed that aggressive media campaigns are not effective, using the Deffuant model with external information, and that individuals should be exposed to the external influence gradually in order to optimise its success. Further, in [17,22], for the Hegselmann-Krause model, it was shown that extreme information causes formation of antagonistic clusters, while mild messages

are more successful. A segregation effect from external information has been also observed for the Axelrod model [15, 32, 31], similar to our observations for extreme information (information in discrete models, such as Axelrod, is equivalent to the extreme information in a continuous model, i.e., promoting one option only).

The initial condition proved to be of large importance in the dynamics, with compact populations resulting in one cluster, while less compact starting points yielding more groupings. A similar large effect of the initial condition has been recently shown in [4], for the Deffuant model. Additionally, for our model, the decrease in the transition point with  $K$  indicated that agreement is easier to obtain for a larger number of choices (with no external information). This is similar to the findings for the model in [25], which shares similarities to our approach. This uses a bounded confidence threshold  $d$ , and it was shown that the critical value of  $d$ , for which complete agreement is obtained, decreases as  $K$  increases. In our model, bounded confidence is not present, however the initial condition plays a similar role in determining the number of clusters, and we observe a very similar effect of  $K$  on the critical  $\bar{o}$  for which complete agreement is obtained.

A scaling analysis showed that results for both the effect of the initial condition and external information are robust with respect to the population size. Additionally, relaxation time (total number of updates), was shown to become linear in the population size as the size increases.

Several further analyses of the model presented are envisioned for the future. These include changing the dynamics to allow individuals to interact on more than one opinion choice and studying different social network topologies (here, all results are presented on a complete network). Additionally, application to real data, in order to simulate observed social processes and be able to make predictions, is required to further validate the model. In the context of the EveryAware project [10], the model will be also applied to simulate behavioural and opinion changes on environmental issues, based on subjective data which will be collected during test cases.

**Acknowledgements** This research has been supported by the EveryAware project funded by the Future and Emerging Technologies program (IST-FET) of the European Commission under the EU RD contract IST-265432 and the EuroUnderstanding Collaborative Research Projects DRUST funded by the European Science Foundation

## References

1. Acemoglu, D., Como, G.: Opinion fluctuations and disagreement in social networks. Massachusetts Institute of Technology Department of Economics Working Paper Series (2010)
2. Axelrod, R.: The dissemination of culture: a model with local convergence and global polarization. *The Journal of Conflict Resolution* **41**(2), 203–226 (1997)
3. Carletti, T., Fanelli, D., Grolli, S., Guarino, A.: How to make an efficient propaganda. *Europhysics Letters (EPL)* **74**(2), 222–228 (2006)
4. Carro, A., Toral, R., San Miguel, M.: The role of noise and initial conditions in the asymptotic solution of a bounded confidence, continuous-opinion model. arXiv preprint arXiv:1208.2618 (2012)

5. Castellano, C., Fortunato, S., Loreto, V.: Statistical physics of social dynamics. *Reviews of modern physics* **81**(2), 591 (2009)
6. Crokidakis, N.: Effects of mass media on opinion spreading in the Sznajd sociophysics model. *Physica A: Statistical Mechanics and its Applications* **391**, 1729–1734 (2011)
7. Deffuant, G., Neau, D., Amblard, F., Weisbuch, G.: Mixing beliefs among interacting agents. *Advances in Complex Systems* **3**(4), 87–98 (2000)
8. Degusta, M.: Are smart phones spreading faster than any technology in human history? *MIT Technology Review* (2012)
9. Duhigg, C.: *The Power of Habit: Why We Do What We Do in Life and Business*. Random House (2012)
10. EveryAware Consortium: EveryAware: Enhance environmental awareness through social information technologies. URL [www.everyaware.eu](http://www.everyaware.eu)
11. Galam, S.: Sociophysics: A review of Galam models. *International Journal of Modern Physics C* **19**, 403–440 (2008)
12. Galam, S.: Public debates driven by incomplete scientific data : the cases of evolution theory, global warming and H1N1 pandemic influenza. *Physica A: Statistical Mechanics and its Applications* **389**(17), 3619–3631 (2010)
13. Galam, S.: Market efficiency , anticipation and the formation of bubbles-crashes. arXiv preprint arXiv:1106.1577 **1106.1577**, 1–23 (2011)
14. Gargiulo, F., Lottini, S., Mazzoni, A.: The saturation threshold of public opinion: are aggressive media campaigns always effective? In: ESSA, pp. 1–4 (2008)
15. González-Avella, J.C., Cosenza, M.G., Eguíluz, V.M., San Miguel, M.: Spontaneous ordering against an external field in non-equilibrium systems. *New Journal of Physics* **12**(1) (2010)
16. Hegselmann, R., Krause, U.: Opinion dynamics and bounded confidence: models, analysis and simulation. *Journal of Artificial Societies and Social Simulation* **5**(3) (2002)
17. Hegselmann, R., Krause, U.: Truth and cognitive division of labour: First steps towards a computer aided social epistemology. *Journal of Artificial Societies and Social Simulation* **9**(3), 1–28 (2006)
18. Hong, H., Strogatz, S.: Conformists and contrarians in a Kuramoto model with identical natural frequencies. *Physical Review E* **84**(4), 1–6 (2011)
19. Huckfeldt, R., Johnson, P., Sprague, J.: *Political disagreement: The survival of diverse opinions within communication networks*. Cambridge University Press (2004)
20. Kondrat, G., Sznajd-Weron, K.: Spontaneous reorientations in a model of opinion dynamics with anticonformists. *International Journal of Modern Physics C* **21**, 559–566 (2010)
21. Kurmyshev, E., Juárez, H., González-Silva, R.: Dynamics of bounded confidence opinion in heterogeneous social networks: concord against partial antagonism. *Physica A: Statistical Mechanics and its Applications* pp. 1–16 (2011)
22. Kurz, S.: On the Hegselmann-Krause conjecture in opinion dynamics. *Journal of Difference Equations and Applications* **13**(6) (2011)
23. Lewenstein, M., Nowak, A., Latané, B.: Statistical mechanics of social impact. *Physical Review A* **45**(2), 763–776 (1992)
24. Lima, F.W.S.: Controlling the Tax Evasion Dynamics via Majority-Vote Model on Various Topologies. *Theoretical Economics Letters* **2**, 87–93 (2012)
25. Lorenz, J.: Continuous opinion dynamics of multidimensional allocation problems under bounded confidence: More dimensions lead to better chances for consensus. *European Journal of Economic and Social Systems* **19**, 213–227 (2006)
26. Manning, C., Schütze, H.: *Foundations of statistical natural language processing*, vol. 999. MIT Press (1999)
27. Martins, A.: Continuous opinions and discrete actions in opinion dynamics problems. *International Journal of Modern Physics C* **19**, 617–624 (2008)
28. Nowak, A., Kuñ, M.: Simulating the coordination of individual economic decisions. *Physica A: Statistical Mechanics and its Applications* **287**, 613–630 (2000)

- 
29. Nowak, A., Lewenstein, M.: Modeling social change with cellular automata. In: *Modelling and Simulation in the Social Sciences From a Philosophy of Science Point of View*, pp. 249–285. Kluwer Academic Publishers (1996)
  30. Nyczka, P., Sznajd-Weron, K., Cislo, J.: Phase transitions in the q-voter model with two types of stochastic driving. *Physical Review E* **86**(1) (2012)
  31. Peres, L., Fontanari, J.: The mass media destabilizes the cultural homogenous regime in Axelrod’s model. *Journal of Physics A: Mathematical and Theoretical* **43** (2010)
  32. Peres, L., Fontanari, J.: The media effect in Axelrod’s model explained. *EPL (Europhysics Letters)* **96**, 38,004 (2011)
  33. Radillo-Díaz, A., Pérez, L.A., del Castillo-Mussot, M.: Axelrod models of social influence with cultural repulsion. *Physical Review E* **80**(6), 1–6 (2009)
  34. Sznajd-Weron, K., Sznajd, J.: Opinion evolution in closed community. *International Journal of Modern Physics C* **11**, 1157–1165 (2000)
  35. Sznajd-Weron, K., Tabiszewski, M., Timpanaro, A.: Phase transition in the Sznajd model with independence. *EPL (Europhysics Letters)* **96**(4), 1–6 (2011)
  36. Vallacher, R.R., Nowak, A.: Dynamical social psychology: on complexity and coordination in human experience. In: M. Uhl-Bien, R. Marion (eds.) *Complexity leadership*, vol. 1, chap. 3, pp. 49–80. Information Age Pub Inc. (2008)
  37. Vaz Martins, T., Pineda, M., Toral, R.: Mass media and repulsive interactions in continuous-opinion dynamics. *EPL (Europhysics Letters)* **91**(4), 48,003 (2010)
  38. Weisbuch, G., Deffuant, G., Amblard, F., Nadal, J.P.: Meet, discuss, and segregate! *Complexity* **7**(3), 55–63 (2002)