A fuzzy set approach for reliability calculation of valve controlling electric actuators

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Abstract. The oil and gas equipment and electric actuators in particular frequently perform in various operational modes and under dynamic environmental conditions. These factors affect equipment reliability measures in a vague, uncertain way. To eliminate the ambiguity, reliability model parameters could be defined as fuzzy numbers. We suggest a technique that allows constructing fundamental fuzzy-valued performance reliability measures based on an analysis of electric actuators failure data in accordance with the amount of work, completed before the failure, instead of failure time. Also, this paper provides a computation example of fuzzy-valued reliability and hazard rate functions, assuming Kumaraswamy complementary Weibull geometric distribution as a lifetime (reliability) model for electric actuators.

1. Introduction

Reliability is the essential property of technical system or equipment. The key aspect of the management and control for any kind of industrial facility is reliability-centered maintenance (RCM) methodology [1-3]. It demands accurate definition of system reliability measures for decision making process concerning equipment maintenance [3, 4].

Failure data analysis allows selecting adequate reliability model in accordance with feasible lifetime distribution and estimating reliability model parameters [5]. In this context, times to failure of identical items are considered as random numbers drawn from the parent population. However, during the period of operation items (including valve controlling electric actuators) perform in various operation modes, subjected to the transient load under diverse environmental conditions [6]. In this regard, the reliability estimation of an electric actuator could be based on either random times or number of OPEN/CLOSE cycles to failure.

Considering the fact of ambiguity in operational conditions of electric actuators, it seems appropriate to apply reliability models with fuzzy parameters [7-10]. With this background, in order to determine joint influence of operating time and amount of work on reliability of an electric actuator, in this study the number of cycles to failure is regarded as a fuzzy value, whilst the reliability itself is considered to be a function of time.

2. Basic reliability measures

As a part of the study we suggest a technique for obtaining 3D-plots of fuzzy-valued functions of reliability and hazard rate [7, 9].
Reliability or survival function is the essential reliability measure defined as the probability that a failure doesn’t occur before specified time $t$:

$$ R(\Theta, t) = \Pr[T > t | \Theta] = 1 - F(\Theta, t), \quad t \geq 0. $$

Here $T$ is a random time to failure and $F(\Theta, t)$ is the cumulative distribution function (cdf) with parameter vector $\Theta$.

Hazard rate function (hrf) is a reliability measure which allows, among other things, defining system lifecycle phases according to the shape of hrf given by

$$ \lambda(\Theta, t) = \frac{F'(\Theta, t)}{1 - F(\Theta, t)}. \quad (1) $$

Generally, hrf has a bathtub-shaped plot depicted in figure 1.

![Bathtub-shaped hazard rate function](image)

**Figure 1.** Bathtub-shaped hazard rate function.

Burn-in period is specified by the failures of substandard equipment and by the failures caused by design flaws, construction and start-up operation errors. Useful life period is characterized by the lowest and nearly constant values of the hazard rate function. Finally, the third period represents failures, occurred due to wear-out or ageing [1-4].

3. Failure Data Preparation and Results

Electric actuator failure data is represented as failure times and amount of work completed before the failure, i.e. number of OPEN/CLOSE cycles to failure. Failure data on 286 electric actuators, received for the study, were grouped into 7 equally sized bins. Since the number of cycles to failure varies between 0 and 17500, the width of each bin is 2500.

Figure 2 shows the histogram of number of cycles, provided that each bin is assigned with normalized membership value, proportional to its height. Then, times of failures, associated with $i^{th}$ bin, should be analyzed to estimate parameters of the lifetime model [11].

For this study Kumaraswamy complementary Weibull geometric (Kw-CWG) distribution, introduced in [12], was selected as a reliability model. This distribution has five parameters, providing opportunities for a fine tuning the shape of reliability function which is essential in parameter estimation. Moreover, the reliability model based on the Kw-CWG distribution is very flexible and contains 23 sub-models as special cases [12], including Weibull distribution, widely used in reliability engineering and survival analysis [13]. It is possible to obtain constant, increasing, decreasing, bathtub and unimodal shape of hrf by changing the set of distribution parameter values. According to Kw-CWG model the reliability function is determined as

$$ R(t) = \left\{ 1 - \alpha^a \left( 1 - \exp \left[ -\left( \gamma t \right)^\beta \right] \right)^a \left( \alpha + (1 - \alpha) \exp \left[ -\left( \gamma t \right)^\beta \right] \right)^{-a} \right\}^b, \quad (2) $$
where $\gamma > 0$ is a scale parameter and $\alpha > 0$, $\beta > 0$, $a > 0$, $b > 0$ are shape parameters.

\[ \lambda(t) = \frac{\alpha^a \beta \gamma a b (\gamma t)^{\beta-1} \exp[-(\gamma t)^{\beta}]}{\alpha + (1-\alpha) \exp[-(\gamma t)^{\beta}]} \left\{ \frac{\alpha - \alpha \exp[-(\gamma t)^{\beta}]}{\alpha + (1-\alpha) \exp[-(\gamma t)^{\beta}]} \right\}^{-1}. \]  

(3)

Figure 2. Histogram of number of OPEN/CLOSE cycles to failure.

The hrf of Kw-CWG reliability model is expressed as

\[
\lambda(t) = \frac{\alpha^a \beta \gamma ab (\gamma t)^{\beta-1} \exp[-(\gamma t)^{\beta}]}{\alpha + (1-\alpha) \exp[-(\gamma t)^{\beta}] \left\{ \frac{\alpha - \alpha \exp[-(\gamma t)^{\beta}]}{\alpha + (1-\alpha) \exp[-(\gamma t)^{\beta}]} \right\}^{-1}}.
\]

We assume that electric actuators operate under similar conditions if their respective numbers of cycles to failure fall into the same bin of the histogram (figure 2). Proceeding from this assumption, seven reliability functions were obtained, each corresponding to the relevant bin (figure 3).

Figure 3. Reliability functions for the different bins.

Point estimations of parameters $\hat{\Theta}_i$ for the reliability functions, obtained by maximum likelihood estimation [11-13], are accumulated in table 1.

Since each bin of the histogram is attributed with the membership value, we plot these reliability functions in 3D space, assigning each one with the corresponding value along Z-axis.
Table 1. Estimated parameters for Kw-CWG distribution

<table>
<thead>
<tr>
<th>Parameter</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>1.28</td>
<td>0.16</td>
<td>0.03</td>
<td>0.72</td>
<td>0.06</td>
<td>0.09</td>
<td>0.18</td>
</tr>
<tr>
<td>$\beta$</td>
<td>10.59</td>
<td>4.94</td>
<td>5.19</td>
<td>5.27</td>
<td>2.93</td>
<td>2.71</td>
<td>1.52</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$2.63 \cdot 10^{-5}$</td>
<td>$3.95 \cdot 10^{-5}$</td>
<td>$5.18 \cdot 10^{-5}$</td>
<td>$5.25 \cdot 10^{-5}$</td>
<td>$7.77 \cdot 10^{-5}$</td>
<td>$8.57 \cdot 10^{-5}$</td>
<td>$1.4 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>a</td>
<td>1.76</td>
<td>1.33</td>
<td>0.59</td>
<td>1.82</td>
<td>0.99</td>
<td>0.79</td>
<td>1.79</td>
</tr>
<tr>
<td>b</td>
<td>0.49</td>
<td>0.35</td>
<td>0.10</td>
<td>0.13</td>
<td>0.25</td>
<td>0.43</td>
<td>0.68</td>
</tr>
</tbody>
</table>

The result is a fuzzy-valued reliability function $\tilde{R}(\Theta, t)$ presented itself as a 3D surface (figures 4, 5).

![Figure 4](image1.png)

**Figure 4.** Plane view of fuzzy-valued reliability function of electric actuator.

![Figure 5](image2.png)

**Figure 5.** Isometric view of fuzzy-valued reliability function of electric actuator.

Fuzzy-valued hrf (figures 6, 7) was obtained by a similar way, provided that estimated parameters were substituted into equation (3). Figure 6 shows the absence of decreasing hazard rate segment for electric actuators, indicating that there were no errors committed during construction and start-up operations and that all flawed items were rejected earlier.
Useful results can be obtained by cutting the surface plot of the fuzzy-valued reliability function with planes, perpendicular to the coordinate axes. For example, if the cutting plane is defined as $t = t^*$, the trace of the surface can be interpreted as a fuzzy value of reliability for a mission time $t^*$. Similarly, cutting $\hat{R}(\Theta, t)$ with a plane $R = \gamma$ gives us a fuzzy value of $100(1-\gamma)th$ percentile life [1, 7]. Figure 8 shows the examples of such cuts for the electric actuators under study.

These results can be defuzzified by Centre of Gravity (or any other) method [14, 15] in order to obtain the crisp values of reliability and percentile life.

Figure 8. Traces of fuzzy-valued reliability function for electric actuators:

a – fuzzy percentile life; b – fuzzy reliability.
4. Conclusion
In this paper the approach for determining of fuzzy reliability measures was suggested. Within this framework, fuzzy-valued reliability function of an electric actuator was obtained, as well as its hazard rate function. These measures can be applied along with material approach of reliability assessment, based on the analysis of electrophysical effects and mechanical friction and wear-out processes, in order to increase fault tolerance of electric actuators.

Informative and examined in detail performance reliability measures are essential for RCM analysis of technical systems. It is necessary to identify the causes of equipment failures and to acquire adequate reliability models in order to enhance the efficiency of preventive maintenance management program. In particular, fuzzy-valued reliability measures of an electric actuator, acquired in this paper, facilitate the integration of time and work completed before the failure and, hence, the assessment of their joint influence on system performance.

References