Orthogonal Code Design for MIMO Amplify-and-Forward Cooperative Networks

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***Abstract*— This paper is on the design of practical distributed space-time codes for wireless relay networks with multiple antennas terminals. The amplify-and-forward scheme is used in a way that each relay transmits a scaled version of the linear combination of the received symbols. We propose distributed orthogonal space-time codes which are distributed among the source node's antennas and relays. Using linear orthogonal decoding in the destination makes it feasible to employ large number of potential relays to improve the diversity order. Assuming multiple amplitude modulation, we derive a formula for the symbol error probability of the investigated scheme over Rayleigh fading channels. Our analytical results have been confirmed by simulation results, using full-rate, full-diversity distributed codes.**

I. Introduction

In [1], a cooperative strategy was proposed which achieves a diversity factor of R in a R-relay wireless network, using the so-called distributed space-time codes (DSTC). A two-phase protocol is used for this strategy. In phase one, the transmitter sends the information signal to the relays and in phase two, relays send information to the receiver. The signal sent by every relay in the second phase is designed as a linear function of its received signal. It was shown that the relays can generate a linear space-time codeword at the receiver, as in a multiple antenna system, although they only cooperate distributively. This method does not require decoding at the relays and for high SNR it achieves the optimal diversity factor [1]. Although distributed space-time coding does not need instantaneous channel information in the relays, it requires full channel information at the receiver, i.e., both the channels from the transmitter to relays and the channels from relays to the receiver, need to be known at the receiver. This requires that training symbols be sent from both the transmitter and relays. Recently, the design of practical DSTC in amplify-and-forward (A&F) mode, that lead to reliable communication in wireless relay networks, has been presented in [2] and [3].

Distributed space-time coding in A&F mode was generalized to networks with multiple-antenna nodes in [4]. It is shown that in a wireless network with N_s antennas at the transmit node, N_d antennas at the receive node, and a total of R antennas at all relay nodes, the diversity order of $R\min\{N_s, N_d\}$ is achievable [4], [5]. However, design of the appropriate space-time codes is *not* investigated in [4]. In [6], an algebraic coding scheme is introduced for A&F DSTC. Note that in decode-and-forward (D&F) based space-time codes, we can simply use the same space-time codes in the context of MIMO in multiple antenna source and relays [7]. Compared with D&F, since no decoding is needed at the relays, A&F DSTC saves both time and energy and more importantly, there is no rate constraint on the transmission.

In this paper, we focus on the design of the MIMO orthogonal DSTCs for multiple antenna terminals with A&F relays, which are systematically constructed, orthogonally decodable, full-rate, full-diversity distributed codes. The proposed space-time codes are distributed among the *source antennas* and the *relays*. Using these distributed codes with linear orthogonal decoding, we can employ large number of antennas and potential relays to improve diversity order. Furthermore, we derive the approximate average symbol error rate (SER) for multiple antenna A&F DSTC with multiple amplitude modulation (M-AM) over Rayleigh-fading channels. The method of moment generating function (MGF) is used for performance analysis, which is valid for *any* full-diversity, full-rate space-time block codes, such as orthogonal DSTCs.

The rest of this paper is organized as follows. In Section II, the system model is given. The formulation of A&F DSTC in matrix form is considered in Section III. The orthogonal DSTC for multiple antenna nodes is presented in Section IV. In Section V, the average SER of DSTC under M-AM modulation is derived. In Section VI, the overall system performance is presented, and the correctness of the analytical formula is confirmed by simulation results. Conclusions are presented in Section VII.

Notations: The superscripts ^{t} and ^{H} stand for transposition and conjugate transposition, respectively. The expectation operation is denoted by $\mathbb{E}\{\cdot\}$. The symbol $\boldsymbol{I}_T$ stands for the $T \times T$ identity matrix. $\|\boldsymbol{A}\|$ denotes the Frobenius norm of the matrix $\boldsymbol{A}$. The trace of the matrix $\boldsymbol{A}$ is denoted by $\mathrm{tr}\{\boldsymbol{A}\}$. $\mathrm{diag}\{\boldsymbol{A}_1, \ldots, \boldsymbol{A}_R\}$ denotes the block diagonal matrix.

II. System Model

Consider a wireless communication scenario where the source node s transmits information to the destination node d with the assistance of one or more relays denoted Relay $r = 1, 2, \ldots, R$. The source and destination nodes are equipped with N_s and N_d antennas, respectively (see Fig. 1). Without

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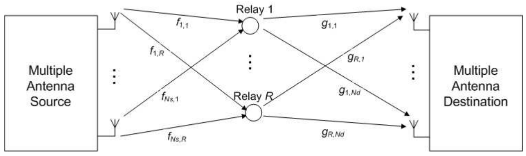

Fig. 1. Wireless relay network including one source with N_s antennas, R relays, and one destination with N_d antennas.

loss of generality, it is assumed that each relay node is equipped with a single antenna. Note that this network can be transformed to relays with multiple antenna, since the transmit and receive signals at different antennas of the same relay can be processed and designed independently.

We denote the links from N_s antennas of the source to the rth relay as $f_{1,r}, f_{2,r}, \ldots, f_{N_s,r}$, and the links from the rth relay to the N_d antennas at the destination as $g_{r,1}, g_{r,2}, \ldots, g_{r,N_d}$. Under the assumption that each link undergoes independent Rayleigh process $f_{i,r}$, and $g_{r,j}$ are independent complex Gaussian random variables with zero-mean and variances $\sigma^2_{f_r}$, and $\sigma^2_{g_r}$, respectively. Since multiple antennas in source and destination are co-located, and the co-located antennas have the same distances to relays, we skipped the i and j indices of $\sigma^2_{f_r}$ and $\sigma^2_{g_r}$.

Assume that the source wants to send K symbols s_1, s_2, $\ldots$, s_K to the destination during T time slots. T should be less than the coherent interval, that is, the time duration among which channels $f_{i,r}$, and $g_{r,j}$ keep constant. Henceforth, we assume using full-rate space-time codes, and thus, $K = T$. Similar to [1], our scheme requires two phases of transmission. During the first phase, the source should transmit a $T \times N_s$ dimensional orthogonal code matrix $\boldsymbol{S}_1$ to *all* relays. We can represent $\boldsymbol{S}_1$ in terms of the vector $\boldsymbol{s} = [s_1, s_2, \ldots, s_T]^t$, consisting of T information symbols as

$$\boldsymbol{S}_1 = [\boldsymbol{A}_1\boldsymbol{s}\, \boldsymbol{A}_2\boldsymbol{s}\, \ldots\, \boldsymbol{A}_{N_s}\boldsymbol{s}], \tag{1}$$

where $\boldsymbol{A}_i$, $i = 1, \ldots, N_s$, are $T \times T$ unitary matrices, and $\boldsymbol{s}_i = \boldsymbol{A}_i\boldsymbol{s}$ describes the ith column of a $T \times N_s$ orthogonal space-time code. We assume the following normalization

$$\mathbb{E}\left[\operatorname{tr}\{\boldsymbol{S}_1^H\boldsymbol{S}_1\}\right] = \mathbb{E}\left[\operatorname{tr}\left\{\sum_{k=1}^{T}|s_k|^2\boldsymbol{I}_{N_s}\right\}\right] = N_s. \tag{2}$$

The source transmits $\sqrt{P_1T/N_s}\boldsymbol{S}_1$ where P_1T is the average total power used at the source during the first phase. Thus, $\sqrt{P_1T/N_s}\boldsymbol{s}_i$, $i = 1, \ldots, N_s$, is the signal sent by the ith antenna with the average power of P_1T/N_s. Assuming that $f_{i,r}$ does not vary during T successive intervals, the $T \times 1$ receive signal vector at the rth relay is

$$\boldsymbol{x}_r = \sqrt{P_1T}\boldsymbol{S}_1\boldsymbol{f}_r + \boldsymbol{v}_r, \tag{3}$$

where $\boldsymbol{f}_r = [f_{1,r}\, f_{2,r}\, \ldots\, f_{N_s,r}]^t$, and $\boldsymbol{v}_r$ is a $T \times 1$ complex zero-mean white Gaussian noise vector with variance $\mathcal{N}_1$.

In the second phase of the transmission, all relays simultaneously transmit linear functions of their received signals $\boldsymbol{x}_r$. In order to construct a distributed space-time codes, the received signal at the jth antenna of the *destination* is collected inside the $T \times 1$ vector $\boldsymbol{y}_j$ as

$$\boldsymbol{y}_j = \sum_{r=1}^{R} g_{r,j}\, \rho_r\boldsymbol{C}_r\boldsymbol{x}_r + \boldsymbol{w}_j, \tag{4}$$

for $j = 1, 2, \ldots, N_r$, where $\boldsymbol{w}_j$ is a $T \times 1$ complex zero-mean white Gaussian noise vector with component-wise variance $\mathcal{N}_2$, ρ_r is the scaling factor at relay r, and $\boldsymbol{C}_r$, of size $T \times T$, are obtained by representing the rth column of an appropriate $T \times R$ dimensional space-time code matrix as $\boldsymbol{C}_r\boldsymbol{s}$. This construction method originates from the construction of a space-time code for co-located multiple-antenna systems, where the transmitted signal vector from the kth antenna is $\boldsymbol{C}_k\boldsymbol{s}$ [8]. When there is no instantaneous channel state information (CSI) at the relays, but statistical CSI is known, a useful constraint is to ensure that a given average transmitted power is maintained. That is, $\rho_r = \sqrt{\frac{P_{2,r}}{\sigma^2_{f_r}P_1 + \mathcal{N}_1}}$, where $P_{2,r}$ is the average transmit power from relay r.

III. Distributed Space-Time Codes in Amplify-and-Forward Mode

In this section, we formulate the A&F DSTC system in matrix-form, and calculate the received SNR at the destination. We can represent input-output relationship of the DSTC as the space-time code in a multiple-antenna system. By setting the $T \times N_sR$ space-time encoded signal

$$\boldsymbol{S} = [\boldsymbol{C}_1\boldsymbol{S}_1,\, \boldsymbol{C}_2\boldsymbol{S}_1,\, \ldots,\, \boldsymbol{C}_R\boldsymbol{S}_1], \tag{5}$$

and by concatenating the received signals of the destination antennas, i.e., $\boldsymbol{Y} = [\boldsymbol{y}_1\, \boldsymbol{y}_2\, \ldots\, \boldsymbol{y}_{N_d}]$, from (3)-(4), we have

$$\boldsymbol{Y} = \sqrt{\frac{P_1T}{N_s}}\boldsymbol{S}\boldsymbol{H} + \boldsymbol{W}_T, \tag{6}$$

where the $N_sR \times N_d$ channel matrix $\boldsymbol{H}$ is defined as

$$\boldsymbol{H} = \boldsymbol{F}\boldsymbol{\Lambda}\boldsymbol{G},$$

$$\boldsymbol{F} = \operatorname{diag}\{\boldsymbol{f}_1, \ldots, \boldsymbol{f}_R\}, \qquad \boldsymbol{\Lambda} = \operatorname{diag}\{\rho_1, \ldots, \rho_R\},$$

$$\boldsymbol{g}_r = [g_{r,1}\, g_{r,2}\, \ldots\, g_{r,N_d}], \qquad \boldsymbol{G} = \left[\boldsymbol{g}_1^t\, \ldots\, \boldsymbol{g}_R^t\right]^t,$$

and the noise is collected into the $T \times N_d$ matrix

$$\boldsymbol{W}_T = \boldsymbol{V}\boldsymbol{\Lambda}\boldsymbol{G} + \boldsymbol{W}. \tag{7}$$

where $\boldsymbol{V} = [\boldsymbol{C}_1\boldsymbol{v}_1\, \boldsymbol{C}_2\boldsymbol{v}_2\, \ldots\, \boldsymbol{C}_R\boldsymbol{v}_R]$ and $\boldsymbol{W} = [\boldsymbol{w}_1\, \boldsymbol{w}_2\, \ldots\, \boldsymbol{w}_{N_d}]$. Now, we derive the covariance of $\boldsymbol{W}_T$ which will be used for calculating the received SNR at the destination. Since $g_{r,j}$, $\boldsymbol{v}_r$, and $\boldsymbol{w}_j$ are zero-mean complex Gaussian random variables and mutually independent, the covariance matrix of $\boldsymbol{W}_T$ can be shown to be

$$\begin{aligned}\operatorname{Cov}(\boldsymbol{W}_T) &= \mathbb{E}[\boldsymbol{V}\boldsymbol{\Lambda}\boldsymbol{G}\boldsymbol{G}^H\boldsymbol{\Lambda}^H\boldsymbol{V}^H] + \mathbb{E}[\boldsymbol{W}\boldsymbol{W}^H] \\ &= \sum_{r=1}^{R}\rho_r^2\sigma^2_{g_r}N_d\mathcal{N}_1\boldsymbol{C}_r\boldsymbol{C}_r^H + N_d\mathcal{N}_2\boldsymbol{I}_T \\ &= N_d\left(\sum_{r=1}^{R}\rho_r^2\sigma^2_{g_r}\mathcal{N}_1 + \mathcal{N}_2\right)\boldsymbol{I}_T. \end{aligned} \tag{8}$$

Thus, the noise vector $\boldsymbol{W}_T$ is white. The third equality in (8) follows from the fact that for each relay, $\boldsymbol{C}_r$ is a unitary matrix.

Since, in this paper, we focus on orthogonal design, the maximum likelihood (ML) detection is decomposed to single-symbol detection, and by the fact that noise in (8) is white, maximal-ratio combining (MRC) can be applied at the destination. To calculate the post detection SNR at the output of the ML DSTC decoder, we need to compute the received signal power. Hence, using (6), we have

$$\eta_{s_d} = \frac{P_1 T}{N_s}\mathbb{E}_s\left[\text{tr}\{\boldsymbol{S}\boldsymbol{H}\boldsymbol{H}^H\boldsymbol{S}^H\}\right] = \frac{P_1 T}{N_s}\mathbb{E}_s\left[\text{tr}\{\boldsymbol{H}\boldsymbol{H}^H\boldsymbol{S}^H\boldsymbol{S}\}\right]$$
$$= \frac{P_1 T}{N_s}\,\text{tr}\{\boldsymbol{H}\boldsymbol{H}^H\,\mathbb{E}_s[\boldsymbol{S}^H\boldsymbol{S}]\}. \quad (9)$$

To have a linear orthogonal ML detection, we should design distributed DSTC, such that

$$\boldsymbol{S}^H\boldsymbol{S} = (|s_1|^2 + |s_2|^2 + \ldots + |s_T|^2)\boldsymbol{I}_{N_sR}, \quad (10)$$

and using the normalization assumed in (2), we have

$$\mathbb{E}_s[\boldsymbol{S}^H\boldsymbol{S}] = \boldsymbol{I}_{N_sR}. \quad (11)$$

Thus, η_{s_d} in (9) can be evaluated as

$$\eta_{s_d} = \frac{P_1 T}{N_s}\,\text{tr}\{\boldsymbol{H}\boldsymbol{H}^H\} = \frac{P_1 T}{N_s}\sum_{i=1}^{N_sR}\left[\boldsymbol{H}\boldsymbol{H}^H\right]_{i,i}$$
$$= \frac{P_1 T}{N_s}\sum_{r=1}^{R}\sum_{n=1}^{N_s}|f_{n,r}|^2\rho_r^2\sum_{j=1}^{N_d}|g_{r,j}|^2 = \frac{P_1 T}{N_s}\sum_{r=1}^{R}\rho_r^2\|\boldsymbol{f}_r\|^2\|\boldsymbol{g}_r\|^2. \quad (12)$$

From (8), the total noise power at the destination can be written as

$$\eta_{w_T} = \mathbb{E}\text{tr}\{\boldsymbol{W}_T\boldsymbol{W}_T^H\} = N_d T\left(\sum_{r=1}^{R}\rho_r^2\sigma_{g_r}^2\mathcal{N}_1 + \mathcal{N}_2\right). \quad (13)$$

Combining (12) and (13), the received SNR at the destination can be written as $\text{SNR}_d = \sum_{r=1}^{R}\gamma_r$, where

$$\gamma_r = \frac{P_1\rho_r^2\|\boldsymbol{f}_r\|^2\|\boldsymbol{g}_r\|^2}{N_sN_d\sum_{k=1}^{R}\rho_k^2\sigma_{g_k}^2\mathcal{N}_1 + N_sN_d\mathcal{N}_2}. \quad (14)$$

It is important to note that in (14), we approximated the conditional variance of the noise vector $\boldsymbol{W}_T$ in (7), which is obtained in (8), with its expected value.

IV. Real Orthogonal Design for A&F MIMO DSTC

In this section, we propose a systematic orthogonal design to construct DSTC in A&F relay networks consisting of multiple antennas, which achieve full-diversity and full-rate. In addition, these codes are generalized to any number of transmit antennas or relays.

The distributed space-time code matrix $\boldsymbol{S}$ should be appropriately designed to achieve full diversity. Combining (1) and (5), we can further rewrite $\boldsymbol{S}$ as

$$\boldsymbol{S} = [\boldsymbol{C}_1\boldsymbol{A}_1\boldsymbol{s},\ \ldots, \boldsymbol{C}_1\boldsymbol{A}_{N_s}\boldsymbol{s},\ \ldots, \boldsymbol{C}_R\boldsymbol{A}_1\boldsymbol{s}, \ldots, \boldsymbol{C}_R\boldsymbol{A}_{N_s}\boldsymbol{s}]. \quad (15)$$

Since the distributed space-time code $\boldsymbol{S}$ has size $T \times N_sR$, there is no point in having N_sR larger than the coherence interval T and the diversity is determined by T in this case (see [9]). Thus, in the following, we will always assume $T \geq N_sR$. For symbols with real modulations, there exists full-rate, full-diversity space-time codes with real orthogonal design [10]. Since each component of the code matrix is a linear combination of symbols $\{s_1, \ldots, s_T\}$, we can represent the $(rN_s - N_s + i)$th column of the matrix code as $\boldsymbol{A}_i\boldsymbol{C}_r\boldsymbol{s}$, where $\boldsymbol{A}_i\boldsymbol{C}_r$ is again a unitary matrix. The coding problem consists of designing unitary matrices $\boldsymbol{A}_i$, $i = 1, \ldots, N_s$, and $\boldsymbol{C}_r$, $r = 1, \ldots, R$, such that $\boldsymbol{S}$ as given in (15) is full rank, or equivalently, condition in (10) is satisfied.

In the following, we systematically construct orthogonal DSTC. A subset of the orthogonal DSTCs is proposed, whose associated matrices $\boldsymbol{A}_i$ and $\boldsymbol{C}_r$ have the structure of a permutation matrix whose entries can be 1, 0, or -1. We consider square matrices of size $T = N_sR = 2^N$, $N = 1, 2, 3$. If one needs a rectangular space-time code, one can always pick some columns of a square code. If the codebook is fully diverse, then the codebook obtained by removing columns will be fully diverse too (see, e.g., [1] where this phenomenon has been considered in the context of node failures).

First, using the Hurwitz-Radon theory [11], we can construct $T \times T$ orthogonal matrices with real-valued components. For example, the 2×2 matrix is $\boldsymbol{S}^{(1)} = \begin{bmatrix} s_1 & -s_2 \\ s_2 & s_1 \end{bmatrix}$. Then, we construct $\boldsymbol{A}_1, \boldsymbol{A}_2, \ldots, \boldsymbol{A}_{N_s}$ such that $\boldsymbol{A}_i\boldsymbol{s}$, $i = 1, \ldots, N_s$, are associated with the first N_s columns of $\boldsymbol{S}^{(1)}$. We set $\boldsymbol{C}_1 = \boldsymbol{I}_N$, and we construct $\boldsymbol{C}_r$, $r = 2, \ldots, R$, such that $\boldsymbol{C}_r\boldsymbol{s}$ is the $(rN_s - N_s + 1)$th column of $\boldsymbol{S}^{(1)}$. Then, we modify matrix $\boldsymbol{S}^{(1)}$ by replacing the consecutive columns $rN_s - N_s + 1$ to rN_s with $\boldsymbol{C}_r\boldsymbol{A}_i\boldsymbol{s}$, $r = 2, \ldots, R$, $i = 1, \ldots, N_s$, and hence, the orthogonal matrix $\boldsymbol{S}$ becomes in the form of (15).

The orthogonal DSTC of any size for arbitrary number of N_s and R can be constructed by simply removing $T - N_sR$ columns of the $T \times T$ matrix $\boldsymbol{S}^{(1)}$. As an example, the application of the proposed 4×4 and 8×8 orthogonal DSTCs is demonstrated.

The 4×4 DSTC matrix consisting of real symbols, which is obtained by procedure given above, can be shown as

$$\boldsymbol{S} = \begin{bmatrix} s_1 & -s_2 & -s_3 & s_4 \\ s_2 & s_1 & s_4 & s_3 \\ s_3 & -s_4 & s_1 & -s_2 \\ s_4 & s_3 & -s_2 & -s_1 \end{bmatrix}. \quad (16)$$

For the case of $N_s = 2, R = 2$, the matrices used at the source and relays are $\boldsymbol{A}_1 = \boldsymbol{C}_1 = \boldsymbol{I}_4$, and

$$\boldsymbol{A}_2 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \boldsymbol{C}_2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}. \quad (17)$$

Moreover, when $N_s = 1$ ($R = 4$) or $R = 1$ ($N_s = 8$), it is straightforward to find the corresponding $\boldsymbol{A}_i$ and $\boldsymbol{C}_r$ from (16).

Using the Hurwitz-Radon theory [11], a 8×8 DSTC matrix with orthogonal design can be shown as

$$\boldsymbol{S}=\begin{bmatrix} s_1 & -s_2 & -s_3 & s_4 & -s_5 & s_6 & -s_7 & -s_8 \\ s_2 & s_1 & s_4 & s_3 & s_6 & s_5 & -s_8 & s_7 \\ s_3 & -s_4 & s_1 & -s_2 & -s_7 & -s_8 & s_5 & -s_6 \\ s_4 & s_3 & -s_2 & -s_1 & -s_8 & s_7 & -s_6 & -s_5 \\ s_5 & -s_6 & s_7 & s_8 & s_1 & -s_2 & -s_3 & s_4 \\ s_6 & s_5 & s_8 & -s_7 & -s_2 & -s_1 & s_4 & s_3 \\ s_7 & s_8 & -s_5 & s_6 & s_3 & -s_4 & s_1 & -s_2 \\ s_8 & -s_7 & -s_6 & -s_5 & s_4 & s_3 & s_2 & s_1 \end{bmatrix}, \quad (18)$$

which is valid for all cases of $N_s = 4$ ($R = 2$), $N_s = 2$ ($R = 4$), $N_s = 8$ ($R = 1$), and $N_s = 1$ ($R = 8$). Since $\boldsymbol{A}_i\boldsymbol{C}_r\boldsymbol{s}$ corresponds to the $(rN_s - N_s + 1)$th column of the code matrix $\boldsymbol{S}$, one can construct the integer matrices - matrices that all of their elements are -1, 0, or 1 - $\boldsymbol{A}_1, \boldsymbol{A}_2, \ldots, \boldsymbol{A}_{N_s}$ and $\boldsymbol{C}_1, \boldsymbol{C}_2, \ldots, \boldsymbol{C}_R$ to be used at the source's antennas and the relays, respectively. Due to lack of space, however, we omit to write the corresponding matrices $\boldsymbol{A}_i$ and $\boldsymbol{C}_r$.

V. Performance Analysis

In this section, we will derive the SER formulas of DSTC in A&F relay networks with multiple antennas nodes. We assume real constellations like multiple amplitude modulation (AM) signals, and our analysis is valid for any full-rate full-diversity DSTC such as the codes proposed in Section III.

The conditional SER of the protocol described in Section II, with R relays, one source, and one destination with multiple antennas, when M-AM signals are used, can be written as [12, Eq. (8.3)]

$$P_e\left(R|\boldsymbol{F}, \boldsymbol{G}\right) = 2\left(\frac{M-1}{M}\right) Q\left(\sqrt{\frac{6}{M^2-1}\sum_{r=1}^{R}\gamma_r}\right), \quad (19)$$

where $Q(x) = 1/\sqrt{2\pi}\int_x^{\infty} e^{-u^2/2}\,du$. For BPSK, (8.3) becomes the $P_e\left(R|\boldsymbol{F}, \boldsymbol{G}\right) = Q\left(\sqrt{2\sum_{r=1}^{R}\gamma_r}\right)$. The average value of γ_r in (14) can be written as

$$\overline{\gamma}_r = \mathbb{E}[\gamma_r] = \frac{P_1\rho_r^2\sigma_{f_r}^2\sigma_{g_r}^2}{\sum_{k=1}^{R}\rho_k^2\sigma_{g_k}^2\mathcal{N}_1 + \mathcal{N}_2}. \quad (20)$$

For calculating the average value of conditional SER in (14) we need to find probability density function (PDF) of γ_r.

Theorem *1:* For γ_r in (14), the probability density function $p_r(\gamma_r)$ can be written as

$$p_r(\gamma) = \frac{2}{\gamma\,(N_d-1)!\,(N_s-1)!}\left(\frac{\gamma}{\overline{\gamma}_r}\right)^{\frac{N_d+N_s}{2}} K_{N_d-N_s}\left(2\sqrt{\frac{\gamma}{\overline{\gamma}_r}}\right), \quad (21)$$

where $K_n(x)$ is the modified Bessel function of the second kind of order n.

Proof: Suppose $X = \frac{\|\boldsymbol{f}_r\|^2}{N_s\sigma_{f_r}^2}$ and $Y = \frac{\overline{\gamma}_r\|\boldsymbol{g}_r\|^2}{N_d\sigma_{g_r}^2}$, where X and Y have gamma distribution with mean of $\overline{X} = 1$ and $\overline{Y} = \overline{\gamma}_r$, respectively. Therefore, $\gamma_r = X\,Y$ and its cumulative density function can be presented to be

$$\begin{aligned} \Pr\{\gamma_r < \gamma\} &= \Pr\{XY < \gamma\} = \int_0^{\infty}\Pr\{Xy < \gamma\}p_Y(y)dy \\ &= \int_0^{\infty}\left(1 - \frac{\Gamma\left(N_s, \frac{\gamma}{y}\right)}{\Gamma(N_s)}\right)\frac{y^{N_d-1}}{(N_d-1)!\overline{\gamma}_r^{N_d}}e^{-\frac{y}{\overline{\gamma}_r}}dy \\ &= 1 - \int_0^{\infty}\frac{\Gamma\left(N_s, \frac{\gamma}{y}\right)}{\Gamma(N_s)}\frac{y^{N_d-1}}{(N_d-1)!\overline{\gamma}_r^{N_d}}e^{-\frac{y}{\overline{\gamma}_r}}dy, \end{aligned} \quad (22)$$

where we have used [13, Eq. (3.324)] for the third equality, $\Gamma(\alpha, x)$ is the incomplete gamma function of order α [3, Eq. (8.350)], and $p_Y(y) = \frac{y^{N_d-1}}{(N_d-1)!\overline{\gamma}_r^{N_d}}e^{-\frac{y}{\overline{\gamma}_r}}$ [12, Eq. (5.14)]. Then, using (22), $\Gamma(N_s) = (N_s-1)!$, and $\frac{-d\,\Gamma(\alpha,x)}{dx} = x^{\alpha-1}e^{-x}$ [3, Eq. (8.356)], the PDF of γ_r can be written as

$$\begin{aligned} p_r(\gamma) &= \frac{d}{d\gamma}\Pr\{\gamma_r < \gamma\} = \frac{\gamma^{N_s-1}}{\overline{\gamma}_r^{N_d}(N_d-1)!\,(N_s-1)!} \\ &\times \int_0^{\infty} y^{N_d-N_s-1}e^{-\left(\frac{\gamma}{y}+\frac{y}{\overline{\gamma}_r}\right)}dy. \end{aligned} \quad (23)$$

Thus, the PDF of γ_i can be found by solving the integral in (4) using [3, Eq. (3.471)], yielding (21). ■

Since the γ_rs are independent, using the MGF approach, the average SER would be

$$\begin{aligned} P_e(R) &= \int_{0;\,R-\text{fold}}^{\infty} P_e\left(R|\boldsymbol{F}, \boldsymbol{G}\right)\prod_{r=1}^{R}\left(p(\gamma_r)\,d\gamma_r\right) \\ &= \int_{0;\,R-\text{fold}}^{\infty}\frac{2(M-1)}{M}Q\left(\sqrt{\frac{6\sum_{r=1}^{R}\gamma_r}{M^2-1}}\right)\prod_{r=1}^{R}\left(p(\gamma_r)\,d\gamma_r\right) \\ &= \int_{0;\,R-\text{fold}}^{\infty}\frac{2(M-1)}{\pi M}\int_0^{\frac{\pi}{2}}\prod_{r=1}^{R}e^{\frac{-3\gamma_r}{(M^2-1)\sin^2\phi}}\,d\phi\prod_{r=1}^{R}\left(p(\gamma_r)\,d\gamma_r\right) \\ &= \frac{2(M-1)}{\pi M}\int_0^{\frac{\pi}{2}}\prod_{r=1}^{R}M_r\left(\frac{-3}{(M^2-1)\sin^2\phi}\right)d\phi, \end{aligned} \quad (24)$$

where $M_r(s) = \mathbb{E}_\gamma\{e^{\gamma_r s}\}$ is the MGF of γ_r. In the following theorem, we derive a closed-form solution for $M_r(s)$.

Theorem *2:* The MGF of random variable γ_r, i.e., $M_r(s) = \mathbb{E}_\gamma\{e^{\gamma_r s}\}$, is given by

$$M_r(-s) = (\overline{\gamma}_r s)^{-\mu+\frac{1}{2}}\,e^{\frac{1}{2\overline{\gamma}_r s}}W_{-\mu+\frac{1}{2},\frac{\nu}{2}}\left(\frac{1}{\overline{\gamma}_r s}\right), \quad (25)$$

where $W_{a,b}(x)$ is Whittaker's function of orders a and b [13, Eq. (9.224)], $\mu = \frac{N_s+N_d}{2}$, and $\nu = |N_d - N_s|$.

Proof: We can express $M_r(-s)$ as

$$M_r(-s) = \int_0^{\infty} e^{-s\gamma}p_r(\gamma)d\gamma. \quad (26)$$

Using (23) and [13, Eq. (6.643)], after some manipulations we obtain (25). ■

VI. Simulation Results

In this section, the performances of distributed orthogonal space-time codes are studied through simulations. The error event is bit error rate (BER). The signal symbols are modulated as BPSK. We fixed the total power consumed in the whole network as P and use the equal power allocation, i.e, $P_1 = \frac{P}{2}$ and $P_2 = \frac{P}{2R}$. Assume the relays and the destination have the

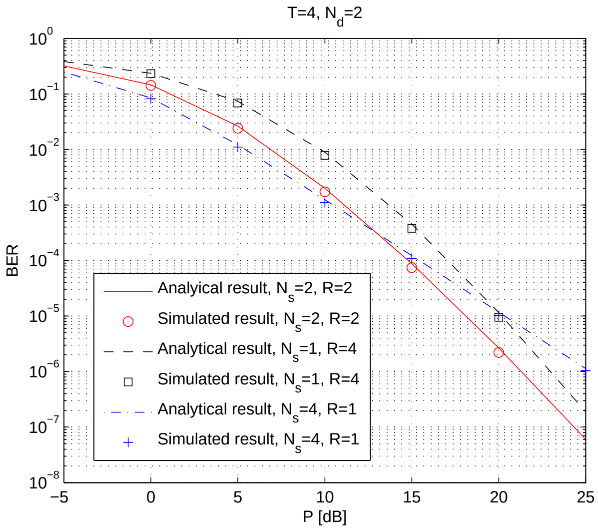

Fig. 2. Performance comparison of analytical and simulated results of a relay network with BPSK signals, $T = 4$, and $N_d = 2$.

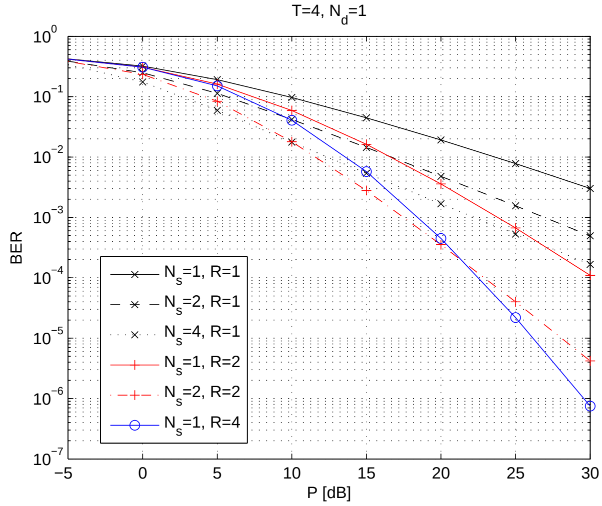

Fig. 3. The average BER curves of a relay network employing orthogonal DSTC with BPSK signals, $T = 4$, and $N_d = 1$.

same value of noise power, i.e., $\mathcal{N}_1 = \mathcal{N}_2$, and all the links have unit-variance Rayleigh flat fading. The orthogonal DSTC of (16) is employed where $T = 4$, and the analytical results are based on (24).

Fig. 2 confirms that the analytical results attained in Section V for the SER have the same performance as practical full-rate, full-diversity distributed space-time codes, such as the proposed codes. It is assumed that destination has two antennas. We compare all possible cases with $T = N_s R = 4$, i.e., $N_s = 2$ ($R = 2$), $N_s = 2$ ($R = 2$), and $N_s = 2$ ($R = 2$).

Fig. 3 compares the performance of the proposed DSTCs for different values of N_s and R when $N_d = 1$. Observing the curves at high SNR conditions, it can be seen that the diversity order of the system becomes $R \min\{N_s, N_d\}$. For instance, for the case of $R = 1$, it is shown that increasing the number of source's antennas from 1 to 2 and 4, the diversity gain would not be varying since $N_d = 1$. However, substantial coding gain is obtainable by increasing the number of N_s, for fixed R and N_d. For example, one can observe that at BER $= 10^{-4}$, a gain of 7 dB is obtained using $N_s = 2$ comparing to $N_s = 1$, when $R = 2$. In addition, it can we observed from Fig. 3 that a higher number of antennas at source, and thus, lower number of relays are preferable in low SNR scenario, due to the accumulation of noise at A&F-based relays.

VII. Conclusion

In this paper, we proposed systematically constructed, full-rate, full-diversity distributed orthogonal space-time codes for a R-relay MIMO cooperative system. A part of the space-time codes is done at the source multiple antennas, and the remaining part is performed at relays. The relays do not require to obtain channel coefficients, and simply transmit the scaled version of the linear combinations of the received signals. We analyzed the performance of the system with M-AM signals. Simulation are in accordance with the analytical results. Furthermore, simulations show that using linear decoder, we can extend the network size with acceptable performance.

VIII. Acknowledgment

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