45TH TURBOMACHINERY & 32ND PUMP SYMPOSIA
HOUSTON, TEXAS | SEPTEMBER 12 – 15, 2016
GEORGE R. BROWN CONVENTION CENTER

ROTOR BALANCING TUTORIAL

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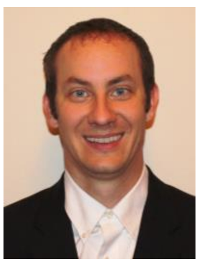

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ABSTRACT

The purpose of this tutorial is to describe details related to shop and field balancing of turbomachinery. The target machinery is API type turbomachinery such as steam turbines, compressors, and other rotating machinery that is common in refining and petrochemical plants.

An introduction to shop balancing includes a review of current practice as it relates to shop balancing of API rotors. Included in the description and discussion is a review of API shop balancing methods as well as a review of ISO standards related to shop and field balancing that are referenced by API standards.

Greater emphasis is presented in this tutorial on field balancing, which applies to balance correction in situ on rotating machinery and similarly applies to methods and techniques used when conducting high speed shop balancing.

INTRODUCTION

Rotor unbalance is a common cause of synchronous rotor vibration that is detected using non-contacting proximity probes or with bearing housing vibration. The causes of unbalance can be varied with the actual causes depending on manufacturing methods and procedures, repair practices, as well as balance condition changes during operation. Some operational causes of unbalance include rotor fouling (dirt or other deposits on the rotor), bowing of rotors due to uneven heating or shaft damage, loss of rotor material possibly from rubs, or other causes.

Balance correction is most effective when it is applied at or on the component that actually has the unbalance. In most cases the rotor is manufactured from a number of components (impellers, balance disks, thrust collars, etc.) that will each have some level of unbalance during assembly of the rotor. During operation, the unbalance state of each mounted component could change due to reasons stated above. As a result, the actual balance condition of an assembled rotor is never fully known prior to, during, or after a successful balance procedure is executed either in the shop or field.

DEFINITION OF UNBALANCE

Unbalance will occur any time a rotor or a component mounted on a rotor has a mass center (or center of gravity, i.e. cg.) that is not coincident with the axis of rotation. When this occurs, a force is generated due to rotation of the shaft that is defined by the following equation and shown graphically in Figure 1:

$$F_{balance} = m \times e \times \omega^2$$

Where

$$F_{balance} = Force\ caused\ by\ the\ imbalance$$
$$m = Mass\ of\ the\ eccentric\ part\ or\ rotor$$
$$e = Eccentricity\ of\ the\ part\ or\ rotor$$
$$\omega = Shaft\ speed, rad/\sec = \frac{2\pi N}{60}$$
$$N = Shaft\ speed, RPM$$

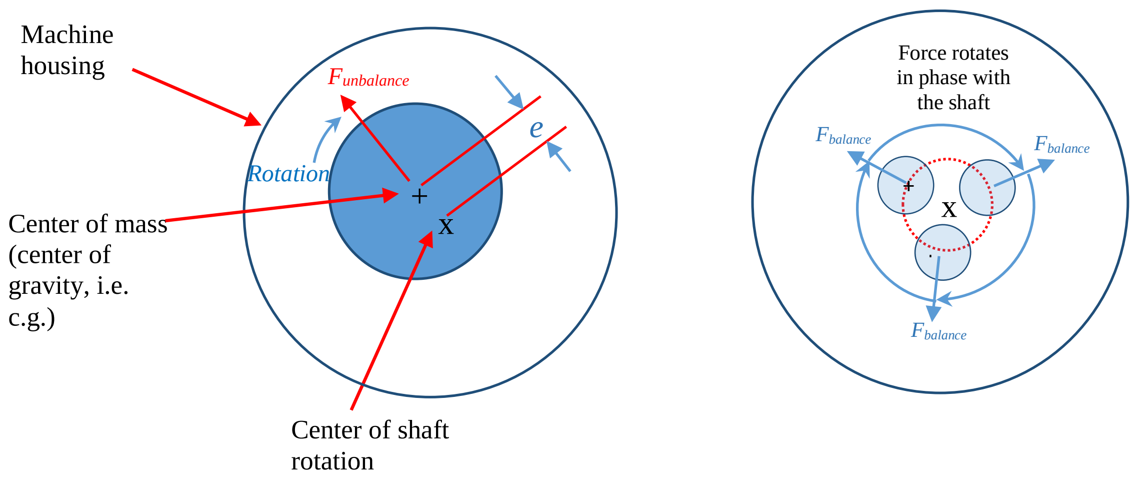

Figure 1 - Unbalance Force

This force rotates about the shaft that is phased to the shaft which results in vibration at 1xRPM. It is possible to measure the dynamic force in a balance machine, but not possible when operating the machine on the rotor support bearings as is the case for either high speed shop balancing or field balancing. As the machine operates, the balance condition of the rotor or components on the rotor can

be evaluated at least in part by measuring and assessing the vibration characteristics of the machine. Many other machine faults also produce vibration at 1xRPM such as bearing or shaft misalignment, loose components, rubs, or a host of other sources.
There are two general conventions for defining an amount of unbalance for a component or a shaft including eccentricity and unbalance magnitude. Both conventions can be used to define or describe the balance state of a component or a shaft and are often used interchangeably.

The term balance eccentricity, or permissible residual specific unbalance as detailed in ISO 1940 (see References), is defined by the amount of unbalance present divided by the mass of the rotor or component. When this term is used, it can be physically related to the runout of a previously balanced component as follows:

$$e = \frac{U}{M} = \frac{TIR}{2}$$

$$U = Unbalance\ of\ the\ part$$
$$M = Mass\ of\ component$$
$$e = balance\ eccentricity$$
$$TIR = Total\ Indicated\ Runout$$

In more detail, the balance of a component can be defined either by the mass of the component and the amount of unbalance (*M* and *U*), or by the mass and the eccentricity (*M* and *e*). This distinction is helpful for the purpose of understanding and evaluating the significance of how components are attached to a shaft. In particular, API 617 defines that individual components should be individually dynamically balanced before assembly to ISO 1940 G1 levels. For a component that is installed on a 5000 RPM shaft, the allowable eccentricity (*e*) at G1 is 0.0019 mm (0.000075”), or 0.15 mils TIR. Consistently maintaining less than 0.15 mils TIR for mounted components on most shafts for industrial machinery will be impractical, making the mounting process a likely larger contributor to the resulting unbalance of the mounted component than the balance condition of the part itself. This very issue is the driving factor behind the use of the incremental balance procedure defined in ISO 11342 and generally specified by API standards (i.e. API 617 and others).

ISO 1940 defines the balance quality of rotors for a variety of services by defining a host of balance quality grades for different types of rotors. The residual unbalance for a rotor is defined in this standard using velocity magnitudes starting at 0.4 mm/sec and increasing in factors of 2.5 (0.4, 1.0, 2.5, 6.3, etc.). The velocity magnitude is defined using the eccentricity concept from above combined with operating speed of the rotor as follows:

$$G = e \times \omega\ , mm/\sec pk$$
$$G = Balance\ Grade\ (i.e. G1 = 1\ mm/\sec pk)$$
$$\omega = Shaft\ speed, rad/\sec = \frac{2\pi RPM}{60}$$

This definition specifies the eccentricity that would result in the rotor vibrating at 1 mm/sec assuming there is no dynamic amplification due to natural frequencies and the rotor is operating well above a critical speed. In reality, the balance grade is much less related to observed vibration on operating machinery due to various natural frequencies.

API standards, such as API 617, specify unbalance tolerances generally by specifying *U* as follows:

$$U = 6350\left({}^{W}/_{N}\right), g - mm\ \ (SI\ units)$$
$$U = 4\left({}^{W}/_{N}\right), Oz - in\ (US\ customary\ units)$$
$$W = Journal\ static\ load, kg\ or\ Lb$$
$$N = Shaft\ speed, RPM$$

It should be noted that for API standards *U* is specified referenced to the journal reaction force (due to static weight) and is generally assumed to be one correction plane per bearing (two for most machines). For comparison, the allowable unbalance for the rotor

(commonly split between two planes) can be calculated from the equation above using *W* as the total rotor weight.
Although the API standards generally specify the rotor unbalance using *U*, API 617 has a minimum limit on eccentricity that is invoked for rotor speeds in excess of 25,000 RPM where the balance tolerance is limited at 250 μm or 10 μinch. This limit is established in general by the capabilities of shop balance machines.

SHOP BALANCING METHODS

The methods employed in shop balancing can have a profound impact on the resulting balance condition of the rotor. The impact of shop balance technique is most important when the rotor is relatively flexible and/or long as is common with most turbomachinery.

To improve the balance condition of most high speed flexible rotors, the following procedure is generally followed:

1. Balance the bare shaft without added components
    a. Assure that any keyways are fitted with half keys in accordance with ISO 8821 unless two keys are located at the same axial position and are 180^0 apart
2. Balance the attached components separately to ISO 1940 grade G1 or better
    a. Balance should be accomplished normally using shop mandrels or other balance hardware. Mandrels should be precision balanced and have eccentricity <0.0001 in and a mass of <25% of the component to be balanced.
    b. Concentricity of mounting hardware such as mandrels during the shop balancing should be adequate to prevent mounting eccentricity that can result in component balance error that exceeds the intended balance tolerance
3. Mount no more than 2 components to the shaft at a time and re-check balance, and if corrections are required only correct on the added components
4. Perform a check balance on the fully assembled rotor after the component assembly procedure above, with final corrections normally on two correction planes near the ends of the rotor (near bearings)
    a. Check balance process will normally allow limited balance correction, since major corrections would have been completed during assembly
    b. "Large" corrections to an assembled rotor pose a nontrivial risk of vibration above the first critical speed due to modal balance, since large unbalance (that could be located away from balance correction planes) can be corrected to minimize shop balance machine output to comply with the 4W/N rule at the two planes defined but not correct the modal balance condition for higher flexible rotor modes

The motivation for following this incremental balance procedure is to minimize the unbalance of the rotor in general, but to specifically reduce the modal unbalance that can result if this method is not followed. If the rotor is fully assembled and balanced after being fully assembled (opposed to the incremental balance), unbalance of components or more specifically the mounting eccentricity of the components can result in very large modal unbalance even though a low speed balance machine may indicate that the rotor is successfully balanced.

API Residual Unbalance Verification

API Describes a method for verifying that the residual unbalance of a rotor falls within the standard 4W/N tolerance. This test is conducted after final low-speed balance of a rotor assembly is completed in a shop. A trial weight equal to twice the 4W/N tolerance for hard bearing balance machines (four times 4W/N for soft bearing machines) is applied at six equally spaced angular positions at each balance planes with the rotor operated at the balance speed between each application of the trial weight. The vibration amplitude vs. trial weight location is plotted on polar graph paper. The plot should approximate a circle with the center of the circle indicating the residual unbalance in the rotor. The API procedure, outlined in API 684 and other API standards, provides additional details.

UNBALANCE DISTRIBUTION

For most turbomachine rotors, the shaft is fabricated from a number of parts (impellers, blade rows, etc.). As the rotor is assembled, an accumulation of unbalance vectors that result from each component being added can produce an unbalance state that will excite different rotor modes at different levels.

For a rigid rotor, the unbalance state can be fully defined by a static unbalance and one balance couple as shown in Figure 2. Based on the rigid rotor theory, the static correction can be made at any location along the shaft although it is generally preferred to correct the static near the center of the rotor. The dynamic correction is applied with equal and opposite (180^0 out of phase) balance correction at two separated planes. When a larger separation between the two dynamic planes is used it results in a lower required dynamic correction weight.

This balance approach is generally acceptable for “rigid” rotors, or rotors that do not demonstrate critical speeds or significant flexibility in operation. This is also the practical limit of most low speed shop balancing machines in that they can correct for the static correction for the rotor and for dynamic couple in two planes.

When a rotor is balanced on a low speed shop balancing machine, the actual unbalance distribution is not known. If the rotor was component balanced per normal API procedures, the modal unbalance will hopefully be under reasonable control for most rotors. However, for rotors that are repaired or refurbished and sent to a shop balance, the actual balance state is not known, and shop balancing may or may not improve the balance condition of the rotor at normal operating speed.

One common practice during shop balancing is to make static corrections near the center of a rotor using the low speed shop balance method by correcting for about 50% of the static near the center of the rotor. Final corrections (remaining 50% of static and the required dynamic) are made at the balance correction planes on the end of the rotor. Since the low speed balance machine cannot distinguish the actual axial position of the unbalance, the selection of the amount of static to correct at the rotor center is based on experience and the intuition of the balance machine operator. If the rotor is repetitively shop balanced using this method, it is possible that each balance procedure will progressively move more static correction either to the center of the rotor or away from the center depending on what the actual balance distribution is for the rotor. Therefore, making large balance corrections to assembled rotors is a risky proposition.

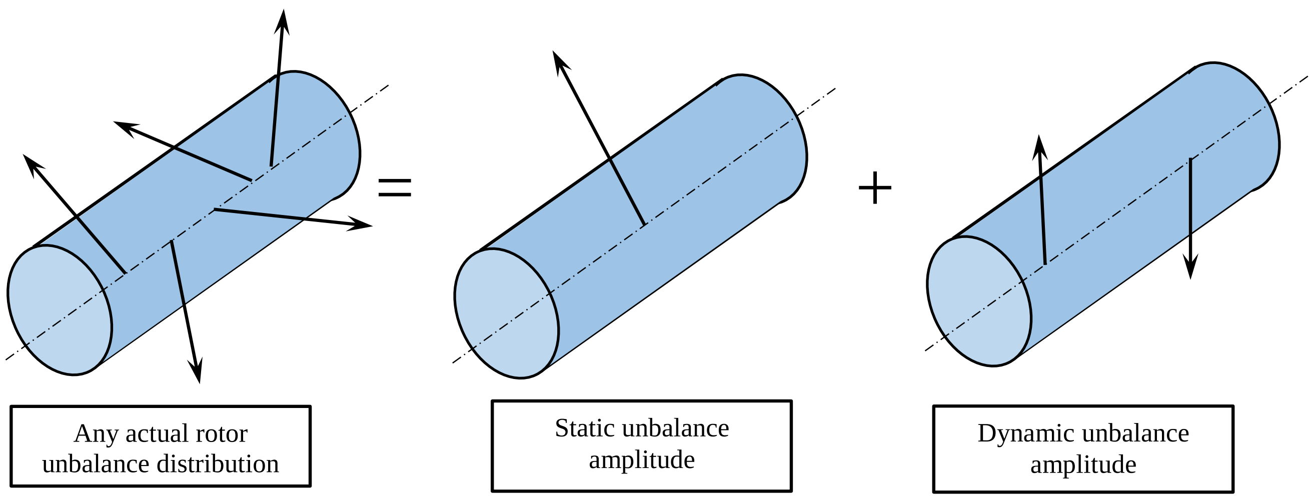

Figure 2 - Static and Dynamic Balance Concept

MODAL UNBALANCE CONCEPT

Since many turbomachinery rotors are in fact flexible from a rotordynamics and balancing perspective, it is helpful to discuss balancing using a modal balance concept. This is important since rotors can generally only be balanced using the rigid rotor method (static and one dynamic couple) using a shop balance machine. This becomes a challenge since it is common for most turbomachinery rotors to operate above a first critical speed.

As rotors pass through critical speeds, the sensitivity of the rotor to unbalance depends on the deflected mode shape at each shaft speed. For a first critical speed of a between-bearing machine (center of gravity between the bearings), the rotor is generally most sensitive to unbalance near the center of the rotor, or unbalance on the ends that are in phase. As the rotor approaches a 2nd critical speed, the rotor will then have a node point near the middle of the rotor. Unbalance near a node point for any mode will not impact the vibration response of that mode.

The concept of modal unbalance is to take the actual unbalance in the rotor (or more commonly used when selecting trial weight locations and phasing for field balancing) and separate it into balance magnitudes for each rotor mode shape that may exist (i.e. 1st critical and 2nd critical speeds). For example, a rotor with a 1st and 2nd critical speed will have rotor mode shapes as shown in Figure 3.

In general, an appropriate use of shop balancing with the incremental balance method can reduce the resulting unbalance for both a 1st and 2nd critical speed for most rotors. However, once a rotor has been in service and/or repaired, the use of a low speed balance method, although frequently successful, cannot assure that high modal unbalance is prevented. This is a possible cause of elevated vibration after repair, and particularly likely when the first critical speed vibration is acceptable and vibration increases dramatically above the first critical speed. Use of a modal balancing concept is therefore essential for the balance of flexible rotors.

Field balance corrections, although intended to reduce vibration at all speeds due to unbalance, can often reduce the response to one mode (i.e. 1st critical speed) while making another (i.e. 2nd critical speed) far worse, or vice versa. Limited access to correction planes in the field can also complicate the process since ISO 11342 details that at least N planes and usually N+2 planes are required for proper balance of a flexible rotor where N is the number of modes either passing through or approaching. Since most turbomachine rotors operate above the first and approach the 2nd, as many as 4 balance correction planes may be warranted for proper balance at all shaft speeds.

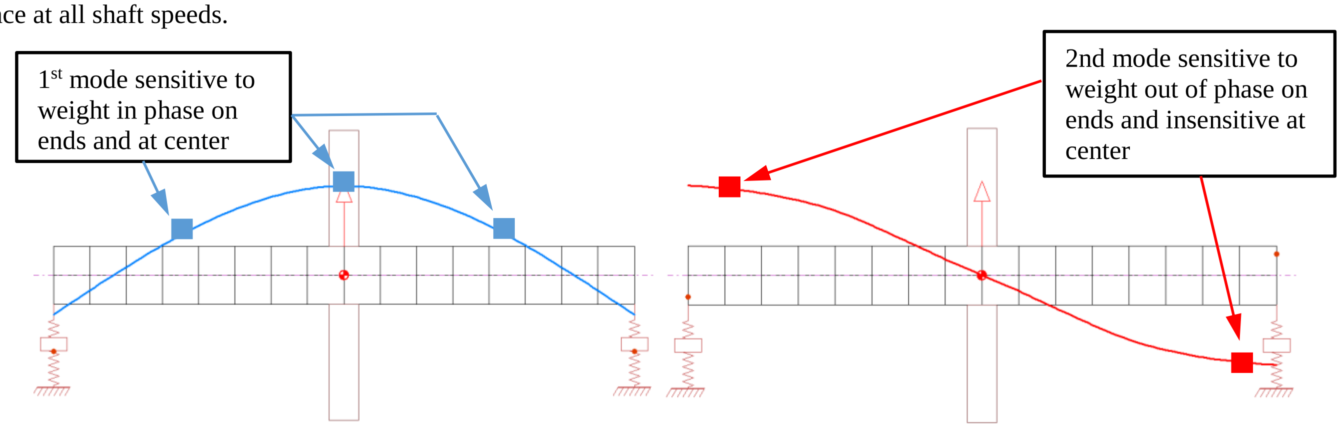

Figure 3 – Modal Unbalance Distribution

It should be noted that high speed shop balance (“at speed balance”) is normally accomplished by making balance corrections to more than 2 correction planes for the purpose of reducing unbalance response at or approaching multiple critical speeds. Therefore, 4 shop correction planes for high speed rotor balancing may be required for appropriate balance for the entire speed range. Additional information on high speed rotor balancing may be found in the reference by Ehrich (1980).

FIELD BALANCING

Once a rotor is installed in the rotating machine, the unbalance condition cannot be directly determined. The only way to assess the unbalance state is to add balance correction weights at various locations and using some calculation tool such as the influence coefficient method, and if the vibration is greatly reduced at most (or all) speeds, then a reasonable estimate of residual unbalance can be calculated based on observed vibration response to multiple trial weights added to the shaft.

When this is done, the balance options include several different methods including:

- Single plane balancing
    - Corrections are only added at a single correction plane (or equal weights are installed on both ends of an impeller or rotor at the same phase angle)
- Two plane balancing
    - Correction weights are added to two correction planes normally with one plane near each support bearing
- Multiplane and/or multispeed balancing
    - Calculation technique where more than two planes and or multiple speeds are considered in the calculations

The selection of the field balance technique is dependent on the dynamic characteristics of the rotor combined with available balance correction planes and knowledge of the machine operating requirements.

SINGLE BALANCE METHODS

Graphical Method

The most basic method for rotor balancing is the single plane balance using a graphical approach. The biggest advantages to being proficient in applying this method are that it requires no specialized software, it can be successfully applied to the majority of balance-of-plant equipment, and it is the basis for and can be used as a sanity check of more advanced multiplane methods or software tools. The general approach is to collect reference synchronous vibration amplitude and phase data, apply a trial weight, collect response data, plot the initial and trial vibration vectors, and determine a correction weight graphically. The only tools needed for successfully applying the single plane graphical balancing method are a vibration transducer, a tachometer, a scale and some polar plotting paper.

The machine to be balanced is operated at full or reduced speed and the synchronous vibration amplitude and phase are measured and recorded. This trial run vibration vector, normally called the *O* vector (original), is plotted on polar graph paper. It is helpful to define a scale on the graph paper appropriate for the vibration amplitudes considered. Since the graphical method relies on an accurate plot, the larger the vectors on the plot, the easier it will be to determine the amplitude and angle for correction weights.

A trial weight is applied to the rotor and a new vibration vector is recorded from operation at the same speed as the first run. This vector, called the *O+T* (original + trial) vector, is plotted on the polar graph paper. Now a line is drawn from the tip of the *O* vector to the tip of the *O+T* vector. This vector will be called the *T* vector (very creative!). The *T* vector is measured and then translated so that it starts from the center of the plot instead of the tip of the *O* vector. Since the *T* vector represents the change in synchronous vibration amplitude and phase caused by the application of the trial weight, it may also be referred to as the Effect vector. Since the field balancing goal is to minimize vibration, the plot can be inspected to determine the direction and angle needed to rotate the trial weight so that the *T* or Effect vector is 180° opposed to the *O* vector. The amount of the trial weight is scaled up or down by the ratio of the length of the O vector to the Effect vector. Appendix A contains additional discussion on phase conventions for balancing that have been applied in this procedure.

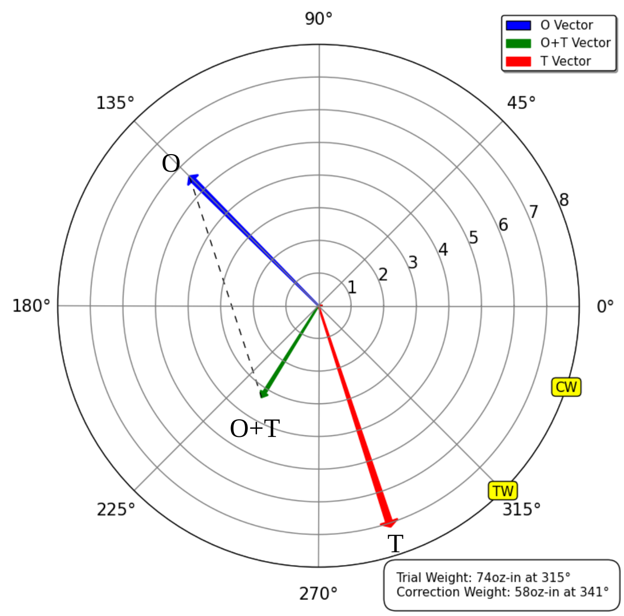

Figure 4 - Single Plane Balance Vector Diagram

As an example of single plane balancing using the graphical method, we will review balance information from a large centrifugal induced draft fan in a coal fired power plant. We ran the fan to full speed and measure our synchronous vibration amplitude and phase. Our analyzer showed us that the vibration is 5.6 mils pk-pk at an angle of 135°. We then plotted this on our polar plot paper (shown in Figure 4) and labeled it as the *O* vector using an appropriate scale. Our next step was to apply a trial weight to the rotor and re-run the fan to get our trial vibration vector. In this case, we chose to install a 74 oz-in weight at an angle of 315° from our phase reference mark as measured in the direction opposite rotation and mark this location on our polar plot. Since we were installing our trial weight at exactly 180° from the high spot (our *O* vector), we were assuming that this rotor ran well below the first critical speed and that our lag angle was 0°. In the majority of cases (including this one) we will see this was a poor assumption. Once the machine was run up to the same speed, we measured the synchronous vibration amplitude and phase to be 3.3 mils pk-pk at 238° and we plotted this on our polar graph and labeled it as our *O+T* vector. We drew a line from the tip of our *O* vector to the tip of the *O+T* vector and then translated this from the tip of the *O* vector to the origin on our plot giving us our *T* vector. This represented the vector change of the vibration with the application of our trial weight. Since our goal in balancing is to cancel out our original vibration (*O* vector) we want to shift the angle of our trial weight so that our *T* vector is oriented 180° from our *O* vector or, in this example, we rotated our trial weight by 26° in the positive direction (opposite rotation). We also want to make our *T* vector equal in length to our *O* vector. Since our *T* vector was longer than our *O* vector in this example, we knew we need to reduce the amount of weight. We divided the length of the *O* vector by the length of the *T* vector and multiplied by the amount of the trial weight to get 58 oz-in. So, from our vector diagram we graphically determined our correction weight for this fan.

Once the correction weight was applied to the rotor, data was collected and evaluated against the vibration acceptance criterion for the subject rotor. In this example, the measured vibration was acceptable and no further corrections were required. If the vibration was still above acceptable levels, it may be necessary to apply an additional correction weight. This is typically done by drawing a new vector diagram considering the *T* vector from the first trial weight as the original or reference run and considering the first correction weight to be a trial weight. It is recommended that this be plotted on a new polar plot so an appropriate scale can be used for the (hopefully) much lower vibration amplitudes. This method is often referred to as "taking a new O".

Influence Coefficient Method

Single plane balancing using the influence coefficient method takes the graphical method and applies math to get the same results. Since the synchronous (1x) vibration is described by a vector with magnitude and phase, vector math is used to manipulate the results. The general procedure for measurement is identical to the graphical method. The reference vibration is measured and recorded (O vector). A trial weight (TW) is applied to the rotor and the response vector is measured (O+T vector). Vector math is then used to calculate the T vector and the *influence coefficient*. Using the *influence coefficient*, a correction weight is calculated and applied to the rotor.

Looking at the graphical example, the O vector was 5.6 mils pk-pk at 135° and our O+T vector was 3.3 mils pk-pk at 238° after applying a 74 oz-in trial weight at 315°. To calculate the T vector, we subtract the O vector from the O+T vector. This is done by converting the vectors from polar coordinates (magnitude and phase) to Cartesian coordinates (real and imaginary), subtracting the components, and then converting them back to polar coordinates (magnitude and phase). An influence coefficient vector is calculated from the trial weight and the T vector. The influence coefficient is a system property that describes how a rotor reacts to a balance correction weight. Multiplying the O vector by the influence coefficient vector reveals to heavy spot on the rotor. In this example, balance corrections are being made by adding weights to the rotor so the correction weight is calculated by adding 180° to the heavy spot vector to determine to correction location for the correction weight. The influence coefficient method gives the same correction weight location as the graphical method, as expected.

Single Plane Influence Coefficient Balancing Math

$$\vec{O} = 5.6\ mils_{pk-pk}@135°$$

$$\overrightarrow{O+T} = 3.3\ mils_{pk-pk}@238°$$

1. ***Calculate T Vector***

$$Re(\vec{O}) = 5.6 * \cos\left(135 * \frac{\pi}{180}\right) = -3.96$$
$$Im(\vec{O}) = 5.6 * \sin\left(135 * \frac{\pi}{180}\right) = 3.96$$

$$Re(\overrightarrow{O+T}) = 3.3 * \cos\left(238 * \frac{\pi}{180}\right) = -1.749$$
$$Im(\overrightarrow{O+T}) = 3.3 * \sin\left(238 * \frac{\pi}{180}\right) = 2.799$$

$$Re(\vec{T}) = Re(\overrightarrow{O+T}) - Re(\vec{O}) = 2.211$$
$$Im(\vec{T}) = Im(\overrightarrow{O+T}) - Im(\vec{O}) = -6.758$$

$$Magnitude(\vec{T}) = \sqrt{[Re(\vec{T})]^2 + [Im(\vec{T})]^2} = 7.11$$

$$Phase(\vec{T}) = \frac{180}{\pi} * \tan^{-1}\frac{Im(\vec{T})}{Re(\vec{T})} = 288.1$$

$$\vec{T} = 7.11\ mils_{pk-pk}@288.1°$$

2. ***Calculate Influence Coefficient***

$$\overrightarrow{TW} = 74 oz - in\ @315°$$

$$\overrightarrow{IC} = \frac{\overrightarrow{TW}}{\vec{T}} = \frac{74\ oz - in}{7.11\ mils_{pk-pk}}\ @\ 315° - 288.1°$$

$$\overrightarrow{IC} = 10.4\frac{oz - in}{mil_{pk-pk}}@26.9°$$

3. ***Calculate Heavy Spot***

$$\overrightarrow{HS} = \vec{O} * \overrightarrow{IC} = 5.6\ mils_{pk-pk} * 10.4\frac{oz_{in}}{mil_{pk-pk}}\ @\ 135° + 26.9°$$

$$\overrightarrow{HS} = 58.24\ oz_{in}@161.9°$$

4. ***Calculate Correction Weight***

$$\overrightarrow{CW} = \overrightarrow{HS} + 180° = 58.24\ oz_{in}@341.9°$$

PRACTICAL SINGLE PLANE CONSIDERATIONS

Trial Weight Selection

One of the keys to a successful balance job is selecting an appropriate trial weight in both magnitude and location. An appropriate trial weight selection means that the effect of the trial weight provides at least a 10% vector shift from the reference vibration. In the polar plotting method, the T vector should be at least 10% of the length of the O vector. This will provide sufficient resolution on the polar plot to accurately calculate the influence coefficient. It is not a requirement that the trial weight result in a reduction in vibration amplitude.

To select an appropriate trial weight magnitude, several methods have been suggested by others. Jackson (1991) recommends that the trial weight magnitude be selected to produce a dynamic force equal to 10% of the rotor static journal weight or:

$$TW = 56{,}333 * \frac{W}{N^2}\, oz - in$$

Other trial weight magnitude selection methods include using residual balance limits from API standards or ISO 1940. The typical residual unbalance limit per API standards is:

$$U = 4 * \frac{W}{N}\, oz - in$$

ISO 1940 residual unbalance limits are specified using:

$$U = 6.015 * G * \frac{W}{N}\, oz - in$$

The ISO 1940 G level varies depending on the class of the machine to be balanced. Common levels are G0.67 (equivalent to the API limit), G2.5 (typical for large motors), and G6.3 (typical for pumps and fans). At low speeds, the 10% static weight method results in a much larger trial weight magnitude. At moderate shaft speeds, the 10% rule becomes more conservative and is a good rule of thumb to follow for trial weight selection.

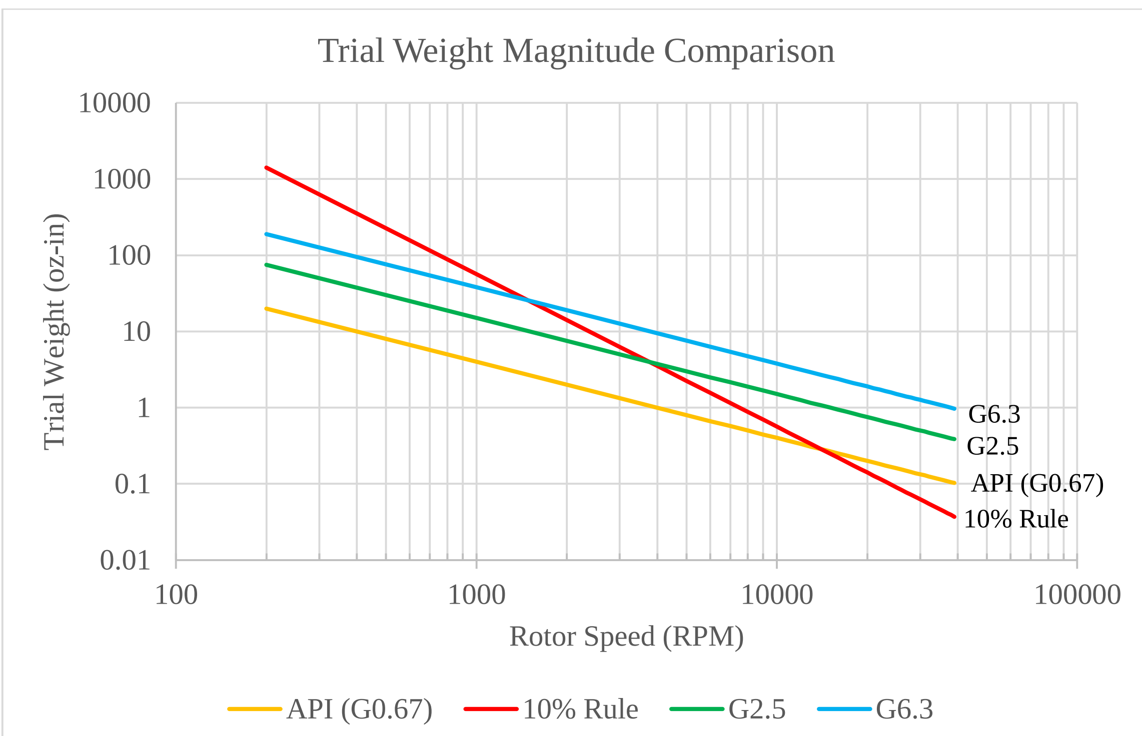

Figure 5 - Comparison of Trial Weight Selection Methods vs. Rotor Speed. API and ISO 1940 and 10% Rule

The angle of the trial weight should also be determined with care. As shown in Figure 6, at low speeds well below the first critical speed, the vibration response (high spot) will be in phase with the unbalance vector (heavy spot) and the lag angle will be zero. As the speed approaches a rotor resonance, the high spot will lag the heavy spot by some angle and at resonance the lag angle will be 90°. Rotors that can be balanced using single plane methods will operate below their first critical speed will have a lag angle between 0-90°. The rate of change of the lag angle as the critical speed is traversed has an inverse relationship with the amount of system damping. A sample synchronous response amplitude and phase plot (Bode plot), shown in Figure 6, shows this lag angle as the critical speed is traversed. Selecting a moderate lag angle of 45-60° for a trial weight for a rotor operating below the first critical speed will generally result in a reduction in vibration amplitude as long as the trial weight magnitude is not excessive.

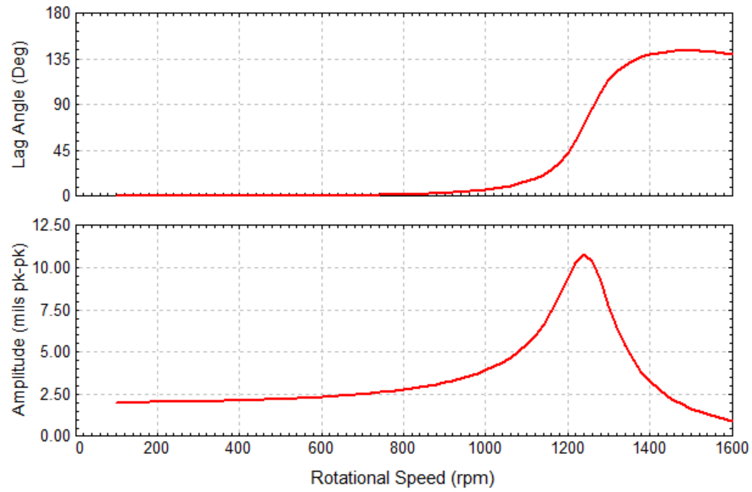

Figure 6 - Vibration Amplitude and Phase Lag

Angle Corrections

Locating the heavy spot is the key to balancing. There are some practical issues that must be accounted for to find this correctly. The first thing is the lag angle that has already been discussed. The sensor angle must also be considered. This is the physical angle between where the vibration sensor is mounted and where the phase reference is measuring (for a laser tachometer, this is where the red dot shows up on the shaft). Per convention, this angle is positive in the direction opposite rotation. An example is shown in Figure 7. Additional discussion on recommended phase angle conventions is included in Appendix A of this tutorial.

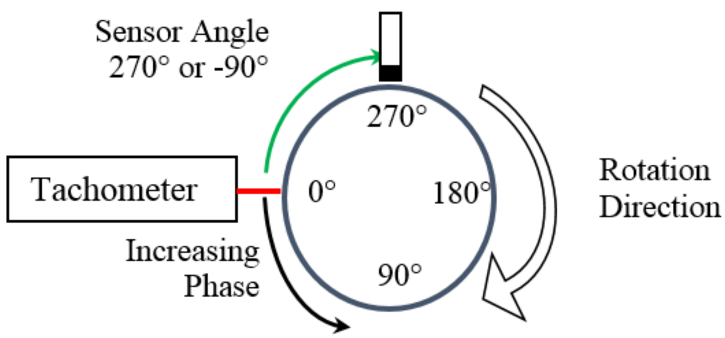

Figure 7 - Sensor Angle Diagram

A word of caution here on what we are calling sensor angle is appropriate. Many resources available for balancing procedures define something called "sensor lag" that must be accounted for. This refers to the lag introduced by a sensor such as a velocity pickup with a spring and mass inside it and does not really refer to the physical angle where the sensor is mounted relative to your phase sensor. Since it is fairly uncommon to find a reason to use these for balancing these days, it is not relevant to include any additional consideration for this lag.

The integration angle must also be considered. Many times, balancing will be done using displacement measurements in units of mils pk-pk. Other times, it may make sense to use velocity measurements in units of in/sec-pk (or IPS-pk) or even G's-pk. This may make sense when the acceptance criterion is in units of velocity or acceleration. It must be noted though that the lag angle previously discussed is the angle between the high spot and the heavy spot and the high spot is defined in terms of displacement. All this means in practice is that you have to consider what we'll call the integration angle. For displacement measurements this is 0°, for velocity measurements this is 90° and for acceleration measurements this is 180°.

Now that the terms have been defined, a standard equation can used to determine where the heavy spot is from a measurement. The formula for calculating the angle of the heavy spot is:

(Heavy Spot)° = (Vibration Phase)° - (Lag Angle)° + (Sensor Angle)° + (Integration Angle)°

As long as the sign conventions are followed as defined above, this formula will always give the physical location of the heavy spot in degrees relative to the phase reference mark on the shaft in the direction opposite shaft rotation.

As an example of applying this, consider a machine monitored via proximity probes. It is being balanced with both these and the addition of magnet-mount accelerometers that are single-integrated to velocity (IPS-pk) and assume a portable tachometer reference. The setup is shown in Figure 8.

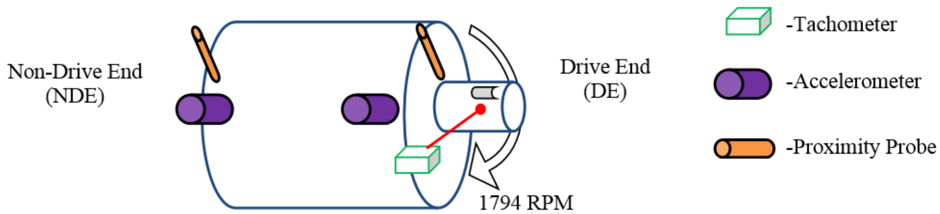

Figure 8 - Measurement Setup for Balancing

Based on the diagram, it can be determined that the rotation direction is CW and the tachometer is oriented at 90° Left. This means it is rotated 90° to the left of vertical when looking from driver to driven. The drive-end (DE) or inboard (IB) and non-drive end (NDE) or outboard (OB) accelerometers are also located at 90° Left. There are also two proximity probes oriented at 45° Left.

For this example, assume that this machine runs below the first critical speed. Based on the previous discussion, assume a lag angle of 45°. For the proximity probes, the sensor angle would be 315° or -45° since they are oriented 45° away from our tachometer in the direction of rotation or 315° from the tachometer in the direction of positive phase (opposite rotation). The integration angle for the proximity probes is 0° since they measure displacement. The integration angle for our accelerometers is 90° since the have already been single integrated to get velocity readings (IPS-pk).

Now assume that a reference set of data was taken where this machine was operated up to the normal operating speed and 1X amplitude and phase data from all of the sensors has been recorded and are shown in Table 1.

Table 1 - Example Trial Measurement Data

Measurement Location	1X Amplitude	1X Phase
DE-Prox. Probe	2.74 mils p-p	227°
DE-Accelerometer	0.251 in/s-pk	91
NDE-Prox. Probe	2.59 mils p-p	234°
NDE-Accelerometer	0.223 in/s-pk	93

Now the rules discussed above can be applied to calculate the location of the heavy spot on the rotor using the assumed lag angle. This is shown in Table 2.

Table 2 - Heavy Spot Calculations for Example Data

Measurement Location	Vibration Phase	−	Assumed Lag Angle	+	Sensor Angle	+	Integration Angle	=	Heavy Spot
DE-Prox. Probe	227°		45°		-45°		0°		137°
DE-Accelerometer	91°		45°		0°		90°		136°
NDE-Prox. Probe	234°		45°		-45°		0°		144°
NDE-Accelerometer	93°		45°		0°		90°		138°

The results in Table 2 show that all four of our measurement locations show that heavy spot is around 139°. This means that the appropriate location for a trial weight added to the rotor 180° from where the heavy spot is with the assumed lag angle of 45° would be 319°.

In practice, not all measurement locations will suggest that the heavy spot is at the same angle on the rotor. If the rotor has just static imbalance (there is a heavy spot at the same angle on both ends of the rotor), all the calculated heavy spot angles should be close to the same. If there is one outlier, verify that the sign conventions were applied correctly and the measurement is good. If different angles for the heavy spot are calculated on opposite ends of the machine, the unbalance is not pure static and it is more of a couple imbalance (there are heavy spots on both ends of the machine that are at different angles on the rotor).

Several commercial tools are available for carrying out single plane balancing including free or low-cost applications for portable devices. The authors have not tested any of the available applications and prefer to use in-house software for single and multi-plane balancing. Readers are encouraged to use this tutorial as a guide for developing balancing tools using spreadsheets or other suitable programming languages. At a minimum, readers should validate any commercial tools using hand or spreadsheet calculations or by testing the tools by balancing a rotor kit.

Runout Corrections

Any field balance effort using proximity probes for response measurement should include corrections for rotor runout. The runout measured at low speed by proximity probes will include any residual rotor bow as well as any eccentricity or non-circular surface and any electrical runout at the probe target area. While it is certainly possibly to leave the rotor runout in the vibration data when completing a field or shop balance and getting acceptable amplitudes at the proximity probes, the consequence is that the unbalance forces transmitted to the bearings will be much higher than if the runout is subtracted. This will be obvious is bearing housing vibration is also measured since the displacement measured by proximity probes may be minimized but the bearing housing vibration will increase due to the transmitted force.

Most, if not all, vibration data acquisition systems will allow users to designate a particular sample as a slow-roll or runout reference sample. The software saves the 1X amplitude and phase at this sample and allows users to correct 1X vibration amplitude and phase at other speeds by vector subtraction of the runout from the vibration. The slow-roll or runout compensated data should be used in the balance calculations. The slow-roll reference data should be selected at the lowest speed where the amplitude and phase is stable and well below any speed where the measured vibration is increasing due to unbalance forces or amplified by the first critical speed.

SINGLE PLANE BALANCING USING MULTIPLE MEASUREMENT POINTS

The use of multiple measurement points described in the previous section shows that an analytical approach may be required to best minimize the vibration at all points simultaneously. The graphical technique can be used, but it is common that different points (i.e. vertical vs. horizontal) will have different reaction to a balance weight. Since the different points will have different effects and initial vibration, each measurement point will individually calculate a different correction weight. With more measurement points than balance planes, the calculation for correction weight is over specified. That is, there is not necessarily a correction weight that can be calculated to drive all the vibration to exactly zero. Instead a numerical method must be used to minimize the vibration.

The most common method used to calculated balance correction weights is the influence coefficient method. This method generates an influence coefficient matrix using measured vibration data (amplitude and phase) as well as a known weight change. Once the influence coefficients are known (vibration per unbalance), the residual vibration can be minimized by selecting a balance weight. The most common minimization routine uses the least squares numerical method.

Before the numerical process can be described, the concept of vibration and weight changes must be well understood. With a single plane method using one sensor, the original vibration is measured (O), a trial weight is added (TW), and the response with the trial weight is measured (O+T). The original vibration run will normally have no trial weight, so the weight change equals the trial weight. The vibration change is called the trial response (T), and is found by determining the amplitude change from O to O+T from a vector plot.

When this is shifted over to a numerical process, terms are defined for each of the values described above but by using subscripts to help accommodate conversion to a computer program:

- **Point i** – Measurement points will be used with i = 1,2,3,… based on the number of measurement points used (no limit to the number of points)
- **Run j** – Runs include the reference run (Run 1), as well as the trial run (Run 2), followed by any additional sets of vibration data (Run 3, Run 4, ….)
- $\mathbf{V_{i,j}}$ – Vibration measurement (amplitude and phase) measured at point i during run j
- $\mathbf{W_j}$ – Weight vector (amplitude and phase) installed when Run j was recorded.
    - Note that W_1 should normally be zero (no weights added on initial reference)
    - Weight addition can become a complicated accounting process with resulting confusion between trial/final weights added as well as trim weights. Recommendation is to document weights using clear notes regarding weights that have been added and removed throughout the process to prevent confusion.
- $\mathbf{\Delta V_{i,j\text{-}k}}$ – Vibration vector change at point i between run j and run k.
    - $\mathbf{\Delta V_{i,j\text{-}k} = V_{i,j} - V_{i,k}}$ (Using vector difference)
- $\mathbf{\Delta W_{j\text{-}k}}$ – Weight vector change between Runs j and k.
    - $\mathbf{\Delta W_{j\text{-}k} = W_j - W_k}$ (Using vector difference)

For a single balance plane, these definitions can be used to calculate the influence coefficients for each measurement point. Two vibration runs are required (j and k) that have a known weight shift. All other influences on the vibration are assumed to be unchanged.

$C_{i,j\text{-}k}$ = Influence coefficient at measurement point i that is based on the vibration/weight shift from run j to run k.

$$C_{i,j-k} = \frac{\Delta V_{i,j-k}}{\Delta W_{j-k}}$$

Applying these definitions to a single measurement and single plane example illustrates the physical relevance of the vectors.

Single Plane, Single Sensor Example

Initial weight	$W_1 = 0$ (no initial weight)
Initial vibration	$V_{1,1}$ = Original amplitude and phase (“O”) at point 1 for run 1
Trial weight	W_2 = Trial weight magnitude and phase (“TW”) installed for run 2
Trial run vibration	$V_{1,2}$ = Vibration with added weight (“O+T”) at point 1 for run 2

$\Delta W_{2\text{-}1} = W_2 - W_1 = W_2$ (“TW”) is the weight change from run 1 to run 2
$\Delta V_{1,2\text{-}1} = V_{1,2} - V_{1,1}$ (“T”) is the vibration change from run 1 to run 2 at point 1

Influence Coefficient = $\Delta V_{1,2\text{-}1} / \Delta W_{2\text{-}1}$ = “T”/”TW” is the influence coefficient for point 1

Final Weight = FW (remove trial and add final weight)

$$FW = \frac{V_{1,1}}{\Delta V_{1,2-1}/\Delta W_{2-1}} = \frac{V_{1,1} \times \Delta W_{2-1}}{\Delta V_{1,2-1}} = \frac{O \times TW}{T}$$

The influence coefficient for each sensor can be determined by calculating the vector vibration change and the change in the trial weight as shown above with the results configured into an influence coefficient vector as shown below:

$$\mathbf{C_{j-k}} = \begin{bmatrix} C_{1,j-k} \\ C_{2,j-k} \\ C_{3,j-k} \\ \vdots \end{bmatrix} \quad \text{where} \quad C_{i,j-k} = \frac{\Delta V_{i,j-k}}{\Delta W_{j-k}}$$

Alternatively, the vibration and weight data can be stacked into a vertical vectors and the influence coefficients can be calculated using matrix operations.

LEAST SQUARES SOLUTION METHOD

The influence coefficients and initial vibration provide an overdetermined system when multiple vibration measurements are used for balancing at each plane. That is, there are more known data points (vibration measurements) than degrees of freedom (balance planes). Since the system is overdetermined, generally there is not an exact solution. Instead of an "exact" solution, a best fit solution must be used. There are several criteria for finding a best fit solution and they all have the objective of minimizing the residual vibration by placing a final correction weight or weights.

A standard numerical method for solving overdetermined systems is to use Linear Least Squares. With Linear Least Squares, the minimized value is the sum of each sensor's residual vibration squared. This minimization objective is convenient for solving with linear algebra because it has a single unique solution. There is only one best fit solution to minimize the objective given all the available data. Additional discussion on the application of the least squares minimization technique as applied to rotor balancing may be found in the reference by Goodman (1964).

LEAST SQUARES WITH MULTIPLE BALANCE PLANES

Having multiple balance planes changes the procedure for calculating the Influence Coefficients, but it does not affect the least squares method for determining the solution. With multiple balance planes, the influence coefficients are placed in a matrix of size # channels x # balance planes, and must be calculated using linear algebra. The vibration change and weight change data must be arranged and solved to provide the correct influence coefficient matrix.

LEAST SQUARES WITH MULTIPLE SPEEDS

Vibration data at multiple speeds allows provides additional data points that can be used to calculate balance correction weights. Essentially, vibration at an additional speed can be thought of as additional data at new sensors. A simple way to configure the calculation is to stack the vibration data for each additional speed in to the same vertical array.

WEIGHTED/SCALED LEAST SQUARES

In some scenarios it can be beneficial to give a preference to reducing the vibration at certain sensors compared to other sensors. This can be accomplished by using what is traditionally called Weighted Least Squares. For this paper, to avoid confusing the term Weighted with unbalance correction weight we will use the term Scaled Least Squares. This calculation method gives a scale value for each vibration measurement. This allows for certain measurements to be given more/less importance for determining the correction weight.

There are several examples where using Weighted/Scaled Least Squares may be beneficial. For example, if you are balancing a rotor with bearings that have different clearances the acceptable vibration may be different at each bearing. Also if you are balancing a machine using both proximity probes (shaft vibration) and accelerometers (casing vibration), then the acceptable vibration for each sensor may be different. Another scenario is that the data may show high error/uncertainty for measuring the 1x amplitude or phase for a particular sensor. If the analyst would like to calculate the results ignoring a suspected problem channel, then the calculation can be done by discounting that channel.

When the input data has known error or uncertainty, then the scale factors can be selected to minimize the influence of the error. The best linear unbiased estimator uses scale factors that are equal to the inverse of the variance for the measurement. When collecting the 1x amplitude and phase it is typical to measure many samples to ensure that the amplitude and phase are statistically consistent. In doing this process, the measurement variance can also be calculated and provides a direct measurement of whether certain channels should be discounted due to measurement variance.

ALTERNATIVE OBJECTIVE FUNCTIONS

Using linear least squares has many benefits, but there are other possible numerical tools to calculate a balance correction weight. It is possible to minimize any objective function by using Non-Linear Least Squares.

With the standard linear least squares method, the calculated balance correction weight may give a solution with a high vibration at one location and very low vibration at all other sensors. Since the standard objective is the sum of the squares, the method does not care whether a single channel is higher than the other channels. An alternative to this approach would be to minimize the maximum vibration amplitude. With this objective function, the best fit solution would be found to minimize the maximum measured amplitude. With the standard linear least squares method, there is no penalty for adding additional weight. This is not an issue if a rotor is very out of balance. However, if the error and noise in the vibration data has a significant influence compared to the residual vibration then the "best fit" solution may try to add more and more weight to cancel out any noise in the measurement. This phenomenon is only relevant when two or more balance planes are used. Measurement noise should be assessed by the analyst in terms of phase stability and amplitude. An alternative to address this would be to use an objective function that seeks to minimize both the residual vibration and the weight added or to set an objective for the least squares minimization to set the maximum projected amplitude to some value less than the acceptable vibration amplitude.

CASE STUDY 1: MULTISTAGE COMPRESSOR FIELD BALANCE

A seven stage centrifugal compressor was overhauled including un-stacking the rotor and re-assembling. The impellers were component balanced during the re-stacking procedure and the rotor assembly was shop balanced on a balance stand prior to installation in the compressor case. During commissioning, overall vibration amplitudes exceeded the acceptance limit of 1 mil pk-pk as measured on the four available proximity probes with the 1X component exceeding 2 mils pk-pk on one of the probes. Field balancing was recommended to reduce amplitudes to acceptable levels. The normal operating speed for the compressor is 8,610 RPM.

Measurement Selection and Reference Vibration Collection

The compressor was monitored by two proximity probes physically located at 45° left of vertical (Y Probes) and 45° right of vertical (X Probes) as viewed from the drive end of the compressor at each bearing housing. No keyphasor probe was available so a temporary laser tachometer was located at the horizontal split-line on the left side as viewed from the drive end of the compressor. A small piece of reflective tape was applied to an exposed shaft location to trigger the tachometer. Using the convention outlined above, the X Probe sensor angle is 225° and the Y Probe sensor angle is 315° for both ends of the rotor.

The rotor was operated up to full speed and data was collected from all four probes. From the data, it was clear that the rotor operated above the first critical speed. It was also noted that there was a fair amount of runout based on the 1X vibration at relatively low speed (0.21-0.44 mils pk-pk).

Slow Roll Compensation

Since proximity probes were used for balancing, slow roll vibration vectors were recorded and all reference and response 1X vibration amplitudes used on the balancing procedure were slow roll compensated. If uncompensated displacement values were used in the balance process, the final uncompensated displacement values may be reduced to amplitudes less than the rotor runout but the consequence is that the unbalance forces transmitted to the bearings may be much greater. This will show up as higher vibration on the bearing housing from seismic measurements in the final data compared to the reference seismic data. Using slow roll compensation for proximity probes, the lowest theoretical final displacement data would be equal to the rotor runout and will result in no unbalance force transmitted to the bearings. The lowest practical final displacement data will be something greater than the rotor runout.

Trial Weight Additions and Response Vibration

The reference data showed two key pieces of information. First, the vibration was higher on the outboard end of the rotor. Second, there was a significant phase difference across the rotor from outboard to inboard. This information was valuable for guiding the selection of trial weight locations and also suggested that a two plane calculation would be required. Since the highest amplitude was on the outboard end, the first trial weight was installed on this end of the machine. The rotor had provisions for installing set screws into balance rings with 20 equally spaced holes on either end of the machine. A trial weight of 6.8 gm was installed at 252° on the outboard end balance ring for the first trial. The machine was returned to full speed and vibration response vectors were recorded.

Since a two plane balance procedure was being used, a second trial weight run was required with a trial weight installed on the inboard end of the rotor. In the interest of time, the first trial weight on the outboard end of the rotor was left in place for this run. This is normally an option in most two plane balancing programs. Since the reference vibration phase data indicated that the couple unbalance was more significant than the static unbalance in the rotor and the response to the first trial weight on the outboard end was favorable, the second trial weight of 6.8 gm was installed at 72°, 180° from the first trial weight location. The 1X vibration data collected for the three runs is summarized in Table 3. Note that the phase data in this table is not corrected using the sensor angles and must be corrected prior to calculating influence coefficients.

Table 3 - Reference and Trial Weight Response Vectors

	Reference Vibration		First Trial - OB Weight		Second Trial - OB/IB Weight	
	Amp	**Phase**	**Amp**	**Phase**	**Amp**	**Phase**
OBX	0.97	310	0.80	324	0.89	312
OBY	2.20	213	2.00	210	2.00	200
IBX	0.84	102	1.05	92	0.95	84
IBY	0.58	3.5	0.53	331	0.33	315.1

Influence Coefficient Calculation

The vibration data from the three runs along with the trial weight vectors were used to calculate a set of influence coefficients using the procedure outlined above. The only significant difference is that the first trial weight on the outboard end of the rotor was left in place for the second trial run and this had to be accounted for in the influence coefficient calculation. If the first trial weight is removed, the effect of the second trial weight is calculated from the vector difference of the second trial response and the reference vibration vectors. If the first trial weight is left in place for the second trial run, the effect of the second trial weight is calculated from the vector difference of the second trial response and the first trial response.

With four measurements and two balance planes, eight influence coefficients were calculated and are shown in Table 4. Note that the units for the influence coefficient magnitude used were displacement per mass instead of the typical displacement per unbalance. Since the balance weights were installed in rings with a constant radius, there was no ability to vary the balance weight radius so referencing the mass only was sufficient. Reviewing the influence coefficients, it is clear that there is significant cross-coupling since the influence coefficients for the inboard measurement response to outboard weight addition (and vice versa) are of similar magnitude to the inboard measurement response to inboard weight addition.

Table 4 - Calculated Influence Coefficients

	Outboard		Inboard	
	mils pk-pk/gm	degrees	mils pk-pk/gm	degrees
OBX	0.040	58.0	0.029	47.9
OBY	0.034	123.3	0.051	358.0
IBX	0.039	31.8	0.025	115.4
IBY	0.046	311.5	0.034	57.0

Correction Weight Calculation

The aforementioned least squares minimization was used to calculate an optimal correction weight location from the reference vibration data and the calculated influence coefficients. The results called for more weight on both ends of the machine with relatively small changes to the weight locations relative to the trial weights. This shows that appropriate trial weight locations were selected and that the trial weight magnitudes were conservative. Since the locations for available holes for correction weights did not match the calculated correction weight locations and the amount of weight in each hole was limited, the correction weights were installed in more than one hole with the weight vectors added to match the calculated correction as close as possible. The trial weights were removed prior to installing the correction weights.

Table 5 - Calculated and Installed Final Correction Weights

	Calculated		Installed	
	gm	degrees	gm	degrees
OB Correction	17.5	230	18.6	233
IB Correction	30.3	0	28.2	0

Vector diagrams for the reference vibration, trial responses, and final vibration response are shown in Figure 9.

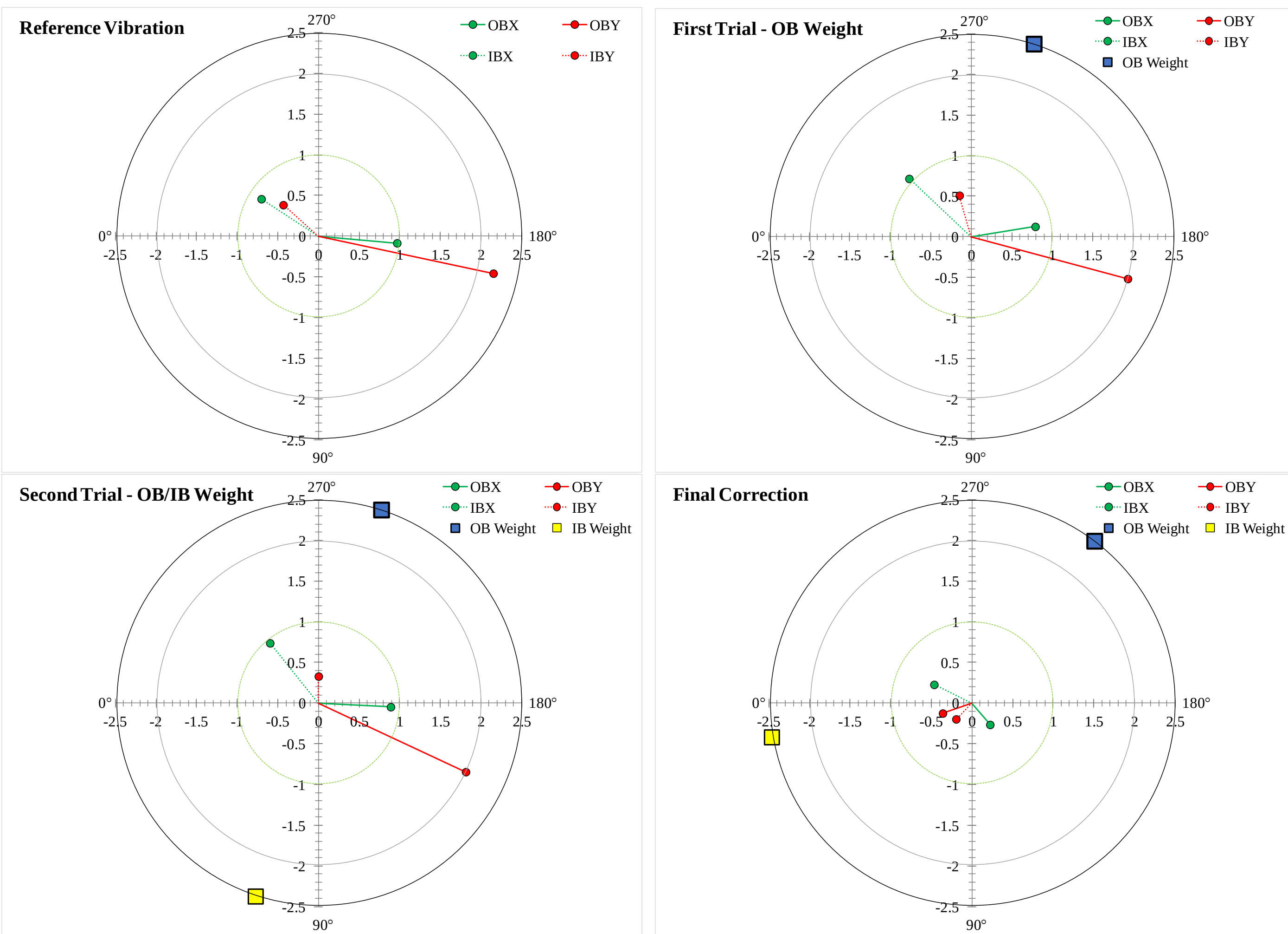

Figure 9 - Vector Diagrams for Two Plane Influence Coefficient Balance

Lessons Learned

While the results of this field balance show that the procedure was successful, there were issues that could have potentially derailed the job. During the measurement setup, the probes on one end of the machine were found to be wired incorrectly. Fortunately, this was identified prior to making any weight corrections. If the probe orientation is incorrect, the influence coefficient method will still result in the correct calculated balance correction angle. However, this can cause problems when attempting to identify a trial weight location.

Another potential issue was identified with the balance rings on the rotor. The rings were identical in that they both had 20 threaded holes and were numbered one through 20 with the numbers stamped into the balance rings. However, numbering of the holes increased with rotation on one end and opposite rotation on the other end. Many balance programs do not allow the user to specify this convention. This highlights the need to use care in noting the rotation direction, phase convention, and hole numbering convention on both ends.

CASE STUDY 2: TURBINE GENERATOR FIELD BALANCE

The turbine generator set is a 5 MW back pressure turbine in a three bearing configuration. Each bearing is equipped with two X-Y type proximity probes in the traditional API configuration with the balance/measurement planes identified in Figure 10.

The turbine was initially unable to reach full speed of 3600 RPM due to elevated vibration primarily on the generator end. Previous balance attempts by others included removing all the existing weights from the generator and adding a single plane shot on the outboard end of the generator. Excessive static and couple combination on the generator was the cause for excessive vibration.

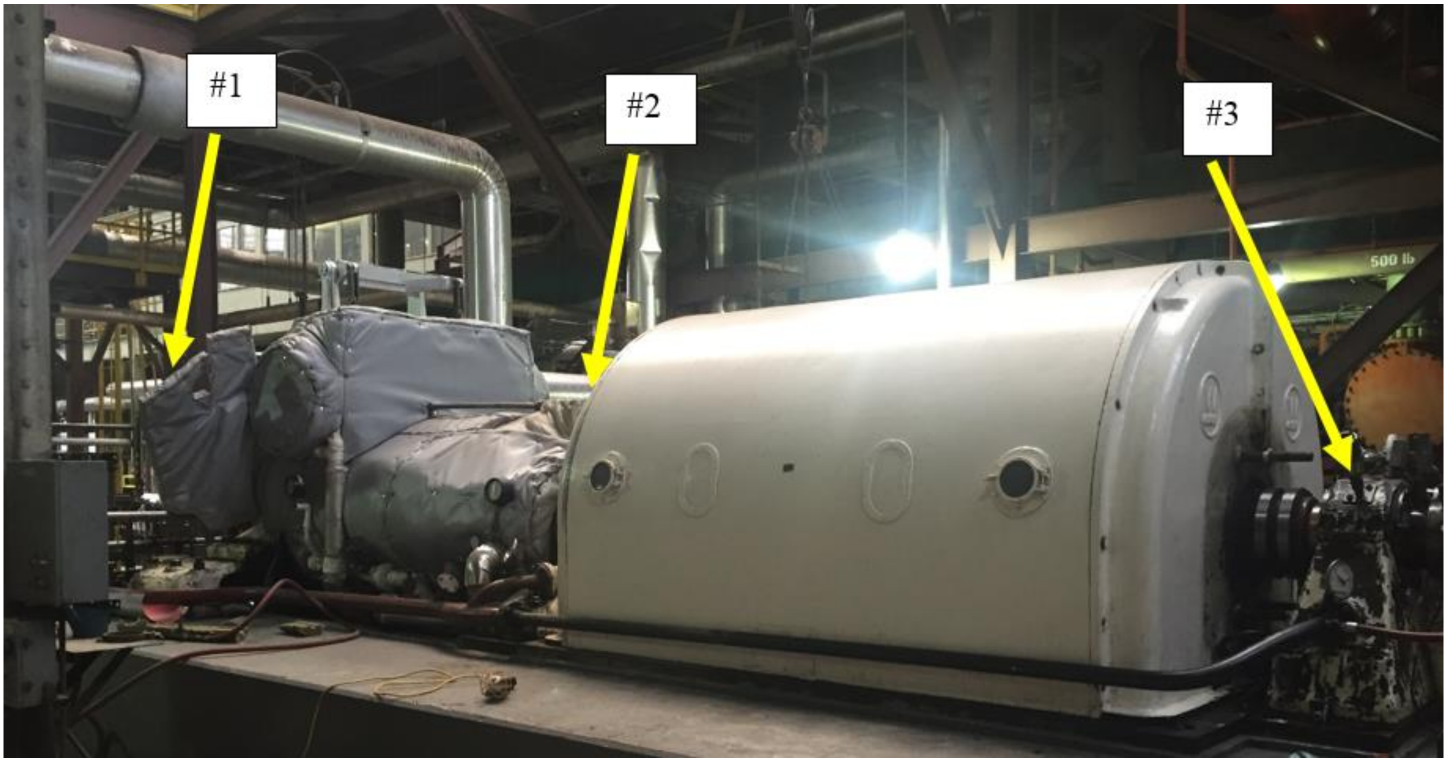

Figure 10 - Turbine Generator Balance/Measurement Planes

Balance Method

The first shot included taking the weight added on the outboard end of the generator and distributing that between the two ends of the generator based on the assumption that the added weight may have been approximately the amount needed for static correction of the generator. A second generator shot was conducted to define the two plane calculation for the generator using a calculation speed of 1500 RPM which was the speed slightly below the generator 1st critical speed.

Once the generator 1st critical was reduced with the combined static/couple shot, the turbine could be run at full speed of 3600 RPM.

Generator Balance Data
Generator balance data at 1500 RPM is as follows:

Location/Description	Reference	Trial 1	Trial 2	Trial 3
#1X	0.49@346°	0.54@350°	0.45@331°	0.59@357°
#1Y	0.37@108°	0.39@113°	0.33@93°	0.40@121°
#2X	0.72@105°	0.64@103°	1.42 @76°	0.03@122°
#2Y	0.45@224°	0.38@217°	0.85@193°	0.01@170°
#3X	0.47@180°	1.07 @176°	1.24 @110°	0.11@194°
#3Y	0.31@304°	0.71@297°	0.73@232°	0.10@332°
Installed weights #2 End, grams #3 End, grams	 109@338° 685@351°	 386@345° 386@345°	 525@359° 520@358°	 532@279° 672@12°

Once the Trial 3 run above was complete, the #2 and #3 bearings were well controlled with #3 below 1 mil through 3600 RPM. The #1 and #2 bearings reacted to the turbine critical speed near 2500-2800 RPM, so the next shot was to reduce turbine vibration.

Turbine Balance Data
Turbine balance data at 2300 RPM is as follows:

Location/Description	Trial 3 (turbine reference)	Trial 4	Trial 5
#1X	2.91@22°	1.32@348°	0.52@345°
#1Y	2.41@155°	0.83@108°	0.39@114°
#2X	1.53@44°	0.25@359°	0.26 @5°
#2Y	0.92@172°	0.15@132°	0.19@129°
#3X	0.61@322°	0.46 @315°	0.27 @177°
#3Y	0.53@80°	0.35@51°	0.21@293°
Installed weights #1 End, grams #2 End, grams	 None None	 75@202° 75@202°	 98@184° 98@184°

Once the Trial 5 run above was complete, the highest vibration at 3600 RPM was at the #3 end with a maximum amplitude of 1.09 mils. Vibration during the startup was never above 1.09 mil on any probe.

Lessons Learned
This case study describes an example where a combination of single plane and static-couple balance procedures were required to correct the balance on a rotor. Additionally, balancing at reduced speeds may be required when excessive vibration amplitudes at critical speeds restrict operation at full speed.

ONE SHOT BALANCING USING PREDICTED INFLUENCE COEFFICIENTS

All the previous discussion focused on methods to calculate influence coefficients from measured vibration data for single or multi-plane balancing. The discussion showed that a reference run and trial weight runs for each balance plane used are required to calculate influence coefficients. For large machinery with flexible rotors where multi-plane balancing is required and multiple critical speeds are encountered below normal operating speed, the standard influence coefficient method will require many runs to define the influence coefficient matrix. This becomes costly when considering the lost production and resources required to start and stop machinery in many plants. This is especially concerning when an incorrectly located or sized trial weight has an adverse effect on vibration or, even worse, no effect. Knowledge of the rotor mode shapes or previous field balance data can be used to help guide the location of trial weights and the trial weight sizing suggestions in this tutorial can help determine a trial weight magnitude. Another approach that can save a very significant amount of time is to use a set of predicted influence coefficients to determine a one shot balance correction.

Description of Method

This method requires the development of a rotordynamic model of the system. Unbalance response analysis is included in a typical lateral rotordynamic analysis according to API. This analysis is normally used to predict the location of critical speeds and the vibration amplitude at probe locations when the rotor model is subjected to various theoretical unbalance distributions. However, this can also be used to analytically determine influence coefficients. The procedure is as follows:

- Develop model of rotor-bearing system.
  - Normal modelling procedures outlined in API 684 are appropriate.
  - Include stations at each available balance plane and measurement location.
  - Include substructure (bearing housing) models where appropriate.
- Apply an unbalance (theoretical trial weight) at the first available balance plane. The size and location is not important but should be documented.
- Calculate the synchronous vibration response at each probe location with the locations matching those available in the field.
- Repeat the application of a theoretical trial weight at the second available balance plane, removing the first theoretical trial weight and re-calculated the synchronous response. Repeat for all available planes.
- Calculate influence coefficients from the vector of all theoretical trial weights and the calculated response at each measurement location.
- Record actual synchronous vibration vectors from field measurements.
- Use a least squares minimization to calculate a one shot balance correction from field measured reference vibration data and predicted influence coefficients.

Advantages

This procedure offers several advantages over a traditional multi-plane influence coefficient balance. The most obvious is that production loss during the process of applying trial weights for calculating influence coefficients is eliminated since this is done analytically from the rotor model.

A second significant advantage is that influence coefficients can easily be calculated for all measurement positions and balance planes at any speed from the predicted response data. The output of the synchronous vibration response calculation for each application of a theoretical trial weight is the predicted trial weight response vector at each speed. The least squares minimization can be applied not only to the normal operating speed field-measured reference data, but to a set of coast-down data from all measurement positions using the calculated influence coefficients at all speeds. The weighting concepts outlined in this tutorial can also be applied to particular speeds (or speed ranges) such as critical speeds or normal operating speed to optimize the correction weight move.

Challenges

There are several challenges to implementing this method. As they say, there is no free lunch. The first challenge is developing an accurate model of the rotor bearing system. Fortunately, commercial rotordynamic software including bearing codes are fairly advanced and following good practice in model development will often result in a theoretical model that accurately reflects field data. For large, critical turbomachinery where such an effort is warranted, structural dynamics are often significant and should be included in the model. Another analytical challenge is correctly documenting phase conventions used by the selected rotordynamics software. In the same way a single plane balance trial weight can be incorrectly located when sensor position or rotation direction is incorrectly applied, the calculated one shot balance correction will be incorrect if the analyst does not correctly account for the machines actual sensor positions and rotation direction in the model.

Challenges with field measurement of reference data can also impact the calculated one shot correction. As with any balance job, incorrectly labeled/wired or non-functional probes will introduce error in the calculation. The reference data collected from a machine coast-down will likely not be sampled at equally spaced speed intervals. When calculating a correction weight from transient reference data, spline curve fitting the measured data can be used to correct for this.

Machine behavior that does not match the analytical model will introduce error in the calculation. The amplitude of response at critical speeds may be non-linear or may vary due to thermal effects or from a coast-down to a startup. This can be addressed by weighting speed ranges and applying the least squares optimization technique to the calculated influence coefficients with more than one set of reference data (hot and cold or coast-down and startup).

While the challenges appear to be a deterrent to applying this method, they are really no different than more traditional methods. Phase conventions must be correctly applied. Sensor orientation must be accounted for. Thermal and non-linear effects may be encountered with a single plane balance. The only (non-trivial) difference with this method is the development of an accurate rotordynamic model of the system.

Field Data

The proposed method has been applied to a six bearing steam turbine generator. The results show that the method, when correctly applied, gives excellent predictions of post- balancing response. In this particular application, accessible balance planes in the field were limited so the technique was used to select a balance correction that would reduce critical speed and operating speed vibration amplitudes using only the balance planes that were easily accessible. Additional complexity was involved in this application since the vibration amplitude was significantly different during startup and coastdown.

Figure 11(left) shows and overlay of the **measured** vibration amplitude and phase vs. speed during both a startup and coastdown as well as the **predicted** vibration amplitude and phase during startup and coastdown after the application of balance corrections for one of the 12 proximity probes. Figure 11 (right) shows the **measured** vibration amplitude and phase vs. speed after the application of the balance corrections. The predicted vs. measured results show excellent agreement at all probes. Ideally, the results would have shown more reduction in the first critical speed amplitude. However, further reduction of the response of this mode was not possible with the accessible balance planes in the field without a significant increase in the amplitude at operating speed.

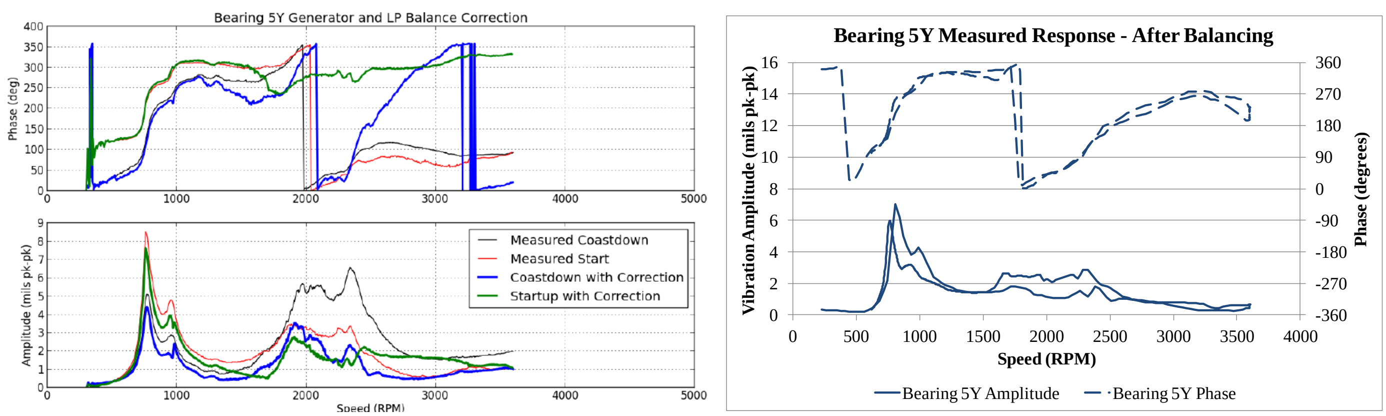

Figure 11 - Original Measured Vibration Amplitude/Phase during Coastdown and Startup with Predicted Post-Balancing Amplitude/Phase vs. Speed (left) and Measured Post-Balancing Amplitude/Phase vs. Speed (right)

Method Summary

The one shot balance method using calculated influence coefficients can be used to reduce machine downtime when compared to a traditional influence coefficient multiplane/multispeed balancing procedure. It is essentially a combination of an influence coefficient and a modal balancing method with both the influence coefficients and mode shapes being determined analytically. Like with other methods, the application of this method requires care in development of the rotor model, careful documentation of phase conventions, and a detailed review of the operational characteristics of the machine behavior to be successful. The time required for the analytical effort to develop the model and predict influence coefficients is significant but is may be easily justified when compared to the avoided lost production over other balancing methods.

SAFETY DISCUSSION

The discussion above about field balancing is a technical description of the methods and practices used to improve the balance condition of rotation machinery in situ. Conducting the balancing process as described obviously requires operation of large rotating machinery while varying the balance condition of the rotating assembly. This process is not without risk. Therefore, it is prudent to describe some important items and concerns related to the field balancing process.

First off, the whole proposition of field balancing implies (and virtually assures) that the machine is already operating at vibration levels in excess of desired amplitudes. In some cases, the entire purpose of the balancing effort is to prevent or prolong rotor repairs resulting from obvious rotor damage or other sources of unbalance such as loss of rotating components, severe erosion or damage to the rotor, and in some unfortunate cases to mask the cause of vibration such as misalignment by intentionally adding unbalance to the rotor to reduce the vibration (the symptom) opposed to reducing the actual forces in the machine.

Operating machinery at elevated vibration amplitudes is inherently risky or else the vibration limits would not be defined. Adding trial weights to rotors with elevated vibration can therefore be inherently riskier. Addition of the trial weights is intended to be completed so that the correction weight is located opposite the actual unbalance, with the purposeful intent of reducing the vibration after the trial weight is added.

In some cases, it is not possible to adequately predict the proper location of required trial weights, and if the weight is added on the heavy side rather than to opposite that, the vibration may in fact increase. Causes for the improper location of the trial weight include simple causes such as human error or complicated ones such as three dimensional damped rotor modes resulting in non-planer mode shapes. Whatever the cause, even higher vibration can result from addition of trial weights or even final weights.

Rotor response should be linear and predictable but often is not observed to be so. Once the rotor sensitivity to balance is determined with influence coefficients or balance effects, weight calculation and placement is determined by simple vector graphics or by computer calculation. However, it is possible for the expected influence to be faulty such as can occur if too small of a trial weight is added or if the system is highly non-linear. When this occurs, it is possible to add excessive weight so that resulting vibration can be much higher than expected.

With these considerations in mind, it is wise to consider and discuss safety around the machinery during the process. The primary items to consider are to manage the risk by proper assessment and placement of the balance correction weights applied and to limit personnel exposure to the machinery during the balancing process.

Controlling the risk for adding the weights can include the following:

- Verify that the initial trial weight will produce no more than 10% of journal reaction force due to the unbalance
- Verify the balance lag angle with review of coastdown data (not always 100% effective but better than nothing)
- Conduct a technical review of the weight type and mounting location to assure that the weight will not damage the component it is attached to and will not come off
    - Clamp on weights (C-clamp style) are normally adding to the inside of blades so that centrifugal force pushes the weight against the blade and the locking set screw is there primarily to keep the weight in position axially opposed to clamping on the outside where the set screw is the only mechanism to keep the weight in position
    - Weights added to holes in fan blades should only be added when the blades are designed to accommodate the weights, and weights do not exceed the intended design. Excessive balance weight can overstress blades and result in blade failure.
    - Welded weights (common for large fans) require technical review of the amount of weld metal used and the location of the added weldment. The force generated by the weight must be restrained by the weld, making the design of the weld critical.
    - Using engineered weights intended for the purpose of field balancing in engineered weight mounting locations (drilled balance holes or dovetail type balance slots) is always preferred to other methods

Limiting personnel exposure sounds simple, but there is an inherent draw for many people to approach machinery under test (particularly to feel how “bad” it is) and also to stand around the machinery during the process and especially during machine startup. It is always wise to use vibration sensors with long cables to allow the analyst to position him or herself in a relatively safe location during the operation of the machine. In particular, the locations depicted in Figure 12 include concerns for components flying off (such as trial weights), possible failure of components (such as couplings), chemical exposure (seals), and electrical faults (such as arcing of terminal boxes). All these items are at much higher risk during starting of large machinery, and at elevated risk due to the probability of sequential starts for balance shots.

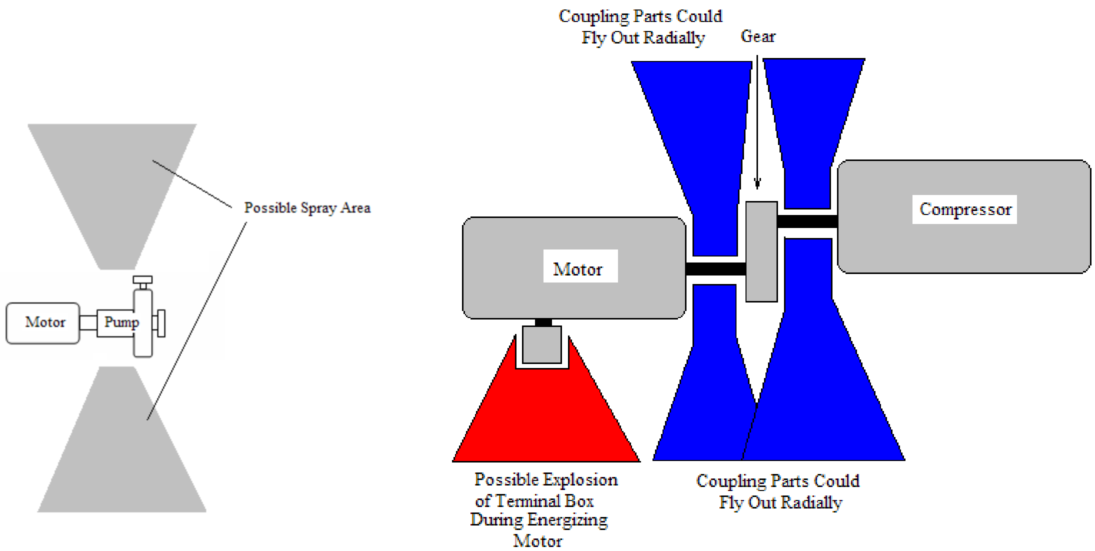

Figure 12 - Locations to Avoid During Testing

Vibration personnel are often called to assess high vibration amplitudes on a variety of processing equipment. In the case of pumps and compressors, there will always be the possibility that that seals could be leaking or could occur during the testing. Since the shaft is rotating where seals exit the casing, it is quite possible that product could be leaking and be slung away from the shaft and not be a visible drip or spray. With compressors, gas leaks will at times not be detectable without appropriate monitoring equipment.

In many cases where possible faults of larger, critical processing equipment and particularly during starting of high horsepower equipment, it is common to have a number of people that choose to be in the area because of the heightened concern for plant production impact, or simply because it appears to be an interesting experience. The crowd seems to generally be larger when the risk is higher or the potential faults are more severe, which can easily produce heightened excitement for the vibration analyst, and possible relaxation of concern for possible chemical exposure.

Therefore, any pump or compressor that has known high vibration should be approached as though it has product leaking from every seal. The thought process should be the same as used in defensive driving training where you are always expecting the other guy to put you at risk. Some simple defensive strategies should include:

1. Always take note of the wind direction prior to approaching the machine, and approach from the upwind side. In the case where the equipment is located on an elevated platform, climb stairs or a ladder cage on the upwind side or as a minimum verify that egress can be made from the upwind side.
2. Take note of the nearest safety shower prior to approaching the machine.
3. NEVER stand perpendicular to the shaft at a process seal (see Figure 12).
4. If the machine becomes unstable or the noise/vibration becomes dramatically more intense while standing next to it, stop vibration sampling and exit the area on the upwind side. As a minimum, go to stand on the upwind end of the machine until transient events stop.
5. NEVER enter an area around a machine that has suspected faults and walk around to the side of the machine where you have no route of escape from. In the event of failure, you should always have an unobstructed route of escape.
6. Spend a minimum amount of time near the machine with the possible fault, and in cases where the risk of chemical exposure is higher (lethal or explosive products), always mount sensors and use longer cables so that the risk is reduced by simply placing your body further away from the source of the potential leak.
7. NEVER attempt to record vibration data on a pump that is leaking or slinging liquid due to a seal leak or other damage unless contact with potentially hazardous materials can be prevented.
8. NEVER congregate or linger around machinery that may have mechanical damage or is being misapplied (surging compressor, deadheaded pump, etc.). Take vibration readings and make visual observations in the shortest time possible, then leave the area. If other personnel are in the area, recommend to them that non-essential people leave the area.

Damage to machinery can easily occur during plant or machine startup/upset conditions. Some situations that indicate higher risk would include:

- Any time a compressor is surging. Compressor surging can quickly cause heating and sudden failure of aluminum impellers. In addition, suction and discharge piping will often experience significant movement due to the high fluid loads associated with a compressor surge.
- Any time that relief devices are opened such as during flaring in plants with flammable materials. Use of the startup and flare piping will usually produce high noise and vibration around the relief and dump valves that feed the flare header. This noise and vibration can cause fatigue failures in rather short time.
- Operation of control valves with the actuator mostly closed and at high differential pressure. These conditions can produce excessive vibration and pipe fatigue very quickly.
- Water hammer can occur when liquid systems are started or when check valves or other control valves are quickly opened or closed. Water hammer events can easily produce much higher pressure surges than the equipment is designed for, and can cause sudden leakage of flanges, distortion of pressure vessel components, and high deflection of piping.

Vibration analysis is frequently required on machinery with known faults. In many cases the machine is kept on-line, or is restarted for vibration analysis prior to tagging the unit out for repair. The vibration analyst will arrive on the site and be expected to proceed directly to the machine for vibration testing followed by quick review to determine if it should be shut down immediately or if it is believed to be safe to continue operating. When this occurs, the risk of damage to machinery and injury to personnel is certainly higher than normal.

Many machinery areas are not equipped with adequate means of exit in the event of major equipment failure. In some cases, the testing must be done from temporary scaffolds or from platforms with only one exit. These situations are certainly ones that taking an extra minute to consider the exit options would be wise.

Some general recommendations should be considered when vibration is sampled on equipment with known faults:

1. Never stand next to drive couplings, or other locations where components would likely come out in the event of failure.
2. If temporary test equipment is setup for extended monitoring, locate the equipment on the end of the machine train, usually on the drive end.
3. Plan an escape route when approaching the machine.
4. Determine a threshold vibration level above which continued testing will not be performed. Discuss this level with plant personnel prior to testing if necessary so that appropriate action can be quickly taken to shut the machine off if the threshold values are exceeded.
5. Be prepared at all times to stop testing, move to a lower risk area, and possibly shut the machine down if conditions change so that noise or vibration levels obviously increase.
6. NEVER stay around a machine that has known faults with increasing severity.
7. NEVER continue testing once the pre-determined safe vibration threshold has been identified to be exceeded on any sample point.
8. NEVER continue operating a machine with an obvious mechanical fault such as loose hold down bolts, coupling element progressing damage (rubber material falling under coupling), metal shavings or bolts falling from the machine, etc.

CONCLUSIONS

In this tutorial, rotor balance definitions and balance tolerances were reviewed and general shop balancing procedures were discussed. Unbalance distribution on a rotor was reviewed and the need for modal balancing procedures for flexible rotors was identified. A thorough discussion of field balancing concepts and procedures was presented with examples of applying single plane graphical and influence coefficient balancing. A discussion of trial weight magnitude and location was presented. A detailed discussion of accounting for the various sensor and integration angles encountered in field balancing was presented with recommended conventions for successfully documenting a measurement setup and applying rules for trial weight locations. The single plane balancing concepts were extended to single plane balancing using multiple measurements. The least squares minimization procedure was discussed for multiple measurement locations, multiple balance planes, and multiple speeds. Concepts for least squares minimization using weighting (or scaling) and other optimization techniques was discussed. The tutorial includes case studies illustrating the application of various balance techniques including relevant data for the reader to replicate the balancing procedure. A method for one shot balancing at multiple planes and multiple speeds using calculated influence coefficients was presented.

NOMENCLATURE

e	= Eccentricity, in
$F_{balance}$	= Force cause by imbalance, lb
IPS	= Vibration velocity, in/sec-pk
M	= Rotor mass
m	= Unbalance mass
N	= Shaft speed, rot/min
T	= Trial vibration effect vector
TIR	= Total indicated runout, mil pk-pk
TW	= Trial weight, oz-in
U	= Residual unbalance, oz-in
V	= Vibration vector
W	= Journal static load, lb
W	= Weight vector
ω	= Angular Velocity, rad/s
C	= Influence coefficient
DE	= Drive end
FW	= Final correction weight
G	= Balance Grade
IB	= Inboard
NDE	= Non-drive end
O	= Original vibration vector
OB	= Outboard

APPENDIX A. PHASE ANGLE CONVENTION

Background

The selection and use of a phase angle convention impacts the polar plotting methods used with balancing and can pose challenging issues with locating the trial weights at optimal locations. In particular, the methods used to determine the lag angle and vibration response direction (phase) as compared to measured amplitudes and phases can be extremely confusing. This appendix is intended to describe and define a phase angle convention that can be used consistently to determine proper locations for balance trial weights in most situations.

Phase Angle Documentation

Phase angles are documented by most vibration measurement devices using phase lag angles. If the equipment you are using report phase lead vs. phase lag, the descriptions below do not apply. The phase lag angle defines the relative position of the peak of a vibration signal relative to a timing reference or tachometer as shown in Figure A. 1 with phase normally reported units of degrees:

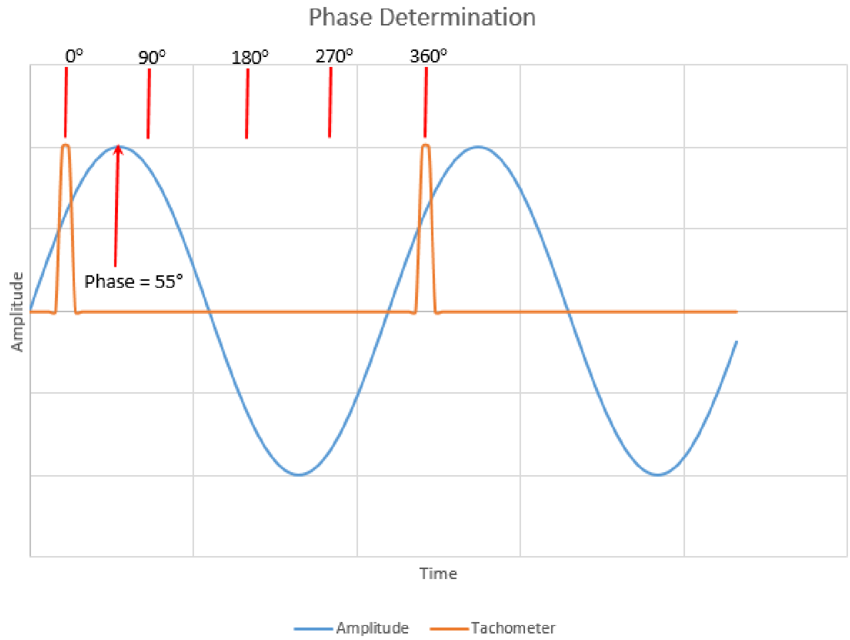

Figure A. 1 - Phase Determination

As shown, the phase is defined by the peak of the signal in a time waveform relative to the tachometer pulse with zero degrees being at the tachometer signal, with phase angle increasing as time passes. For the example above, the phase angle is about 55°.

Use of Vectors

Balance data is normally documented and plotted using vectors on a polar plot. The vector magnitude includes and amplitude and phase at 1xRPM that is measured and reported with appropriate instrumentation. Magnitudes will typically be documented with values such as 1.3 mils pk-pk @ 43°. The vibration measurement units will depend on the job setup. Phase angles are always phase lag as stated above.

The vector amplitudes can be plotted on a polar plot such as the plot below (Figure A. 2). This plot uses a radial distance from the center of the plot for vibration amplitude, and rotational position based on the phase associated with the vibration. The vector shown in the plot is 0.9 mils pk-pk @ 135°.

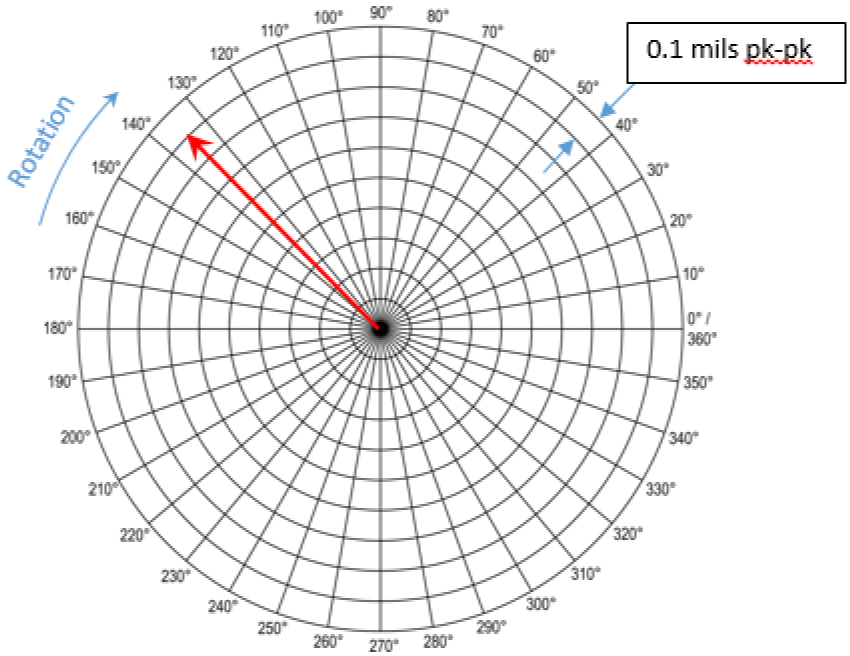

Figure A. 2 - Polar Plot Example

Using the polar plot approach and vectors to represent the vibration amplitudes, the data gathered can be used to determine the location for trial weights when properly documented.

Some details of the polar plot make the data display useful including:

- Shaft rotation direction should always be shown on the plot
- The polar plot should be drawn with angles increasing in the opposite direction to shaft rotation
- 0° on the plot should be drawn in line with the tachometer or trigger location

The plotting of vibration vectors for balancing will typically include X-Y orthogonal probes (i.e. shaft probes for a bearing) for each bearing location. Proper plotting of the vector amplitudes on the polar plot will help with proper location of the initial trial weight position.

Trial Weight Location

The trial weight location is desired to be 180° from the “heavy spot” so that the vibration vector shifts opposite to the existing vibration, and with an amplitude that results in near zero vibration. The actual relationship between vibration response and weight placement is normally not known, but can be estimated based on knowledge of the machine operating conditions.

The vibration response will always lag behind the location of the unbalance (the “heavy spot”) by a phase angle between 0 and 180° when the vibration response is measured near the axial location of the unbalance. The actual phase lag angle is determined by the relative location of the shaft critical speed to the operating speed. Operation well below the critical speed results in near 0° phase lag. Operation at resonance results in 90° lag. Operation well above the critical speed results in near 180° lag.

The vibration response phase can then be used to estimate the angular location of the unbalance at least within 90° by estimating the phase lag angle and locating the trial weight relative to a properly plotted vibration vector.

Angle Plotting

There are several angles that should be defined for the proper plotting of vectors as follows:

- Vibration measurement angle – this is the phase angle in the measured vibration data
    - If displacement units are used for this measurement, the phase angle is the “high spot”, or the angular

- If velocity or acceleration units are used for the measurement, the measured phase angle with have an integration offset described below
- "High Spot" – the phase angle where the shaft or component has its maximum movement in the direction of the sensor = Measured Vibration phase + Integration Angle
  - High spot = measurement angle when using displacement
  - High spot = measurement angle + 90° when using velocity
  - High spot = measurement angle + 180° when using acceleration
- Corrected phase angle = High Spot + sensor angle
- Sensor angle = angle difference between the sensor and the tachometer location

The proper method for plotting the vectors on the plot is to use the corrected phase angle defined above with the phase angle scale based on 0° being located at the tachometer position. This is functionally identical to plotting the vibration vectors with the measured phase angle plotted as the angle from the probe location opposed to the angle relative to the tachometer.

When this convention is used, an X-Y probe pair will have overlaying vectors on the polar plot when the probes are located 90° apart and the vibration response shows a 90° phase angle difference as is typical with a balance response. The method also provides a direct way to apply the phase lag angle for proper location of the first trial weight.

Phase Angle Plotting Example

The example below shows how the vector plotting is executed for a typical balance problem. The original vibration data and configuration are as follows and plotted in Figure A. 3:

- Shaft rotation direction is clockwise when viewing from end of driver towards driven
- Tachometer location is on the left a 90° from the top (9:00 clock position)
  - Tachometer angle = 0°
- X probe is on the left at 45° (10:30 clock position)
  - X Probe sensor angle = 315°
- Y probe is on the right at 45° (1:30 clock position)
  - Y Probe sensor angle = 225°
- Probes are proximity probes (displacement type so integration angle = 0°)
- X probe vibration is 1.3 mils pk-pk at 47°
- Y probe vibration is 2.1 mils pk-pk at 125°

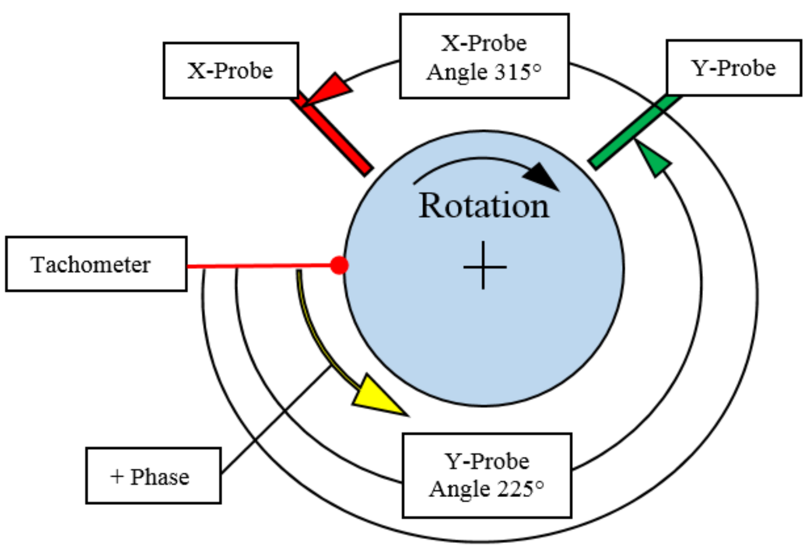

Figure A. 3 - Probe Angle Conventions

Using the method described above, the corrected angles used for plotting are as follows:

- X probe = 47° + 315° = 2°
- Y probe = 125° + 225° = 350°

Based on an estimated phase lag angle of 120° (operating above first critical speed), the heavy spot leads the corrected phase angles above by about 120°. That puts the heavy spot near 116°. The proper location of the first trial weight would then be 180° from there, or 296° relative to the tachometer location.

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ACKNOWLEDGEMENTS

The authors would like to thank the Turbomachinery and Pump Symposia Advisory Committees for the opportunity to present this tutorial. Thanks especially to Turbomachinery Symposium Advisory Committee members Malcolm Leader, P.E. and Dr. Luis San Andres for reviewing the drafts of the tutorial and providing excellent feedback.