International Journal of Innovative Research in Engineering & Management (IJIREM)
ISSN: 2350-0557, Volume-2, Issue-5, September- 2015

GEOMETRIC ANALYSIS AND COMPUTER AIDED DESIGN OF CYLINDRICAL WORM GEAR DRIVE HAVING ARCHED PROFILE

Prof. Dr. Illés Dudás
D.Sc., professor emeritus
Institute of Manufacturing Science, University of Miskolc, H-3515 Miskolc, Egyetemváros, Hungary,

Dr. Sándor Bodzás
Ph.D., associate professor
Department of Mechanical Engineering, University of Debrecen, H-4028 Debrecen, Ótemető str. 2-4.,

Dr. Zsuzsanna Balajti
Ph.D., vice rector, associate professor
Institute of Descriptive Geometry, University of Miskolc, H-3515 Miskolc, Egyetemváros, Hungary,

ABSTRACT
The objectives of this publication are geometric establishment and creation of computer geometric models (CAD) of developed in Diósgyőri Machine Factory and using exact production geometry first produced (grinded) cylindrical worm gear drive having arched profile by Illés Dudás [4, 5, 6, 7] for connection analysis, Finite Element Method analysis, etc.
Since the worm and worm gear wrap each other, that is why the worm gear has to be produced by a tool geometry of which is similar to the worm geometry. Knowing of the Connection I. Statement tooth surface of driven element which is wrapped by driver element is defined by numerical calculations.
The CAD model of the worm gear could be designed by adaptation of interpolating B – spline surface to the tooth surface points of worm gear.

Keywords
profile, worm, worm gear, hob, CAD, modelling

1. INTRODUCTION
One the most modern types of cylindrical helicoidal surfaces is the worm generated using a circular profile tool.
Contact surfaces between worms having ruled surfaces (Archimedian, convolute, involute types) and mated worm gears do not allow the formation of a continuous, high pressure bearing oil film. It is best to build up an oil film between mated surfaces so that the direction of the relative velocity of the drive faces into the direction normal to the common contact curve or very close to it [3, 4, 5].
A further advantage of mated elements having curved profile is that radius of mating flank surfaces are situated on the same side of the contact point common tangent, that is concave and convex surfaces are in contact, generating relatively small Hertz stress [3, 10].
As a consequence of smaller contact pressure, the load-carrying oil film can form easier.
As a result of arched profile teething the tooth shape and the suitable positioning of the centre of curvature of tooth flank (the position of engagement line) the dedendum tooth thicknesses, both on the worm $\overline{S}_{1F}$ and worm gear $\overline{S}_{2F}$, are significantly wider [3, 8].

2. ANALYSIS AND EQUATION OF HELICOIDAL SURFACE HAVING CIRCULAR PROFILE IN AXIAL SECTION
The cylindrical helicoidal surface is generated by a circle with radius ρ_{ax} situated in axial section. The arc is rotated round axis z_1 parallel to it; an axial displacement is realized in accordance with parameter p (Figure 1).

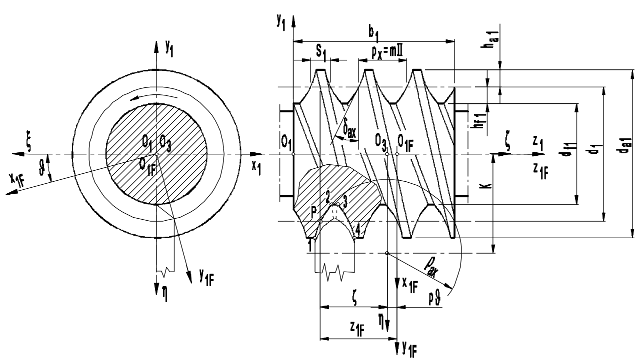

Figure 1. Profile of worm having circular profile in axial section (sketch of its generation)[4]

Displacement (z_1), angular displacement ϑ and parameter p correlate each other as:

$$z_1 = p \cdot \hat{\vartheta} \cdot \qquad (1)$$

The parameter p is the axial displacement belonging to unit angular displacement (ϑ=1 rad). The points of the generating circle during one complete revolution follow different thread lines, having equal leads p_z, which is the lead of the helicoidal surface as well

$$p_z = 2 \cdot \pi \cdot p \cdot \qquad (2)$$

The value of parameter p is:

$$p = \frac{p_z}{2 \cdot \pi} = \frac{d_{01}}{2} \cdot tg\gamma_0 = \frac{m \cdot z_1}{2} \cdot \qquad (3)$$

The helicoidal surface, having a circular generator in axial section, can be manufactured by lathe turning. The parameters required for generation and the geometry can be seen in Figure 1.

The face of the tool fits on plane $\eta O_3 \zeta$, its cutting edge being determined by the circle of radius ρ_{ax}. One side of the worm surface is prepared using the 1-2 edge of the moving tool. As the generator of the helicoid, the cutting edge 1-2 follows a worm path relative to the workpiece; this is why the origin (O_{1F}) of coordinate system K_{1F} (x_{1F}, y_{1F}, z_{1F}) performs a translational motion along the centre line of the workpiece within the coordinate system K_{sz} (ξ, η, ζ).

The sign of parameter p can be positive or negative depending on whether a right- or left-hand worm path is followed, ie whether it is a right- or left-hand lead helicoid. Applying coordinate transformation the correlations between systems K_{1F} (x_{1F}, y_{1F}. z_{1F}) and K_{sz} (ξ, η, ζ) based on Figure 1 and Figure 2 can be derived.

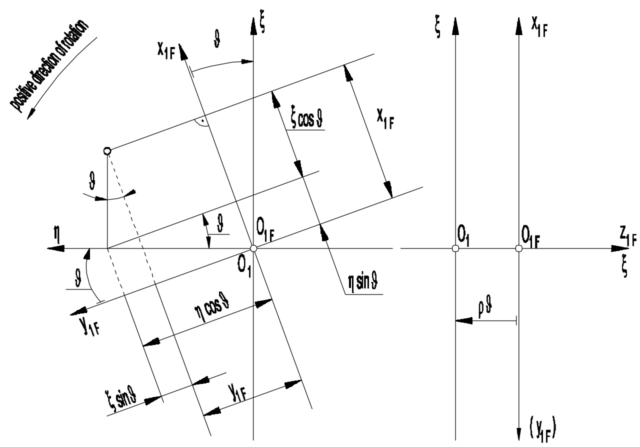

Figure 2. Correlation between coordinate systems the K_{1F} (x_{1F}, y_{1F}. z_{1F}) rotating, and K_{sz} (ξ, η, ζ) jointed to tool [4]

To investigate teeth surfaces the kinematic method is applied. For the coordinate transformation needed for use in the kinematic method, homogeneous coordinates should be used (Litvin, 1972). That is, necessary because the origins of the coordinate systems are at different points. That is, the transformation consists of two parallel movements, rotational and translational (thread motion) [4].

According to the above, the matrix of transformation from system K_{sz} (ξ, η, ζ) to system K_{1F} (x_{1F}, y_{1F}. z_{1F}) is:

$$\underline{M}_{1F,sz} = \begin{bmatrix} \cos\vartheta & \sin\vartheta & 0 & 0 \\ -\sin\vartheta & \cos\vartheta & 0 & 0 \\ 0 & 0 & 1 & -p\cdot\vartheta \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (4)$$

As the generator of the profile fits the plane η, O_3, ζ (it is the axial section) the equation of the profile generator can be written using Figure 3.

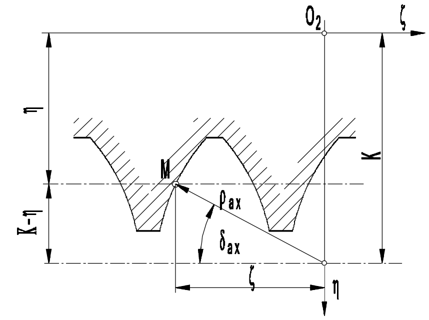

Figure 3. Determination of generator in axial section [4]

The coordinates of an oblique point of the profile generator can be expressed as:

$$\left.\begin{aligned} &M_j\ \left[0, \eta_M, -\sqrt{\rho_{ax}^2 - (K-\eta)^2}\right] \text{ on the right hand side of the tooth} \\ &M_b\ \left[0, \eta_M, +\sqrt{\rho_{ax}^2 - (K-\eta)^2}\right] \text{ on the left hand side of the tooth} \end{aligned}\right\} \qquad (5)$$

Transformation into the coordinate system jointed to the worm can be written as:

$$\vec{r}_{1F} = \underline{M}_{1F,sz} \cdot \vec{r}_{sz} = \begin{bmatrix} \cos\vartheta & \sin\vartheta & 0 & 0 \\ -\sin\vartheta & \cos\vartheta & 0 & 0 \\ 0 & 0 & 1 & -p\cdot\vartheta \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ \eta \\ -\sqrt{\rho_{ax}^2 - (K-\eta)^2} \\ t_{sz} \end{bmatrix} \qquad (6)$$

The right-hand side surface of the worm helicoid is obtained in the rotating coordinate system. Taking into consideration the direction of rotation ϑ in Figure 2, in this case it is negative, and the following equation are obtained [1, 4, 6]:

$$\left.\begin{aligned} &\left.\begin{aligned} x_{1F} &= -\eta\cdot\sin\vartheta; \\ y_{1F} &= \eta\cdot\cos\vartheta; \\ z_{1F} &= p\cdot\vartheta - \sqrt{\rho_{ax}^2 - (K-\eta)^2} \\ t_{1F} &= t_{sz} = 1. \end{aligned}\right\} \text{profile on the right hand side} \\ &\left.\begin{aligned} x_{1F} &= -\eta\cdot\sin\vartheta; \\ y_{1F} &= \eta\cdot\cos\vartheta; \\ z_{1F} &= p\cdot\vartheta + \sqrt{\rho_{ax}^2 - (K-\eta)^2} \\ t_{1F} &= t_{sz} = 1. \end{aligned}\right\} \text{profile on the left hand side} \end{aligned}\right\} \qquad (7)$$

3. DEFINING OF COORDINATE SYSTEMS FOR CAD MODELLING

The Illés Dudás type general mathematical model which is appropriate for mathematical modelling of production technologies methods is considered for defining of the necessary coordinate systems for modelling [4, 6].

For the description of the motion relations we define the own motion of each coordinate system. Thus K_{1F} (x_{1F}, y_{1F}, z_{1F}) coordinate system rotates with

$$\vec{\omega}_1 = \frac{d\varphi_1}{dt} = const. \tag{8}$$

angular velocity in K_{1cs} (x_{1cs}, y_{1cs}, z_{1cs}) stationary coordinate system [4, 6, 9].
The K_{2F} (x_{2F}, y_{2F}, z_{2F}) coordinate system in the K_2 (x_2, y_2, z_2) coordinate system rotates with

$$\vec{\omega}_2 = \frac{d\varphi_2}{dt} = const. \tag{9}$$

angular velocity [4, 6, 9].

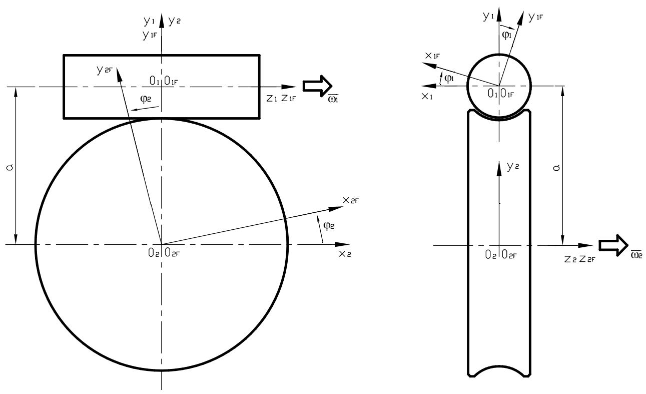

Figure 4. Coordinate systems dispositions for modelling of cylindrical worm gear drive

Transformation matrices between the K_{1F} (x_{1F}, y_{1F}, z_{1F}) rotational coordinate system fixed to member 1 and the K_{2F} (x_{2F}, y_{2F}, z_{2F}) rotational coordinate system fixed to member 2 are (Figure 4):

$$M_{2F,1F} = M_{2F,2} \cdot M_{2,1} \cdot M_{1,1F} =$$

$$= \begin{bmatrix} \sin\varphi_2 \cdot \sin\varphi_1 & \sin\varphi_2 \cdot \cos\varphi_1 & \cos\varphi_2 & a \cdot \sin\varphi_2 \\ \sin\varphi_1 \cdot \cos\varphi_2 & \cos\varphi_2 \cdot \cos\varphi_1 & -\sin\varphi_2 & a \cdot \cos\varphi_2 \\ -\cos\varphi_1 & \sin\varphi_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

$$M_{1F,2F} = M_{1F,1} \cdot M_{1,2} \cdot M_{2,2F} =$$

$$= \begin{bmatrix} \sin\varphi_2 \cdot \sin\varphi_1 & \sin\varphi_1 \cdot \cos\varphi_2 & -\cos\varphi_1 & -a \cdot \sin\varphi_1 \\ \cos\varphi_1 \cdot \sin\varphi_2 & \cos\varphi_1 \cdot \cos\varphi_2 & \sin\varphi_1 & -a \cdot \cos\varphi_1 \\ \cos\varphi_2 & -\sin\varphi_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{11}$$

4. GENERATION OF THE TOOTH SURFACE POINTS OF WORM GEAR

The dentation of worm gear is produced by hob which has similar geometry to worm connecting with worm gear based on the direct motion mapping [1, 2, 3, 4, 5, 6].
That is why we have to know the two parametric equations of the cutting edge of hob (7) for modelling.
The tooth surface points of worm gear are determined by direct kinematic method. Knowing the cutting edge of hob and the Connection I. Statement we search the tooth surface points of worm gear which are generated by mutual wrapping.
The relative velocity vector is in the K_{1F} system:

$$\vec{v}_{1F}^{(12)} = M_{1F,2F} \cdot \frac{dM_{2F,1F}}{dt} \cdot \vec{r}_{1F} = P_{1k} \cdot \vec{r}_{1F} \, . \tag{12}$$

Based on (12) the derived matrix is:

$$\frac{d}{dt} M_{2F,1F} = \begin{bmatrix} \begin{matrix} i_{21} \cdot \cos\varphi_2 \cdot \sin\varphi_1 \\ + \sin\varphi_2 \cdot \cos\varphi_1 \end{matrix} & \begin{matrix} i_{21} \cdot \cos\varphi_2 \cdot \cos\varphi_1 \\ - \sin\varphi_2 \cdot \sin\varphi_1 \end{matrix} & -i_{21} \cdot \sin\varphi_2 & a \cdot i_{21} \cdot \cos\varphi_2 \\ \begin{matrix} \cos\varphi_1 \cdot \cos\varphi_2 \\ -i_{21} \cdot \sin\varphi_2 \cdot \sin\varphi_1 \end{matrix} & \begin{matrix} -i_{21} \cdot \sin\varphi_2 \cdot \cos\varphi_1 \\ - \cos\varphi_2 \cdot \sin\varphi_1 \end{matrix} & -i_{21} \cdot \cos\varphi_2 & -a \cdot i_{21} \cdot \sin\varphi_2 \\ \sin\varphi_1 & \cos\varphi_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{13}$$

The P_{1k} matrix of the kinematic motion mapping is:

$$P_{1k} = \begin{bmatrix} 0 & -1 & -i_{21} \cdot \sin\varphi_1 & 0 \\ 1 & 0 & -i_{21} \cdot \cos\varphi_1 & 0 \\ i_{21} \cdot \sin\varphi_1 & i_{21} \cdot \cos\varphi_1 & 0 & a \cdot i_{21} \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{14}$$

If η and ϑ parameters are independent from one another, then the normal vector can be calculated by the following way [3, 6]:

$$\vec{n}_{1F} = \frac{\partial \vec{r}_{1F}}{\partial \eta} \times \frac{\partial \vec{r}_{1F}}{\partial \vartheta} \quad . \tag{15}$$

The equations of the tooth surface of member 2 which can be defined as the mashing surface of the group of contact lines in the K_{2F} coordinate system [4, 9]:

$$\left. \begin{aligned} \vec{n}_{1F} \cdot \vec{v}_{1F}^{(12)} &= 0 \\ \vec{r}_{1F} &= \vec{r}_{1F}(\eta, \vartheta) \\ \vec{r}_{2F} &= M_{2F,1F} \cdot \vec{r}_{1F} \end{aligned} \right\} \quad . \tag{16}$$

5. COMPUTER AIDED DESIGN (CAD) OF WORM GEAR DRIVE AND HOB

Knowing of the (16) equations we have worked out a computer program in case of a concrete cylindrical worm gear drive [3, 4, 6] for producing of the tooth surface points of worm gear (Figure 5, Figure 6).

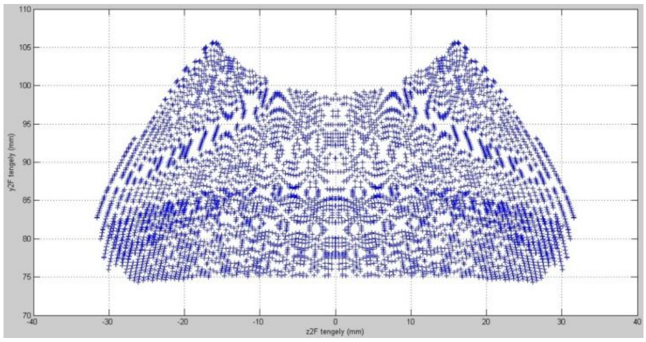

Figure 5. Determination of tooth surface points of worm gear

On Figure 5 it could be seen the touched tooth surface points by the cutting edge of hob namely the tooth surface points of worm gear in the $y_{2F} - z_{2F}$ plane.

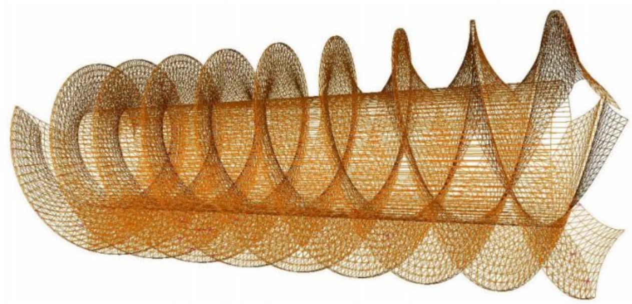

Figure 6. Theoretical model of cylindrical worm having arched profile [1]

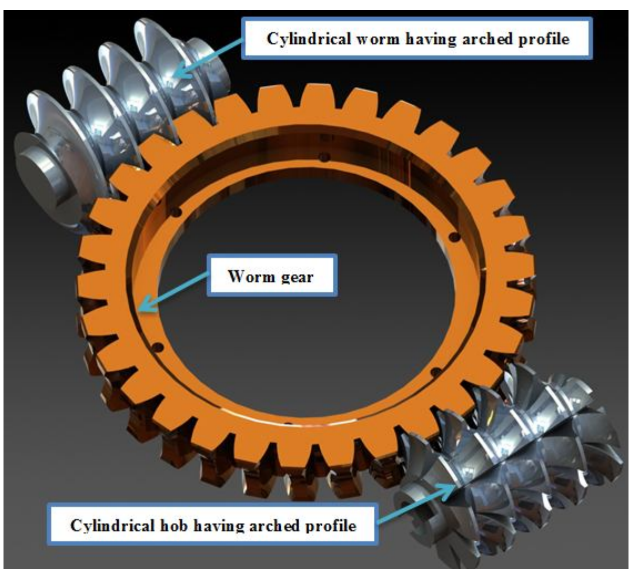

Figure 7. CAD models of cylindrical worm, hob and worm gear

The generated TXT file by computer program is imported to Solid Works designer software.

On the profile points interpolating B spline was drawn, then tooth profile, along the circumference of the worm gear, was determined according to number of the gear cogs [2]. Limiting of the dentation territory has been occurred based on geometric data and technical drawing of the worm gear drive [3, 4].

Using the stated mathematical method for determination of the tooth surface points of worm gear, the worked out computer program and the Solid Works designer software the CAD models of cylindrical worm gear drive (worm, worm gear and hob) could be produced (Figure 7).

6. CONCLUSION

The tooth surface of worm gear is determined by the worm dentation based on mutual wrapping. That is why in case of toothing the worm gear has to be produced by hob which has similar geometry to worm. We have determined the profile equations of cylindrical worm having arched profile.

Knowing the cutting edges of the hob we have determined the wrapped surface by the hob by numerical way namely the tooth surface of the worm gear.

We have connected interpolating B – spline surface for the tooth surface points of worm gear and after produced the CAD models of cylindrical worm, worm gear and hob.

The received CAD model is suitable for other Finite Element Analysis and connection analysis.

REFERENCES

[1] Balajti Zs.: *Kinematikai hajtópárok gyártásgeometriájának fejlesztése*, Ph.D. dissertation, University of Miskolc, 2007., Research Leader: Prof. Dr. Illés Dudás

[2] Bodzás, S.: *Kúpos csiga-, tányérkerék- és szerszám felületek kapcsolódásának elemzése,* Ph.D. dissertation, University of Miskolc, 2014., p. 154., Research Leader: Prof. Dr. Illés Dudás, DOI 10.14750/ME.2014.006

[3] Dudás I.: *Ívelt profilú csigahajtás egyszerűsített gyártása és minősítése*, University doctoral dissertation, Miskolc, 1973.

[4] Dudás, I.: *The Theory & Practice of Worm Gear Drives.* Kogan Page US, Sterling, USA, 2004. ISBN 1 9039 96619 9.

[5] Dudás, I.: *Ívelt profilú csigahajtások szerszámozásának és gyártásának fejlesztése,* Candidate dissertation, Miskolc, 1980. p.153+30 attachment

[6] Dudás, I.: *„Csavarfelületek gyártásának elmélete”.* Academic doctoral dissertation, Miskolc, 1991.

[7] Dudás I., Drobni J., Ankli J., Garamvölgyi T.: *Berendezés és eljárás főmetszetben ívelt profilú csigahajtópár geometriailag helyes gyártására alkalmas köszörűkorong profilozására,* Patent number: 170118, The day of the patent registration: 1983. 12. 27.

[8] Krivenko, I. Sz.: *Novüe tipü cservjacsnüh peredacs na szudah,* Izd. Szudoszrovenie, Leningrád, 1967.

[9] Litvin, F. L., Fuentes, A.: *Gear Geometry and Applied Theory,* Cambridge University Press, 2004., ISBN 978 0 521 81517 8

[10] Niemann, G.: *Maschienenelemente.* Band. 2., Sprienger-Verlag, Berlin, Heidelberg, New York, Tokyo, 1965.

APPENDIX

φ_1		Angular displacement of the helicoid (parameter for angular displacement, for meshing and for movement);
φ_2		Angular displacement of the tool (milling cutter or grinding wheel);
d_{a1}	(mm)	Addendum cylinder diameter of the worm
d_{g1}	(mm)	Pitch cylinder diameter of the worm
d_{fl}	(mm)	Root cylinder diameter of the worm
h_{f1}	(mm)	Dedendum height of the worm tooth
h_{a1}	(mm)	Addendum height of the worm tooth
K	(mm)	The distance of origin of profile radius and worm centerline
K_0 (x_0, y_0, z_0)		Stationary coordinate system affixed to machine tool
K_{1F} (x_{1F}, y_{1F},z_{1F})		Rotating coordinate system affixed helicoid surface
K_{2F} (x_{2F}, y_{2F},z_{2F})		Rotating coordinate system fixed to worm gear
Ks (ξ,η,ζ)		Tool coordinate system of generating curve of helicoid surface
m	(mm)	Axial module
$M_{lF,2F}$		Coordinate transformation matrix (transforms K_{2F} to K_{1F})
$M_{2F,1F}$		Coordinate transformation matrix (transforms K_{1F} to K_{2F})
n_{1F}		Unit normal vector of helicoid surface in coordinate system K_{1F}
n_{2F}		Unit normal vector of tool surface in coordinate system K_{2F}
O_0, O_1, O_2, O_{1F}, O_{2F}		Origins of coordinate systems related to their subscripts
p		Screw parameter of the helix on worm
p_t		Tangential screw parameter
p_a		Axial screw parameter
P_{1h}, P_{1k}, P_{1s}, P_{1a}		Kinematic projection matrix, for direct method (cylindrical, conical, general)
p_x	(mm)	Axial pitch of the worm
p_z	(mm)	Lead of thread
S_1	(mm)	Tooth thickness of the worm
S_{1F}	(mm)	Tooth thickness of dedendum of the tooth of the worm
$v_{1F(1,2)}$	(m/ min-1)	Velocity vector of helicoid and tool surfaces in the K_{1F} coordinate system
$v_{2F(1,2)}$	(m/ min-1)	Velocity vector of helicoid and tool surfaces in the K_{2F} coordinate system
ρ_{ax}	(mm)	Radius of tooth profile of worm having circular profile in axial section
ζ, η, ξ		Axes of the coordinate system (K_{sz}) of the tool
i_{21}		$i_{21} = \varphi_2/\varphi_1$ gearing ratio;
$\vec{r}_{sz}$		The equation of the tool edge or generating curve in the tool coordinate system K_{sz}
$\vec{r}_{1F}$		The position vector of a point fitted on worm surface
$\vec{r}_{2F}$		The position vector of an oblique point fitted on tool surface
γ_o		Lead angle on worm reference cylinder
z_1		Number of teeth on worm