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Fractal descriptors based on the probability dimension: a texture analysis and classification approach

Fractal descriptors based on the probability dimension: A texture analysis and classification approach ☆

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A B S T R A C T

In this work, we propose a novel technique for obtaining descriptors of gray-level texture images. The descriptors are provided by applying a multiscale transform to the fractal dimension of the image estimated through the probability (Voss) method. The effectiveness of the descriptors is verified in a classification task using benchmark over texture datasets. The results obtained demonstrate the efficiency of the proposed method as a tool for the description and discrimination of texture images.

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1. Introduction

Fractals have played an important role in many areas with applications related to computer vision and pattern recognition [1–6], owing to their flexibility in representing structures usually found in nature. In such objects, we observe different levels of detail at different scales, which are described in a straightforward manner by fractals, rather than through classical Euclidean geometry.

Most fractal-based techniques are based on the concept of fractal dimension. Although this concept was originally defined only for mathematical fractal objects, it contains some properties that make it a very interesting descriptor for any object in the real world. Indeed, fractal dimension measures how the complexity (level of detail) of an object varies with scale, an effective and flexible means of quantifying how much space an object occupies, as well as important physical and visual properties of the object, such as luminance and roughness.

Fractal techniques include the use of Multifractals [7–9], Multiscale Fractal Dimension [10,11] and fractal descriptors [12–16]. Here we are focused on the last approach, which has demonstrated the best results in texture classification [17]. The main idea of fractal descriptors theory is to provide descriptors of an object represented in a digital image from the relation among fractal dimensions taken at different observation scales, thus these values provide a valuable information on the complexity of the object, in the sense that they capture the degree of detail at each scale. In this way, fractal descriptors are capable of quantifying important physical characteristics of the structure, as the fractal dimension, but presenting a richer information than can be provided by a single number (fractal dimension).

Although fractal descriptors have demonstrated to be a promising technique, we observe that they are defined mostly on well-known methods to estimate the fractal dimension. Here, we propose fractal descriptors based on a less known definition of fractal dimension: the probability dimension. This is a statistical approach, which measures the distribution of pixel intensities along the image. In this way, such descriptors can express how the statistical arrangement of pixels in the image changes with the scale and how much such correlation approximates a fractal behavior. In this sense, our descriptor also measure the self-similarity and complexity of the image but upon a statistical viewpoint. This is a rich and not explored perspective, which is studied in depth in this work.

We use the whole power-law curve of the dimension and apply a time-scale transform to emphasize the multiscale aspect of the features. Finally, we test the proposed method over three well-known datasets, that is, Brodatz, Outex and UIUC, comparing the results with another fractal descriptor approach showed in [13] and other conventional texture analysis methods. The results demonstrate that probability descriptors achieve a more precise classification than other classical techniques.

2. Fractal theory

In recent years, fractal geometry concepts have been applied to the solution of a wide range of problems [1–6], mainly because

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conventional Euclidean geometry has severe limitations in providing accurate measures of real-world objects.

2.1. Fractal dimension

The first definition of fractal dimension provided in [18], is the Hausdorff dimension. In this definition, a fractal object is a set of points immersed in a topological space. Thus one can use results from measure theory to define a measure over this object. This is the Hausdorff measure expressed by

$$H_\delta^s(X) = \inf \sum_{i=1}^{\infty} |U_i|^s : U_i \text{ is a } \delta\text{-cover of } X, \tag{1}$$

where $|X|$ denotes the diameter of X, that is, the maximum possible distance among any elements of X:

$$|X| = \sup\{|x - y| : x, y \in X\}. \tag{2}$$

Here, a countable collection of sets U_i, with $|U_i| \leqslant \delta$, is a δ-cover of X if $X \subset \cup_{i=1}^{\infty} U_i$.

Notice that H also depends on a parameter δ, which expresses the scale at which the measure is taken. We can eliminate such dependence by applying a limit over δ, defining in this way the s-dimensional Hausdorff measure:

$$H^s(X) = im_{\delta \to 0} H_\delta^s(X). \tag{3}$$

The plot of $H^s(X)$ as a function of s shows a similar behavior in any fractal object analyzed. The value of H is ∞ for any $s < D$ and it is 0 for any $s > D$, where D always is a non-negative real value. D is the Hausdorff fractal dimension of X. More formally,

$$D(X) = \{s\} | \inf \{s : H^s(X) = 0\} = \sup \{H^s(X) = \infty\}. \tag{4}$$

In most practical situations, the Hausdorff dimension is difficult or even impossible to calculate. Thus assuming that any fractal object is intrinsically self-similar, the literature shows a simplified version, also known as the similarity dimension or capacity dimension:

$$D = -\frac{\log(N)}{\log(r)}, \tag{5}$$

where N is the number of rules with linear length r used to cover the object.

In practice, the above expression may be generalized by considering N to be any kind of self-similarity measure and r to be any scale parameter. This generalization has given rise many methods for estimating fractal dimension, with widespread applications to the analysis of objects that are not real fractals (mathematically defined) but that present some degree of self-similarity in specific intervals. An example of such a method is the probability dimension, used in this work and described in the following section.

2.2. Probability dimension

The probability dimension, also known as the information dimension, is derived from the information function. This function is defined for any situation in which we have an object occupying a physical space. We can divide this space into a grid of squares with side-length δ and compute the probability p_m of m points of the object pertaining to some square of the grid. The probability function is given by

$$N_P(\delta) = \sum_{m=1}^{N} \frac{1}{m} p_m(\delta), \tag{6}$$

where N is the maximum possible number of points of the object inside a unique square. Here we use a generalization of the above expression defined in the multifractal theory [19]:

$$N_P(\delta) = \sum_{m=1}^{N} m^\alpha p_m(\delta), \tag{7}$$

where α is any real number.

The dimension itself is given as

$$D = -\lim_{\delta \to 0} \frac{\ln N_P}{\ln \delta}. \tag{8}$$

When this dimension is estimated over a gray-level digital image $I : [M, N] \to \Re$, a common approach is to map it onto a three-dimensional surface S as

$$S = \{i, j, I(i, j) | (i, j) \in [1 : M] \times [1 : N]\}. \tag{9}$$

In this case, we construct a three-dimensional grid of 3D cubes also with side-length δ. The probability p_m is therefore given by the number of grid cubes containing m points on the surface divided by the maximum number of points inside a grid cube (see Fig. 1).

3. Fractal descriptors

Fractal descriptors are values extracted from the log–log relationship common to most methods of estimating fractal dimension. Actually, any fractal dimension method derived from the concept of the Hausdorff dimension obeys a power-law relation, which may be expressed as

$$D = -\frac{\log(\mathfrak{M})}{\log(\epsilon)}, \tag{10}$$

where $\mathfrak{M}$ is a measure depending on the fractal dimension method and ϵ is the scale at which this measure is taken.

Therefore Fractal descriptors are provided from the function u:

$$u : \log(\epsilon) \to \log(\mathfrak{M}). \tag{11}$$

We call the independent variable t to simplify the notation. Thus $t = \log \epsilon$ and our fractal descriptor function is denoted $u(t)$. For the probability dimension used in this work, we have

$$u(t) = -\frac{\log(N_V(\delta))}{\log(\delta)}. \tag{12}$$

The values of $u(t)$ may be directly used as descriptors of the analyzed image or may be post-processed by some kind of operation aimed at emphasizing some specific aspects of that function. Here, we apply a multiscale transform to $u(t)$ and obtain a bi-dimensional function $U(b, a)$, in which the variable b is related to t and a is related to the scale at which the function is observed. A common means of obtaining U is through a wavelet transform:

$$U(b, a) = \frac{1}{a} \int_{\Re} \psi\left(\frac{t - b}{a}\right) u(t) dt, \tag{13}$$

where ψ is a wavelet basis function and a is the scale parameter [20]. Fig. 2 shows an example where two textures with the same dimension, but visually distinct, provide different descriptors.

4. Proposed method

This work proposes to obtain fractal descriptors from textures by using the probability fractal dimension. At first, the values on the curve $u(t) : \log(N_P(\delta))$ in Eq. (7) are computed for each image. Therefore we apply a multiscale transform to u.

The multiscale process employs a wavelet transform of $u(t)$, as described in the previous section:

$$U(b, a) = \frac{1}{a} \int_{\Re} \psi\left(\frac{t - b}{a}\right) u(t) dt. \tag{14}$$
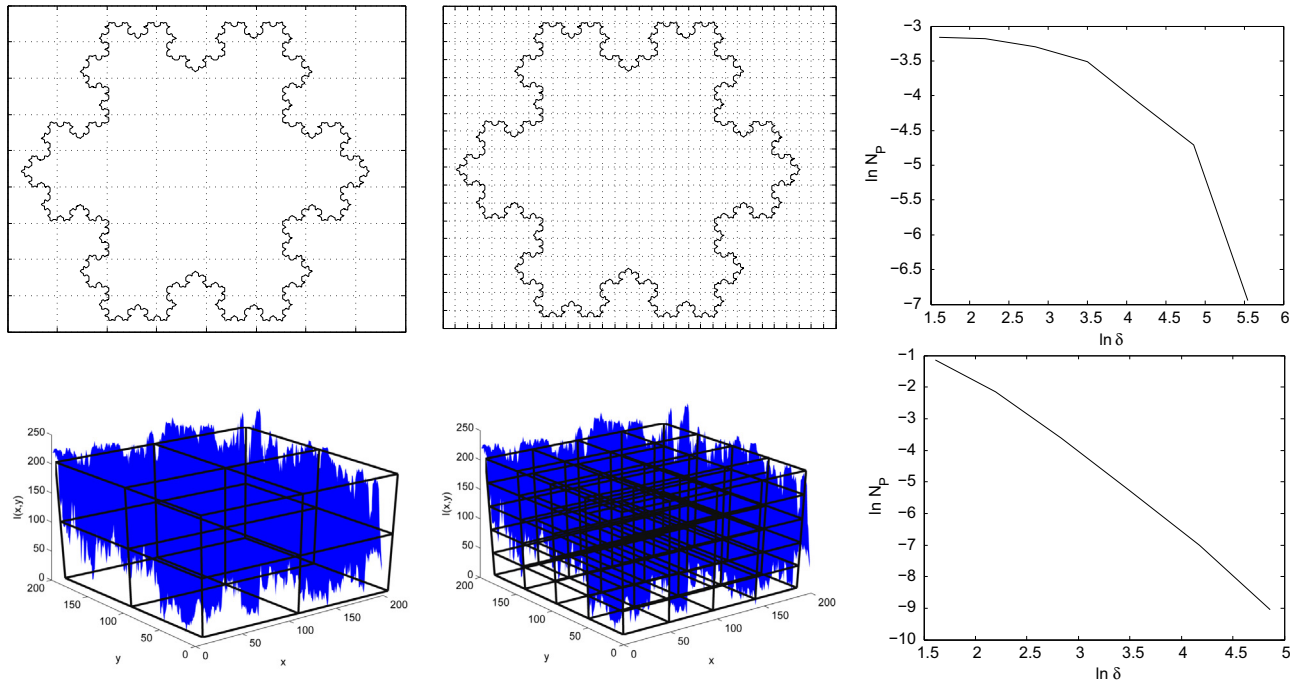
Fig. 1. Two estimates of the probability dimension. Above, the 2D version used for shape analysis. Below, the 3D version used for gray-level images. In the 3D case, the original image is represented by a surface.
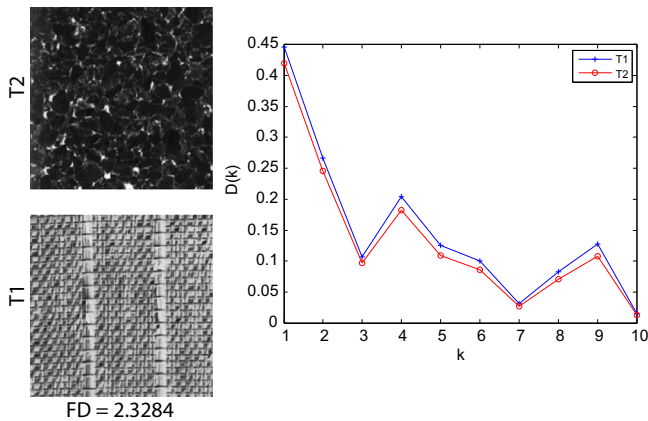

Fig. 2. Two textures with the same dimension (FD), but with different descriptors.

As the multiscale transform maps a one-dimensional signal onto a bi-dimensional function, it is a process that generates intrinsic redundancies. There are different approaches to eliminating such redundancies and keeping only the relevant information [11]. Here, we adopt a simple method, fine-tuning smoothing, in which $U(b, a)$ is projected onto a specific value a_0 of the wavelet parameter. We tested values of a ranging between 0.1 and 5 and used the values that provided the best performance in the training experiments.

Empirically, we observed that the initial points in this multi-scaled curve provided better performance in our application. Then, we established a threshold t after which all the points in the convolution curve are disregarded. Finally, different values of α in Eq. (7) can be used and the multiscale transforms resulting from each value are combined to provide the proposed descriptors.

The proposed method is different from any other fractal method proposed in the literature in the sense that it is the first proposal of a multiscale fractal analysis under a statistical viewpoint. While other fractal-based descriptors employ geometrical and/or spectral information of the pixel arrangement in the entire image, here the analysis relies on the number of regions with a specific distribution, in this way reducing the sensitivity to distortions in the pixel distribution within a particular scale and leading to a representation of the texture more robust to noises and other artifacts. Fig. 3 shows a visual example illustrating the discriminative capability of the proposed method on texture images.

5. Experiments

In order to verify the efficiency of the proposed technique, we applied our probability descriptors to the classification of three benchmark datasets and compared our results to the performance of other well-known and state-of-the-art methods for texture analysis.

The first classification task used the Brodatz dataset, a classic set of natural gray-level textures photographed and assembled in an architecture book [21]. This dataset is composed by 111 classes
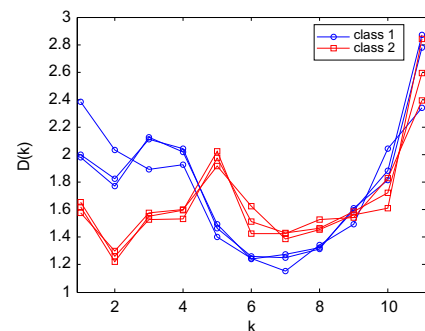

Fig. 3. Discrimination of texture image by the proposed descriptors. We have six images from two classes and the respective descriptors. Notice that the classes are substantially separated by the curves.

with 10 textures in each class. Each image has a pixel dimension of 200×200.

The second data set was Outex, a set of color textures extracted from natural scenes [22]. Here, we used the first 20 classes, each one having 20 images with a 128×128 pixel dimension, and converted them to gray-level images.

The third database is UIUC [23]. This is composed by 68 classes with 20 gray-level textures in each one. Such textures have a strong variation on acquisition parameters, like albedo, illumination and viewpoint, making its classification a challenging task.

We compared our probability descriptors to eight other techniques, namely, Local Binary Patterns (LBP) [24], shift Local Binary Patterns [25], fuzzy Local Binary Patterns [25], Gabor-wavelets [26], Gray-Level Difference Method (GLDM) [27], a multifractal approach described in [19] and Bouligand–Minkowski fractal descriptors [13,17].

The proposed method has three important parameters that are the multiscale threshold t and scale a and the coefficient (s) α in the probability dimension. These parameters are set empirically by scanning within a range of values and considering those values that provide the best classification of a training set of the database. Particularly, a varies between 0.1 and 5.0 (with increments of 0.1), t varies between 1 and 150 (with increments of 1) and α varies between −2 and 2 (with increments of 0.1). These ranges were chosen as there is no significant contribution outside this boundary. Table 1 shows the best parameters found for each database.

For the literature methods, the setup described in each reference was followed.

Since most of the compared methods provide feature vectors with large dimension, we applied a Principal Component Analysis (PCA) over the data to extract the most meaningful information and avoid problems such as the dimensionality curse [28]. Finally to verify the accuracy of each compared approach, the dataset was split following a cross-validation scheme, that is, the data is partitioned into K complementary subsets and the classification is performed in K rounds. In each round, one subset is used for testing and the $K − 1$ remaining subsets for training. The value of K usually ranges between 5 and 10 and is obtained empirically for each database. The classification itself is carried out by a Support Vector Machine (SVM) classifier [29]. The classification also has two parameters set empirically in the same way as the method settings, e. g., the K of the K-fold and the maximum number of features (descriptors) per image. Table 2 shows the parameters used in each data set.

6. Results

Table 3 shows the correctness rate in the classification of the Brodatz dataset using the compared descriptors. The proposed method obtained the best result, outperforming the powerful Bouligand–Minkowski fractal descriptors and taking substantial advantage over other state-of-the art techniques such as Gabor and LBP. A particularly important aspect of our method with this data set is the reduced number of descriptors needed to provide a precise classification. This point is especially important in large databases, for which computational performance is more relevant.

Table 1
Empirical parameters used for the proposed method.

Database	a	t	α
Brodatz	0.1	8	0.2
Outex	0.1	30	−0.4, −0.5, −0.8
UIUC	0.1	110	−1, −1.1, −1.3

Table 2
Parameters used for the classification. K is the K-fold parameter and n is the maximum number of features per image.

Database	K	n
Brodatz	5	10
Outex	10	20
UIUC	5	11

Table 3
Correctness rate for Brodatz dataset.

Method	Correctness rate (%)	Number of descriptors
LBP	82.5 ± 0.2	10
Shift LBP	92.0 ± 0.1	9
Fuzzy LBP	87.7 ± 0.1	10
Gabor	86.8 ± 0.1	8
GLDM	79.6 ± 0.2	8
Multifractal	54.9 ± 0.5	9
Bouligand–Minkowski	90.8 ± 0.1	10
Proposed method	93.0 ± 0.1	8

Furthermore, the small number of features avoids the curse of dimensionality, which impairs the reliability of the global result.

Table 4 shows the results for the Outex textures. The proposed approach achieved a success rate of 100%, despite the challenge of applying a gray-scale-based method to color analysis. In fact, Outex textures exhibit nuances which are better expressed in the color information, such as the changes in the lighting perspective and the images from different classes presenting similarities in the intensity distribution, though distinguished by color. Based on this result, our method demonstrates that although it does not use any color information, it is powerful also for color image analysis.

In Table 5, the success rates for UIUC database is showed. The proposed method outperformed even state-of-the-art approaches like the variants of LBP and Gabor-wavelets. This database is characterized by its significant variation in viewpoint, albedo, 3D shape, scale and illumination conditions. Such variation makes the categorization of those textures a challenging task. Again, our method provided the greatest success rate confirming the richness of the information enclosed in the multiscale transform of the probability fractal dimension. In fact, the multiscale view of the texture captures the particular behavior of each material concerning the environment changings, whereas the probability dimension describes the essential morphology of the image. The combination of both tools explains the best results obtained in these experiments.

Fig. 4 shows how the success rate varies according to the number of descriptors used in both datasets. The graphs show a well-known property of Karhunen–Loève transform enclosed in PCA. The most expressive information is concentrated in the initial descriptors, so that the success curves show a quick growing and then tend to stabilize at a constant rate. The larger size and the native gray-scale format of Brodatz data leads to a clearer advantage

Table 4
Correctness rate for Outex dataset.

Method	Correctness rate (%)	Number of descriptors
LBP	100.0 ± 0.0	13
Shift LBP	100.0 ± 0.0	6
Fuzzy LBP	99.3 ± 0.0	11
Gabor	98.8 ± 0.0	12
GLDM	92.5 ± 0.1	14
Multifractal	84.5 ± 0.2	11
Bouligand–Minkowski	99.8 ± 0.0	11
Proposed method	100.0 ± 0.0	17

Table 5
Correctness rate for UIUC dataset.

Method	Correctness rate (%)	Number of descriptors
LBP	74.1 ± 0.3	11
Shift LBP	69.9 ± 0.3	11
Fuzzy LBP	67.9 ± 0.3	11
Gabor	76.2 ± 0.2	11
GLDM	64.5 ± 0.3	11
Multifractal	64.1 ± 0.4	10
Bouligand–Minkowski	83.9 ± 0.2	11
Proposed method	84.9 ± 0.1	11

of probability descriptors in that database. In Outex, the first descriptors, corresponding to the PCA components with higher variance, do not have as much significance for the classification purpose. However, the sum of all of them provide the best result. This is a specific property of fractal descriptors, as can be observed in the Bouligand–Minkowski descriptors for the Brodatz data as well. Fractal descriptors are tightly correlated among themselves, thus we do not have a large significance carried only in a few descriptors. Finally, the correctness rates for UIUC demonstrates the efficiency of fractal approaches in this kind of textures. The intrinsic multiscalar property of fractal descriptors attenuate the effect of condition variations in that data set, by focusing on the morphology of the texture instead of the simple pixel intensities.

Finally, Figs. 5 and 6 show the confusion matrices of the methods with the best performances. In this kind of representation, a good descriptor must produce a matrix with a diagonal as lighter and continuous as possible and the minimum of dark points outside the diagonal.

As can be seen, in Brodatz data, the probability descriptors clearly presented these characteristics, with almost no "gap" in the diagonal and with a few dark points outside. Both gaps and

gray points indicate the confusion of the classifier, that is, elements classified incorrectly in some way. This confusion is caused mostly by the high similarity inter-class and low similarity intra-class. A precise descriptor, like the proposed, avoids such confusion by providing measures capable of faithfully representing the most complex structures.

In the case of Outex, all the best methods (proposed and variants of LBP) achieved 100% of success in the classification, resulting in a confusion matrix with the entire diagonal in red. In the same experiment, Bouligand–Minkowski also provided a good result even though its matrix shows a slight misclassification on class 13 (class 13 and 14 are hardly distinguished without using color).

Fig. 7 shows the confusion matrices for UIUC database. The picture illustrates visually the behavior of each method in each class. For instance, although LBP and the proposed approach have similar overall performance, the binary patterns present a significant misclassification on class 14, confused with class 12. The proposed method exhibits a more homogeneous result with a good prediction skill on all the classes.

An overall analysis of the results demonstrates that the proposed method outperformed the compared ones in all the tested datasets, using a small number of descriptors. Such results were expected from fractal theory given its wide applicability to the analysis of natural textures. Actually, fractal geometry presents a remarkable flexibility in the modeling of objects that cannot be well represented by Euclidean rules. The fractal dimension is a powerful metric for the complex patterns and spatial arrangements usually found in nature. Fractal descriptors provide a way of capturing multiscale variations and nuances that could not be measured by conventional methods. More specifically, the probability descriptors proposed here combine a statistical approach with fractal analysis, comprising a framework that supports a precise and reliable discrimination technique, as confirmed in the above results.
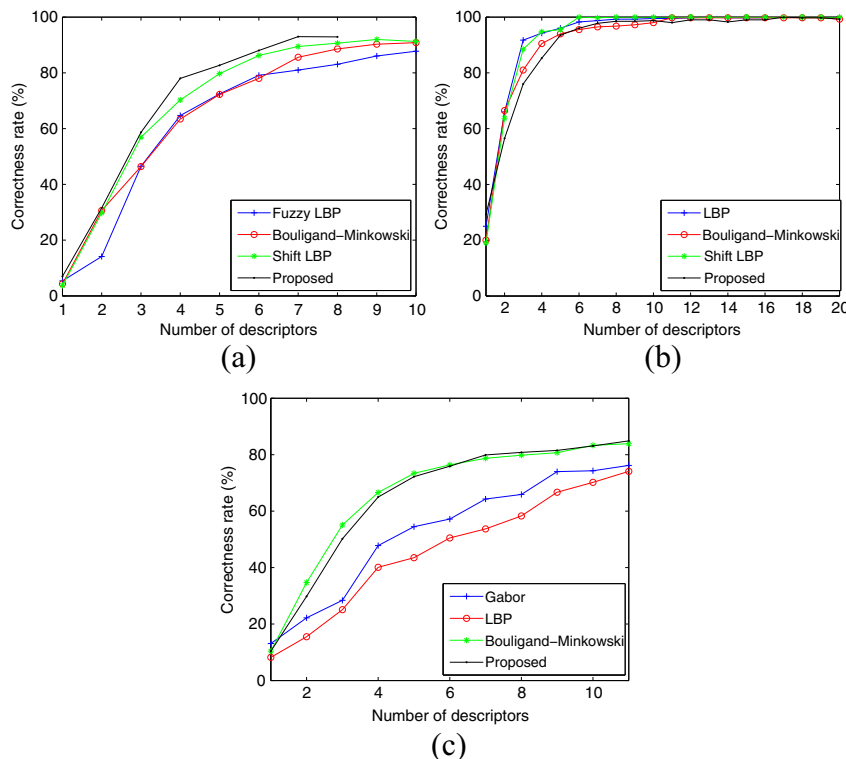
Fig. 4. Success rate against the number of descriptors in each dataset. (a) Brodatz, (b) Outex and (c) UIUC.
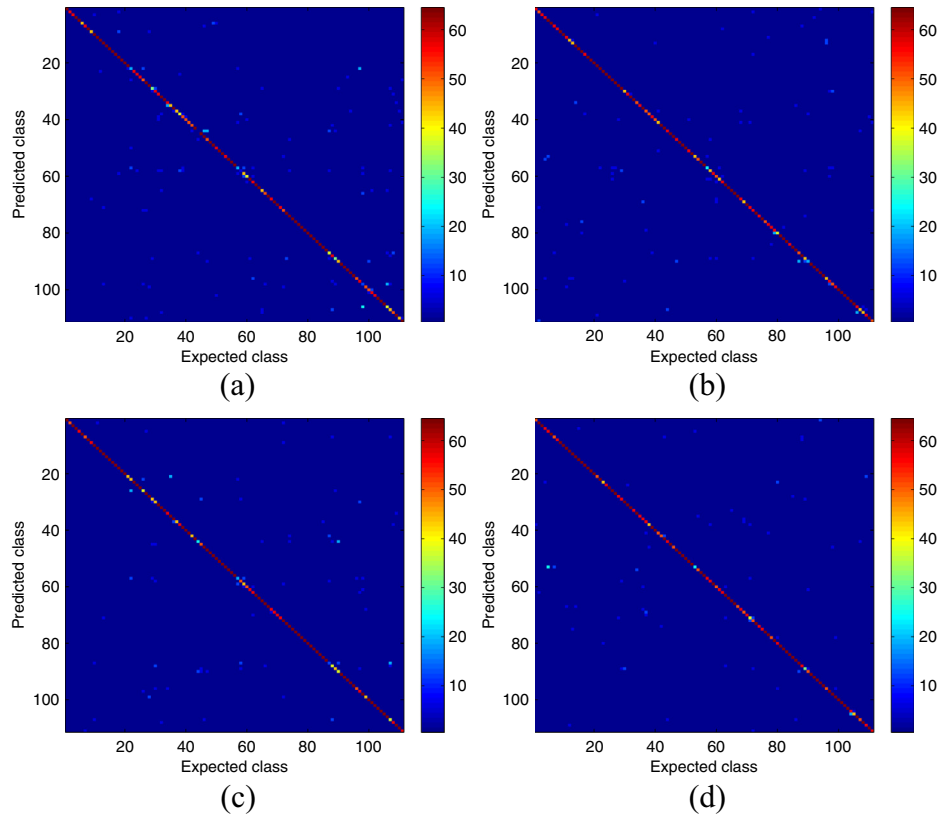
Fig. 5. Confusion matrices in Brodatz dataset. In such figures, we have the predicted classes in the rows and the expected ones in the columns. The number of images expected/predicted in each class is given by the color at each point (red points correspond to large number of images). (a) Fuzzy LBP, (b) Bouligand–Minkowski, (c) Shift LBP and (d) proposed method. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
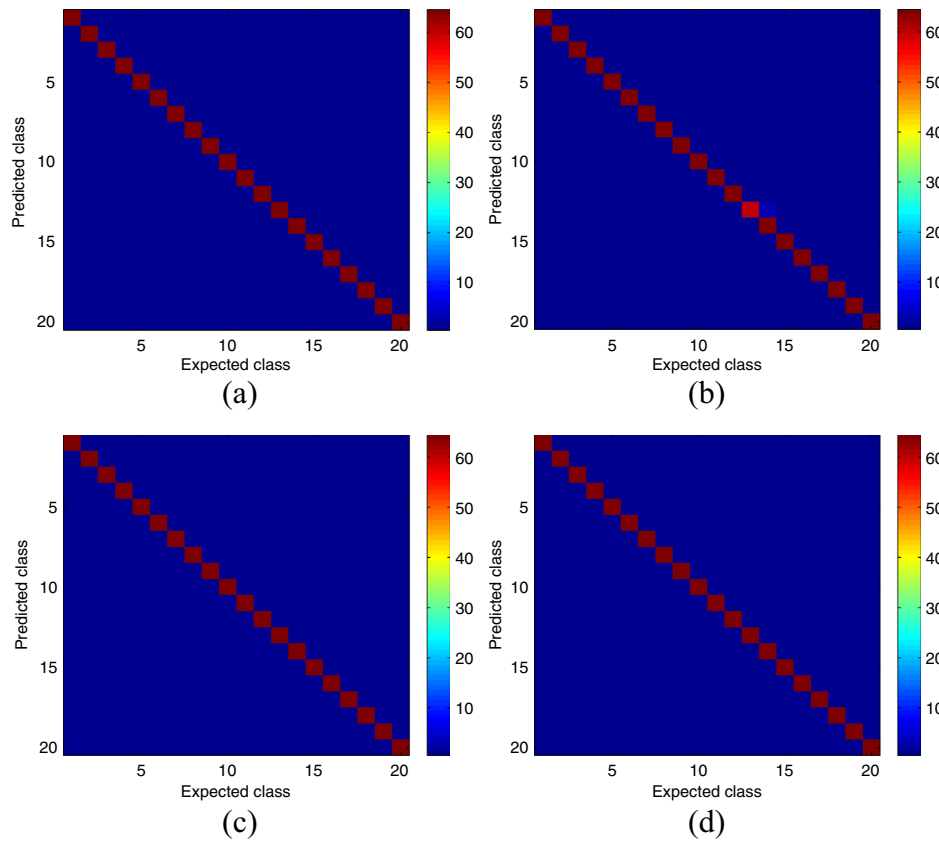

Fig. 6. Confusion matrices in Outex dataset. (a) LBP, (b) Bouligand–Minkowski, (c) Shift LBP and (d) proposed method.
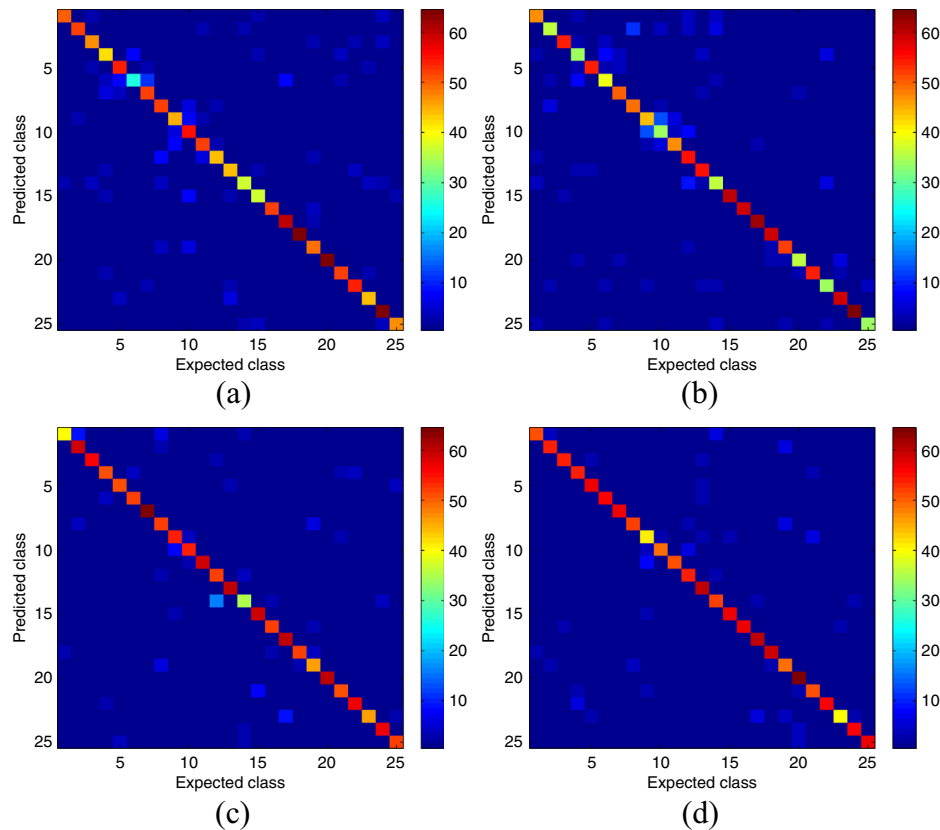
Fig. 7. Confusion matrices in UIUC dataset. (a) Gabor, (b) LBP, (c) Bouligand–Minkowski and (d) proposed method.

7. Conclusion

We have proposed a novel method for extracting descriptors by applying a multiscale transform over the power-law relation of the fractal dimension estimated by the probability method.

We tested the efficiency of the proposed technique in the classification of three well-known benchmark texture datasets and compared its performance to that of other classical texture analysis methods. The results demonstrated that probability fractal descriptors are a powerful tool for modeling such textures. The proposed method achieved a high success rate in the classification of the benchmark data sets, using a few descriptors in this task. These results demonstrate that the proposed method is capable of combining precision, low computational cost and robustness.

As a consequence, our method offers a reliable approach to solve a large class of problems involving the analysis of texture images.

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