Trajectory Tracking Passivity-based Control for Marine Vehicles Subject to Disturbances

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Abstract

In this paper we present a dynamic model of marine vehicles in both body-fixed and inertial momentum coordinates using port-Hamiltonian framework. The dynamics in body-fixed coordinates have a particular structure of the mass matrix that allows the application of passivity-based control design developed for robust energy shaping stabilisation of mechanical systems described in terms of generalised coordinates. As an example of application, we follow this methodology to design a passivity-based controller with integral action for fully actuated vehicles in six degrees of freedom that tracks time-varying references and rejects disturbances. We illustrate the performance of this controller in a simulation example of an open-frame unmanned underwater vehicle subject to both constant and time-varying disturbances. We also describe a momentum transformation that allows an alternative model representation of marine craft dynamics that resembles general port-Hamiltonian mechanical systems with a coordinate dependent mass matrix.

Keywords: Marine robotics, Port-Hamiltonian Systems, Integral Control, Disturbance Rejection.

1. Introduction

Interconnection and damping assignment passivity-based control (IDA-PBC) is an attractive technique for designing motion-control strategies for physical dynamic systems. This technique uses the control action to transform the open-loop system into a closed-loop system in port-Hamiltonian (pH) form \cite{1, 2}. The closed-loop potential energy is chosen such that it attains its minimum at the desired configuration of the system—this determines
the closed-loop equilibrium. Under certain conditions on different terms of the closed-loop model, stability can be proven using the closed-loop energy as a Lyapunov function. The design also exhibits passivity properties with respect to force inputs and velocity outputs.

The passivity properties of the hydrodynamic and rigid-body models of marine craft have been exploited for design of motion control systems. For example, in [3] (see also the summary in [4]), the authors use the concept of vectorial-integrator backstepping for the design of dynamic positioning for ships—a technique that uses control-Lyapunov functions and can be related to feedback passivation [5]. In [6], the authors use passivity-based techniques to design a ride controller (reduction of roll and pitch) for a surface-effect ship. In [7], the authors consider the dynamics of fully-actuated underwater vehicles as a Hamiltonian system, and address the problem of stabilisation of underwater vehicles using internal rotors as actuators. This control approach involves shaping the kinetic energy of the vehicle preserving the Hamiltonian structure and adding dissipation to ensure asymptotic stability of the closed-loop. The use of IDA-PBC for positioning with integral action of open-frame fully-actuated underwater vehicles is addressed in [8] and this work is extended to tracking in [9] in three degrees of freedom (DOF). Considerations of actuator saturation and the addition of remedial anti-windup is addressed in [10] for the problem of dynamic positioning of offshore vessels. In [11], the authors consider the problem of stabilisation of the under-actuated Kirchhoff equations for an underwater vehicle moving in an ideal fluid, that is, neglecting hydrodynamic dissipative forces. The latter authors apply IDA-PBC to deal with the stabilisation problems of the steady longitudinal motion and the steady rising/diving with forward/reverse motion. In [12], the authors solve the attitude and speed regulation problem for a slender underwater vehicle with a full hydrodynamic model (potential plus viscous effects) and focus on both forward speed and attitude (roll, pitch, and yaw) tracking based on energy shaping and damping assignment such that the closed-loop system retains a port-Hamiltonian form. The unactuated channels of the system are left in open loop, and a suppression control is used to completely remove the uncontrolled behaviour from the target (closed-loop) dynamics.

In this paper, we present Hamiltonian models of marine vehicle dynamics in six DOF in both body-fixed and inertial momentum coordinates. The model in body-fixed coordinates exhibits a particular structure of the mass matrix that allows the adaptation and application of a change of coordinates to assign a full-rank dissipation matrix first proposed in [10], and then generalised for mechanical systems in [13, 14]. We follow this approach to design a passivity-based tracking controller with integral action for fully actuated vehicles in six DOF. This extends the work in [10] that only considers the set-point regulation problem in 3DOF to trajectory tracking in six DOF. The work in [9] consider the tracking problem, however, the proof only ensure asymptotically stability, and rejection of constant disturbance is not theoretically ensured. In this paper, we prove exponential stability of the tracking error as well as rejection of constant disturbances and bounded-input-bounded-state of the closed loop. In this paper we expand the development of the model and the numerical simulations performed in our preliminary work [15]. We also describe a momentum transformation that allows an alternative model representation that resembles general port-Hamiltonian mechanical systems with a coordinate dependent mass matrix. This similarities can be
exploited to adapt the theory of control of port-Hamiltonian systems developed in robotic manipulators and mechanical systems to the case of marine craft dynamics.

**Notation.** In this paper we use the following notation: we note the standard 2 norm of a vector $v$ as $|v| = \sqrt{v^T v}$, and its induced norm for a matrix $A$ and $|A|$.

2. Port-Hamiltonian systems

2.1. Modelling

The dynamics of mechanical systems can be described using a a set of $2n$ first-order differential equations known as a Hamilton’s canonical equations of motion [16]:

\[
\begin{align*}
\dot{q} &= \nabla_p H(p, q), \\
\dot{p} &= -\nabla_q H(p, q) + \tau,
\end{align*}
\]

where $q$ and $p$ are the $n$-dimensional vectors of generalised coordinates and momenta respectively, and $\tau$ is the vector of generalised forces. The Hamiltonian $H(p, q)$ is the sum of the kinetic energy and the potential energy:

\[
H(p, q) = \frac{1}{2} p^T M^{-1}(q)p + V(q),
\]

This function represents the total energy of the system. The Hamilton equations (1)-(2) is a particular state-space representation of the system dynamics equivalent to the classical Euler-Lagrange models. We should note, however, that there are systems that admit Hamiltonian but not Lagrangian representations [16].

In the control literature, the Hamiltonian model (1)-(2) has been generalised to what is known as a port-Hamiltonian system [17]:

\[
\begin{align*}
\dot{x} &= [J_o(x) - R_o(x)] \nabla H(x) + G_o(x) u, \\
y &= G_o^T(x) \nabla H(x),
\end{align*}
\]

where $x$ is the state vector and the pair $u, y$ are the input and output $m$-dimensional vectors. These are conjugate variables in the sense that their inner product represents (or is akin to) the power exchanged between the system and its environment. The function $J_o(x) = -J_o^T(x)$ describes the power preserving interconnection structure through which the components of the system exchange energy. The symmetric function $R_o(x) \geq 0$ captures dissipative phenomena in the system. The function $G_o(x)$ weigts the action of the input on the system and defines the conjugate output. From (4)-(5), it follows that

\[
\dot{H}(x) = y^T u - [\nabla H(x)]^T R_o(x) \nabla H(x) \leq y^T u,
\]

which shows passivity of the pH model [2].
2.2. Interconnection and damping assignment passivity-based control

Following [18], consider the open-loop system of the form

$$\dot{x} = f(x) + g(x)u.$$  \hfill (7)

The idea of IDA-PBC is to use the state-feedback control law \( u = K(x) \) to re-shape the system (7) into the desired closed-loop system or target dynamics

$$\dot{x} = [J_d(x) - R_d(x)] \nabla H_d(x),$$  \hfill (8)

where \( J_d(x) = -J_d^T(x) \), \( R_d(x) > 0 \), and the desired Hamiltonian attains its minimum at the desired equilibrium state:

$$x^* = \arg \min_x H_d(x).$$  \hfill (9)

The stability of the equilibrium \( x^* \) can be established using \( H_d(x) \) as Lyapunov function.

Such design specifies the interconnection \( J_d \), injects damping through the dissipation function \( R_d \), and shapes the system energy such that the minimum is at the equilibrium \( x^* \).

A control law of the form \( u = K(x) \) exists provided that the following PDE can be solved:

$$g^\perp(x)[f(x) - (J_d(x) - R_d(x))\nabla H_d(x)] = 0,$$  \hfill (10)

where \( g^\perp(x) \) is the full-rank left annihilator of \( g(x) \). This leads to the general form

$$u = K(x) = [g^T(x)g(x)]^{-1}g^T(x)[(J_d(x) - R_d(x))\nabla H_d(x) - f(x)].$$  \hfill (11)

If the control is for a fully-actuated open loop system, it is not necessary to solve the matching PDE (10) since one can propose the desired Hamiltonian and constructively determine the functions \( J_d \) and \( R_d \). This is the procedure we follow in Section 4 of this paper. The problem of tracking is addressed in a similar fashion as described above, but the target dynamics is associated with the tracking error. This design requires not only the desired reference \( x^*(t) \) but also its time derivatives. For further details on IDA-PBC, see for example [2, 1, 19, 18].

3. Dynamics of marine craft in port-Hamiltonian form

The design of an IDA-PBC control strategy can be aided in some cases if the open-loop control system itself is described as a pH system. Such description exhibits energy properties of the open-loop system that a control design can exploit and seek to preserve. Therefore, in this section, we formulate the open-loop models of marine craft in pH form.

The classical equations of motion used for marine craft can be written as follows [20]:

$$\dot{\eta} = J(\eta)\nu,$$  \hfill (12)

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + G(\eta) = \tau_c + \tau_d,$$  \hfill (13)

where \( \eta \) describe the pose of the vehicle (position and orientation) (North, East, Down, roll, pitch, yaw), \( \nu \) is the body-fixed linear-angular velocity (surge, sway, heave, roll, pitch, yaw).
The vector $\tau_c$ represents the force and torque control inputs (surge, sway, heave, roll, pitch, yaw), and $\tau_d$ represent the force and torque disturbance inputs. The constant matrix $M = M^T > 0$ is the total generalised mass matrix due to rigid-body mass distribution and fluid added mass, $C(\nu) = -C^T(\nu)$ is the total Coriolis-centripetal matrix, and $D(\nu) = D^T(\nu) > 0$ is the total hydrodynamic damping matrix, and $G(\eta)$ is the vector of hydrostatic forces and torques due to gravity and buoyancy. The function $J(\eta)$ is a $6 \times 6$ kinematic transformation matrix, which is well-defined if the pitch angle $\theta \neq \pm \frac{\pi}{2}$ (see e.g. [4] for details on this model).

Following on of the work in [10], we will write the dynamics (12)-(13) in pH form. We first make the following assumption:

**Assumption 1 (A1).** There exist a potential function $V(\eta) : \mathbb{R}^3 \times S^3 \rightarrow \mathbb{R}$ that satisfies

$$J^T(\eta) \nabla_\eta V(\eta) = G(\eta). \quad (14)$$

By Poincare’s Lemma, a necessary and sufficient condition for the existence of $V(\eta)$ is that

$$\nabla_\eta \left[ J^{-T}(\eta)G(\eta) \right] = \left( \nabla_\eta \left[ J^{-T}(\eta)G(\eta) \right] \right)^T. \quad (15)$$

Note that this equation is satisfied for example by neutrally buoyant underwater vehicles. For the latter, the function $V$ has the form

$$V(\eta) = -W \sin(\theta) X + W \cos(\theta) \sin(\phi) Y + W \cos(\theta) \cos(\phi) Z, \quad (16)$$

where $W = mg$ is the submerged weigh of the vehicle, and $(X, Y, Z)$ are the cartesian coordinates of the centre of buoyancy relative to the centre of gravity.

### 3.1. pH Model in Body-fixed Coordinates

The following proposition establishes the pH model in terms of a transformation of the body-fixed velocity.

**Proposition 3.1.** Consider the dynamics (12)-(13). Then under assumption A1, the dynamics of the marine craft can be written in port-Hamiltonian form as follows

$$\begin{bmatrix} \dot{\eta} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & J(\eta) \\ -J^T(\eta) & -J_2(p) \end{bmatrix} \nabla H + \begin{bmatrix} 0 \\ I_n \end{bmatrix}(\tau_c + \tau_d), \quad (17)$$

where

$$H(\eta, p) = \frac{1}{2} p^T M^{-1} p + V(\eta), \quad (18)$$

the momentum is defined through the following transformation of the body-fixed velocities$^1$:

$$p = M \nu, \quad (19)$$

and $J_2(p) = C(\nu) + D(\nu) \big|_{\nu=M^{-1}p}$, which satisfies $J_2(p) + J^T_2(p) > 0$.

---

$^1$Note that the momentum (19) is not the conjugate momentum of the generalised coordinate vector $\eta$ since $\nu$ has as components the body-fixed angular velocity (quasi-coordinates) [21, p193]. These do not equate to the time-derivative of the Euler angles—which are part of $\eta$. 

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Proof The proof follows from construction of the state equations for $\eta$ and $p$. First, we note that

$$
\dot{\eta} = J(\eta)\nu = J(\eta)M^{-1}p = J(\eta)\nabla_p H, \tag{20}
$$

which is the first row of (17). Then, from (13) we obtain

$$
\dot{p} = M\dot{\nu} = -G(\eta) - C(\nu)\nu - D(\nu)\nu + \tau_c + \tau_d = -J^T(\eta)\nabla_\eta V(\eta) - J_2(p)M^{-1}p + \tau_c + \tau_d = -J^T(\eta)\nabla_\eta H - J_2(p)\nabla_p H + \tau_c + \tau_d, \tag{21}
$$

which is the second row of (17). The fact the $J_2(p)$ is positive definite follows readily from the properties of $C(\nu) = -C^T(\nu)$ and $D(\nu) = D^T(\nu) > 0$. \hfill \Box

3.2. pH Model in Inertial Coordinates

An alternative port-Hamiltonian model, still under assumption A1, can be built by defining a new momentum vector as follows

$$
\ell = J^{-T}(\eta)M\nu. \tag{22}
$$

Then, using $\eta$ and $\ell$ as states, the marine craft dynamics (17) can be written as follows

$$
\begin{bmatrix}
\dot{\eta} \\
\dot{\ell}
\end{bmatrix} =
\begin{bmatrix}
0 & I_n \\
-I_n & -L(\eta,\ell)
\end{bmatrix}
\nabla H_\eta +
\begin{bmatrix}
0 \\
J^{-T}(\eta)
\end{bmatrix}
(\tau_c + \tau_d), \tag{23}
$$

where

$$
H_\eta(\eta,\ell) = \frac{1}{2} \ell^T J(\eta)M^{-1}J^T(\eta)\ell + V(\eta), \tag{24}
$$

$$
= \frac{1}{2} \ell^T M^{-1}(\eta)\ell + V(\eta), \tag{25}
$$

and

$$
L(\eta,\ell) = \left( \sum_{i=1}^n \nabla_{\eta_i}[J^{-T}]M\nu e_i^T \right)^T - \sum_{i=1}^n \nabla_{\eta_i}[J^{-T}]M\nu e_i + J^{-T}C(\nu)J^{-1} + J^{-T}D(\nu)J^{-1} \bigg|_{\nu=M^{-1}J^T\ell}, \tag{26}
$$

The matrix $J^{-1}(\eta)$ is well-defined if the pitch angle $\theta \neq \pm \frac{\pi}{2}$. 

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where \( e_i \in \mathbb{R}^n \) is the \( i \)-th vector of the Euclidean orthonormal basis. The first three terms of the matrix \( L(\eta, \ell) \) in (26) determine the skew-symmetric component of the matrix and accounts for the gyroscopic forces, whilst the last term describes the damping and implies that \( L(\eta, \ell) + L^T(\eta, \ell) > 0 \). Note that the expression of Hamiltonian function (24) and (25) are equivalent, but (24) uses a factorisation of the mass matrix in terms of \( J \). This factorisation, which also arises from expressing the kinetic co-energy in terms of \( \dot{\eta} \), inspires a change of momenta to obtain a pH model with constant mass matrix as in (17). This type of change of coordinates has also been used in the literature to obtain an identity mass matrix in body-fixed momentum coordinate and been exploited to design controllers and observers for general mechanical systems, see for example [13, 22].

The dynamics (23) is obtained in two steps. First, we notice that we can write

\[
\dot{\eta} = J(\eta)\nu = J(\eta)M^{-1}J^T(\eta)\ell = M^{-1}(\eta)\ell,
\]

which is the first row of (23). Second, we compute the derivative of (22) with respect to time as follows

\[
\dot{\ell} = \frac{d}{dt} [J^{-T}(\eta)M\nu] = J^{-T}(\eta)p + J^{-T}(\eta)\dot{p}
\]

\[
= \sum_{i=1}^{n} \nabla_{\eta_i}[J^{-T}]M\nu e_i^T \dot{\eta} - \nabla_{\eta} V - J^{-T}J_2M^{-1}J^T\ell + J^{-T}(\tau_c + \tau_d)
\]

\[
= \sum_{i=1}^{n} \nabla_{\eta_i}[J^{-T}]M\nu e_i^T \nabla_\ell H_\eta - \nabla_{\eta} V - J^{-T}J_2M^{-1}J^T\ell + J^{-T}(\tau_c + \tau_d)
\]

\[
+ \left[ \sum_{i=1}^{n} \nabla_{\eta_i}[J^{-T}]M\nu e_i^T \right]^T \nabla_\ell H_\eta - \left[ \sum_{i=1}^{n} \nabla_{\eta_i}[J^{-T}]M\nu e_i^T \right]^T \nabla_\ell H_\eta
\]

(28)

Then, using the fact that

\[
-\frac{1}{2} \nabla_{\eta} [\ell^TJM^{-1}J^T\ell] = \left[ \sum_{i=1}^{n} \nabla_{\eta_i}[J^{-T}]M\nu e_i^T \right]^T \nabla_\ell H_\eta
\]

(29)

in (28), we obtain

\[
\dot{\ell} = -\frac{1}{2} \nabla_{\eta} [\ell^TJM^{-1}J^T\ell] - \nabla_{\eta} V - J^{-T}J_2J^{-1}\nabla_\ell H_\eta + J^{-T}(\tau_c + \tau_d)
\]

\[
+ \sum_{i=1}^{n} \nabla_{\eta_i}[J^{-T}]M\nu e_i^T \nabla_\ell H_\eta - \left[ \sum_{i=1}^{n} \nabla_{\eta_i}[J^{-T}]M\nu e_i^T \right]^T \nabla_\ell H_\eta
\]

\[
= -\nabla_{\eta} H_\eta - \left\{ J^{-T}J_2J^{-1} - \sum_{i=1}^{n} \nabla_{\eta_i}[J^{-T}]M\nu e_i^T \nabla_\ell H_\eta + \left[ \sum_{i=1}^{n} \nabla_{\eta_i}[J^{-T}]M\nu e_i^T \right]^T \right\} \nabla_\ell H_\eta
\]

\[
+ J^{-T}(\tau_c + \tau_d)
\]

\[
= -\nabla_{\eta} H_\eta - L(\eta, \ell)\nabla_\ell H_\eta + J^{-T}(\tau_c + \tau_d),
\]

(30)
which is the second row of (23).

Notice that while the pH model (17) is related to the body-fixed representation (12)-(13), the pH model (23) is related to the NED representation presented in [4, p171]:

\[ M_\eta(\eta)\ddot{\eta} + C_\eta(\eta, \dot{\eta})\dot{\eta} + D_\eta(\eta, \dot{\eta})\dot{\eta} + g_\eta(\eta) = J^{-T}(\eta)(\tau_c + \tau_d). \]

The model (23) resemble the dynamics of mechanical systems in the pH form (see [18]).

4. Robust tracking control of fully-actuated marine craft

We consider the marine craft dynamics (17) and a time-varying references \( \eta^*(t), \dot{\eta}^*(t) \) and \( \ddot{\eta}^*(t) \). In this section, we propose a robust PBC tracking controller that ensures

\[ \lim_{t \to \infty} \eta(t) = \eta^*(t). \]

We consider that the disturbance has a constant and a time-varying component, i.e. \( \tau_d(t) = d + d(t) \). Therefore, the controller should ensure robust properties with respect to both components. Specifically, it is desirable that the controller ensures tracking in spite of constant disturbances and bounded state trajectories if there are time-varying disturbances.

First, we define the tracking errors

\[ \tilde{\eta} = \eta - \eta^*, \quad \tilde{p} = p - p^*, \]

where \( \eta^*(t) \) is the position reference and \( p^* \) is a function to be selected. We will also extend the state vector with a new state \( \zeta \), which is the state of the integrator that compensates the constant disturbance.

We will design a controller such that the closed-loop dynamics have the desired pH form

\[
\begin{bmatrix}
\dot{\tilde{\eta}} \\
\dot{\tilde{p}} \\
\dot{\zeta}
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} & S_{13} \\
-S_{12}^T & S_{22} & S_{23} \\
-S_{13}^T & -S_{23}^T & S_{33}
\end{bmatrix}
\nabla H_d +
\begin{bmatrix}
0 \\
d(t) \\
0
\end{bmatrix}
\]

with

\[ H_d(\tilde{\eta}, \tilde{p}, \zeta) = \frac{1}{2} \tilde{p}^T M^{-1} \tilde{p} + V_d(\tilde{\eta}) + \frac{1}{2} (\zeta - \alpha)^T K_I (\zeta - \alpha). \]

The matrices \( S_{ij} \) with \( i, j = 1, 2, 3 \) are functions to be selected. The constant vector \( \alpha \) will be properly defined during the design, and the constant matrix \( K_I \) is symmetric and positive definite. The function \( V_d \) should have a strict minimum at \( \tilde{\eta} = 0 \). In addition, the matrices \( S_{11}, S_{22} \) and \( S_{33} \) should satisfy

\[ S_{11} + S_{11}^T < -\epsilon_1 I_n < 0 \]
\[ S_{22} + S_{22}^T < -\epsilon_2 I_n < 0 \]
\[ S_{33} + S_{33}^T < -\epsilon_3 I_n < 0 \]

with \( \epsilon_1, \epsilon_2, \epsilon_3 \in \mathbb{R}_{>0} \) and \( I_n \) is the \( n \times n \)-identity matrix. The next proposition shows that the closed loop (34) has the desirable stability features to achieve the control objective.
Property 3 (P3). Under the action of constant disturbance $\bar{d}$ with $\theta$ implements a disturbance rejection. This is further discussed in the following proof.

\[
\lim_{t \to \infty} \eta(t) = \eta^*(t).
\]

Notice that the controller (39) and its dynamics (40), which provided the integral action, are independent of the disturbance $\bar{d}$. Yet, it is by construction that the closed loop attains the minimum of the Hamiltonian (35), where $\bar{p}, \bar{\eta} \to 0$ and $\zeta \to \alpha$. Therefore, the controller implements a disturbance rejection. This is further discussed in the following proof.
To design the control law, we first start by writing the dynamics of the position error (32), and we substitute the time derivatives of $\eta$ and $\dot{\eta}$ by the state equation (17) and the desired state equation (34), respectively, as follows

$$
\dot{\eta} = \dot{\eta} - \dot{\eta}^* = J(\eta)M^{-1}p - \dot{\eta}^* \\
\equiv S_{11}\nabla V_d + S_{12}M^{-1}\dot{p} + S_{13}K_I(\zeta - \alpha),
$$

(45)

from which we obtain $p^*$ that ensures the desired dynamics for $\dot{\eta}$ as in (34). That is,

$$
p^* = MJ^{-1}(\eta)S_{11}\nabla V_d + MK_I(\zeta - \alpha) + MJ^{-1}(\eta)\dot{\eta}^*,
$$

(46)

where we have chosen $S_{12} = S_{13} = J(\eta)$. For ease of notation, we will drop the dependence on $\eta$ in remaining of the derivations.

In the second step of the design, we need to ensure that the dynamics of $\dot{p}$ is as the desired dynamics in (34). We compute the time derivative of $\dot{p}$ as follow

$$
\dot{\dot{p}} = \dot{p} - \dot{p}^* \\
= -J^T\nabla V - J_2(p)M^{-1}p + \tau_c + \bar{d} + d(t) - \frac{d}{dt} [MJ^{-1}S_{11}] \nabla V_d - MJ^{-1}S_{11}\nabla^2 V_d \dot{\eta} \\
- MK_I\dot{\zeta} - \frac{d}{dt} [MJ^{-1}] \dot{\eta}^* - MJ^{-1}\dot{\eta}^* \\
\equiv -J^T\nabla V_d + S_{22}M^{-1}\dot{p} + S_{23}K_I(\zeta - \alpha) + d(t),
$$

(47)

where in the second equality we have substituted $\dot{p}$ by its state equation in (17), and $\dot{p}^*$ by the time derivative of (46).

Then, the control law can be obtained by isolating $\tau_c$ from (47) as follows

$$
\tau_c = J^T\nabla V + J_2M^{-1}p + \frac{d}{dt} [MJ^{-1}S_{11}] \nabla V_d + MJ^{-1}S_{11}\nabla^2 V_d (JM^{-1}p - \dot{\eta}^*) + MK_I\dot{\zeta} \\
+ \frac{d}{dt} [MJ^{-1}] \dot{\eta}^* + MJ^{-1}\dot{\eta}^* - J^T\nabla V_d + S_{22}M^{-1}p - S_{22}J^{-1}S_{11}\nabla V_d - S_{22}J^{-1}\dot{\eta}^* \\
- S_{22}K_I(\zeta - \alpha) + S_{23}K_I(\zeta - \alpha) - \bar{d}.
$$

(48)

The control law (48) is independent of the disturbance $\bar{d}$ if $\dot{\zeta}$ does not dependent on $\bar{d}$ and if $\alpha$ is chosen as

$$
\alpha = K_I^{-1}(S_{22} - S_{23})^{-1}\bar{d}.
$$

(49)

The dynamics of the integral action is given by

$$
\dot{\zeta} = -S_{13}^T\nabla V_d - S_{23}^T M^{-1}\dot{p} + S_{33}K_I(\zeta - \alpha) \\
= -J^T\nabla V_d - S_{23}^T M^{-1} [p - MJ^{-1}S_{11}\nabla V_d - MK_I(\zeta - \alpha) - MJ^{-1}\dot{\eta}^*] + S_{33}K_I(\zeta - \alpha) \\
= -J^T\nabla V_d - S_{23}^T M^{-1}p + S_{23}^T J^{-1}S_{11}\nabla V_d + S_{23}^T J^{-1}\dot{\eta}^* + (S_{23}^T + S_{33})K_I(\zeta - \alpha),
$$

(50)

which is independent on the disturbance $\bar{d}$—see (49)—if $S_{23} = -S_{33}^T$.  

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The exponential convergence of the marine craft position vector η to the time-varying reference η* follows from the exponential stability of the tracking errors to zero. To show that, we will study the exponential stability of the equilibrium (\(\bar{\eta}, \bar{\rho}, \zeta\)) = (0, 0, \(\alpha\)) of the error dynamics (41). The Hamiltonian \(H_d\) has a minimum at the equilibrium, and since \(M^{-1}\) and \(K_1\) are positive definite and \(V_d\) satisfies (43), then \(H_d\) qualify as a Lyapunov candidate function and can be bounded as follows

\[
c_1 |\chi|^2 \leq H_d(\bar{\eta}, \bar{\rho}, \zeta) \leq c_2 |\chi|^2
\]  

(51)

where \(c_1, c_2 \in \mathbb{R}_{>0}\) and \(\chi = \text{vec}(\bar{\eta}, \bar{\rho}, \zeta - \alpha)\), that is the arrangement of \(\bar{\eta}, \bar{\rho}\) and \(\zeta - \alpha\) in a vector noted \(\chi\). We, then compute the derivative of \(H_d\) respect to time along the trajectories of the dynamics (41) as follows

\[
\dot{H}_d = \left[(\nabla_\eta H_d)^T (\nabla_\rho H_d)^T (\nabla_\zeta H_d)^T\right] \begin{bmatrix} \dot{\eta} \\ \dot{\rho} \\ \dot{\zeta} \end{bmatrix} \\
= (\nabla_\eta H_d)^T S_{11} \nabla_\eta H_d + (\nabla_\eta H_d)^T J \nabla_\rho H_d + (\nabla_\eta H_d)^T J \nabla_\zeta H_d - (\nabla_\rho H_d)^T J^T \nabla_\eta H_d \\
+ (\nabla_\rho H_d)^T S_{22} \nabla_\rho H_d - (\nabla_\rho H_d)^T S_{33} \nabla_\zeta H_d - (\nabla_\zeta H_d)^T J^T \nabla_\eta H_d \\
+ (\nabla_\zeta H_d)^T S_{33} \nabla_\zeta H_d + (\nabla_\rho H_d)^T J^T d(t) \\
= \frac{1}{2}(\nabla_\eta V_d)^T (S_{11} + S_{11}^T) \nabla_\eta V_d + \frac{1}{2}(\nabla_\rho H_d)^T (S_{22} + S_{22}^T) \nabla_\rho H_d \\
+ \frac{1}{2}(\nabla_\zeta H_d)^T (S_{33} + S_{33}^T) \nabla_\zeta H_d + (\nabla_\rho W)^T d(t) \\
\leq -\frac{\epsilon_1}{2} |\dot{\eta}|^2 - \frac{\epsilon_2}{2} |\dot{\rho}|^2 - \frac{\epsilon_3}{2} |\dot{\zeta}|^2 + (\nabla_\rho H_d)^T d(t) \\
\leq -\frac{\epsilon_1 k_3}{2} |\dot{\eta}|^2 - \frac{\epsilon_2 k_3}{4} |\dot{\rho}|^2 - \frac{\epsilon_3 k_3}{2} |\dot{\zeta}|^2 + \frac{\epsilon_3}{\epsilon_2} |d(t)|^2 \\
\leq -\frac{\epsilon_1 k_3}{2} |\dot{\eta}|^2 - \frac{\epsilon_2 k_2}{4} |\dot{\rho}|^2 - \frac{\epsilon_3 k_3}{2} |\dot{\zeta}|^2 + \frac{1}{\epsilon_2} |d(t)|^2 \\
\leq -\frac{\gamma}{c_2} H_d(\bar{\eta}, \bar{\rho}, \zeta) + \frac{1}{\epsilon_2} |d(t)|^2,
\]  

(52)

where \(\gamma = \min\{\epsilon_1 k_3, \epsilon_2 k_3, \epsilon_3 k_3\}\), \(k_2 = |M^{-1}|^2\) and \(k_3 = |K_1|^2\).

Exponential stability of the closed loop with constant disturbances \(\bar{d}\) and without time-varying disturbances \(d(t) = 0\) follows directly from (52). Indeed, the inequality

\[
\dot{H}_d(\bar{\eta}, \bar{\rho}, \zeta) \leq -\frac{\gamma}{c_2} H_d(\bar{\eta}, \bar{\rho}, \zeta)
\]

and the bound in the Lyapunov function (51) ensure exponential stability of the equilibrium \((\bar{\eta}, \bar{\rho}, \zeta) = (0, 0, \alpha)\) (see e.g. [23]). Exponential stability of the equilibrium implies that \(\dot{\eta}(t)\) exponential converge to the reference \(\eta^*(t)\), in spite of the presence of unknown constant disturbances, which shows P2.
The bounded-input-bounded-state property follow from (52) with \( d(t) \neq 0 \), which yields
\[
\dot{H}_d(\tilde{\eta}, \tilde{\rho}, \zeta) \leq -\frac{c_1 \gamma}{c_2} |\chi|^2 + \frac{1}{c_2} |d(t)|^2 \leq -\frac{c_1 \gamma (1 - \rho)}{c_2} |\chi|^2 < 0
\] (53)
for all \(|d(t)|^2 < \rho \frac{c_1 \gamma^2}{c_2} |\chi|^2 \) and \( \rho \in (0, 1) \), which shows P3 [23]. Notice that the proof is valid for every trajectory that remains away from the singularity of the model (\( \theta \neq \pm \frac{\pi}{2} \)). □

5. Case Study: Open-frame UUV

We consider an open-frame underwater vehicle with a dry mass of 140kg with the tracking control law (39) for motion control in the horizontal plane. The vehicle has four thrusters in an x-type configuration, which provides full actuation in all the DOF of interest (surge, sway and yaw). The mass, damping and Coriolis matrices of the model are
\[
M = \begin{bmatrix} 290 & 0 & 0 \\ 0 & 404 & 50 \\ 0 & 50 & 132 \end{bmatrix}, \quad D = \begin{bmatrix} 95 + 268 |v| & 0 & 0 \\ 0 & 613 + 164 |u| & 0 \\ 0 & 0 & 105 \end{bmatrix}
\]
\[
C = \begin{bmatrix} 0 & 0 & -404v - 50r \\ 0 & 0 & 290u \\ 404v + 50r & -290u & 0 \end{bmatrix}
\]
The controller parameters are \( S_{11} = -\text{diag}(0.3, 0.3, 1), \) \( S_{22} = -\text{diag}(30, 40, 40), \) \( S_{33} = -\text{diag}(5, 5, 1), \) \( K_I = \text{diag}(0.5, 0.5, 0.5), \) and \( \dot{V}_d = \tilde{\eta}^T K_d \tilde{\eta}, \) with \( K_d = \text{diag}(0.3, 0.5, 0.1). \)

In the first simulation scenario, the vehicle must follow a desired circular trajectory in the horizontal plane. The state measurements have an additive uncorrelated noise component with statistical features characteristics of practical navigation and data fusion systems.

To test the disturbance rejection properties, the following disturbance is implemented
\[
\tau_d(t) = \mu(t - 20) \begin{bmatrix} 100 \\ 200 \\ 20 \end{bmatrix} + [\mu(t - 70) - \mu(t - 120)] \begin{bmatrix} 50 \\ 100 \\ 10 \end{bmatrix} \sin(0.5 \pi t), \quad (54)
\]
where \( \mu(\cdot) \) is the standard Heaviside step function.

Figure 1 present three plots that correspond to three time intervals of the simulation. The first interval correspond to the simulation time from 0 to 63 seconds, the second interval correspond to time from 55 to 110 seconds, and the third interval from 95 to 150 seconds. The left-hand-side plot of Figure 1 shows that the controller recover tracking of the desired trajectory under the action of the constant disturbance. The middle plot of Figure 1 shows that the states of the system remain bounded under the action of bounded disturbances, which is is ensured by the bounded-input-bounded-state property of the control system (P3 of proposition 4.1). The right-hand-side plot of Figure 1 shows that the trajectory tracking is recovered when the bounded time-varying disturbance vanishes, although the constant
disturbance is still acting on the vehicle. Figures 2 and left column of Figure 3 show the displacements, displacement errors, and velocities in the DOF of interest. As we can see, the actual position and velocities of the vehicle track their reference as per the control design objectives. The controller recover the trajectory tracking even under the action of a constant disturbance, which acts on the vehicle from $t = 20s$ until the end of the simulation. The sinusoidal disturbance produces a bounded error, which vanishes with exponential decay as the disturbance subsides. The control forces and torque are shown in the right column of Figure 3. It can be seen that the control actions are bounded within admissible values.

In the second simulation scenario, we consider a sinusoidal trajectory as shown in Figure 4. The vehicle starts at an initial position given by $x(0) = 1m$, $y(0) = 0.5m$ and $\psi(0) = \pi/2$rad and with no disturbance acting on it. A disturbance is added in the simulation at $t = 70sec$. This disturbance is constant in the NED frame but time varying in body-fixed frame. The values of the disturbance forces are set to $F_N = -10N$ and $F_E = -10N$, which are the disturbance components in the North and East directions respectively.

Figure 4 shows that the vehicle follows the desired trajectory. Figures 5 and left column of Figure 6 show the displacements, displacement errors, and velocities in the DOF of interest. As in the first scenario, the controller shows good performance tracking the references as desired, and ensures ensures exponential convergency of the trajectory error, and bounded error under the action of the disturbance. The control forces and torque are shown in the right column of Figure 6. It can be seen that the control actions are bounded within admissible values.

As shown in simulations, the designed controller demonstrate the stability properties proven and perform satisfactorily for both tracking and disturbance rejection tasks.

6. Conclusions

This paper presents port-Hamiltonian models of marine vehicle dynamics in six DOF in both body-fixed and inertial momentum coordinates. The model in inertial coordinates resembles general port-Hamiltonian model of mechanical systems with a coordinate dependent mass matrix. This model opens the possibility of specialising passivity-based control
Figure 2: Reference and measured motion variables for forward motion (left column) and lateral motion (right column).

Figure 3: Reference and measured motion variables for heading (left column), and control forces and torque in the DOF of interest.
Figure 4: Reference and actual position and heading of the vehicle in the $xy$-plane. A disturbance acts on the system at time $t = 70\text{s}$, which correspond to the position $x = 14\text{m}$ and $y = 1\text{m}$.

Figure 5: Reference and measured motion variables for forward motion (left column) and lateral motion (right column).
strategies developed for mechanical systems for the motion control of marine vehicles, which will be a topic of future research.

The body-fixed coordinate model generalises our previous work in this field. We show how the two models in the two momentum coordinates are related via a momentum transformation, and we indicate how these models relate to vector models commonly used in the literature of motion control of marine vehicle dynamics. The body-fixed coordinate model is then used to design a robust IDA-PBC tracking controller. This is a very attractive technique for designing motion controllers for mechanical systems. Having the models in pH form allows to see energy properties of the system, which the designer can choose to preserve. Indeed, in the control design developed in the paper, we choose to maintain the open-loop mass matrix in the kinetic energy component of the error dynamics. We also show how to augment the state and the target Hamiltonian in order to achieve disturbance rejection of fully actuated vehicles. We prove exponential stability in the case of constant disturbances and bounded errors in the case of bounded time-varying disturbances.

Finally, we use simulation example to illustrate the stability properties and the performance of the controller.

References