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ANALYSIS OF ROLLOVER SHAPE AND ENERGY STORAGE AND RETURN IN CANTILEVER BEAM-TYPE PROSTHETIC FEET

Kathryn M. Olesnavage[*]
MIT GEAR Lab
Massachusetts Institute of Technology
Cambridge, MA, USA
Email: kolesnav@mit.edu

Amos G. Winter, V
MIT GEAR Lab
Massachusetts Institute of Technology
Cambridge, MA, USA
Email: awinter@mit.edu

ABSTRACT

This paper presents an analysis of the rollover shape and energy storage and return in a prosthetic foot made from a compliant cantilevered beam. The rollover shape of a prosthetic foot is defined as the path of the center of pressure along the bottom of the foot during stance phase of gait, from heel strike to toe off. This path is rotated into the reference frame of the ankle-knee segment of the leg, which is held fixed. In order to achieve correct limb loading and gait kinematics, it is important that a prosthetic foot both mimic the physiological rollover shape and maximize energy storage and return.

The majority of prosthetic feet available on the market are cantilever beam-type feet that emulate ankle dorsiflexion through beam bending. In this study, we show analytically that a prosthetic foot consisting of a beam with constant or monotonically decreasing cross-section cannot replicate physiological rollover shape; the foot is either too stiff when the ground reaction force (GRF) acts near the ankle, or too compliant when the GRF acts near the toe. A rigid constraint is required to prevent the foot from over-deflecting.

Using finite element analysis (FEA), we investigated how closely a cantilever beam with constrained maximum deflection could mimic physiological rollover shape and energy storage/return during stance phase. A constrained beam with constant cross-section is able to replicate physiological rollover shape with $R^2 = 0.86$. The ratio of the strain energy stored and returned by the beam compared to the ideal energy storage and return is 0.504. This paper determines that there is a trade off between rollover shape and energy storage and return in cantilever beam-type prosthetic feet. The method and results presented in this paper demonstrate a useful tool in early stage prosthetic foot design that can be used to predict the rollover shape and energy storage of any type of prosthetic foot.

1 INTRODUCTION

In this work we present an analysis of cantilever beam-type prosthetic feet. The motivation behind this work is ultimately to design a low cost, high performance, mass-manufacturable prosthetic foot for the developing world. The need for a prosthetic foot designed with the developing world context in mind is huge; in India alone there are nearly 600,000 lower limb amputees [1]. Amputees in these settings introduce a unique set of culturally specific requirements, such as being able to squat, sit cross-legged, and walk barefoot through mud, in water and over uneven terrain. These together with a low price point preclude the use of any feet available in the western market.

Literature on prosthetic feet exhibits a lack in theoretical understanding of how form affects function. The ultimate goal of any prosthesis is to enable a user to achieve a symmetric gait pattern. This can only be evaluated after a prosthetic foot has been designed and built. Additionally, because gait varies significantly between individuals even when both limbs are intact, it is very difficult to draw conclusions about specific prosthetic feet. Even when a problem is identified in a particular prosthetic foot, it is difficult to determine how the design of the foot can be altered to correct it.

Several studies have proposed a variety of mechanical, or "amputee-independent", metrics of prosthetic feet that can be used either to predict the performance of a foot or to draw

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[*] Address all correspondence to this author

comparisons between different feet to aid in prescribing the best foot for a particular patient [2-5]. The most widely used of these is the categories defined by the American Orthotics and Prosthetics Association (AOPA), which group feet based on a series of mechanical tests. The idea is that the feet in a given category will behave somewhat similarly, so prosthetists can determine what type of foot to prescribe a given patient based on his age, activity level, etc. However, studies that have attempted to compare the performance of feet within a single one of these categories have failed to find commonalities through gait analysis [6-8]. The biggest contribution from the AOPA tests is defining "dynamic keel," or energy storage and return (ESAR) feet. As the name suggests, feet in this category behave like springs – they store strain energy during stance phase, which is then released during push off, simulating the power generation that occurs in the ankle muscles in intact limbs.

Another important predictor of prosthetic foot performance is the rollover shape of a foot [9]. The rollover shape is defined as the path of the center of pressure along the bottom of a foot from the time the heel hits the ground (heel strike) until heel strike on the opposite foot. This path is then rotated from the lab reference frame into the ankle-knee based reference frame, as shown in Fig. 1. The rollover shape shown here and all other gait data used in this work comes from Winter's *Biomechanics and Motor Control of Human Movement* [10].

In this paper, we investigate cantilever beam-type feet and the theoretical limitations in simultaneously mimicking a physiological rollover shape and maximizing energy storage and return. We apply beam bending principles to predict the rollover shape of a prosthetic foot model consisting of a simple cantilever beam. We then perform a finite element analysis (FEA) to investigate the rollover shape and ESAR properties of a cantilever beam with a rigid constraint that limits deflection to the correct RO shape, similar to the experimental "Shape" foot conceived by Knox [3]. The work concludes with a discussion of the implications of this work for designing a prosthetic foot for use in developing countries.
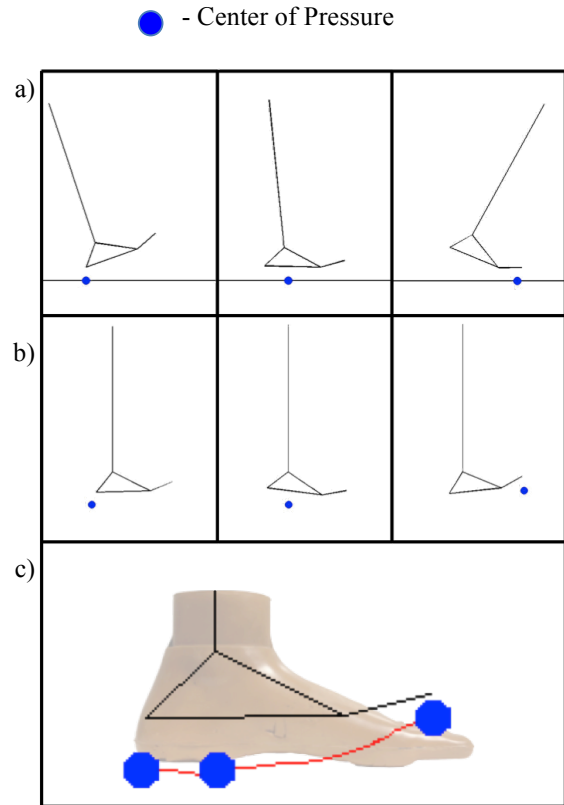

— Center of Pressure

Figure 1. ILLUSTRATION OF OBTAINING THE ROLLOVER SHAPE OF A FOOT. The rollover shape of a foot is defined as the path of the center of pressure along the foot from heel strike to opposite foot heel strike, rotated from (a) the laboratory reference frame to (b) the ankle-knee reference frame. The rollover shape (c) comes from following this center of pressure through an entire step, and is shown here in red.

2 UNCONSTRAINED CANTILEVER BEAM MODEL

In gait analysis, the ground reaction force, location of the center of pressure, and the upward motion of the toe with respect to the axis defined by the ankle and knee joints are all directly measured. These can be equated to the force, length, and deflection of a cantilever beam, as depicted in Fig. 2.
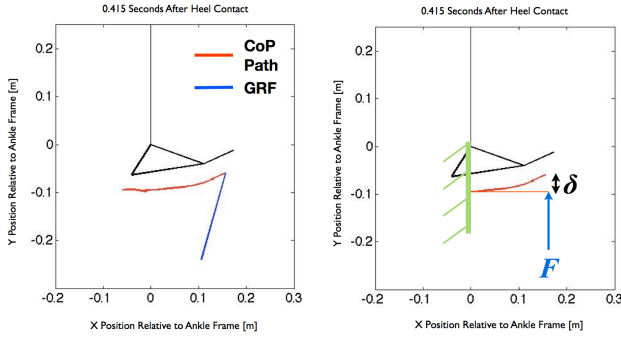
Figure 2. DATA MEASURED DURING GAIT ANALYSIS (LEFT) TRANSLATED INTO A CANTILEVER BEAM MODEL OF A PROSTHETIC FOOT (RIGHT)

Given the force and location of the center of pressure, the stiffness of a cantilever beam can be found such that it deflects the desired amount, using Eqn. 1

$$-M = EI \frac{d^2x}{dy^2} \qquad (1)$$

where M is the internal bending moment, x is the distance along the beam, y is the deflection of the beam, E is the elastic modulus, and I is the area moment of inertia. The quantity EI is also referred to as the bending stiffness of the beam.

In the simple case of a cantilever beam with a point load, the equation for the deflection at the point where the load is applied is

$$\delta = \frac{FL^3}{3EI} \qquad (2)$$

where δ is the deflection at that point, F is the load applied perpendicular to the beam, and L is the distance from the supported end of the beam to the application point of the load.

The strain energy stored in a cantilever beam in bending is given by Eqn. 3.

$$U = \int F d\delta \qquad (3)$$

where U is the strain energy stored and F and δ are as previously defined.

This is a very simple calculation given one static load. What makes this difficult for prosthetic feet is that each of these three quantities changes with time over the course of a step.

From the published gait data of a subject with both lower limbs intact [10], we have the ground reaction force and the lab reference frame coordinates of the center of pressure as well as every joint. This data was collected at a time interval of approximately .014 seconds throughout a step. At each of these time intervals, we translate this data to a measured force applied at a measured distance from the ankle, and we use Eqn. 2 to predict the deflection. Ideally, that deflection is equal to the

measured deflection due to dorsiflexion at that time. If that's the case, then the rollover shape is matched exactly at that point. The resulting energy storage is shown in Fig. 3. Because energy is conserved in this model, the strain energy that is stored is available for release later in stance.
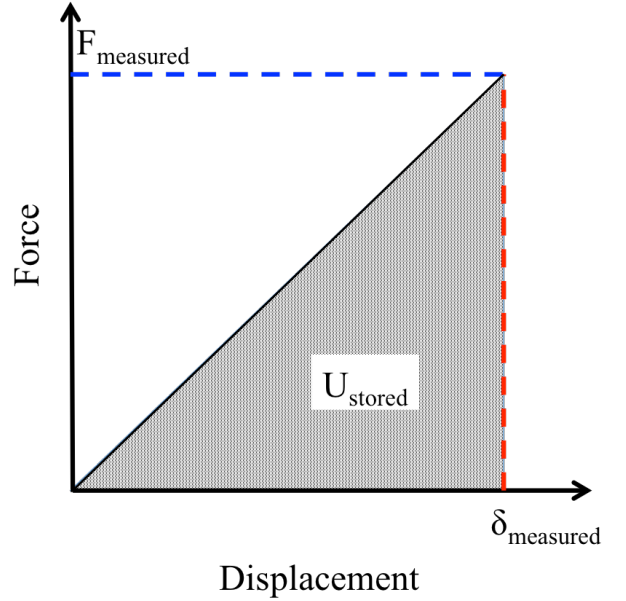

Figure 3. IDEAL FORCE VERSUS DEFLECTION CURVE FOR CANTILEVER BEAM-TYPE PROSTHETIC FOOT AT ANY GIVEN TIME DURING STANCE PHASE. At the ground reaction force measured during gait analysis for an individual with both limbs in tact, ideally the resulting deflection of the beam should be equal to the measured upward motion due to dorsiflexion at the ankle (ie the rollover shape coordinate at that time). The energy stored in the beam is given by the shaded area under the curve.

Combining the deflections at each time step results in the rollover shape of the foot. Note that in calculating the deflection in each time step individually, we are treating the foot as quasistatic. This is a common assumption, as the loading frequency of walking is well below the normal frequencies of feet. It has been found that there is little difference between rollover shapes measured quasistatically and dynamically [9]. We are also neglecting both the horizontal component of the ground reaction force and the horizontal displacement of the beam.

If a prosthetic foot is made up of a cantilever beam of uniform cross-section, the resulting rollover shapes are as shown in Fig. 4. When the center of pressure is directly under the ankle, the beams tend to not deflect enough. However, when the center of pressure moves out towards the toes, the beams deflect too much. If prosthetic feet are not stiff enough,

then when the amputee's weight is over their toes, the amputee is forced to take an abbreviated step on the prosthetic side, then "falls" onto the other limb. This results in asymmetric gait and higher impact on the intact limb, both of which could lead to long term injury [5].
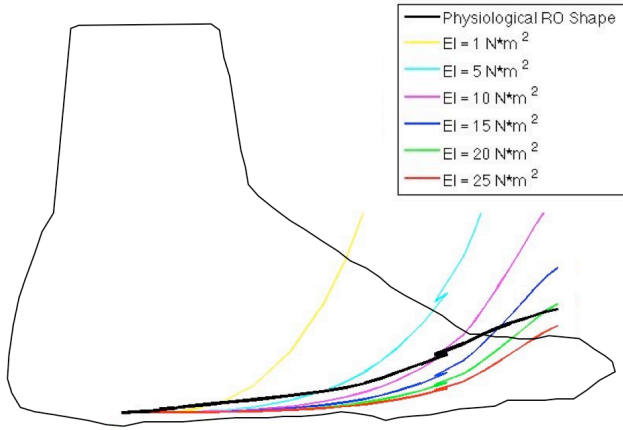

Figure 4. ANALYTICAL ROLLOVER SHAPES FOR CANTILEVER BEAM WITH UNIFORM CROSS SECTION AND VARYING BENDING STIFFNESS, *EI*

3 CONSTRAINED CANTILEVER BEAM MODEL

The analysis of a cantilever beam prosthetic foot shows that while energy recovery is ideal, a simple cantilever beam is unable to replicate a physiological rollover shape. A beam of constant cross section is too stiff near the ankle and too compliant near the toe. In order to keep the beam from over-deflecting, a rigid constraint was added. The resulting model is nearly identical to the Shape foot, used in experiments that were instrumental in defining the rollover shape of a foot and its effectiveness as a predictor of prosthetic performance [3]. This foot is convenient for testing in that it decouples the energy storage and return component (the cantilever beam) from the rollover shape (the rigid constraint). In discussing the design of the Shape foot, Knox states that a beam must be chosen that is compliant enough to conform to the rollover shape; thus the amount of energy stored is limited. This paper builds on Knox's work in determining analytically how close to the physiological rollover shape this foot can come and how much energy it is able to store compared to an unconstrained beam.

3.1 FINITE ELEMENT MODEL

To answer the above questions, a finite element model of the foot was created. From Fig. 4, it appears that a bending stiffness of 10 – 20 N*m^2 fits the physiological rollover shape

the best. A bending stiffness of 15 N*m^2 was chosen for the finite element model. The thickness and width of the beam were set to 0.01 m and 0.07 m respectively. To achieve the desired bending stiffness, the elastic modulus of the material was selected as 2.571 GPa, which falls in the appropriate range for materials in consideration, such as Nylon.

The prototype was approximated as a 2D solid in plane stress. The rigid constraint and the floor were both modeled as contact surfaces, paired with the top and bottom surfaces of the beam respectively. Figure 5 shows the FE model in detail together with a physical prototype of the foot.
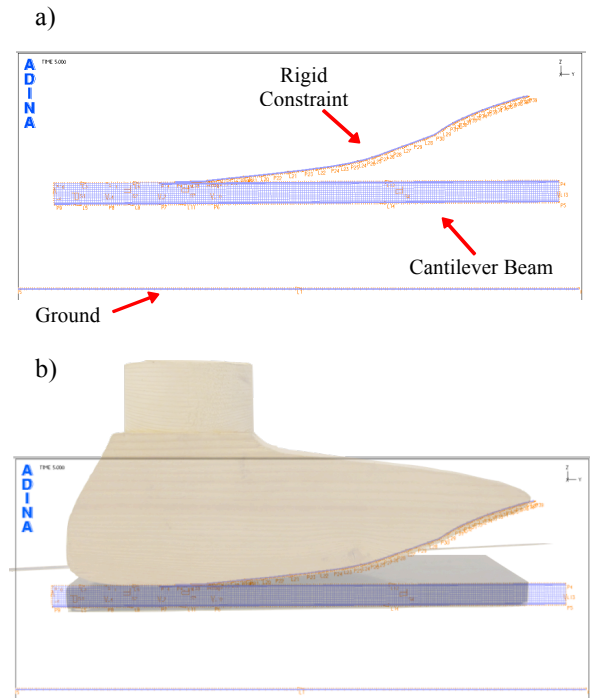

Figure 5. (A) DETAILED FEA MODEL AND (B) FEA MODEL OVERLAID ON PICTURE OF ACTUAL PROTOTYPE PROSTHETIC FOOT

3.2 METHOD

As with the unconstrained beam in Section 2, the foot was modeled quasistatically at several time intervals. The smallest natural frequency of the beam as found by FEA was 63.9 Hz, which is significantly greater than the loading frequency during walking of approximately 1 Hz, so this is again verified as a safe assumption.

The portion of the cantilever beam which would be rigidly attached to the ankle was oriented with respect to the ground in 12 different positions covering the range of orientations from heel strike to toe-off. Three of these positions are depicted in Annex A in the physical situation (a person walking with the foot), the analytical situation (MATLAB simulation using measured gait data), and the FE model.

In each of these orientations, a vertical displacement was imposed in 1 mm increments on the portion of the beam that would be fixed to the foot and ankle, as well as on the rigid constraint. The vertical ground reaction force was recorded from the solution at each step, producing a force versus displacement curve in each orientation. The vertical displacement at which the ground reaction force was closest to, but not exceeding, the measured physiological ground reaction force at that point during stance phase was taken as the best prediction of the actual deflection of the beam at that point. The center of pressure on the bottom surface of the beam and the final ankle position were noted for each orientation. The center of pressure was then rotated into the ankle-knee reference frame to produce a data point in the predicted rollover shape, as shown (Fig. 6).
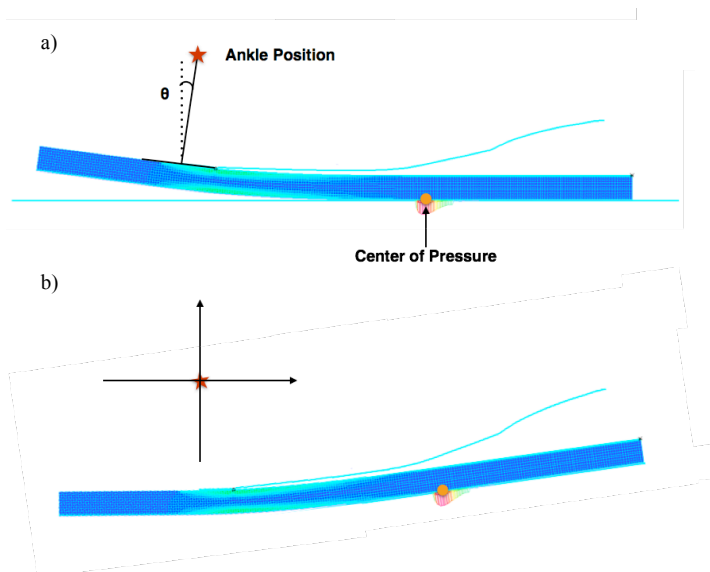

Figure 6. ILLUSTRATION OF OBTAINING A POINT ON THE ROLLOVER SHAPE PLOT USING AN FE MODEL OF A SINGLE INSTANT DURING WALKING. The center of pressure and the ankle position coordinates are measured in the lab reference frame (a), then rotated and translated into an ankle-knee reference frame (b). When this is done for several time steps and plotted on the ankle-knee reference frame, the resulting curve is the rollover shape.

3.3 RESULTS

The resulting rollover shape from the FEA is shown in Fig. 7. Compared with the physiological rollover shape, it produces an R^2 value of 0.86.
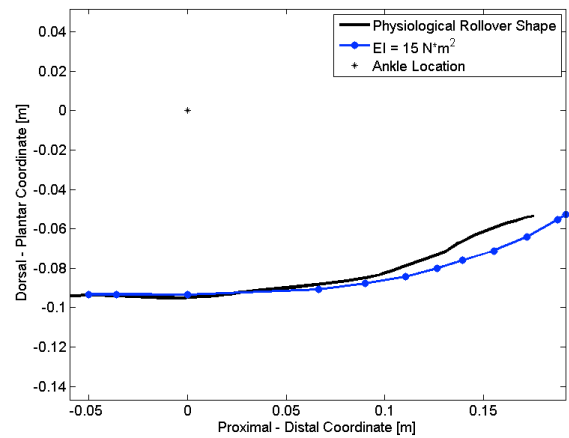

Figure 7. ROLLOVER SHAPE OF CONSTRAINED CANTILEVER BEAM PROSTHETIC FOOT MODEL AS PREDICTED BY FEA COMPARED TO PHYSIOLOGICAL ROLLOVER SHAPE

If the beam is too compliant, it comes in contact with the constraint at a ground reaction force less than the measured physiological ground reaction force magnitude. If the beam is too stiff, the predicted deflection of the model is less than the deflection required to match the gait data, and the rollover shape is not achieved. Neither of these situations are ideal energetically, as both store less energy than a beam that deflects the desired amount under the specified load. Figure 8 illustrates this concept.

A force versus deflection curve can be generated for any of the orientations of the foot. A common method for measuring the amount of energy storage of which a particular prosthetic foot is capable is measuring the energy storage just before pushing off the ground. Up until this point, as the beam is increasingly deflected, the amount of energy stored increases. At push off, the prosthesis returns the energy stored to the user, facilitating his or her next step.

AOPA determines whether a prosthetic foot is classified as a dynamic keel (*i.e.* ESAR foot) by using a compression testing machine to load a foot onto a fixed platform inclined 20 degrees relative to the foot, and measuring the difference in the areas under the force versus displacement curves during loading and unloading. We can similarly use the amount of energy stored in the foot at this orientation to compare the value to the ideal case of an unconstrained cantilever beam.

Just before toe-off, when the foot is aligned at approximately 20 degrees with respect to the ground such that the toes are lower than the heel, the force versus displacement curve solved through FEA is displayed in Fig. 9. Note that the slope of the curve changes as the beam makes contact with the rigid constraint at different points along its length and subsequently stiffens. By numerically integrating, we find that this curve predicts 6.37 J of strain energy stored. If we had a constant cross-section cantilever beam that fit the measured force and deflection at that instant, it would store 12.6 J (also

shown in Fig. 9). This means the constrained cantilever beam is storing 50.4% of what a theoretical, ideal cantilever beam-type foot could store.
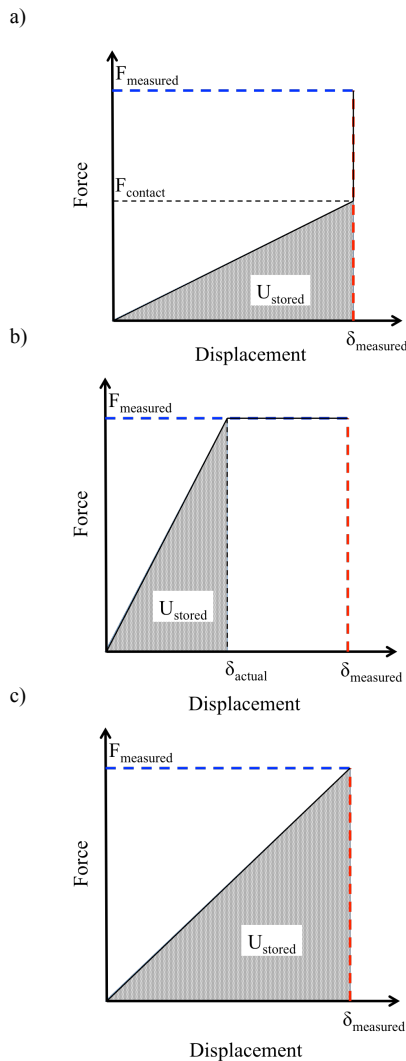

Figure 8. COMPARISON OF STRAIN ENERGY STORED FOR BEAM THAT IS TOO STIFF, TOO COMPLIANT, AND IDEAL. If the beam is either too stiff or too compliant, less strain energy is stored than if the beam were the exact stiffness. If the beam is too compliant (a), it makes contact with the constraint before the full expected load has been applied. After making contact, the force continues to increase, but the deflection does not. If the beam is too stiff (b), it never reaches the necessary deflection for the physiological rollover shape. If the beam has the correct compliance (c), the constraint is not necessary, and the maximum possible energy is stored. Note that the subscript "measured" here refers to values that were obtained in gait analysis of an individual with all limbs intact [10].
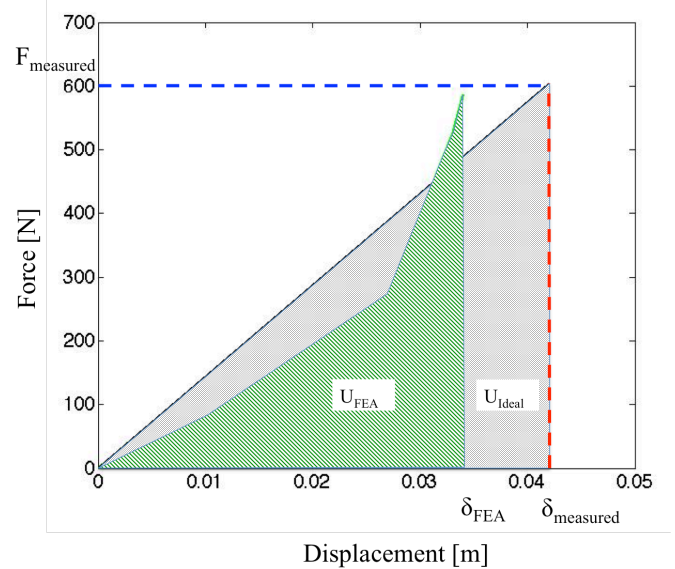

Displacement [m]

Figure 9. FORCE VERSUS DISPLACEMENT CURVE PREDICTED BY FEA WHEN MODEL FOOT IS ORIENTED IN POSITION JUST PRIOR TO PUSH OFF. Note that the slope of the line changes as the beam comes in contact with the constraint and the stiffness consequently increases. The area under the FEA curve shows the predicted strain energy storage of 6.37 J. This is overlaying the ideal force versus displacement curve for a beam of constant stiffness that exactly matches the force and deflection measured in gait analysis of a subject with both limbs intact, which would store 12.6 J of strain energy. The subscript "measured" refers to values that come from gait analysis of an individual with both lower limbs intact [10].

4 DISCUSSION

The analysis of an unconstrained cantilever beam prosthetic foot in Section 2 shows that, while ideal from an energy recovery perspective, an unconstrained cantilever beam is inadequate in reproducing a physiological rollover shape. With a constant cross-section cantilever beam, the prosthetic foot will tend to deflect too much when the center of pressure is near the toe. This could lead to asymmetrical gait and long-term injury, as the user tends to fall onto his or her sound limb rather than switch weight smoothly between feet [5]. A rollover shape that does not provide ample support in the forefoot was found to be a commonality in nearly all prosthetic feet used in developing countries [12]. Many prosthetic feet have a monotonically decreasing keel thickness. This only exacerbates the problem of over-deflection at the toe.

The FEA of a constrained cantilever beam prosthetic foot model produced an R^2 value of 0.86. In the orientation of the foot just before push off, it was predicted to store 6.37 J of strain energy. This is only 50.4% of the ideal energy storage of

a cantilever beam with no constraint. For comparison, the Flex Foot, a prosthetic foot that has since been replaced by newer models, has been observed to enable very symmetric gait and stores and returns approximately 9 J [3].

An important takeaway from this analysis is that there is a trade off between rollover shape and energy storage for cantilever beam-type feet – a beam could be made stiff enough to deflect the correct amount with no constraint immediately prior to push off, but such a beam would be too stiff earlier in stance phase and would not replicate the physiological rollover shape well.

The value of these results lies in the ability to apply the same method to various foot designs and quantitative metrics that can be compared early in the design process. For example, the FEA methods described herein could be applied to a series of constrained cantilever beam foot models with various bending stiffnesses and used to optimize energy storage while staying within a given R^2 value of a physiological rollover shape. Alternatively it can be used to compare two feet of completely different mechanical designs. Further investigation must be done to identify any specific correlations that exist between these predictive metrics and performance of prosthetic feet, both in terms of gait symmetry and subjective feedback from prosthesis users.

It should also be noted that the work herein addresses only the technical factors of walking on smooth ground. There are many other factors, both social and technical, that must drive the design of a prosthetic foot in the developing world.

5 CONCLUSION

We have presented an analysis of cantilever beam-type prosthetic feet, with and without rigid constraints, in the context of rollover shape and energy storage and return. We have shown that while both a physiological rollover shape and maximum energy storage and return are desirable for good prosthetic foot performance, there is a trade-off between the two for cantilever beam-type feet.

In order to create the next generation of high performance, low cost prosthetics, it is essential that we gain a deeper understanding of the function of a prosthetic foot both theoretically and in practice. The work herein offers a method to evaluate prosthetic feet from early stage design, greatly reducing the time and expenses required design, test and iterate from the conventional approach of testing a prosthetic foot only after its been built. Future work should focus on applying these methods to other categories of prosthetic feet and to identifying trends between the results of these methods and the performance of prosthetic feet in the field.

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PHYSIOLOGICAL PROBLEM, MATHEMATICAL PROBLEM, AND FINITE ELEMENT ANALYSIS AT THREE OF THE TWELVE STAGES OF STANCE PHASE MODELED

t = 0.029 s after heel strike	t = 0.343 s after heel strike	t = 0.472s after heel strike