A Simplified Approach to Determine α and the Penguin Amplitude in $B_d \rightarrow \pi\pi$

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Abstract

The effect of inelastic final-state interactions (IFSI's) on the determination of the weak phase α from the isospin triangles of $B \rightarrow \pi\pi$ is qualitatively illustrated. Neglecting the electroweak penguins and IFSI's and assuming the dominance of the top-quark loop in strong penguin diagrams, we propose an experimentally accessible way to approximately determine α and the penguin amplitude in $B_d \rightarrow \pi\pi$. This approach relies on a simplified isospin consideration and the factorization approximation, and its feasibility is irrelevant to the time-dependent measurements of $B_d \rightarrow \pi\pi$.

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1. Introduction

Today the most promising way to test the Cabibbo-Kobayashi-Maskawa (CKM) mechanism of quark mixing and CP violation is to measure CP asymmetries in neutral B-meson decays to CP eigenstates [1]. For this purpose, the decay mode $B_d \to \pi^+\pi^-$ is a good candidate in addition to the gold-plated channel $B_d \to J/\psi K_S$. However, the penguin-induced effect on $B_d \to \pi^+\pi^-$ may be significant enough so that a reliable prediction for the CP asymmetry is theoretically difficult [2]. Using the isospin triangle relations among the decay amplitudes of $B_u^\pm \to \pi^\pm\pi^0$, $\overset{(-)}{B}{}^0_d \to \pi^+\pi^-$ and $\overset{(-)}{B}{}^0_d \to \pi^0\pi^0$, Gronau and London have shown that the tree-level and penguin contributions can be disentangled by measuring the relevant decay rates [3]. Their work provides a relatively clean way to extract α, an angle of the CKM unitarity triangle in the complex plane [4], although its feasibility depends closely upon the time-dependent measurements of $B_d \to \pi\pi$.

The naive isospin analysis of $B \to \pi\pi$ has to be modified, if these decay modes involve non-negligible electroweak penguin (EWP) diagrams [5] or inelastic final-state interactions (IFSI's) [6]. It has recently been shown that the overall effect of EWP's on the determination of α from the isospin triangles is merely at the level of $O(\lambda^2)$ ($\lambda = 0.22$) or smaller [7], although the individual EWP amplitude may be $O(\lambda)$ of the strong penguin amplitude in $B_d \to \pi^0\pi^0$. The IFSI's are possible to induce channel mixing (e.g., between the direct reactions $B_d \to \pi\pi$ and the two-step processes $B_d \to D\bar{D} \to \pi\pi$), such that both the branching ratios and CP asymmetries of $B_d \to \pi\pi$ are difficult to be predicted. An illustration of the effect of IFSI's on the extraction of α from $B \to \pi\pi$ is still lacking in the literature.

In this work, we first illustrate the influence of $\pi\pi \rightleftharpoons D\bar{D}$ scattering on the isospin triangles of $B \to \pi\pi$ qualitatively. We find that the determination of α may suffer from large mixing between the $I = 0$ states of $\pi\pi$ and $D\bar{D}$. The approach proposed in ref. [3] can work only if the IFSI's are insignificant enough to be negligible. Second, we develop an approximate but more practical approach to isolate the weak phase α and the penguin amplitude in $B_d \to \pi\pi$. Neglecting the effects of EWP's and IFSI's and assuming the dominance of the top-quark loop in strong penguin diagrams, we relate the tree-level and penguin amplitudes of $B \to \pi\pi$ under a simplified isospin consideration and the factorization approximation. Provided that the decay rates of $B_u^\pm \to \pi^\pm\pi^0$, $\overset{(-)}{B}{}^0_d \to \pi^+\pi^-$ and $\overset{(-)}{B}{}^0_d \to \pi^0\pi^0$ are measured, we show that it is possible to determine the magnitudes of the penguin and tree-level amplitudes and to extract the weak and strong phase shifts between them. This approximate method has two obvious advantages: (1) its feasibility depends only upon the

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time-independent measurements of the relevant decay rates, which can be carried out at either B-meson factories or high-luminosity hadron machines; and (2) it can confront the nearest data of $B \to \pi\pi$ and give a ballpark number to be expected for α, before a delicate determination of α is available in experiments.

2. Effect of IFSI's on the isospin triangles

In the absence of EWP's and IFSI's, the isospin amplitudes of $B_d^0 \to \pi^+\pi^-$, $B_u^+ \to \pi^+\pi^0$, $B_d^0 \to \pi^0\pi^0$ and their CP-conjugate counterparts are given as [3]

$$
\begin{aligned}
A_{+-} &= \langle \pi^+\pi^- | \mathcal{H} | B_d^0 \rangle &= \sqrt{2} A_2^{\pi\pi} - \sqrt{2} A_0^{\pi\pi} , \\
A_{+0} &= \langle \pi^+\pi^0 | \mathcal{H} | B_u^+ \rangle &= 3 A_2^{\pi\pi} , \\
A_{00} &= \langle \pi^0\pi^0 | \mathcal{H} | B_d^0 \rangle &= 2 A_2^{\pi\pi} + A_0^{\pi\pi} ;
\end{aligned}
\tag{1a}
$$

and

$$
\begin{aligned}
\bar{A}_{+-} &= \langle \pi^+\pi^- | \mathcal{H} | \bar{B}_d^0 \rangle &= \sqrt{2} \bar{A}_2^{\pi\pi} - \sqrt{2} \bar{A}_0^{\pi\pi} , \\
\bar{A}_{-0} &= \langle \pi^-\pi^0 | \mathcal{H} | B_u^- \rangle &= 3 \bar{A}_2^{\pi\pi} , \\
\bar{A}_{00} &= \langle \pi^0\pi^0 | \mathcal{H} | \bar{B}_d^0 \rangle &= 2 \bar{A}_2^{\pi\pi} + \bar{A}_0^{\pi\pi} ,
\end{aligned}
\tag{1b}
$$

where some Clebsch-Gordan coefficients have been absorbed into the definitions of $A_{0,2}^{\pi\pi}$ and $\bar{A}_{0,2}^{\pi\pi}$. Clearly the above relations form two isospin triangles in the complex plane:

$$
A_{+-} + \sqrt{2} A_{00} = \sqrt{2} A_{+0} , \qquad \bar{A}_{+-} + \sqrt{2} \bar{A}_{00} = \sqrt{2} \bar{A}_{-0} .
\tag{2}
$$

Since the decay modes $B_u^\pm \to \pi^\pm\pi^0$ occur solely through the tree-level quark diagrams, the ratio $\bar{A}_2^{\pi\pi}/A_2^{\pi\pi} = \bar{A}_{-0}/A_{+0}$ is purely governed by the CKM factor $(V_{ub}V_{ud}^*)/(V_{ub}^*V_{ud}) = e^{-2i\gamma}$ (γ is an angle of the unitarity triangle [8]). To probe CP violation induced by the interplay of decay and $B_d^0 - \bar{B}_d^0$ mixing in $B_d \to \pi^+\pi^-$, we need to measure the interference term

$$
\mathrm{Im}\xi_{+-} = \mathrm{Im}\left(e^{-2i\beta} \frac{\bar{A}_{+-}}{A_{+-}} \right) = \mathrm{Im}\left(e^{2i\alpha} \frac{1-\bar{z}}{1-z} \right) ,
\tag{3}
$$

where β and α are also the angles of the CKM unitarity triangle ($\alpha + \beta + \gamma = 180^0$), $z = A_0^{\pi\pi}/A_2^{\pi\pi}$ and $\bar{z} = \bar{A}_0^{\pi\pi}/\bar{A}_2^{\pi\pi}$. As shown in ref. [3], the complex parameter z ($\bar{z}$) is determinable from the isospin triangles up to a two-fold ambiguity in the sign of its phase. In a similar way one can discuss the CP-violating term $\mathrm{Im}\xi_{00}$ in $B_d \to \pi^0\pi^0$. Then the weak angle α can be extracted from $\mathrm{Im}\xi_{+-}$ and $\mathrm{Im}\xi_{00}$. This approach relies on the time-dependent measurements of $B_d \to \pi\pi$.

Now let us illustrate the effect of IFSI's on the isospin triangles and the CP-violating measurable $\mathrm{Im}\xi_{+-}$ ($\mathrm{Im}\xi_{00}$), which are crucial for the extraction of α. Due to IFSI's the $I = 0$

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state of $\pi\pi$ can mix with that of $D\bar{D}$, and this leads to the two-step decays $B_d \to D\bar{D} \to \pi\pi$. The final states $\pi^\pm\pi^0$ cannot mix with $D\bar{D}$ because of their different isospin configurations ($I = 2$ for $\pi^\pm\pi^0$, $I = 0$ and $I = 1$ for $D\bar{D}$). But in principle $\pi^\pm\pi^0$ could mix with $\rho^\pm\rho^0$, and $D\bar{D}$ could mix with $D^*\bar{D}^*$ [6]. For simplicity, here we only consider the influence of $\pi\pi \rightleftharpoons D\bar{D}$ scattering on $B_d \to \pi\pi$, leaving the $I = 2$ state of $\pi\pi$ unmixed with others. As a result, the "bare" isospin amplitudes of $B_d \to \pi\pi$ in eq. (1) are modified as

$$
\begin{aligned}
A'_{+-} &= \sqrt{2}S_2^{\pi\pi}A_2^{\pi\pi} - \sqrt{2}\left(S_0^{\pi\pi}A_0^{\pi\pi} + S_0^{\pi D}A_0^{DD}\right) , \\
A'_{00} &= 2S_2^{\pi\pi}A_2^{\pi\pi} + \left(S_0^{\pi\pi}A_0^{\pi\pi} + S_0^{\pi D}A_0^{DD}\right) ;
\end{aligned}
\tag{4a}
$$

and

$$
\begin{aligned}
\bar{A}'_{+-} &= \sqrt{2}S_2^{\pi\pi}\bar{A}_2^{\pi\pi} - \sqrt{2}\left(S_0^{\pi\pi}\bar{A}_0^{\pi\pi} + S_0^{\pi D}\bar{A}_0^{DD}\right) , \\
\bar{A}'_{00} &= 2S_2^{\pi\pi}\bar{A}_2^{\pi\pi} + \left(S_0^{\pi\pi}\bar{A}_0^{\pi\pi} + S_0^{\pi D}\bar{A}_0^{DD}\right) .
\end{aligned}
\tag{4b}
$$

In the above equations, A_0^{DD} denote the $I = 0$ amplitude component of $B_d \to D\bar{D}$, $S_{0,2}$ are complex matrix elements connecting the unitarized isospin amplitudes to the bare ones [6]. Of course, $S_0^{\pi D} = 0$ and $S_2^{\pi\pi} = S_0^{\pi\pi} = 1$ if there is no mixture between the $I = 0$ states of $\pi\pi$ and $D\bar{D}$. From eq. (4), we obtain the following triangle relations:

$$
A'_{+-} + \sqrt{2}A'_{00} = \sqrt{2}S_2^{\pi\pi}A_{+0} , \qquad \bar{A}'_{+-} + \sqrt{2}\bar{A}'_{00} = \sqrt{2}S_2^{\pi\pi}\bar{A}_{-0} .
\tag{5}
$$

Comparing this result with eq. (2), one can observe an overall change of the bare isospin triangles due to IFSI's. In this case, the CP-violating measurable $\mathrm{Im}\xi_{+-}$ becomes

$$
\mathrm{Im}\xi'_{+-} = \mathrm{Im}\left(e^{-2i\beta}\frac{\bar{A}'_{+-}}{A'_{+-}}\right) = \mathrm{Im}\left(e^{2i\alpha}\frac{1-\bar{z}'}{1-z'}\right) ,
\tag{6}
$$

where

$$
z' = z\frac{S_0^{\pi\pi}}{S_2^{\pi\pi}} + \frac{A_0^{DD}}{A_2^{\pi\pi}}\frac{S_0^{\pi D}}{S_2^{\pi\pi}} , \qquad \bar{z}' = \bar{z}\frac{S_0^{\pi\pi}}{S_2^{\pi\pi}} + \frac{\bar{A}_0^{DD}}{\bar{A}_2^{\pi\pi}}\frac{S_0^{\pi D}}{S_2^{\pi\pi}} .
\tag{7}
$$

We find that in principle the angle α can still be extracted from $\mathrm{Im}\xi'_{+-}$ (and $\mathrm{Im}\xi'_{00}$). However, the determination of z' and $\bar{z}'$ from eq. (5) needs the knowledge of $S_2^{\pi\pi}$, since the measured branching ratio of $B_u^+ \to \pi^+\pi^0$ can only fix $|A_{+0}|$ other than $|S_2^{\pi\pi}A_{+0}|$. Before we obtain definite information on the mixing of $\pi\pi$ and $D\bar{D}$, it is impossible to determine the values of $|S_0^{\pi D}|$ and $|S_{0,2}^{\pi\pi}|$ reliably [3]. Thus the isospin approach of ref. [3] cannot work well in the presence of significant IFSI's.

If the $\pi\pi \rightleftharpoons D\bar{D}$ scattering effect is insignificant in the problem under discussion, one expects that $|S_0^{\pi D}|$ may be small enough and $|S_2^{\pi\pi}|$ does not deviate too much from unity. In

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[3]From ref. [6], one can obtain a rough feeling of the possible effect of IFSI's on the magnitudes of decay rates and CP asymmetries.

this case, the two triangles in eq. (5) are able to be approximately determined from direct measurements, then one can isolate the unknow parameters z' and $\bar{z}'$ to an acceptable degree of accuracy and extract α from $\mathrm{Im}\xi'_{+-}$ and $\mathrm{Im}\xi'_{00}$. In the following we shall assume this simple case and present an approximate approach to determine α and the penguin amplitude in the decay modes $B_d \to \pi\pi$.

3. Approximate isolation of the penguin amplitude

To lowest order in weak interactions, all two-body mesonic B decays can be generally described by ten topologically different quark diagrams [9]. In this language, we assume that the effect of IFSI's on every decay mode is insignificant enough to be negligible. Neglecting the EWP diagrams and those exchange- or annihilation-type channels, the dominant quark-diagram amplitudes for $B_d^0 \to \pi^+\pi^-$, $B_u^+ \to \pi^+\pi^0$ and $B_d^0 \to \pi^0\pi^0$ are illustrated in fig. 1. Another reasonable assumption to be used is that the strong penguin diagram is dominated by the top-quark loop [10, 11]. This implies that the weak phase of the penguin amplitude comes only from $V_{tb}^* V_{td}$ in $B_d^0 \to \pi^+\pi^-$ or $B_d^0 \to \pi^0\pi^0$. In terms of the angles of the CKM unitarity triangle [8], we parametrize the decay amplitudes of the three modes and their CP-conjugate counterparts as follows [4]:

$$
\begin{aligned}
A_{+-} &= -Te^{i\gamma} - Pe^{i(\delta-\beta)}\,, \\
A_{+0} &= -\frac{1+a}{\sqrt{2}}Te^{i\gamma}\,, \\
A_{00} &= -\frac{a}{\sqrt{2}}Te^{i\gamma} + \frac{1}{\sqrt{2}}Pe^{i(\delta-\beta)}\,;
\end{aligned}
\tag{8a}
$$

and

$$
\begin{aligned}
\bar{A}_{+-} &= -Te^{-i\gamma} - Pe^{i(\delta+\beta)}\,, \\
\bar{A}_{-0} &= -\frac{1+a}{\sqrt{2}}Te^{-i\gamma}\,, \\
\bar{A}_{00} &= -\frac{a}{\sqrt{2}}Te^{-i\gamma} + \frac{1}{\sqrt{2}}Pe^{i(\delta+\beta)}\,.
\end{aligned}
\tag{8b}
$$

The above expressions are based on a simple isospin consideration and the factorization approximation for the final states $\pi\pi$, as we shall explain below. Here δ denotes the strong phase shift between the penguin and tree-level amplitudes of $B_d \to \pi\pi$. Thus without loss of any generality, T and P can be assumed to be real and positive. The parameter a represents the color-mismatched suppression in the tree-level amplitudes of $B_u^+ \to \pi^+\pi^0$, $B_d^0 \to \pi^0\pi^0$ and their CP-conjugate processes. We shall see later that a can be either calculated using

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[4]Here we use the same valence-quark notations for the pions as refs. [12, 13]: $|\pi^+\rangle = |u\bar{d}\rangle$, $|\pi^0\rangle = |d\bar{d} - u\bar{u}\rangle/\sqrt{2}$, and $|\pi^-\rangle = -|d\bar{u}\rangle$. This convention guarantees that the three pion mesons belong to an isotriplet. Also, we adopt $|B_d^0\rangle = |d\bar{b}\rangle$, $|\bar{B}_d^0\rangle = |b\bar{d}\rangle$, $|B_u^+\rangle = |u\bar{b}\rangle$, and $|B_u^-\rangle = -|b\bar{u}\rangle$.

the factorization approximation or measured directly from the decay rates. Clearly the relations in eq. (8) can form the same isospin triangles as those in eq. (2). This implies that the quark-diagram language used for describing $B \to \pi\pi$ is consistent with the isospin analysis, as shown first in ref. [13]. In the factorization approximation, we can assume that the difference between the interactions of $I = 0$ and $I = 2$ final states is negligible. Thus the two quark-diagram amplitudes of $B_u^+ \to \pi^+\pi^0$, which occurs solely through the $\Delta I = 3/2$ transition, can be parametrized as eq. (8a) with a real factor a. This accordingly implies that the tree-level amplitude of $B_d^0 \to \pi^+\pi^-$ and that of $B_d^0 \to \pi^0\pi^0$ also have negligible strong phase shift, to guarantee the isospin triangle relations in eq. (2). So do the penguin amplitudes of $B_d^0 \to \pi^+\pi^-$ and $B_d^0 \to \pi^0\pi^0$. Therefore, we expect that contributions from the penguin diagrams of $B_d \to \pi\pi$ are crucial for the isospin triangles to have nonvanishing area. For a detailed discussion about translation of the isospin amplitudes of $B \to \pi\pi$ into the corresponding combinations of quark-diagram amplitudes, we refer the reader to ref. [13]. Based on the correspondence between the quark-diagram description and the isospin analysis, we subsequently use eq. (8) to relate the weak and strong phases in $B_d \to \pi\pi$ to the time-independent measurables.

From eqs. (8a) and (8b), we have $|A_{+0}| = |\bar{A}_{-0}|$. The decay rates of the above six processes should be measured in the near future at either B-meson factories or high-luminosity hadron machines. Let us define the following four measurables:

$$R_{+-} = \frac{|A_{+-}|^2}{|A_{+0}|^2 + |\bar{A}_{-0}|^2} , \qquad \bar{R}_{+-} = \frac{|\bar{A}_{+-}|^2}{|A_{+0}|^2 + |\bar{A}_{-0}|^2} ; \qquad (9a)$$

$$R_{00} = \frac{|A_{00}|^2}{|A_{+0}|^2 + |\bar{A}_{-0}|^2} , \qquad \bar{R}_{00} = \frac{|\bar{A}_{00}|^2}{|A_{+0}|^2 + |\bar{A}_{-0}|^2} . \qquad (9b)$$

It is clear that T can be directly obtained from $|A_{+0}|^2$ or $|\bar{A}_{-0}|^2$. On the other hand, the magnitude of the penguin amplitude P is only determined once the data on R_{+-} ($\bar{R}_{+-}$) and R_{00} ($\bar{R}_{00}$) become available. By use of eqs. (8) and (9), the ratio $P/T \equiv \chi$ is given as

$$\chi = \sqrt{(1+a)(aR_{+-} + 2R_{00}) - a} = \sqrt{(1+a)(a\bar{R}_{+-} + 2\bar{R}_{00}) - a} . \qquad (10)$$

It should be noted that the size of χ is sensitive to the uncertainty associated with the color-suppression factor a. There are two ways to determine a. One way is to use the effective weak Hamiltonian $\mathcal{H}(\Delta B = 1)$ [14] and the factorization approximation [15]. It is easy to obtain

$$a = \frac{C_2 + N_c C_1}{C_1 + N_c C_2} , \qquad (11)$$

where N_c is the number of colors, $C_1 \approx -0.291$ and $C_2 \approx 1.133$ are the Wilson coefficients at the scale $\mu = O(m_b)$ [14]. We obtain $a \approx 0.23$ for $N_c \approx 2.2$ and $a \approx -0.26$ for $N_c = \infty$.

The current data on exclusive hadronic B decays favor the former, i.e. a positive a with the magnitude of $O(\lambda)$ [16]. The other way to determine a is through measurements of the decay modes $B_d \to \pi^+\pi^-$ and $B_d \to \pi^0\pi^0$. From eqs. (8) and (9) one can find

$$a = -2 \cdot \frac{\bar{R}_{00} - R_{00}}{\bar{R}_{+-} - R_{+-}} . \tag{12}$$

Indeed the rate differences $\bar{R}_{00} - R_{00}$ and $\bar{R}_{+-} - R_{+-}$ indicate the signals of direct CP violation. The assumptions made in eq. (8) imply that the direct CP asymmetries in $B_d \to \pi^+\pi^-$ and $B_d \to \pi^0\pi^0$ are governed by the same weak phase difference $(\beta + \gamma)$ and the same strong phase shift (δ), but their magnitudes are different. A comparison between the results of a obtained in eqs. (11) and (12) can serve as a rough test of the factorization approximation. In practice, however, measuring the rate difference $\bar{R}_{00} - R_{00}$ might be very difficult due to the expected smallness of the branching ratio of $B_d \to \pi^0\pi^0$. Considering $SU(3)$ symmetry and its small breaking effect, Deshpande and He [17] have shown that the rate difference of $B_d^0 \to \pi^+\pi^-$ ($\pi^0\pi^0$) vs $\bar{B}_d^0 \to \pi^+\pi^-$ ($\pi^0\pi^0$) is related to that of $B_d^0 \to \pi^-K^+$ (π^0K^0) vs $\bar{B}_d^0 \to \pi^+K^-$ ($\pi^0\bar{K}^0$). The latter can be more easily measured in experiments, thus can be used to determine a through eq. (12). It should be noted that our main results still hold even in the special case $a = 0$. If δ happens to be 0^0 or 180^0, however, eq. (12) will not work. In this case, we have to use the data on other B decays to determine a [16].

4. Approximate determination of α

Now we look at how to extract the weak phase α without resorting to measurements of the decay-time distribution of $B_d \to \pi\pi$. Since the weak phase shift between the penguin and tree-level amplitudes of $B_d \to \pi^+\pi^-$ is $\beta+\gamma$, by use of the unitarity condition $\alpha+\beta+\gamma = 180^0$ we obtain two concise equations connecting the weak and strong phases to the measurables:

$$\cos(\alpha + \delta) = \frac{1}{2\chi}\left[1 + \chi^2 - (1+a)^2 R_{+-}\right] , \tag{13a}$$

$$\cos(\alpha - \delta) = \frac{1}{2\chi}\left[1 + \chi^2 - (1+a)^2 \bar{R}_{+-}\right] . \tag{13b}$$

Similarly, α and δ can be related to R_{00} and $\bar{R}_{00}$. Because χ is determined from eq. (10), both α and δ can be extracted from (13) with four-fold ambiguity. Due to the assumptions made in eq. (8), this ambiguity cannot be reduced by the relations among α, δ, R_{00} and $\bar{R}_{00}$. To the accuracy that R_{+-} and $\bar{R}_{+-}$ cannot be distinguished in experiments, we expect that the strong phase shift δ might be close to 0^0 or 180^0 (i.e., vanishingly small direct CP violation in $B_d \to \pi\pi$). Note that eq. (13) would become invalid if the magnitude of the

penguin amplitude P were too small to be measured. In this case, the two isospin triangles in eq. (2) would collapse to lines. Within the standard model, however, several calculations have given that χ is at the level of $O(\lambda)$ or larger [2]. Thus our results (10) and (13) are feasible to probe the weak phase α as well as the significant penguin amplitude in $B_d \to \pi\pi$.

It is worth remarking that our approach to determine the angle α does not require the measurement of the CP-violating term $\text{Im}\xi_{+-}$:

$$\text{Im}\xi_{+-} \approx \sin(2\alpha) - 2\chi \sin\alpha \cos\delta \tag{14}$$

(to lowest-order approximation of χ), which is induced by the interplay of decay and $B_d^0 - \bar{B}_d^0$ mixing. Thus the feasibility of our approach is independent of the time-dependent measurements of neutral B decays to be carried out at the asymmetric B factories of KEK and SLAC [18]. Nonetheless, it should be noted that the determination of $|A_{+-}|$ and $|\bar{A}_{+-}|$ from any experiment is indeed influenced by $B_d^0 - \bar{B}_d^0$ mixing. To overcome this problem, one available way is through the time-integrated measurements of $B_d \to \pi^+\pi^-$ on the $\Upsilon(4S)$ resonance, where the produced two B_d mesons are in a coherent state (with odd charge-conjugation parity) until one of them decays. Using the semileptonic decay of one B_d meson to tag the flavor of the other meson decaying to $\pi^+\pi^-$, the probability for observing such a joint decay has been give as [19]:

$$\Pr(l^+X^-; \pi^+\pi^-) = |A_l|^2 \left[\frac{|A_{+-}|^2 + |\bar{A}_{+-}|^2}{2} - \frac{1}{1+x_d^2} \cdot \frac{|A_{+-}|^2 - |\bar{A}_{+-}|^2}{2} \right], \tag{15a}$$

$$\Pr(l^-X^+; \pi^+\pi^-) = |A_l|^2 \left[\frac{|A_{+-}|^2 + |\bar{A}_{+-}|^2}{2} + \frac{1}{1+x_d^2} \cdot \frac{|A_{+-}|^2 - |\bar{A}_{+-}|^2}{2} \right], \tag{15b}$$

where $|A_l| = |\langle l^+X^-|\mathcal{H}|B_d^0\rangle| \overset{CPT}{=} |\langle l^-X^+|\mathcal{H}|\bar{B}_d^0\rangle|$, and $x_d = \Delta m/\Gamma \approx 0.71$ is a measure of $B_d^0 - \bar{B}_d^0$ mixing [8]. At present, the semileptonic decays $B_d^0 \to l^+X^-$ and $\bar{B}_d^0 \to l^-X^+$ have been well reconstructed [8, 20], thus $|A_l|$ is determinable independent of the above joint decays. Once $\Pr(l^\pm X^\mp; \pi^+\pi^-)$ are measured, we shall be able to determine $|A_{+-}|$ and $|\bar{A}_{+-}|$ through eq. (15). In practice, the measurements of $\Pr(l^\pm X^\mp; \pi^+\pi^-)$ can be realized either by the symmetric e^+e^- collider running at Cornell [20] or by the forthcoming asymmetric B factories. Similarly, $|A_{00}|$ and $|\bar{A}_{00}|$ can be determined.

5. Discussions and conclusion

In the first part of this work, we have qualitatively illustrated the effect of IFSI's on the isospin triangles of $B \to \pi\pi$. The determination of α from these decay modes may suffer

from large mixing between the $I = 0$ states of $\pi\pi$ and $D\bar{D}$. A quantitative estimation of IFSI's needs much progress in theory and more data in experiments.

In the assumption that the IFSI's in $B \to \pi\pi$ are insignificant enough to be negligible, we have presented a self-consistent and time-independent approach to approximately isolate the weak phase α and the penguin amplitude of $B_d \to \pi\pi$. Here it is worthwhile to comment briefly on the uncertainties with this approach. In the first approximation, we neglected the electroweak penguins in $B \to \pi\pi$. As pointed out in ref. [7], the effects of electroweak penguins on the decays via $\bar{b} \to \bar{u}u\bar{d}$ or $\bar{b} \to \bar{d}$ are at most $O(\lambda^2)$ of the dominant tree-level amplitude T or $O(\lambda)$ of the strong penguin amplitude P. Hence it is unlikely that the validity of the isospin analysis in ref. [3] and our present method are significantly affected by the neglect of the electroweak penguins. On the other hand, the approximation of neglecting those exchange- and annihilation-type topologies in $B \to \pi\pi$ is expected to be safe for some dynamical reasons [13]. The approach to test the reliability of this kind of approximations has been suggested in ref. [11]. The factorization approximation used to parametrize the decay amplitudes of $B \to \pi\pi$ as eq. (8) is equivalent to neglecting the difference between the $I = 0$ and $I = 2$ interactions in the final states. The validity of this assumption can be examined by the forthcoming experimental data. The assumption that the strong penguin diagram is dominated by the top-quark loop can be interpreted as follows. On the basis of the standard model with three families of quarks, the penguin contributions with u and c quarks in the loop are equal in magnitude up to $O(m_c^2/m_b^2)$ apart from their CKM factors. Thus the top-quark loop dominates the penguin amplitude to the same degree of accuracy, i.e. $O(\leq 10\%)$, guaranteed by the CKM unitarity. Indeed this is a good approximation to obtain the lowest order magnitudes of α, δ and P. When the time-dependent measurements of $B_d \to \pi\pi$ become available at an asymmetric B factory to allow for the more precise isospin analysis proposed by Gronau and London [3], one can check the difference between our results and theirs. The corresponding deviation should definitely constrain the reliability of the top-quark dominance and the factorization approximation assumed in our work.

By now the experimental sensitivity to the penguin contribution in $B_d \to \pi\pi$ has been analyzed based on the isospin triangle relations [3] and under the circumstance of asymmetric B-meson factories [21]. Such an analysis is also applicable to the simplified approach proposed in this work. It is concluded that determining the weak and strong phases associated with the penguin amplitude to a satisfactory accuracy is experimentally feasible in the near future. Finally we stress again that our approach is time-independent, and thus it can be confronted with all the forthcoming measurements of $B \to \pi\pi$, no matter whether they

are time-integrated or time-dependent.

Acknowledgements

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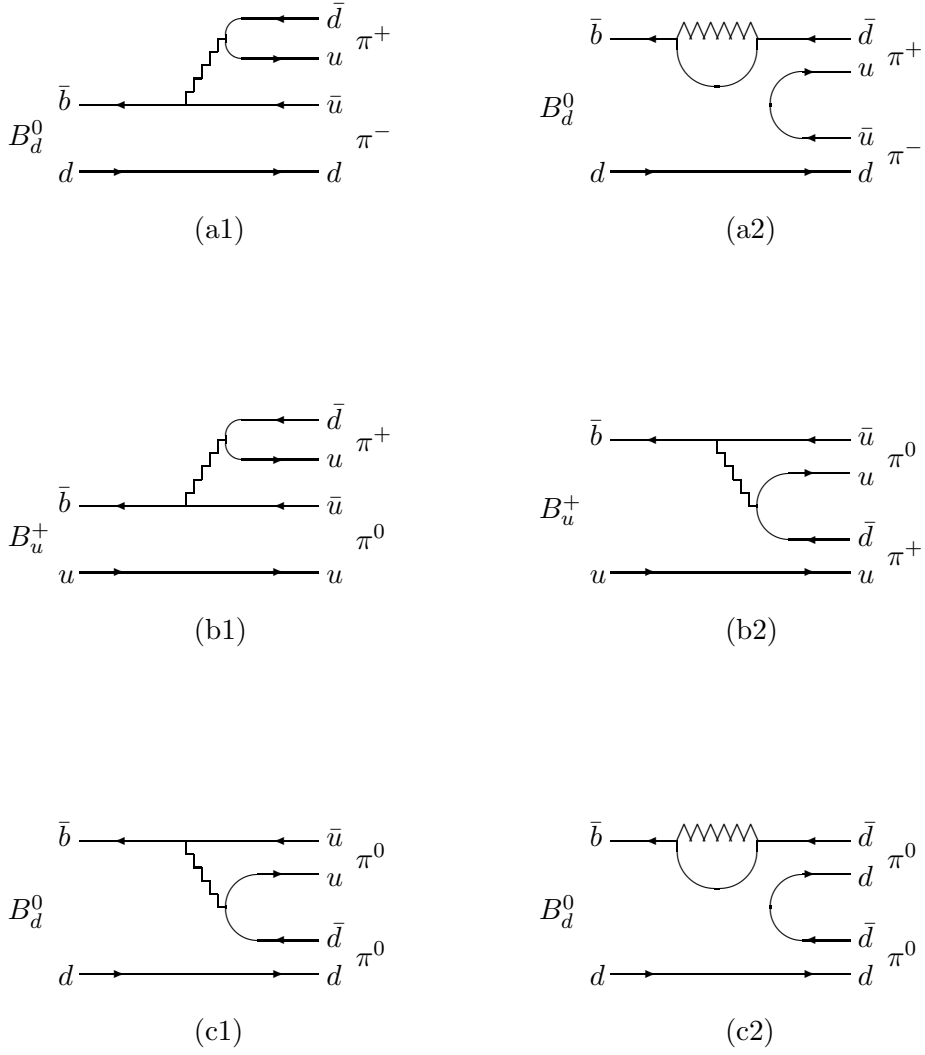
Figure 1: The dominant tree-level and strong penguin diagrams for $B_d^0 \to \pi^+\pi^-$, $B_u^+ \to \pi^+\pi^0$, and $B_d^0 \to \pi^0\pi^0$.