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How natural is a small $\bar{\theta}$ in left-right SUSY models ?

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Abstract

In the world without an axion the smallness of $\bar{\theta}$ may be achieved due to a spontaneously broken discrete left-right symmetry. We analyze the radiatively induced $\bar{\theta}$ in the context of generic left-right symmetric SUSY models without assuming flavor degeneracy in the squark sector. Left-right symmetry allows to keep $\bar{\theta}$ within its present bound only if the inter-generational mass splitting in the squark sector at the scale of the left-right symmetry breaking is smaller than 0.5%. We also consider the naturalness of $m_u = 0$ solution to the strong CP problem in the context of horizontal flavor symmetries. A strong bound on the combination of the horizontal charges in the Up quark sector is found in this case.

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1 Introduction

The strong CP problem remains an important and open issue in particle physics. The QCD Lagrangian has a fundamental parameter θ which labels different super-selection sectors. Its effect can be accounted by an additional term in the QCD Lagrangian,

$$\mathcal{L} = \theta \frac{g_3^2}{16\pi^2} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}, \tag{1}$$

which violates P and CP symmetries [1].

In a full theory, the effective low energy value for the theta term is modified by possible complex phases in the quark mass matrices:

$$\bar{\theta} = \theta + \arg(\det M_u M_d) + ... \tag{2}$$

The ellipsis stands for other possible contributions from yet unknown fermions, charged with respect to $SU(3)$ color gauge group (for example, gluino).

Current experimental limits on the electric dipole moment (EDM) of the neutron put severe constraints on the allowed size of the $\bar{\theta}$-angle. Among different ways of calculating the EDM of the neutron, induced by $\bar{\theta}$, the most reliable ones use chiral perturbation theory [2] or QCD sum rules [3, 4, 5]. Here we use the numerical result of ref. [3] which gives the following prediction for $d_n(\bar{\theta})$:

$$d_n \simeq 1.2 \times 10^{-16} \bar{\theta}\, e \cdot cm. \tag{3}$$

Together with the current experimental limits on the neutron EDM [6] it implies a stringent bound on the theta term, $\bar{\theta} < 6 \times 10^{-10}$. Similar bounds are provided by the limits on the electric dipole moment of ^{199}Hg atom [7, 8, 9].

The puzzling smallness of $\bar{\theta}$ in comparison with a natural expectation of $\bar{\theta} \sim 1$ is usually referred to as the strong CP problem. There are several theoretical possibilities of removing $\bar{\theta}$ from the theory, none of which are free from their own intrinsic problems. The most popular solution to the strong CP problem uses the dynamical relaxation of $\bar{\theta}$ through the axion mechanism [10]. Perhaps, it is the most elegant and universal way of removing the theta term. However, the negative results of all experimental searches of axions and very restrictive astrophysical and cosmological considerations which place the axion coupling constant into a relatively narrow range, $10^{10} - 10^{12}$ GeV, suggest to look for other alternative solutions.

In principle, one can speculate on the vanishing of the Yukawa coupling for the up quark, hoping that the hadronic phenomenology would still allow for $m_u = 0$ [11, 12]. The vanishing of this coupling may be a consequence of the horizontal flavor symmetries, supposedly responsible for the hierarchy of masses and mixing angles in the fermion sector [13, 14, 15]. We will comment on this possibility in supersymmetric models and show that m_u is highly susceptible to the supersymmetric threshold corrections. $m_u = 0$ is unnatural unless there exists large differences in horizontal charges in the Up sector.

The main goal of this paper is to consider an important class of solutions where $\bar{\theta}$ is small *by construction.* This can occur if parity or CP symmetry are exact symmetries of the full theory. Here we assume that these symmetries are spontaneously broken at some energy scale above the electroweak scale. In the absence of the axion mechanism, the radiative corrections to $\bar{\theta}$ which are induced below this scale, will be the main source of the EDMs in the hadronic sector. Therefore, these corrections have to be within the experimental bounds. This provides severe restrictions on the amount of CP-violation that one can have in this class of theories [16].

The models with spontaneously broken CP, constructed to solve the strong CP problem [17], normally fail to keep $\bar{\theta}$ within the experimental bound after the radiative corrections are taken into account. In the supersymmetric framework this problem was emphasized in Ref. [19], where it was shown that the non-universality in the squark sector generates θ_{rad} considerably larger than the experimental bound. In order to keep the radiative corrections to $\bar{\theta}$ small, one has to suppress all CP-violating phases in the theory, including the phase in the Kobayashi-Maskawa matrix. In general, it is hard to achieve and these are of the superweak type models. Recent confirmation of the non-zero result on ϵ'/ϵ [20] casts strong doubts on the viability of the superweak framework and disfavors most of the models with small $\bar{\theta}$ due to spontaneous breaking of CP. There is, however, a recent model-building proposal which combines the low energy SUSY breaking and strong complex renormalization of the quark wave functions at high scale with unbroken SUSY in order to get zero $\bar{\theta}$ *and* generate sufficiently large Kobayashi-Maskawa phase [18].

The idea of the spontaneous breaking of parity, initially introduced in the framework of Pati-Salam unification [21], was thoroughly studied in the case of $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)$ gauge group [22]. Among different appealing features of these models is the possibility to have $\theta = 0$ at the tree level as a consequence of exact left-right symmetry under which $\theta \to -\theta$ [23]. Several years ago, this idea was discussed again in the context of the supersymmetric left-right models [24, 25]. The vanishing of $\bar{\theta}$ at the tree level does not necessarily lead to a solution to the strong CP problem as loop corrections below the scale of left-right breaking can generate θ_{rad} above the required bound. Careful analysis of the radiative corrections to the theta term performed in Ref. [26] showed that θ_{rad} can be kept within the experimental limits, *assuming* the initial universality in the squark sector at the scale of the SUSY breaking. However, it is not generally expected that the universality in the squark sector is exact, and therefore it is not clear how large the radiative corrections to $\bar{\theta}$ in a generic left-right SUSY model can be. One can hope that a moderate splitting among the squark masses would allow to keep $\bar{\theta}$ within the experimental bound. Examples of $\sim 1\%$ splitting in the squark mass sector could be attained in some variants of the free fermionic superstring models [27, 28].

In this paper we explore the necessary conditions for naturally small $\bar{\theta}$ in a generic supersymmetric theory which has the following features. Below some scale Λ, the field content of the theory is that of the Minimal Supersymmetric Standard Model with the SM gauge group. Above Λ, the theory has the built-in discrete left-right symmetry and more complicated gauge structure compatible with it. It is important that the vanishing of the $\bar{\theta}$ parameter at the tree level in these models relies on the existence of the discrete left-right symmetry rather than on the particular choice of the gauge group. We assume that the Higgs structure below the scale of the left-right breaking is minimal in the spirit of ref. [29]. We analyze radiative corrections to the theta term without assuming squark degeneracy and find the allowed degree of the non-universality in the soft-breaking sector consistent with the bounds on theta. It turns out that EDMs require 0.5% degeneracy in the squark sector at the scale Λ as well as a strong alignment of squark masses and Yukawa couplings in the Down quark sector.

2 Theta term and left-right symmetry in SUSY

Previous works on the the theta problem in $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)$ left-right symmetric theories [24, 25, 26] used very specific ansatz of proportionality and degeneracy in the squark sector. By these conditions we understand the following requirements imposed on the soft-breaking sector at the scale of the SUSY breaking:

$$\mathbf{M}_Q^2 = m_Q^2 \mathbf{1}; \;\; \mathbf{M}_D^2 = m_D^2 \mathbf{1}; \;\; \mathbf{M}_U^2 = m_U^2 \mathbf{1} \quad \text{"degeneracy"} \tag{4}$$

$$\mathbf{A}_u = A_u \mathbf{Y}_u; \;\; \mathbf{A}_d = A_d \mathbf{Y}_d \quad \text{"proportionality".} \tag{5}$$

It is known, however, that in the models which use a spontaneously broken CP symmetry to solve strong CP problem, the departure from the exact universality should be at a very tiny level, 10^{-6} or so [19]. To determine allowed squark mass splittings in LR SUSY models, we relax the conditions of the universality and proportionality, while keeping explicit left-right symmetry at the scale Λ. This symmetry requires the hermiticity of the Yukawa matrices $\mathbf{Y}$ and trilinear scalar coupling matrices $\mathbf{A}$, as well as the identity of the left and right handed squark mass matrices,

$$\mathbf{Y}_u = \mathbf{Y}_u^\dagger, \quad \mathbf{Y}_d = \mathbf{Y}_d^\dagger, \quad \mathbf{A}_u = \mathbf{A}_u^\dagger, \quad \mathbf{A}_d = \mathbf{A}_d^\dagger$$
$$\mathbf{M}_Q^2 = \mathbf{M}_D^2 = \mathbf{M}_U^2 = \mathbf{M}^2 \equiv m^2 \mathbf{1} + \mathbf{S}. \tag{6}$$

Matrix $\mathbf{S}$ parametrizes the departure from the degeneracy condition, and we choose $\mathbf{S}$ in such a way that $\mathrm{Tr}\mathbf{S} = 0$. Another important set of conditions is the absence of the explicit CP-violating phases in the gaugino masses and B-parameter in the Higgs potential,

$$B = B^*, \quad m_{\lambda_i} = m_{\lambda_i}^*, \quad \mu = \mu^*. \tag{7}$$

As noted in [24, 25], the reality of the $SU(2)$ gaugino mass does not follow from the left-right symmetry. We consider it as a consequence of a higher unification scheme which makes all gaugino phases equal.

Conditions (6) and (7) ensure the absence of the theta term at the tree level. At the loop level one has to consider radiative corrections to the quark and gluino mass matrices.

These corrections must be proportional to CP-violating phases present in the theory. When squarks are degenerate, this source is just the complexity of the Yukawa matrices, which is the Kobayashi-Maskawa (KM) phase in this case. The latter provides a *minimal* content of CP-violation. If the contribution to $\bar{\theta}$ from KM phase happens to be large, this means that one cannot obtain a viable solution to the strong CP problem without fine tuning. This question was studied in the framework of pure SM [30, 31], where radiative corrections to $\bar{\theta}$ arise first in the order $\alpha_s G_F^2 m_c^2 m_s^2$ times the CP-odd KM invariant I_{KM} [31], and in the MSSM with the KM mechanism of CP-violation [32] where the result is also found to be much smaller than 10^{-9}. The main reason behind the suppression of θ_{rad} is the smallness of the Yukawa couplings and mixing angles contained in the so–called Jarlskog factor,

$$J_{SM} = I_{KM}(\lambda_t^2 - \lambda_c^2)(\lambda_c^2 - \lambda_u^2)(\lambda_u^2 - \lambda_t^2)(\lambda_b^2 - \lambda_s^2)(\lambda_s^2 - \lambda_d^2)(\lambda_d^2 - \lambda_b^2) \tag{8}$$

Here I_{KM} is the imaginary part of the invariant quartic combination of the KM mixing elements, $I_{KM} = \mathrm{Im}(V_{td}^* V_{tb} V_{cb}^* V_{cd})$.

When the squark degeneracy is abandoned, an analog of Jarlskog–type factor may come from the squark sector. If violation of degeneracy is "moderate", i.e. proportional to the small factor $\epsilon_{ij} = (s_i - s_j)/m^2$, there is a chance that the experimental bound on $\bar{\theta}$ can be satisfied without fine tuning.

In the case of MSSM the leading contributions to $\bar{\theta}$ come from corrections to the quark mass matrix, gluino mass term and the radiative correction to the Higgs potential, Fig. 1.

As it will become clear shortly, the corrections to the quark masses are by far more important than other types of contributions. In order to solve strong CP problem, the corrections to Yukawa matrices which can complexify their determinants should be small and thus we expand eq. (2) to obtain

$$\bar{\theta} = \mathrm{Im}\left[\mathrm{Tr}(\mathbf{Y}_u^{-1}\Delta\mathbf{Y}_u) + \mathrm{Tr}(\mathbf{Y}_d^{-1}\Delta\mathbf{Y}_d)\right]. \tag{9}$$

Below the scale Λ, the general form of the squark mass matrices becomes very complicated. The renormalization group equations are no longer left-right symmetric and induce the splitting between left- and right-handed squarks; the Yukawa matrices enter into the running of the squark masses, etc. To analyze SUSY threshold corrections to $\bar{\theta}$ we choose the following strategy. We treat separately the cprrections to $\bar{\theta}$ induced by the resulting left-right asymmetry in the squark masses and those induced by the intergenerational splitting which is induced by Yukawa interactions. In both cases we derive constraints on **S**-matrix and require **S** to satsify all of them, thus neglecting possible cancellations among different mechanisms of inducing θ. Since we are interested in *natural* solution to the strong CP problem, possible cancellations between different mechanisms should be rightfully ignored as they represent a fine tuning which this solution wants to avoid. In addition to that, we treat μ and **A**-proportional corrections separately.

The gauge evolution of the squark masses from the scale Λ to the electroweak scale induce the left and right-handed squark mass splitting, $m_L^2 - m_R^2 \simeq m_2^2(3\alpha_2/2\pi)\ln(\Lambda^2/m^2)$. In our

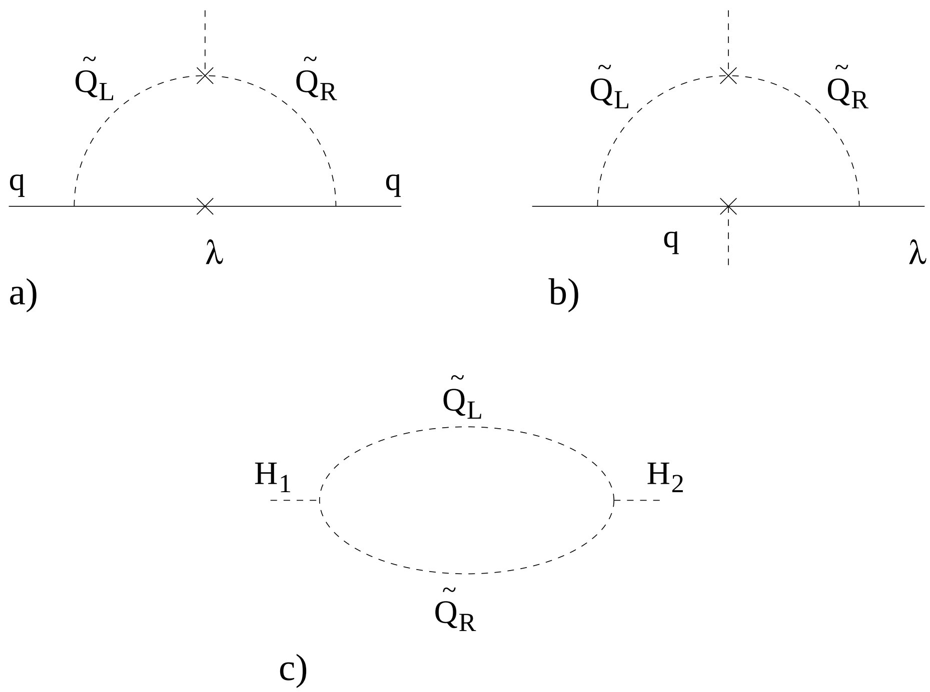

Figure 1: SUSY threshold corrections to *a)* quark Yukawa couplings, *b)* gluino mass, *c)* m_{12}^2 soft breaking parameter which can complexify $v_1 v_2$

final result we take this splitting to be $O(1)$. The value of θ induced by this effect can be estimated by expanding the diagram, given by Fig. 1a, in $\mathbf{S}$ matrix and keeping only the lowest possible terms in $\mathbf{S}$. This procedure is justified as long as mass splittings in the squark sector are smaller than characteristic momenta in the loop. Thus we have

$$\mathrm{Im Tr}(\mathbf{Y}_d^{-1}\Delta\mathbf{Y}_d) = i\frac{8g_3^2}{3}\int\frac{d^4p}{(2\pi)^4}\frac{\mu m_\lambda \tan\beta}{p^2-m_\lambda^2}\sum_{k,l}\frac{\mathrm{Im Tr}(\mathbf{Y}_d^{-1}\mathbf{S}^k\mathbf{Y}_d\mathbf{S}^l)}{(p^2-m_L^2)^{k+1}(p^2-m_R^2)^{l+1}} \tag{10}$$

$$\mathrm{Im Tr}(\mathbf{Y}_u^{-1}\Delta\mathbf{Y}_u) = i\frac{8g_3^2}{3}\int\frac{d^4p}{(2\pi)^4}\frac{\mu m_\lambda \cot\beta}{p^2-m_\lambda^2}\sum_{k,l}\frac{\mathrm{Im Tr}(\mathbf{Y}_u^{-1}\mathbf{S}^k\mathbf{Y}_u\mathbf{S}^l)}{(p^2-m_L^2)^{k+1}(p^2-m_R^2)^{l+1}} \tag{11}$$

It is easy to see that the lowest possible order in which the CP violation does not vanish is $k+l=3$. It is convenient to choose the basis in which squark masses are diagonal:

$$\mathbf{S} = diag(s_1, s_2, s_3), \quad \mathbf{Y}_u = U\mathbf{Y}_u^{diag}U^\dagger, \quad \mathbf{Y}_d = V\mathbf{Y}_d^{diag}V^\dagger. \tag{12}$$

The product of U and V matrices gives the Kobayashi-Maskawa mixing matrix, $K = U^\dagger V$. Then the two lowest order structures, naturally arising from eqs. (10-11), are

$$J_u = \mathrm{Im}(U_{21}^* U_{23} U_{33}^* U_{31})(s_1-s_2)(s_2-s_3)(s_3-s_1)\frac{(m_u-m_c)(m_c-m_t)(m_t-m_u)}{m_u m_c m_t}$$

$$J_d = \mathrm{Im}(V_{21}^* V_{23} V_{33}^* V_{31})(s_1-s_2)(s_2-s_3)(s_3-s_1)\frac{(m_d-m_s)(m_s-m_b)(m_b-m_d)}{m_d m_s m_b} \tag{13}$$

To estimate the size of the integral we take $m_L \simeq m_R \simeq m_\lambda$ and arrive at the following expression for $\bar\theta$

$$\bar\theta \simeq \frac{2\alpha_s}{90\pi}\;\frac{(m_L^2-m_R^2)\mu}{m^3}\;\frac{J_u\cot\beta+J_d\tan\beta}{m^6}\simeq$$

$$8.5\cdot 10^{-4}\;\frac{(m_L^2-m_R^2)\mu}{m^3}\;\epsilon_{12}\epsilon_{23}\epsilon_{31}\;\left(\frac{m_t}{m_u}\mathrm{Im}(U_{21}^*U_{23}U_{33}^*U_{31})\cot\beta+\frac{m_b}{m_d}\mathrm{Im}(V_{21}^*V_{23}V_{33}^*V_{31})\tan\beta\right). \tag{14}$$

Here, only the leading terms enhanced by the ratios m_t/m_u or m_b/m_d are retained in eqs. (13). We note in passing that the answer in this form does not allow to take the limit $m_{u(d)}\to 0$ because it relies on $\Delta m_u \sim \mathrm{Im}(U...U)m_t \ll m_u$. The numerical bound on theta is satisfied as long as

$$\frac{(m_L^2-m_R^2)\mu\tan\beta}{m^3}\;\mathrm{Im}(V_{21}^*V_{23}V_{33}^*V_{31})\;\epsilon_{12}\epsilon_{23}\epsilon_{31}<10^{-9} \tag{15}$$

$$\frac{(m_L^2-m_R^2)\mu\cot\beta}{m^3}\;\mathrm{Im}(U_{21}^*U_{23}U_{33}^*U_{31})\;\epsilon_{12}\epsilon_{23}\epsilon_{31}<10^{-11} \tag{16}$$

Equations (15-16) suggest that the squarks have to be degenerate at 1% level in the basis in which mixing angles of U and V are on the order of CKM mixing angles, $U_{ij}\sim V_{ij}\sim K_{ij}$. Indeed, with $\mathrm{Im}(V_{21}^*V_{23}V_{33}^*V_{31})\sim \mathrm{Im}(V_{21}^*V_{23}V_{33}^*V_{31})\sim I_{KM}\sim 10^{-5}$ and $\epsilon_{ij}\sim 0.01$, both constraints (15) and (16) can be satisfied. The most relaxed constraints on ϵ_{ij}, $\epsilon_{ij}<0.05$ are for the case of $U=\mathbf{1}$ and $V=K$, which corresponds to the squark matrix, $\mathbf{Y}_u$ being diagonal in the same basis.

The inclusion of the squark mass renormalization group running, generated by the Yukawa interaction, introduce additional "dangerous" corrections to $\bar\theta$. The change of the mass matrices at one loop level is given by the following set of expressions (See, e.g. ref. [33]), linearized in the renormalization group coefficients:

$$M_{uLL}^2 = m^2\mathbf{1}+\mathbf{S}+\mathbf{M}_u^\dagger\mathbf{M}_u+c_1\mathbf{Y}_u^\dagger(m^2\mathbf{1}+\mathbf{S})\mathbf{Y}_u+c_2\mathbf{Y}_d^\dagger(m^2\mathbf{1}+\mathbf{S})\mathbf{Y}_d \tag{17}$$

$$M_{uRR}^2 = m^2\mathbf{1}+\mathbf{S}+\mathbf{M}_u\mathbf{M}_u^\dagger+c_3\mathbf{Y}_u(m^2\mathbf{1}+\mathbf{S})\mathbf{Y}_u^\dagger$$

For the down squark matrices, u and d indices should be interchanged. The renormalization group coefficients $c_i\sim(16\pi^2)^{-1}\ln(\lambda^2/m^2)$ can be found in refs. [34, 33]. Their particular forms are not important for our purposes as we take them to be $O(1)$. Using these matrices we calculate the theta term, again expanding the propagators in the Yukawa couplings. In principle, for the top quark one should retain all orders in this expansion and the correct way of doing this was given in ref. [33]. However, for the present discussion it is sufficient to keep only the first-order term. The most important contributions to the $\bar\theta$ parameter are coming from the expansion of the right-handed squark line in $\mathbf{S}$ and left-handed squark line in $\mathbf{Y}_{u(d)}$:

$$\mathrm{ImTr}(\mathbf{Y}_d^{-1}\Delta\mathbf{Y}_d) = i\frac{8g_3^2}{3}\int\frac{d^4p}{(2\pi)^4}\;\frac{m^2\mu m_\lambda\tan\beta}{p^2-m_\lambda^2}\;\frac{c_2\mathrm{ImTr}(\mathbf{Y}_d^{-1}\mathbf{Y}_u^2\mathbf{Y}_d\mathbf{S})}{(p^2-m^2)^4} \tag{18}$$

$$\mathrm{ImTr}(\mathbf{Y}_u^{-1}\Delta\mathbf{Y}_u) = i\frac{8g_3^2}{3}\int\frac{d^4p}{(2\pi)^4}\;\frac{m^2\mu m_\lambda\cot\beta}{p^2-m_\lambda^2}\;\frac{c_2\mathrm{ImTr}(\mathbf{Y}_u^{-1}\mathbf{Y}_d^2\mathbf{Y}_u\mathbf{S})}{(p^2-m^2)^4} \tag{19}$$

Taking advantage of large mass ratios in the quark sector, we reduce these expressions to a simpler form given by the combination of K, V and U matrix elements:

$$\frac{\alpha_s}{18\pi}\frac{\mu\tan\beta}{m}c_2 y_t^2\frac{m_b}{m_d}\mathrm{Im}\sum_{ij}K_{id}K_{td}^*K_{tb}K_{jb}^*(U\frac{S^{diag}}{m^2}U^\dagger)_{ji} \tag{20}$$

$$\frac{\alpha_s}{18\pi}\frac{\mu\cot\beta}{m}c_2 y_b^2\frac{m_t}{m_u}\mathrm{Im}\sum_{ij}K_{ui}^*K_{ub}K_{tb}^*K_{tj}(V\frac{S^{diag}}{m^2}V^\dagger)_{ji} \tag{21}$$

In a general case, allowed by previous constraints (16), $|U_{ij}|$; $|V_{ij}|\sim|K_{ij}|$ and $\epsilon_{ij}\sim 0.01$, the new contributions (20-21) to $\bar\theta$ would violate the experimental bound. To satisfy it, we would have to impose strong restrictions on the splitting in the squark sector, $\epsilon_{ij}<10^{-4}$. There are, however, two important exceptions from this constraint, which we should treat separately.

Case 1: this is when the squarks and Yukawa couplings of down quarks are diagonal in the same basis. This corresponds to $V=\mathbf{1}$ and $U^\dagger=K$ and leads to the vanishing of the expression in eq. (20). The theta term is given by

$$\bar\theta=\frac{\alpha_s}{18\pi}\frac{\mu\cot\beta}{m}c_2 y_b^2 I_{KM}\epsilon_{12}\simeq 10^{-7}\epsilon_{12} \tag{22}$$

As we can see, the value of $\epsilon_{12}=0.005$ is consistent with the theta-constraint if the value of $\tan^\beta\sim O(1)$.

Case 2: this is when the deviation from universality is expressed as the function of traceless bilinear combinations of Yukawa matrices.

$$\mathbf{S}=am^2(\mathbf{Y}_u^2-\frac{1}{3}\mathrm{Tr}\left(\mathbf{Y}_u^2)\right)+bm^2\left(\mathbf{Y}_d^2-\frac{1}{3}\mathrm{Tr}(\mathbf{Y}_d^2)\right) \tag{23}$$

This is the generalization of the model discussed in ref. [26]. Indeed, this form of the mass matrix at the scale Λ can be viewed as the result of the squark universality at the Plank scale, modified by the renormalization group flow *above* the scale Λ. Due to a higher left-right symmetric group, more Higgses are present and this explains the appearance of both $\mathbf{Y}_d$ and $\mathbf{Y}_u$ in Eq. (23). In this case the resulting value of $\bar\theta$ was estimatyed in ref. [26],

$$\bar\theta=I_{KM}\frac{\alpha_s}{90\pi}\frac{(a+c_2)ac_2\mu\tan\beta}{m}y_t^4y_c^2\frac{m_b}{m_d}\sim 10^{-10}\tan\beta \tag{24}$$

This value is within a desirable bound if $\tan\beta$ is not too large.

Next we include the corrections coming from the violation of proportionality in $\mathbf{A}$-matrices. The hermiticity of $\mathbf{A}_{u(d)}$ at the scale Λ is violated at lower scales so that

$$\mathbf{A}_u\to(1+c_4\mathbf{Y}_u^2+c_5\mathbf{Y}_d^2)\mathbf{A}_u(1+c_6\mathbf{Y}_u^2), \tag{25}$$

with the similar transformation for $\mathbf{A}_d$. Insertion of these structures into the squark line leads to the following potentially dangerous corrections to the Yukawa matrices:

$$\mathrm{ImTr}(\mathbf{Y}_d^{-1}\Delta\mathbf{Y}_d) = i\frac{8g_3^2}{3}\int\frac{d^4p}{(2\pi)^4}\,\frac{m_\lambda}{p^2-m_\lambda^2}\,\frac{c_5\mathrm{ImTr}(\mathbf{Y}_d^{-1}\mathbf{Y}_u^2\mathbf{A}_d)}{(p^2-m^2)^3} \tag{26}$$

$$\mathrm{ImTr}(\mathbf{Y}_u^{-1}\Delta\mathbf{Y}_u) = i\frac{8g_3^2}{3}\int\frac{d^4p}{(2\pi)^4}\,\frac{m_\lambda}{p^2-m_\lambda^2}\,\frac{c_5\mathrm{ImTr}(\mathbf{Y}_u^{-1}\mathbf{Y}_d^2\mathbf{A}_u)}{(p^2-m^2)^3} \tag{27}$$

Let us parametrize $\mathbf{A}$-matrices in the basis where the same type (up or down) Yukawa matrices are diagonal:

$$\mathbf{Y}_d = \mathbf{Y}_d^{diag},\;\; \mathbf{A}_d = V_A\mathbf{A}_d^{diag}V_A^\dagger; \quad \mathbf{Y}_u = \mathbf{Y}_u^{diag},\;\; \mathbf{A}_u = U_A\mathbf{A}_u^{diag}U_A^\dagger \tag{28}$$

Then the equations (26-27) provide severe restrictions either on the allowed form of the V_A and U_A matrices or on the magnitude of the eigenvalues of A matrices at the scale Λ. In particular, we find that for $A_b^{diag}\sim my_b$, the allowed values of element 13 of V_A matrix should be of the order $O(10^{-7})$ or smaller. This suggests that the allowed departure from proportionality may occur only at the level of eigenvalues, i.e. $A_b/y_b \neq A_s/y_s \neq A_d/y_d$, while $V_A = U_A = 1$ must be preserved. Another, more radical assumption, would be to take the matrices $\mathbf{A}_u$ and $\mathbf{A}_d$ at the scale Λ to be arbitrary but all entries suppressed to the level of 10^{-7}.

3 Strong CP, $m_u = 0$ and horizontal symmetries

Horizontal symmetries have a potential to explain the hierarchical patterns among the quark masses and mixing angles [35]. Recently there have been some activities in supersymmetric models supplemented by horizontal symmetries. These symmetries might be behind an approximate "alignment" between squark and quark mass matrices [13].

The basic idea in this approach is to relate the smallness of some entries in the Yukawa matrices with certain powers of the order parameter $\lambda = \langle s\rangle/M \ll 1$, characterizing the breaking of the horizontal symmetry. In other words, below the scale M, the superpotential is the sum of different operators, classified by the dimension of λ,

$$W = \sum_{ij} Q_iU_jH_2c_i\left(\frac{\langle s\rangle}{M}\right)^{p_{ij}} + ... \tag{29}$$

Coefficients c_i are of the order 1. Let us assume that the H_2 field is not charged with respect to the horizontal group and that s-field carries unit negative charge, $X_s = -1$. Then the selection rule for p_{ij} can be formulated in the following form:

$$p_{ij} = \begin{cases} X_{Q_i} + X_{U_J}; & \text{for } X_{Q_i} + X_{U_j} \geq 0 \\ 0 & \text{for } X_{Q_i} + X_{U_j} < 0. \end{cases} \tag{30}$$

Thus the holomorphic properties of the superpotential, i.e. the absence of terms like QUH_2s^*, can be used to decouple right-handed u-quark from the rest of the MSSM chiral superfields. In other words, an accidental $detY_u = 0$ can be a natural consequence of horizontal symmetries in the SUSY framework [14], thus reviving an $m_u = 0$ solution to the strong CP problem.

Does this solution withstand radiative corrections to m_u which may arise due to the same SUSY threshold correction, Fig. 1a? To answer this question we have to check whether it is natural to have radiatively induced $m_u(1\text{ GeV}) < 10^{-9} \cdot 5MeV$. As it was pointed out in [13], the selection rule for the squark mass matrix elements, $M^2_{ij} \sim m^2(\langle s\rangle/M)^{q_{ij}}$ is quite different from (30),

$$q_{ij} = |X_{Q_i} - X_{Q_j}|, \;\; |X_{U_i} - X_{U_j}|, \;\; |X_{D_i} - X_{D_j}|. \tag{31}$$

The arguments based on holomorphy do not apply and therefore u-squark cannot be decoupled from the rest of the squarks. This is sufficient to generate possibly small but non-vanishing m_u at the SUSY threshold which turns out to be

$$m_u \simeq \eta \; m_t \; \frac{\alpha_s}{18\pi} \; \frac{A - \mu\cot\beta}{m} \; \lambda^{|X_{Q_1}-X_{Q_3}|+|X_{U_1}-X_{U_3}|}. \tag{32}$$

Here again we take advantage of the possibility to "import" large m_t through the flavour-changing along the squark line. $\eta \sim 2.5$ accounts for the QCD renormalization change of m_u from SUSY threshold to 1 GeV. Assuming that λ is equal to the Wolfenstein's parameter $\lambda_W = 0.22$, we arrive at the following bound on the combination of the horizontal charges in the Up quark sector:

$$|X_{Q_1} - X_{Q_3}| + |X_{U_1} - X_{U_3}| > 17. \tag{33}$$

Is such a hierarchy of horizontal charges natural? At the very least it is another serious model-building problem.

4 Conclusion

The vanishing of the theta term at the tree level might be achieved via an exact left-right symmetry, acting above certain high energy scale Λ. The particular form of the gauge group which permits such a symmetry is not important for the solution of the strong CP problem.

What is crucial, however, is the degree of the universality in the soft-breaking sector which influences the value of the radiatively induced $\bar{\theta}$ term. Assuming the MSSM field structure below a certain scale Λ and exact left-right symmetry above this scale, we studied the allowed departure from the flavour universality in the squark mass sector. We find that the 0.5% mass splitting among squarks at the scale Λ can be consistent with the bounds on $\bar{\theta}$, but only in two very specific cases. The first case corresponds to a situation when the squark mass matrix can be diagonalized in the same basis as the Down-quark Yukawa matrix. The second one is when the departure from the universality is proportional to the combination of Up and Down Yukawa matrices. The latter form of the mass matrix for squarks may result from the renormalization group evolution of the initially universal squark masses between Plank scale and Λ. Thus any model-building effort which tries to explain the smallness of $\bar{\theta}$ by a restoration of parity has to ensure that squark masses fall into one of these two patterns.

We have also shown that a possibility of $m_u = 0$ type of solution to the strong CP problem in the context of horizontal symmetries depends upon the size of M_u generated radiatively at SUSY threshold. The squark-gluino exchange diagram would typically induce a nonzero value for m_u which can be expressed in terms of the differences between horizontal charges of the quark superfields from the first and third generation. Assuming that the order parameter governing the hierarchical structure is of the order of Wolfenstein's λ, we obtain the constraint $|X_{Q_1} - X_{Q_3}| + |X_{U_1} - X_{U_3}| > 17$ which represents a serious model-building challenge.

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