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THE DISTRIBUTION OF TAX PAYMENTS IN A LÉVY INSURANCE RISK MODEL WITH A SURPLUS-DEPENDENT TAXATION STRUCTURE

JEAN-FRANÇOIS RENAUD

ABSTRACT. We study the distribution of tax payments in the Lévy insurance risk model of Kyprianou and Zhou [6], that is a Lévy insurance risk model with a surplus-dependent tax rate. More precisely, after a short discussion on the so-called tax identity, we derive a recursive formula for arbitrary moments of the discounted tax payments until ruin and we identify the distribution of the tax payments when there is no force of interest.

1. Introduction

In a recent paper, Kyprianou and Zhou [6] extended both the models studied by Albrecher, Borst, Boxma and Resing [1] and by Albrecher, Renaud and Zhou [3]. Indeed, they introduced a Lévy insurance risk process with a surplus-dependent tax rate and obtained the following three fundamental results: a solution to the two-sided exit problem, an expression of the expectation of the present value of tax paid until ruin, as well as an expression for a generalized Gerber-Shiu function. In this follow-up paper, we further analyze their model by studying the distribution of the tax payments made over the lifetime of the company.

In their paper, Kyprianou and Zhou [6] have provided elegant proofs of the abovementioned results using Itô's excursion theory. This theory being a rather sophisticated technology, we will use instead a methodology first introduced by Zhou [8] and that can be considered as a Lévy analogue of the *conditioning on the time and size of the first claim technique*, which is widely used in Cramér-Lundberg type models (i.e., renewal risk models). Our contributions consist of a new derivation of the expression for the expectation of the present value of tax paid until ruin, as obtained in [6], and more importantly of a recursive formula for arbitrary moments of those tax payments. We also identify the distribution of the tax payments until ruin when there is no force of interest. Moreover, this paper strenghtens the fact that Zhou's methodology is now a well-established technique for the study

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of Lévy insurance risk models. Finally, as we are using only elementary arguments (assuming the solution to the two-sided exit problem), we hope to bring our results, as well as those in [6], to a wider audience.

2. A Lévy insurance risk process under a general tax structure

Let $X = (X(t))_{t\geq 0}$ be a spectrally negative Lévy process or, in other words, a Lévy process with no positive jumps. The law of X such that $X(0) = u$ will be denoted by $\mathbb{P}_u$ and the corresponding expectation by $\mathbb{E}_u$. We assume that X has differentiable scale functions $W^{(q)}$, which are the unique, strictly increasing and continuous functions $W^{(q)} \colon [0,\infty) \to [0,\infty)$ with Laplace transform

$$\int_0^\infty e^{-\lambda z} W^{(q)}(z)\, dz = \frac{1}{\psi(\lambda) - q},$$

for $\lambda > \Phi(q)$. Here, ψ is the Laplace exponent of X given through

$$\mathbb{E}_0\left[e^{\lambda X(t)}\right] = e^{t\psi(\lambda)}$$

for $\lambda \geq 0$ and $t \geq 0$, and, since it is strictly convex and such that $\lim_{\lambda\to\infty} \psi(\lambda) = \infty$, there exists a function $\Phi \colon [0,\infty) \to [0,\infty)$ given by

$$\psi(\Phi(\lambda)) = \lambda, \lambda \quad \geq 0.$$

We write $W = W^{(0)}$ when $q = 0$. Note that the assumption on the differentiability of the scale functions is satisfied for most Lévy insurance risk processes. For more details on Lévy processes and scale functions, the reader is refered to [5].

Let $S = (S(t))_{t\geq 0}$ denote the running maximum of X, i.e.,

$$S(t) = \sup_{0\leq s\leq t} X(s).$$

For a function $\gamma \colon \mathbb{R}_+ \to [0,1)$, which stands for the surplus-dependent tax rate, Kyprianou and Zhou [6] defined the taxed process $U_\gamma = (U_\gamma(t))_{t\geq 0}$ as follows:

$$U_\gamma(t) = X(t) - \int_0^t \gamma(S(u))\, dS(u), \tag{1}$$

for $t \geq 0$, i.e., a process with the following dynamic:

$$dU_\gamma(t) = dX(t) - \gamma(S(t))\, dS(t).$$

The process U_γ indeed models the surplus process of an insurance company that pays out taxes according to a loss-carried-forward tax scheme, using a surplus-dependent rate $\gamma(\cdot)$. In other words, tax are collected when the company has *recovered* from its previous losses, i.e., is in a so-called profitable situation.

Finally, note that when $\gamma(\cdot) \equiv \gamma \in [0,1)$, this model amounts to the situation studied in [3] where the tax rate is constant, and when $\gamma(\cdot) \equiv 1$, we retrieve the model where the company pays out as dividends any capital above its initial value u as in a risk model with an horizontal barrier strategy (see e.g. [7]).

For practical purposes, one would expect the function γ to be increasing; however, this condition is not needed for the forthcoming analysis. On the other hand, one should emphasize the fact that the range of the function γ must lie in $[0, 1)$: it would be rather unrealistic to have a tax rate greater than the rate of increase of the company's surplus. Mathematically speaking, this means that the function

$$\bar{\gamma}_u(x) = x - \int_u^x \gamma(y)\, dy, \tag{2}$$

defined for $x \geq u$, must be a strictly increasing function since at an increase time t, we have $X(t) = S(t)$ (see [6], in particular Lemma 1). Hence, we want $\bar{\gamma}'_u(x) > 0$, or equivalently, $\gamma(x) < 1$, for all x. Note that in this case $\bar{\gamma}_u^{-1}$ is well-defined.

2.1. **The tax identity.** One of the main results derived in [6] is the solution to the two-sided exit problem. Let $q \geq 0$ and define the following two stopping times: $\tau_a^+ = \inf\{t > 0 \colon U_\gamma(t) = a\}$ and $\tau_0^- = \inf\{t > 0 \colon U_\gamma(t) < 0\}$. Then, if $0 < u < a$, we have that the *discounted* probability of hitting a (the process being skip-free upward) before getting ruined is given by

$$\mathbb{E}_u\left[e^{-q\tau_a^+}\mathbb{I}_{\{\tau_a^+<\tau_0^-\}}\right] = \exp\left\{-\int_u^a \left(1-\gamma(\bar{\gamma}_u^{-1}(s))\right)^{-1}\frac{W^{(q)\prime}(s)}{W^{(q)}(s)}\, ds\right\}, \tag{3}$$

where $\bar{\gamma}_u$ is the function defined in Equation (2). First note that, if $q = 0$, then $\mathbb{E}_u\left[e^{-q\tau_a^+}\mathbb{I}_{\{\tau_a^+<\tau_0^-\}}\right] = \mathbb{P}_u\{\tau_a^+ < \tau_0^-\}$ is the *genuine* probability that the surplus process U_γ will reach level a before getting ruined.

A direct application of this result gives us the probability of ruin in this model with a surplus-dependent taxation scheme. Indeed, let

$$\phi_\gamma(u) = \mathbb{P}_u\left\{\inf_{t\geq 0} U_\gamma(t) \geq 0\right\}$$

denote the survival probability when the initial surplus is u, meaning that $\phi_0(u)$ is the survival probability in the risk model without taxation. If X satisfies the net profit condition, which in our setup amounts to $\psi'(0+) > 0$, it is known that $\phi_0(u) = W(u)/W(\infty)$, where $W(\infty) := \lim_{x\to\infty} W(x)$ (see [5]). Thus,

$$\frac{d}{du}\ln(\phi_0(u)) = \frac{W'(u)}{W(u)}.$$

Consequently, as τ_0^- is the time of ruin in this risk model, taking the limit when a goes to infinity in Equation (3) yields the following relationship between the survival probabilities of the risk models with and without taxation:

$$\phi_\gamma(u) = \exp\left\{-\int_u^\infty \left(1-\gamma(\bar{\gamma}_u^{-1}(s))\right)^{-1}\frac{d}{ds}\ln(\phi_0(s))\, ds\right\}.$$

This is an extension of the so-called tax identity (see [1], [2] and [3]). Even though this result was not explicitely stated in [6], one clearly sees that this is a direct consequence of Equation (3).

Remark 2.1. *The model defined through Equation* (1) *has a taxation rate depending on the underlying risk process X, not on the taxed process U_γ. Therefore, the use of the function $\bar{\gamma}_u^{-1}$ in the results takes care of this change of scaling. Note that the rate used in [1] depends on the taxed process.*

3. The distribution of tax payments

Within the model introduced in Equation (1), the process

$$t \mapsto \int_0^t e^{-\delta s} \gamma(S(s))\, dS(s)$$

represents the cumulative discounted (at the force of interest $\delta \geq 0$) tax payments made until time t. In [6], the following result was derived:

(4)
$$\mathbb{E}_u \left[\int_0^{\tau_0^-} e^{-\delta s} \gamma(S(s))\, dS(s) \right] = \int_u^\infty \exp\left\{ - \int_u^t \frac{W^{(\delta)\prime}(\bar{\gamma}_u(s))}{W^{(\delta)}(\bar{\gamma}_u(s))}\, ds \right\} \gamma(t)\, dt.$$

3.1. **Moments of the discounted tax payments.** Let the discounted amount of tax payments made until ruin be denoted by

$$T_{\gamma,\delta} = \int_0^{\tau_0^-} e^{-\delta s} \gamma(S(s))\, dS(s)$$

and, for an integer $k \geq 0$, define

$$v_{\gamma,\delta}^{(k)}(u) = \mathbb{E}_u \left[(T_{\gamma,\delta})^k \right].$$

Clearly, we have $v_{\gamma,\delta}^{(0)}(u) = 1$ and, to lighten the notation , we set $v_{\gamma,\delta}(u) = v_{\gamma,\delta}^{(1)}(u)$.

Theorem 3.1. *If $\delta > 0$, then*

(5)
$$v_{\gamma,\delta}(u) = \int_u^\infty \exp\left\{ - \int_u^s \frac{W^{(\delta)\prime}(t)}{W^{(\delta)}(t) \left(1 - \gamma(\bar{\gamma}_u^{-1}(t))\right)}\, dt \right\} \frac{\gamma(\bar{\gamma}_u^{-1}(s))}{1 - \gamma(\bar{\gamma}_u^{-1}(s))}\, ds.$$

Proof. We first collect intermediate results. Define $\tau_n := \tau_{u+1/n}^+ = \inf\{t > 0 \colon U_\gamma(t) = u + 1/n\}$. Note that if $X(0) = u$, then on the event $\{\tau_n < \infty\}$, we have that

$$\int_0^{\tau_n} \gamma(S(s))\, dS(s) = \bar{\gamma}_u^{-1}(u + 1/n) - u - \frac{1}{n}. \tag{6}$$

This is the amount of tax paid until the taxed surplus process reaches $u+1/n$. Also, we have that

$$\lim_{n\to\infty} \frac{\bar{\gamma}_u^{-1}(u+1/n) - u - \frac{1}{n}}{1/n} = \left(\bar{\gamma}_u^{-1}(u)\right)' - 1 = \frac{\gamma(\bar{\gamma}_u^{-1}(u))}{1-\gamma(\bar{\gamma}_u^{-1}(u))}. \tag{7}$$

One has to be careful with the latter differential quotient and resist the temptation of using the fact that $\bar{\gamma}_u(u) = u$; the index u in the expression of the function $\bar{\gamma}_u^{-1}$ is fixed and must be seen as a parameter.

Also, from Equation (3), it is clear that

$$\lim_{n\to\infty} \mathbb{E}_u \left[e^{-\delta\tau_n} \mathbb{I}_{\{\tau_n < \tau_0^-\}} \right] = 1. \tag{8}$$

Finally, using the Lebesgue-Stieltjes integration by parts formula (see Theorem 18.4 in Billingsley [4]), we get

$$\begin{aligned} \int_0^t e^{-\delta s} \gamma(S(s))\, dS_s \\ = e^{-\delta t} \int_0^t \gamma(S(s))\, dS_s + \delta \int_0^t e^{-\delta s} \left(\int_0^s \gamma(S(r))\, dS_r \right) ds. \end{aligned} \tag{9}$$

The rest of the proof follows the proof of Theorem 3.2 in [3], adapting the arguments to take into account that the tax rate is surplus-dependent. Considering whether $\tau_n < \tau_0^-$ or $\tau_n > \tau_0^-$, one can write

$$\begin{aligned} v_{\gamma,\delta}(u) &= \mathbb{E}_u \left[\int_0^{\tau_n} e^{-\delta s} \gamma(S(s))\, dS(s)\, \mathbb{I}_{\{\tau_n < \tau_0^-\}} \right] \\ &\quad + \mathbb{E}_u \left[\int_{\tau_n}^{\tau_0^-} e^{-\delta s} \gamma(S(s))\, dS(s)\, \mathbb{I}_{\{\tau_n < \tau_0^-\}} \right] + o(1/n). \end{aligned}$$

Intuitively, for large values of n, if $\tau_n > \tau_0^-$, almost no taxes will be paid before ruin.

We treat the other two expectations one at the time. Using Equation (9) and Equation (6), one gets

$$\begin{aligned} &\mathbb{E}_u \left[\int_0^{\tau_n} e^{-\delta s} \gamma(S(s))\, dS(s)\, \mathbb{I}_{\{\tau_n < \tau_0^-\}} \right] \\ &\quad = \left(\bar{\gamma}_u^{-1}(u+1/n) - u - \frac{1}{n} \right) \mathbb{E}_u \left[e^{-\delta\tau_n} \mathbb{I}_{\{\tau_n < \tau_0^-\}} \right] \\ &\qquad + \mathbb{E}_u \left[\delta \int_0^{\tau_n} e^{-\delta s} \int_0^s \gamma(S(r))\, dS_r\, ds\, \mathbb{I}_{\{\tau_n < \tau_0^-\}} \right], \end{aligned}$$

where

$$\begin{aligned} &\mathbb{E}_u \left[\delta \int_0^{\tau_n} e^{-\delta s} \int_0^s \gamma(S(r))\, dS_r\, ds\, \mathbb{I}_{\{\tau_n < \tau_0^-\}} \right] \\ &\quad \le \left(\bar{\gamma}_u^{-1}(u+1/n) - u - \frac{1}{n} \right) \mathbb{E}_u \left[\left(1 - e^{-\delta\tau_n} \right) \mathbb{I}_{\{\tau_n < \tau_0^-\}} \right]. \end{aligned}$$

Using Equation (7) and Equation (8), we have that

$$\mathbb{E}_u\left[\delta\int_0^{\tau_n} e^{-\delta s}\int_0^s \gamma(S(r))\,dS_r\,ds\,\mathbb{I}_{\{\tau_n<\tau_0^-\}}\right] = o(1/n) \tag{10}$$

and hence

$$\begin{aligned}\mathbb{E}_u&\left[\int_0^{\tau_n} e^{-\delta s}\gamma(S(s))\,dS(s)\,\mathbb{I}_{\{\tau_n<\tau_0^-\}}\right]\\ &= \left(\bar{\gamma}_u^{-1}(u+1/n)-u-\frac{1}{n}\right)\mathbb{E}_u\left[e^{-\delta\tau_n}\mathbb{I}_{\{\tau_n<\tau_0^-\}}\right]+o(1/n).\end{aligned}$$

On the other hand, using the strong Markov property, we get

$$\mathbb{E}_u\left[\int_{\tau_n}^{\tau_0^-} e^{-\delta s}\gamma(S(s))\,dS(s)\,\mathbb{I}_{\{\tau_n<\tau_0^-\}}\right] = \mathbb{E}_u\left[e^{-\delta\tau_n}\mathbb{I}_{\{\tau_n<\tau_0^-\}}\right]v_{\gamma,\delta}(u+1/n).$$

In other words, we start collecting taxes only at time τ_n, at which time the surplus process will be at level $u+1/n$, but we must take into account the *discounted* probability of reaching that level.

Finally, putting all of the above together, we obtain

$$\begin{aligned}v_{\gamma,\delta}(u) &= o(1/n)\\ &+\mathbb{E}_u\left[e^{-\delta\tau_n}\mathbb{I}_{\{\tau_n<\tau_0^-\}}\right]\left(v_{\gamma,\delta}(u+1/n)+\left(\bar{\gamma}_u^{-1}(u+1/n)-u-\frac{1}{n}\right)\right).\end{aligned}$$

Using Equation (7) once again, we get the following differential equation

$$v'_{\gamma,\delta}(u) = \frac{W^{(\delta)\prime}(u)}{W^{(\delta)}(u)(1-\gamma(\bar{\gamma}_u^{-1}(u)))}v_{\gamma,\delta}(u)-\frac{\gamma(\bar{\gamma}_u^{-1}(u))}{1-\gamma(\bar{\gamma}_u^{-1}(u))}.$$

Its solution is then given by

$$v_{\gamma,\delta}(u) = \int_u^\infty \exp\left\{-\int_u^s \frac{W^{(\delta)\prime}(t)}{W^{(\delta)}(t)\left(1-\gamma(\bar{\gamma}_u^{-1}(t))\right)}\,dt\right\}\frac{\gamma(\bar{\gamma}_u^{-1}(s))}{1-\gamma(\bar{\gamma}_u^{-1}(s))}\,ds.$$

□

Note that changing variables twice (letting $z=\bar{\gamma}_u(x)$), we get

$$\begin{aligned}\int_u^\infty &\exp\left\{-\int_u^s \frac{W^{(\delta)\prime}(t)}{W^{(\delta)}(t)\left(1-\gamma(\bar{\gamma}_u^{-1}(t))\right)}\,dt\right\}\frac{\gamma(\bar{\gamma}_u^{-1}(s))}{1-\gamma(\bar{\gamma}_u^{-1}(s))}\,ds\\ &= \int_u^\infty \exp\left\{-\int_u^t \frac{W^{(\delta)\prime}(\bar{\gamma}_u(s))}{W^{(\delta)}(\bar{\gamma}_u(s))}\,ds\right\}\gamma(t)\,dt,\end{aligned}$$

therefore recovering Equation (4).

We now look at the higher moments of $T_{\gamma,\delta}$. Note that the previous result/proof was presented separately from the next one for the sake of clarity only; it is clearly embedded in the following:

Theorem 3.2. *If $\delta > 0$, then the k-th moment of the present value of tax payments made until ruin can be expressed in term of the $(k-1)$-th moment:*

$$v_{\gamma,\delta}^{(k)}(u) = k \int_u^\infty \exp\left\{ - \int_u^s \frac{W^{(k\delta)\prime}(t)}{W^{(k\delta)}(t)\left(1-\gamma(\bar{\gamma}_u^{-1}(t))\right)}\,dt \right\} \times v_{\gamma,\delta}^{(k-1)}(s) \frac{\gamma(\bar{\gamma}_u^{-1}(s))}{1-\gamma(\bar{\gamma}_u^{-1}(s))}\,ds.$$

Proof. After having used the Binomial Theorem twice to expand powers (as in [7]), we proceed as in the proof of Theorem 5 and, in particular, we use the *intermediate results* and as well as the estimates to identify terms of order $o(1/n)$ (e.g., Equation (10)). For example, for an integer $j \geq 2$, using Equation (7), we clearly have

$$\left(\bar{\gamma}_u^{-1}(u+1/n) - u - \frac{1}{n} \right)^j = o(1/n).$$

Consequently, one obtains

$$v_{\gamma,\delta}^{(k)}(u) = o(1/n) + \mathbb{E}_u \left[e^{-k\delta\tau_n} \mathbb{I}_{\{\tau_n < \tau_0^-\}} \right] \times \left(v_{\gamma,\delta}^{(k)}(u+1/n) + k v_{\gamma,\delta}^{(k-1)}(u+1/n) \left(\bar{\gamma}_u^{-1}(u+1/n) - u - \frac{1}{n} \right) \right).$$

Then, we get that

$$v_{\gamma,\delta}^{(k)\prime}(u) = \frac{W^{(k\delta)\prime}(u)}{W^{(k\delta)}(u)(1-\gamma(\bar{\gamma}_u^{-1}(u)))} v_{\gamma,\delta}^{(k)}(u) - k v_{\gamma,\delta}^{(k-1)}(u) \frac{\gamma(\bar{\gamma}_u^{-1}(u))}{1-\gamma(\bar{\gamma}_u^{-1}(u))}.$$

Solving this ordinary differential equation leads to

$$v_{\gamma,\delta}^{(k)}(u) = k \int_u^\infty \exp\left\{ - \int_u^s \frac{W^{(k\delta)\prime}(t)}{W^{(k\delta)}(t)\left(1-\gamma(\bar{\gamma}_u^{-1}(t))\right)}\,dt \right\} \times v_{\gamma,\delta}^{(k-1)}(s) \frac{\gamma(\bar{\gamma}_u^{-1}(s))}{1-\gamma(\bar{\gamma}_u^{-1}(s))}\,ds.$$

□

When $\gamma(\cdot) \equiv \gamma \in [0,1)$, and up to some elementary algebraic manipulations, these expressions for the moments agree with those in [3].

Finally, note that both the proofs of Theorem 3.1 and Theorem 3.2 used the fact that $\delta > 0$.

3.2. **Tax payments with no force of interest.** In [6], the expected discounted tax payments until ruin are computed even if there is no force of interest, i.e., if $\delta = 0$. We now consider this case.

First, define $\alpha_u(x) = \int_u^x \gamma(y)\,dy$ for $x \geq u$. Note that if $\gamma(y) > 0$ for all y, then α_u is strictly increasing and, therefore, α_u^{-1} is well-defined.

Proposition 3.1. *If the range of the tax rate function γ lies in $(0,1)$, then, for all $x \geq 0$,*

$$\mathbb{P}_u \left\{ \int_0^{\tau_0^-} \gamma(S_s)\, dS_s \geq x \right\} = \exp\left\{ - \int_u^{\bar{\gamma}_u(\alpha_u^{-1}(x))} \frac{W'(s)}{W(s)\left(1-\gamma(\bar{\gamma}_u^{-1}(s))\right)}\, ds \right\}$$

and, consequently, Equation (4) *holds for* $\delta = 0$.

Proof. Note that the sum of tax payments first reaches level x when X is at a new maximum at level $\alpha_u^{-1}(x)$, which means that $\bar{\gamma}_u\left(\alpha_u^{-1}(x)\right)$ represents the level of the corresponding new maximum attained by U_γ (see Lemma 1 in [6]). Consequently,

$$\left\{ \int_0^{\tau_0^-} \gamma(S_s)\, dS_s \geq x \right\} = \left\{ \tau^+_{\bar{\gamma}_u\left(\alpha_u^{-1}(x)\right)} < \tau_0^- \right\}$$

and, by the solution of the two-sided exit problem given in Equation (3), we have

$$\mathbb{P}_u \left\{ \int_0^{\tau_0^-} \gamma(S_s)\, dS_s \geq x \right\} = \exp\left\{ - \int_u^{\bar{\gamma}_u(\alpha_u^{-1}(x))} \frac{W'(s)}{W(s)\left(1-\gamma(\bar{\gamma}_u^{-1}(s))\right)}\, ds \right\}.$$

Henceforth, we compute the expectation of the tax payments as follows:

$$\begin{aligned}
\mathbb{E}_u \left[\int_0^{\tau_0^-} \gamma(S_s)\, dS_s \right] &= \int_0^\infty \exp\left\{ - \int_u^{\bar{\gamma}_u(\alpha_u^{-1}(x))} \frac{W'(s)}{W(s)\left(1-\gamma(\bar{\gamma}_u^{-1}(s))\right)}\, ds \right\} dx \\
&= \int_u^\infty \exp\left\{ - \int_u^{\bar{\gamma}_u(\alpha_u^{-1}(x-u))} \frac{W'(s)}{W(s)\left(1-\gamma(\bar{\gamma}_u^{-1}(s))\right)}\, ds \right\} dx \\
&= \int_u^\infty \exp\left\{ - \int_u^{\alpha_u^{-1}(x-u)} \frac{W'(\bar{\gamma}_u(s))}{W(\bar{\gamma}_u(s))}\, ds \right\} dx \\
&= \int_u^\infty \exp\left\{ - \int_u^{x} \frac{W^{(\delta)\prime}(\bar{\gamma}_u(s))}{W^{(\delta)}(\bar{\gamma}_u(s))}\, ds \right\} \gamma(x)\, dx,
\end{aligned}$$

where we have changed variables first in the inside integral (letting $t = \bar{\gamma}_u^{-1}(s)$) and then in the outside one (letting $t = \alpha_u^{-1}(x-u)$). $\square$

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Department of Statistics and Actuarial Science, University of Waterloo, 200 University Avenue West, Waterloo (Ontario) N2L 3G1, Canada
E-mail address: jf2renaud@math.uwaterloo.ca