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Up-down unification just above the supersymmetric threshold

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Abstract. Large corrections to the quark mass matrices at the supersymmetric threshold allow the theory to have identical Yukawa matrices in the superpotential. We demonstrate that up–down unification can take place in a moderate quark–squark alignment scenario with an average squark mass of the order 1 TeV and with $\tan\beta > 15$.

1 Introduction

The observed hierarchical patterns in the masses of quarks and leptons and in the Kobayashi–Maskawa mixing suggest the existence of new physics beyond the standard model, perhaps in the form of new symmetries. Neither the character of these symmetries nor the scale of the new physics is understood so far. Various theoretical attempts have been made to construct a realistic explanation for the observed mass spectrum and reproduce experimentally observed values as closely as possible. Some of them exploit horizontal symmetries [1,2], gauged or nongauged, anomalous symmetries [3], possible compositeness [4], etc.

The mass problem in supersymmetry has attracted serious theoretical attention in the last few years. Although very helpful for solving some of the shortcomings of the standard model, the supersymmetric framework by itself does not explain the observed patterns displayed by fermion masses. Yet it brings new phenomena in flavor physics related to the possible mismatch between quark and squark mass matrices. This mismatch, if significant, may induce an unacceptable contribution to the neutral kaon mixing and therefore is constrained if the masses of the scalar quarks are not very far from the electroweak scale.

Another theoretical goal, closely related to the mass problem, is the possibility to reduce the number of the free parameters in the fundamental theory in comparison with that of the standard model. The supersymmetric grand unified theories (GUTs) offer this possibility [5]. The scale of the unification for Yukawa couplings in this case is believed to coincide with the scale of the gauge unification and therefore is very high, 10^{16} GeV or so.

In this article, we ask how low the scale of the flavor unification could be. We attempt to construct a model with a minimal number of free parameters in the superpotential. This is a model with identical up and down quark Yukawa matrices in the superpotential and the general form of the soft-breaking parameters, not constrained by the usual assumption of universality and proportionality. The Kobayashi–Maskawa mixing itself in this situation originates from the supersymmetric threshold corrections. We show that at the present stage of knowledge about supersymmetry breaking parameters this possibility is not excluded.

There are several possible physical motivations related to the up-down (U–D) unification. For example, it can be a consequence of a horizontal symmetry responsible for the generation of the Yukawa couplings which "does not feel hypercharge", i.e. which does not distinguish between H_1 and H_2, U and D superfields and therefore can only generate identical Y_u and Y_d. Another example is the case of the supersymmetric left–right theory based on the $\mathrm{SU}(3)\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{SU}(2)_\mathrm{R}\times U(1)_{B-L}$ group [6] where the unification of U and D right-handed superfields is protected by an extra gauge symmetry.

2 U–D unification at low energy

The standard superpotential of the minimal supersymmetric standard model (MSSM):

$$W = \epsilon_{ij}[-Q^I H_2^j \mathbf{Y}_\mathrm{u} U + Q^I H_1^j \mathbf{Y}_\mathrm{d} D + L^I H_1^j \mathbf{Y}_e E + \mu H_1^I \mathrm{H}_2^j], \tag{1}$$

contains the same number of free dimensionless parameters as the Yukawa sector of the standard model.

In addition, in the soft-breaking sector there are other couplings which have a potential influence on flavor physics. Among different scalar masses, the soft-breaking sector has the squark mass terms

$$\tilde{U}^\dagger \mathbf{M}_U^2 \tilde{U} + \tilde{D}^\dagger \mathbf{M}_D^2 \tilde{D} + \tilde{Q}^\dagger \mathbf{M}_Q^2 \tilde{Q}; \tag{2}$$

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and the trilinear terms

$$\epsilon_{ij}\left(-\tilde{Q}^i H_2^j \mathbf{A}_{\mathrm{u}}\tilde{U} + \tilde{Q}^i H_1^j \mathbf{A}_{\mathrm{d}}\tilde{D}\right) + h.c.; \tag{3}$$

as the possible sources of flavor transitions.

Counting all free parameters in the model, one comes to a huge number, 105 [7]. This enormous number of free parameters originates mainly from the soft-breaking sector and cannot be reduced *a priori*, without knowledge of the ways the supersymmetry breaking occurs. It is customary to assume, at the scale of the breaking, that the following, very restrictive conditions are fulfilled:

$$\mathbf{M}_Q^2 = m_Q^2\mathbf{1}; \quad \mathbf{M}_D^2 = m_D^2\mathbf{1}; \quad \mathbf{M}_U^2 = m_U^2\mathbf{1} \quad \text{"degeneracy"} \tag{4}$$

$$\mathbf{A}_{\mathrm{u}} = A_{\mathrm{u}}\mathbf{Y}_{\mathrm{u}}; \quad \mathbf{A}_{\mathrm{d}} = A_{\mathrm{d}}\mathbf{Y}_{\mathrm{d}} \quad \text{"proportionality"}, \tag{5}$$

and similarly for leptons. These conditions, if held, ensure that the physics of flavor comes entirely from the superpotential. But it might not necessarily be the case. For example, these conditions are not held in superstring inspired models (see [8] and references therein). Neither are they, in the simplest flavor models, operating with horizontal symmetries [9].

To this end, it is interesting to abandon strict conditions (4) –(5) in the soft-breaking sector and to explore the possibility of having a smaller number of free parameters in the superpotential. As an ultimate example of this, let us analyze the theory with the low-energy unification of up and down Yukawa matrices. Instead of working with the superpotential in the form (1) with two independent matrices $\mathbf{Y}_{\mathrm{u}}$ and $\mathbf{Y}_{\mathrm{d}}$, we consider the theory with $\mathbf{Y}_{\mathrm{u}} \equiv \mathbf{Y}_{\mathrm{d}}$ so that the superpotenial can be written in the following compact form:

$$W = \epsilon^{ij}\epsilon^{kl}[Q^i \Phi^{jk}\mathbf{Y}Q_{\mathrm{R}}^l + \frac{1}{2}\mu\Phi^{jk}\Phi^{il}] \tag{6}$$

where $Q_{\mathrm{R}} = (U, D)^T$ and $\Phi = (H_1; H_2)$. From here on we consistently omit the leptonic part. At the same time we assume similar U–D unification in the soft-breaking sector as well:

$$\begin{aligned}\ldots &+ \tilde{Q}^\dagger \mathbf{M}_{\mathrm{L}}^2\tilde{Q} + \tilde{Q}_{\mathrm{R}}^\dagger \mathbf{M}_{\mathrm{R}}^2\tilde{Q}_{\mathrm{R}} \\ &+\epsilon^{ij}\epsilon^{kl}\tilde{Q}^i\Phi^{jk}\mathbf{A}\tilde{Q}_{\mathrm{R}}^l + H.c\ldots.\end{aligned} \tag{7}$$

At the scale of the breaking, matrix $\mathbf{Y}$ can be chosen in the diagonal form and it will not develop any off-diagonal elements in the course of renormalization down to the supersymmetric threshold scale. This means that, at the tree level plus logarithmic renormalization, $M_{\mathrm{u}} \sim M_{\mathrm{d}}$ and all Kobayashi–Maskawa mixing angles are zero. This might be considered, with a certain degree of optimism, as a good zeroth-order approximation to the realistic mass matrices and mixing. In this case the observed mixing angles and masses come from the supersymmetric threshold corrections. These corrections induce additional terms in the Yukawa interaction containing the conjugated Higgs fields, H_1^*, H_2^*, and in our case Φ^*. As a result, below the threshold, the effective interaction of fermions with Higgs doublets can be written in the form

$$\mathcal{L}_{\mathrm{eff}} = \bar{Q}\mathbf{Y}_1\tau_2\Phi\tau_2 Q_{\mathrm{R}} + \bar{Q}\mathbf{Y}_2\Phi^* Q_{\mathrm{R}} + \ldots. \tag{8}$$

The ellipsis stands here for the possible terms of bigger dimension which may also influence the fermion spectrum (for example, $M_{\mathrm{sq}}^{-2}\bar{Q}_{\mathrm{R}}\mathbf{Y}_1\tau_2\Phi\tau_2 Q\mathrm{Tr}(\Phi^\dagger\Phi)$). In Fig. 1 we list the diagrams which contribute to the matrices $\mathbf{Y}_1$ and $\mathbf{Y}_2$.

To some extent, one can view this unification as being inspired by supersymmetric left–right models where the relevant part of the superpotential is usually written in the following form:

$$\begin{aligned}W &= Q\mathbf{Y}_1\tau_2\Phi_1\tau_2 Q_{\mathrm{R}} + Q\mathbf{Y}_2\tau_2\Phi_2\tau_2 Q_{\mathrm{R}} \\ &+ \sum_{i,j=1,2}\mu_{ij}\mathrm{Tr}\tau_2\Phi_i\tau_2\Phi_j^T.\end{aligned} \tag{9}$$

If the analysis of the threshold correction can generate realistic $\mathbf{M}_{\mathrm{u}}$ and $\mathbf{M}_{\mathrm{d}}$ then one can eliminate one of the bi-doublets, reducing (9) to the more economic form (6).

The supersymmetric threshold corrections to the mass matrices and mixing have been considered before in a number of papers [10]. In most of these analyses, the conditions (4)–(5) were imposed on the soft-breaking sector. In some specific variants of the unified models [11], however, the departure from these conditions can reproduce masses of first generation and the Cabbibo angle. For a recent discussion of the radiative mechanism for fermion masses in the presence of chiral symmetry violating soft-breaking terms see also [12].

The typical size of the corrections to the mass of the b-quark when the mass of the gluino is equal to the masses of the squarks is of the order of m_{b} itself, $\Delta m_{\mathrm{b}}/m_{\mathrm{b}} \sim 0.4\mu/m_{\mathrm{squark}}$. The ratio μ/m_{squark} is presumably of the order 1 but can be larger. For $m_{\mathrm{squark}} \sim 1\mathrm{TeV}$, μ/m_{sq} can be as large as $\sqrt{2}m_{\mathrm{sq}}/v \sim 5$. In this case m_{b}, as well as the other quark masses and mixing angles, can be completely of radiative origin. In the following we consider the possibility of the low-energy unification of Yukawa couplings, not confining our analysis to the case of $\mu \simeq m_{\mathrm{squark}}$ and not specifying the details of a particular model which lies behind the origin of hierarchy. In particular, we have to check if the following three conditions are satisfied:

1. The matrix $\mathbf{A}$ is consistent with scale-independent constraints resulting from the absence of color-breaking minima.
2. Radiatively generated masses and mixing angles correspond to the observed hierarchy.
3. The predictions for FCNC are acceptable.

3 Phenomenological consequences of U–D unification

It is convenient to choose the basis in which matrix $\mathbf{Y}$ is diagonal. In this case, superpotential contains only three

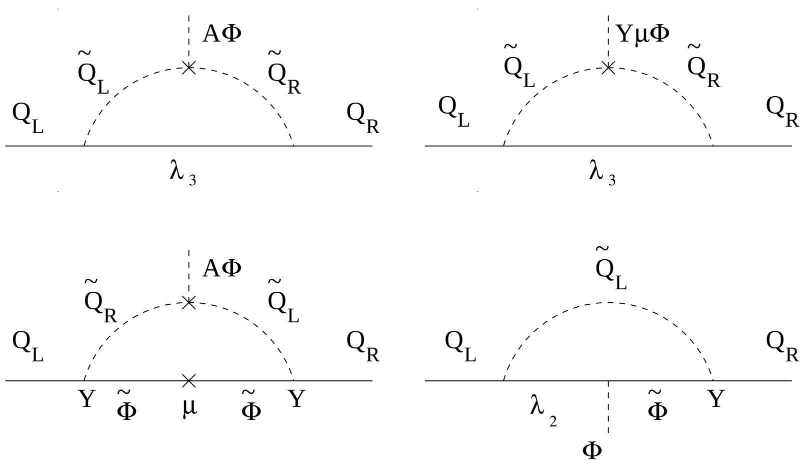

Fig. 1. The diagrams generating threshold corrections for $\mathbf{M}_{\rm u}$ and $\mathbf{M}_{\rm d}$.

dimensionless parameters in the quark sector

$$\mathbf{Y} = \mathrm{diag}(y_1;\, y_2;\, y_3) \tag{10}$$

and y_3 basically coincides with the top quark Yukawa coupling, $y_3 \simeq m_{\rm t}\sqrt{2}/v \simeq 1$.

The value of $\tan\beta \equiv v_{\rm u}/v_{\rm d}$ has to be large but it is not fixed to $m_{\rm t}/m_{\rm b}$ due to the substantial renormalization of $m_{\rm b}$. Therefore, to $\mathrm{O}(\tan^{-1}\beta)$ accuracy we can adopt the following approximation for the squark mass matrices:

$$M^2_{\tilde u} = \begin{pmatrix} \mathbf{M}^2_{\rm L} + \frac{v^2}{2}\mathbf{Y}^\dagger\mathbf{Y} & \frac{v}{\sqrt{2}}\mathbf{A}^\dagger \\ \frac{v}{\sqrt{2}}\mathbf{A} & \mathbf{M}^2_{\rm R} + \frac{v^2}{2}\mathbf{Y}\mathbf{Y}^\dagger \end{pmatrix} \tag{11}$$

$$M^2_{\tilde d} = \begin{pmatrix} \mathbf{M}^2_{\rm L} + \frac{v^2}{2}\mathbf{Y}^\dagger\mathbf{Y} & \frac{v}{\sqrt{2}}\mu^*\mathbf{Y}^\dagger \\ \frac{v}{\sqrt{2}}\mu\mathbf{Y} & \mathbf{M}^2_{\rm R} + \frac{v^2}{2}\mathbf{Y}\mathbf{Y}^\dagger \end{pmatrix} \tag{12}$$

The general formulae for the mass matrices, tree-level plus radiative corrections, can be presented in the following form:

$$\begin{aligned} \frac{\sqrt{2}}{v}\mathbf{M}_{\mathbf{u}ij} &= \mathbf{Y}_{ij} + \frac{2\alpha_{\rm s} m_\lambda}{3\pi}\int \frac{d^4p}{(2\pi)^4(p^2-m^2_\lambda)} \\ &\quad\times \left[\frac{1}{p^2-\mathbf{M}^2_{\rm L}}\mathbf{A}\frac{1}{p^2-\mathbf{M}^2_{\rm R}}\right]_{ij} + \ldots . \\ \frac{\sqrt{2}}{v}\mathbf{M}_{\mathbf{d}ij} &= \frac{\mathbf{Y}_{ij}}{\tan\beta} + \frac{2\alpha_{\rm s}\mu m_\lambda}{3\pi}\int \frac{\mathrm{d}^4p}{(2\pi)^4(p^2-m^2_\lambda)} \\ &\quad\times \left[\frac{1}{p^2-\mathbf{M}^2_{\rm L}}\mathbf{Y}\frac{1}{p^2-\mathbf{M}^2_{\rm R}}\right]_{ij} + \ldots . \end{aligned} \tag{13}$$

Here, ellipses stand for the chargino and neutralino corrections and next- order corrections in v^2/m^2 which we neglect at the moment.

The form of the matrix $\mathbf{A}$ suggested by the absence of unwanted directions in the field space along which color-breaking minima can emerge is the following [13]:

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & A_{13} \\ 0 & 0 & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix} \tag{14}$$

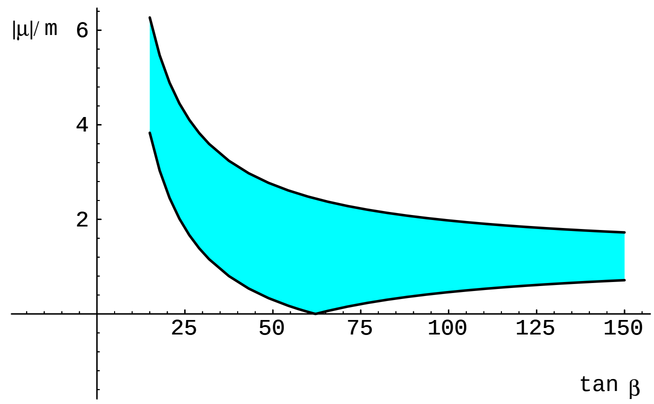

Fig. 2. Allowed area on the $|\mu| - \tan\beta$ plane in the case of maximal threshold corrections.

In this way it can easily satisfy the required bound:

$$|A_{ij}|^2 \le |y_k|^2\left((\mathbf{M}^2_{\rm L})_{ii} + (\mathbf{M}^2_{\rm R})_{jj}\right), \quad k = \max(i,j). \tag{15}$$

The same form is suggested by the analysis of the FCNC processes [14]. Restricting our analysis only to this form of matrix $\mathbf{A}$, we satisfy the first condition on the absence of "wrong" vacua mentioned in the previous section.

Let us consider the corrections to the mass of the b-quark. Neglecting for a moment all flavor-changing effects in the squark sector, we can identify $m_{\rm b}$ with the $(\mathbf{M}_{\rm d})_{33}$ matrix element of $\mathbf{M}_{\rm d}$ and write the relation among observable $m_{\rm b}$ taken at the scale of the squark mass, $\tan\beta$ and dimensionless ratios $\mu/m_{\rm sq}$, $x = m_\lambda/m_{\rm sq}$. For simplicity we assume also the approximate equality of the left- and right-handed squark masses:

$$\begin{aligned} \frac{2m^2_{\rm b}}{v^2} &= |y_3|^2\left|\frac{1}{\tan\beta} + \frac{2\alpha_{\rm s}}{3\pi}\mathrm{e}^{\mathrm{i}\phi}\frac{|\mu|}{m_{\rm sq}}F(m_\lambda/m_{\rm sq})\right|^2 \\ F(x) &= \frac{x}{1-x^2} + \frac{x^3\ln x^2}{(1-x^2)^2}. \end{aligned} \tag{16}$$

Taking $y_3 \sim 1$, $m_{\rm b}(1\,\mathrm{TeV}) \sim 2.8\,\mathrm{GeV}$ and $F = F_{\max} = F(2.1) = 0.57$, we plot the allowed values of $\tan\beta$ and $|\mu|$ in Fig. 2. The phase ϕ between the tree-level contribution and the radiative corrections to m_b is unknown[1] which leads to a certain allowed area in the $\tan\beta$–$|\mu|$ plane. The case of relatively low $\tan\beta$ corresponds to the destructive interference between the tree-level contribution and the loop correction. $\tan\beta \sim 15$ corresponds to the 80% mutual cancellation between the tree-level value and the radiative correction which we take as the maximally allowed degree of fine- tuning.

The necessity for the off-diagonal entries in the squark mass matrices to be nonzero comes from two reasons. First, they can be the only source for the off-diagonal mass matrices leading to nonvanishing Kobayashi–Maskawa mixing angles. Second, their existence leads to the substantial renormalization of u, d, s, c quark masses which is needed to account for the relations $m_\mathrm{s}/m_\mathrm{c} > m_\mathrm{b}/m_\mathrm{t}$; $m_\mathrm{d}/m_\mathrm{u} \gg m_\mathrm{b}/m_\mathrm{t}$. The 100% renormalization of the charm mass, for example, may result from the combination of the flavor- changing entries in $\mathbf{A}$ and $\mathbf{M}^2_\mathrm{R}$, and so on. At the same time, it is preferable to keep the off-diagonal elements of the squark mass matrices at the lowest possible level to avoid large FCNC contributions from the box diagrams. Treating these flavor transitions as the mass insertions, i.e. assuming that they are small in some sense, we arrive at the following set of order-of-magnitude relations connecting observable masses and mixing angles to the flavor structure of the soft-breaking sector:

$$\begin{aligned}
\frac{\sqrt{2}}{v}\Delta m_\mathrm{c} &\sim \eta_\mathrm{c}\frac{A_{23}(\mathbf{M}^2_\mathrm{R})_{23}}{m^3_\mathrm{sq}} \\
\frac{\sqrt{2}}{v}\Delta m_\mathrm{s} &\sim \eta_\mathrm{s}\frac{(\mathbf{M}^2_\mathrm{L})_{23}(\mathbf{M}^2_\mathrm{R})_{23}}{m^4_\mathrm{sq}} \\
\frac{\sqrt{2}}{v}\Delta m_\mathrm{d} &\sim \eta_\mathrm{d}\frac{(\mathbf{M}^2_\mathrm{L})_{13}(\mathbf{M}^2_\mathrm{R})_{13}}{m^4_\mathrm{sq}} \\
\theta_{23} &\sim \frac{(\mathbf{M}^2_\mathrm{L})_{23}}{m^2_\mathrm{sq}} \\
\theta_{13} &\sim \frac{(\mathbf{M}^2_\mathrm{L})_{13}}{m^2_\mathrm{sq}}.
\end{aligned} \tag{17}$$

Here m_sq is the average squark mass and numerical coefficients η_i represent loop factors. For our estimates we take η_i to be of the order of $\eta_\mathrm{b} \sim \sqrt{2}m_\mathrm{b}/v$. As to the mixing angle between first and second generations, it can be generated either in the down sector [11], or in the up sector through the correction to the matrix element $(\mathbf{M}_\mathrm{u})_{12} \sim \frac{\sqrt{2}}{v}\eta A_{13}(\mathbf{M}^2_\mathrm{R})_{23}/m^3_\mathrm{sq}$.

If the squark masses were diagonal, one would observe an approximate relation $m_\mathrm{b}/m_\mathrm{t} \simeq m_\mathrm{s}/m_\mathrm{c}$ which is violated in reality. There are several possible ways to avoid this problem depending on which part of the allowed values in the μ - $\tan\beta$ plane we choose. If $\tan\beta$ is in the neighbourhood of $m_\mathrm{t}/m_\mathrm{b}$, it is preferable to have $y_2 v/\sqrt{2} \simeq m_\mathrm{c}$ and the strange quark mass being completely of radiative origin. The latter condition requires $(\mathbf{M}^2_\mathrm{L})_{23}/m^2_\mathrm{sq} \sim \lambda^2$ and $(\mathbf{M}^2_\mathrm{R})_{23}/m^2_\mathrm{sq} \sim 1$. For lower values of $\tan\beta$, when the tree-level contribution to the mass of the bottom quark is compensated by loop corrections to give observable m_b, the element $(\mathbf{M}^2_\mathrm{R})_{23}$ can be made smaller and $y_2 v/\sqrt{2} \simeq m_\mathrm{s}$.

Even with the assumptions of the low-energy U–D unification, the number of free parameters is large enough to reproduce the observed masses and mixings. Combining relations (18), we write the phenomenologically acceptable form of the squark mass matrices which can produce the correct flavor physics through the loop mechanism:

$$\begin{aligned}
\mathbf{M}^2_\mathrm{L} &\sim m^2\begin{pmatrix} 1 & \lambda^4 & \lambda^2 \\ \lambda^4 & 1 & \lambda^2 \\ \lambda^2 & \lambda^2 & 1 \end{pmatrix}; \\
\mathbf{M}^2_\mathrm{R} &\sim m^2\begin{pmatrix} 1 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix}.
\end{aligned} \tag{18}$$

Every entry in (18) denotes an order of magnitude of the corresponding matrix element in terms of the power of the Wolfenstein parameter λ. Squark masses chosen in the form (18), plugged in the general formula (13), reproduce correctly the hierarchy among quark masses and mixing angles and therefore satisfy the second condition formulated at the end of the previous section. $(\mathbf{M}^2_\mathrm{L(R)})_{12}$ is not fixed by (17) and for its value we take $(\mathbf{M}^2_\mathrm{L(R)})_{12} \sim (\mathbf{M}^2_{L(R)})_{13}(\mathbf{M}^2_\mathrm{L(R)})_{23}/m^2_\mathrm{sq}$. One should note also that the choice of the squark mass matrices (18) is not unique and other possibilities are plausible.

Similar forms of the squark mass matrices can appear in supersymmetric theories with horizontal symmetries responsible for mass hierarchy [9] (quark–squark alignment). We can invert the set of arguments and conclude that in the quark–squark alignment picture with large $\tan\beta$ and $\mu m_\lambda \sim m^2_\mathrm{sq}$ the radiative corrections to the mass matrices and mixings are important. At the same time, the bare superpotential of the model can be of the form (6), i.e. simpler than that of the conventional MSSM.

The significant FCNC processes are generated in the model by various SUSY diagrams. When the FCNC contribution of the supersymmetric box diagrams with the squark mass matrices (18) is considered, the constraints [15] implies that the lowest possible value for the squark mass has to be at least 1 TeV. The splitting in the neutral B-meson sector requires $(\mathbf{M}^2_\mathrm{L(R)})_{13}$ to be smaller than ~ 0.2 for a 1 TeV squark and gluino mass [15], which is fulfilled in the *ansatz* (18). The splitting in the neutral K-mesons is barely acceptable, whereas the ϵ parameter is too large even for 1 TeV squarks. Therefore, the ϵ-constraint in the *ansatz* (18) requires either accidental cancellation among different contributions or the scale of the soft-breaking masses to be in the region of 3- -5 TeV. A similar mass scale arises from the analysis of the $b \to s\gamma$ process [16] due to the large value for $(\mathbf{M}^2_\mathrm{R})_{23}$ in (18).

Perhaps the most serious consequence of the low-energy unification of up and uown Yukawa matrices and the radiative mechanism for Kobayashi–Maskawa mixing is the appearence of the FCNC transitions mediated by the H_d field. Since the resulting Yukawa interaction resembles that of a generic left–right model with the right-

[1] Possible constraints on ϕ from the limits on the neutron EDM are relaxed when squark mass $\sim$1 TeV

handed mixing angles not smaller than left-handed ones, $|V_{\mathrm{R}ij}| > |V_{\mathrm{L}ij}|$, we can use the limits on the FCNC Higgs masses obtained in [17]:

$$M_A > 10\,\mathrm{TeV} \tag{19}$$

We would like to emphasize here that in the generic MSSM with large $\tan\beta$ the tree-level FCNC induced by the threshold corrections to the mass matrices may exceed contributions from box diagrams and therefore should be properly accounted for in the general analysis [18].

We believe that an elaborate calculation of the FCNC amplitudes in the model is not instructive, because the results will depend on too many details of the soft-breaking sector, which we cannot uniquely determine from the observable quark masses and mixings. Instead, one can ask the question of what acceptable forms of the squark matrices can bring FCNC amplitudes to the lowest possible value and therefore would allow us to lower a common squark mass. The answer is that we have to minimize the presence of the off-diagonal elements in $\mathbf{M}^2_{\mathrm{L(R)}}$, fixing the "problems" in the observable quark mass sector at the expense of the off-diagonal entries in the $\mathbf{A}$ matrix:

$$\mathbf{M}^2_{\mathrm{L}} \sim m^2 \begin{pmatrix} 1 & \lambda^4 & \lambda^2 \\ \lambda^4 & 1 & \lambda^2 \\ \lambda^2 & \lambda^2 & 1 \end{pmatrix}; \quad \mathbf{M}^2_{\mathrm{R}} \sim m^2 \begin{pmatrix} 1 & 0 & \lambda^2 \\ 0 & 1 & 0 \\ \lambda^2 & 0 & 1 \end{pmatrix};$$
$$\mathbf{A} \sim m \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}, \tag{20}$$

where again every entry denotes an order of magnitude. Such an *ansatz* minimizes FCNC amplitudes, opening the possibility for squarks to be of the order of 1 TeV and, perhaps, less than that. A realistic relation between $m_{\mathrm{b}}/m_{\mathrm{t}}$ and $m_{\mathrm{s}}/m_{\mathrm{c}}$ in this case can be obtained through the large radiative corrections compensating tree levels m_{b} and/or m_{c}. For m_{d} we reserve again the radiative mechanism of generation.

We turn now to the "effective supersymmetry" picture [19] which has many fewer degrees of freedom and where flavor physics can be formulated in a more definite way. In the squark mass matrices, diagonalized by unitary transformations U and V,

$$\mathbf{M}^2_{\mathrm{L}} = U^\dagger \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U;$$
$$\mathbf{M}^2_{\mathrm{R}} = V^\dagger \begin{pmatrix} m_1'^2 & 0 & 0 \\ 0 & m_2'^2 & 0 \\ 0 & 0 & m_3'^2 \end{pmatrix} V, \tag{21}$$

the eigenvalues m_1^2, m_2^2, $m_1'^2$, $m_2'^2$ are taken to be in the multi-TeV scale and eventually decoupled from the rest of the particles. At the same time, the squark from the third generation is believed to be not heavier than 1 TeV and weakly coupled to the first and second generations of quarks to avoid the excessive fine-tuning in the radiative corrections to the Higgs potential and suppress FCNC contributions to the kaon mixing. The advantage of this approach in our case is that the loop integrals in (13) can be parametrized by one number η. Moreover, the up quark Yukawa matrix keeps its nearly diagonal form and the Kobayashi–Maskawa mixing results entirely from $\mathbf{M}_{\mathrm{d}}$. Introducing the notations

$$U^*_{13} = \omega, \quad U^*_{23} = \rho, \quad U^*_{33} \simeq 1,$$
$$V_{13} = \gamma, V_{23} = \delta, \quad V_{33} = \sqrt{1 - |\gamma|^2 - |\delta|^2}, \tag{22}$$

we write the resulting form of the mass matrix:

$$\frac{\sqrt{2}}{v}\mathbf{M}_{\mathrm{d}} \tag{23}$$
$$= \eta \begin{pmatrix} \omega\gamma & \omega\delta & \omega\sqrt{1-|\gamma|^2-|\delta|^2} \\ \rho\gamma & \frac{y_2}{\eta\tan\beta} + \rho\delta & \rho\sqrt{1-|\gamma|^2-|\delta|^2} \\ \gamma & \delta & \frac{y_3}{\eta\tan\beta} + \sqrt{1-|\gamma|^2-|\delta|^2} \end{pmatrix}.$$

Here we take into account that $y_3 \simeq 1$ and $\rho, \sim \omega \ll 1$.

Using this form of the mass matrix, we calculate $H_{\mathrm{d}} = \frac{2}{v^2}\mathbf{M}_{\mathrm{d}}\mathbf{M}^\dagger_{\mathrm{d}}$, its eigenvalues and the Kobayashi–Maskawa matrix which diagonalizes H_{d}. The determinant of H_{d} has a simple form:

$$det H_{\mathrm{d}} = |\eta|^6 |\omega|^2 |\gamma|^2 \frac{|y_2|^2}{|x|^4}, \tag{24}$$

where $x = \eta\tan\beta$. The presence of y_2 in $det H_d$ suggests that we should choose $y_2 v/\sqrt{2} \simeq m_{\mathrm{s}}$. This requires the partial compensation of tree-level and radiative corrections in m_{b} and as a result one has $x \simeq -1$ and $\delta \sim \mathrm{O}(\lambda)$.

The analysis of $|V_{\mathrm{us}}|$ yields an even smaller value for the parameter γ. It turns out that to sufficient accuracy one has

$$|V_{\mathrm{us}}|^2 = \left(\frac{m_{\mathrm{d}}^2}{m_{\mathrm{s}}^2}\right)\left(\frac{2m_{\mathrm{s}}^2}{v^2|y_2|^2}\right)\frac{|\delta|^2 + |\gamma|^2}{|\gamma|^2} \tag{25}$$

and therefore $\gamma \sim \mathrm{O}(\lambda^2)$. At the same time

$$|V_{ub}| = |\omega| \frac{\left|1 + \frac{1}{x}\sqrt{1-|\delta|^2}\right|}{\left|1 + \frac{1}{x}\sqrt{1-|\delta|^2}\right|^2 + |\delta|^2} \tag{26}$$

which makes $|\omega|$ be of the order of λ^3. A similar relation for $|V_{cb}|$ shows that $\rho \sim \mathrm{O}(\lambda^2)$. Thus the analysis of the mass matrices generated by the loop with a third generation of squarks inside gives $\rho,\ \omega,\ \gamma,\ \delta \ll 1$, consistent with similar requirement imposed by the absence of large FCNC contributions.

4 Conclusions

The supersymmetric mass problem has one interesting aspect, which is not always properly emphasized. With the general flavor-changing soft-breaking terms, it is hard to interpret the dimensionless coefficient in the superpotential in terms of the observed fermionic masses and mixing angles. This opens up the possibility of a supersymmetric theory with many fewer free parameters in the superpotential than is assumed in the conventional MSSM. We have shown the phenomenological possibility of the low-energy U–D unification in the supersymmetric models. The Kobayashi–Maskawa mixing in this case is the result of the supersymmetric threshold corrections. The analysis of these corrections shows that $\tan\beta$ is not fixed by the requirement of the unification. U–D unification allows it to be in a large range $15 \leq \tan\beta \leq \infty$.

Among different quark masses only m_{t} is clearly of tree-level origin and m_{d} is of radiative origin. The mass of the bottom quark can either come from the radiative corrections ($\tan\beta \to \infty$), or from the tree-level ($\tan\beta \sim 65$) or be the combination of both including the possibility of a destructive interference between the tree-level and radiative correction terms. This applies to the other quark masses as well ($\mathrm{u}, \mathrm{s}, \mathrm{c}$). The radiative origin of m_{s} normally implies large entries in the right-handed squark mass matrix leading to 3–5 TeV quark mass scale to satisfy FCNC requirements (ϵ-parameter and $b \to s\gamma$).

It was shown in [2] that the scale of the physics responsible for the flavor hierarchy can be as low as a few TeV. Our conclusion is that the condition of the unification of all Yukawa couplings does not require us to raise this scale significantly. The scale of the unification, i.e. the supersymmetric threshold in the case considered, can be as low as 1 TeV without causing unacceptable FCNC amplitudes.

An obvious advantage of the "effective supersymmetry" approach in connection with the low-energy U–D unification is that the resulting mass matrix (23) can be parametrized by a small number of parameters i.e. the mixing angles between the third-generation squark and quark fields. At the same time all loop integrals are expressed by one number η and this significantly simplifies the analysis. The condition imposed on this mass matrix to reproduce observable masses and mixings automatically leads to the smallness of the mixing angles between third-generation squarks and first- and second-generation quarks, which coincides with the similar requirements imposed by the absence of FCNC.

The possibility of U–D unification at low energies with the radiative mechanism for Kobayashi–Maskawa mixing has an interesting application to the left–right supersymmetric models. It allows one to reduce the number of Higgs bi-doublets, making it more similar to the conventional (nonsupersymmetric) left–right models. Unfortunately, in the case of manifest left–right symmetry, where left-handed and right-handed squark masses are equal, the radiatively induced fermion masses and mixings do not correspond to the observables.

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References

1. C.D. Froggatt, H.B. Nielsen: Nucl. Phys. B **147**, 277 (1979)
2. M. Leurer, Y. Nir, N. Seiberg: Nucl. Phys. B **398**, 319 (1993)
3. P. Binutruy, S. Lavignac, P. Ramond: Nucl. Phys. B **477**, 353 (1996); A.N. Nelson, D. Wright: Phys. Rev. D **56**, 1598 (1997)
4. M.L. Luty, R.N. Mohapatra: Phys. Lett. B **396**, 161 (1997); D.B. Kaplan, F. Lepeintre, M. Shmaltz: Phys. Rev. D **56**, 7193 (1997)
5. S. Dimopoulos, L.J. Hall, S. Raby: Phys. Rev. Lett. **68**, 1984 (1992); Phys. Rev. D **45**, 4192 (1992); P. Ramond, R.G. Roberts, G.G. Ross: Nucl. Phys. B **406**, 19 (1993)
6. R. Mohapatra, A. Razin: Phys. Rev. D **54**, 5835 (1996)
7. M. Dine: hep-ph/9612389
8. M. Dine, A. Kagan, R. Leigh: Phys. Rev. D **48**, 2214 (1993)
9. Y. Nir, N. Seiberg: Phys. Lett. B **309**, 337 (1993); M. Leurer, Y. Nir, N. Seiberg: Nucl. Phys. B **420**, 468 (1994)
10. L. Hall, R. Rattazzi, U. Sarid: Phys. Rev. D **50**, 7048 (1994); M. Carena et al.: Nucl. Phys. B **426**, 269 (1995); T. Blazek, S. Raby, S. Pokorski: Phys. Rev. D **52**, 4151 (1995)
11. R. Hemplfing: Phys. Rev. D **49**, 6168 (1994)
12. F.M. Borzumati et al.: hep-ph/9712428
13. J.A. Casas, S. Dimopoulos: Phys. Lett. B **387**, 107 (1996)
14. D. Choudhury et al.: Phys. Lett. B **342**, 180 (1995)
15. F. Gabbiani et al.: Nucl. Phys. B **447**, 321 (1996); P.H. Chankowski, S. Pokorski: hep-ph/9703442
16. T. Blazek, S. Raby: hep-ph 9712257; H. Baer et al.: hep-ph 9712305
17. R.N. Mohapatra, G. Senjanovich, M. Tran: Phys. Rev. D **28**, 546 (1983); G. Ecker, W. Grimus, H. Neufeld: Phys. Lett. B **127**, 365 (1983); F.G. Gilman, M.H. Reno: Phys. Lett. B **127**, 426 (1983); F.G. Gilman, M.H. Reno: Phys. Rev. D **29**, 937 (1983): J.F. Gunion, J. Grifols, A. Mendez, B. Kayser, F. Olness: Phys. Rev. D **40**, 1546 (1989); M.E. Pospelov: Phys. Rev. D **56**, 259 (1997)
18. C. Hamzaoui, M. Pospelov, M. Toharia: hep-ph9807350
19. A.G. Cohen, D.B. Kaplan, A.E. Nelson: Phys. Lett. B **388**, 588 (1996); A.G. Cohen, D.B. Kaplan, F. Lepeintre, A.E. Nelson: Phys. Rev. Lett. **78**, 2300 (1997)