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Wave-Induced Vertical Bending Moment Estimation by Onboard Tiltmeters units on Container Ship

M. Li, E. Boulougouris, I. Lazakis & G. Theotokatos
University of Strathclyde, Glasgow, U.K.

ABSTRACT: Full-scale measurements in oceangoing ships have shown that the relationship between bending moment with the curvature curve of hull girder. As part of the INCASS (Inspection Capabilities for Enhanced Ship Safety) EU FP7 project, this paper carried out an estimation of wave-induced vertical bending moment for cargo hold of the 4250 TEU container ship, based on the data of pitch angles processing from the Tiltmeter units placed on board. The results are enable to be processed to the Decision Support System (DSS), in order to assist to monitoring and risk analysis for ship structure and machinery the towards enhanced and efficient ship operations (Konstantinos, et al., 2015). The prediction values also provide a reference for the trend analysis of the past record signals (Ulrik Dam et al, 2015) for evaluation of longitudinal strength of container ship. The advance in different pitch angle response (deformation curvature) of hull girder can be as a development of modern decision support systems for guidance to the ship's master (Lloyd's Register, 2016).

Keywords: wave-induced, vertical bending moment, onboard, container ship

1 INTRODUCTION

Most new advanced ships have extensive data collection systems to be used for continuous monitoring of engine and hull performance, for voyage performance evaluation (Nielsen et al., 2011). It is important to have a measure of the variability in the wave-induced extreme response for ships due to this statistical uncertainty for sea states in various services (Moan et al, 2005). Therefore, it is necessary to ensure safety, reliability, and integrity of ship structures for avoiding major accidents.

INCASS project targets to enable more efficient coordination of inspection scheduling and monitoring practices, so as to address emergency (short term) decision making in case of accidents occurring as well as supporting long term decisions.

As an innovative framework, INCASS project develops the system for monitoring ship structures, machinery and equipment (Michala, L., 2016), and aims to avoid ship accidents, promote maritime safety and protect the environment via structural and machinery monitoring, data gathering, reliability analysis and decision support (EC 2009). Monitoring is an important part that it is costly for ship companies with regard to either inspection or maintenance (Lazakis et al., 2016). From the initially sensors utilized for data acquisition, eventually, the DSS sys-

tem presented and processed the stored data in order to assist predicting the probabilities of upcoming failures and taking timely and effective maintenance actions (Michala et al, 2016). Most new advanced ships have extensive data collection systems to be used for continuous monitoring of engine and hull performance, for voyage performance evaluation (Nielsen et al., 2011), where the most critical wave-induced ship extreme response can be estimated. To illustrate the VBM variability during navigation, amount of database, like pitch angle, obtained from the tiltmeter installed on board, can be recorded into the log files and transfer the data into VBM.

2 REVIEW

2.1 Strip theory

Based on the concept of the slender body theory in aerodynamic, the applied hydrodynamic models are the Strip theories (Kouvin-Kroukovdky 1955), as the most appropriate tool, first developed for accurate prediction of the wave loads and ship responses to account for the forward speed. Due to the linear strip theories are computational simplicity and the generally satisfactory agreement with experiments, so far the theories have been widely accepted and used as the main tool for estimating the performance of a

ship in waves, especially very straightforward in irregular waves (Wang, Z., 2000).

Based on a perturbation procedure, a non-linear quadratic strip theory formulated in the frequency domain predicts wave loads and ship responses in moderate seas (Jensen et al., 1996. Schlachter et al, 1989) used the non-linear strip theory of higher order differential (Soding, 1982) Predicted vertical ship motions and wave loads. ITTC (1987) concluded that the slender body theory is not seem to possess advanced over the strip theory for the vertical motions of a ship at forward speed, whereas it seems to provide better predictions for sway and yaw motions.

2.2 Long term prediction of bending moment

The St. Denis-Pierson method is originated to predict the ship motions at other conditions by the contributions of various wave frequencies to response motions in regular or random seas (St. Denis and Pierson, 1953). The short term response of wave induced motions and loads in frequency-domain are obtained as the product of an input wave spectrum by a transfer function. It is discussed the mainly choice of wave scatter diagram and calculation of the transfer functions, on the definition of the design requirement for wave induced vertical bending moments (Soares et al., 1990). Variability in the wave-induced extreme response for marine structure is important due to the statistical uncertainty for sea states in various service periods. (Moan et al, 2005). The adequacy of the Pierson-Moskowitz spectrum for long-term formulations of wave-induced load effects has been tested for ships of different characteristics (Soares et al., 1990). IACS has issued the standard wave data (IACS Recommendation 34) which is recommended to use a return period of at 20 years, ship's service life $T_0=20$ year, corresponding to about 10^{-8} probability of exceedance (p.o.e) per cycle. Lee et al. (2011a, 2011b) report the non-linear hydroelastic analysis of a container ship (336m), by comparing the calculated vertical bending moments with results from model tests with flexible model in head seas.

2.3 Hydroelastic Method

The treatment of ships as rigid bodies responding to waves has largely continued to use the traditional approach, however, bodily motions imply that the structure experiences neither strains nor stresses. Also, concepts such as mode shapes, natural and resonant frequencies, fatigue, etc. are not encompassed by the rigidity restriction (Hirdaris, 2005). Heller and Abramson (1959) proposed the following definition: 'Hydroelasticity is concerned with the phenomena involving mutual interactions among inertial, hydrodynamic and elastic forces'. Bishop and Price established the basic principles of a theory for flexible

beam-like hulls subject to steady state and transient (e.g. slamming) wave-induced loads, combining Timoshenko beam and strip theories.

The global hydro-elastic structural responses, i.e. whipping and springing, are important issues in the container ship design and operation. A description of the dynamic response of a ship considered as a Bernoulli-Euler beam is given, and the emphasis here is on the 2-node vertical bending mode. (Andersen, I.M.V., 2014). The 2-node vertical bending mode shape of the 9,400 TEU ship is determined using the in-house non-linear strip-theory code SHIPSTAR (see Xia et al., 1998) with the ship hull modelled as an elastic non-uniform Timoshenko beam with the actual stiffness $EI(x)$ and mass $m(x)$ variation along the length of the hull including the added mass of water. The mode shape compared to that determined by Bureau Veritas (Kapsenberg et al., 2010) by the use of a finite element beam model, and the two are found to be almost identical.

3 METHODOLOGY

3.1 Hull longitudinal curvature with different VBM

The governing equation for the vertical deflection, $w(x, t)$, of a Bernoulli-Euler beam as function of location x and time t is:

$$(EI(x)w''(x,t))'' = p(x,t) - m(x)\ddot{w}(x,t) \quad (1)$$

Where

$$\dot{()}' = \frac{\partial}{\partial t} \quad \text{and} \quad ()' = \frac{\partial}{\partial x}$$

$EI(x)$ is the bending rigidity, $m(x)$ is the mass per length non-including the added mass of water; $p(x)$ represented the external load. This paper only consider the 2-node vertical bending mode, shown in Fig. 1, and the relationship between curvatures κ loaded with different VBM in mid ship. The symbol κ is the hull longitudinal curvature when the hull girder is subjected to the mixed complexed forces, i.e. gravity, wave hydrostatic and wave hydrodynamic pressure.

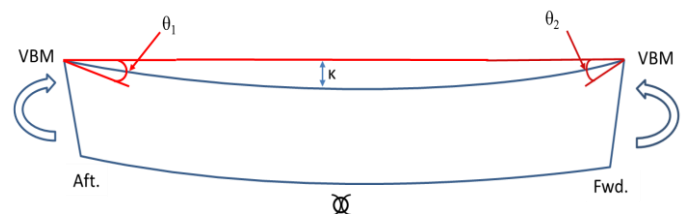


Figure 1. Curvature κ of the hull girder in wave.

To obtain the VBM variance with longitudinal curvature, using a simplified method based on an incremental-iterative approach, using the relation between the angle of the resultant bending moment

vector φ and the angle curvature vector θ in the linear elastic range (Özgüç et al, 2008).

$$\tan \varphi = \frac{I_y}{I_x} \tan \theta \quad (2)$$

Following procedure is discussing the curvature of the bulwark edge at the top of cargo hold. In Fig. 19, angles 1 and 2, mean that the cargo hold being subjected to the VBM, has a pitch angle at the two ends of the top bulwark edge, the total pitch angle = $\theta_1 + \theta_2$.

To obtain the relationship of the total pitch angle and vertical bending moment, the cargo hold is loaded different VBM at two sides, like Fig. 16. Following graphs show the bulwark top edges vertical deformation. Due to we only need the factor of VBM, however, other conditions are also impact the curvature of the cargo hold. This total deformation ω_t consists of two parts, 1. The deformation ω_{vbm} caused by VBM; 2. The other deformation ω_{other} . To explicitly explain the VBM affection, it should subtract the deformation when the VBM value = 0.

The equation of the deflection curve of continuous beam in simple support (Hughes et al, 2010):

$$\varepsilon = -\frac{l^3}{6EI} \frac{x}{l} \left(1 - \frac{x}{l}\right) \times \left[m_1 \left(2 - \frac{x}{l}\right) + m_2 \left(1 + \frac{x}{l}\right) \right] \quad (3)$$

And the beam-end rotation angle

$$\theta_1 = -\frac{m_1 l}{3EI} - \frac{m_2 l}{6EI} \quad (4)$$

And

$$\theta_2 = \frac{m_1 l}{6EI} + \frac{m_2 l}{3EI} \quad (5)$$

Where ε is deflection of the beam; x is the coordinate in beam; l is the length of the beam; m_1 and m_2 are vertical bending moment loading in beam-end; θ_1 and θ_2 is the rotation angle on the beam-end.

In the general beam theory, the basic equation (Özgüç and Barltrop, 2008) that relate the applied VBM and horizontal bending moment to the longitudinal stress are very simple:

$$\sigma_i = \frac{M_x y_i}{I_x} + \frac{M_y x_i}{I_y} \quad (6)$$

4 CASE STUDY

4.1 Numerical analysis of cargo hold

Normally, a cargo hold model is only the parallel mid-body region. From the loading manual and the structural arrangement of the mid ship section and

support bulkhead document of 4,250 TEU Container Vessel, the cargo holds FE model of container ship are established by ANSYS workbench for static analysis, and the cross section is shown in Fig. 2. Based on the assumption, a reasonable model is chosen to extend from halfway hold No.5 (Frame. 97) up to halfway hold No. 7 (Frame. 205) lengthwise, i.e. (1/2+2+1/2) cargo holds. Symmetry about the centre-plane is also considered.

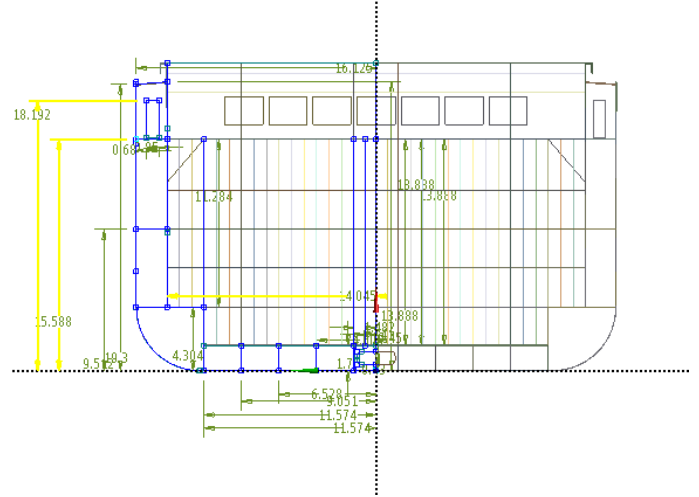


Figure 2. Cross section of cargo hold.

4.2 Material models and meshing method

Only one kind of steel is used in this model. The material considered is Isotropic Elasticity, Young's Modulus $E = 2E+11$ Pa; Poisson's Ratio is 0.3, Tensile Yield Strength $\sigma_t = 2.5E+08$ Pa, as well as the Compressive Yield Stress $\sigma_c = 4.6E + 08$ Pa, and Tensile Ultimate Strength $\sigma_u = 4.6E + 08$ Pa. This case model is adopted the line, surface, and solid, three type of elements. This finite element modelling consists of choosing the appropriate element type and mesh size. Even today, rapid increase in computer processing power and memory has not eliminated computational cost and time constrains. The meshing size is adopted the automatic sizing method in software, minimum to maximum size is 0.13 to 0.64 m, shown in Fig. 3.

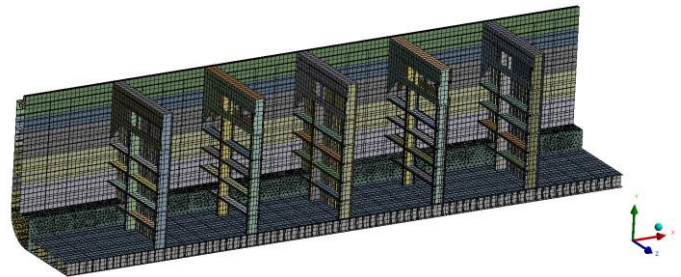


Figure 3. Final mesh model

4.3 Boundary Conditions

Generally, to obtain the accuracies in results, boundary condition effects in the light-ballast condition is particular important. In this case, it is considered the impact of VBM on longitudinal strength. Then the design load cases of extreme vertical bending moment that be applied to the cargo hold model, shown in Fig. 4, with the probability level 10^{-8} (Lloyd's Register, 2014a), in which the design regular wave ($H = 15.6$ m). Both vertical bending moments are positive (+) sagging condition.

Apart from the loading boundary, it is hard to define the displacement boundary condition. The BC (boundary condition) rules (DNV-RP-C205 4.3, 2013), plane A and B are restricted the in the horizontal X direction and rotation of the Y, Z direction. All the stiffness in plates is fixed rotation in X direction. To simulate the approximate symmetry at the two end of the structure, two sides need to be restricted in X displacement and Y, Z rotation displacement.

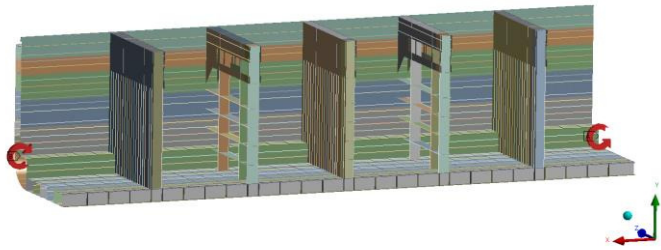


Figure 4. Sagging VBM at two sides.

The Total Deformation, von-Mises Equivalent Stress and equivalent strain graph loaded by the extreme wave-induced VBM, shown in Fig. 5, 6, 7.

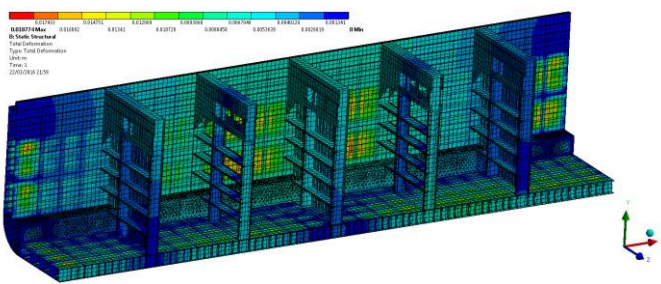


Figure 5. Total deformation of cargo hold.

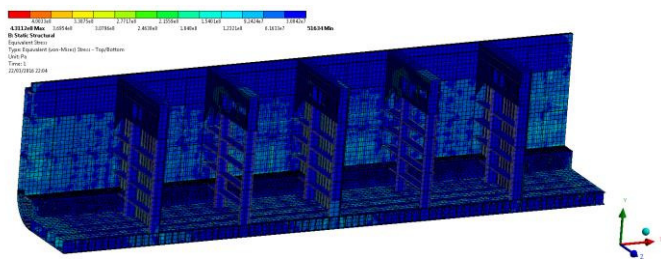


Figure 6. Von-Mises stress of cargo hold.

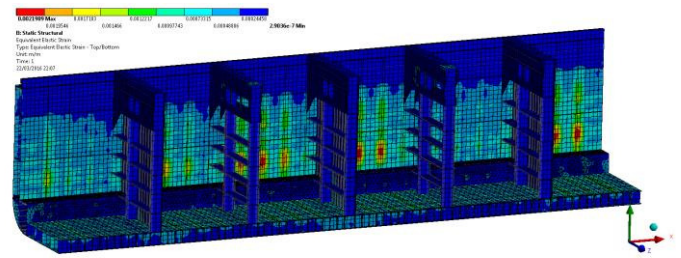


Figure 7. Equivalent strain of cargo hold.

4.4 Hull longitudinal curvature with Different VBM

This chapter studies the relationship between curvatures κ loaded with different VBM in mid ship.

To obtain the approximate deflection curve equation of the neutral axial vertical bending shape of the 2-node shape of hull girder, first step is important to analyze the deflection equation of the cargo hold. From the equation (6),

$$\omega(x) = ax^n + b \quad (n \geq 3) \quad (6)$$

where a and b is the equation coefficient of the deflection of the simply support beam loaded by VBM is linear with the x^n . From the Fig. 8, it is the deflection curve of 2-node vertical bending shape with VBM value $= 10^{10}$ Nm. This paper presents a deflection curve equation of this vessel.

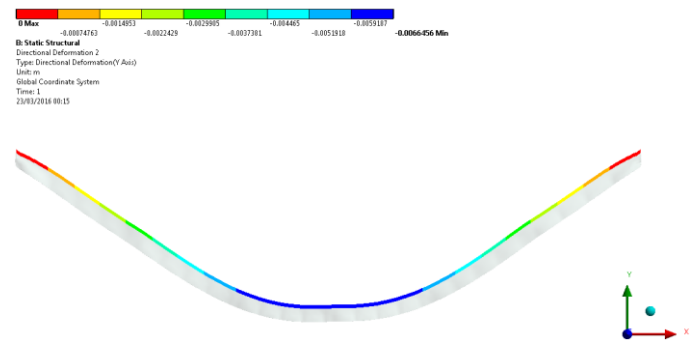


Figure 8. Deflection curve of cargo hold.

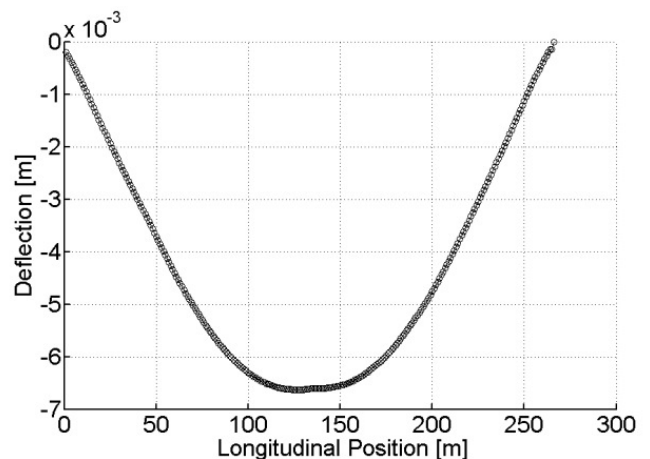


Figure 9. Deflection curve of cargo hold.

Andersen (2014) using the in-house non-linear strip-theory code SHIPSTAR (Xia et al., 1998) with the hull model to compare the mode shape with that determined by Bureau Veritas, and the result is identical. Using the statistical analysis software of MATLAB (Gao, 2011), for neutral axial 2-node shape of vertical bending shape with wave-induced VBM value is equal to 1010 Nm. This paper using least square fit and a 6th order polynomial being approximated by:

$$\varepsilon(x) = -3.966e^{-13}x^6 + 1.716e^{-12}x^5 + 4.834e^{-10}x^4 - 2.297e^{-09}x^3 + 4.081e^{-06}x^2 + 2.764e^{-07}x - 0.006695 \quad (7)$$

where x is the distance from the end of the model. Furthermore, with this equation of the deflection curve of the cargo hold, due to the deflection of longitudinal of the hull girder is very small correspond to the principle of the container ship and the continuity, here the equation of cargo hold can be promoted to the 2-node shape of hull girder deflection in vertical direction.

Table 1. Vertical deformation variation with different wave-induced VBM.

Deformation (m)	different VBM (10 ⁹ Nm)			
	0	2.1	4	10
ε_t	6.67E-03	6.679E-03	6.684E-03	6.71E-03
ε_s	6.67E-03	6.67E-03	6.67E-03	6.67E-03
ε_w	0.0E+00	9.0E-06	1.4E-05	4.0E-05

With the different VBM values, it is obtained the deflection curve equations correspondingly. The Neutral surface 2-node vertical bending shape shown in Table 1, where ε_t is total deflect with both the still and wave-induced vertical bending moment, ε_s is deflect only active by still water, ε_w is deflect only active by wave-induced vertical bending moment. According to the define of the deflect, there is presented that the deflection $\varepsilon_t = \varepsilon_s + \varepsilon_w$.

Then the pitch angle θ of the hull girder shown in Fig. 9 is obtained from the slope value $\tan \theta$ of deflection curve rotation angle at the two end of the hull girder. Due to pitch angle is a minimum value witch is approached 0, further more $\theta \approx 0^\circ$ means that $\theta \approx \tan \theta$. Above paragraph, it is presented that the deflection $\varepsilon_t = \varepsilon_s + \varepsilon_w$, in the same way, $\theta = \theta_1 + \theta_0$ where θ is the pitch angle by VBM; and θ_0 is pitch angle of other loading condition. The different deflection curve equation and curvature are comparison with the study result for bulk carrier (Özgüç and Barltrop, 2008), shown in Table 2. Fig. 18 is showing the linear relationship refer to these VBM and curvature of the vessel.

Table 2. Vertical deformation variation with different wave-induced VBM.

VBM [10 ⁹ Nm]	a [10 ⁻¹¹]	2 tan θ [deg]	2(tan $\theta - \tan \theta_0$) [deg]	curvature
0	4.537	7.424	0	0
2.1	4.543	7.434	0.0095	7.27E-05
4	4.547	7.440	0.0155	0.000119
10	4.565	7.469	0.0450	0.000346

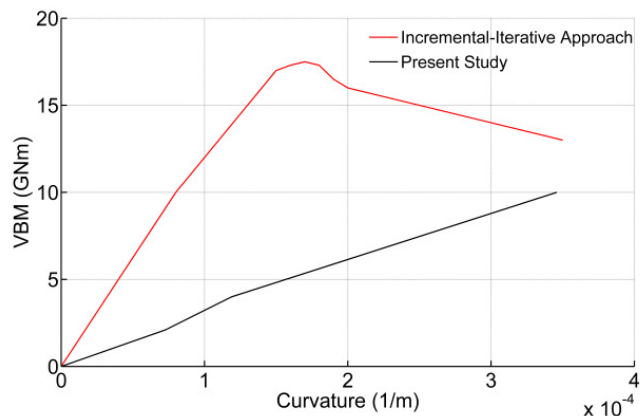


Figure 10. Curvature varied with VBM.

It can be seen that before the VBM of vessel approach to the extreme bending moment, the curvature linearly increases with the VBM, and the slope between these two parameters is depend on the transverse flexural rigidity.

And we only consider the vertical bending moment influence, bonding with the equation (7). This paper presents a VBM- curvature relation equation:

$$\frac{M_{VBM}}{D_{eff}} = \frac{1}{\kappa} \quad (8)$$

Where D_{eff} is the effective vertical bending rigidity, this symbol reflect the longitudinal strength between the VBM and curvature of hull girder. From the Table 2, the approximate values of the effective vertical bending rigidity $D_{eff} = 28877.94N/m^2$.

5 RESULTS

The recorded data have been separated in log files 12 hours per day, about 10Hz per record. For this vessel cross-Atlantic trip from Europe to America, the records of the different pitch angle at the two end of the vessel is known by the Tiltmeter units shown in Fig. 11.

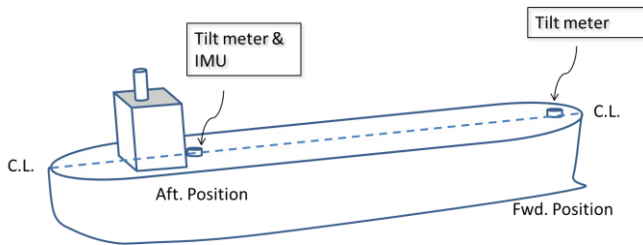


Figure 11. Installed unites and corresponding location on the ship.

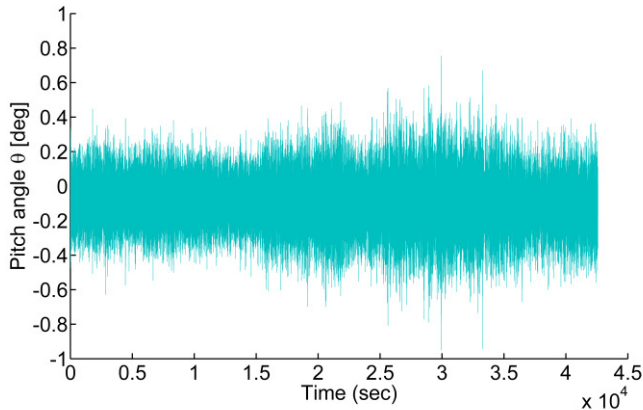


Figure 12. Differential Pitch angle at the end of vessel in time-domain.

With the equation (8) presented by the paper, and the estimate of rigidity of D_{eff} , the visual value above can be converted to VBM in time-domain during this vessel trip, shown in Fig.13. The extreme VBM in level of probability occurrence of 10^{-5} is equal to 1.8×10^9 Nm. Comparing with the Fig. 21, VBM-LOG curve, VBM of probability of 10^{-5} values is 1.6×10^9 Nm. The reason of the different between these two results is due to the other loading like, horizontal bending moment, torsion moment and the variable transverse section along with the longitudinal length of the ship, also influence the effective vertical bending rigidity D_{eff} .

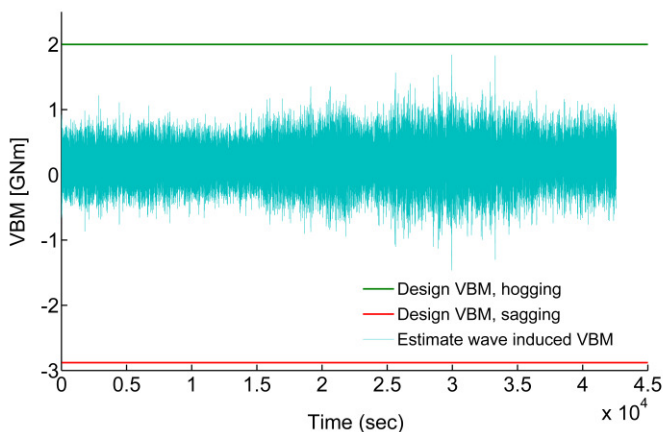


Figure 13. VBM during the vessel trip in time-domain.

6 CONCLUSION

From the database collected from the sensors of tilt-meter units installing in the deck at the forward and aft ward end of the ship, record of the two angles from the ship motion can be obtained, in case of that the curvature can got from the pitch angle (θ). The overall research of this paper is as follow:

Due to numerical simulation results for obtaining the relationship between VBM and longitudinal curvature of the hull girder, the curve of VBM variance in time domain can be obtained from the database obtained from the tiltmeters installed on board during the navigation of the container ship. During this period cycle, the 10^{-5} exceedance of possibility of VBM value is high than the long-term prediction results above, because in realistic, other loading, like horizontal bending moment, torsion bending moment, or other hydrodynamic coefficient have impact on the bending stiffness of the hull girder.

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REFERENCES

- Andersen, IMV, Jensen, JJ & Nielsen, UD 2014, Full Scale Measurements of the Hydro-Elastic Response of Large Container Ships for Decision Support. Ph.D. thesis, Technical University of Denmark (DTU), Kgs. Lyngby. DCAMM Special Report, no. S166 APA
- Bishop, R.E.D, Price, W.G. and Wu, Y. A General Linear Hydroelasticity Theory of Floating Structures Moving in a Seaway, Phil. Trans. Royal Soc. London, A316: 375-426 (1986). DNV Classification Notes No.34.1, CSA - Direct Analysis of Ship Structures, August 2015
- DNV Recommended Practice DNV-RP-C205, Environmental Conditions and Environmental Loads, April 2014
- DNV Recommended Practice DNV-RP-H103, Modelling and Analysis of Marine Operations, April 2011
- DNV Rules for Classification of Ships Pt.3 Ch.1, Hull structural design - Ships with length 100 metres and above, January 2016
- Dikis, K., Lazakis, I., Taheri, A. and Theotokatos, G., 2015, May. Risk and Reliability Analysis Tool Development for Ship Machinery Maintenance. In International Symposium on Ship Operations, Management and Economics (pp. 28-29).
- EC (2009). Regulation (EC) No 391/2009 of the European Parliament and of the Council of 23 April 2009 – (Common Rules and Standards for Ship Inspection and Survey Organisations). Official Journal of the European Union.
- Gao, F., 2011. Applications of matlab in mathematical analysis. Journal of Software, 6(7), pp.1225-1229.

- Soares, C.G., Fonseca, N. and Pascoal, R., 2004. Long term prediction of non-linear vertical bending moments on a fast monohull. *Applied Ocean Research*, 26(6), pp.288-297.
- Heller, S.R. and ABRAMSON, H.N., 1959. Hydroelasticity: a new naval science. *Journal of the American Society for Naval Engineers*, 71(2), pp.205-209.
- Hirdaris, S.E. and Chunhua, G., 2005. Review and introduction to hydroelasticity of ships. *Lloyd's Register*.
- IACS, 2001. Standard Wave Data, IACS Recommendation No.34.
- Kapsenberg, G. K., M. Schiere, and I. Drummen (2010). TULCS Work Package 6: Model test. Specifications for experiments (confidential). Maritime Research Institute Netherlands.
- Korvin-Kroukovsky, B.V., 1955. Investigation of ship motions in regular waves. SNAME.
- Lazakis, I., Dikis, K., Michala, A.L. and Theotokatos, G., 2016. Advanced ship systems condition monitoring for enhanced inspection, maintenance and decision making in ship operations. *Transportation Research Procedia*, 14, pp.1679-1688..
- Lee, Y., White, N., Wang, Z., Zhang, S. and Hirdaris, S.E., 2012. Comparison of springing and whipping responses of model tests with predicted nonlinear hydroelastic analyses. *International Journal of Offshore and Polar Engineering*, 22(03).
- Lee, Y., Wang, Z., White, N. and Hirdaris, S.E., 2011, January. Time domain analysis of springing and whipping responses acting on a large container ship. In *ASME 2011 30th International Conference on Ocean, Offshore and Arctic Engineering* (pp. 139-147). American Society of Mechanical Engineers.
- Lloyd, A.R.J.M., (1989). *General Ship Dynamics Aspects of Different Vehicle Types*, 13th WEGEMT School.
- Lloyd's Register (2016). *Rules and Regulations for the Classification of Ships*, Pt 4, Ch 8, Sec 15.3.
- Lloyd's Register (2014a). *Guidance Notes on the Assessment of Global Design Loads of Large Container Ships and Other Ships Prone to Whipping and Springing*, Version 1.4.
- Michala, A.L., Lazakis, I. and Theotokatos, G., *Predictive Maintenance Decision Support System For Enhanced Energy Efficiency Of Ship Machinery*.
- Moan, T., Gao, Z. and Ayala-Uraga, E., 2005. Uncertainty of wave-induced response of marine structures due to long-term variation of extratropical wave conditions. *Marine Structures*, 18(4), pp.359-382.
- Nielsen UD. (2015). *Response-based sea state estimation for onboard DSS - Safe and Efficient Marine Operations*. [Sound/Visual production (digital)].
- Nielsen, U.D., Jensen, J.J., Pedersen, P.T. and Ito, Y., 2011. Onboard monitoring of fatigue damage rates in the hull girder. *Marine Structures*, 24(2), pp.182-206.
- Nikolaidis, E. and Kaplan, P., 1991. Uncertainties in stress analysis on marine structures; final rept (no. Ssc-363)..
- Özgüç, Ö. and Barltrop, N.D., 2008. Analysis on the hull girder ultimate strength of a Bulk Carrier using simplified method based on an incremental-iterative approach. *Journal of Offshore Mechanics and Arctic Engineering*, 130(2), p.021013.
- Soares, C.G., 1984. Representation of double-peaked sea wave spectra. *Ocean Engineering*, 11(2), pp.185-207.
- Söding, H., 1982. *Leckstabilität im seegang*
- St Dinis, M. and Pierson Jr, W.J., 1953. *On the motions of ships in confused seas*. NEW YORK UNIV BRONX SCHOOL OF ENGINEERING AND SCIENCE.
- Jensen, J.J. and Dogliani, M., 1996. Wave-induced ship full vibrations in stochastic seaways. *Marine Structures*, 9(3), pp.353-387.
- Wang, Z., 2000. *Hydroelasticity of high speed ships*.