On the temporal stability of the coda of ambient noise correlations

Andrea Colombi\textsuperscript{a)}

Julien Chaput\textsuperscript{b)}

Florent Brenguier\textsuperscript{c)}

Gregor Hillers\textsuperscript{d)}

Philippe Roux\textsuperscript{e)}

Michel Campillo\textsuperscript{f)}

\textsuperscript{a)}\textit{e-mail: andree.colombi@gmail.com}
\textsuperscript{b)}\textit{e-mail: julien.chaput@ujf-grenoble.fr}
\textsuperscript{c)}\textit{e-mail: florent.brenguier@ujf-grenoble.fr}
\textsuperscript{d)}\textit{e-mail: gregor.hillers@ujf-grenoble.fr}
\textsuperscript{e)}\textit{e-mail: Philippe.Roux@ujf-grenoble.fr}
\textsuperscript{f)}\textit{e-mail: Michel.Campillo@ujf-grenoble.fr}

ISTerre

Université Joseph Fourier, Grenoble,

BP 53 38041 Grenoble CEDEX 9
Abstract We analyze the sensitivity of cross correlations to the anisotropic intensity of the incident field in the context of ambient noise monitoring of slight seismic velocity changes. We perform numerical simulations of elastic waves on a 2D scattering plate to study the sensitivity to noise anisotropy of direct and coda waves in the cross-correlation. In presence of strong anisotropy, we show that correlation coda waves exhibit a much weaker sensitivity to anisotropy than the direct waves. We observe similar behavior with real data recorded on Erebus volcano, where a database of icequakes is used to simulate an anisotropic source field. We propose a simplified approach to evaluate the sensitivity of scattered waves to noise anisotropy. We rely on previous results obtained for direct waves and on intrinsic properties of scattered waves to predict the errors produced by a strong anisotropy for numerical experiments. These results also yield realistic values for monitoring precision to be expected at crustal scales with real data. Our analysis shows that high precision noise-based monitoring could be performed with correlation coda waves even in the presence of unknown variations in the distribution of ambient noise intensity.

1 Introduction

It has recently been shown that correlations of ambient seismic noise can be used to retrieve the elastic response of the Earth between two receivers [Shapiro and Campillo, 2004]. This
property has consequently become a widely used tool in seismic imaging efforts [Shapiro et al., 2005, Lin and Ritzwoller, 2011]. Furthermore, the inherent temporal stability of noise correlation measurements [e.g. Stehly et al., 2006] has led to time dependent analyses to detect variations in elastic parameters in the Earth crust. This has successfully been performed in various contexts and media [Wegler and Sens-Schonfelder, 2007, Brenguier et al., 2008a,b, Renalier et al., 2010, Clarke et al., 2011, Mainsant et al., 2012, Rivet et al., 2011].

To maximize the sensitivity of these measurements, it is convenient to analyze the temporal changes of the coda of the correlations rather than those of the direct waves [Poupinet et al. [1982], Snieder et al. [2002]. It is important to note that the coda of correlation exhibits some properties characteristic of the actual coda. Indeed, Sens-Schnfelder and Wegler [2006] showed that the envelope of the late part of the correlations presents a decay similar to that of actual coda. Using correlations of the coda of correlations (C3), Stehly et al. [2008] further showed that the coda of the correlations consists of multiply reflected arrivals. This C3 function actually converges towards the Green function, as does the coda of actual seismograms [Campillo and Paul [2003]. This argument provides a firm basis for the use of the coda of correlations in monitoring efforts, given that the late part of the correlation contains multiply scattered physical arrivals, just as the early part contains the direct waves. That being said, the degree of precision or robustness required in the reconstructed phases of the correlation varies as a function of the application. In the case of standard surface wave to-
mography, where strong lateral variations are expected and where a posteriori uncertainties on the models are significant, a precision in the velocity of the reconstruction of the direct waves of less than 1% is generally acceptable. For monitoring, the temporal variations of relative velocities that are observed are far smaller than this value. The typical variations prior to volcanic eruptions are measured to be at most of the order of 0.1% [e.g. Brenguier et al., 2008b]. Post seismic responses of the same order have also been detected [e.g. Brenguier et al., 2008a] Chen et al., 2010). In a recent study of the continuous records of the HiNet network in Japan, Brenguier et al. [2014] observed fluctuations of the seismic velocity of the order of $10^{-4}$ in absence of large tectonic disturbance but without specific corrections for external forcing effects. The large fluctuations of seismic velocities that have been reported are unambiguously related to seismic or volcanic events, both in terms of causality [Brenguier et al., 2008a] and spatial location [Obermann et al., 2014]. There are therefore clear observations that confirm the feasibility of ambient noise monitoring.

The practical use of this type of monitoring for the forecast of events such as volcanic eruptions requires a complete understanding of the the origin of the observed velocity changes, including the weak ones. Such fluctuations can have various origins, and can be associated with instrumental issues [e.g. Stehly et al., 2007], with external forcing such as precipitations [Hillers et al., 2014, Sens-Schnfelder and Wegler, 2006], regional water load [Froment et al., 2013], tides [Yamamura et al., 2003, Hillers et al., 2014], or thermo elasticity
Since they are correlated with external parameters that are commonly recorded and have a seasonal component, there is a prospect that they can be detected and eventually accounted for.

A more difficult issue is the sensitivity of the cross-correlation to temporal variations in the noise source. The theoretical requirement for a perfect reconstruction of the Green function includes the isotropy of the field incident on the receivers (a complete discussion is beyond the scope of this paper and we refer here to Campillo and Roux, 2014 and the references herein). When the incident field is non-isotropic, a possible bias in the arrival times has to be considered. Tsai [2009], van der Neut and Bakulin [2009] and Yao and Van Der Hilst [2009] discussed the travel time bias produced in noise-based measurement by a non-isotropic distribution of noise intensity.

Weaver et al. [2009] argued that the anisotropy of a smooth azimuthal distribution of noise intensity does not impair the quality of measurement of direct wave arrival time in the limit when two receivers are separated by a distance that is much larger than the wavelength. In the more realistic configuration when the distance is finite, they gave approximate analytical results for the amplitude of the bias produced by the anisotropic character of the azimuthal distribution of noise intensity. Froment et al. [2010] verified this theory with actual data to demonstrate that these effects can be quantified and eventually corrected. Their conclusion was that the biases expected from a smooth distribution of intensity are
predictable, and, in most case small when considered in the context of tomography. We note here that the smoothness of the azimuthal distribution of noise intensity is guaranteed by the presence of scattering for frequencies larger than 0.1 Hz. The importance of scattering was illustrated by Froment et al. [2010] who considered a very unfavorable incident intensity anisotropy to show that the bias is widely reduced when they correlate coda waves with respect to the case when they correlate direct arrival. As expected from the role of scattering in diversifying the directions of propagation [Campillo, 2006], correlating coda waves yields almost exact reconstructions of the direct waves. Nonetheless, selecting scattered waves is impossible with continuous noise records and the high amplitude direct waves in the noise result in biased arrival times for the direct waves measured in the correlation. Since the noise source (mostly oceanic gravity waves) is evolving with time, it is natural to be mindful of non-physical apparent velocity variations purely due to the changes in the distribution of intensity. Here, we are not referring to possible changes in spectral content that are easy to control. As already noted, actual noise correlations display remarkable stability in absence of events like earthquakes or other external forcing. In this paper we aim to test the stability of the correlation function and to evaluate the robustness of relative velocity change measurements made in coda windows. We use numerical simulations, and a further test performed with real data. The numerical simulations are performed in a medium with significant scattering for the waves at the frequency considered, and the real-world experiment
is performed with data recorded on a volcano in under scattering conditions.

2 Numerical simulation of coda signals in a scattering unbounded 2D medium

2.1 Simulations setup

We extend the work of Colombi et al. [2014] who simulated flexural waves on a 2D plate to investigate the role of reverberations and source distribution on the quality of the cross-correlation reconstruction in a multiple scattering medium. We simulate the propagation in a 2.5 m × 2.5 m × 2 mm thick plate that contains a circular region (radius 0.6 m), where 220 scatterers are randomly distributed (Fig. 1a). The material properties of the homogeneous plate are those of aluminium, characterised by $V_p = 6100$ m/s, $V_s = 2900$ m/s and $\rho = 2700$ kg/m$^3$. The scatterers are either fast or slow circular inclusions with diameters varying between 2 and 3 cm. The $P$ and $S$ are either halved for slower inclusions or 50% faster for fast inclusions, while $\rho$ is set at 2000 kg/m$^3$. A pair of receivers separated by 0.5 m is centered in the scatterer region. Both are kept sufficiently distant from the inclusions to avoid near field effects (e.g. resonance inside the scatterers). Spectral element simulations were chosen for this numerical analysis. The plate motion is simulated using the
SPECFEM3D software [Peter et al., 2011]. The discretization of the plate and scatterers is straightforward with the software CUBIT. We choose model the boundary of the scatterers explicitly resulting in an adaptive paving scheme for the meshing. The smallest element edge (down to 2 mm) drives the cost of the simulation by defining the size of the integration time-step according to the stability condition. By using the source-receiver reciprocity as in Colombi et al. [2014], only two forward simulations are required to calculate the wavefield. Computational cost is therefore limited.

PML conditions are applied to the four lateral boundaries while the top and bottom sides are modelled as free surfaces. Such requirements render this type of study difficult to implement in the laboratory unless dynamic environments such as Vasmel et al. [2013] are employed.

∼200 sources are distributed among 4 rings surrounding the scattering region (Fig. 1a). By choosing such an annular surface of sources instead of a simple circular distribution, we minimize the non-physical contributions as shown in [e.g. Colombi et al., 2014]. The sources consist of vertical point forces with a Ricker wavelet centered at 35 kHz acting perpendicularly to the plate surface, resulting in mainly $A_0$ Lamb waves in the plate [Colombi et al., 2014, Larose et al., 2007]. For this reason, only the vertical component of the displacement is used for the cross-correlation. The computational domain is assumed to be non-dissipative and linear elastic. Signals are sampled at 500 kHz, and a band-pass filter between 15 kHz
and 45 kHz is applied to the signal to eliminate numerical noise. Lamb wave dispersion in the 15 kHz to 45 kHz frequency band leads to wavelengths ranging from 1.5 to 5 cm. With a phase velocity \( C \) of the \( A_0 \) mode reaching 2000 m/s at 40 kHz, the direct wave (with shorter travel time) and the scattered coda are contained in a 2.5 ms long signal (Fig. 1b). The displacement \( u_z \) for the two simulations is uniformly computed over the top surface using a grid of 3 mm spaced receivers in both \( x \) and \( y \) directions. This provides us a full description of the wave propagation in the plate (e.g., Fig. 1b).

We compute the cross-correlations for each individual source at the two receivers R1 and R2 (as represented in Fig. 1a) and we stack the results. The causal part of the stacked cross-correlation is shown in Fig. 1c. The reference ”band-limited Green function” is the true Green function obtained by a single vertical force at one of the receivers. The two traces are nearly identical, even in their late part. Here, the individual correlations have been normalized by their energies to simulate an isotropic noise distribution and therefore to maximize the quality of the reconstruction.

### 2.2 Beam forming and scattering mean free path in the plate

We first investigate the nature of the scattered wavefield via beam forming (hereafter abbreviated with BF). This technique provides directional and velocity information about the elastic energy propagating in the plate by mapping records from a 2D array of receivers to
a 2D plane wave domain [Boué et al., 2013, Rost and Thomas, 2002]. BF is used here to analyze the field content in windows corresponding to direct and coda waves. Fig. 2a depicts BF results obtained for an antenna with an aperture of 30 cm, containing 200 receivers and a single source. The center of the antenna corresponds to the receiver R1 in Fig. 1a. The ideal approach discussed in [Boué et al., 2013] has been followed, and we compute the BF for a 0.3 ms long moving window. Fig. 2b depicts the strong directivity of the energy when the window is centered on the first arrival. First order effects of scattering are already visible with a clear spread of energy incidence away from the direct path. Nevertheless, the energy comes prevalently from the right side. The weak spot to the left is attributed to the small imperfection of the PML boundary layer in the simulation. The BF is then applied to a coda window, resulting in the field illustrated in Fig 1(c). In this case, the intensity distribution presents a wide azimuth (Fig. 2c), and it is difficult to recognize a dominant direction associated with the source. The characteristics of the two regimes illustrated in Figs. 2 b and c are used in our interpretation of the effects of noise anisotropy.

For scattering problems, a key parameter is the scattering mean free path (mfp) $l$, or conversely the scattering mean free time (mft) $t_l = l/C$ [e.g. SATO, 1978]. Following the work of De Rosny and Roux [2001] the mfp is evaluated from the coherent and incoherent energies. The first represents the energy of the average seismogram while the second the average energy of the seismograms. The estimation of $l$ is finally obtained by linear regression
when plotting the ratio as a function of the distance from the source [see De Rosny and Roux, 2001, Eq. 3]. Here, we consider displacements recorded for a 2D array centered on a source in the middle of the scattering region. We then bin these recordings by distance from the source. In the frequency band considered, the mfp of the $A_0$ mode varies between 0.4 and 0.7 m, hence of the order of the distance between receivers (0.5 m) in our correlation experiment.

3 Coda stability to noise anisotropy

To test the sensitivity of the direct arrival reconstructed via cross-correlation with respect to noise anisotropy, [Froment et al., 2010] have considered noise intensity distributions $B(\theta)$ of the following type:

$$B(\theta) = B_0 + B_1 \cos(\theta) + B_2 \cos(2\theta) + B_3 \cos(3\theta) + ...$$  \hspace{1cm} (1)

Here $\theta$ is the source azimuth measured as indicated in Fig. 1a, $B_0, B_1...$ are coefficients with values between 0 and 1. Notice that only cosine distributions are considered. This parity follows the fact that the correlation between the two receivers does not distinguish between arrivals with incidence $\theta$ or $-\theta$. An isotropic source distribution will result from taking $B(\theta) = B_0$. The case where $B(\theta) = B_2 \cos(2\theta)$ with $B_2 = 0.6$ represents a rather extreme case of anisotropy. This is the case we consider here to illustrate the effect of a drastic
change in the intensity distribution with respect to the isotropic case. Fig. 1a represents the corresponding source intensity of the synthetic 'noise' on a color scale. The arrow shows the dominant direction of the energy, being in this case perpendicular to the receiver pair strike and to the end-fire lobes. For receiver distances larger than the wavelength, the travel time error $\delta t$ of the anisotropic noise correlation can be calculated for positive correlation times following Weaver et al. [2009]:

$$
\delta t \sim \frac{B''(0)}{2\tau \omega_0^2 B(0)},
$$

(2)

where $\omega_0$ is the angular frequency, $\tau$ is the nominal travel time of the phase, and $B''(\cdot)$ the second derivative of the noise intensity. Note that $\delta t(\tau)$ decreases with $\tau$ and vanishes for large travel times. Equation 2 is only valid for distances larger than the wavelength and therefore the divergence of $\delta t(\tau)$ for $\tau$ approaches zero is non-physical. The natural limit is obviously 0. In the following, we apply this anisotropic distribution to cross-correlations of direct waves and compare the subsequent error in reconstruction for the direct and coda waves.

Given that the Green’s function between two stations is never available in practice, monitoring studies are based on variations with respect to an arbitrarily chosen reference stacked cross correlation stack. In our case, we construct a reference cross correlation function from an isotropic intensity distribution $B(\theta) = B_0$. To simulate correlations of anisotropic
noise fields, the cross correlation for each individual source is modulated according to $B(\theta)$ (Fig 1a) before stacking. The analysis is limited to the causal part of the correlation. Given the reference and the perturbed signals, the lag times are calculated using the doublet (or multiple window cross spectral) method of Poupinet et al. [1982].

Froment et al. [2010] used real data records and considered in detail the case where only the coherent part of the energy (i.e. that obtained by muting the signal after the ballistic wave) is used. The same has been done with our synthetic data using only the first arrival (between 0 and 0.5 ms in Fig. 1c). We measure values of $\delta t$ when anisotropy is present that are similar to their study and well predicted by equation 2. This result is not shown as we simply used it to test our procedure.

We then compute the cross-correlation using the full 2.5 ms signals that include both direct and coda waves. To further compute the lag-time, a moving window of 0.3 ms has been used. The windows partially overlap and the value of $\delta t$, obtained via the doublet method, (dots in Fig. 3a) are plotted for the center of the moving window. The correlations for isotropy ($B(\theta) = B_0$) and anisotropy ($B_2 = 0.6$) are plotted in Fig 3b, and a time lag bias is clearly visible on the direct wave. In Fig. 3a we show the evolution of the bias along the time axis of the cross-correlation. The peak value of $d\tau$ for the direct wave can be comparatively estimated by Eq. 2, (green line in Fig. 3a), by setting $\omega_0 = 2\pi \cdot 35000$ rad/s, and $\tau = 0.25$ ms. We note that this theoretical prediction is not reached in the numerical
experiment. This is most likely due to the effect of scattering, not included in the theory, which generates a variety of incidence angles for each individual source (Fig 2b and c). The scattering thus has a smoothing effect on the imposed anisotropy of the source intensity.

$\delta t$ drops rapidly after the arrival of the direct wave, then fluctuates at a level that does not seem to evolve in time and is on the order of one third of the direct wave bias. As is commonplace in monitoring studies, a fractional delay is defined as $\delta t(\tau)/\tau$. It is the opposite of the fractional velocity change for a homogeneous change in the medium, and is represented by the dashed blue line in percent in Fig. 3. $\delta t(\tau)/\tau$ calculations for arrivals following the direct wave converge to very small error values, on the order of $\leq 10^{-4}$. Notice that the spectral coherency of the signal remains always above 90% suggesting that the introduction of source anisotropy does not provoke major changes in the structure of the complex coda waves reconstructed by correlation.

This numerical result indicates a weaker sensitivity to source anisotropy of correlation coda waves when compared to direct waves. For monitoring applications, this means that correlation coda are expected to be weakly sensitive to changes in the ambient noise source distribution. We note that the relative delay $\delta t(\tau)/\tau$ measured in the late coda of our synthetic correlations for a large change in source distribution is of the same order ($10^{-4}$) as the actual fluctuations measured by Brenguier et al. [2014]. We observe a rapid drop of the bias after the direct waves and a high coherency between isotropic and anisotropic
scenarios. In the next section, we perform a similar test with real data, where a database of small impulsive signals allows us to replicate the synthetic setup fairly accurately.

4 Real data from Erebus volcano

We use a set of small impulsive icequakes to replicate the role of the vertical forces in the previous numerical experiment. A large database of such records is available from the broadband portion of a temporary network of seismometers deployed on the upper edifice of Erebus volcano, Antarctica [Zandomeneghi et al., 2013, Knox, 2012]. Given the rapid decay of the direct waves for icequake peak frequencies of 1-3 Hz, we compute vertical component cross-correlation gathers for inter-station pairs with distances short enough for the Rayleigh wave to be both visible and distinct from other phases. Furthermore, given the extremely short scattering mean free path (several hundred meters to a few kilometers) computed for these frequencies on Erebus volcano [Chaput et al., in review], we only compute cross-correlations using events for which the direct ballistic arrivals show high signal to noise ratios. Naturally, if the event is too far from the station pair, the recorded envelope rapidly becomes cigar-shaped, and the multiple scattering influence on the recovery of a stable correlation function will tend to dominate any de-phasing of the Rayleigh wave due to anisotropy in the illumination. We compute the correlations using a 7.5 s window encompassing the direct
waves and a part of coda for all events within 3 kms of the station pair. Contributions from
sources that lie within a circular region between the station pairs are removed to better
match the numerical experiment. We first compute a reference using all events in this range,
representing an isotropic illumination of the receivers, and then progressively limit the source
distribution away from the inter-station tangent by a factor of $2\theta$. For the reconstruction to
be effective, a sufficiently large number of sources must remain in the limited distribution
to allow for coda convergence. Figure 4 shows an example of this for 2 stations (6s of
correlation shown) and an angular opening of 70 degrees, clearly showing the de-phasing
of the Rayleigh wave followed by an abrupt recovery in phase for the coda portion of the
correlations. It should be noted that the corresponding $\delta t(\tau)/\tau$ for the direct wave is on
the order of a few percent, thus making this effect worthy of consideration if this wave is
used to infer temporal changes in media. $\delta t$ values in the coda fluctuate with amplitudes on
the order of one fourth of the direct wave bias. Despite unfavorable experimental conditions
including source variability and locations in the vicinity of the receivers, we observe that the
average $\delta t(\tau)/\tau$ measured in the early coda (2-6s) is roughly 0.001, more than 30-50 times
less than for direct waves. Here, we have presented a single station pair to replicate the
numerical results as closely as possible, and we are consequently dealing with issues arising
from poor signal to noise ratios later in the coda. Where many equidistant station pairs are
available, averaging the $\delta t(\tau)/\tau$ measurements over similarly spaced pairs will necessarily
reduce coda fluctuations, and we can therefore expect far more accurate and smooth results. Furthermore, studies using ambient noise benefit from a nearly limitless quantity of data, whereas in our case we have a finite number of sources. Therefore, the error estimate for the coda given here should be considered a practical upper bound scenario.

5 Discussion and physical interpretation.

In the following we discuss the sensitivity of coda waves to noise anisotropic as derived from the basic properties of scattered waves. Firstly, we elaborate on the specific portions of the propagation paths that are sensitive to anisotropy of the noise source. Figure 5 presents a depiction of a scattered wave path. Let us consider the correlation of signal A with signal in B, that is, the propagation from A to B. The direct path between A and B is sensitive to the anisotropy of the noise around the direction $\theta = 0$. The coda waves consist of the superposition of the contributions of various scattered paths, as illustrate the beam forming results in Figure 2. A given multiply scattered path is commonly thought of as a random walk process [e.g. Gusev and Abubakirov, 1987, Margerin et al., 2000]. For such a random path, the bias produced by the anisotropy of the incident intensity exclusively affects the part of the path between the receiver acting as a virtual source (A here) and the first scatterer (S1, Fig. 5.). The rest of the path (the black trace in Fig. 5) is not affected if we assume
that the scatterers do not move. The bias $\delta t/\tau$ is therefore independent of the number of scattering events and of the total length of the path. We note that we observe this behavior in our numerical experiment and in the real data with fluctuations of the bias independent of the lapse time (Figures 3 and 4). Note that in practical applications of monitoring, the detection of a homogeneous velocity change assumes a constant fractional delay $\delta t(\tau)/\tau$.

The bias due to anisotropy of the noise does not produce such a constant fractional delay.

We use the expression of Eq. 2, valid for a direct wave bias only, to evaluate the sensitivity of the path between the virtual source and the first scatterer. We call $l_f$ the distance A-S1 for this particular path of Figure 5. The travel time between A and S1 is given by $t_f = l_f/V$. When using the expression 2, the relative error produced by anisotropy for a single scattering path can be written as:

$$\delta t \sim \frac{B''(\theta)}{2 t_f \omega^2_0 B(\theta)}.$$  \hspace{1cm} (3)

where $\theta$ is the azimuth of the path A-S1.

This result is however only valid for a single path. The characteristic average distance between the source and the first scatterer is given by the scattering mean free path $l$, and the corresponding mean free time, $t_l = l/V$. For our purpose, $l$ can be considered as the distance for which, on average, the wave behaves as a direct wave. We know that the average travel time between scattering events $t$ is given by $t_l$, the mean free time, but the summed average
timing error has to be computed in accordance with the underlying statistical distribution of the propagation time between two scattering events.

In a random walk scenario, it is intuitive to consider a fixed step represented by the mean free time. A discussion of the statistics of distance (or equivalently of time) between scattering events in the diffusion regime is out of the scope of this paper. We refer to Heiderich et al. [1994], who showed that the diffusion in a medium with anisotropic scattering can be described by a random walk process when the step length follows an exponential distribution. In our case, this means that the statistical distribution of $l_f$ is of the form:

$$P(t) = \frac{1}{t_l} \exp\left(-\frac{t}{t_l}\right).$$

(4)

The average of $\delta t$ cannot be computed directly because of the divergence of the equation 2 when $t$ approaches 0. Indeed, this limit is not in the validity range of the results of Weaver et al. [2009]. To identify a realistic and conservative upper bound of the average value $\langle \delta t \rangle$, we add the constraint that $\delta t$ must be smaller than $t$. This leads to the upper bound $\langle \delta t \rangle_{\text{max}}$ that we compare with the value of $\delta t$ computed for $t = t_l$ through the ratio:

$$R = \frac{\langle \delta t \rangle_{\text{max}}}{\delta t|_{t_l}}.$$  

(5)

For the source anisotropy depicted in Figure 1a ($B_2 = -0.6$), and choosing parameters characteristic of monitoring at the crustal scale (frequency $\sim 1\text{Hz}$), we obtain $R = 3.9$ for
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\( t_l = 30 \text{s} \) and \( R = 2.9 \) for \( t_l = 10 \text{s} \). We now make use of the approximation \(< \delta t > - \delta t|_{t_l}\)

and we apply the correction \( R \).

We extend this analysis for a single path and a single azimuth to all the different paths that contribute to the coda by averaging the bias \( \delta t \) over the azimuth \( \theta \) of the first scatterer.

Coda waves consist of multiple arrivals following pseudo-random paths. This implies that the analysis we propose for a single path must be generalized to an ensemble of paths, or, in other words, to a distribution of first scatters such as \( S1' \) on Fig. 5. We assume that these scatterers are evenly distributed with an average distance to \( A \) of \( l \). For each path, the coda of the correlation is affected by anisotropy of the incident intensity by a bias \( \delta t \). If we assume that the contributions of each path to the final result are equal and that their time shift \( \delta t \) are small \( (\omega_0 \delta t < 1) \), the total contribution is subject to a delay \(< \delta t > \) that is simply:

\[
< \delta t > = \frac{R}{2\pi} \int_0^{2\pi} \delta t(\theta) d\theta = \frac{R}{4\pi t_l \omega_0^2} \int_0^{2\pi} \frac{B''(\theta)}{B(\theta)} d\theta
\]

(6)

Whereas the bias expected for direct waves is the maximum value of the term \( \frac{B''(\theta)}{B(\theta)} \), we note that the coda could not suffer from such an elevated bias since it is represented by the average value of the term that is governing the error associated with anisotropy. When considering the same extreme case of anisotropy as used for the numerical simulations in section 3, we note that the term \( B''/B \) has a max value of 6, leading to the theoretical error in green of Fig. 3a. The average value \(< B''/B >\), however, is 1. This contributes to the
weaker sensitivity of coda waves and the rapid drop of $< \delta t >$ when passing from direct to
coda waves that has been noted in both numerical simulations and real data.

We have tested the relation 6 with two examples. It is important to recall here that the
physical parameter that is monitored is the seismic velocity $C$, through its relative variation:

$$\frac{\delta C}{C} = -\frac{\delta t(\tau_m)}{\tau_m} \tag{7}$$

where $\tau - m$ is the lapse time at which the measure is performed. The first case we consider
is the numerical example presented in this paper. We evaluate the relative error due to the
strong source anisotropy, and we further set the parameters as: $l = 0.5m$, $C = 2000m/s$,
$f_0 = 30000Hz$, $B_2 = -0.6$ and $\tau_m = 0.002s$. The equation 6 gives a fractional error $\frac{\delta t(\tau_m)}{\tau_m}$ of
$10^{-4}$, similar to what has been observed with the synthetics. The second case we consider
is the monitoring of velocity changes in Japan proposed by Brenguier et al. [2014]. In this
case the parameters are: $l=60 \times 10^3m$, $C = 3000m/s$, $f_0 = 0.3Hz$ and $\tau_m = 60s$. If we assume
extreme source anisotropy, setting $B_2 = -0.6$ as in the synthetic case, the equation 6 yields
a fractional error of $2 \times 10^{-4}$, that is slightly larger than what is observed. A less severe but
significant source anisotropy with $B_2 = -0.3$ implies a level of fractional error similar to
the one observed, on the order of $0.5 \times 10^{-4}$. Notice that Brenguier et al. [2014] averaged $\delta t$
for a set of station pairs, that results in further reducing the intrinsic uncertainties of the
measurements (Weaver et al. [2011]).

These agreements show that our simplified approach allows for predicting the expected
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level of error. It is nevertheless worth considering that our expression returns an upper bounds estimate. It is therefore likely that precisions better than $10^{-4}$ will be achieved when using waves in the late coda. We expect very good precision for the two end members of scattering regimes observed in practice. In the case of weak scattering, i.e. mean free path larger than inter station distance, and measurements in the late coda, formulas 6 and 6 indicate that large $t_l$ and $\tau_m$ imply small fractional errors. In the opposite case of strong scattering, the effect of scattering on the incoming source wavefield cannot be neglected. Given that the nature of the scattered wave field rapidly converges to a smooth and isotropic intensity distribution, the expected $B''(\theta)$ tends naturally to 0 and a perfect reconstruction is expected for the correlation functions [e.g. Colombi et al., 2014], leading to an absence of a bias even for direct waves as shown by Froment et al. [2010].

6 Conclusions

We show that ambient noise monitoring is subject to very small errors even in the presence of a temporally evolving source distribution. This is illustrated with numerical experiments as well as real data from a volcano. The measurements of fractional delay are far more stable for coda waves than for direct waves, due to intrinsic properties of scattered waves. We propose a simplified approach to evaluate the sensitivity of correlation functions to noise anisotropy
based on previous results obtained for direct waves. With our simple analysis we are able
to correctly estimate the errors produced by a strong anisotropy in numerical experiments,
and such effects are further documented for real data on a volcano. Our generalization also
yields realistic values for the ambient monitoring precision observed at the crustal scale with
real data. Our analysis shows that high precision noise-based monitoring could be performed
with coda waves even in presence of unknown variations in the intensity distribution of the
noise source. Furthermore in practice, averaging over roughly equidistant stations pairs
would reduce the bias associated with noise temporal evolutions.

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Figure 1: (Color online) (a) The source-receiver layout on the plate reproduced in the simulations. The four rings of sources (∼200), forming an annular region, are placed just outside the area containing scatterer and receivers (bounded by the light green circle). The angle $0 \leq \theta \leq 180^\circ$ is measured counter clockwise according to the convention. The four boundaries are modelled with PML condition to suppress reflections. When anisotropy is turned on, the intensity is distributed as shown by the colormap (for $B_2 = 0.6$). This is equivalent to work with a noise directivity as that indicated by the black arrows. (b) One snapshot of the propagation in the scatterers region with the source located in R1. (c) Comparison between the reference signal (thick grey line) and the causal part of the stacked cross-correlation (black line) computed using a uniform intensity distribution. Amplitudes are normalised.
Figure 2: (Color online). (a) The layout used for the beamforming analysis. The antenna is $30 \times 30$ cm large with $\sim 200$ receivers. The source is located near the edge of the scatterers region, 50 cm away from the center of the array. (b-c) Snapshots of the BF corresponding to the time windows indicated in (d).
Figure 3: (Color online) (a) The relative error $\delta t/\tau$ normalised over lapse time $\tau$ in blue and that normalised over a constant time $\delta t/T$ in black for the traces in (b). The scale is in percent (left axe). Each dot represents the center of the window used to estimate $\delta t$. The green line represent the relative error predicted using Eq. 2 for the first arrival. The gray line depict the value of the coherency inside each moving window. (value are in %). (b) Comparison between the unperturbed cross-correlation and the perturbed one.
Figure 4: A) Top panel, comparison of the vertical-vertical reference cross-correlation function with the cross-correlation function obtained from the limited distribution shown in B) (causal portion shown, pre-Rayleigh signal removed), at station MAC and ETB22. A) Bottom panel, $dt$ (black line), associated coherence values (colored dots), and $dt/t$ (red line) for a 0.5 s window and an overlap of 0.2 s. Note that the de-phasing that occurs in the Rayleigh wave falls off very rapidly in the coda.
Figure 5: (Color online) Sketch of one of the scattering paths produced a wave front (thick arrow) generated by a distant noise source.