Improved FE Simulation of Ultrasound in Plastics

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Abstract. Some UK and US nuclear power stations have begun introducing high-density polyethylene (HDPE) pipes to certain cooling water circuits. HDPE offers improved performance over existing pipe materials, such as cast iron, by not corroding in-ternally or externally, yet occasional defects form in HDPE pipe fusion joints at the production stage. This necessitates suitable volumetric NDE to safely and reliably assess joint integrity. Ultrasonic NDE is the most viable current technique, but improved inspection capability is needed, given that the challenges of NDE of plastics differ significantly from those of metals. This also necessitates an accurate and reliable wave propagation simulation technique, such as finite-element (FE) modelling. Accurate FE modelling of ultrasound in high-density polyethylene (HDPE) must account for frequency-dependent behaviour but, the most ap-parent way to do so – frequency domain FE modelling – is prohibitively computationally expensive and potentially impossible to solve for all but the smallest models. Here we present a multiband time domain FE simulation technique to address this. The proposed multiband technique is a computationally efficient and accurate approach to time domain FE modelling of ultrasonic wave propagation. It could, for example, be used to validate the NDE of a large range of candidate fusion joint defects in HDPE. The proposed model uses a small number of time domain FE simulations at individual frequency bands that together cover the bandwidth of interest. The frequency dependence of acoustic properties of ultrasound is accurately represented for HDPE and could readily be applied to other media.

INTRODUCTION

EDF Energy provides a large proportion of the UK’s energy via nuclear-generated electricity as well as gas. They are in the process of replacing some of the cast iron pipework in their tertiary cooling circuits with high-density polyethylene (HDPE) pipes that do not degrade via the internal and external corrosion, fouling, and microbial attack that currently reduce the service life of cast iron pipes [1]. When HDPE pipes are heat fused there is the potential for defects to occur on the fusion service [2], shown in figure 1. The defects can be either planar (lack of fusion, cold fusion) or volumetric (inclusions, porosity). The current most viable approach to direct NDE of these fusion surface defects is considered to be ultrasonic array inspection.

Similar defects as to those above, if found in other engineering materials such as steel, might be straightforward to inspect using current ultrasonic data acquisition and analysis techniques. However, waves propagating in HDPE pipes are highly attenuated via viscoelastic damping, which results in an especially low signal amplitude after propagation through larger path lengths. A material properties study is therefore necessary to accurately and reliably characterise HDPE. Other influences on HDPE material properties specific to the pipe geometry and inspection environmental conditions, such as temperature dependence, anisotropy and radial inhomogeneity, are also to be necessarily investigated in this ongoing study but will not be covered here.

To accurately simulate ultrasonic wave propagation in HDPE, its phase velocity and attenuation (dispersion relation) must be described with frequency dependence and, specifically, this must agree with the viscoelastic theo-retical description applied to polymers that describes both phase velocity and attenuation as varying via a frequency-dependent power law [3]. Further, the real and imaginary components of the dispersion relation must adhere to the Kramers-Kronig relation [4].

Current suitable techniques for describing frequency-dependent ultrasonic wave interaction with general defect geometries involve a numerical method such as finite element (FE) analysis. Wave propagation can be described with time domain FE or with frequency domain FE. The former is significantly more computationally efficient, espe-
volumetric planar HDPE weld defects

FIGURE 1. Cartoon of cross-section of HDPE pipe fusion joint with potential volumetric and planar defects found on the fusion surface.

specially for large modelled regions such as HDPE pipe fusion joints, but it only accurately describes material properties for a small frequency range, such as at the centre frequency. Frequency domain FE accurately describes frequency-dependent material properties for all frequencies modelled, so consequently it is significantly more accurate at describing the frequency-dependent propagation found in viscoelastic media such as HDPE. This undesired compromise between computational efficiency and accuracy can be avoided via the proposed multiband technique described here.

HDPE MATERIAL PROPERTIES

Discussed below is the approach to accurately simulate wave propagation in materials with a frequency-dependent dispersion relation (equation (2)), such as EDF Energy’s chosen pipe material, PE-100 – which will be referred to as the more general ‘HDPE’. This approach is realised through implementation of empirical descriptions of the phase velocity and attenuation obtained over a broad frequency range.

Experiment

The experimental apparatus used to obtain the material properties of HDPE is shown in figure 2. The experimental procedure involves immersing in water a test sample cut from HDPE pipe parent material. A broadband ultrasonic transducer is immersed at normal incidence to the surface of the sample and used to transmit and receive ultrasound such that frontwall and backwall reflections are obtained as amplitude-time traces (A-scans). The phase difference between the frontwall and backwall traces yields the compressional phase velocity, \( v_p \), and the amplitude ratio yields the attenuation \( \alpha \). Shear waves are too highly attenuated in HDPE and are therefore neglected, so only mode conversion loss rates are significant. Both 2.25 and 5MHz transducers are used to obtain independent measurements with partially coincident bandwidths, such that data can be proved consistent between multiple transducers.

Analysis

A 1D harmonic plane wave pressure field, \( p(x, t) \), can be defined in terms of the inverse Fourier transform of its spectrum,

\[
p(x, t) = \mathcal{F}^{-1} \{ S(x, f) \} = \mathcal{F}^{-1} \left\{ S(0, f)e^{ikx} \right\}
\]

(1)

where the spectrum, \( S(x, f) \), is the product of a spatially invariant term, \( S(0, f) \), and a propagation term, \( e^{ikx} \). The propagation term includes the dispersion relation,
which is defined in terms of both phase velocity and attenuation. Further, empirical descriptions of phase velocity and attenuation are obtained using

\[ v_p(f) = \frac{2\pi f x_s}{\phi_{B1}(f) - \phi_{F1}(f)} \]  \hspace{1cm} (3)

and

\[ \alpha(f) = -\frac{1}{x_s} \ln \left| \frac{B_1}{F_1}(f) \frac{1}{T(f)} \frac{D_{F1}}{D_{B1}}(f) \right| \]  \hspace{1cm} (4)

where \( x_s \) is the path length through the sample, \( B_1/F_1 \), is the ratio of backwall and frontwall spectra, \( T \) is the plane wave normal incidence transmittance, and \( D_{F1}/D_{B1} \) is the diffraction correction ratio for the frontwall and backwall reflections.

It is required for a viscoelastic material that attenuation obeys the following frequency power-law [3],

\[ \alpha(f) = \alpha_0 f^{y_\alpha}, \]  \hspace{1cm} (5)

where \( \alpha_0 \) and \( y_\alpha \) are constant. Similarly, phase velocity must obey the power-law [5],

\[ v_p(f) = v_{p0} + v_{p1} f^{y_{vp}}, \]  \hspace{1cm} (6)

where \( v_{p0} \), \( v_{p1} \), and \( y_{vp} \) are constant. Attenuation and phase velocity must also obey the Kramers-Kronig relation via [4],

\[ \alpha(f) = \frac{\pi^2 f^2}{v_{p0}} \frac{dv_p(f)}{df}. \]  \hspace{1cm} (7)

From which it is implied that

\[ y_{vp} = y_\alpha - 1. \]  \hspace{1cm} (8)

Plotted in figure 3 and 4 are frequency-dependent attenuation and phase velocity normalised to their respective values at 2.5MHz. In these figures the usable bandwidths of the 2.25 and 5MHz transducers define the dataset bandwidths, shown respectively as lines with crosses and dashed lines. A power law fit derived from the 5MHz attenuation dataset that has a functional form, \( f(x) = ax^y \), where \( a \) and \( y \) are constant, is plotted in figure 3 with a solid
line. In figure 4 a power law fit is also plotted with a solid line. This fit was constrained by equation (8) such that the fit had a functional form, $g(x) = c + bx^{\beta-1}$, where $b$ and $c$ are constant.

In both figure 3 and 4 the 2.25 and 5MHz datasets agree where they overlap, within an approximate 50% of the usable bandwidth. The increased discrepancy in the outer frequency quartiles is caused by the lack of usable frequency content transmitted and received by the transducers. Equivalently, in the spectral ratio, $B_1/F_1$, of equation (4), these outer quartiles are where one or both the spectra have a low amplitude relative to their respective maxima and consequently the relative uncertainty from dividing the two spectra is large. Further, in the regions of higher accuracy the power law fits are adhered to, as required by established theories of viscoelastic wave propagation (equation (5) and (6)).

**MULTIBAND TIME DOMAIN FE MODEL**

The empirical phase velocity, $v_p(f)$, and attenuation, $\alpha(f)$, obtained above are applicable to equation (1) via equation (2), such that analytical wave propagation in HDPE can be accurately simulated. It is necessary also to model ultrasonic interactions with general defect geometries. This added complexity necessitates a numerical method. Finite element (FE) analysis is considered to be an approach well suited to this requirement. General geometries can be modelled via a variety of techniques to optimise for accuracy and efficiency. The current availability of many commercial FE software codes adds to the versatility of this approach.

FE simulations can be computed either by explicit time integration in the time domain, or by frequency response solutions in the frequency domain. The advantage of frequency domain FE is that solutions can be obtained accurately by separate solutions at each frequency over the bandwidth of interest, which is critical when the validity of the simulation relies on the accurate description of frequency-dependent material properties. It is, however, much less computationally efficient than time domain FE, especially for large modelled regions such as HDPE pipe fusion joints where greatest paths can be of the order of 200 wavelengths at input centre frequency. It might also be impossible to solve in the frequency domain for the largest modelled regions. Accuracy is crucial to a valid model; computational efficiency is crucial when simulating large modelled regions, and more so when applied to the example of array
FIGURE 4. Normalised phase velocity as a function of frequency over the usable bandwidths of 5MHz (dashed line) and 2.25MHz (solid line with crosses) transducers with a power law fit (solid line) derived from the 5MHz dataset using the attenuation power law fit exponent, y.

imaging where multiple solutions are obtained for transmitting and receiving elements. Using either frequency or time domain FE results in a compromise that can potentially be avoided via the multiband time domain FE model.

Method

The multiband procedure requires multiple time domain FE simulations to be performed, each of which has a centre frequency equal to that of the centre frequency of each frequency ‘band’. This is because a time domain FE simulation is most accurate at its centre frequency. At the band centre frequencies alone, these time domain simulations provide a mapping between initial and analytical propagated pulses known as the transfer function,

\[ H(f) = \frac{S_p(f)}{S_{p,\text{init}}(f)}, \tag{9} \]

where \( S_p(f) \) is the spectrum of the analytical propagated pulse and \( S_{p,\text{init}}(f) \) is the spectrum of the initial pulse.

The multiband spectrum is now defined at its band centre frequencies. To define it for the whole bandwidth of interest, interpolation between the band centres is required. This is where our several devised multiband schemes differ, where only one is discussed here. Either the transfer function, \( H(f) \), or the spectrum, \( S(f) \), can be interpolated. The scheme we adopt for illustration in this article is ‘tanh filtering’. First, all values of the transfer function within each band are set to that of its respective centre frequency transfer function, obtained via each single time domain FE simulation. This results in a stepped, discontinuous transfer function. The propaged spectrum resulting from this transfer function features sharp gradients and discontinuities near the band edges. These are subsequently smoothed by scaling each band independently by \( \tanh(f) \) to yield spectra such as that of figure 5.

The locations of the band edges (and centres) have been optimised for highest accuracy for a given number of bands. The band centre with the highest amplitude – the second of three in figure 5 – is set as the peak amplitude of the analytical propagated pulse. This shall be the definition of the centre frequency, \( f_{0,p} \), noting that the skew of the spectrum causes this modal value to differ significantly from the mean or median. To define the other band centres and corresponding edges the area under the analytical spectrum is divided into equal bands such that equal frequency content is given to each band. This avoids the bias of, for example, dividing into equal bandwidth sections,
where evidently the outer bands (1st and 3rd here) would be solved at centre frequencies where minimal pulse frequency content exists. The analytical pulse after propagation, \( p \), necessarily describes only ray-like behaviour including (specular) reflections from large flat regions. Consequently, interaction with general defects including scattering and other frequency-dependent behaviours would alter the shape of the propagated spectrum. This means that, while the band locations are optimal, they can never be perfect. Surpassing this limit would require \textit{a priori} knowledge of the propagated spectrum after defect interaction and consequently negate the need for any such simulation technique as multiband.

In general, the more bands, the higher the accuracy of the multiband method but, given that the FE solution will asymptote to the analytical solution until the number of bands equals the sample-rate of the analytical spectrum, there is consequently a decreased improvement in accuracy between higher numbers of bands. Increasing the number of bands increases the total computational execution duration, given that the product of the number of bands and the duration of a single time domain simulation equals the total run time. This is where the compromise between accuracy and efficiency can be optimised, yet it will be shown that in practice there is little compromise, because as few as three bands can greatly increase accuracy over one time domain simulation.

**Analysis**

Figure 5 shows spectral amplitudes as a function of frequency for a single time domain run (a) and for the tanh filtering multiband scheme (b) where here the multiband spectrum is split into three bands.

The initial pulses input into the FE simulation, \( p_{\text{init}} \), are the solid lines associated with the left-hand y-axes. These are the same for (a) and (b). The lines with circles are associated with the right-hand y-axes, \( p \) are the pulses after analytical propagation of 100 wavelengths through HDPE, as governed by equation (1). The spectra are again the same in (a) and (b). The spectrum has shifted down in frequency and dropped significantly in amplitude, owing to the frequency-dependent damping that has attenuated it. The FE approximation to these propagated pulses, \( p_{\text{FD}} \) and \( p_{0} \), are the lines with crosses in (a) and (b) whose scales are also shown on the right-hand y-axes. The input pulses are 5-cycle tone bursts centred at \( f_{0} = 5 \text{MHz} \). The y-axes are necessarily separate for input and propagated pulses as the ratios of maximum amplitudes are of the order \( 10^{3} \), given that the path here was chosen to be the greatest expected median inspection path with of the order of 100 wavelength at 5MHz. This is because greatest distance was chosen so as to show the most demanding case, given that multiband accuracy decreases with path length and therefore larger paths more clearly exhibit subtle differences in accuracy.

It is clear that a single time domain simulation exhibits poor agreement with the analytical pulse. A single time domain FE simulation cannot describe the frequency dependence of the attenuation and therefore its centre frequency remains at 5MHz rather than shifting to below 4MHz. The amplitude of the pulse is also less than half of that required. By contrast, tanh filtering multiband, with just three bands, greatly increases accuracy. More bands can be added to achieve a greater accuracy still. However, the discontinuities at the band edges result in some low-amplitude artefacts in the time domain of this multiband propagated pulse, the significance of which will be discussed below.

Figure 6 shows the pulse amplitude as a function of time for a single time domain run (a) and for the tanh filtering multiband scheme (b). The initial pulses input into the FE simulation, \( p_{\text{init}} \), are the lines with circles associated with the left-hand y-axes; the pulses after analytical propagation through HDPE, \( p \), are the dashed lines whose scales are shown on the right-hand y-axes; and the FE approximation to these propagated pulses, \( p_{\text{FD}} \) and \( p_{0} \), are the solid lines whose scales are also on the right-hand y-axes. Two y-axis scales are used because the attenuation is very high after such a large propagation. The time axes are broken to remove the regions where no pulses exists between 1.5 and 23.5\( \mu \text{s} \).

The poor agreement between the single time domain FE simulation and the analytical propagated pulse is again evident. The amplitude is less than half of that required but, also, the time of arrival is close to a microsecond (4\%) early. This is a consequence of the single time domain simulation not accounting for the frequency dependence of the phase velocity. The multiband approach can be seen to deliver a signal of very similar amplitude to the required pulse. Mentioned above, the discontinuities at the edges of the bands of the amplitude spectrum cause the long-duration low-amplitude ringing artefacts, yet the amplitude would in practice be near or below the noise floor of a signal that had propagated such long distances; and pulses travelling shorter paths, where the noise floor would be lower relative to the pulse amplitude, would have a more accurate multiband representation, such that these artefacts would probably never reduce accuracy, even with as few as three bands. The time of arrival is also close to exactly that of the analytical pulse. With tanh filtering the phase velocity is constant with frequency but taken to be that predicted empirically via the analytical description at the centre frequency of the propagated pulse, \( v_{p}(f = f_{0,p} = 4 \text{MHz}) \), while the single time
CONCLUSION

The multiband time domain FE model has been shown to improve accuracy significantly over a single time domain FE simulation while only taking a total computation run time equal to the product of the number of bands with the run time of a single time domain simulation. This is favourable when compared also with frequency domain FE, where accuracy is high but at significant cost to computational efficiency, especially when modelling large regions such as the HDPE pipe fusion joints exemplified here. Such large models might also be impossible to realise in frequency domain FE.

The multiband technique can not only be applied to HDPE but any media where the requirement is to accurately and efficiently describe the frequency dependence of ultrasonic (or acoustic) wave propagation. While only compressional waves are considered here, shear wave dispersion could be simulated without alteration to the multiband method.

In this ongoing study, other influences on HDPE material properties specific to the pipe geometry and inspection environmental conditions, such as temperature dependence, anisotropy and radial inhomogeneity, are to be necessarily investigated. Further, alternative multiband schemes will be quantitatively analysed and compared with tanh filtering and a single time domain FE simulation.

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FIGURE 6. Time traces for (a) single time domain FE and (b) tanh filtering multiband FE with three bands. The input pulses, \(p_{\text{init}}\), (lines with circles) are 5-cycle tone bursts centred at \(f_0 = 5\text{MHz}\) that are associated with the left-hand y-axes. The empirical analytically propagated pulses, \(p\), after 100 wavelengths of propagation (dashed lines), and the FE approximations to these propagated pulses, \(p_{f0}\) and \(p_0\), (solid lines) are on the right-hand y-axes. The time axes are broken to remove the regions where no pulses exists between 1.5 and 23.5\(\mu\)s.

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