Option Value of Soft Open Points in Distribution Networks

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***Abstract*—We propose a novel stochastic planning model that considers investment in conventional assets as well as in Soft Open Points, as a means of treating voltage and thermal constraints caused by the increased penetration of renewable distributed generation (DG) sources. Soft Open Points are shown to hold significant option value under uncertainty; however, their multiple value streams remain undetected under traditional deterministic planning approaches, potentially undervaluing this technology and leading to a higher risk of stranded assets.**

***Index Terms*--Active Network Management, Option Value, Soft Open Point (SOP), Stochastic Optimization**

NOMENCLATURE

Sets and indices

Ω_C	Set of normally-open points, indexed c
Ω_E	Set of epochs, indexed e
Ω_{DG}	Set of distributed generation units, indexed g
Ω_L	Set of distribution lines, indexed l
Ω_M	Set of scenario-tree nodes, indexed m
Ω_N	Set of system buses, indexed n
Ω_{PS}	Set of primary substations, indexed g
Ω_q	Set of typical days, indexed q
Ω_T	Set of demand periods, indexed t
ε_m	Epoch to which scenario-tree node m belongs
$\Phi_k(m)$	Time-ordered set containing all parent nodes of scenario-tree node m, from the first epoch up to epoch $\varepsilon_m - k$

Input Variables

γ_B	Annuitized investment cost for reconductoring a distribution line l (£/year)
γ_S	Annuitized SOP investment cost (£/year)
δ_t	Duration of one period (hours). Here it is 1 hour.
η_f	SOP efficiency factor
π_m	Probability of scenario-tree node m occurring
$\Psi_{n,t}$	Tangent of the load angle at bus n at period t
$\zeta_{t,g}$	Percentage output of intermittent generator g at period t relative to its installed capacity
b_l^{O}	Line susceptance before reinforcement (pu)
b_l^{N}	Line susceptance after reinforcement (pu)
c^{c}	Cost of curtailing DG output ($£/pu \cdot h$)
F_{max}	Extra capacity, obtained from reconductoring, relative to the existing capacity (pu)
F_l	Existing capacity of line l (pu)
g_l^{O}	Line conductance before reinforcement (pu)
g_l^{N}	Line conductance after reinforcement (pu)
$I_{n,g}$	Signifies if generator g is connected to bus n
$d_{t,n}$	Real power demand at bus n, period t (pu)
k_L	Build time for distribution line l (epochs)
k_s	Build time for SOP (epochs)
N_q	Frequency of typical day q in a year (days)
n_c^x	The two terminals $(x = a, b)$ of SOP which is installed at normally-open point c
$P_{m,g}^{max}$	Max real power stable generation of g (pu)
$Q_{m,g}^{max}$	Max reactive power stable generation of g (pu)
$r_{\varepsilon_m}^{I}$	Cumulative discount factor for investment cost
$r_{\varepsilon_m}^{O}$	Cumulative discount factor for operational cost
P_c^{max}	Real power capacity of SOP installed at c (pu)
Q_c^{max}	Reactive capacity of SOP installed at c (pu)
u_l	Sending bus of line l
v_l	Receiving bus of line l
V_{set}	Voltage setpoint value at primary substation (pu)
V_{min}	Minimum voltage statutory limit (pu)
V_{max}	Maximum voltage statutory limit (pu)

Decision Variables

$\theta_{m,t,n}$	Voltage angle corresponding to bus n (rad)
$B_{m,l}$	Binary variable for deciding to reconductor l
$\tilde{B}_{m,l}$	State variable of reconductoring line l
$\tilde{F}_{m,l}$	State variable representing the extra capacity due to reconductoring of line l (pu)
$H_{m,t,c}$	Real power drawn by SOP at terminal n_c^a (pu)
$H_{m,t,c,n}^{Q}$	Reactive power drawn by SOP at terminal n (pu)
$R_{m,t,c}$	Real power drawn by SOP at terminal n_c^b (pu)
$P_{m,t,g}$	Real power output of unit g (pu)

$P^{s}_{m,t,l}$	Real power flow at sending bus of line l (pu)
$P^{r}_{m,t,l}$	Real power flow at receiving bus of line l (pu)
$Q_{m,t,g}$	Reactive power output of unit g (pu)
$Q^{s}_{m,t,l}$	Reactive power flow at sending bus of l (pu)
$Q^{r}_{m,t,l}$	Reactive power flow at receiving bus of l (pu)
$S_{m,c}$	Binary variable for deciding to invest in SOP
$\tilde{S}_{m,c}$	State variable of SOP investment
$V_{m,t,n}$	Voltage magnitude at bus n (pu)

I. Introduction

A larger penetration of distributed generation (DG) sources is expected over the next decades, potentially leading to significant network reinforcements so as to cope with ensuing voltage and thermal constraints; however, the increased uncertainty that surrounds future generation connections may prevent network planners from making fully informed decisions. This uncertainty rests on the fact that DG installations proceed without prior coordination with the network planners, thereby making it impossible for Distribution Network Operators (DNOs) to accurately determine in advance where voltage and/or thermal violations may occur. As a result, conventional network reinforcements run the risk of turning into stranded assets, thus potentially limiting the effectiveness and rate of DG deployment due to increased integration costs.

Active distribution network management is an alternative to conventional reinforcements [1] including technologies such as Soft-Open Points (SOPs) [2] and techniques such as curtailment of active distributed generation. While the regulatory basis for curtailment differs across Europe [1], it is a generally accepted measure of last resort. With regards to SOP, although it is characterized as a mature technology given the commercial availability of power electronic converters [3], at present there are limited, if any, operational examples of its deployment across European networks. The benefits stemming from the ability for real-time network reconfiguration are well recognized. It has also been suggested that SOPs can constitute valuable strategic solutions when facing uncertainty [4]. However, there is currently a gap in existing modelling capabilities for the strategic evaluation of such flexible assets [5]. The paper's contributions are as follows:

- Presentation of a multistage stochastic cost-benefit framework allowing investment in conventional and flexible assets while also accommodating active network management principles such as active generation curtailment of PV units.
- Development of a methodology to quantify the option value of investing in SOPs in distribution networks.
- Demonstration of the inherent inability of deterministic approaches to discover cost-efficient strategic opportunities for SOP deployment through a comprehensive case study.

II. Planning Under Uncertainty

In this work, when we characterize SOPs as flexible assets we refer to two types of flexibility. First, SOPs have non-localized effects, meaning that their operation can not only improve the DG hosting capability of a single line or busbar but it can affect a broader network area by enhancing the utilization of the existing assets. Secondly, when facing uncertainty, SOPs can be deployed on an interim basis, thereby rendering 'wait-and-see' investment strategies cost-effective and viable. This is because SOPs typically have faster build times than conventional reinforcements since time-consuming planning permissions or asset reinforcement activities are not necessary. As a result, investment in SOPs can enable the deferral of conventional reinforcements until their need is justified, ultimately leading to system cost reductions. We incorporate the aforementioned flexibility of SOPs in the term option value [6]. It is imperative to highlight that the increasing relevance of incorporating option value in investment appraisals is gaining traction with industry and institutions worldwide (for example see [7]).

Numerous techniques have been suggested for quantifying the option value of flexible assets, with the majority of efforts focusing on Real Options Analysis [8]. Nevertheless, the application scope of such valuation frameworks is limited to a small number of candidate investment strategies defined a priori. In reality, however, a very large number of competing strategic opportunities can arise in distribution system design due to the planner's ability to dynamically adjust his strategy according to the uncertainty evolution across a wide range of investment technologies, locations and timings [9]. To this end, the use of an optimization framework is essential for the identification of the optimal investment strategy across all candidate solutions [10].

III. Mathematical Formulation

A multi-stage scenario tree consisting of $|\Omega_{\mathrm{M}}|$ nodes spanning $|\Omega_{\mathrm{E}}|$ epochs is used to model the uncertainty around penetration of distributed resources. The planner can choose to either invest in reconductoring of distribution lines, deploy SOPs at any of the available normally-open points or resort to DG generation curtailment. The planning problem is formulated as a stochastic mixed integer nonlinear problem.

$$z = \min_{B,C,D,S} \left\{ \sum_{m\in\Omega_M} \pi_m \left(r^{I}_{\varepsilon_m} \omega^{I}_{m} + r^{O}_{\varepsilon_m} \omega^{O}_{m} \right) \right\} \quad (1)$$

$$\omega^{I}_{m} = \sum_{l\in\Omega_L} B_{m,l}\gamma_B + \sum_{c\in\Omega_C} S_{m,c}\gamma_S \quad (2)$$

$$\omega^{O}_{m} = \sum_{t\in\Omega_T}\sum_{q\in\Omega_q} N_q \delta_t c^c \sum_{g\in\Omega_{DG}} \left(P^{max}_{m,g}\zeta_{t,g} - P_{m,t,g}\right) \quad (3)$$

$$\tilde{B}_{m,l} = \sum_{\varphi\in\Phi_{k_L}(m)} B_{\varphi,l} \qquad \forall\, m,l \quad (4)$$

$$\tilde{F}_{m,l} = \sum_{\varphi\in\Phi_{k_L}(m)} B_{\varphi,l}F_{max} \qquad \forall\, m,l \quad (5)$$

$$\tilde{S}_{m,c} = \sum_{\varphi\in\Phi_{k_S}(m)} S_{\varphi,c} \qquad \forall\, m,c \quad (6)$$

$$P_{m,t,g} \le P^{max}_{m,g} \qquad \forall\, m,t,g\in\Omega_{PS} \quad (7)$$

$$Q_{m,t,g} \le Q^{max}_{m,g} \qquad \forall\, m,t,g\in\Omega_{PS} \quad (8)$$

$$P_{m,t,g} \le P^{max}_{m,g}\cdot\zeta_{t,g} \qquad \forall\, m,t,g\in\Omega_{DG} \quad (9)$$

$$Q_{m,t,g} \le Q^{max}_{m,g}\cdot\zeta_{t,g} \qquad \forall\, m,t,g\in\Omega_{DG} \quad (10)$$

$$P_{m,t,l}^{s} = (1-\tilde{B}_{m,l})[V_{m,t,u_l}^2 g_l^o - V_{m,t,u_l}V_{m,t,v_l}g_l^o \cdot \cos(\theta_{m,t,u_l}-\theta_{m,t,v_l}) - V_{m,t,u_l}V_{m,t,v_l}b_l^o \sin(\theta_{m,t,u_l}-\theta_{m,t,v_l})] + \tilde{B}_{m,l}[V_{m,t,u_l}^2 g_l^N - V_{m,t,u_l}V_{m,t,v_l}g_l^N \cos(\theta_{m,t,u_l}-\theta_{m,t,v_l}) - V_{m,t,u_l}V_{m,t,v_l}b_l^N \cdot \sin(\theta_{m,t,u_l}-\theta_{m,t,v_l})] \quad \forall\, m,t,l \tag{11}$$

$$P_{m,t,l}^{r} = (1-\tilde{B}_{m,l})[V_{m,t,v_l}^2 g_l^o - V_{m,t,v_l}V_{m,t,u_l}g_l^o \cdot \cos(\theta_{m,t,v_l}-\theta_{m,t,u_l}) - V_{m,t,u_l}V_{m,t,v_l}b_l^o \sin(\theta_{m,t,v_l}-\theta_{m,t,u_l})] + \tilde{B}_{m,l}[V_{m,t,v_l}^2 g_l^N - V_{m,t,u_l}V_{m,t,v_l}g_l^N \cos(\theta_{m,t,v_l}-\theta_{m,t,u_l}) - V_{m,t,u_l}V_{m,t,v_l}b_l^N \cdot \sin(\theta_{m,t,v_l}-\theta_{m,t,u_l})] \quad \forall\, m,t,l \tag{12}$$

$$Q_{m,t,l}^{s} = (1-\tilde{B}_{m,l})[-V_{m,t,u_l}^2 b_l^o - V_{m,t,u_l}V_{m,t,v_l}g_l^o \cdot \sin(\theta_{m,t,u_l}-\theta_{m,t,v_l}) + V_{m,t,u_l}V_{m,t,v_l}b_l^o \cos(\theta_{m,t,u_l}-\theta_{m,t,v_l})] + \tilde{B}_{m,l}[-V_{m,t,u_l}^2 b_l^N - V_{m,t,u_l}V_{m,t,v_l}g_l^N \sin(\theta_{m,t,u_l}-\theta_{m,t,v_l}) + V_{m,t,u_l}V_{m,t,v_l}b_l^N \cos(\theta_{m,t,u_l}-\theta_{m,t,v_l})] \quad \forall\, m,t,l \tag{13}$$

$$Q_{m,t,l}^{r} = (1-\tilde{B}_{m,l})[-V_{m,t,v_l}^2 b_l^o - V_{m,t,u_l}V_{m,t,v_l}g_l^o \cdot \sin(\theta_{m,t,v_l}-\theta_{m,t,u_l}) + V_{m,t,u_l}V_{m,t,v_l}b_l^o \cos(\theta_{m,t,v_l}-\theta_{m,t,u_l})] + \tilde{B}_{m,l}[-V_{m,t,v_l}^2 b_l^N - V_{m,t,u_l}V_{m,t,v_l}g_l^N \sin(\theta_{m,t,v_l}-\theta_{m,t,u_l}) + V_{m,t,u_l}V_{m,t,v_l}b_l^N \cos(\theta_{m,t,v_l}-\theta_{m,t,u_l})] \quad \forall\, m,t,l \tag{14}$$

$$\left(P_{m,t,l}^{s,r}\right)^2 + \left(Q_{m,t,l}^{s,r}\right)^2 \le \left[F_l + \tilde{F}_{m,l}\right]^2 \quad \forall\, m,t,l \tag{15}$$

$$V_{min} \le V_{m,t,n} \le V_{max} \quad \forall\, m,t,n-\{1\} \tag{16}$$

$$V_{m,t,1} = V_{set} \quad \forall\, m,t \tag{17}$$

$$R_{m,t,c} \le P_c^{max} \cdot \tilde{S}_{m,c} \quad \forall\, m,t,c \tag{18}$$

$$H_{m,t,c} \le P_c^{max} \cdot \tilde{S}_{m,c} \quad \forall\, m,t,c \tag{19}$$

$$|H_{m,t,c,n}^{Q}| \le Q_c^{max} \cdot \tilde{S}_{m,c} \quad \forall\, m,t,c \tag{20}$$

$$\sum_{g\in\Omega_g} P_{m,t,g} I_{n,g} - \sum_{l\in\{\Omega_L|v_l=n\}} P_{m,t,l}^{r} - \sum_{l\in\{\Omega_L|u_l=n\}} P_{m,t,l}^{s} = +d_{t,n} + \sum_{c\in\{\Omega_C|n=n_c^a\}} (H_{m,t,c} - R_{m,t,c}\eta_f) + \sum_{c\in\{\Omega_C|n=n_c^b\}} (R_{m,t,c} - H_{m,t,c}\eta_f) \quad \forall m,t,n \tag{21}$$

$$\sum_{g\in\Omega_g} Q_{m,t,g} I_{n,g} - \sum_{l\in\{\Omega_L|v_l=n\}} Q_{m,t,l}^{r} - \sum_{l\in\{\Omega_L|u_l=n\}} Q_{m,t,l}^{s} = +\Psi_{n,t} d_{t,n} + \sum_{c\in\{\Omega_C|n=n_c^a \text{ or } n=n_c^b\}} H_{m,t,c,n}^{Q} \quad \forall m,t,n \tag{22}$$

The objective function is given by (1) describing the minimization of the discounted expected investment (2) and operational (3) cost. Constraints (4) and (6) define the state variables that aggregate all investment decisions taken in previous epochs while also considering the corresponding commissioning delays. Constraints (7) and (8) set the upper limits for the real and reactive power that flow through the primary substation transformer, while (9) and (10) represent the real and reactive generation of DG units. Constraints (11)-(14) express the AC power flow equations in the form of a disjunctive formulation dependent on state variable $\widetilde{B}_{m,l}$ in order to capture the effect that reconductoring has on a line's electrical characteristics b_l and g_l. Note that different variables are used to model the flow at the sending and receiving ends of each line similar to [12]; differences between these variables represent losses over the line. Constraint (15) states that real and reactive power flows cannot exceed the line's thermal rating. This constraint can be relaxed and expressed linearly [12] or approximated in a piecewise-linear form [13]. Constraint (16) defines the statutory voltage limits for all system buses, with the exception of the substation busbar ($n = 1$) where the OLTC keeps the voltage at a setpoint V_{set}, as in (17). Modelling the OLTC in this manner guarantees that the optimal value of the substation voltage will not be affected by any other bus voltage across the network, given that the OLTC does not have visibility of network parameters. Constraints (18) - (20) impose the upper bounds for the real and reactive power that a SOP can absorb or generate. The position of the SOP in the network is defined by its two terminals (or ports) n_c^a and n_c^b corresponding to the normally open point c. Then, the variables $R_{m,t,c}$ and $H_{m,t,c}$ that can only assume positive values, are used to model the ability of a SOP to transfer active power in any direction between its two terminals with efficiency η_f. The SOP can also absorb or generate reactive power at any of its two terminals. Finally, (21) and (22) ensure application of the second Kirchhoff law at every system bus.

IV. Case Study

We present a case study where the prospect of large PV penetration can lead to voltage rise complications, thus driving investments in the distribution network or requiring active generation curtailment of PV units. We illustrate how radically the optimal investment strategy of a stochastic planner can change when considering SOPs as a candidate investment alternative in addition to reconductoring. We also demonstrate the shortcomings of traditional deterministic methodologies in undervaluing the flexibility benefits of SOPs.

A. Description

The Medium Voltage (11kV) semi-urban overhead distribution network is depicted in Fig. 1 where the six normally-open points are marked by dotted lines. As we can observe, a total of 6 buses may accommodate some PV capacity, but this happens only in a stochastic manner. That is, the amount of distributed PV generation to be connected over the six-year horizon (three epochs/stages each of 2-year duration) is uncertain in time, size and location of connection. This uncertainty is captured by the scenario tree (Fig. 2), constructed based on expert opinion.

Network operation should take place within statutory voltage limits defined at all buses to be 1.1 and 0.9 pu. To achieve this, a benchmark value of £100/MWh is selected for the cost of curtailing active generation of PV units. In addition, the planner has two alternatives for investment, as shown in Table I where the respective investment costs have been estimated according to relevant sources ([14] [15]). Note that the term 'build time' refers to the number of epochs starting from the epoch at which the decision to invest is taken, up to the epoch at which the investment becomes operational. The difference in build time between SOPs and the conventional investment (reconductoring) can be attributed to the fact that the latter involve greater network intervention that can be subject to lengthy permissioning processes.

The SOP technology allows optimal control of active power flow through its two terminals (or ports) and optimal reactive compensation at any of its two terminals; 90% efficiency (in transporting active power from one terminal to the other) and 130 kW / 130kVAr capacity are used in the case study. The other candidate technology is the reconductoring of a distribution line, which involves the replacement of an existing line with a new lower-resistance conductor. All existing lines have R/X factor equal to two, with a cross-sectional area of $40mm^2$ and R=0.6 Ω/km; new lines have R/X factor equal to one, $200mm^2$cross-section and R=0.12 Ω/km.

Given that each scenario node in Fig. 2 covers a two-year duration, we need to express the investment and operational costs in annual terms. The former can be easily done by dividing the investment cost (Table I) by the corresponding number of years comprising an epoch and discounting appropriately. The latter could be ideally calculated by considering the network operation across 8760 hourly periods. However, in a nonlinear setting this method leads to intractability. Hence, we resort to approximating the seasonal variations across a year by three typical days, each characterized by a combination of demand and solar insolation as shown in Fig. 3 and Fig. 4. This is a typical approach taken to alleviate the computational load of planning studies without compromising solution integrity (e.g. see [16]). Two of the days correspond to the summer season, while the 'average' day represents the other three seasons of a year. 0displays the number of days in a year represented by the three typical days. Special focus is placed on the summer as the subject of study is the voltage rise effect, and summer in the UK is characterized by high PV generation and low demand levels. Note that the 'summer-high' day, having the highest insolation and lowest demand levels, exhibits the most appropriate conditions for the creation of the voltage rise effect. Regarding the load and PV power factors, we assume a constant value equal to 0.9 and 1 respectively.

Table II also shows the assumed substation voltage setpoints; these values are traditionally selected above 1 pu as a means of preventing voltage drop at remote buses, with lower setpoints selected for summer due to lower likelihood of voltage drop occurring. We utilize the model presented in Section III to perform a number of deterministic and stochastic programming studies. All models were developed using FICO Xpress 7.6 [17].

TABLE I. AVAILABLE TECHNOLOGIES FOR INVESTMENT

Technology	Build Time (epochs)	Investment Cost
SOP	0	£90,000
Reconductoring	1	£65,000 / km

TABLE II. CHARACTERISTICS OF TYPICAL DAYS

Typical Day	Substation Setpoint (pu)	Annual occurrence (days)
'Summer-high'	1.01	45
'Summer-low'	1.01	45
'Average'	1.03	275

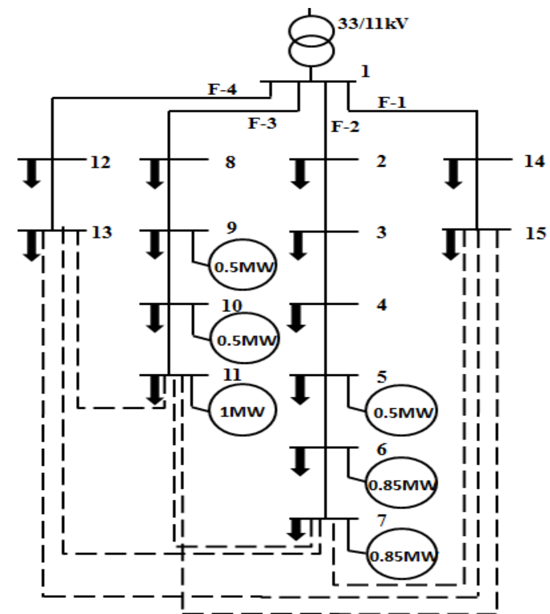

Figure 1. Diagram of the semi-urban 11kV distribution network, showing prospective DG connections. Any section between two buses is considered to be a one-km length distribution line.

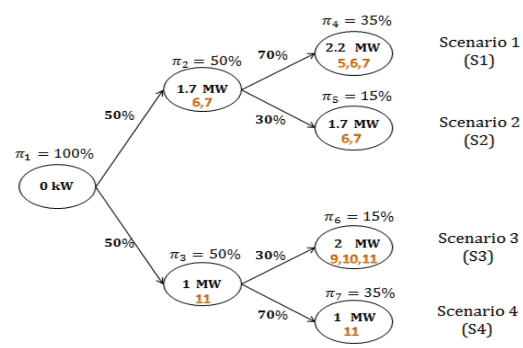

Figure 2. Scenario tree, with 7 nodes across 4 scenarios, capturing the uncertainty of PV capacity (MW). Transition probabilities are shown above each arc, while π_m is the probability of node m occurring. Inside each node we show the aggregate PV capacity installed and the buses to which the PV units connect (red font).

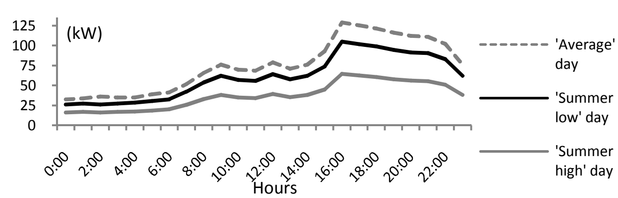

Figure 3. Demand pattern for each typical day, assuming that all load buses have an identical electricity consumption profile.

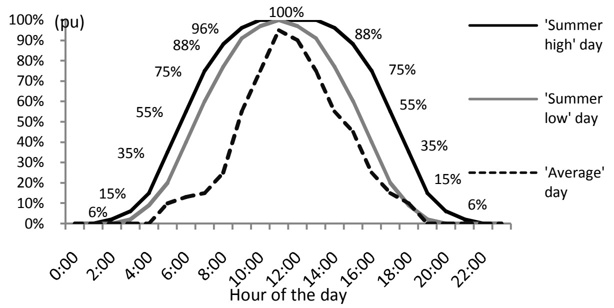

Figure 4. Normalized PV generation profiles for each typical day.

B. *Deterministic planning*

We find the optimal investment schedule for each of the four scenarios (S1– S4) depicted in Fig. 2 by applying the model described in Section III and setting the relevant π_m probabilities equal to one. Note that the planner can invest in both technologies shown in Table I. The resulting output is shown in Table III where [a-b] represents the decision to invest in reconductoring line a - b, while TIC and TOC represent total investment and operational cost respectively.

TABLE III. DETERMINISTIC STUDY

	Investment Decisions			**Costs (£k)**		
	Epoch 1	Epoch 2	Epoch 3	*TIC*	*TOC*	Total
S1	[2-3],[3-4], [4-5],[5-6]	-	-	*127.9*	*110.9*	*238.8*
S2	[2-3],[3-4], [4-5],[5-6]	-	-	*127.9*	*39.8*	*167.7*
S3	[8-9]	[1-8]	-	*52.3*	*46.7*	*99.0*
S4	[9-10]	-	-	*32.0*	*0*	*32.0*

It is remarkable that SOP technology is fully ignored because a deterministic planner does not consider the possibility of conditional adjustments to the optimal investment policy, thus neglecting the strategic benefits that accompany SOP technology. In addition, although the planner considers deterministic growth in PV capacity, in reality the eventual scenario realization is uncertain. Hence, in the event that some PV connections do not materialize according to the scenario considered, some of the capital decisions may prove to be inefficient. Note that first-stage commitments are particularly risky since they forego the possibility for strategically exploiting the uncertainty resolution that occurs in the second stage. For example, if the planner decides to follow the optimal investment schedule for scenario 3 and scenario 2 materializes instead, then line [8-9] will have been a stranded investment; the planner will need to re-adjust his capital commitments while also incurring PV curtailment costs in the interim.

C. *Stochastic planning*

In this section, two stochastic planning studies are carried out. In the first, line reconductoring is the sole available investment option. In the second, we allow the planner to consider both reconductoring and SOPs. The optimal investment strategies for both studies are shown in Table IV and Table V respectively. Note that E{TC} represents the expected sum of investment (E{IC}) and operational (PV curtailment) cost (E{OC}), while S(a-b) represents the decision to invest in a SOP at the normally-open point between buses a - b.

We observe that the availability of SOP to the planner leads to reduced investment in reconductoring; only lines 4-5 and 8-9 are chosen for reconductoring, while lines 2-3, 3-4 and 5-6 are no longer reconductored. In addition, only one line per scenario is reconductored as opposed to minimum four in Table IV. Finally, no-first stage investment decisions are made, leading to the substantial reduction of stranding risk. Note also the similarity between investment solutions for S1 and S2 in Table III and Table IV. This is because line reconductoring is a commitment that entails little strategic potential for agility; stochastic planning solely based on this technology may lead to strategies that are similar to deterministic ones, foregoing the possibility for adopting a 'wait-and-see' approach.

TABLE IV. STOCHASTIC STUDY (RECONDUCTORING)

	Investment Decisions			**Costs (£k)**			
	Epoch 1	Epoch 2	Epoch 3	*TIC*	*TOC*	Total	**E{*}**
S1	[2-3],[3-4], [4-5],[5-6]	-	-	*127.9*	*110.9*	*238.8*	E{TC}= 209
S2	[2-3],[3-4], [4-5],[5-6]	-	-	*127.9*	*39.8*	*167.7*	E{IC}= 138
S3	[2-3],[3-4], [4-5],[5-6]	[8-9]	-	*148.2*	*126.8*	*275*	E{OC}= 71
S4	[2-3],[3-4], [4-5],[5-6]	[8-9]	-	*148.2*	*20.6*	*168.7*	

TABLE V. STOCHASTIC STUDY (RECONDUCTORING, SOP)

	Investment Decisions			**Costs (£k)**			
	Epoch 1	Epoch 2	Epoch 3	*TIC*	*TOC*	Total	**E{*}**
S1	-	[4-5], S(7-11), S(7-13), S(7-15)	-	*192.5*	*106.0*	*298.5*	E{TC}= 172.7
S2	-	[4-5], S(7-11), S(7-13), S(7-15)	-	*192.5*	*39.1*	*231.6*	E{IC}= 110.5
S3	-	[8-9]	S(11-15)	*47.6*	*80.3*	*127.9*	E{OC}= 61.9
S4	-	[8-9]	-	*20.3*	*20.6*	*40.9*	

Operation of SOP for tackling voltage rise is depicted in Fig. 5. The figure focuses on feeders F-2 and F-3. As can be seen, in the absence of SOPs there is voltage rise above 1.1 pu. In order to keep the voltage magnitude at bus 7 within limits, the SOPs at 7-11, 7-13 and 7-15 are engaged to draw active power from bus 7 and release it to buses 11, 13, 15. This leads to an increase in total real demand of feeder F-2 while simultaneously reducing that of F-3, F-1 and F-4. In addition, the three SOPs absorb reactive power at bus 7 (negative values in the figure), as an extra measure of voltage regulation. When the voltage rise effect is no longer an issue, the system topology returns to its initial configuration state. Fig. 6 shows that without SOPs, the amount of PV curtailment required to keep voltages within statutory limits in scenario-tree node 2, will be higher by 35%. Similar situations characterize other scenario-tree nodes as well.

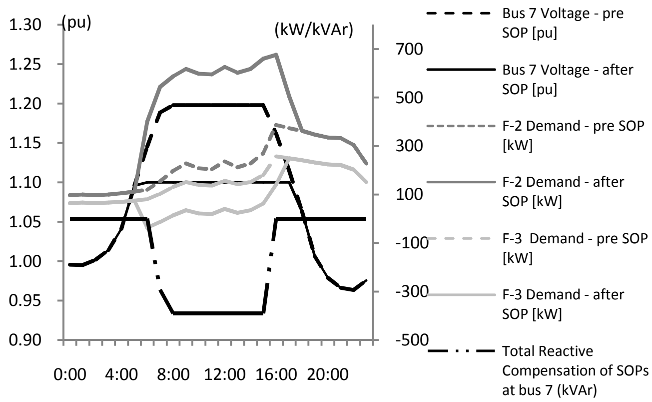

Figure 5. Impact of SOP operation on voltage profile (left axis) of bus 7, on load profile of feeders 2 & 3 (kW, right axis) and on reactive compensation of bus 7 (kVAr, right axis) for node 2, summer-high day.

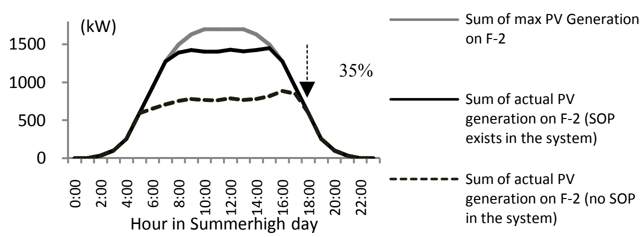

Figure 6. Impact of SOP on PV curtailment, node 2, summer-high day.

TABLE VI. OPTION VALUE & NET BENEFIT OF SOPS (£K)

Net Benefit	S1	-59.7
	S2	-63.9
	S3	147.1
	S4	127.8
Option Value		36.3

By comparing the individual scenarios of Table IV and Table V we can quantify the net benefit of SOP under each scenario, which represents investment and operation cost savings. As shown in Table VI, while the net benefit is negative in scenarios S1 - S2, we can observe that it is substantial for S3 - S4, underlining the significance of SOP for hedging against unfavourable realizations. For instance, scenarios S3-S4 in Table IV entail significant number of fist-stage conventional investments to cope with PV deployment in F-2, while these scenarios assume that PV deployment will only take place in F-3; SOPs allow hedging against this stranding risk. By comparing Table IV with Table V we also can quantify the option value of SOP (shown in Table VI), representing the expected net benefit accrued from investing in this technology. This term amounts to £209k – £172.7k = £36.3k and reflects a 20% reduction in expected investment cost and a 13% reduction in expected operational cost as the flexibility of SOPs led to 13% lower curtailment of active PV generation.

V. CONCLUSIONS

The paper proposes a multistage stochastic framework for quantifying the option value of investing in SOPs and provides a mathematical formulation of SOP operation.

A significant conclusion that can be drawn from the present paper is that investment frameworks that can comprehensively accommodate uncertainty and decision flexibility are necessary to evaluate the strategic benefits of smart technologies and enable the cost-efficient transition to the smart grid era.

Future work includes the investigation of decomposition methods for nonlinear programming for achieving more efficient solution times. Also, we aim at incorporating risk-averse decision criteria in the formulation to model the planner's attitude towards risk of stranded assets.

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