Abstract
We analyse the effect of distinct levels of interest rates on the stability of the financial network under our modelling framework. We demonstrate that banking failures are likely to emerge early on under sustained high interest rates, and at much later stage - with higher probability - under a sustained low interest rate scenario. Moreover, we demonstrate that those bank failures are of a different nature: high interest rates tend to result in significantly more bankruptcies associated to credit losses whereas lack of liquidity tends to be the primary cause of failures under lower rates.

Keywords: Crisis, Finance, Modelling, Interest rates

1. Introduction
Little is known about the long term effects in the financial system of different levels of interest rates, since it is not feasible to conduct long term experiments with such a key monetary policy instrument. In addition, financial markets and the economy as a whole are complex systems with a multitude of evolving and interacting agents that do not necessarily respond to changes in rates in a mechanistic manner [1,2]. This makes the development of reliable, yet simple, theoretical modelling environments particularly important as a means to develop an understanding of the key mechanisms that can lead to financial instability.

In this paper we analyse the effect of interest rates on the stability of the financial markets under the modelling framework developed in [3] in which endogenous systemic crashes are shown to occur due to the evolutionary dynamics arising from copy of business models and benchmarking. Given the evolutionary dynamics inherent to the system, large and concentrated levels of financial failures will occur regardless of the levels of interest rates. However, the likelihood and timing of those events will depend on such levels.

The model is underpinned by five key principles (i) the relationship between risk and returns on the investment of an agent is always rationally maintained so that at the time of an investment, the returns are always higher than the expected losses; (ii) the funding costs of a bank increase as a function of its risk profile; (iii) in order to replicate basic regulatory rules, banks cannot obtain funding above a certain leverage threshold, and cannot operate below a determined capital ratio threshold; (iv) investors diversify their portfolios, therefore, concentration limits on a single counter-party are required; and (v) a basic asset and liability management strategy is used so that banks do not have maturity or interest rate mismatches. Under this framework, we find that sustained low interest rates result in a higher likelihood of crashes after prolonged periods of time. The magnitude of the crashes will be of a maximum magnitude comparable to the failures observed at higher interest rates. However, high rates result in a very quick market saturation and a substantial proportion of failures earlier on, albeit, with a lower probability.

**This is a preliminary title of the paper**

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2. Historical Outlook

Looking at the historical data of the US Federal Funds Interest Rates ("FED rates")[4] and the number of FDIC regulated institutions [5] (see Fig. 1) one can note that the FED rates reached historically high levels before the "Loans and Savings Crisis" in 80-90s [6] and unusually low ones, nearing zero, since the beginning of the "Subprime Crisis" which originated in 2007 [7].

![Figure 1: Total number of banks that have failed in a given year versus base interest rate in U.S. where highlighted are two periods of crisis.](image)

The peak of the FED rates was reached in 1981 in response to inflationary pressures, whereas, a large number of bank failures only started occurring after 1985, at a time that the FED rates had already returned closer to the long term trend. By 1989, when the crisis reached its summit, the time FED rates had already dropped by a factor of two as compared to the peak reached in 1981. Since the mid-nineties, interest rates tended to drop. At the time the "Subprime Crisis" has started, a period of historically low interest rates was reached. Such observations suggest there is a non-trivial relationship between interest rates and bank failures, which can be better understood using a model environment driven by fundamental market dynamics.

3. Model

Our analysis is underpinned by the agent-based model developed in [3], and all details can be found in Appendix A. The model successfully reproduces crashes in financial markets by supplementing conventional dynamics with two evolutionary elements: the Cultural Dissemination of investor strategies and Infection of Bank Business Strategies. Crashes occur without introducing any exogenous shocks.

The basic description of the model can be summarised in Fig. 2 where types of agents and cash flows are indicated.
The model is based on 3 types of agents: investors, banks and borrowers. Banks act as the connecting part between the other two agent types in a tripartite interconnected network. The investors choose which banks they want to invest in based on their risk appetite at a given moment. Banks can then lend that money out in a form of loans. All the unallocated money will go to market placements, which represent the central bank. For simplicity interbank lending is ignored. However, the potential contagion effects on defaults arising from interbank lending and borrowing network are recognized in the model through the interbank market losses mechanism described in Eq. (A.26) whereby losses are shared between all of the banks within the same rating category. This is to recognize that losses are likely to be clustered within a group of banks without the need to generate a model for a full interbank network. Finally, in the model, some of the loans will default, thus triggering losses for banks and investors.

We emphasise that under the conventional dynamics, i.e. without any of the aspects of the evolutionary dynamics, crashes do not emerge, in line with the principles of rational expectations. Crashes do however occur when the model supplements the conventional dynamics with the following two modifications: (1) culture dissemination in the investors’ community and (2) strategy infection among the banks which supports arguments seminally laid out by Minsky that crashes are inherent to the structure of the system and occur endogenously without any exogenous input.
Investors performing under the benchmark, i.e. their returns are at the 40th percentile between the best and worst performers, can adjust their Investment Return Expectations with a fixed probability towards the benchmark. To do that we are using a modified version of the Axelrod model of cultural dissemination\cite{12} and modify Return Expectations using

\[ \text{Re}_x(t) = \text{Re}_x(t-1) + \frac{a}{4} e^{b r(t)} \]  

(1)

where \( a \) and \( b \) regulate how quick the movement towards the benchmark is.

Banks can change their Target Shareholder Returns and Bonus Ratio parameters by adopting those used by the top performing bank, if these parameters are lower. We refer to this behaviour as the infection of the business strategy which happens with a fixed probability for every bank at any given time step and is inspired by the process of bacterial conjugation \cite{13}.

These two modifications are sufficient to support statements by Lo \cite{14} and Shiller \cite{15}. Lo highlights that individuals tend to be risk averse in the face of gains and risk seeking in the face of loses; this behaviour can lead to some very poor financial decisions \cite{14}. In his book "Irrational Exuberance" Shiller refers to contagion of investor thinking, which implies that at any given moment there could be an idea, strategy or principle which becomes more and more popular among increasingly larger fractions of investors, leading to bubbles and crashes. \cite{15}.

Banks experience failure when any of the following two conditions are met:

1. Credit Failure, when the Capital Ratio \( \frac{\text{capital}}{\text{assets}} < 8\% \)
2. Liquidity failure, when the Bank Deposits = 0

4. Method

In the original paper \cite{3} the sole input to the model was the actual US interest rate during the period from 1973 to 2011. Here our aim is to use the same framework and replace the actual input data with theoretical interest rate levels so that we could build an insight into the behaviour of the model dynamics for different levels of fixed interest rate. We study fixed interest rates \( r \in [0.01, 0.2] \) with steps of 0.01 for a theoretical time span of 60 years and longer. All other parameters are the same as those used in \cite{3}. For each \( r \), simulated for 60 years, the analysis is based on averaging over \( \Omega = 1000 \) realisations, unless otherwise specified. For each \( r \) over 60 years it is based over \( \Omega = 100 \) realisations.

The total number of banks at time \( t \) is \( N(t) \) and \( n \) is the number of banks that have failed over a given period. We use superscripts \( \text{tot}, c, l \) to refer to total, credit and liquidity failures respectively.

4.1. Number of failures

We study \( \langle N(t) \rangle \), the average at time \( t \) across realisations, for different interest rates.
Fig. 3: The plot shows the mean number of active banks in the system averaged over 100 runs for flat interest rates for extended periods of time. Due to the clustering arising from the evolutionary dynamics, the banking sector reaches crisis at any level of interest rates, albeit at different timelines.

Fig. 3 shows that for all interest rates $\langle N(t) \rangle$ initially exhibits only a weak $t$ dependence. Gentle increase (decrease) for low (high) interest rates. This is followed by an abrupt drop for low interest at a characteristic time which increases with increasing interest rate. High interest exhibits a very different behaviour over the simulated timespan. The initial gentle increase is followed by a gradual decline of $\langle N(t) \rangle$. In Fig. 3 about two-thirds of banks are lost in the abrupt drop. Over the same time period (60 years) only 10% of banks are lost for low interest levels. It could be also noted that the time and rate of collapse are also increasing with the interest rate. This shows that under such a framework large systemic failure can only be delayed and the speed of collapse controlled, but not completely avoided.

This long term behaviour can be explained by the fact that investors' evolutionary dynamics are such that the higher the interest rate, the slower the movements towards the benchmark. Equally, the lower the interest rates, the faster are the movements. This means that the clustering of bank choices will be achieved faster under low interest rates. These movements also link to the propositions such as the unintentional herding effect as explored by [16] or overconfidence according to [17,18] because all agents do the rational thing at any given moment, but limited choices lead to an unavoidable overlap in their decisions.

Next we consider the total number of monthly bank failures, see Fig. Fig. 4, defined as:

$$\langle n^{\text{tot}}(t) \rangle = \frac{1}{\Omega} \sum_{i=1}^{\Omega} n^c_i(t) + n^l_i(t).$$

To understand the evolution of the system in the early stages we take the timeframe of only 60 years where the difference between low and high interest rates is more distinct for the early onsets of failures.
Figure 4: The plot shows the mean number of monthly bank failures averaged over 1000 runs for flat interest rates. Low interest rate leads to a high mean number of failures at a relatively late time - for our parameters the onset of massive failures occur after more than 27 years. High interest rates lead to a much earlier onset of bursts in the number of failures, but at a significantly lower level of losses. These results are summarised in Fig. 5.

Figure 5: The plot shows the mean number of monthly bank failures averaged over 1000 runs for flat interest rates. Colours indicate the size of the mean avalanche size.
4.2. Liquidity vs Credit Failures

In this section we analyse the nature of micro-crashes in order to check the hypothesis that lower interest rates result in quicker clustering of investors’ choices. Fig. 6 demonstrates that crashes vary substantially in timing, size and shape. Credit driven failures behave differently as a class from liquidity driven failures as is evident in Fig. 6.

Credit failures are concentrated during a relatively brief time span, which occurs earlier for the higher interest levels. Liquidity failures occur also in a concentrated time span but late in the simulated time interval. For high interest levels failures are not localised in time.

We conclude that the interest level strongly influences the type of failures in the early times and that a high interest level leads to an early burst of failures followed by prolonged period of stability. Overall, high interest leads to few failures over the period of 60 years.

4.3. Rate of Failures

In this section we study the rate of failures

\[ R^{(i)} = \frac{n^{(i)}(t)}{N^{(i)}(t-1)} \]  

which measures the size of the micro-crash relative to the number of healthy banks in the previous step. This measure is important because crises are declared when a specific fraction of banks fail over a specified period of time which is normally a year.

We use \( \langle R^{(i)} \rangle \) to study the relative differences between liquidity and credit failures for very high, medium and low interest rates. We chose three scenarios where interest rates are set to 1%, 10% 20%. Figure Fig. 7 shows the mean yearly rate of failures.
Figure 7: Mean rate of failures on the left and representative samples of individual runs on the right. Shaded regions represent periods of crisis where more than 2% of banks fail in a period of one year. The figures are not at the same vertical scale so that differences between credit and liquidity failure patterns for higher interest rates could be identified.

It is clear from these plots that for low rates, the average size is high, where 8% of banks fail for low interest rates compared with 1% of banks for high interest rates. Liquidity failures dominate in a low interest rate environment, compared to credit failures in high interest rate environment.

It is important to note that there are significant differences underlying each of those 3 plots. In a low interest environment all the simulations result in very large and long lasting crises which take out the majority of the banks and the shape of onsets of failures at each individual run is roughly the same as the plot for the mean rate of failures at $r = 0.01$. For $r = 0.10$ the underlying mean rate of failures is
completely different. The multi-peak structure appearing there is due to two or more crashes happening in each simulation at different times as illustrated by a sample of a single run. For \( r = 0.2 \) the double large peak structure comes from the fact that crises happen either after 15 or 24 years. For 10% and 20% rates onsets of failures are very short and last only a year, but for low interest rates once the failures start happening, they last for a prolonged period of time so the cumulative number of banks that fail in a low rate environment is much larger.

By looking at the probability of being above a given threshold \( R_{th} \)

\[
P(R > R_{th}) = \frac{\sum \Omega_i \theta(R(i) - R_{th})}{\Omega}
\]  

(4)

we can see from Fig. 8 that at early times multiple crashes happen in a high interest environment with a 30% probability. On the other hand, low interest environment exhibiting no crashes early on, results in a certain crisis after about 28 years for 1% rates and progressively later for higher rates.

**Figure 8:** The figure shows the probability of the fraction of failures exceeding different thresholds for credit (top) and liquidity (bottom). Leftmost plots show the distributions of any failure of a given type happening. Clear structural differences can be seen. These plots suggest that there are a lot of small scale bankruptcies occurring due to the random nature of the model and they start at different times for varying interest rates but never exceed higher thresholds. Liquidity failures on the other hand preserve the same distribution of failures even for the ‘crisis’ threshold of 2% and above.

Fig. 8 also shows that credit failures never go above the 2% threshold and majority stay under 1%, which is to be expected in a static interest rate environment where liquidity failures should dominate.

To sum up, low interest rates result in late bank failures as well as a single but very lengthy crisis and
high interest rates result in early bank failures as well as potentially multiple crises which last for short periods of time.

5. Shocks

In order to assess the effect of changing interest rates we applied a permanent shock after 100 months. Fig. 9 shows that by increasing interest rates, it is possible to decrease the probability of failures significantly in the long run. The opposite effect can be seen for going from high to low rates. Plots show the results for two thresholds of 1% and 2% of banks failing.

![Figure 9](image)

**Figure 9:** The plots show the probability of more than 1% (top) and 2% (bottom) of banks failing over a 12 month period for simulations starting at rates $r = 0.01$ (left), $r = 0.10$ (centre) and $r = 0.20$ (right) and shock being applied after 100 months (white dashed line).

When a significant downward move occurs, it can trigger an oscillatory behaviour of crashes after 40 simulated years, but it cannot prevent failures in the short term. This is noted by the ridges at year 16 and further scattered onsets later on with probabilities of about 30%.

If we consider what would be the best interest rate to end up with (see Fig. 10), it is clear that low rates are not the best option no matter where we start off. Higher rates appear to be a better option in this regard, offering the lowest probabilities of experiencing crisis identified by more than 2% of banks failing in a year.
Figure 10: The plots show the probability of more than 1% (top) and 2% (bottom) of banks failing over 12 month period for simulations where rates jump to $r = 0.01$ (left), $r = 0.10$ (centre) and $r = 0.20$ (right) where the shock is being applied after 100 months (white dashed line).

Now if we look at the effect on liquidity and credit failures we can see (Fig. 11) that as expected, increasing rates lead to an overall larger fraction of credit failures as compared to liquidity failures. In the case of movement from 1% to 20%, liquidity failures are almost completely avoided at the expense of a fractionally higher possibility of bankruptcies. However, if we compare this result with what happens with a static 1% interest rates, we see that the overall amount of credit failures is comparable.

Lowering rates on the other hand has more dramatic effect. By lowering the rates from 20% to 1% the early credit failure onset is prevented at the expense of increased liquidity problems.
There is also a clear periodicity visible suggesting that the investment time $t_{inv} = 48$ months has an effect on the exact timing of late crisis and once the first rapid onset of liquidity failures occurs it leads to a second one soon after, accompanied by a significant number of bankruptcies.

So to sum up, increasing rates lead to a much larger amount of credit failures compared to liquidity failures, whereas dropping the rates results in prevention of early credit failures, but results in unpredictable behaviour at later times with a much more significant number of bankruptcies.

6. Conclusions

Within this framework we suggest that the link between interest rate levels and banking failures is of an indirect nature. This means that whereas failures are the result of clustering of business strategies and investors’ returns appetite, the levels of interest rate may speed up or slow down the clustering formation. As a result we tentatively start to address a key question in monetary policy making and macro prudential regulation: what are the sustainable levels of interest rates to be set and how long can they be sustainably maintained. We suggest that such levels of interest rate should not only be a function of the economic activity and price stability, the traditional mandate of monetary activities, but should also be dependent on the structure of the financial sector in particular with regards to diversity of business models and investment appetite. Governments and monetary agents always have to achieve a balance between competing priorities, for example, to increase the production and spending in order to stimulate growth of the wider economy but to avoid high levels of inflation. In recent years, as a response to the financial crisis, most governments have
adopted extreme low levels of interest rate in order to maintain economic growth. The effect of maintaining such a policy for very long periods is unknown, since there is no equivalent parallel in modern history. Our findings, however, suggest that there might be fundamental risks stored for the future due to such a policy. Having said that, we believe further detailed work is required to validate, refute or potentially clarify the interaction between interest rates and expected - not actual - returns.

Moreover, consistent with our previous findings through analysis of actual data for banking mergers and acquisitions, we suggest in this theoretical framework that the evolutionary dynamics of the markets will lead to failures and crashes in the system regardless of the levels of interest rates. Within such a framework the choice between low and high interest rate is a choice between late or early failures, single (multiple) and long (short) crises.

Within the framework, in general terms by raising interest rates the overall number of failures is reduced significantly at the expense of early moderate crises. In contrast, maintaining low interest rates for very long periods may lead to large number of failures at short time spans. Given the fact that this is the current situation most advanced economies find themselves in today, we believe that further work on this area is fundamental.

Appendix A. Details of the Model

Below are the steps performed for a single period of the simulation. The notation used here differs from the original paper where we change the indices corresponding to different types of agents: Investors $i$, Banks $b$ and Loans $l$. Once we refer to an agent or any particular type we write $\text{Variable}_i$ to refer to a variable corresponding to an investor $i$. Tranches $T_{i,n}$ are denoted with two indices, the first one indicating the type of agent and the second one the tranche number $n$. The list of variables used is summarised in the Table A.1. One period of simulation is split into 4 parts P1-P4 to reflect processes corresponding to the cash flows as per Fig. 2.
Appendix A.1. P1: Investors-Banks

We start by computing Risk Appetite for each investor $i$ as a standard deviation of the downside risk over the investment period $t_{inv}$ which in our simulations was set to 24 months.

$$q_i(t) = \sqrt{\frac{1}{t_{inv}} \sum_{\tau = t - t_{inv} + 1}^t \left( \min \left[ R_i(\tau) - Rex_i(\tau), 0 \right] \right)^2} \quad (A.1)$$

Investors are then ranked by $q_i$ and assigned an appetite group

$$AG_i(t) \in [1, 11]$$

where $AG_i = 1$ for investors with lowest $q_i$.

Banks are ranked by $TSR_b$ and assigned rating $RT_b = 1$ corresponding to the highest TSR

$$RT_b(t) \in [1, 11]$$

and their rating category modified if they are below the mean $TSR$

$$RT_b(t) = RT_b(t) + 1 \quad \text{if } TCR_b < E[TSR]$$

Investors divide their funds available to invest $\Delta F_i(t)$ into equal tranches $T_{i,n}$ controlled by the concen-
tration limit $CL$ which in our case is 10%.

$$\Delta F_i(t) = F_i(t) - \sum_{\tau=t-(t_{inv}-1)}^{t-1} \Delta F(\tau) \quad (A.2)$$

$$T_{i,n} = \frac{\Delta F_i(t)}{nt_i} \quad \text{where } nt_i = \frac{1}{CL} \quad (A.3)$$

Banks now compute their Borrowing Limits $Lim_b$ in such a way that the Target Capital Ratio is preserved.

$$Lim_b(t) = C_b(t) \left( \frac{1}{TCR_b(t)} - 1 \right) \quad (A.4)$$

The cost of borrowing is set as the sum of the base interest rate and funding spread for the banks within the same rating category $k = RT_b$.

$$CB_b(t) = r(t) + FS(k,t) \quad (A.5)$$

Utilising the fact that there is a direct mapping between appetite groups and rating categories, we can compute the funding spread $FS(k,t)$ as a function of the offers from investors $A(k) = \{i \mid AG_i = k\}$ in appetite group $k$ and demand from the banks $R(k) = \{i \mid RT_i = k\}$ in the corresponding category. By construction, it is bounded by investors’ expectations.

$$FS(k,t) = r(y-z) + w \quad (A.6)$$

$$FS(k,t) \in [\min(Re_x[i \in A(k)]), \max(Re_x[i \in A(k)])] \quad (A.7)$$

where the terms of this equation are defined in [A.9]. Here we introduce $\Phi(k)$ which checks whether offers from investors are larger than the overall demand from the banks.

$$\Phi(k) = \sum_{b \in R(k)} \Delta Lim_b(t) - \sum_{i \in A(k)} \Delta F_i(t) \quad (A.8)$$

<table>
<thead>
<tr>
<th>$\Phi(k) &lt; 0$</th>
<th>$\Phi(k) &gt; 0$</th>
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<tbody>
<tr>
<td>$r$</td>
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<tr>
<td>$\frac{\sum_{b \in R(k)} \Delta Lim_b(t)}{\sum_{i \in A(k)} \Delta F_i(t)}$</td>
<td>$\frac{\sum_{i \in A(k)} \Delta F_i(t)}{\sum_{b \in R(k)} \Delta Lim_b(t)}$</td>
</tr>
<tr>
<td>$y$</td>
<td>$\max [Re_x[i \in A(k)]]$</td>
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<tr>
<td>$Re_{wa}(k)$</td>
<td>$Re_{wa}(k)$</td>
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<tr>
<td>$z$</td>
<td>$\min [Re_x[i \in A(k)]]$</td>
</tr>
<tr>
<td>$w$</td>
<td>$\max [Re_x[i \in A(k)]]$</td>
</tr>
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where

$$\Delta Lim_b(t) = Lim_b(t) - TD_b(t)(t-1) \quad (A.10)$$

$$Re_{wa}(k) = \frac{\sum_{i \in A(k)} Re_x(i) \Delta F_i(t)}{\sum_{i \in A(k)} \Delta F_i(t)} \quad \text{where } A(k) = \{i \mid AG_i = k\} \quad (A.11)$$

All the available tranches $T_{i,n}$ are now shuffled and each of them is invested if $T_{i,n}(t) < Lim_b(t) - TD_b(t-1)$.
with probability

\[
P = \begin{cases} 
0.8 & \text{if } |RT_b(t) - AG_i(t)| = 0 \\
0.2 & \text{if } |RT_b(t) - AG_i(t)| = 1 \\
0.1 & \text{if } |RT_b(t) - AG_i(t)| = 2 
\end{cases}
\] (A.12)

Otherwise, the tranche is not invested.

The Total Funding Spread \(TFS_b\) is now recomputed for each bank

\[
TFS_b(t) = \frac{1}{TD_b(t)} \sum_{\tau = t - (t_{inv} - 1)}^{t} FS(RT_b, \tau) \Delta TD_b(\tau)
\] (A.13)

where \(\Delta TD_b(t) = TD_b(t) - TD_b(t-1)\).

Finally, each bank updates its Benchmark Return \(BK_b\)

\[
BK_b(t) = \frac{1}{C_b(t) + TD_b(t)} \left( \frac{C_b(t)TSR_b(t)}{1 - TBN_b(t)} + TD_b(t)CB_b(t) \right) - r(t)
\] (A.14)

**Appendix A.2. P2: Banks-Loans**

All of the new deposits \(\Delta TD_b\) are then assigned as funds available to be lent

\[
\Delta L_b(t) = \Delta TD_b(t)
\] (A.15)

and then divided into several tranches \(T_{b,n}\) controlled by the thickness of the tranche \(TT\) (in our simulations set to 10%)

\[
T_{b,n} = \frac{\Delta L_b}{nt_b} \quad \text{where } nt_b = \frac{1}{TT}
\] (A.16)

The pricing of a loan \(LP_l\) is determined by the relative performance parameter \(q_l\) of each loan cluster \(l \in [1, 41]\), prime rate \(pr = 0.03\) and volatility factor \(vol = 0.2\) and is given by

\[
LP_l(t) = (pr + q_l)(1 + vol)
\] (A.17)

where \(q_l\) is a cumulative distribution function of a log-normal distribution

\[
q_l(t) = \frac{1}{2} \text{erfc} \left( \frac{\ln \left( \frac{5 \max(l) + 1 - t}{\max(l)} \right) - \mu(t)}{\sqrt{2\sigma^2}} \right)
\] (A.18)

with \(\sigma^2 = 0.5\) and mean of the distribution \(\mu(t)\) equal to

\[
\mu(t) = \ln[r(t) + 1]c
\] (A.19)

where \(c = 2.71\) is a scaling parameter.

The probability of \(T_{b,n}\) allocation into a loan cluster as a function of the distance between benchmark return \(BK_b(t)\) and pricing of the loan cluster \(LP_l(t)\) is normally distributed

\[
p = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\left| BK_b(t) - LP_l(t) \right| - \mu}{\sigma} \right)^2 \right]
\] (A.20)

with \(\mu = 0\) and \(\sigma = 1\), and allocation happens only if

\[
T_{b,n} < mrk_l - TL_l(t-1)
\] (A.21)
where $mrk_l$ is the maximum potential market of a selected loan cluster. If the tranche is not assigned, it is then put to the interbank market.

**Appendix A.3. P3: Loans-Banks**

When loans expire after $t_{inv}$, income $Inc_b$ and losses $Loss_b$ are calculated for each bank taking into account whether tranches were lent or placed into the interbank market.

$$Inc_b(t) = \sum_{n=1}^{nt_b} T_{b,n}(t) Rem \frac{t_{inv}}{12}$$

(A.22)

$$Loss_b(t) = \sum_{n=1}^{nt_b} T_{b,n}(t) q(t)(1 - rec)$$

(A.23)

where the recovery level $rec$ is set to 40% and

$$Rem = \begin{cases} LP_1(t - t_{inv}) + r(t - t_{inv}) & \text{if } T_{b,n} \text{ in Loans} \\ Spr + r(t - t_{inv}) & \text{if } T_{b,n} \text{ in Interbank with } Spr = 1% \end{cases}$$

(A.24)

The bankruptcy check looks at the Actual Capital Ratio $ACR_b$ defined as

$$ACR_b(t) = \frac{C_b(t)}{C_b(t) + TD_b(t)}$$

(A.25)

If $ACR_b(t) < 8\%$ bank is declared bankrupt and removed form the system, if $ACR_b(t) \geq 8\%$ but $TD_b(t) = 0$ bank requires financial assistance, but keeps trading.

If the bankruptcy occurs, losses are shared between all of the banks within the same rating category $RT$ via the Interbank Market Losses $Lsib_b$

$$Lsib_b(t) = \frac{\sum_{k \in bankrupt} \delta(RT_b, RT_k)IB_k}{\sum_{k \in banks} \delta(RT_b, RT_k)IB_k} IB_b(t)$$

(A.26)

where $bankrupt$ is the set of banks that went bankrupt and

$$IB_b(t) = TD_b(t) - L_b(t)$$

(A.27)

**Appendix A.4. P4: Banks-Investors**

After $t_{inv}$ funds borrowed by banks from investors can be redeemed. For each tranche the cost for banks $Bor$ can be computed by

$$Bor_b(t) = \sum_{n=1}^{nt_b} T_{b,n}(t) CB_b(t - t_{inv}) \frac{t_{inv}}{12}$$

(A.28)

Similarly, income for investors $Incv_i$ is given by

$$Incv_i(t) = \sum_{n=1}^{nt_i} T_{i,n}(t) CB_b(t - t_{inv}) \frac{t_{inv}}{12}$$

(A.29)

and losses for investors $Lossv_i$

$$Lossv_i(t) = \sum_{n=1}^{nt_i} T_{i,n}(t) \delta(n, bankrupt)$$

(A.30)
where we are summing all of the tranches which have gone into banks that went bankrupt and have been lost as a result.

Before computing the net result for the bank we have to evaluate capital remuneration where \( S_{pr} \) is the same as in Eq. (A.24)

\[
C_{inc_b}(t) = \frac{C_b(t)(S_{pr} + r(t))}{12}
\]  
(A.31)

The net result \( NetRes_b \) for the banks in each period is now given by

\[
NetRes_b(t) = Ninc_b(t) \left( 1 - \theta(Ninc_b(TBN_b(t))) \right)
\]  
(A.32)

where \( Ninc_b(t) \) is given by

\[
Ninc_b(t) = Inc_b(t) - Loss_b(t) - Cinc_b(t) - Lsib_b(t) - Bor_b(t)
\]  
(A.33)

and \( \theta(x) \) is just the Heaviside step function

\[
\theta(x) = \begin{cases} 
1 & \text{if } x \geq 0 \\
0 & \text{otherwise} 
\end{cases}
\]  
(A.34)

The overall capital of the bank is then given by

\[
C_b(t) = C_b(t-1) + NetRes_b(t)(1 - Div)
\]  
(A.35)

where dividends distribution \( Div \) is given by

\[
Div = \begin{cases} 
0.95 & \text{if } NetRes > 0 \\
0 & \text{otherwise} 
\end{cases}
\]  
(A.36)

Next, investors’ wealth is updated

\[
F_i(t) = F_i(t-1) + (Inc_v_i(t) - Loss_v_i(t))(1 - Dis)
\]  
(A.37)

where distribution ratio \( Dis \) is given by

\[
Dis = \begin{cases} 
0.9 & \text{if } Inc_v_i(t) > Loss_v_i(t) \\
0 & \text{otherwise} 
\end{cases}
\]  
(A.38)

Finally, all the investors are ranked by their returns ranging from the highest to the lowest. Every investor whose returns fall below the 40th centile increase their return expectations for the next step towards this benchmark according to the following expression

\[
Rex(t) = Rex(t-1) + \frac{a}{4} e^{b \cdot r(t)}
\]  
(A.39)

Accordingly, every bank copies \( TSR_b \) and \( TBN_b \) from the top performing bank with a probability of 1% per simulated year and this completes a single period of the simulation.

References

