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PIV measurements of convection velocities in a turbulent mixing layer

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Abstract. Particle image velocimetry (PIV) experiments that resolve a range of scales from 1.5$L$ to 10.4$\eta$, where $L$ and $\eta$ are the integral and Kolmogorov length scales respectively, are performed in the self similar region of a nominally two dimensional planar mixing layer. Data is acquired at 7.25Hz with a field of view that extends to 1.5$L$, thereby permitting spatio-temporal correlations of large and intermediate scale velocity fluctuations to be performed. The overall convection of velocity fluctuations is found to be similar to the mean flow, although fluctuations on the low speed side of the mixing layer on average convect at speeds greater than the mean and fluctuations on the high speed side of the mixing layer are observed to convect at speeds less than the mean. The convection of fluctuations of different length scales in the flow is then observed. It is found that large-scale structures, of the order of the integral scale, convect at speeds that are greater than the mean in the low speed side of the mixing layer and that small scale fluctuations convect at speeds significantly lower than the mean. A crossover wavenumber is also found at which the fluctuations in the low speed side of the mixing layer convect at greater velocities than those on the high speed side of the mixing layer. This wavenumber is found to be Reynolds number dependent.

1. Introduction

In turbulent flows with a dominant velocity component, such as a turbulent boundary layers and jets, eddies are observed to propagate downstream at speeds that are usually close to the local mean velocity and are referred to as convection velocities, $u_c$ (Townsend, 1976). By making use of Taylor’s frozen flow field hypothesis (Taylor, 1938) and using a convection velocity experimentalists have been able to obtain pseudo three-dimensional velocity fields from time resolved, planar data. For example Matsuda & Sakakibara (2005), van Doorne & Westerweel (2007), Ganapathisubramani et al. (2007) used stereoscopic particle image velocimetry (PIV) data acquired at a sufficiently high rate to create three-dimensional velocity fields. Despite its use in experimental studies and in the analysis of turbulence dynamics at large a complete characterisation of the turbulent convection velocity in many turbulent flows is lacking in the literature. Taylor’s hypothesis is also only considered to be valid when the turbulence level is low, viscous forces are negligible and the mean shear is small (Wills, 1964). This clearly presents a serious problem for the study of turbulent shear flows in which the convection velocities are not well documented and all three of the assumptions of Taylor’s hypothesis, above, are violated. Additionally, Kim & Hussain (1993) state that “various flow properties and their different scales are not expected to propagate at the speed of the mean flow, nor are they always expected to propagate at identical speeds”. It is thus of great importance to more closely examine the scale dependence of convection velocities in turbulent shear flows, and to examine the convection of different quantities, $q$.

The convection velocity of turbulent fluctuations has often been computed by finding the maximum of two point space - time correlations of a variable, $q$, within a turbulent flow, or by correlating between time delayed flow field images (Smith & Dutton, 1999\textit{b},\textit{a}). However, several studies have reported that the measured convection velocity varies significantly with the probe separation and time delays employed due to a scale dependence of the propagation velocity (Wills, 1964, Hussain & Clark, 1981,\textit{b},\textit{a}).
Kim & Hussain, 1993). The dependence upon probe separation can further be explained by considering the short eddy turnover times of smaller scale structures, due to the higher mean rate of dissipation at small scales, leading to poor correlations (Kim & Hussain, 1993). Zhao & He (2009) further state that in turbulent shear flows the large scales induce distortions of the small scales thereby accelerating the decorrelation process of the small eddies.

The work of Wills (1964) has inspired a closer look at the convection of quantities at different length scales. Several studies have reported that smaller scales are convected at slower speeds than the larger scales, such as Wills (1964), Hussain & Clark (1981) in an axisymmetric jet, Kim & Hussain (1993), del Álamo & Jiménez (2009) in a channel and Krogstad et al. (1998) in a boundary layer. This paper examines the scale dependence of the convection velocity in a planar turbulent mixing layer. Winant & Browand (1974) stated that “the region between two parallel streams moving at different speeds is the simplest free shear flow which can be considered”. Specifically, the experiments are performed far downstream in which the mixing layer attains a self similar state (Rogers & Moser, 1994) in which a broad range of fluctuations, from the large-scale rollers down to the dissipative length scales are present (Zohar & Ho, 1996).

The simplicity of the planar mixing layer makes it the ideal flow in which to examine the convection of turbulent fluctuations in free shear flows, as opposed to wall bounded shear flows such as channels and boundary layers. This paper, therefore, seeks to more closely examine the effects of probe separation, time delay, length scale and Reynolds number on convection velocities in a planar mixing layer. The data used in this study is two dimensional PIV, which grants access to the velocity field across the entire cross-stream ($x_2$) extent of the mixing layer. It is therefore possible to examine the different convection behaviour in the high speed side as well as the low speed side of the mixing layer at the range of wavenumbers captured in the PIV experiments. Comment is subsequently given on the validity of using the Taylor’s frozen flow field hypothesis in the creation of a pseudo three-dimensional dataset from planar mixing layer experimental data.

2. Experimental methods

The PIV experiments in this study were conducted in the recirculating water tunnel facility in the Department of Aeronautics at Imperial College London. The facility has a working section of width 700 mm, length 9 m and the water was filled to a depth of 600 mm. A nominally two dimensional planar mixing layer was produced by means of placing a perforated metal sheet, 50% open area ratio, on one side of a splitter plate of length 1.25 m and thickness, $h = 20$ mm, that was placed just downstream of the water tunnel’s contraction. Honeycomb aluminium was placed downstream of the perforated sheet in order to straighten the flow behind the obstruction. This honeycomb was placed on both sides of the splitter plate. Both boundary layers along the splitter plate were tripped with a 1 mm diameter wire downstream of the honeycomb and the boundary layers were given a streamwise distance of 800 mm over which to develop along the splitter plate. The splitter plate had a 4° triangular trailing edge appended to it in order to generate the mixing layer.

The boundary layers at the trailing edge of the splitter plate and the development of the mixing layer from the trailing edge to the self similar region were documented and presented in Buxton (2011). Additional PIV experiments performed in the $x_1 - x_3$ plane confirmed that the flow behaves as a nominally two dimensional planar mixing layer. The measurement location chosen for this study is 2m downstream of the trailing edge and is located in the self similar region of the mixing layer, meaning that the turbulence is fully developed, and the mixing layer is not constrained by the sidewall boundary layers, which is the case further downstream in the facility.

The PIV experiments used two cameras that were both synchronised by the same synchroniser and were connected to the same master PC. The two cameras were aligned adjacent to one another such that they were capable of imaging a streamwise domain of 16.6$h$, with a crossover region of $\approx h$. The cameras were fitted with Sigma 50 mm lenses.
Figure 1. Six successive PIV vector fields showing contours of $u_1/U_c$. The sequence runs from (a) to (f), with successive vector fields captured $7.25^{-1}$ s apart. The convection of a large-scale positive fluctuation through the domain can be seen in the sequence, as can the effect of the mean shear.

Images were acquired at a rate of 7.25 Hz; the rate being limited by the bus speed of the PC and the ability of the frame grabber to transfer the images to RAM. In order to acquire data at this rate it was necessary to leave the images in the PC’s memory. A total of 725 images were captured during twenty runs, in which images were captured until the memory was full. A $\Delta t$ of 2.4 ms was used giving a maximum pixel displacement of 11 pixels and a mean displacement of 9 pixels in the $x_1$ direction and a maximum magnitude of 2 pixels in the $x_2$ direction. The image pairs were processed recursively with TSI’s Insight software with a final interrogation window size of $32 \times 32$ pixels with a 50% overlap, thereby giving an uncertainty floor, $\sigma(\epsilon_{int,floor})$ of 0.06 pixels (Herpin et al., 2008). An in house post processing code was then applied to replace spurious vectors with valid secondary peaks or the local $3 \times 3$ mean displacement and then map pixel locations and displacements into spatial locations and displacements using a third order polynomial. The $32 \times 32$ pixel final interrogation windows corresponded to $2.82 \times 2.82$ mm, with adjacent vectors separated by 1.41 mm ($5.2\eta$) due to the 50% oversampling. The final size of the overall domain was $16.6h \times 7.9h$. The vector fields from the two cameras were then interpolated onto a regular Cartesian grid using a bi-linear interpolation method, with a sharp cut-off made in the crossover region. Figure 1 illustrates six successive PIV vector fields (a) to (f) that were captured $\tau = 7.25^{-1}$s apart. The contours in the figure are of the streamwise velocity.
fluctuation, \( u_1/U_c \), where \( U_c = (U_{\infty}^{HS} + U_{\infty}^{LS})/2 \) is defined as the mixing layer convection velocity (Townsend, 1976). The sequence shows the convection of a large-scale positive fluctuation through the domain and the mean shear of the flow. The uncertainty thus equates to 0.67% in the measurement of the mean of \( U_1 \) and 5.1% in the measurement of the r.m.s. fluctuation. Further details of the experimental methods can be found in Buxton (2011).

3. Results and discussion

This data was used to observe the convection of the large-scale velocity fluctuations through the PIV field of view by computing the autocorrelation function of the velocity fluctuations from PIV vector fields separated by an integer number, \( n \), of PIV captures in the time sequence. The autocorrelation function is computed in the streamwise direction using the discrete formulation shown in equation 1 below:

\[
R_{XX}(r \Delta x_1, x_2, n\tau) = \frac{1}{(N_x - r)(N_t - n)} \sum_{r=1}^{N_t-n} \sum_{i=1}^{N_x-r} \left\{ u_1(x_1^i, x_2, t^i)u_1(x_1^i + r \Delta x_1, x_2, t^i + n\tau) \right\} 
\]

where \( \Delta x_1 \) is the spacing between successive vectors in the PIV vector field, \( N_x \) is the number of vectors in the grid of the PIV vector fields (in the streamwise direction), \( r \) is an integer offset of PIV vectors in the streamwise direction and \( N_t \) is the number of vector fields captured in the time sequence. The autocorrelation function was computed at all locations of \( x_2 \), the direction of the mean velocity gradient in the flow and at fixed values of \( n \). Figure 1 shows that a large-scale structure convecting through the PIV field of view is captured in approximately six successive vector fields, and hence the upper limit on \( n \) is evidently 6.

Figure 2(a) shows examples of the autocorrelation function, \( R_{XX}(\Delta x_1, x_2, n\tau) \), at \( x_2 \) locations of \( x_2 = 4.21h \) and \( x_2 = 0.73h \) and figure 2(b) shows the autocorrelation function at these two \( x_2 \) locations normalised by the peak value for the case of \( Re_\lambda \approx 260 \), where \( \lambda \) is the Taylor micro scale. These correspond to locations close to the high speed freestream and close to the location of the peak Reynolds stresses respectively. The autocorrelation functions at these two \( x_2 \) locations are presented for \( n = 1 \) and \( n = 2 \). It can be seen in figure 2 that there are distinct peak values for the autocorrelation function, but that these peaks are fairly broad. These peaks are at a significantly higher value close to the location of peak Reynolds stresses than at the location close to the high speed freestream, reflecting the higher magnitude fluctuations present in the centre of the mixing layer. When the autocorrelation function is then normalised by the peak value in figure 2(b) it can be seen that the peaks are broader in the location close to the centre of the mixing layer than that closer to the extremity of the mixing layer. Additionally, it can be seen that the \( \Delta x_1 \) location of the peak values are higher, indicating a faster convection velocity, for the autocorrelations computed at the \( x_2 \) location closer to the high speed freestream.

Figures 2(c) and (d) are the equivalent plots to (a) and (b), respectively, for the mixing layer at the higher Reynolds number of \( Re_\lambda \approx 470 \). It can be seen that the peaks of all the autocorrelations become significantly broader and shift to a higher streamwise displacement, \( \Delta x_1 \). This is evidently connected to the higher convection velocity, \( U_c \), of the mixing layer at the higher Reynolds number case. The peak in the autocorrelations for the \( x_2 \) location closer to the high speed freestream is again located at a greater value of \( \Delta x_1 \), indicating a greater convection velocity. Autocorrelation functions in which \( u_2 \) and \( \omega_3 \), the spanwise component of vorticity, were used as the correlation variable produce identical peak locations to those presented in figure 2 which are computed from the streamwise velocity fluctuations. These, however, are not shown for brevity.

A fluctuation convection velocity, \( u_c(x_2) \), can thus be defined as \( \left( f_s \Delta x_1 \right)_{R_{XX}(x_2, n\tau)\text{=max}} / n \), where \( f_s \) is the sampling frequency (7.25 Hz). A three-point Gaussian peak was fitted through the discrete point located at the peak value of \( R_{XX}(\Delta x_1, x_2, n\tau) \) (from figure 2) and those immediately either side of this peak location. This three-point Gaussian fitting is identical to that used in the sub-pixel interpolation.
in the PIV processing algorithm. The fluctuation convection velocity, \( u_c \), was thus computed using the location of this fitted Gaussian peak. Figure 3(a) shows \( u_c \) computed from the \( Re_\lambda \approx 260 \) case for \( n = 1 \) (dashed line) and \( n = 2 \) (crosses), in addition to the mean flow computed as a function of \( x_2 \) for the whole domain (solid line) for comparison, normalised by the global convection velocity, \( U_c \). Figure 3(b) shows \( u_c \) computed from the \( Re_\lambda \approx 470 \) case with the same symbol definitions as for (a) with the \( n = 1 \) case from the lower Reynolds number case also plotted (the circles) for comparison. It can be seen that as \( n \) is increased in the higher Reynolds number case \( u_c \) becomes increasingly noisy. This is also the case for the lower Reynolds number case, although it only becomes evident for \( n = 3, 4, 5, 6 \), which are not shown for brevity. Nevertheless, it can be seen that the choice of \( n \) does not unduly affect the computation of \( u_c \), and henceforth only the least noisy \( n = 1 \) cases will be used. Keeping \( n = 1 \) also reduces the time interval between PIV vector fields being correlated, and hence the number of eddy turnover times for the smallest scale fluctuations, which should also improve the correlations (c.f. Kim & Hussain (1993)).

It can be seen that the fluctuations tend to convect at a lower velocity than the mean in the high speed
side of the mixing layer and at a higher velocity than the mean in the low speed side of the mixing layer. It can also be seen in figure 3(b) that the profiles of \(u_c\), computed from both Reynolds numbers, collapse onto each other when normalised by \(U_c\), the “global convection velocity”. Figure 2 shows that the peaks of the autocorrelation function are actually quite broad, particularly near the centre of the mixing layer where the peak Reynolds stresses are located, and that an increase in Reynolds number tends to broaden the peaks further. The mean fluctuation convection velocity is thus a mean over a broad band of correlations. The fact that higher Reynolds number and location close to the peak Reynolds stresses, where the turbulence intensities are higher, broadens these peaks suggests that different scale fluctuations convect at different convection velocities.

Figure 3. (a) Convection velocity for the \(Re_\lambda \approx 260\) case. (b) Convection velocity for the \(Re_\lambda \approx 470\) case. The solid lines are the mean velocity for the PIV field of view, the dashed lines are computed from \(n = 1\), the crosses are computed from \(n = 2\) and the circles in (b) are the \(n = 1\) case from the lower Reynolds number for comparison.

Figure 4. Single wavenumber content of a signal at time \(t\) (dashed line) and at time \(t + \tau\) (solid line). The convection in time \(\tau\) is marked as \(\Delta x_1\) on the figure.
The effect of scale dependence on the fluctuation convection velocity is examined by means of one-dimensionally spectrally filtering the streamwise velocity fluctuations in the streamwise direction with a narrow band pass filter that only passes a single wavenumber. The wavenumber is defined as $k_1 = 2\pi/\Lambda$, where $\Lambda$ is the length scale associated to that particular wavenumber in the streamwise direction. The output of the single-wavenumber-pass filter is thus a pure sinusoid, that is periodic over a length scale of $\Lambda$. Each sinusoid has a magnitude and phase which can be written as:

$$u_1(k, x_2, t) = \mathcal{F} \{ u_1(x_1, x_2, t) \} |_{k_1=k} = A_1 e^{i\phi_1} e^{ikx_1} = A_1 e^{i(kx_1+\phi_1)}$$ (2)

$$\hat{u}_1(k, x_2, t + \tau) = \mathcal{F} \{ u_1(x_1, x_2, t + \tau) \} |_{k_1=k} = A_2 e^{i\phi_2} e^{ikx_1} = A_2 e^{i(kx_1+\phi_2)}$$ (3)

where $A$ is the amplitude of the sinusoid and $\phi$ is the phase. The convection distance and velocity can thus be given by:

$$\Delta x_1 = \frac{\phi_2 - \phi_1}{2\pi}$$ (4)

$$u_c = f_S \langle \Delta x_1 \rangle$$ (5)

**Figure 5.** (a) Profiles of $u_c(x_2)$ for the first 15 non-zero wavenumbers with the mean profile $\langle U_1 \rangle(x_2)$ marked as the dashed line for comparison. (b) Bands centred on the corresponding wavenumber, $k_1$, showing contours of $\langle u_c(x_2) - \langle U_1 \rangle(x_2) \rangle/U_c$. Both (a) and (b) are from the $Re_\lambda \approx 260$ case.

Figure 5(a) shows the $u_c(x_2)$ profiles for the first 15 non-zero wavenumbers of the $Re_\lambda \approx 260$ case with the mean profile for the whole domain marked on as the dashed line for comparison. Only the first 15 non-zero wavenumbers are show in order to ensure that the convection distance, $\Delta x_1$, is not greater than the wavelength of the sinusoid. Figure 4 clearly shows that the phase shift between the two sinusoids is less than $\pi$. However, when the wavenumber increases, and $\Lambda$ decreases, it is possible that the convection distance $\Delta x_1$ is greater than $\Lambda$, i.e. the phase shift is greater than $2\pi$, such that the first peak of the dashed line sinusoid actually corresponds to the second peak of the solid line sinusoid in figure 4. The first 15 non-zero wavenumbers, however, provide a range of fluctuations from $\approx 31\lambda$, which is greater than the integral scale, down to $\approx 2\lambda$.  


It can clearly be seen that the large-scale (low wavenumber) fluctuations convect at a greater velocity than the small-scale (high wavenumber fluctuations). Although profiles for the low wavenumber fluctuations can be seen to be clustered together it can be seen that there is a great variation in the convection velocity as a function of the cross stream location, $x_2$, for these fluctuations. These fluctuations are observed to convect at a convection velocity that is slightly greater than the mean velocity in the low speed side of the mixing layer, whereas they convect at a speed that is less than the local mean in the high speed side of the mixing layer. This is further illustrated in figure 5(b) which shows bands, centred on the corresponding wavenumber $k_1 = 2\pi/\Lambda$, of contours of $u_c(x_2) - \langle U_1 \rangle(x_2)$. The dark red bands at the low wavenumber/low speed side of the figure highlight this convection velocity that is greater than the mean. There is, however, a much greater spread in the low wavenumber convection velocity profiles in the high speed side of the mixing layer, in which these fluctuations convect at velocities that are less than the mean. It can be seen that in fact the range of scales that convect at the greatest convection velocities are not the largest scales examined here but those in the range $8\lambda - 10\lambda$.

The smaller scale fluctuations (higher wavenumbers) are all observed to convect at velocities that are significantly less than the local mean and the convection velocities of the larger scale structures. Additionally, after a critical length scale which is $\approx 4\lambda$, the cross-stream gradient of the convection velocity profiles changes sign with respect to that for the mean profile and the larger scale fluctuations such that these intermediate scale fluctuations actually convect more quickly in the low speed side of the mixing layer than the high speed side. This phenomenon is further illustrated in figure 5(b) which shows a strong colour gradient from the high speed side of the mixing layer to the low speed side of the mixing layer. This same effect is also visible for the mixing layer at $Re_\lambda \approx 470$, illustrated in figure 6, although the convection of the large/intermediate scales examined is slower with respect to the mean than for the lower Reynolds number case of figure 5(b). The length scale at which the gradient changes sign from the same as the mean flow to the opposite sign is observed to be $\Lambda \approx 7.2\lambda$, which is larger than for the lower Reynolds number case.
4. Summary

PIV experiments were performed at a relatively high data acquisition rate of 7.25 Hz in order to examine the convection velocities of turbulent fluctuations in a nominally two dimensional planar mixing layer. These convection velocities were computed from the discretely formed autocorrelation function of equation 1. The peaks of this autocorrelation function can be seen to be broad, and become increasingly broad at higher Reynolds number, indicating that there is a scale dependence on this autocorrelation due to different convection velocities for different length scale fluctuations. The overall convection velocity profile for the mixing layer, computed from the autocorrelation function, shows that fluctuations in the high speed side of the mixing layer tend to convect at speeds less than the mean flow whereas fluctuations on the low speed side of the mixing layer convect faster than the mean. This is a similar finding to that of Wills (1964) which was conducted in a turbulent jet.

Convection velocities associated to fluctuations of the first 15 non-zero wavenumbers in the flow were computed by finding the mean phase difference between the sinusoids of wavenumber $k$ at time $t$ and at time $t + \tau$. It was shown that the low wavenumber fluctuations convect at speeds that are greater than the mean flow in the low speed side of the mixing layer, which is a similar finding to studies conducted in other shear flows (Wills, 1964, Hussain & Clark, 1981, Kim & Hussain, 1993, Krogstad et al., 1998, del Álamo & Jiménez, 2009). The higher wavenumber fluctuations convected at much lower velocities. It was found that there is a particular wavenumber at which the cross stream gradient of $u_c(x, y)$ changes to the opposite sign of the mean flow’s gradient, meaning that the fluctuations in the low speed side of the mixing layer convect at a greater speed than those in the high speed side of the mixing layer. This wavenumber was found to be Reynolds number dependent, with a higher value for the higher Reynolds number case investigated.

It is shown that the “overall” convection velocity profile is similar, but crucially more uniform, than the mean velocity profile. This, and the clear scale dependence of the convection velocity, casts further doubt on the validity of using the mean streamwise velocity as the “convection velocity” when applying Taylor’s frozen flow field hypothesis to planar, high acquisition rate experimental data in order to compute a pseudo three dimensional dataset.

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