Non-linear stability of under-deck cable-stayed bridge decks

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Abstract
The stability of comparatively more slender decks of under-deck cable-stayed bridges is studied, by considering both the critical loads and the post-buckling behaviour. A potential energy approach is applied to a simplified discrete link and spring model that allows for an exact nonlinear formulation of the equilibrium equations. The physical response is found to be dependent on the ratio of the axial stiffness of the cable-staying system to the flexural stiffness of the deck. The influence of several parameters is analysed and unstable mode interaction is observed to occur under certain geometric conditions. The presented analytical model is compared with a nonlinear finite element model that shows good correlation. Finally, some design criteria and recommendations are suggested, which are relevant for designers of this innovative typology of cable-stayed bridges.

Keywords:
cable-supported structures, under-deck cable-stayed bridges, non-linear buckling, mode interaction, energy methods, analytical modelling

1. Introduction
Under-deck cable-stayed bridges (UDCSBs) are an innovative typology of cable-stayed bridges [1], in which the stay cables are located underneath the deck [2, 3]. The stay cables, which are initially prestressed, are self-anchored to the deck and follow a polygonal layout (Fig. 1). The deviation forces generated in the edges of this layout are introduced into the deck by means of struts, consequently providing additional elastic supports to the deck. Hence, depending on the number of struts employed and the initial prestress force, the bending moments acting on the deck can be reduced substantially when compared to a bridge with no cable-supporting system [4].

UDCSBs have been designed and built since the late 1970s, an example of which is shown in Fig. 2. Research focused on these bridges has demonstrated their advantages for medium spans when compared with conventional bridges without cable-staying systems [4].

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These advantages include: (1) higher structural efficiency by reducing the flexural response of the deck and enhancing the axial response; (2) significantly higher deck slendernesses leading to a reduction in the structural self-weight, allowing for more sustainable construction; (3) multiple construction solutions; and (4) arguably, more attractive aesthetic characteristics.
for setting the design load higher than the critical buckling load. However, an unstable response, which is usually a signature for high sensitivity to initial imperfections, implies that the design load has to be set lower than the critical buckling load [13, 14]. Nevertheless, critical loads would not normally be reached during the construction of UDCSBs, and the additional load allowance of stable post-buckling paths would lead to greater safety factors under unexpected loading scenarios.

An analytical approach is presented in the current work that allows for an exact formulation of simplified bridge behaviour, by employing a methodology based on energy principles [15, 14]. A model comprising discrete rigid-links and springs is employed. These rigid-link models have successfully mimicked the behaviour of prestressed stayed columns, in which mode interaction phenomena can be observed under certain circumstances [16, 13, 17]. The principal advantage of these rigid-link models is that the relatively simple, but non-linear, formulation allows the determination of the influence of various parameters on the response.

After an initial formulation of the perfect case, a particular solution is presented with the aim of demonstrating the practical application of the model. The presented model allows for multiple initial and boundary conditions that can replicate different construction methods. The results are then compared with the results obtained with a finite element model formulated in the commercial code Abaqus [18]. Finally, the discussion of results, some design criteria and general conclusions are presented.

2. Analytical model development

A single-span UDCSB with two struts and a stay cable eccentricity of 10% of the total span is studied due to its structural efficiency [2, 4], the total span being consequently divided into three subspans. The struts, which are pinned to the deck to avoid the introduction of moments (as recommended by [4]), bisect the angle between the stay cables at the edges of the polygonal layout, see Fig. 3. Hence, the deviation force generated by the prestressed stay cable follows the direction of the struts introducing, in turn, an axial and a lateral force into the deck. Following the trend from the research into the buckling of

![Figure 3: UDCSB with two struts, a stay cable eccentricity of 10% and the corresponding subspans. The deviation force is a consequence of the prestressing force \( T \) in the stay cables.](image)

columns, the term ‘lateral’ is employed to refer to any load that acts perpendicularly to the axis of the deck, such as the vertical component of the deviation forces. The following assumptions are made in the simplified analytical model:

1. The axial deformation of the deck and struts is considered to be negligible.
2. A constant flexural rigidity is considered for the entire length of the deck.

3. The cable-staying system is anchored at the centroid of the cross-section of the deck at support sections and therefore no bending moments are introduced into the deck at these sections.

4. All materials and springs are considered to be linearly elastic.

A three degree-of-freedom (3-DOF) link model is presented that allows for the exact formulation of the total potential energy of the system $V$. Equilibrium equations, which are deduced from $V$, are solved numerically by means of Auto [19], a powerful well-established numerical continuation package that can compute the bifurcation points as well as the solution branches. The model comprises four rigid links, linear longitudinal springs of stiffness $k$ at pins B and D, and rotational springs of stiffness $c$ at pins B, C and D (Fig. 4). Rotational springs account for the flexural stiffness of the deck, while longitudinal springs represent the cable-staying system. The length of the rigid links is dependent on the parameter $\alpha$, which is introduced to consider different subspan length distributions.

Figure 4: Rigid-link and springs model with the corresponding rotational and longitudinal springs being of stiffness $c$ and $k$ respectively. Generalized coordinates $Q_1, Q_2$ and $Q_3$ define the system kinematics and represent the non-dimensional lateral displacements of nodes B, C and D respectively; $\mathcal{E}$ is the end-shortening of the deck.

Even if the stay cables are located purely on one side of the deck, the effect of these can be modelled by means of longitudinal springs such that:

- If a downward perturbation is introduced in the deck, the axial force in the stay cables would increase; consequently, the upward component of the deviation force would increase.

- If an upward perturbation is introduced in the deck, the axial force in the stay cables would decrease; consequently, the upward component of the deviation force would decrease.

There may also be a case of an upward perturbation value where the stay cables slacken, which would diminish the stiffness of the stay cables in turn [16]. However, this effect is neglected in the current work. Hence, stay cables are considered to be under tension at any given position.

Initially, a perfect case with forces acting purely in the direction of the deck is analysed. The compression force introduced by the stay cables into the deck at the support sections
is modelled as an external load $P$; this is the horizontal component of the axial load in the inclined stay cables. The expression for the total potential energy $V$ is:

$$V = U - P\mathcal{E},$$

(1)

where $U$ is the strain energy stored in the system and the term $P\mathcal{E}$ represents the work done by the external load, which is obtained by multiplying the magnitude of the force $P$ by the distance that the load moves parallel to the loading displacement $\mathcal{E}$, i.e. the end-shortening of the deck. The strain energy $U$ is given by the energy stored in both the rotational and longitudinal springs:

$$U = \frac{1}{2} c (\theta_B^2 + \theta_C^2 + \theta_D^2) + \frac{1}{2} k [(Q_1 l)^2 + (Q_3 l)^2],$$

(2)

where $\theta_B$, $\theta_C$ and $\theta_D$ are the rotation angles of the pins B, C and D respectively. These angles can be expressed as:

$$\begin{align*}
\theta_B &= \arcsin \left( \frac{Q_1}{2 - \alpha} \right) + \arcsin \left( \frac{Q_1 - Q_2}{\alpha} \right), \\
\theta_C &= \arcsin \left( \frac{Q_2 - Q_1}{\alpha} \right) + \arcsin \left( \frac{Q_2 - Q_3}{\alpha} \right), \\
\theta_D &= \arcsin \left( \frac{Q_3 - Q_2}{2 - \alpha} \right) + \arcsin \left( \frac{Q_3}{\alpha} \right).
\end{align*}$$

(3)

The end-shortening of the deck to obtain the work done term is:

$$\mathcal{E} = l \left[ 4 - \sqrt{(2 - \alpha)^2 - Q_1^2} - \sqrt{\alpha^2 - (Q_2 - Q_1)^2} - \sqrt{\alpha^2 - (Q_2 - Q_3)^2} - \sqrt{(2 - \alpha)^2 - Q_3^2} \right].$$

(4)

The total potential energy can be non-dimensionalized by dividing through by the stiffness of the rotational springs $c$:

$$\tilde{V} = \frac{1}{2} (\theta_B^2 + \theta_C^2 + \theta_D^2) + \frac{1}{2} k (Q_1^2 + Q_3^2) - p\tilde{\mathcal{E}},$$

(5)

where $\tilde{V} = V/c$, $K = kl^2/c$, $p = Pl/c$ and $\tilde{\mathcal{E}} = \mathcal{E}/l$.

The inclined struts introduce an axial force into the deck at nodes B and D that needs to be considered. Owing to the proportionality of these axial forces $P_B$ and $P_D$ with the external load $P$ through the stay cable prestress:

$$P_B = P_D = \gamma P,$$

(6)

where $\gamma$ depends on the current bridge geometry and consequently it would be affected by the deflected shape of the bridge. However, for simplification purposes, a constant value
dependent purely on the initial bridge geometry is considered currently. These inclined forces lead to a modified work done term in the total potential energy \( \tilde{V} \):

\[
\tilde{V} = \frac{1}{2} (\theta_B^2 + \theta_C^2 + \theta_D^2) + \frac{1}{2} K (Q_1^2 + Q_3^2) - p \left( \tilde{\mathcal{E}} + \gamma \tilde{\mathcal{E}}_D - \gamma \tilde{\mathcal{E}}_B \right),
\]

where:

\[
\tilde{\mathcal{E}}_D = 2 + \alpha - \sqrt{(2 - \alpha)^2 - Q_1^2} - \sqrt{\alpha^2 - (Q_2 - Q_1)^2} - \sqrt{\alpha^2 - (Q_2 - Q_3)^2},
\]

\[
\tilde{\mathcal{E}}_B = 2 - \alpha - \sqrt{(2 - \alpha)^2 - Q_1^2}.
\]

2.1. Lateral loads and initial conditions

During the construction of the bridge, lateral loads are also present, as well as some initial deflections from the following sources: self-weight, initial geometric imperfections and the precamber introduced to compensate in-service deflections. Additionally, the employment of temporary supports and props defines some initial conditions that need to be accounted. The benefit of the analytical model presented is that these initial conditions can be introduced with relative ease to consider different construction methods and stages.

In the current analysis, two types of lateral loads are considered: the self-weight \( w \) as a distributed load along the entire deck and the vertical components of the deviation forces introduced by the stay cables into the deck by means of struts at nodes B and D, \( F_B \) and \( F_D \) respectively. In the analytical model presented in the current work, temporary props are not considered and the defined self-weight \( w \) acts on the deck to define an initially deflected shape. However, this methodology allows for the obtention of the same final stage as that reached in a construction process that employs temporary supports. Moreover, not considering props defines a more severe scenario without compromising the safety of the erection process since props may also introduce uncertain boundary conditions into the model. With this approach, the stability of the deck during construction could not be compromised because of the potential low stiffness of the temporary towers. Hence, the model comprises the following stages, as represented in Fig. 5:

1. The deck is located in place and vertically supported by pinned abutments.
2. The self-weight \( w \) acts on the deck causing an initially deflected shape.
3. The cable-staying system is located in place.
4. The stay cables are prestressed.
5. As the prestress force \( T \) increases the deck is erected and hence straightens.
6. The desired deflected shape, which is given by the compensation level of the permanent load [20], is reached.

The prestressing sequence is generally divided into different steps [1], in which different stay cables are prestressed at each stage. The stiffness of the longitudinal springs in this context represents the stay cables that have already been prestressed.
The vertical components of the deviation forces ($F_B$ and $F_D$), are determined by the prestressing force, and are also proportional to the axial load in the deck $P$, consequently:

$$F_B = F_D = \eta P,$$

(9)

where $\eta$ is the proportionality factor dependent on the geometry of the bridge. The initial bridge geometry is considered when obtaining $\eta$. If buckling is not triggered, a unique equilibrium position can be defined at each step. This equilibrium position $Q_{Ei}$ is given by:

$$Q_{Ei} = Q_{Ei,w} - Q_{Ei,T},$$

(10)

where the $w$ and $T$ superscripts refer to the equilibrium position for a given self-weight $w$ and prestress force $T$, respectively as represented in Fig. 6.

These equilibrium positions are easily obtained by establishing the static equilibrium equations from Fig. 6. If second order effects of horizontal forces are neglected, this leads
to following relationships:
\[
\begin{align*}
Q_{1}^{E,w} &= Q_{3}^{E,w} = (2 - \alpha) \sin \left( \frac{6 - \alpha^2}{2} W \right), \\
Q_{2}^{E,w} &= (2 - \alpha) \sin \left( \frac{6 - \alpha^2}{2} W \right) + \alpha \sin (W), \\
Q_{1}^{E,T} &= Q_{3}^{E,T} = (2 - \alpha) \sin \left( (2 - \alpha) \frac{3\eta p}{2} \right), \\
Q_{2}^{E,T} &= (2 - \alpha) \sin \left( (2 - \alpha) \frac{3\eta p}{2} \right) + \alpha \sin \left( (2 - \alpha) \frac{\eta p}{2} \right),
\end{align*}
\]  
(11)

where \( W \) is the non-dimensional self-weight, \( W = \frac{wl^2}{c} \). Hence, the longitudinal springs are active when they are displaced from these equilibrium positions. The consideration of these lateral loads and the initial conditions leads to a modified potential energy expression, thus:
\[
\tilde{V} = \frac{1}{2} \left( \theta_B^2 + \theta_C^2 + \theta_D^2 \right) + \frac{1}{2} K \left[ (Q_1 - Q_1^E)^2 + (Q_3 - Q_3^E)^2 \right] - p \left( \tilde{\epsilon} + \gamma \tilde{\epsilon}_D - \gamma \tilde{\epsilon}_B \right) \\
+ \eta p \left( Q_1 - Q_1^{E,w} + Q_3 - Q_3^{E,w} \right) - W (Q_1 + \alpha Q_2 + Q_3). 
\]  
(12)

3. Equivalent spring stiffnesses

Rotational springs represent the flexural stiffness of the deck, while longitudinal springs represent the stiffness of the cable-staying system. As a consequence, the equivalent stiffnesses of the springs need to be deduced from an actual bridge geometry.

3.1. Rotational springs

The rotational spring stiffness \( c \) is obtained by equating the flexural strain energy of the link model with the continuous bridge deck. These flexural strain energies need to be computed for the same deflection magnitudes in both models, and the area enclosed by the deflected shapes is considered for this purpose. Moreover, Mode 1 is considered to calculate the previous magnitudes, and the stiffness of the longitudinal springs is considered to be zero.

The flexural strain energy in the link model \( U_L \) is given by:
\[
U_L = \frac{1}{2} c \left( \theta_B^2 + \theta_C^2 + \theta_D^2 \right). 
\]  
(13)

For the case in which \( \alpha = 1.0 \), the generalized coordinates \( Q_i \) for Mode 1 as functions of \( Q_2 \) are (see Fig. 7):
\[
Q_1 = Q_3 = \frac{\sqrt{2}}{2} Q_2, 
\]  
(14)
which leads to the following flexural strain energy expression:

$$U_L = 2 \left( 3 - 2\sqrt{2} \right) cQ_2^2, \quad (15)$$

to leading order. The area enclosed by the deflected shape is:

$$A_L = Q_1 (2 - \alpha) \frac{l^2}{2} + (Q_1 + Q_2) \frac{\alpha l^2}{2} + (Q_2 + Q_3) \frac{\alpha l^2}{2} + Q_3 (2 - \alpha) \frac{l^2}{2} = \left( 1 + \sqrt{2} \right) Q_2 l^2. \quad (16)$$

A sinusoidal deflected shape is considered for the continuous bridge deck (see Fig. 7):

$$y = Q_m l \sin \frac{\pi x}{4l}, \quad (17)$$

where $Q_m$ is the non-dimensional maximum deflection at midspan. The flexural strain energy of the deck $U_C$ to leading order is:

$$U_C = \frac{1}{2} E_D I_D \int_0^{4l} \left( \frac{d^2 y}{dx^2} \right)^2 \, dx, \quad (18)$$

where $E_D$ and $I_D$ are the Young’s modulus and the second moment of area of the deck respectively. Hence, $U_C$ for Mode 1 can be expressed as:

$$U_C = \frac{\pi^4 E_D I_D}{256l} Q_m^2. \quad (19)$$

Equivalently, the area enclosed by the continuous bridge deck is:

$$A_C = Q_m l \int_0^{4l} \sin \frac{\pi x}{4l} = \frac{8l^2 Q_m}{\pi}. \quad (20)$$

By equating $A_C$ with $A_L$, the following relationship is found between $Q_m$ and $Q_2$:

$$Q_m = \left( \frac{1 + \sqrt{2}}{8} \right) \pi Q_2. \quad (21)$$

Finally, by equating the flexural energy terms $U_L$ and $U_C$, the stiffness of the rotational
spring (for $\alpha = 1.0$) is obtained:

$$c = \frac{(17 + 12\sqrt{2}) \pi^6 E_D I_D}{32768} \approx 0.997 \frac{E_D I_D}{l}.$$  

Equivalently, the stiffness of the rotational spring can be obtained for different $\alpha$ values. The stiffness $c$ can be expressed as:

$$c = C_\alpha \frac{E_D I_D}{l},$$  

where the value of the $C_\alpha$ coefficient is given graphically in Fig. 8. This coefficient $C_\alpha$, for the case where all the links are of the same length, tends to unity when increasing the number of degrees of freedom [21]. In the current work, $\alpha$ values ranging from 0.5 to 1 are studied, since other values do not provide practically realistic subspan distributions, as common values in UDCSBs have been in the range between 0.67 and 0.77, i.e. $\alpha$ values corresponding to lateral subspan lengths equal to 100% and 80% of that of the middle subspan respectively [4, 22].

![Figure 8: Coefficient $C_\alpha$ for the calculation of the equivalent stiffness of the rotational springs $c$ for different $\alpha$ values that define the subspan lengths.](image-url)

3.2. Longitudinal springs

Longitudinal springs account for the effect of the cable-staying system. The flexibility method is employed to calculate the equivalent stiffness of the stay cables, i.e. the vertical component of the deviation force induced in the stay cables $F_v$ is computed when a known vertical deflection $\delta_v$ is imposed to the deck–strut connection section, hence:

$$F_v = k \delta_v,$$  

where $k$ is the equivalent spring stiffness. The struts, which are pinned to the deck, are considered to be infinitely rigid due to their significantly higher stiffness compared with the stay cables. Initially, the deflected shape corresponding to Mode 1 is analysed.
A horizontal and a vertical deflection of the bottom tip, $\Delta_x$ and $\Delta_y$ respectively, originate from a vertical deflection of the top tip of the strut $\delta_v$ (Fig. 9). Owing to the lack of elongation of the strut, and by neglecting the horizontal displacement of the deck, the relationship between the aforementioned deflections is:

$$\frac{H}{D} = \frac{\Delta_x}{\delta_v - \Delta_y},$$

(25)

where $H$ and $D$ are the eccentricity of the cable-staying system at midspan and the horizontal projection of each strut, respectively (Fig. 10). The axial strains in the inclined ($\epsilon_i$) and horizontal ($\epsilon_h$) stay cables are:

$$\epsilon_i = \frac{\Delta_x}{\alpha l + D},$$

$$\epsilon_h = \frac{\Delta_y \sin (\pi - 2\beta) - \Delta_x \cos (\pi - 2\beta)}{\sqrt{H^2 + [(2 - \alpha)l - D]^2}},$$

(26)

where $\beta$ is the angle between the struts and the stay cables (Fig. 10). The axial forces in the inclined and horizontal stay cable, $T_i$ and $T_h$ are given by:

$$T_i = EA\epsilon_i, \quad T_h = EA\epsilon_h,$$

(27)

where $E$ and $A$ respectively are the Young’s modulus and the cross-sectional area of the stay cables, respectively. If the sag effect of the stay cables is accounted, which is negligible in the span lengths and prestressing levels considered in the current work, $E$ would become the Ernst’s modulus [23]. Since the forces in all the stay cables must be equal, the following
The relationship holds:

\[ T_i = T_h = T. \]  

(28)

The vertical component of the deviation force introduced by the stay cables into the deck by means of the struts is:

\[ F_v = 2T \cos \beta \sin \beta. \]  

(29)

Finally, by combining Equations (24)–(29), the expression for the equivalent spring stiffness \( k \) is obtained, thus:

\[
    k = \frac{4EA \sin^2 \beta \cos^2 \beta}{2(\alpha l + D)(\sin \beta \cos^2 \beta + \sin \beta - \cos \beta) + \sqrt{H^2 + [(2 - \alpha)l - D]^2}}.
\]  

(30)

However, if Mode 2 is considered when evaluating the stiffness of the longitudinal springs \( k \), a lack of tension can be observed in the stays for an imposed deflection. The pinned joints between the deck and struts allow the stay cables to adopt the shape of Mode 2 without any elongation. Hence, the spring stiffness \( k \) depends on the relative displacements of nodes B and D. Any given buckled shape \( \Phi \) is decomposed into the previously defined equilibrium shape \( (Q_i^E, \text{defining Mode ‘E’}) \) and three non-orthogonal shapes: Mode ‘F’ (‘F’ for flat), Mode 2 and Mode ‘P’ (‘P’ for peak), as shown in Fig. 11:

\[
    \Phi = \Phi_E + q_F \Phi_F + q_2 \Phi_2 + q_P \Phi_P,
\]  

(31)

where \( \Phi_E \) is the equilibrium shape for a given self-weight and prestress level \( i.e. \text{Mode ‘E’} \); \( \Phi_F, \Phi_2 \) and \( \Phi_P \) are the unitary shapes relative to Modes ‘F’, 2 and ‘P’ respectively; with \( q_F, q_2 \) and \( q_P \) being the amplitudes of the previous modes respectively. The equilibrium shape has been shown not to activate the longitudinal springs. Furthermore, while Mode 2 does not generate any force in the stay cables, Mode ‘P’ does not activate the cable-staying system since the vertical deflection of the nodes B and D, where the springs are located, is zero. Hence, the only Mode that contributes to the activation of the stay cables is ‘F’, and the amplitude of this is:

\[
    q_F = \left( Q_1 - Q_1^E + Q_3 - Q_3^E \right) \frac{l}{2},
\]  

(32)

with the force in each spring being:

\[
    F_{k1} = F_{k3} = \left( Q_1 - Q_1^E + Q_3 - Q_3^E \right) \frac{kl}{2}.
\]  

(33)
The forces in both springs are equal, this is consistent with the fact that the same force in the entire stay cable induces the same deviation forces in both struts.

4. Results and discussion

From the potential energy of the system, the critical loads $p_i^C$ can be obtained from the condition that the Hessian matrix $\tilde{V}$ becomes singular:

$$\det(\tilde{V}) = \begin{vmatrix} \tilde{V}_{11} & \tilde{V}_{12} & \tilde{V}_{13} \\ \tilde{V}_{21} & \tilde{V}_{22} & \tilde{V}_{23} \\ \tilde{V}_{31} & \tilde{V}_{32} & \tilde{V}_{33} \end{vmatrix} = 0,$$

where:

$$\tilde{V}_{ij} = \frac{\partial^2 \tilde{V}}{\partial Q_i \partial Q_j}.$$  \hspace{1cm} (34)

This gives three eigenvalue solutions $p_i^C$:

$$p_1^C = \frac{\alpha (2 - \alpha)^2 (1 + \gamma) K - \alpha (2 - \gamma) + 6 - \chi}{2\alpha (2 - \alpha) (1 + \gamma)},$$

$$p_2^C = \frac{4}{\alpha (2 - \alpha) [2 + (2 - \alpha) \gamma]},$$

$$p_3^C = \frac{\alpha (2 - \alpha)^2 (1 + \gamma) K - \alpha (2 - \gamma) + 6 + \chi}{2\alpha (2 - \alpha) (1 + \gamma)},$$

where:

$$\chi = \sqrt{d_0 + d_1 K + d_2 K^2},$$

$$d_0 = (\gamma^2 + 4\gamma + 12) \alpha^2 - 4 (10 + \gamma) \alpha + 36,$$

$$d_1 = \frac{2\alpha (2 - \alpha)^2 (1 + \gamma) (\alpha \gamma + 4\alpha - 6)}{\alpha^2 (2 - \alpha)^4 (1 + \gamma)^2}.$$  \hspace{1cm} (37)

The following condition provides the equilibrium relationships:

$$\frac{\partial \tilde{V}}{\partial Q_i} = 0,$$  \hspace{1cm} (38)

where $i = \{1, 2, 3\}$ defines the appropriate generalized coordinate. When considering the exact solution, this equation leads to three non-linear equations that are solved numerically in AUTO [19] and define the system equilibrium paths.

4.1. Critical buckling

The critical loads have been obtained in Equation (36) corresponding to the three modes. Each mode is characterized by a shape, which is represented in Fig. 12(a). These
modes, which are orthogonal for $K = 0$, vary with $K$. Results show that Mode 1 tends towards Mode ‘P’ defined in Section 3, since the higher stiffness of longitudinal springs tends to reduce deflections at nodes B and D. Analogously, Mode 3 approximates to Mode ‘F’, but Mode 2 remains the same.

The normalized values of the critical loads are plotted against the normalized lateral stiffness in Fig. 12(b). Increasing the stay cable stiffness $K$ increases the critical loads corresponding to Modes 1 and 3. On one hand, $p_{1}^{C}$ increases approximately linearly with $K$ for lower stiffness values, but this load becomes approximately invariant with higher $K$ values. On the other hand, $p_{3}^{C}$ does not vary significantly for lower $K$ values, but it increases at a higher rate and approximately linearly for higher $K$ values. However, critical loads for Mode 2 do not vary with $K$, as already explained in Section 3: antisymmetric modes do not change the length of the stay cables, and so no additional stay cable forces are introduced.

Mode 1 is the lowest critical load for lower $K$ values, but for stiffness values $K > 2$, Mode 2 becomes the lowest critical load. This response has been also observed in the dynamic behaviour of UDCSBs [24, 25], in which the second flexural mode is the natural mode when the axial stiffness of the cable-staying system is sufficiently high when compared with the bending stiffness of the deck.

The influence of the ratio between the lengths of the lateral and central subspans, through the $\alpha$ parameter, is plotted in Fig. 13(a). The non-dimensional critical loads $p_{i}^{C}$ tend to increase when decreasing $\alpha$, except for very low $K$ values in which $p_{1}^{C}$ decreases minimally.

Increasing the horizontal component of the force introduced by the struts into the deck (i.e. increasing $\gamma$) decreases the critical loads due to the higher effective load acting within the deck, see Fig. 13(b). Owing to the changing buckling shapes with $K$, the reduction of the critical load also varies with $K$, except for the antisymmetric Mode 2. In Modes 1

![Figure 12: Critical loads corresponding to the perfect case. (a) Buckling modes for $\alpha = 1.0$, $K = 0$, $\gamma = 0.0$. (b) Critical loads corresponding to the three modes for different $K$ values ($\alpha = 1.0$, $\gamma = 0.0$).](image-url)
and 3, the lever arm of the horizontal component of the forces introduced by the struts can be defined as the relative vertical distance between nodes B and C. As $K$ increases, the lever arm for Mode 1 increases since this mode tends to Mode ‘P’, consequently causing a greater reduction in the critical load. Conversely, for Mode 3 the lever arm decreases as $K$ increases since this mode tends to Mode ‘F’, resulting in a less severe load reduction.

![Figure 13: Variation in critical loads while varying: (a) $\alpha$ ($\gamma = 0.0$, $\alpha = 1.0, 0.8, 0.6$); (b) $\gamma$ ($\alpha = 1.0$, $\gamma = 0.00, 0.05, 0.10, 0.15, 0.20$).](image)

4.2. Post-buckling behaviour

Equilibrium paths for the perfect case were obtained by solving the system of non-linear algebraic equations derived from Equation (38) in Auto; these are plotted in Fig. 14. The graphs show that Mode 1 presents a weakly stable post-buckling response when stay cables are not installed, i.e. $K = 0$ as shown in Fig. 14(a), and a weakly unstable response for $K > 0$. However, there is a further transition point when $K \approx 3$, and higher $K$ values tend to stabilize the post-buckling path. The response for Mode 2 is weakly stable and independent of the $K$ value. The response for Mode 3 is stable until $K \approx 2$, after which it becomes unstable. Even though these limit values are calculated for $\alpha = 1.0$, they do not vary from these significantly for other $\alpha$ values studied in the current work.

The critical loads corresponding to Modes 1 and 2 coincide when $K = 2$ (for $\alpha = 1$). For $K$ values slightly higher, the mode interaction or ‘mode jumping’ phenomenon [26] is observed. Figure 14(b) shows the elliptical mode interaction relationship that is observed when $K \approx 2.0$ (the example shown is $K = 2.06$). Initially, the response hits the S1 bifurcation point in which antisymmetric buckling (Mode 2) is triggered. However, when $Q_1$ and $Q_3$ are sufficiently high, a secondary bifurcation point is reached (i.e. S2) that breaks the antisymmetry, leading to an asymmetric interactive buckling shape, which is highly unstable in turn. Finally, the buckled shape becomes symmetric after bifurcation point S3 is encountered (i.e. Mode 1), which is inherently unstable for that value of $K$. 
Figure 14(c) shows the elliptical interaction loop relating the generalized coordinates $Q_1$ and $Q_2$.

Increasing $K$ further increases the difference between $p_{C1}^C$ and $p_{C2}^C$, with $p_{C1}^C$ being higher, and the interaction loop increases in size consequently. Hence, higher deflections are needed to reach secondary bifurcation points, and as a consequence, mode interaction is less likely to be triggered. If $K$ is increased sufficiently, the interaction loop distorts and eventually breaks, which causes the deck to remain in the interactive mode without triggering a subsequent symmetric mode; this effect is illustrated in [16].

Mode interaction is observed when the post-buckling path corresponding to the lowest critical load is stable, and that of the higher buckling load is unstable. Despite the stable path of the lower load, the mode interaction introduces a highly unstable path. This same effect has already been observed in stayed columns [16, 13].

The critical loads $p_{C1}^C$ and $p_{C2}^C$ can also coincide for lower $\alpha$ values, and hence interactive buckling can also be triggered, an example of which is presented in Fig. 14(d).

However, if $K$ is reduced to lower values than that in which $p_{C1}^C$ and $p_{C2}^C$ are identical, $p_{C1}^C$ becomes the lowest load and the interaction is not observed. Mode 1 dominates the response and the deck does not exhibit any asymmetry.

4.3. Effect of lateral loads

The self-weight $W$ introduces an initial deflection. Hence, when the stay cables are not installed (i.e. $K = 0$), the equilibrium paths are asymptotic to the perfect equilibrium paths, although they are further from the perfect case as $W$ is increased. This effect is identical to that of introducing deck imperfections with a quartic polynomial shape. In this case, $W$ introduces downward deflections, so when the stay cables are prestressed in the first stage, the axial load in the deck tends to amplify the deflections.

However, in UDCSBs, the effect of the vertical component of the deviation force must be considered through the $\eta$ parameter. This force, which is in the upward direction, can compensate the downward deflection caused by the combined action of $W$ and the second order effect from the axial force in the deck. Nevertheless, the effect of $\eta$ does not always fully compensate $W$, see Fig. 15; for a given bridge geometry and $\eta$ value, a transition value of $W$ exists, termed $W^T$. The vertical force in the struts is not able to compensate the downward deflections caused by the self-weight fully for $W > W^T$; while for $W < W^T$, the vertical force in the struts fully compensates the downward effect, consequently causing the deck to deflect upwards.

Equivalently, for a given bridge geometry and $W$ value, $\eta$ can be increased sufficiently to compensate the downward deflections of the combined action of $W$ and the axial force on the deck, see Fig. 15(b).

If the prestressing stage is not at the beginning, which is to say that some stay cables are already installed (i.e. $K \neq 0$), the previously explained phenomenon does not occur. The second order effect from the axial force in the deck, by means of the downward deflection, activates the cable-staying system, and hence prevents the deck from developing further downward deflections, see Fig. 16(a). As previously explained, the higher the $K$ value the
Figure 14: Equilibrium paths for the perfect case: (a) shows distinct modal post-buckling behaviours; (b)–(d) show mode interaction. (a) $p$–$Q_1$ plot for the three modes when $\alpha = 1.0$, $\gamma = 0.0$, $K = 0.0$; (b) $p$–$Q_1$ plot when $\alpha = 1.0$, $\gamma = 0.0$, $K = 2.06$; (c) $Q_2$–$Q_1$ plot when $\alpha = 1.0$, $\gamma = 0.0$, $K = 2.06$; and (d) $p$–$Q_1$ plot when $\alpha = 0.6$, $\gamma = 0.0$, $K = 1.53$. 
Figure 15: Equilibrium paths under lateral loads: (a) $p$–$Q_1$ plot for varying self-weight $W$ when $\alpha = 1.0$, $K = 0.0$, $\eta = 0.15$; and (b) $p$–$Q_1$ plot for varying $\eta$ when $\alpha = 1.0$, $K = 0.0$, $W = 0.05$. Higher the $p_1^C$ critical load, and therefore the imperfect equilibrium paths tend to a higher asymptote.

Figure 16: Equilibrium paths under lateral loads: (a) $p$–$Q_1$ plot for varying stay cable stiffness $K$ when $\alpha = 1.0$, $\eta = 0.20$, $W = 0.07$; and (b) $p$–$Q_1$ plot for varying $\gamma$ when $\alpha = 1.0$, $K = 0.0$, $W = 0.085$, $\eta = 0.25$.

The relative axial force factor $\gamma$ tends to amplify the downward deflections through the second order effect from the horizontal component of the axial force in the struts acting on the deflected shape. In a given bridge configuration, even if $W < W^T$, $\gamma$ can cause the same effects of having $W$ values higher than the transition value $W^T$ with the self-weight deflections being unable to be fully compensated.
5. Validation

The analytical model presented in the current work has been validated by employing the commercial Finite Element (FE) software *Abaqus* [18]. A two dimensional (2D) model of an 80 m span UDCSB has been employed, in which three main elements can be distinguished: the deck, the struts and the stay cables. The deck (Fig. 17) is formed by beam elements, while the struts and the stay cables are modelled with truss elements that do not allow for bending moments—the struts being pinned to the deck. The influence of the axial stiffness of the struts has been observed to be negligible when considering realistic values, and hence, for simplicity the struts have been considered to be axially rigid by providing them with a cross-sectional area of several orders of magnitude greater than both the deck and stay cables. The stay cables, that present an 8 m eccentricity at midspan (*i.e.* 10% of the total span), are formed by a varying number of strands of 150 mm$^2$ each. Even though all the FE results are non-dimensionalized when they are compared with the analytical results, the mechanical properties of the elements are summarized in Table 1.

![Figure 17: Cross-section of the deck formed by two steel I-beams and a reinforced concrete slab.](image)

**Table 1: Geometrical and mechanical properties of the elements that form the FE model of the UDCSB.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total span</td>
<td>$L = 4l = 80$ m</td>
</tr>
<tr>
<td>Stay cable eccentricity</td>
<td>$H = 8$ m</td>
</tr>
<tr>
<td>Homogenized deck</td>
<td></td>
</tr>
<tr>
<td>2nd moment of area</td>
<td>$I_D = 0.0411$ m$^4$</td>
</tr>
<tr>
<td>Area</td>
<td>$A_D = 0.5164$ m$^2$</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>$E_D = 210$ GPa</td>
</tr>
<tr>
<td>Stay cables</td>
<td></td>
</tr>
<tr>
<td>Number of strands</td>
<td>$n_{str} = \text{variable}$</td>
</tr>
<tr>
<td>Area of each strand</td>
<td>$A_{str} = 150$ mm$^2$</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>$E = 195$ GPa</td>
</tr>
</tbody>
</table>
Critical buckling loads are obtained by means of a linear eigenvalue analysis. The non-linear equilibrium paths are obtained by employing the static Riks method \[\text{(27)}\], in which the prestressing action is introduced as an imposed deformation (through a thermal load) in multiple steps and the geometrical non-linearities are taken into account. Even though the prestressing force is the load acting in UDCSBs, the horizontal component of the axial load acting on the deck at the support sections is accounted, which is compared with the external load \(P\) of the analytical model. The three different self-weight values \(w\) are also considered when studying the equilibrium paths: 0, 50 and 100 kN/m. The results are non-dimensionalized with the following relationships:

\[
p = \frac{Pl}{c} = \frac{Pl^2}{C_\alpha E_D I_D}, \quad K = \frac{kl^2}{c} = \frac{kl^3}{C_\alpha E_D I_D},
\]

\[
W = \frac{wl^2}{c} = \frac{wl^3}{C_\alpha E_D I_D}, \quad Q_i = \frac{\delta_i}{l},
\]

where \(\delta_i\) represent the deflections of the characteristic points considered in the deck.

Additionally, the vertical and horizontal components of the force introduced by the inclined stay cables are considered by means of the previously defined \(\eta\) and \(\gamma\) parameters, thus:

\[
\eta = -\tan 2\beta, \quad \gamma = -\frac{2\cos^2 \beta}{\cos 2\beta},
\]

where \(\beta\) has been defined as the angle between the struts and the stay cables (Fig. 10), and \(\pi > 2\beta > \pi/2\) radians. By geometric considerations the value of \(\beta\) is obtained by solving the non-linear equation:

\[
\frac{H}{(2 - \alpha) L \tan \beta} - 1 = \cos 2\beta.
\]

The analysis of the influence of \(K\) on the buckling shapes shows that the analytical model correlates with the trends of the FE model very well. The shape for Mode 2 is not affected by the stay cable stiffness at all; Mode 1 tends to approximate Mode ‘P’ and Mode 3 approximates to Mode ‘F’.

Figure 18 shows the comparison of the critical loads obtained by the analytical and the FE models. The qualitative trend in both models is observed to be nearly the same. While \(p_1^C\) increases linearly with \(K\) for lower stiffness values, it does not vary significantly for larger \(K\); \(p_3^C\) increases linearly with \(K\) for the higher stiffness values and does not vary significantly for the lower \(K\) range. Mode 2 is shown not to be affected by the stay cable stiffness, as previously predicted. Analytical critical loads for Modes 2 and 3 are unconservative for \(\alpha = 0\).

An alternative approach to obtain more accurate results would be to equate the non-dimensional critical load for Mode 2 obtained in Equation (36) when \(K = 0\) with the critical load of the FE model. As a result, a new rotational spring stiffness \(c\) would be obtained and critical loads evaluated with the analytical model would be more accurate for the particular case of \(\alpha = 0.6\). However, this change is unlikely to affect the non-linear behaviour significantly.

Stay cables in UDCSBs are usually prestressed to compensate 100% of the permanent...
load \[20\], \textit{i.e.} the vertical component of the deviation force is equal to the reaction force found in a 3-span bridge with the same subspan distribution. Hence, the deflection of nodes B and D, represented by \(Q_1\) and \(Q_3\), under permanent load is zero. The range of interest for the deflections in the current study therefore varies from positive values (\textit{i.e.} corresponding to the self-weight \(W\)) to negative values in close proximity to zero to compensate the further deflections due to the difference between the permanent load and the self-weight \(W\). Hence, the correlation between the analytical and the FE model is good in the moderately large deflection range (Fig. 19).

Regarding the influence of the stay cable stiffness \(K\) for very large deflections, the analytical model predicts different equilibrium paths, while the FE model provides a common one, see Figs. 19(a) and (c). This difference is owing to the second order effects when defining the equilibrium positions \(Q^E_i\) for a given self-weight \(W\) value and prestress level; positions \(Q^E_i\) are obtained by assuming that second order effects are small, which is valid in the deflection range of interest. However, the potential energy approach intrinsically considers these second order effects in the equilibrium equations, leading to an incongruence that is only noticeable for very large deflections.

The self-weight \(W\) defines the initial deflection of the deck, given by the value \(Q_1\) when \(p = 0\), see Figs. 19(b) and (d). Increasing \(W\) increases the initial deflection, an effect predicted by both models. Equivalently to \(K\), the correlation between both models is good for the moderately large deflection range.

Finally, the difference between the slopes of the equilibrium paths for both models for the highest deflection values is due to the non-linear nature of the stay cables. The analytical model employs linear longitudinal springs, while the stiffness of these springs depends on the deflected geometry of the UDCSB. The considered assumption is once again valid in the deflection range of interest, but when very large deflections are analysed, the good correlation between the analytical and the FE model diminishes, as perhaps would
Figure 19: Equilibrium paths ($p$–$Q_1$) when the longitudinal spring stiffness $K$ and the self-weight $W$ are varied: (a) $\alpha = 1.0$ with varying $K$ and $w = 50$ kN/m; (b) $\alpha = 1.0$ with varying $W$ and 12 strands/stay cable have been installed; (c) $\alpha = 0.6$ with varying $K$ and $w = 50$ kN/m; (d) $\alpha = 0.6$ with varying $W$ and 12 strands/stay cable have been installed. The $K$ values are obtained for the cases in which 6, 12 and 18 strands are employed in each of the six stay cables. The self-weight $w$ values are 0, 50 and 100 kN/m.
be expected.

6. Design implications

The axial load acting on the deck, which is proportional to the pretressing force of the stay cables, depends on the self-weight $w$ that needs to be compensated by the cable-staying system. If a UDCSB with two struts is considered, the non-dimensional axial load $p$ at the support sections of the deck is obtained by cancelling the vertical deflections of nodes B and D:

$$p = -\frac{W_c}{\tan 2\beta} \left[ \frac{64 - (6 + \alpha)(2 - \alpha)^2}{64 - (2 + \alpha)^2 - (4 + \alpha)(2 - \alpha)^2} \right],$$

(42)

where $p = Pl/c$ and $W_c = wc^2/c$. In this expression, $W_c$ represents the non-dimensional load compensated by the cable-staying system, and it may comprise the permanent load (i.e. self-weight and dead load) and part of the live load, depending on the compensation level $\rho$ [20], the dimensional total compensated load intensity being $wc$.

In the following list, the findings are summarized and some design recommendations are presented:

- Critical loads corresponding to symmetric modes depend on the ratio between the flexural stiffness of the cable-staying system to that of the deck. However, the stay cables do not affect the antisymmetric buckling modes (Fig. 13).

- A transitional stay cable stiffness $K^T$ can be defined as the $K$ value in which the critical loads corresponding to the lowest symmetric and antisymmetric modes coincide, see Fig. 20. The response in terms of $K$ can be defined with the following points:
  - When the first stay cable set is being prestressed, the symmetric mode is critical with a weakly stable post-buckling response.
  - If $K<K^T$, the symmetric mode is still critical, but with a weakly unstable post-buckling response.
  - If $K$ is slightly higher than $K^T$, the highly unstable mode interaction may be triggered, in which the buckling mode jumps from the antisymmetric to the symmetric mode through an interactive mode. In a slender structure, such as the deck of a UDCSB, this jump could introduce large dynamic effects that may compromise the safety of the construction workers and equipment.
  - For $K$ being sufficiently higher than $K^T$, the antisymmetric mode is the dominant mode leading to a stable post-buckling response.

- The prestressing sequence may be represented in a $p-K$ plot as sketched in Fig. 20. In this, the solid vertical lines represent the pretensioning of a certain stay cable family, while the dashed horizontal lines represent the anchoring of these stay cable
families that increase the stiffness of the cable-staying system. Further pretensioning of existing stay cable families can be represented by a series of parallel horizontal lines, in which a stay cable family is released to introduce a further pretension. Hence, the whole stay cable pretensioning sequence leads to the pretensioning path, which starts at point ‘S_T’ and finishes at ‘F_T’. The following recommendations can be made:

– The pretensioning path should be sufficiently far from the buckling load lines to avoid any instability phenomenon.

– The point ‘F_T’ should be out of the mode interaction region $K = [K^T, K^S]$ where the post-buckling path is unstable and preferably should be located in the region where Mode 2 is dominant, where the post-buckling path is stable.

– It is preferable to divide the pretension of each stay cable family in different steps. As a result, in the first prestressing steps the stiffness of the system would be increased to reach the region in which Mode 2 is dominant without increasing the axial load in the deck considerably. Further prestressing steps would increase the prestressing force in the stay cables to reach the desired final level. As a consequence, the final and most significant prestressing steps would be located in the region where the critical load is highest and the post-buckling path is stable.

– Two conditions have been identified that are required to be satisfied to trigger mode interaction:

  – The stiffness of the cable-staying system would need to be equal to or slightly greater than $K^T$. For the example bridge geometry that is studied in the current work, Table 2 shows the total number of strands that would need to be installed in the bridge to trigger mode interaction (out of a total of 186 that would be necessary from the static viewpoint).
The axial load in the deck needs to be sufficiently large to be approximately the load of the critical eigenmode. This condition can be easily stated by equating the axial load in the deck, \( i.e. \) Equation (42), and the critical load corresponding to Mode 2. As a result, the non-dimensional load \( W_c \) that needs to be compensated to trigger mode interaction is obtained:

\[
W_c = \left[ \frac{4\eta}{\alpha (2 - \alpha) [2 + (2 - \alpha) \gamma]} \right] \frac{64 - (2 + \alpha)^3 - (4 + \alpha) (2 - \alpha)^2}{64 - (6 + \alpha) (2 - \alpha)^2}.
\] (43)

Figure 21 shows the load intensities \( w_c \) that need to be compensated for different realistic span lengths and cross-sectional geometries of the deck. It can be observed that the longer the span, the lower \( w_c \) needs to be. Moreover, the stiffer the deck, the higher \( w_c \) has to be. In the bridge geometry analysed in the current work, which has a span of 80 m and a I-beam depth of \( d = 750 \) mm, the load compensated by the cable-staying system is around 125 kN/m. Hence, the axial load induced in the deck is lower than the critical value and the buckling phenomenon consequently will not be present. Given these bridge properties (\( i.e. \) \( d = 750 \) mm and \( w_c = 125 \) kN/m), instability problems would begin arising for spans of 120 m or above. However, since longer spans would need deeper decks, the limit of 120 m would also increase in practice.

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>1.0</th>
<th>0.80</th>
<th>0.70</th>
<th>0.67</th>
<th>0.60</th>
<th>0.50</th>
</tr>
</thead>
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<tr>
<td>Analytical</td>
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<td>30</td>
<td>33</td>
<td>36</td>
<td>40</td>
<td>48</td>
</tr>
<tr>
<td>FE</td>
<td>24</td>
<td>25</td>
<td>25</td>
<td>26</td>
<td>27</td>
<td>29</td>
</tr>
</tbody>
</table>
7. Concluding remarks

The stability of the deck of UDCSBs during the construction stages has been investigated by formulating an analytical model based on total potential energy principles comprising discrete links and springs. Critical buckling loads and the post-buckling behaviour have been shown to be intrinsically connected to the ratio of the axial stiffness of the cable-staying system to the flexural stiffness of the deck. However, the stay cables do not influence the response relative to the antisymmetric mode due to the particular bridge configuration considered (i.e. struts are pinned to the deck to avoid the introduction of concentrated moments). For a particular cable-staying system stiffness, mode interaction is triggered, in which the buckling mode rapidly jumps from the antisymmetric to the symmetric mode, consequently leading to unstable post-buckling behaviour. Moreover, this mode jumping could also induce strong dynamic effects in the slender decks of UDCSBs.

The influence of several parameters such as the relative length of subspans, the horizontal and vertical components of the deviation forces introduced by the struts and the self-weight has been analysed. The analytical model presented allows for multiple erection conditions, even if only a simple methodology is studied in the current work. The horizontal component of the deviation force reduces the critical loads, while the vertical component is responsible for compensating the deflections due to the initial self-weight. A transitional self-weight value has been defined: for higher self-weight values, the desired geometry of the deck would not be feasible during the prestressing stages; for lower self-weight values that are close to the transition load, the response becomes highly non-linear, and the desired geometry may be difficult to achieve. These situations must be avoided to ensure a simple and safe construction process.

The analytical model has been validated by employing a non-linear finite element model. Results show good correlation between both models for moderately large deflections. For larger deformations, which are unreasonable from the structural serviceability perspective during the construction stage of a real bridge, the analytical model does not replicate the response accurately. This is due to the linear behaviour considered in the longitudinal springs, which in reality would be non-linear since they depend on the deformed shape of the bridge. However, in the deflection range of interest, the analytical model has been shown to provide good quantitative and excellent qualitative accuracy. This model becomes a powerful tool that allows for the analysis of different parameters in the response.

Finally, some design recommendations are provided; these may be useful for designers aiming to achieve safer, yet more efficient UDCSBs particularly when construction sequences and initial sizings are being devised. It is also demonstrated that the schemes which have been proposed by the authors in previous work [24, 25] as appropriate for medium spans (i.e. spans around 80 m) do not present any instability issues when considering realistic cross-section properties and prestressing loads. The instability issues may begin to be relevant for spans of 120 m or above.

Since the validity of the rigid link models has been demonstrated, further analyses could be performed to find the most appropriate construction sequence. Usually, temporary supports located at the strut–deck connection sections are employed to build UDCSBs.
The consideration of temporary supports in the current methodology would lead to a more favourable scenario in relation to the stability of the deck. Moreover, the optimum time to cast the concrete slab could be found. On one hand, the concrete slab introduces a higher initial self-weight leading to larger deformations. On the other hand, the slab provides the deck with a higher stiffness to resist the compression load of the cable-staying system. Therefore, a parametric study could determine the most appropriate casting and prestressing sequence. The sensitivity to initial imperfections and precambers that are given to steel elements also needs to be established. Finally, with the aim of extending the conclusions of the present work to other UDCSBs, the influence of different stay cable arrangements, by, for example, modifying the number of struts, can also be determined.

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