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Time dependent mechanisms involved in shale barriers sealing around the casing: a poroviscoelastic approach.

By
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A report submitted in partial fulfillment of the requirements for the MSc and/or DIC

September 2011
Declaration of own work

I declare that this thesis

“Time dependent mechanisms involved in shale barriers sealing around the casing: a poroviscoelastic approach”

is entirely my own work and that where any material could be construed as the work of others, it is fully cited and referenced, and/or with appropriate acknowledgement given.

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Abstract

This paper attempts to characterize the time dependent behaviour of shale formations that creates effective secondary annular barriers following the Norwegian Continental Shelf (NCS) regulation. In a recent paper, S. M. Williams et al qualified existing shale annular barriers using wireline logs and pressure testing during Plug and Abandonment operations. This paper tries to focus on the prediction of such annular barrier using the theory of poroelasticity that was first introduced by Y. Abousleiman et al. in [1]

The objective of this paper is to present the time dependent borehole closure for an infinite vertical borehole in a poroviscoelastic formation under isotropic horizontal stress. It corresponds to Detournay and Cheng solution for the borehole problem under loading mode 1 in the case of a poroviscoelastic medium. The solution for an infinite vertical borehole in a poroviscoelastic formation under anisotropic horizontal stress will also be discussed.

First, the relaxation moduli and creep compliance functions developed in viscoelasticity will be introduced. Then the phenomenological model such as the Generalized Kelvin model and the Burger substance will be presented. Following Y. Abousleiman steps, the poroelastic solution for the borehole problem loading mode 1 (by E. Detournay and A. H-D Cheng) will be studied and combined with viscoelasticity to get the poroviscoelastic solution. The Generalized Kelvin model and the Burger substance will be used as viscoelastic models to get the solution in the time domain. Finally the limit of this approach will be discussed and the concept of viscoplasticity will be introduced.
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Time dependent mechanisms involved in shale barriers sealing around the casing: a poroviscoelastic approach.
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This paper attempts to characterize the time dependent behaviour of shale formations that creates effective secondary annular barriers following the Norwegian Continental Shelf (NCS) regulation. In a recent paper, S. M. Williams et al qualified existing shale annular barriers using wireline logs and pressure testing during Plug and Abandonment operations. This paper tries to focus on the prediction of such annular barrier using the theory of poroviscoelasticity that was first introduced by Y. Abousleiman et al. in [1].

The objective of this paper is to present the time dependent borehole closure for an infinite vertical borehole in a poroviscoelastic formation under an isotropic horizontal stress. It corresponds to Detournay and Cheng solution for the borehole problem under loading mode 1 in the case of a poroviscoelastic medium. The solution for an infinite vertical borehole in a poroviscoelastic formation under anisotropic horizontal stress will also be discussed.

First, the relaxation moduli and creep compliance functions developed in viscoelasticity will be introduced. Then the phenomenological model such as the Generalized Kelvin model and the Burger substance will be presented. Following Y. Abousleiman steps, the poroelastic solution for the borehole problem loading mode 1 (by E. Detournay and A. H-D Cheng) will be studied and combined with viscoelasticity to get the poroviscoelastic solution. The Generalized Kelvin model and the Burger substance will be used as viscoelastic models to get the solution in the time domain. Finally the limit of this approach will be discussed and the concept of viscoplasticity will be introduced.
**Introduction**

In the overburden part of the Gullfaks reservoir in the Norwegian Continental Shelf (NCS), the Green Clay of Eocene age has demonstrated some viscous behaviour. The long term formation creep is sometimes so important that it impinges casings. This phenomenon detailed by S. M. Williams et al in [2], creates annular seals. Such seal may be qualified by the Norwegian Continental Shelf regulation as secondary annular barriers for permanent plug and abandonment (P&A) of wells. Until now such phenomena have been observed on wireline logs and qualified as a secondary barrier on a case to case basis, to save plug and abandonment costs. This paper’s objective is to go a step further and try to predict such mechanisms. Being able to predict the shale formation creep would help to design wells. For instance, wells with a tight drilling window require the use of numerous casing strings and therefore a very small clearance between casing strings. Because of such design, the cement return is not always achievable and it results in poor cement jobs. Shale formation creep could supplement cement operations in such cases.

In order to characterize time dependent behaviours of porous rocks, M. Biot developed the theory of poroelasticity [3] and [18]. This theory became fundamental for any study involving time dependent phenomena occurring within a porous rock. It described successfully the coupled effect of an elastic matrix deformation occurring with a fluid diffusion throughout pore pressure readjustments. Several papers [4], [5] have studied this theory. In 1988 E. Detournay and A, H-D Cheng published the poroelastic response of a borehole in a non-hydrostatic stress field [6]. This elegant solution considers the various coupled poroelastic processes triggered by the drilling of a vertical borehole in a saturated formation subjected to a non-hydrostatic in situ stress. However even though this solution gives promising results for elastic porous media, it under predicts the creep associated with rocks that exhibit viscous features.

On the other hand with the advent of polymers in the 70s, a theory of viscoelasticity emerged in order to characterize their time dependent behaviour. Several books from authors such as W. Flügge, R. M. Christensen or N. W. Tschoegl [7], [8] and [9] report the advancements made in this field. A viscoelastic model could be used to represent the viscous behaviour of the Green Clay matrix.

In [10], using poroelastic and viscoelastic theories, Y. Abousleiman et al proposed a poroviscoelastic analysis of the borehole and cylinder problem. Starting at this point, this paper will review Y. Abousleiman publication and focus on the time dependent radial displacement of the borehole in a poroviscoelastic medium. For instance, the borehole problem loading mode 1 and 3 defined by E. Detournay and A. H-D Cheng will be studied in this paper. The borehole problem and its three loading modes are briefly described in Appendix F.

**Outline**

The table below relates the publications/advancements this paper followed to get to the radial displacement of a borehole in a poroviscoelastic formation under hydrostatic stress.

<table>
<thead>
<tr>
<th>Advancement</th>
<th>Year, Publication</th>
<th>Author</th>
<th>Outcome</th>
</tr>
</thead>
</table>
| Theory of poroelasticity                   | [3], 1941 [18], 1962 | M. Biot                 | Constitutive Law for an elastic porous medium: \[
|                                           |                   |                         | \[\sigma_y - \delta_y \sigma_p = 2G e_y + \delta_y \frac{3K - 2G}{3} e\] \[
|                                           |                   |                         | \[C_p = \xi - \alpha e\]                                                                          |
| Correspondence principle in viscoelasticity | [7], 1975          | W. Flügge               | Generalization of Hooke’s law to viscoelastic material: \[
|                                           |                   |                         | \[\sigma(t) = \sigma_0 J_E(t)\] \[
|                                           |                   |                         | \[\varepsilon(t) = \varepsilon_0 E(t)\]                                                          |
| Hereditary integral applied to viscoelasticity | [7], 1975 [8], 1982 | W. Flügge, R.M. Christensen | Generalization of the correspondence principle: \[
|                                           |                   |                         | \[\sigma(t) = \int_0^t \frac{\sigma(\tau)}{d\tau} d\tau\] \[
|                                           |                   |                         | \[\varepsilon(t) = \int_0^t \frac{\varepsilon(\tau)}{d\tau} d\tau\]                           |
| Relationship between the Relaxation modulus and creep compliance of a viscoelastic material | [7], 1975 [8], 1982 | W. Flügge, R.M. Christensen | Relaxation modulus and creep compliance function relationship in the Laplace Domain: \[
|                                           |                   |                         | \[\tilde{J}_E = \left(\frac{1}{2} \tilde{E}\right)^{1/2}\]                                    |
1. Poroviscoelasticity, definitions and governing equations

In order to present the theory of poroviscoelasticity, linear elasticity and poroelasticity will be first presented. The correspondence principle and viscoelasticity will be then introduced in order to integrate the viscous behaviour of the rock frame itself to the theory of poroelasticity. This will define the theory of poroviscoelasticity.

Governing equations for a linear elastic medium

The theory of linear elasticity assumes linear relationships between applied stresses and resulting strains. Following the notation used in [17], for isotropic materials the general relation between stresses and strains may be written:

\[ \sigma_{ij} = 2G\varepsilon_{ij} + \delta_{ij} \frac{3K_d - 2G}{3} e \]  (1.1)

Where \( \sigma_{ij} \) is the total stress tensor, \( \varepsilon_{ij} \) the solid frame strain tensor, \( e = \varepsilon_{ii} \) the solid frame dilatation and \( \delta_{ij} \) the Kronecker delta symbol. Two independent bulk material constants are used: The shear modulus \( G \) which is the sample’s resistance against shear deformation and the drained bulk modulus \( K_d \) which is the measure of the sample’s resistance against hydrostatic compression.

This simplistic approach has been developed further by Maurice Biot in order to account for two-phases medium. The approach Biot followed was to take into account the pore pressure diffusion within a porous medium subject to a stress. His studies led to the theory of poroelasticity presented below.

Governing equations for poroelasticity

The same approach proposed in [17] is followed. Consider an isotropic, porous and permeable medium saturated with a fluid of bulk modulus \( K_f \). Under stress, the change in the mass of fluid depends on the change in pore volume \( V_p \) and the compressibility of the fluid as the pore pressure \( p \) changes.

Therefore a strain parameter, \( \zeta \), defined as the variation of the fluid volume per unit reference volume is expressed as follow:

\[ \zeta = -\phi \left( \frac{\Delta V_p}{V_p} + \frac{p}{K_f} \right) \]  (1.2)

In [18], Biot shows that fundamental linear stress-strain relation (1.1) describing an isotropic, linear elastic material could be expressed in terms of \( \zeta \), \( p \) and \( e \) (solid frame dilatation) for a two-phases system. The introduction of these parameters led to the governing equations for the theory of poroelasticity:

\[ \sigma_{ij} - \delta_{ij} \alpha p = 2G\varepsilon_{ij} + \delta_{ij} \frac{3K_d - 2G}{3} e \]  (1.3)
Two additional bulk material constants are used: Biot's effective stress coefficient $\alpha$, and a compressibility coefficient for the solid-fluid system $C$. In [10] Abousleiman shows that $\alpha$ and $C$ may be expressed using engineering constants such as:

$$\alpha = \frac{K_u - K_d}{BK_u}$$  \hspace{1cm} (1.5)

$$C = \frac{9(v_u - \nu)(1 - 2v_u)}{2GB^2(1 - 2\nu)(1 + v_u)^2}$$  \hspace{1cm} (1.6)

Where $K_u$ is the undrained bulk modulus, B is the skempton coefficient, $\nu$ is the poisson’s ratio and $v_u$ is the undrained poisson’s ratio.

**Correspondence principle**

With the advent of polymers, the theory of viscoelasticity has been developed in order to characterise time dependent behaviour of viscoelastic materials. Based on elasticity principles, viscous deformations are integrated using the correspondence principle. In [7], Flügge defined the correspondence principle by the following way:

Consider first an elastic material under a stress $\sigma_0$. The strain $\varepsilon$ depends on stress $\sigma$ following Hooke’s law:

$$\varepsilon = \frac{\sigma}{E}$$  \hspace{1cm} (1.7)

$E$ is the Young modulus.

Now consider a viscoelastic material under a stress $\sigma_0$ applied at $t=0$. The time dependent stress is defined as follow:

$$\sigma(t) = \sigma_0 H(t)$$  \hspace{1cm} (1.8)

$H(t)$ is the Heaviside step function that equals 0 for strictly negative values of $t$ and 1 for positive or null values of $t$.

The viscoelastic material would be subject to the following time dependent strain $\varepsilon(t)$:

$$\varepsilon(t) = \sigma_0 J_E(t)$$  \hspace{1cm} (1.9)

$J_E(t)$ is defined as the Young creep compliance function.

At any time $t$ the strain would be distributed like the strain in an elastic material of modulus $E=[J_E(t)]^{-1}$. $J_E(t)$ is the inverse of the young modulus at a time $t$ of a rock under a stress $\sigma_0$ since $t=0$.

A symmetric approach can be use but this time, the elongation is fixed and the stress is allowed to change over time.

$$\varepsilon(t) = \varepsilon_0 H(t)$$  \hspace{1cm} (1.10)

The Young relaxation modulus $E(t)$ is then defined:

$$\sigma(t) = \varepsilon_0 E(t)$$  \hspace{1cm} (1.11)

At any time $t$ the stress would be distributed like the stress in an elastic material of modulus $E=E(t)$. $E(t)$, or the Young relaxation modulus, is the Young modulus at a time $t$ of a rock under a stress $\sigma_0$ since $t=0$.

Therefore the correspondence principle defines a Young relaxation modulus $E(t)$ and Young creep compliance function $J_E(t)$ that correspond, at a given time $t$, respectively to the young modulus and the young modulus inverse for a viscoelastic material. The correspondence principle can be applied to torsion problems if $E$ is replaced by the shear modulus $G$, and $E(t)$ and $J_E(t)$ are replaced respectively by the shear relaxation modulus $G(t)$ and the shear creep compliance $J_G(t)$. It will be further extended for all the elastic moduli used in poroelasticity.

**Correspondence principle generalization**

The correspondence principle described above can be generalized by measuring the strain change induced by a changing load. This is achieved analytically by the use of the hereditary integrals defined by Flügge in [7]. It involves the Stieltjes convolution of strain with relaxation moduli and the convolution of stress with creep compliance.

Further explanation of this generalization is detailed by Christensen in [8], Flügge in [7] or Tschoegl in [9]. The notation followed in this paper is the one used by Christensen in [8].
According to the viscoelastic convention a relaxation modulus can be derived from the stress-strain relationship when the current stress is determined by the current value and past history of strain:

\[
\sigma(t) = \int_{-\infty}^{t} X(t - \tau) \frac{d\varepsilon(\tau)}{d\tau} d\tau \quad (1.12)
\]

\(\sigma(t)\) is the time dependent stress tensor. \(X(t)\) is a four-order tensor called relaxation modulus. \(\varepsilon(\tau)\) denotes the time dependent strain tensor.

It should be noted that the stress-strain relation (1.12) can be written using Stieltjes convolution:

\[
\sigma(t) = X(t) * d\varepsilon(t) \quad (1.13)
\]

With

\[
\tilde{\varphi} * d\tilde{\psi} = \int_{-\infty}^{t} \tilde{\varphi}(t - \tau) d\tilde{\psi}(\tau) \quad (1.14)
\]

By symmetry with relaxation functions, creep compliance functions \(J_X(t)\) can be derived from the stress-strain relationship when the current strain is determined by the current value and past history of stress:

\[
\varepsilon(t) = \int_{-\infty}^{t} J_X(t - \tau) \frac{d\sigma(\tau)}{d\tau} d\tau \quad (1.15)
\]

Using the preceding theory, in the particular case of a uniaxial deformation that is typically obtained during relaxation test and creep test, the following constitutive relations are given:

\[
\sigma(t) = \int_{-\infty}^{t} E(t - \tau) \frac{d\varepsilon(\tau)}{d\tau} d\tau \quad (1.16)
\]

\[
\varepsilon(t) = \int_{-\infty}^{t} J_E(t - \tau) \frac{d\sigma(\tau)}{d\tau} d\tau \quad (1.17)
\]

With \(E(t)\) and \(J_E(t)\) respectively the Young relaxation modulus and the Young creep compliance function defined above. By applying the Laplace transform to equation (1.16) and (1.17), it comes:

\[
\tilde{\sigma} = s\tilde{E}\tilde{\epsilon} \quad (1.18)
\]

\[
\tilde{\varepsilon} = s\tilde{J}_E\tilde{\sigma} \quad (1.19)
\]

It can be noticed that \(\tilde{J}_E\) and \(\tilde{E}\) are linked in the Laplace domain by the following relationship:

\[
\tilde{J}_E = \left(s^2 \tilde{E}\right)^{-1} \quad (1.20)
\]

This equation can be generalized to any creep compliance function and its corresponding relaxation modulus:

\[
\tilde{J}_X = \left(s^2 \tilde{X}\right)^{-1} \quad (1.21)
\]

Therefore, using this generalized correspondance principle, by analogy with poroelasticity, four poroviscoelastic relaxation moduli are considered:

- The drained bulk relaxation modulus \(K_d(t)\) obtained through a drained jacketed creep experiment. It is associated with the drained bulk creep compliance function \(J_d(t)\).
- The drained shear relaxation modulus \(G(t)\) obtained through a drained shear creep experiment. It is associated with the drained shear creep compliance function \(J_d(t)\).
- The solid grain bulk relaxation modulus \(K_s(t)\) obtained through a drained unjacketed creep experiment. It is associated with the solid grain bulk creep compliance function \(J_d(t)\).
- The undrained bulk relaxation modulus \(K_u(t)\) obtained through an undrained jacketed creep experiment. It is associated with the undrained bulk creep compliance function \(J_d(t)\).
A methodology to get an expression of these relaxation moduli will be proposed further down in this paper. Since these relaxation moduli are by definition viscoelastic quantities, the following expressions can be written:

\[ \sigma_{Kd}(t) = \int_{-\infty}^{t} K_d(t - \tau) \frac{d\sigma_{Kd}(\tau)}{d\tau} d\tau \quad \text{and} \quad \varepsilon_{Kd}(t) = \int_{-\infty}^{t} J_{Kd}(t - \tau) \frac{d\varepsilon_{Kd}(\tau)}{d\tau} d\tau \]  

(1.22)

\[ \sigma_{\text{shear}}(t) = \int_{-\infty}^{t} G(t - \tau) \frac{d\sigma_{\text{shear}}(\tau)}{d\tau} d\tau \quad \text{and} \quad \varepsilon_{\text{shear}}(t) = \int_{-\infty}^{t} J_{\text{shear}}(t - \tau) \frac{d\varepsilon_{\text{shear}}(\tau)}{d\tau} d\tau \]  

(1.23)

\[ \sigma_{Ks}(t) = \int_{-\infty}^{t} K_s(t - \tau) \frac{d\sigma_{Ks}(\tau)}{d\tau} d\tau \quad \text{and} \quad \varepsilon_{Ks}(t) = \int_{-\infty}^{t} J_{Ks}(t - \tau) \frac{d\varepsilon_{Ks}(\tau)}{d\tau} d\tau \]  

(1.24)

\[ \sigma_{Ku}(t) = \int_{-\infty}^{t} K_u(t - \tau) \frac{d\sigma_{Ku}(\tau)}{d\tau} d\tau \quad \text{and} \quad \varepsilon_{Ku}(t) = \int_{-\infty}^{t} J_{Ku}(t - \tau) \frac{d\varepsilon_{Ku}(\tau)}{d\tau} d\tau \]  

(1.25)

In the Laplace Domain, they respectively become:

\[ \tilde{\sigma}_{Kd} = \tilde{s} \tilde{K}_d \tilde{\varepsilon}_{Kd} \quad \text{and} \quad \tilde{\varepsilon}_{Kd} = \tilde{s} \tilde{J}_{Kd} \tilde{\sigma}_{Kd} \]  

(1.26)

\[ \tilde{\sigma}_{\text{shear}} = \tilde{s} \tilde{G} \tilde{\varepsilon}_{\text{shear}} \quad \text{and} \quad \tilde{\varepsilon}_{\text{shear}} = \tilde{s} \tilde{J}_{\text{shear}} \tilde{\sigma}_{\text{shear}} \]  

(1.27)

\[ \tilde{\sigma}_{Ks} = \tilde{s} \tilde{K}_s \tilde{\varepsilon}_{Ks} \quad \text{and} \quad \tilde{\varepsilon}_{Ks} = \tilde{s} \tilde{J}_{Ks} \tilde{\sigma}_{Ks} \]  

(1.28)

\[ \tilde{\sigma}_{Ku} = \tilde{s} \tilde{K}_u \tilde{\varepsilon}_{Ku} \quad \text{and} \quad \tilde{\varepsilon}_{Ku} = \tilde{s} \tilde{J}_{Ku} \tilde{\sigma}_{Ku} \]  

(1.29)

And finally

\[ \tilde{J}_G = \left( \tilde{s} \tilde{G} \right)^{-1}, \quad \tilde{J}_{Kd} = \left( \tilde{s} \tilde{K}_d \right)^{-1}, \quad \tilde{J}_{Ks} = \left( \tilde{s} \tilde{K}_s \right)^{-1} \quad \text{and} \quad \tilde{J}_{Ku} = \left( \tilde{s} \tilde{K}_u \right)^{-1} \]  

(1.30)

**Governing equation for poroviscoelasticity**

In [1], Abousleiman et al underline the simultaneous existence of the time dependent response of rocks accounting for pore pressure diffusion (poroelasticity) and the time dependent behaviour of the rock matrix itself (viscoelasticity). Following a correspondence principle described earlier, a “micromechanically consistent” poroviscoelastic model was proposed in [1]. The outcome of these studies has been the emergence of a new theory of poroviscoelasticity. The governing equations for this theory are obtained from the preceding theory of poroelasticity. In fact these constitutive equations are obtained by simply replacing all rock moduli defined in poroelasticity by relaxation moduli from viscoelasticity. These relaxation moduli encompass the rheological behaviour of the rock structure itself. In order to convolve two time dependent variables, the Stieltjes convolution (1.14) is used. Therefore in poroviscoelasticity equations (1.3) and (1.4) become:

\[ \sigma_{ij}(t) - \delta_{ij} \alpha(t) * p(t) = 2 \tilde{G}(t) * e_{ij}(t) + \delta_{ij} \frac{3K_d(t) - 2G(t)}{3} \int_0^t e(t) \]  

(1.31)

\[ C(t) * p(t) = \tilde{\zeta}(t) - \alpha(t) * e(t) \]  

(1.32)

For clarity, (1.31) and (1.32) can also be expressed in the Laplace domain,

\[ \tilde{\sigma}_{ij} - \delta_{ij} \tilde{\alpha} \tilde{p} = 2 \tilde{G} \tilde{e}_{ij} + \delta_{ij} \frac{3\tilde{K}_d - 2\tilde{G}}{3} \tilde{e} \]  

(1.33)

\[ \tilde{C} \tilde{p} = \tilde{\zeta} - \tilde{\alpha} \tilde{e} \]  

(1.34)

Equations (1.33) and (1.34) correspond to the “micromechanical” poroviscoelastic model defined by Abousleiman in [1] and [10].

**2. Phenomenological models**

In the literature depending on the type of rock tested and how it is tested, time dependent creep strain response can fit several viscoelastic models. Exponential and Power laws functions of time have been commonly used to model creep behaviour. In [11] P. N. Hagin and M. D. Zoback show that a power law combined with a Maxwell substance reflects best the Wilmington
unconsolidated sand time dependent deformation. In fact such model is also able to predict the bulk modulus dispersion and attenuation when the sample is subject to cycling loading.

In the scope of this paper, considering a constant hydrostatic state of stress around the borehole, only the spring dashpot models are investigated. In fact these models can be represented by exponential functions with a single relaxation time. These models are also called phenomenological models. The advantage of such model, following Hagin and Zoback description in [11], is that the deformational behaviour of rocks can be predicted without any knowledge of the mechanics responsible for the deformation. Therefore these models will provide a mean for bridging laboratory observations of time dependent deformations for Rock samples with observations made in the field.

The Generalized Kelvin Model

Under hydrostatic stress, the time dependent creep behaviour of the system \{isotropic rock; saturating fluid\} is assumed to follow a Generalized Kelvin Model response (Assumption A1). The figure below represents the schematic of such model:

\[ \sigma(t) = \sigma_0 H(t) \]  

(2.1)

H(t) is the Heaviside step function that equals 0 for strictly negative values of t and 1 for positive or null values of t.

Zimmerman showed in [12] that the following differential equation can be used to model such systems:

\[ \eta \frac{d\sigma}{dt} + (z_1 + z_2)\sigma(t) = z_1 \eta \frac{d\varepsilon}{dt} + z_1 z_2 \varepsilon(t) \]  

(2.2)

In [12] Zimmerman shows that \( \varepsilon(t) \) solution of (2.2) can be written:

\[ \varepsilon(t) = \sigma_0 \left( \frac{1}{z_2} + \frac{1}{z_1} - \frac{1}{z_2} e^{-\frac{z_2 t}{\eta}} \right) H(t) \]  

(2.3)

The Generalized Kelvin model deformation can be plotted against time.

If a uniaxial compressive creep test is performed on a rock sample (elongation following a constant stress recorded against time) and if assumption (A1) is correct then there should be a unique match between the creep data and a Generalized Kelvin model type curve. Therefore a set of parameters \( z_1, z_2, \) and \( \eta \) would define uniquely the time dependent deformation behaviour of the rock sample under a specific uniaxial compressive load. By extension this set of parameters will represent the creeping behaviour of the porous rock layer studied under a given stress.

By definition, following (1.9) the Young creep compliance function for this model is given:

\[ J_E(t) = \left( \frac{1}{z_2} + \frac{1}{z_1} - \frac{1}{z_2} e^{-\frac{z_2 t}{\eta}} \right) H(t) \]  

(2.4)
By transforming (2.4) into the Laplace domain, it comes:

$$TL(J_E(t)) = \frac{1}{z_2 s} + \frac{1}{z_1 s} - \frac{1}{z_2 (s + \frac{z_2}{\eta})} = \frac{z_1 + z_2}{s(z_1 z_2)} - \frac{\eta}{z_2 (s \eta + z_2)} = \frac{(z_1 + z_2) + s \eta}{s(z_1 z_2 + s z_2 \eta)} \quad (2.5)$$

Using (1.20) it comes:

$$\tilde{E} = \frac{(z_1 + z_2) + s \eta}{z_1 z_2 + s \eta} = \frac{1 + s \eta}{z_2} \quad (2.6)$$

with $q_1 = \frac{(z_1 + z_2)}{z_1 z_2}$, $q_2 = \frac{\eta}{z_1 z_2}$ and $p_1 = \frac{\eta}{z_2}$

Following the same methodology used by Findley in [13]; after some algebra (Appendix B), it can be shown that the Laplace Young modulus obtained in (2.6) can be inversed in the time domain to get the Young relaxation modulus $E(t)$:

$$E(t) = \frac{1}{q_1} \left( \frac{p_1}{q_2} - \frac{1}{q_1} \right) e^{\frac{q_1}{q_2} t} \quad (2.7)$$

with $q_1 = \frac{(z_1 + z_2)}{z_1 z_2}$, $q_2 = \frac{\eta}{z_1 z_2}$ and $p_1 = \frac{\eta}{z_2}$

It should be noted that

$$E(t = 0) = \frac{p_1}{q_2} = z_1$$

The instantaneous response of the Generalized Kelvin model to a stress $\sigma_0$ is governed by the spring constant $z_1$. Therefore $z_1$ is the young modulus measured through standard triaxial tests.

**The Burger Substance**

Under hydrostatic stress, the time dependent creep behaviour of the system {isotropic rock; saturating fluid} is assumed to follow a Burger Substance response (Assumption A2). The figure below represents the schematic of such model:

[z1, z2 are the spring constants and $\eta_1$ & $\eta_2$ are the dashpot fluid viscosity. The system is initially unstrained and unstressed and a stress $\sigma_0$ is instantaneously imposed at $t=0$: $\sigma(t) = \sigma_0 H(t)$]

H(t) is the Heaviside step function defined above.

Zimmerman showed in [12] the following differential equation can be used to model such system behaviour:

$$\frac{\eta_1}{z_1} \frac{\partial^2 \sigma}{\partial t^2} + \left( \frac{1 + \frac{z_2}{z_1} + \frac{\eta_1}{\eta_2}}{\eta_1} \right) \frac{\partial \sigma}{\partial t} + \frac{z_2}{\eta_2} \sigma(t) = \eta_2 \frac{\partial^2 \varepsilon}{\partial t^2} + z_2 \frac{\partial \varepsilon}{\partial t} \quad (2.8)$$

In [12] Zimmerman shows that $\varepsilon(t)$ solution of (2.8) can be written:
\[ \varepsilon(t) = \sigma_0 \left( \frac{1}{z_2} + \frac{1}{z_1} - \frac{1}{z_2 e^{\eta_1 t} + \frac{t}{\eta_1}} \right) H(t) \] (2.9)

The Burger Substance deformation can be plotted against time.

![Figure 4: Creep Data curve and Burger Substance type curve match](image)

If a uniaxial compressive creep test is performed on a rock sample and if assumption (A2) is correct then there should be a unique match between the creep data and a Burger Substance type curve. Therefore a set of parameters \( z_1, z_2 \) and \( \eta_1, \eta_2 \) would define uniquely the time dependent deformation behaviour of the rock sample under a specific load. By extension this set of parameters will represent the creeping behaviour of the porous rock layer studied under a specific uniaxial compressive stress.

By definition, following (1.9) the young creep compliance function for this model is:

\[ J_E(t) = \left( \frac{1}{z_2} + \frac{1}{z_1} - \frac{1}{z_2 e^{\eta_1 t} + \frac{t}{\eta_1}} \right) H(t) \] (2.10)

By transforming (2.10) into the Laplace domain, It comes:

\[ TL(J_E(t)) = \frac{1}{z_2 s} + \frac{1}{z_2 s} + \frac{1}{z_1 (s + \frac{z_2}{\eta_1})} + \frac{1}{z_1 s^2} = \frac{s \eta_1 (\eta_2 s + z_2) + s \eta_2 z_1 + z_1 (\eta_2 s + z_2)}{s^2 \eta_1 z_1 (\eta_2 s + z_2)} \] (2.11)

Using (1.20) it comes:

\[ \bar{E} = \frac{\eta_1 z_1 (z_2 + \eta_2 s)}{s \eta_1 z_1 (\eta_2 s + z_2) + s \eta_2 z_1 + z_1 (\eta_2 s + z_2)} = \frac{s \eta_1 + s \eta_2}{s \eta_1 + s \eta_2 + s \eta_1} = \frac{q_1 + q_2 s}{1 + p_1 s + p_2 s^2} \] (2.12)

with \( p_1 = \eta_1 \frac{z_1}{z_2}, \quad p_2 = \eta_2 \frac{z_1}{z_2}, \quad q_1 = \eta_1 \quad \text{and} \quad q_2 = \eta_2 \frac{z_1}{z_2} \)

In [13] Findley shows that the young relaxation modulus can be obtained after some algebra (Appendix C):

\[ E(t) = \frac{1}{A} \left[ (q_1 - q_2 r_1) e^{-r_1 t} - (q_1 - q_2 r_2) e^{-r_2 t} \right] \] (2.13)

with \( p_1 = \eta_1 \frac{z_1}{z_2}, \quad p_2 = \eta_2 \frac{z_1}{z_2}, \quad q_1 = \eta_1 \quad \text{and} \quad q_2 = \eta_2 \frac{z_1}{z_2} \)

and \( A = \sqrt{p_1^2 - 4 p_2}, \quad r_1 = \frac{p_1 - A}{2 p_2} \quad \text{and} \quad r_2 = \frac{p_1 + A}{2 p_2} \)

Similarly to the General Kelvin model, it should be noted that

\[ E(t = 0) = \frac{1}{A} \left[ q_1 - q_2 r_1 - q_1 + q_2 r_2 \right] = \frac{q_2 (r_2 - r_1)}{A} = \frac{q_2 (p_1 + A)}{p_2 A} = q_2 \frac{z_1}{z_2} = z_1 \]
The instantaneous response of the Burger substance model to a stress $\sigma_0$ is governed by its spring constant $z_1$. Therefore $z_1$ is the young modulus measured through standard triaxial tests.

3. Poroviscoelastic relaxation moduli

Following the phenomenological approach described earlier, depending on the type of creep experiment performed, an analytical expression of poroviscoelastic relaxation moduli could be obtained. In the next paragraph $X$ is a “variable” that can take the following values:

- Drained Bulk Jacketed
- Undrained Bulk Jacketed
- Drained Bulk unjacketed
- Drained Shear Jacketed

In order to get an analytical expression for a poroviscoelastic relaxation moduli $X(t)$, the methodology to follow is:

- **Step 1:** Perform a creep experiment $X$ on a rock sample by applying a load equivalent to the in situ stress.
- **Step 2:** Plot the resulting strain against time and get the corresponding creep compliance function $J_X(t)$ and its phenomenological model associated that describes the creeping behaviour $X$ of the sample. Here in order to simplify the problem, it is assumed that all the creep compliance functions defined hereafter follow a Generalized Kelvin substance.
- **Step 3:** Move the creep compliance function into the Laplace domain $\tilde{J}_X$ and get the corresponding relaxation modulus $K_X$ in the Laplace domain using (1.21)
- **Step 4:** Inverse it back in the time domain to get the relaxation modulus $X(t)$ corresponding to creep experiment $X$

**Drained bulk relaxation modulus $K_d(t)$**

The methodology described above is applied to get an expression of the drained bulk relaxation modulus $K_d(t)$. In poroelasticity, as it is described in [17], the drained bulk modulus $K_d$ (also called Framework modulus $K_{fr}$) gives the instantaneous response of a drained jacketed rock sample under uniaxial load. Similarly Poroviscoelasticity defines the drained bulk relaxation modulus $K_d(t)$, by being the relaxation modulus obtained through a uniaxial creep experiment performed on a drained jacketed rock sample.

- **Step 1:** A uniaxial creep experiment is performed on a drained jacketed rock sample.
- **Step 2:** The resulting strain is plotted against time and it follows a Generalized Kelvin substance response. Therefore:

$$J_{Kd}(t) = \frac{\varepsilon_{Kd}(t)}{\sigma_{Kd}} = \left( \frac{1}{K_{d2}} + \frac{1}{K_{d1}} - \frac{1}{K_{d2}} e^{-\frac{K_{d2}}{\mu_{Kd}}\sigma_{Kd}} \right) H(t) \quad (3.1)$$

- **Step 3:** (3.1) can be transformed into Laplace to get $\tilde{J}_{Kd}$:

$$\tilde{J}_{Kd} = \frac{K_{d1} + K_{d2}}{s(K_{d1} + K_{d2}) + sK_{d1}\mu_{Kd}} \quad (3.2)$$

And using (1.30) $\tilde{K}_d$ follows:

$$\tilde{K}_d = \frac{K_{d1}K_{d2} + sK_{d1}\mu_{Kd}}{s(K_{d1} + K_{d2}) + s\mu_{Kd}} \quad (3.3)$$

- **Step 4:** Following (2.7) steps the Drained bulk relaxation modulus is obtained by taking the Laplace inverse of (3.3):

$$K_d(t) = \frac{1}{q_1} + \frac{p_1}{q_2} \left( \frac{q_1}{q_2} - \frac{q_1}{q_1} \right) e^{-\frac{q_1}{q_2}t} \quad (3.4)$$

with $q_1 = \frac{(K_{d1} + K_{d2})}{K_{d1}K_{d2}}$, $q_2 = \frac{\mu_{Kd}}{K_{d1}K_{d2}}$, and $p_1 = \frac{\mu_{Kd}}{K_{d2}}$

$K_{d1}$, $K_{d2}$ and $\mu_{Kd}$ are the intrinsic rock parameters determined by a drained uniaxial creep experiment on a jacketed rock sample.
In [8] R.M Christensen reminds that the comparable elasticity relationship between moduli and compliances is \( J = K^{-1} \). Therefore the intuitive extension of this result from elasticity to viscoelasticity would suggest \( J(t) = [K(t)]^{-1} \) which is seen from (3.4) to be incorrect. However, it can be shown that at early and late time, the behaviour of creep compliance function agrees with the inverse of the relaxation modulus.

At early times:

\[
\lim_{t \to 0} J_{Kd}(t) = \frac{1}{K_{d2}} + \frac{1}{K_{d1}} - \frac{1}{K_{d2}} = \frac{1}{K_{d1}}
\]  

(3.5)

On the other hand at early time \( K_d(t) \) becomes:

\[
\lim_{t \to 0} K_d(t) = \frac{1}{q_1} + \frac{p_1}{q_2} - \frac{1}{q_1} = p_1 = K_{d1}
\]  

(3.6)

At late times:

\[
\lim_{t \to \infty} J_{Kd}(t) = \frac{1}{K_{d2}} + \frac{1}{K_{d1}} = \frac{K_{d1} + K_{d2}}{K_{d1}K_{d2}}
\]  

(3.7)

On the other hand at late time \( K_d(t) \) becomes:

\[
\lim_{t \to \infty} K_d(t) = \frac{1}{q_1} = \frac{K_{d1}K_{d2}}{K_{d1} + K_{d2}}
\]  

(3.8)

Drained shear relaxation modulus \( G(t) \)

The same methodology described above is applied to get an expression of the drained shear relaxation modulus \( G(t) \).

In poroelasticity, as it is described in [17], the drained shear modulus gives the instantaneous response of a drained jacketed rock sample under shear stress. Similarly Poroviscoelasticity defines the drained shear relaxation modulus \( G(t) \), by being the relaxation modulus obtained through a shear creep experiment performed on a drained jacketed rock sample.

- **Step 1:** a shear creep experiment is performed on a drained jacketed rock sample.
- **Step 2:** the resulting shear strain is plotted against time and it follows a Generalized Kelvin substance response.

Therefore:

\[
J_G(t) = \frac{\varepsilon_G(t)}{\sigma_G} = \left( \frac{1}{G_2} + \frac{1}{G_1} - \frac{1}{G_2} \right) e^{\frac{G_2}{\mu_G} t} \cdot H(t)
\]  

(3.9)

- **Step 3:** (3.9) can be transformed into Laplace to get \( \tilde{J}_G \):

\[
\tilde{J}_G = \frac{G_1 + G_2 + s \mu_G}{s(G_1 G_2 + s G_1 \mu_G)}
\]  

(3.10)

And using (1.30) \( \tilde{G} \) follows:

\[
\tilde{G} = \frac{G_1 G_2 + s G_1 \mu_G}{s(G_1 + G_2 + s \mu_G)}
\]  

(3.11)

- **Step 4:** Following (2.7) steps the drained shear relaxation modulus is obtained by taking the Laplace inverse of (3.11):

\[
G(t) = \frac{1}{q_1} + \left( \frac{p_1}{q_2} - \frac{1}{q_1} \right) e^{\frac{q_2}{q_1} t} \right)
\]  

(3.12)

with \( q_1 = \frac{G_1 + G_2}{G_1 G_2} \), \( q_2 = \frac{G_2 \mu_G}{G_1 G_2} \) and \( p_1 = \frac{\mu_G}{G_2} \)

\( G_1, G_2 \) and \( \mu_G \) are the intrinsic rock parameters determined by a drained shear creep experiment on a jacketed rock sample.

Other relaxation moduli

The exact same approach can be used to express the relaxation moduli and there associated creep compliance functions stated in 1. Assuming a specific creep compliance function \( J_i(t) \) follows a Generalized Kelvin model, it can be expressed as follows:
After some algebra (Appendix G) the following expression of the radial displacement can be obtained:

\[ J_x(t) = \frac{\varepsilon_x(t)}{\alpha_x} = \left( \frac{1}{X_2} + \frac{1}{X_1} - \frac{X_1}{X_2} e^{-\frac{X_2 t}{\mu_x}} \right) H(t) \]  

(3.13)

and comes subsequently:

\[ J_x = \frac{X_1 + X_2 + s \mu_x}{s(X_1X_2 + sX_1 \mu_x)} \]  

(3.14)

\[ \tilde{X} = \frac{X_1X_2 + sX_1 \mu_x}{s(X_1 + X_2 + s \mu_x)} \]  

(3.15)

\[ X(t) = \frac{1}{q_1} + \left( \frac{p_1}{q_2} - \frac{1}{q_1} \right) e^{-\frac{q_1 t}{q_2}} \]  

(3.16)

with \( q_1 = \frac{(X_1 + X_2)}{X_1X_2} \), \( q_2 = \frac{\mu_G}{X_1X_2} \) and \( p_1 = \frac{\mu_x}{X_2} \)

Where \( X_1, X_2 \) and \( \mu_x \) are the intrinsic rock parameters determined by the creep experiment performed on the rock sample.

4. Radial displacement of an infinite vertical borehole in a poroviscoelastic media under isotropic horizontal stress \( P_0 \) (Borehole problem loading mode 1)

First consider the following poroelastic solution from Detournay and Cheng for a Borehole problem mode 1 loading [6], the radial displacement of the borehole is the classical Lamé solution in elasticity:

\[ G_1 u_1^{(1)} = -\frac{a^2 P_0}{2r} \]  

(4.1)

\( a \) being the wellbore radius, \( r \) the radial coordinate and \( P_0 \) the isotropic horizontal stress.

Following the correspondence principle this poroelastic solution holds in poroviscoelasticity by replacing the poroelastic static rock moduli by poroviscoelastic relaxation moduli. The Stieltjes convolution needs to be used to convolve the two time dependent variables \( G(t) \) and \( u_1(t) \). Therefore, at the wellbore wall (4.1) becomes:

\[ G(t) * u_1^{(1)}(t) = -\frac{a^2 P_0}{2} \]  

(4.2)

It should be noted that (4.2) cannot be solved using the Laplace transform. In fact assuming the shear deformation follows a Generalized Kelvin Behaviour, using (3.11), (4.2) would lead to the inversion of the following equation in the Laplace domain:

\[ \tilde{u}_1^{(1)} = -\frac{aP_0}{2} \frac{s(G_1 + G_2 + s \mu_G)}{G_1G_2 + sG_1 \mu_G} \]  

(4.3)

However, the inverse Laplace transform of (4.3) is not defined. In fact the limit in infinity of \( \tilde{u}_1^{(1)} \) does not converge toward zero and therefore this polynomial fraction does not satisfy the existence condition of its inverse:

\[ \lim_{s \to \infty} \tilde{u}_1^{(1)} = \lim_{s \to \infty} \left( -\frac{aP_0}{2} \frac{s(G_1 + G_2 + s \mu_G)}{G_1G_2 + sG_1 \mu_G} \right) = +\infty \]  

(4.4)

The only option is therefore to solve (4.2) in the time domain. Using the Stieltjes integral notation (1.14), the expression of \( G(t) \) (3.12), and assuming radial displacement is null for negative time, (4.2) becomes:

\[ \int_0^t u_1^{(1)}(\tau) \left( \frac{1}{q_1} + \left( \frac{p_1}{q_2} - \frac{1}{q_1} \right) e^{-\frac{q_1 \tau}{q_2}} \right) d\tau = -\frac{aP_0}{2} \]  

(4.7)

After some algebra (Appendix G) the following expression of the radial displacement can be obtained:
\[ u_r^{(1)}(t) = u_r^{(1)}(0) e^{-\frac{G_2 t}{\mu}} \quad (4.8) \]

\[ u_r^{(1)}(t) = \frac{a P_0}{2 G_1} e^{-\frac{G_2 t}{\mu}} \quad (4.9) \]

Using the same approach it can be shown (Appendix D) that when the shear creep of a rock follows a Burger’s substance behavior, the expression of the radial displacement becomes:

\[ u_r^{(1)}(t) = -\frac{a P_0}{2 G_1} + \frac{\mu G_2}{G_2} u_r^{(1)}(0) \left(1 - e^{-\frac{G_2 t}{\mu G_2}}\right) \quad (4.10) \]

Assuming that the radial displacement gets to zero after an infinite period of time imposes the following expression:

\[ \lim_{t \to \infty} u_r(t) = -\frac{a P_0}{2 G_1} + \frac{\mu G_2}{G_2} u_r^{(1)}(0) = 0 \quad (4.11) \]

\[ u_r^{(1)}(0) = \frac{a P_0 G_2}{2 G_1 \mu G_2} \quad (4.12) \]

Finally, under isotropic horizontal stress, the radial displacement of a borehole wall drilled in a poroviscoelastic medium of which the shear creep follows a Burger substance model is obtained:

\[ u_r^{(1)}(t) = -\frac{a P_0}{2 G_1} + \frac{a P_0}{2 G_1} \left(1 - e^{-\frac{G_2 t}{\mu G_2}}\right) = -\frac{a P_0}{2 G_1} e^{-\frac{G_2 t}{\mu G_2}} \quad (4.13) \]

Both Generalized Kelvin substance and Burger substance give the same solution for the radial displacement. In fact the additional dashpot present in the Burger substance does not contribute in a time dependent behaviour of the radial displacement.

One could think that this artifact is coming from the boundary condition imposed in (4.11). However, if the shear creep compliance function is driven by a simple Maxwell substance, the solution found for the radial displacement of a borehole problem loading mode 1 does not depend on time (Appendix E). In fact, it gives the same solution as the classical Lamé solution found in elasticity where the resulting shear creep compliance function can be modelled by a single spring. Therefore it can be conclude that adding a dashpot in series to a phenomenological substance (used to model creep behaviour) does not impact the radial displacement solution for the borehole mode 1 problem. However it would be interesting to follow up on this unexpected result in a future study.

5. Radial displacement of an infinite vertical borehole in a poroviscoelastic media under anisotropic horizontal stress \( \sigma(\theta) \) (Borehole problem loading mode 1&3).

There are many difficulties associated with these loading modes combined. The main limitation encountered comes from a fundamental difference between elastic rock moduli and viscoelastic rock relaxation moduli. In the theory of poroelasticity the rock moduli are characterized for a range of stress. Indeed the proportionality relationship between the stress and its associated strain response define the elastic rock modulus. In the theory of poroviscoelasticity, since the solutions of the differential equations associated with phenomenological models involve a step function for a single stress impulse, the corresponding relaxation moduli are defined only for a unique state of stress. Therefore the relaxation moduli defined previously cannot be used when the stress is non hydrostatic.

On the other hand, Detournay and Cheng solution for the radial displacement in the case of a poroelastic borehole under loading mode 3 is already time dependent. In fact the expression of the radial displacement is given in the Laplace domain and an analytical solution is given only for early and late time behaviour. Unless some rough approximations are performed it seems extremely difficult to get the full poroviscoelastic description for this loading mode. Indeed the time dependent behaviour due to poro-effects needs to be differentiated with the time dependent behaviour coming from viscoelastic effects.
Nevertheless solving the borehole problem loading mode 1 & 3 combined is crucial to apply the theory for field application. Loading mode 1 oversimplifies real cases when the wells are deviated or when the horizontal stresses are strongly anisotropic. However, as it is mentioned above the relaxation modulus G(t) is computed only for one given value of stress. Since it is not practicable to perform creep experiments for a range of stress, an extrapolation of the relaxation modulus to different values of stress is then needed.

In a first attempt to solve this borehole problem two major assumptions will be made:

a) The time dependent behaviour of the rock resulting from pore pressure diffusion is neglected. Such assumption can be made for shale rocks since pore pressure diffusion relies mainly on rock permeability.

b) Under different stresses, only the elastic response of the rock is affected, the time dependent behaviour of the strain response remains the same. The time derivative strain of the rock is assumed to be constant independently of the level of stress applied. It can be translated graphically as follows:

In order to simplify the borehole problem, when the loading modes 1 & 3 are combined, the anisotropic horizontal stress will be express as a function of the cylindrical coordinate \( \theta \) as follows:

**Borehole Problem**

**Mode (1+3)**
Detournay and Cheng Notation

\[
\sigma(\theta) = P_\sigma + S_\theta
\]

**Mode (1+3)**
viscoelastic notation

\[
\sigma(\theta) = P_\sigma(1-\cos\theta)
\]

Therefore if the creep deformation of the rock behaves like a Generalized Kelvin substance, using assumption made in b) and using the preceding notation, the following simplification (S1) can be made:

Under non-hydrostatic stress, the creep response of a rock of elastic modulus \( Z_1 \) would be similar to the creep response of a pseudo rock of variable elastic modulus \( Z_1(\theta) \) under a hydrostatic stress.
Therefore the solution for the borehole problem loading mode 1 & 3 can be obtained following the same method that the one used in 4. A pseudo elastic shear modulus is introduced and the radial displacement for loading mode 1 and 3 becomes the solution of the following equation:

$$G(\theta,t) \ast u_r^{(1+3)}(\theta,t) = -\frac{a\sigma_0}{2}$$  \hspace{1cm} (5.1)

And it comes:

$$u_r^{(1+3)}(\theta,t) = -\frac{a\sigma_0}{2G_1(\theta)} e^{\frac{G_2}{\mu_0}t}$$  \hspace{1cm} (5.2)

The expression of the pseudo elastic shear modulus $G_1(\theta)$ can be deduced from the expression of the horizontal stress using the cylindrical coordinate $\theta$. Following assumption made in b) and simplification (S1), the response of a Generalized Kelvin substance under changing load gives at $t=0$:

$$\frac{G_1}{\sigma(\theta)} = \frac{G_1(\theta)}{\sigma_0}$$  \hspace{1cm} (5.3)

Using (5.3) the analytical solution for the radial displacement of an infinite borehole under anisotropic stress becomes:

$$u_r^{(1+3)}(t) = -\frac{a\sigma(\theta)}{2G_1} e^{\frac{G_2}{\mu_0}t}$$  \hspace{1cm} (5.4)

$\sigma(\theta)$ is the anisotropic horizontal stress and $\theta$ is the cylindrical coordinate.

Under assumptions a) and b), equation obtained in (4.9) holds also for an anisotropic horizontal stress. It should be noted that the elastic shear modulus $G_1$ is obtained through a shear experiment under the average horizontal stress $\sigma_0$.


The closure $Bc(t)$ of an infinite borehole wall drilled in a poroviscoelastic formation under hydrostatic stress can be estimated from the radial displacement:

$$Bc(t) = -\int_0^t u_r^{(1)}(\tau)d\tau$$  \hspace{1cm} (6.1)

The negative sign in (6.1) is coming from the convention used in poroelasticity: compressive stresses are positive and therefore tensile elongations are negatives. Using (4.9) and (6.1), it comes:

$$Bc(t) = \int_0^t \left( \frac{aP_0}{2G_1} e^{\frac{G_2}{\mu_0}t} (\tau) \right)d\tau$$  \hspace{1cm} (6.2)

Finally:

$$Bc(t) = \frac{aP_0\mu_0}{2G_1G_2} \left( 1 - e^{\frac{G_2}{\mu_0}t} \right)$$  \hspace{1cm} (6.3)

It should be noted that this solution applies as long as the Generalized Kelvin Substance or the Burger substance are used to model the viscoelastic creep compliance functions of the rock. This formula predicts a finite amount of creep for long time behaviour. This feature however does not agree with the Burger substance design. In fact at very long time the Burger substance extrapolates to zero stiffness [11] because it has a dashpot in series with the other mechanical elements (see figure 3) and therefore would behaves like a viscous fluid.

7. Limits of Viscoelasticity and Viscoplasticity

The poroviscoelastic approach used to estimate borehole closure relies on numerous creep experiments performed on rock samples taken from the overburden. In fact such experiments need be performed on in situ core material since Chang and Zoback have showed in [16] the existence of a consolidation state of shale rocks. The creep rate of shale rocks changes drastically if the load applied is above or below the in situ stress at which the rock has been compacted by geological deposition (pre-consolidation stress).
However there are few problems associated with performing experiments on in situ shale plugs. For a long time overburdens have been neglected by the industry and still today few shale cores are available for creep experiments. However with the advent of shale gas reservoir this obstacle is expected to be lifted and more and more knowledge on shale formations will be available in the coming decades. On the other hand even if shale cores are available shale creep experiments may not be easy to implement. First, they are difficult to perform due to the brittleness of the shale structure and second, they are not necessarily representative of the whole shale layer. Shale formations tend to be highly heterogeneous and uncertainties must be therefore taken into account when performing such experiment.

In [11] Hagin and Zoback show that even though a Burger Substance is able to match creep data for an unconsolidated sand deformation, the model does not predict accurately the attenuation data following a cyclic loading on the sample. Worse, the model predicts physically unreasonable results when extrapolated to very long time. In fact as mentioned previously, the Burger Substance extrapolates to zero stiffness at long time. The outcome of this study is to use a power law-Maxwell combination model instead:

\[
\varepsilon(t) = b + \frac{t}{\eta} + At^p \tag{7.1}
\]

In addition, in [14], Sone and Zoback show that the constitutive law governing the time-dependent deformation of a shale gas reservoir rock is visco-plastic. Three stages experiments are performed on dry shale gas reservoir samples in order to isolate the frame deformation of the rock from poro-effects.

- First in the hydrostatic stage, samples are subject to several steps of isotropic confining pressures.
- Then in the triaxial stage, the axial differential load is increased while holding the confining pressure constant.
- Finally in the failure and friction stage, the sample is taken to failure by increasing the axial load on the sample.

This study highlighted that the rock frame deformation is viscoplastic and follows a power-law function of time. The plastic characteristic of the deformation is demonstrated through a further cyclic axial loading experiment.

In addition Sone and Zoback study shows that creep deformation increases with clay content and perpendicular bedding direction. This last point suggests that inclined bedding will be more susceptible to creep deformation than in bedding-parallel directions. Therefore, for practical purposes, creep is more likely to occur in deviated wells than in vertical wells. Finally based on their analysis the static young's modulus proxies fairly well the clay content and the bedding orientation. Therefore the general expression for the creep of shale gas reservoir proposed is:

\[
\varepsilon(t) = A(P_{\text{diff}}, E_{\text{static}}) t^p \tag{7.2}
\]

With \(P_{\text{Diff}}\) being the axial differential load applied on the sample.

### Nomenclature

| \(\varepsilon\) | Strain |
| \(\sigma\) | Stress |
| \(E\) | Young modulus |
| \(H(t)\) | Heavyside step function |
| \(J_0(t)\) | Young creep compliance function |
| \(E(t)\) | Young relaxation modulus |
| \(\sigma(t)\) | Time dependent stress tensor |
| \(\varepsilon(t)\) | Time dependent strain tensor |
| \(K_d\) | Drained bulk modulus |
| \(K_{d}(t)\) | Drained bulk relaxation modulus |
| \(J_{d}(t)\) | Drained bulk creep compliance function |
| \(G\) | Shear modulus |
| \(G(t)\) | Drained shear relaxation modulus |
| \(J_{d}(t)\) | Drained shear creep compliance function |
| \(K_s\) | Solid grain modulus |
| \(K_{s}(t)\) | Solid grain bulk relaxation modulus |
| \(J_{s}(t)\) | Solid grain grain bulk creep compliance function |
| \(K_u\) | Undrained bulk modulus |
| \(K_{u}(t)\) | Undrained bulk relaxation modulus |
| \(J_{u}(t)\) | Undrained bulk creep compliance function |
Conclusion
The theory of poroviscoelasticity has been used to derive the borehole closure of a formation under hydrostatic stress. It has been shown that viscoplasticity would give a more complete approach to analyse the creep behaviour of the rock especially in case of cycling loading. However for overburden sections, this feature is rarely observed, at least for a time scale as short as the life of a field. Therefore poroviscoelasticity seems to be sufficient for borehole closure prediction.

The analytical solution obtained for borehole closure (6.3) decays exponentially with time. Therefore once the equilibrium between the borehole and the pore pressure has been reached, the creep rate is maximal and decreases with time until it reaches a finite amount which may or not close on the casing. If a borehole closes on a casing, the strength of such seal is driven by the horizontal stress of the surrounding formation.

Additional work on this topic should investigate further down the borehole problem loading mode 3 since it plays a non negligible role in the borehole closure. There is also room for other research areas. Besides the fact that other phenomenological models could be examined, chemical and thermal activities of shale formations could be studied. In [15], M. A. Fam, M. B. Dusseault and J.C Fooks have shown that both of these effects may play a big role in the time dependent behaviour of mudrocks.

On the practical side, future work on this topic would be to validate or invalidate this poroviscoelastic approach by making an experiment on a field case. The Gullfaks oilfield is a very good candidate for such experiment. Today many annular seals have been qualified as secondary barrier for P&A wells within this field. If in situ cores of Gullfaks overburden where taken, using the geomechanic and logging data that has been collected until now, a robust analysis of shale formation seals could be performed.
References


APPENDIX
# Appendix A: Critical literature review - Table and Summary

<table>
<thead>
<tr>
<th>Publication</th>
<th>Year</th>
<th>Title</th>
<th>Authors</th>
<th>Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Journal of applied physics, V.12</td>
<td>1941</td>
<td>General Theory of three-dimensional consolidation</td>
<td>Maurice A. Biot</td>
<td>First paper coupling elastic behaviour of the matrix with pore pressure diffusion. Theory of Consolidation is proposed.</td>
</tr>
<tr>
<td>Proceedings of the royal society, 444, 161-184</td>
<td>1994</td>
<td>Swelling of shale around a cylindrical wellbore</td>
<td>J.D. Sherwood and L. Bailey</td>
<td>This paper proposes a chemical model for chemical reactions occurring when shale formation swells. The outcome is unexpected.</td>
</tr>
<tr>
<td>Acta Mechanica 119</td>
<td>1996</td>
<td>Poroviscoelastic analysis of borehole and cylinder problems</td>
<td>Y Abousleiman, A. H.-D Cheng, C. Jiang and J.-C. Roegiers</td>
<td>Borehole deforming under plain condition is studied and a cylinder under generalized plane strain condition is solved</td>
</tr>
<tr>
<td>ARMA 11-417</td>
<td>2001</td>
<td>Visco-plastic properties of shale gas reservoir rocks</td>
<td>H. Sone and M.D. Zoback</td>
<td>This paper shows that viscoelasticity is not appropriate to model formation rock,Visco-plasticity is introduced. A viscoelastic model is detailed for unconsolidated sand. In appendix they show how to go from the creep compliance function to the relaxation modulus.</td>
</tr>
<tr>
<td>Geophysics, vol 69, No3</td>
<td>2003</td>
<td>Drilling in mudrocks: rock behaviour issues</td>
<td>Moheb A. Fam, Maurice B. Dusseault, Jeanette C. Fooks</td>
<td>Time dependent mechano-chemical properties in borehole environment are presented. Changes in engineering properties of mudrock are studied according to the ionic composition of their pore fluids. Reactivity coeff. of mudrock is introduced</td>
</tr>
<tr>
<td>Blackwell publishing</td>
<td>2008</td>
<td>Laboratory Poromechanics Simulation &amp; Modelling of Transversely Isotropic Hollow Cylinders</td>
<td>Hoang S. K., Abousleiman, Y. N</td>
<td>This paper presents an experiment to illustrate the consolidation mechanism for different field scenarios.</td>
</tr>
<tr>
<td>ARMA 08-037</td>
<td>2008</td>
<td>Insights into borehole deformation between wellbore induced stresses, breakouts and in-situ stresses</td>
<td>Al-Tahini A.M. and Abousleiman Y. N.</td>
<td>The verification of the general elastic solution derived by Kirsh is done on 4 different rock samples. The far field in-situ stress is measured and compared with the predicted one.</td>
</tr>
<tr>
<td>Elsevier 53</td>
<td>2008</td>
<td>Petroleum Related rock mechanics</td>
<td>E. Fjaer, R.M. Holt, P.Horsrud, A.M. Raen &amp; R. Risnes</td>
<td>Rock mechanics fundamentals. Time dependent effect such as consolidation and creep are introduced.</td>
</tr>
<tr>
<td>Journal of petroleum science and engineering 69</td>
<td>2009</td>
<td>Viscous creep in room-dried unconsolidated Gulf of Mexico shale (I): Experimental results</td>
<td>Chandong Chang, Mark D. Zoback</td>
<td>In companion to ARMA 08-130 paper- A simple model combining Perzyna’s viscoplasticity and Cambridge clay plastic theory is derived to describe the viscoplastic shale deformation under step-change stress.</td>
</tr>
<tr>
<td>SPE 119321</td>
<td>2009</td>
<td>Identification and Qualification of Shale annular barriers using wireline logs during plug and abandonment operations</td>
<td>Stephen Williams, Truls Carlsen, Kevin Constable and Arne Guldahl</td>
<td>Demonstrate that shale formations may seal around the casing and create an extra barrier that may be used as secondary barrier for P&amp;A wells in North Sea</td>
</tr>
<tr>
<td>Not published - Statoil corporate</td>
<td>2010</td>
<td>Formation as barrier - summer internship report</td>
<td>Thomas belahi</td>
<td>Study how plasticity of shale relates to shale creeping. Investigate the time dependent mechanism involve during shale creeping</td>
</tr>
</tbody>
</table>
General Theory of three-Dimensional Consolidation

Authors:
Maurice A. Biot

Contribution to the understanding of Natural Formation Barriers sealing the casing:
This is the theory of three-dimensional consolidation therefore it is of a crucial importance to understand this paper for the thesis. In fact creep and consolidation are two different mechanisms that play at the same time and have a comparable effect. To understand creep it is therefore very important to understand Consolidation!

Objective of the paper:
The objective of this paper is to complement and enhance K. Terzaghi’s publication on dimensional soil consolidation. A proper mathematic demonstration is proposed to quantify soil consolidation with time.

Methodology used:
First theory’s assumptions are discussed briefly. Rock mechanics tools such as Hooke’s law are presented first and pore pressure / water content variables are included thereafter. With reference to a soil deposition, a vertical axial load case on a porous medium filled with water is then studied. Darcy’s law are introduced to characterise the flow of water and the soil’s consolidation governing equation is derived. To illustrate, an application to a standard soil test is performed. The simplified differential equation is presented, with its initial and boundary conditions. Finally the vertical displacement is computed as a function of time.

Conclusion Reached:
The governing equation for soil’s consolidation is demonstrated and illustrated. K. Terzaghi’s case is generalized to three-dimension.

Comments:
This theory is applicable to any porous medium. The pore pressure, rock elastic modulus and properties disturbances induced by drilling a borehole are not described in this paper.
Swelling of Shale around a cylindrical Wellbore

Authors:
J.D Sherwood and L. Bailey

Contribution to the understanding of Natural Formation Barriers sealing the casing:
This paper gives a scientific approach of shale swelling. Particularly chemical interactions between drilling fluids and shale formation are discussed. In this paper it is demonstrated that shale swelling occurs whenever there is a chemical potential gradient between Drilling fluid and water saturating a shale formation. Biot’s theory is modified to include these effects. An analytical solution for displacement and Stress state around a cylindrical wellbore are presented in this paper.

Objective of the paper:
The objective of this paper is to expand Biot’s poroelastic theory in order to better characterize the shale swelling effect. This theory takes into account chemical reactions between the drilling fluid and shale formation. These reactions are described analytically and integrated into Detournay and Cheng poroelastic response of a borehole in a non – hydrostatic stress field that is based on Biot’s theory.

Methodology used:
First shale is assumed to be a perfect ion exclusion membrane, hence only the chemical potential is playing a role in the swelling process. The chemical potential equation is integrated into Detournay and Cheng analysis for a circular wellbore in an infinite shale. Swelling predictions in finite shale samples are computed and experimental results are presented. The samples are initially drained to model the drilling process. Then undrained response of samples to loading are studied, it corresponds to a wellbore under non-hydrostatic stress. Finally the evolution of swelling with a circulating fluid in the wellbore is predicted analytically and measured in an experiment. In this experiment several fluids with different ionic concentration are used for comparison.

Conclusion Reached:
Shale swelling due to chemical reactions between the drilling fluid and the water that is saturating the shale formation will cause the borehole to increase in size. This phenomenon is dictated by the potential different between both fluids.

Comments:
This theory gives a quite surprising result that I don’t fully understand. Borehole size increases in swelling shale. The author discuss about this and point out that clay particles were found in the circulating system showing that erosion occurred (against initial assumptions). Therefore I don’t know how reliable these conclusions are.
Poroviscoelastic analysis of borehole and cylinder problem

Authors:
Y. Abousleiman, A. H.-D. Cheng, C. Jiang, J.-C. Roegiers

Contribution to the understanding of Natural Formation Barriers sealing the casing:
This paper proposes a theory on poroviscoelasticity and therefore considers both consolidation and creep mechanisms at the same time. This poroviscoelasticity theory is based on Biot’s poroelasticity, generalize to encompass viscoelastic effects (creep) through the correspondence principle. This theory could be used to model the inward movement of shale formation onto the casing. It would therefore consider the consolidation and creep mechanism as one mechanism broken into two steps.

Objective of the paper:
The objective of this paper is to generalize Biot’s poroelasticity theory in order to encompass the viscoelastic effect using the correspondence principle.

Methodology used:
First main results from the poroelasticity theory are presented. Then the correspondence principle is used to encompass the viscoelastic effect into Biot’s consolidation theory. The Laplace transform are used to get the governing equations of the poroviscoelastic theory, describing a fluid saturated poroviscous medium. These equations are then reconstructed using a micromechanics model that is based on the generalized Kelvin model (using springs and dashpot). Then the authors propose laboratory tests to be able to feed the parameters of the constitutive equations of the poroviscoelastic theory. Finally two problems are presented. First the problem of the borehole subject to a non-hydrostatic stress state, but deforming under plane strain condition, is examined. Second, a cylinder under generalized plain strain conditions is solved.

Conclusion Reached:
A new theory is presented. The creep is being identified as being a combination of: the solid frame volumetric deformation, the solid constituent volumetric deformation and the solid frame shear deformation. In addition based on the theoretical prediction, the time dependant behaviour of bulk material coefficient can be complicated.

Comments:
This paper is the thesis milestone.
Visco-plastic properties of shale gas reservoir rocks

Authors:
Sone, H and Zoback, M.D

Contribution to the understanding of Natural Formation Barriers sealing the casing:
This paper uses a visco-plastic approach to characterize shale-gas reservoir rocks creep. It is based upon 3 stage creep experiments performed on 6 type of shale formation representing a wide range of shale-gas reservoir rock. The main outcome of this paper is the finding of a constitutive law that governs the time-dependent deformation of shale gas reservoir rock. It is found to be a visco-plastic law opposed to a visco-elastic approach that is discussed at the last publication’s chapter. Finally the impact of mineralogy and bedding orientation is also discussed.

Objective of the paper:
The objective was to find a constitutive law for shale formation creep in order to better design hydraulic fracturing operations in shale-gas reservoir.

Methodology used:
A typical experiment on a dry sample is described; it is performed in 3-stages: hydrostatic, triaxial and failure & friction. The power-law function and the logarithm approaches are compared. A cyclic loading on a sample is done to show the creep follows a visco-plastic behaviour. It is then compared with visco-elastic approach discussed at the end of the publication.

Conclusion Reached:
Shale formation behaves like a visco-plastic material. Creep tests show that the power law gives a reasonable fit with the data in all case. The creep is function of a constant that depends upon the young modulus and the differential pressure exerted to the sample. It is shown that the young modulus captures both mineralogy (clay content) and bedding orientation of the sample.

Comments:
There is some very useful information in this publication. On top of the results presented there, some very interesting papers are referenced in this publication.
Viscous deformation of unconsolidated reservoir sands – part 2: Linear viscoelastic models

Authors:
Paul N. Hagin and Zoback, M.D

Contribution to the understanding of Natural Formation Barriers sealing the casing:
In this publication, the authors develop the methodology they followed to find the appropriate viscoelastic model for an unconsolidated sand formation. They also come back on the viscoelastic theory explained by Flügge and Christensen in their respective books: Viscoelasticity and Theory of Viscoelasticity, an Introduction. In the appendix of the publication they also derive the transformation from the creep compliance function to the relaxation modulus for a burger substance.

Objective of the paper:
The objective of this paper is to set up a methodology in order to take the appropriate model for a viscoelastic formation. The methodology is explicitly presented for the unconsolidated sand formation from the Wilmington field. The outcome is that a power law combined with a Maxwell substance represent best creep, bulk modulus dispersion and attenuation data.

Methodology used:
They first pointed out that even though several conceptual models could lead to very similar creep law; it was not necessarily the case when dealing with bulk modulus dispersion law and certainly not when looking at attenuation data. They studied creep experiment data on samples (to get the creep law) as well as cyclic loading test to get both bulk modulus dispersion and attenuation. Looking a cyclic loading test allowed them to discriminate the viscoelastic models that were fitting the creep data but not the attenuation or bulk modulus dispersion response.

Conclusion Reached:
A power law function combined with a Maxwell substance gives the best fit for all three experiments. It can therefore be used to represent the viscoelastic behaviour of the formation.

Comments:
The burger substance model which as long been used in the industry to represent the viscoelastic behaviour of saturated rocks do not produce physically reasonable results regarding the attenuation data. If we choose a loading period of 30 years which may correspond to a field life, it extrapolates to a zero stiffness rock meaning it behaves like a viscous fluid at long times.
Authors:
Moheb A. Fam, Maurice B. Dusseault, Jeanette C. Fooks

Contribution to the understanding of Natural Formation Barriers sealing the casing:
This paper proposes a way to classify shale as being reactive or not. This paper relate the relative importance of factors such as physico-chemical coupling, temperature change, in situ fabric, stress field and orientation on mudrocks strength.

Objective of the paper:
Firstly this paper point out all the factors affecting mudrocks strength. Then a characterization of mudrocks using a reactive coefficient is given. Mudrocks are characterized as being reactive (subject to problems during drilling operations) or nonreactive thanks to a reactive coefficient cutoff determined empirically.

Methodology used:
The factors involved in mudrocks strengthening/weakening are found in the literature. The reaction coefficient is determined through empirical experiments performed for this purpose.

Conclusion Reached:
Mudrock strength depends upon the confining stress (pore pressure), the temperature, the moisture content, the chemical activity of the drilling fluid and the anisotropy. Mudrocks with a reactive coefficient greater than 0.1 may interact with the drilling fluid during and should therefore be characterized/understood to get smooth drilling operations.

Comments:
Even though this paper is not directly related to creep, it describes the parameters that can be played with to delay shale creeping.
Contribution to the understanding of Natural Formation Barriers sealing the casing:
This book provides fundamentals in mechanical properties of a porous media. Particularly in chapter 9.8, this book presents the conceptual models for creep based on springs and dashspots developed by Bland (1960). Some examples of boundary-values problems are discussed at the end of the chapter.

Objective of the paper:
(For chapter 9.8) Sets the limit of our understanding regarding creep phenomena and provide the mathematic tools to model creep.

Methodology used:
Experimental creep measurements on alabaster sample immersed in water (Gibbs - 1940) are first presented. Conceptual models based on springs and dashpots are then explained in order to make an introduction on the theory of viscoelasticity.

Conclusion Reached:
Creep strain curves can be represented by the following equation:
ε = εe + ε1(t) + Vt + ε3(t)
where εe is the instantaneous elastic strain, ε1(t) is the transient creep, Vt is the steady state creep and ε3(t) is the accelerating creep. Before rock failure, creep can be modelled using a burger substance composed of a Kelvin substance (η1,k1) and a Maxwell substance (η2,k2) put in series. A rheological law is then developed:
(η1D^2 + k1D)ε = (η1/k2)D^2 + [1 + (k1/k2) + (η1/ η2)]D + (k1/ η2))σ

Comments:
Very comprehensive approach is used.
Contribution to the understanding of Natural Formation Barriers sealing the casing:
This paper presents an experiment performed on a Pierre Shale rock. This paper illustrates the consolidation mechanism and presents the change in radial and tangential stresses over time when subject to different pressure type load. It also shows us that we need to be careful when analysing an experiment results in order to represent field conditions.

Objective of the paper:
The main objective of this paper is to present an experiment to illustrate the consolidation mechanism for different field scenarios.

Methodology used:
The model used is a transversely isotropic material. The inner boundary of the hollow cylinder is permeable and the outer boundary is jacketed. The analytical solution of this problem is presented (using the general theory of three dimensional Consolidation – Biot 1941). It is solved using experiment boundary conditions. For each field scenario the stresses and pore pressure curves are presented as a function of sample’s dimensionless radius.

Conclusion Reached:
Radial and tangential stresses as well as pore pressure results are presented for the four different field scenarios. Heaviside, linear and exponential step loadings give relatively similar results. Oscillating loading gives a more complex response. Uncertainties on the pressure load of a hollow cylinder can have a significant impact on laboratory/field conditions.

Comments:
I am not sure what the exact measurement is! The experiment setup is not very detailed.
Insights into borehole deformation and relationship between wellbore induced stresses, breakouts, and in-situ stresses

Authors:
Al-Tahini, Ashraf M. and Abousleiman, Younane N.

Contribution to the understanding of Natural Formation Barriers sealing the casing:
Verification of the general elastic solution derived by Kirsch is performed thanks to an experiment on four different rock types (one being shale). The far field in-situ stress is measured and compared with the predicted one for each of the rock.

Objective of the paper:
Validate Kirsch elastic solution and point out divergences.

Methodology used:
The experiment is conducted on 6”x 6”x6” rock blocks. A 0.93” hole is drilled through them. Stress is applied parallel to the borehole.
Measure the tangential borehole strain using a strain gage at for angles (0, 90, 180 and 270 degrees from applied stress)
Measure horizontal and vertical displacement using clip gages.
Computed tangential strain using displacement measured in 2 and see how it compares with 1
Compute the borehole induced tangential stress using tangential strain and compare it with Kirsch elastic solution

Conclusion Reached:
The stress magnitude measured at the wellbore wall varies depending on the type of rock being tested. Kirsch solution may over or under estimate the actual induced compressive stresses at the borehole wall in rocks. Four empirical tangential stress correlations have been reported for each rock type.

Comments:
Impact of the grain size is neglected. It seems the sample are filed with air: Can we talk about consolidation in this case?
Contribution to the understanding of Natural Formation Barriers sealing the casing:
Present briefly Biot’s theory on consolidation. It clearly states the difference between consolidation and creep. Consolidation being the result of re-equilibrium of the pore pressure after a change in stress state has been applied. Creep is result from the viscous behaviour of the solid framework. The stresses distribution around the borehole is also discussed in Chapter 4. The hollow cylinder model (equation and solutions) is presented. The general elastic solution derived by Kirsch (1898) is also given. However some assumptions used in those model may not be adequate to model shale creep (Ro >> Rw ; Rw = cste; εz = 0 )

Objective of the paper:
Give fundamental understanding of creep and consolidation.

Methodology used:
The paper present the main equations related to consolidation of a porous medium. Part of the theory of elasticity and consolidation of a porous medium (Biot 1956) is reported. It is illustrated by an experiment on a porous rock sample. The creep theory using conceptual model of dashpot and spring is then briefly presented.
Regarding stresses distribution around the borehole, first a simple model is derived (infinite vertical well, isotropic horizontal stress). The solution for a deviated well in anisotropic horizontal stress given by Kirsch is then presented.

Conclusion Reached:
Conditions are presented to distinguish between a fluid flow governed by Darcy’s law or by the elastic properties of the framework. It depends on elastic moduli of the framework, fluid and porosity.
Radial and tangential displacement can be derived from the radial and tangential stresses given by Kirsch solution. Is this displacement responsible for creep, consolidation or stress deformation?

Comments:
No quantitative approach has been used to differentiate creep from consolidation. Only a porous medium is studied, shale is not particularly discussed.
 Contribution to the understanding of Natural Formation Barriers sealing the casing:
The paper illustrates the viscoplastic behaviour of a shale framework (shale sample being room dried). It demonstrates that room dried unconsolidated GoM shale is subject to a linear viscous rheology when compressed beyond its initial in situ state. The unloading strain experiments are also presented. Although they are small compared to the loading strain observed, we should not neglect them.

Objective of the paper:
The main objective of this paper is to illustrate the viscoplastic behaviour of a shale framework. This paper present hydrostatic compression and triaxial creep experiments on a room dried unconsolidated shale.

Methodology used:
Hydrostatic compression was performed on a room-dried sample using pressure steps of 5 MPa as well as erratic increase. Creep was observed for 6h for each pressure step. Samples were unloaded and relaxation creep has been recorded. Finally triaxial creep experiments have been performed to reproduce the observation made in hydrostatic test. Microscopic observations were made to support author’s experiment.

Conclusion Reached:
Regarding room dried unconsolidated GoM shale sample:
- Creep is the result of pore compaction
- Creep is a viscoplastic phenomenon (not elastic)
- Compaction creep behaves linearly when compresses beyond its initial in situ state.
- Creep is an intrinsic characteristic of the rock frame

Comments:
Since the shale sample is room dried, pore pressure is not taken into account. Is this a linear matter: Can we add pore pressure and frame effects separately?
Authors:
Stephen Williams, Truls Carlsen and Kevin Constable

Contribution to the understanding of Natural Formation Barriers sealing the casing:
First and only paper written on Natural Formation Barriers, the main mechanism ("creep") involved is determined with a qualitative approach. Only shale barriers are investigated in this paper. A clear and detailed methodology to validate a shale barrier for secondary barrier for P&A well is described.

Objective of the paper:
Point out that shale formations may seal around the casing and create an extra barrier that may be used as secondary barriers in case of P&A wells. Barrier integrity are proved using logs and leak off tests using a cased hole formation tester / perforation + retrievable packer pressure test.

Methodology used:
Use wireline logging to prove the bond exists. Use pressure test (leak off test) to prove the bond is acting as a barrier to flow in the annulus.

Conclusion Reached:
Three type of shale in the Norwegian Continental Shelf (Green clay, Shetland clay and Nise clay) have been identified as formations that could impinge the casing and therefore could be considered as a secondary annulus barrier for P&A well. This phenomenon is likely to be expected in other regions of the world. A “calibration” using logs and pressure tests should be performed when a new formation is suspected to creep around the casing in order to be considered as a Natural Formation Barrier.

Comments:
This paper gives a general qualitative approach of Natural Formation Barrier creation.
Contribution to the understanding of Natural Formation Barriers sealing the casing:
This non-published document is the first geotechnical analysis done on shale tendency to seal around the casing in some of the north-sea field. Self-sealing and plasticity of mudrocks are the main topics studied in this paper. It is demonstrated that creep is the principal drive mechanism involved when shale impinges the casing.

Objective of the paper:
Determine what are the processes involved when shale impinges the casing. Investigate the influence of rock plasticity in such phenomenon. Perform some literature review on creep.

Methodology used:
Shale formation are defined and characterized, experiments done by the nuclear industry to dispose their nuclear wastes are presented. Self-sealing and plastic behaviour of shale rock are investigated. The time dependant behaviour of the shale creep is discussed, an experiment on a shale sample is carried out and creep is identified as being the drive mechanism to self-seal around the casing. Finally rock creeping models found in the literature are briefly summarized.

Conclusion Reached:
Creep is the main mechanism involved when a shale formation self-seal around the casing. Formation consolidation takes also part of this phenomenon and it is difficult to differentiate between those two during the impinging process.

Comments:
No specific rock mechanics model is presented or investigated for shale self-sealing around the casing.
Appendix B: Inversion in the time domain of the Laplace relaxation modulus (Generalized Kelvin model)

\[
\tilde{E} = \frac{1 + sp_1}{s(q_1 + q_2s)} = \frac{1}{s(q_1 + q_2s)} + \frac{p_1}{q_1 + q_2s} = \frac{1}{q_1} \left( \frac{q_1}{q_2} \right) \left( \frac{q_2}{s + \frac{q_1}{q_2}} \right) + p_1 \left( \frac{1}{s + \frac{q_1}{q_2}} \right)
\]

Going back to the time domain,

\[
E(t) = \frac{1}{q_1} \left( 1 - e^{-\frac{q_1}{q_2}} \right) + \frac{p_1}{q_2} e^{-\frac{q_1}{q_2}} = \frac{1}{q_1} + \left( \frac{q_1 p_1}{q_2} - q_2 \right) \frac{q_1}{q_2} e^{-\frac{q_1}{q_2}}
\]
Appendix C: Inversion in the time domain of the Laplace relaxation modulus (Burger substance)

Starting from the relaxation modulus $E(t)$

$$E(t) = \frac{1}{A} \left[ (q_1 - q_2 r_1)e^{-r_1 t} - (q_1 - q_2 r_2)e^{-r_2 t} \right]$$

In the Laplace domain it becomes:

$$\tilde{E} = \frac{1}{A} \left[ \frac{q_1 - q_2 r_1}{s + r_1} - \frac{q_1 - q_2 r_2}{s + r_2} \right]$$

This equation can be developed using the definition of $r_1, r_2$ and $A$

$$\tilde{E} = \frac{1}{A} \left[ \frac{q_1 - q_2}{2p_2} \frac{p_1 - A}{2p_2} - \frac{q_1 - q_2}{2p_2} \frac{p_1 + A}{2p_2} \right]$$

$$\tilde{E} = \frac{1}{A} \left[ \frac{2p_2 q_1 - q_2 (p_1 - A)}{2p_2 s + (p_1 - A)} - \frac{2p_2 q_1 - q_2 (p_1 + A)}{2p_2 s + (p_1 + A)} \right]$$

$$\tilde{E} = \frac{1}{A} \left[ \frac{2p_2 s (2q_2 A) + p_1 (2q_2 A) + A (4q_1 p_2 - 2q_2 p_1)}{4(p_2 s^2 + 4p_1 p_2 s + p_1^2 - A^2)} \right]$$

$$\tilde{E} = \left[ \frac{4p_2 q_1 q_2 s + 4p_2 q_1}{4p_2 s^2 + 4p_2 p_1 s + p_1^2 - (4p_2)} \right]$$

$$\tilde{E} = \left[ \frac{q_2 s + q_1}{p_2 s^2 + p_1 s + 1} \right]$$
Appendix D: Derivation of the radial displacement for the borehole problem mode 1 (Burger substance)

The following expression for $G(t)$ can be written:

$$G(t) = ae^{-\gamma_1 t} + be^{-\gamma_2 t}$$

with

$$a = \frac{(q_1 - q_2 r_1)}{A} \quad \text{and} \quad b = -\frac{(q_1 - q_2 r_2)}{A}$$

Starting from equation (5.2), using the Stieltjes integral notation, it comes:

$$\int_{0}^{t} u_r(\tau) \left[ ae^{-\gamma_1(t-\tau)} + be^{-\gamma_2(t-\tau)} \right] d\tau = -\frac{aP_0}{2}$$

Deriving the above equation by $t$ and using the fact that:

$$\frac{d}{dt} f(t) = \int_{0}^{t} f(t, \tau) d\tau + \int_{0}^{t} \frac{\partial}{\partial t} f(t, \tau) d\tau$$

It leads to:

$$u(t)(a + b) + \int_{0}^{t} \frac{\partial}{\partial t} \left[ u_r(\tau) \left( ae^{-\gamma_1(t-\tau)} + be^{-\gamma_2(t-\tau)} \right) \right] d\tau = 0$$

$$u(t)(a + b) = \int_{0}^{t} u_r(\tau) \left( r_1 a e^{-\gamma_1(t-\tau)} + r_2 b e^{-\gamma_2(t-\tau)} \right) d\tau \quad (D.2)$$

Deriving (D.2) by $t$ gives:

$$u'(t)(a + b) = u(t)(r_1 a + r_2 b) - \int_{0}^{t} u_r(\tau) \left( r_1^2 a e^{-\gamma_1(t-\tau)} + r_2^2 b e^{-\gamma_2(t-\tau)} \right) d\tau \quad (D.3)$$

(D.2) can be re-written such as:

$$\int_{0}^{t} u_r(\tau) r_1 a e^{-\gamma_1(t-\tau)} d\tau = u(t)(a + b) - \int_{0}^{t} u_r(\tau) r_2 b e^{-\gamma_2(t-\tau)} d\tau$$

Inserting this expression in (D.3) gives:

$$u'(t)(a + b) = u(t)(r_1 a + r_2 b) - \int_{0}^{t} u_r(\tau) r_2^2 b e^{-\gamma_2(t-\tau)} d\tau - u(t)(a + b) r_1 + \int_{0}^{t} u_r(\tau) r_1 r_2 b e^{-\gamma_2(t-\tau)} d\tau$$

$$u'(t)(a + b) = u(t)(r_2 b - r_1 b) + \left( r_1 - r_2 \right) \int_{0}^{t} u_r(\tau) r_2 b e^{-\gamma_2(t-\tau)} d\tau \quad (D.4)$$

Deriving (D.4) by $t$ and using (D.1) leads to:

$$u''(t)(a + b) = u''(t)b(r_2 - r_1) + u(t) r_2 b \left( r_1 - r_2 \right) - \left( r_1 - r_2 \right) \int_{0}^{t} u_r(\tau) r_2^2 b e^{-\gamma_2(t-\tau)} d\tau \quad (D.5)$$

The operation (D.4)*r2 + (D.5) allows to get rid of the integral:
\[ u'(t)(a + b) + u'(t)r_2(a + b) = u'(t)b(r_2 - r_1) + u'(t)r_2b(r_1 - r_2) + u'(t)r_2b(r_2 - r_1) \]

\[ u'(t)(a + b) + u'(t)[r_2a + r_2b - r_2b + br_1] = u'(t)br_2(r_1 - r_2 + r_2 - r_1) \]

\[ \frac{u'(t)}{u'(t)} = -\frac{ar_2 + br_1}{a + b} \]

\[ u'(t) = u'(0)e^{-\frac{ar_2 + br_1}{a + b}} + \text{Cte} \quad (D.6) \]

At \( t=0 \)

\[ \text{Cte} = u(0) + \frac{a + b}{ar_2 + br_1}u'(0) \quad (D.7) \]

With the boundary condition at \( t=0 \) it comes:

\[ u(t) = -\frac{aP_0}{2G} + \frac{a + b}{ar_2 + br_1}u'(0) \left( 1 - e^{-\frac{ar_2 + br_1}{a + b}} \right) \quad (D.8) \]

Replacing \( a, b, r_1 \) and \( r_2 \) by their respective expression in (3.13) leads to:

\[ u(t) = -\frac{aP_0}{2G} + \frac{u_2}{G_2}u'(0) \left( 1 - e^{-\frac{G_2t}{u_2}} \right) \quad (D.9) \]
Appendix E: Derivation of the radial displacement for the borehole problem mode 1 (Maxwell substance)

Radial displacement of a borehole problem loading mode 1 in case the shear creep compliance function follows a Maxwell substance. The respective expression for the creep compliance function and the relaxation modulus of a burger substance can be found in [13]

![Maxwell substance layout](image)

Figure 8: Maxwell substance layout

\[ J_e(t) = \frac{1}{G} + \frac{t}{\mu} \]

\[ G(t) = Ge \]

By solving (5.2) it comes:

\[ \int_0^t \int_0^t u(\tau)G(t-\tau)\,d\tau = \int_0^t u(\tau)Ge^{\frac{G(t-\tau)}{\mu}}\,d\tau = -\frac{aP_0}{2} \quad (E.1) \]

Deriving the equation above gives:

\[ Gu(t) + \frac{\partial}{\partial t} \left( u(\tau)Ge^{\frac{G(t-\tau)}{\mu}} \right) d\tau = 0 \]

\[ Gu(t) - \frac{G^2}{\mu} \int_0^t u(\tau)e^{\frac{G(t-\tau)}{\mu}}\,d\tau = 0 \quad (E.2) \]

Deriving (E.2) gives:

\[ Gu'(t) - \frac{G^2}{\mu} u(t) + \frac{G^3}{\mu^2} \int_0^t u(\tau)e^{\frac{G(t-\tau)}{\mu}}\,d\tau = 0 \quad (E.3) \]

Performing the operation \( \frac{G}{\mu} \times (E.2) + (E.3) \) gives:

\[ \frac{G^2}{\mu} u(t) + Gu'(t) - \frac{G^2}{\mu} u(t) = 0 \]

\[ u'(t) = 0 \]

\[ u(t) = u(0) = -\frac{aP_0}{2G} \]

The radial displacement found is time independent and the solution is identical to Lamé’s solution in elasticity.
Appendix F: Borehole problem following Detournay and Cheng notation

**Borehole Problem** (Detournay and Cheng 1988)

Figure 9: Borehole problem with the three loading modes
Appendix G: Derivation of the radial displacement for a borehole problem loading mode 1

Starting from (4.7)

\[ \int_0^t u_r^{(1)}(\tau) \left( \frac{1}{q_1} + \frac{p_1}{q_2 - q_1} e^{-\frac{q_2}{q_1} (t-\tau)} \right) d\tau = -\frac{aP_0}{2} \]  \hfill (G.1)

Differentiating (G.1) by \( t \) and using the fact that

\[ \frac{d}{dt} \int_0^t f(t,\tau) d\tau = f(t,t) + \int_0^t \frac{\partial}{\partial t} f(t,\tau) d\tau \]  \hfill (G.2)

It comes:

\[ \frac{p_1}{q_2} u_r^{(1)}(t) + \frac{1}{q_2} \left( 1 - \frac{q_1 p_1}{q_2} \right) \int_0^t u_r^{(1)}(\tau) e^{-\frac{q_2}{q_1} (t-\tau)} d\tau = 0 \]  \hfill (G.3)

Then differentiating (G.3) by \( t \) and using (G.2) gives:

\[ \frac{p_1}{q_2} u_r^{(1)}(t) + \frac{1}{q_2} \left( 1 - \frac{q_1 p_1}{q_2} \right) u_r^{(1)}(t) - \frac{q_1}{q_2} \left( 1 - \frac{q_1 p_1}{q_2} \right) \int_0^t u_r^{(1)}(\tau) e^{-\frac{q_2}{q_1} (t-\tau)} d\tau = 0 \]  \hfill (G.4)

By multiplying (G.3) by \( q_1/q_2 \) and adding it to (G.4), the integral terms cancel out, it comes:

\[ \frac{p_1}{q_2} u_r^{(1)}(t) + \frac{p_1}{q_2} u_r^{(1)}(t) + \frac{1}{q_2} \left( 1 - \frac{q_1 p_1}{q_2} \right) u_r^{(1)}(t) = 0 \]  \hfill (G.5)

And therefore

\[ \frac{u_r^{(1)}(t)}{u_r^{(1)}(t)} = -\frac{1}{p_1} \]  \hfill (G.6)

(G.6) is a differential equation, which has the following solution:

\[ u_r^{(1)}(t) = u_r^{(1)}(0) e^{-\frac{t}{p_1}} \]  \hfill (G.7)

Replacing \( p_1 \) with its expression (4.5) gives:

\[ u_r^{(1)}(t) = u_r^{(1)}(0) e^{-\frac{t}{\mu c}} \]  \hfill (G.8)