IMPERIAL COLLEGE LONDON

Department of Earth Science and Engineering
Centre for Petroleum Studies

Upscaling Issues in Simulations of Thermal Recovery for Heavy Oil

By

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A report submitted in partial fulfillment of the requirements for the MSc and/or the DIC.

September 2013
I declare that this thesis: “Upscaling Issues in Simulations of Thermal Recovery for Heavy Oil” is entirely my own work and that where any material could be construed as the work of others, it is fully cited and referenced, and/or with appropriate acknowledgement given.

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Acknowledgements

This project wouldn’t have been possible without the invaluable support and guidance of my supervisor Olivier Gosselin to whom I express my deepest appreciation and respect.

I would also like to acknowledge all my colleagues and friends with whom I had passionate conversation about this thesis during the last three months.
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Upscaling Issues in Simulations of Thermal Recovery for Heavy Oil

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Imperial College Supervisor: O. Gosselin

Abstract

The world resources of heavy and extra-heavy oil are huge. By supplying heat into the reservoir, we are able to significantly reduce the viscosity of the oil in place, thereby allowing the displacement of the oil from the injector well to the producer. However, building an oil field simulation will be made harder because viscosity needs to be upscaled in this case, contrary to classical simulation. This thesis carries out a study towards greater understanding of viscosity upscaling in the case of steam injection. The study builds several numerical models, with various levels of complexity and accuracy, on STARS and MATLAB and assesses their validity comparing them to data field cross-checked analytical models such as Marx-Langenheim or Mandl-Volek. It highlights the existing upscaling problem and suggests a procedure to improve the results in 1D. An automatic adjustment of the parameters of the viscosity-temperature correlation is made at any moment, allowing deriving a function of time for each parameter. The implementation of those functions shows a better match between the viscosity of the fine and coarse grid. However, those functions are case dependant; therefore we will have to carry out this study every time we will have a new problem. Finally, it was possible to conclude with a first methodology of addressing each problem, which should be considered as one of the first steps towards greater understanding of upscaling issues in simulations of thermal recovery for heavy oil, calling further studies in the future in order to improve the accuracy and range of validity of this study.

Introduction

The world resources of heavy and extra-heavy oil (API less than 22° and 12°, viscosity greater than 1000 and 10000, respectively) are tremendous. With 4700 Gb, which represents 70% of the world’s total oil resources of 9 to 13 trillion bbl (as shown in Fig.1 below), 80% of which are in Canada, Venezuela, and the U.S., this particular kind of oil has aroused the interest of the Oil & Gas industry for many years now. However, those resources are hard to recover. The main obstacle rests upon the viscosity which is so high that the oil is unable to flow properly. In fact, in many reservoirs, primary recovery is negligible. With solvent injection, one of the most simple and widely used way to overcome the problem is to take advantage of the effect of heat on oil viscosity, especially for the lower gravity oils. For instance, an increase of temperature from 100°F to 500°F can bring down the viscosity with a factor 1000 (see Fig.2 below).
Thermal recovery processes rely on this general idea. By supplying heat into the reservoir, we are able to significantly reduce the viscosity of the oil in place, thereby allowing the displacement of the oil from the injector well to the producer. There are mainly three processes which have been developed over the years: forward in-situ combustion, steam-drive, and cyclic steam stimulation. To choose between the three can be difficult but some criteria exist, taking into account considerations on reservoir (depth, pressure, temperature, thickness), petrophysical properties (permeability, porosity, oil saturation at start), crude oil (specific gravity, in-situ viscosity, nature) and economics. In the first process listed, the oil in place serves as fuel and we inject a fluid containing oxygen into the reservoir to ensure the combustion and thus the generation of heat. For the two other processes, the heat is provided by injecting steam into the reservoir. Although any hot fluid could be used, steam is the most commonly injected, since it is widely available, has a high heat-carrying capacity, and a large amount of available heat which may be released to the formation when condensing. It is worth noticing that the steam-drive process or steamflood allows a much higher recovery than cyclic steam stimulation alone and thus is replacing more and more old cyclic steam stimulated wells nowadays. The steam injection process is represented in Fig.3.

When building fine geological grids for reservoir simulation, we generate highly accurate and detailed geological realizations on our software in order to catch at best the fine scale heterogeneity of the formation. This can have a huge impact on the reservoir flow performance and therefore has to be done with great care. However, due to the scale of a typical oil field, this leads to a fine grid with millions of blocks. Thus, simulating the model on the fine grid will outrun any computing resources available. The solution is to approximate our fine grid, being made up of tiny blocks (1-10m long and 0.1-1m high) into a coarse grid consisting of bigger blocks (50-200m long and 2-5m high), a process also known as upscaling. Yet, this method introduces some bias. Although many ways of by pass this difficulty have been developed for conventional recovery, it is hard to find something on the subject when speaking about thermal recovery.

Objectives

The main objectives of this thesis are the following:

- To report the current upscaling problem in one dimension,

- To develop simplified and quick running numerical models providing a rough but reliable temperature and viscosity distribution within the formation during steam injection in 1D,

- To develop a procedure to follow enabling a better match of viscosity between fine and coarse grid in 1D.
Methodology

The methodology which has been followed to reach the objectives stated above constitutes also the plan of this thesis, and can be split up in XX steps.

1. Presentation of the Steam Injection problem.
The goal is to explain what is at stake during the process of steam injection, which mechanisms are involved, what kind of temperature and viscosity distribution are observed on the field, and which analytical models can describe this particular behavior.

2. Numerical model built on STARS (CMG software)
In order to test the performance of the procedure developed through this thesis, we must build a base case to simulate the steam injection problem.

3. Validity of the STARS model assessed comparing to data field cross-checked analytical model of Marx-Langenheim
We must assure that our numerical model is valid and reliable. Therefore, the results of the analytical models developed in the first part are compared to the results obtained with the simulation built on STARS in part 2. The Marx-Langenheim model has been chosen for its significant good fit with the data collected on the field.

4. Report of the current upscaling problem
Thanks to our now proven realistic numerical model, we have been able to observe the current upscaling problem.

5. Simplified numerical models built on MATLAB according to the time of injection
The objective here, is to provide an easy and straight forward tool to adjust any parameters we could think about to improve the results of the upscaling. They are built based on the theory of Mandl-Volek, an upgrade of the Marx-Langenheim theory, which distinguishes two time interval and therefore two corresponding models.

6. Optimisation: creation of a pseudo-viscosity
On the appropriate MATLAB model according to the time of injection, a code has been implemented providing a modified value of the viscosity-temperature correlation parameters for the coarse grid such as the upscaling results would be enhanced. The new viscosity-temperature correlation is no longer representative of the physical features of the fluid but has become a mathematical artifact meant to fit the purpose of the upscaling, which is why we call it “pseudo”.

7. Implementation of the newly pseudo viscosity-temperature correlation on STARS
We have to test our results on our base case to conclude on the relevance of the method and to advise on further work.

Steam Injection Problem

In order to aid in oil recovery, steam is injected into the reservoir, heating both fluids and rock formation and displacing oil toward producing wells. The main advantage of using steam rather than water lies in the fact that the heat content of a unit mass of steam is much higher than that of water for the same parameters of temperature and pressure (for instance, at 401°F and 250 psi, the energy of one pound of steam is 1133 Btu, compared to 308 for water). Moreover, steam introduces far less water into the reservoir for a given amount of heat than water. Thus less the water-cut is lower, which means that more heat remains underground. The resulting increase in temperature will enhance dramatically the oil mobility through the reduction of its viscosity and thus the recovery. This effect is the predominant one but other mechanisms operate and have been described by Willman et al. (1961). The relative importance of these mechanisms on light and heavy oil is shown in Table 1.

<table>
<thead>
<tr>
<th>Source: Willman et al.</th>
<th>Torpedo SST Core</th>
<th>Torpedo SST Core</th>
</tr>
</thead>
<tbody>
<tr>
<td>37°API Crude</td>
<td>12.2°API Crude</td>
<td></td>
</tr>
<tr>
<td>Steam injection pressure, psig</td>
<td>800 (520°F) 84 (327°F)</td>
<td>800 (520°F) 84 (327°F)</td>
</tr>
<tr>
<td>Hot waterflooding recovery (includes viscosity reduction and swelling)</td>
<td>71.0</td>
<td>68.7</td>
</tr>
<tr>
<td>Recovery from gas drive</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Extra recovery from steam distillation</td>
<td>18.9</td>
<td>15.6</td>
</tr>
<tr>
<td>Recovery improvements from solvent/extraction effects</td>
<td>4.7</td>
<td>4.6</td>
</tr>
<tr>
<td>Total recovery by steam</td>
<td>97.6</td>
<td>91.9</td>
</tr>
</tbody>
</table>

Table 1: Relative importance of recovery mechanisms on light and heavy oil

On top of the already rather complicated heat and mass transfer problem of steam injection, there are also a significant number of changes in fluids and the solid matrix related to this particular process (Burger et al, 1985). Physical properties such as viscosity, density, specific volume, heat capacity, thermal conductivity are to change during the injection. In a certain extent, interfacial tension, wettability and capillary pressure will be modified too and phenomenon like distillation of liquid phases, pyrolysis or oxidation of hydrocarbons, as well as combustion in the gas phase can happen in diverse places within the reservoir, allowing numerous different flow regimes. All things considered, the steam injection problem is one of the most complicated. Therefore, based on the field experience or the observation in the laboratory of the relative importance...
of these various effects, assumptions have been made in different analytical model to describe the problem and compared to the field. These different models found in the literature will permit the validation of our simplified numerical models used later to work on the upscaling in this particular problem.

**Different zones in the displacement of oil by steam**

After a certain amount of time, four different zones can be distinguished (Baker, 1969).

**ZONE 1** – Steam Zone: In a zone directly around the well, a steam zone appears. Within this zone, the temperature is high and stays approximately uniform. As for the oil saturation, oil has been displaced and vaporized for the volatile fractions, remaining very few behind.

**ZONE 2** – Condensation Zone: steam as well as the more volatile oil compounds condense upon contact with the cold matrix and release the latent heat carried ahead of the zone 1.

**ZONE 3** – Hot Condensate Zone: formation of a bank, acting the same as what would happen in a hot water displacement but with a higher velocity, pushed by the steam zone which volume per unit mass is much greater than that of the liquid water. This bank is responsible for high oil production rates when steam breakthrough is close.

**ZONE 4** – Unaffected Zone: original temperature and oil saturation.

**Temperature distribution**

The experimental curves of the temperature distribution in a Steam-drive process looks like the solid curve in Fig.5.
The dashed line is the temperature distribution proposed by Marx and Langenheim. To build their model, they have assumed the heat is injected into a pay zone bounded by two neighboring formations allowing losses. The temperature distribution within the steam zone is uniform and the steam is assumed to drop all its latent heat at the condensation front, the flow of heat from the steam zone into the liquid zone ahead of the condensation front being neglected. Thus, the advance of the steam front is represented by a moving step function perpendicular to the base/cap rock. This is a strong assumption but the equations derived taking the heat balance from the simplifying step idealization described above, are still valid for the more realistic temperature distribution shown in Fig.6, if we redefine the thermal invasion radius as being the distance from injection well to the mid-point of the temperature distribution.

For further use in this thesis, it is also important to present another study, carried out by Mandl and Volek (1969). Through extensive laboratory experiment, they observed in many cases an earlier breakthrough of hot water, due to heat flowing across the condensation front. Consequently they built a model by generalizing the Marx-Langenheim one, allowing heat to be transferred from the steam zone to the liquid zone through the condensation front. During their experiment they also noticed that the actual heated area tends to be a bit lower than that predicted by the Marx-Langenheim model after a certain critical time \( t_c \) and explained this by a change in the heat flow regime at \( t_c \). For \( t < t_c \), the regime is conductive (see Fig.7), for \( t > t_c \) it is convective (see Fig.8).
Numerical Modelisation of the problem

Several numerical models have been made during this thesis, using the CMG simulator STARS and MATLAB. All those models have been made for a one dimensional problem, first step for every study about upscaling issues before eventual extensions to higher dimension.

Simulation on STARS

The model consists of a 1D reservoir with a length L=125m, linking an injector at x=0 and a producer at x=L. We consider a steam injection at constant pressure and the presence of three phases: water, dead oil and Solution Gas (made up of steam and vaporized oil for its more volatile compounds). The size of the cells forming the grid for the base case simulation is 0.5m. Therefore, we get a grid 250x1x1. The Table 2 below sums up the main settings implemented in the software.

<table>
<thead>
<tr>
<th>Reservoir Thickness</th>
<th>1m</th>
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<tr>
<td>Top of Reservoir</td>
<td>500m</td>
</tr>
<tr>
<td>OWC</td>
<td>501m</td>
</tr>
<tr>
<td>GOC</td>
<td>500m</td>
</tr>
<tr>
<td>Ref Depth</td>
<td>500m</td>
</tr>
<tr>
<td>Reservoir Temperature</td>
<td>37°C</td>
</tr>
<tr>
<td>Steam Temperature</td>
<td>325°C</td>
</tr>
<tr>
<td>Steam Quality</td>
<td>0.8</td>
</tr>
<tr>
<td>Bubble Point Pressure</td>
<td>8576 KPa</td>
</tr>
<tr>
<td>Oil Viscosity at Bubble Point</td>
<td>443 cp</td>
</tr>
<tr>
<td>Stock Tank Oil Gravity</td>
<td>21 API</td>
</tr>
<tr>
<td>Gas Gravity</td>
<td>0.65</td>
</tr>
<tr>
<td>Porosity</td>
<td>0.3</td>
</tr>
<tr>
<td>Permeability</td>
<td>400 mD</td>
</tr>
<tr>
<td>Formation Compressibility</td>
<td>1.8e-5 1/KPa</td>
</tr>
<tr>
<td>Thermal Conductivity of Reservoir Rock</td>
<td>6.6e5 J/(m – day - °C)</td>
</tr>
<tr>
<td>Thermal Conductivity of Water Phase</td>
<td>5.35e5 J/(m – day - °C)</td>
</tr>
<tr>
<td>Thermal Conductivity of Oil Phase</td>
<td>8035 J/(m – day - °C)</td>
</tr>
<tr>
<td>Thermal Conductivity of Gas Phase</td>
<td>2000 J/(m – day - °C)</td>
</tr>
<tr>
<td>Thermal Conductivity of Overburden</td>
<td>1.5E+05 J/(m – day - °C)</td>
</tr>
<tr>
<td>Thermal Conductivity of Underburden</td>
<td>1.5E+05 J/(m – day - °C)</td>
</tr>
<tr>
<td>Volumetric Heat Capacity of Rock</td>
<td>2.35e6 J/(m^3 - °C)</td>
</tr>
<tr>
<td>Volumetric Heat Capacity of Overburden</td>
<td>2.35E+06 J/(m^3 - °C)</td>
</tr>
<tr>
<td>Volumetric Heat Capacity of Underburden</td>
<td>2.35E+06 J/(m^3 - °C)</td>
</tr>
</tbody>
</table>

| Swr | 0.3 | 0.4 |
| Sor | 0.4 | 0.2 |
| Sgr | 0.45 | 0.05 |

Table 2 Input Settings used to build the numerical model on STARS

The results for the temperature distribution are shown Fig.9 and plotted Fig.10. We can see that the shape of the temperature distribution is very close to what we had in Fig.4 and Fig.8. Moreover, we can identify without any doubt the different zones defined in Fig.4. Therefore, we see that we have a qualitative match between numerical simulation and analytical model.
Temperature Distribution at different time of Injection

Figure 1 Temperature Distribution and zonation obtained with STARS at the end of injection

Figure 2 Temperature Distribution obtained with STARS at different time of injection (middle and end)

In order to have a more quantitative comparison, we will plot together in Fig1 the cumulative oil production during the time of injection simulated with STARS and calculated with the same parameters through the Marx-Langenheim analytical model:

\[
Cumulative \ oil, \ bbl = \sum_{day} 4.275 \left[ \frac{H_0 \phi(S_{oil} - S_{orr})}{M\Delta T} \right] \exp(t_0 \ erfc(\sqrt{t_0}))
\]
A pretty good agreement is observed between analytical and numerical solutions. Thus, we can conclude that our numerical model is valid and we will use it from now to test the optimization procedure developed further. Below are the evolution of several parameters such as steam, water and oil saturations and viscosity at the end of injection (respectively Fig.12, 13, 14, and 15).
Observation of the Upscaling Problem and Optimisation

Now that we know that our model is reliable, we can carry out our study on it. The 125m separating the injector from the producer are meshed in different grids of respectively 250, 50, 20 and 5 cells. The results for all the cases are plotted together for the temperature distribution and the viscosity distribution in Fig. 16 and 17. We can see that as the grid coarsen, the temperature distribution is shifted to the top, indicating that the temperature is overestimated. This will lead to an underestimated viscosity and therefore a higher cumulative recovery than observed. Therefore, there is an upscaling problem which needs to be address.

![Comparison of Temperature distribution at end of injection between different size of grid cells](image1.png)

**Figure 1** Comparison of Temperature distribution at end of injection between different size of grid cells

![Comparison of Viscosity distribution at end of injection between different size of grid cells](image2.png)

**Figure 2** Comparison of Viscosity distribution at end of injection between different size of grid cells

To understand what happens, it is convenient to ask STARS to provide with a representation of the temperature via a scale of colour. If we consider for instance our base case as a fine grid of our simulation and a case with a coarser grid, with a rapport of 12.5 between the number of cells/size of cells, we obtain the kind of temperature distribution shown in Fig. 18 and 19. What we observe is that some colors, corresponding to some temperature, are missing in the coarse grid (orange for instance, representing the range of temperature between 239°C and 269°C).
Upscaling Issues in Simulations of Thermal Recovery for Heavy Oil

What probably happened is that some “red” and “orange” fine cells have been merged into a coarse cell, and the temperature which has been hold has been the “red” one. In Fig. 20 have been represented fine cells in fine black line, merged into a coarse cell in thick black line. For each fine cell, there is a temperature, represented in fine line (either orange or red, depending on the value of the temperature, forming the “red” and “orange” cells we were talking about above). The more logical and accurate way to upscale it would be to take the arithmetic mean and it would give us the light-blue dotted line. But what is more likely to happen is that the average temperature taken for the coarse grid will be closer than the one of the red cells (resulting in the dark-blue dotted line). So there is a shift toward the high temperature.

From here we begin to feel that somehow, we will have to find a way to adjust the dark-blue dotted line to the level of the light-blue dotted line. We can take action on the temperature or directly on the viscosity since a correlation links the two of them and this is this latter solution which will be used in the following part.

Figure 3 Temperature for 250*0.5 simulation, fine grid

Figure 4 Temperature for 20*6.25 simulation, coarse grid

Figure 5 Difference in the Temperature Distribution in the fine grid and the coarse grid
Building of simplified MATLAB model

In order to be able to work on the upscaling issue of the Steam Injection problem, very simplified models have been developed on MATLAB. Although the level of complexity of those models and the strong assumptions related to each one of them do not allow a very accurate representation of the reality, at least they provide an easy and straightforward tool to adjust any parameters we could think about to improve the results of the upscaling as well as some leads to address the problem with their simple analytical solution.

The viscosity-temperature correlation used will be the default equation used in STARS. It’s an exponential relationship expressing the absolute viscosity \( \eta \), in cp, with the temperature T, in °K such as: \( \eta = a \exp(b \frac{T}{T}) \). We set the value of \( a \) and \( b \) as 1.2215249 cp and 460.46 °K respectively. This equation is known as the first equation of Andrade. Not only this equation appears convenient to use because of its wide validity and its simplicity, but also because of the theoretical justification behind it, contrary to other arbitrary correlations. Indeed, a strong link exists between this equation and the Arrhenius model, based on the assumption that the fluid obeys the Arrhenius equation for molecular kinetics \( \eta = \eta_0 \exp \left( \frac{E}{RT} \right) \) where T is the temperature, \( \eta_0 \) a constant, E the activation energy, and R, the universal gas constant.

On each one of the models, we have assume a 1D configuration with a problem of heat transfer in porous media, using dimensionless variables to be able to compare a wide range of problem. We have considered only one phase, within which we observe the propagation of heat through different processes. Each time, the respective system of equation is solved on a fine grid and on a coarse grid (the coarsening factor between the two being a setting) and the temperature distribution curves are compared, as well as the viscosity-temperature curves.

The first process is purely conductive, and aims to reproduce the behavior of the temperature distribution in the reservoir for \( t < t_c \) as described and verified by Mandl and Volek (1969). It is represented by a heat diffusion equation. The second one is predominantly convective, but retains some conduction as well. This time the goal is to reproduce the behavior of the temperature distribution in the reservoir for \( t < t_c \). It is represented by an advection-diffusion equation with a Darcy’s velocity. This last one implies a coupled system of equation since \( \eta \) appears in Darcy’s law.

The two models developed, correspond to a particular time/location within the reservoir while injecting steam. As \( t_c \) can be a bit challenging to find, the distinction is made thanks to the number of Peclet, which is the ratio of advective temperature transfer rate by the diffusive temperature transfer rate:

\[
Pe = \frac{Lv}{D_{th}}
\]

Heat Conduction

We consider the following system of equation, with dimensionless variables:

\[
\begin{align*}
\frac{\partial \theta_D}{\partial t_D} &= \frac{\partial^2 \theta_D}{\partial x_D^2} \\
\forall x_D \in [0,1], \quad \theta_D(x_D, t_D = 0) &= 0 \quad (IC) \\
\forall t_D > 0, \quad \theta_D(x_D = 0, t_D) &= 1 \quad \text{and} \quad \frac{\partial \theta_D}{\partial x_D}(x_D = 1, t_D) &= 0 \quad (BC)
\end{align*}
\]

It has been numerically implemented in MATLAB with a finite difference method of Crank-Nicholson. It is of second order in space and time and implicit in time, thus unconditionally stable and significantly accurate. We obtain the following evolution of temperature distribution while \( t_D \) increases (see Fig.21). It appears to be consistent with the Mandl-Volek theory qualitatively.

![Figure 1 Qualitative Comparison between the Temperature Distribution obtained analytically with MATLAB and theoretically with Mandl-Volek (Conduction case)](source: Mandl-Volek)
Advection-Diffusion with a Darcy’s velocity

The velocity given by Darcy’s law, with a constant pressure gradient. We have implemented a method which is of second order in space, first order in time, implicit in temperature and explicit in velocity. The following system of equations is considered:

\[
\begin{align*}
\frac{\partial \theta_D}{\partial t_D} &= \frac{\partial^2 \theta_D}{\partial x_D^2} - P_e \frac{\partial \theta_D}{\partial x_D} \\
P_e &= \frac{R_m L}{D_m} a_0 \exp \left( \frac{b_0}{T_{inj} - T_0} \right) \left( T_{inj} - T_0 \right) \\
\forall x_D \in [0, 1], \quad &\theta_D(x_D, t_D = 0) = 0 \quad \text{and} \quad P_e(x_D, t_D = 0) = 0 \quad (IC) \\
\forall t_D > 0, \quad &\theta_D(x_D = 0, t_D) = 1 \quad \text{and} \quad \frac{\partial \theta_D}{\partial x_D}(x_D = 1, t_D) = 0 \quad (BC)
\end{align*}
\]

We obtain the following evolution of temperature distribution while \( t_D \) increases. It appears to be consistent with the Mandl-Volek theory qualitatively.

Figure 2 Qualitative Comparison between the Temperature Distribution obtained analytically with MATLAB and theoretically with Mandl-Volek (Advection-Diffusion case)

Optimisation: creation of a pseudo-viscosity

If we observe that for our particular problem the number of Peclet \( (P_e = \frac{1}{D_{th}}) \), is big enough (resp. little enough), we can legitimately assume that the steam injection process can be described accurately enough with our simplified advection-conduction model (resp. conduction model) on MATLAB.

The model fixed, thanks to the corresponding program, we can run a sensitivity analysis of the correlation between the curves of viscosity corresponding to the fine grid and the coarse grid with respect to the variation of the coefficient \( a_0 \) and \( b_0 \) of the viscosity-temperature correlation \( \eta = a_0 \exp \left( \frac{b_0}{T} \right) \). It will take a reasonable amount of time, considering the fact that the complexity of the model remains as low as possible. We use the covariance to perform this step.

Figure 3 Temperature Distribution for the fine grid and the coarse grid on MATLAB (Conduction case)

Figure 4 Viscosity Distribution for the fine grid and the coarse grid on MATLAB (Conduction case)
In other words, the code allows us to find the best \(a\) and \(b\) such as the blue curve (corresponding to the fine grid) and the red curve (corresponding to the coarse grid) match as well as possible for every time step on Fig. 23 to Fig. 26.

From then on, we are able to choose \(a\) and \(b\) optimum for each time \(t\), thus making \(a\) and \(b\) function of \(t\). We call them \(a^*\) and \(b^*\), optimal parameters of the viscosity for the coarse grid. \(\eta^*=a^* \exp(b^*/T)\) is our pseudo viscosity. It’s worth noticing that \(a^*\) and \(b^*\) are function of time and case dependant. They are plotted below.

As we can see, the evolution of \(a^*/a_0\) is contrary to the one of \(b^*/b_0\) whatever the case, and the evolution of \(a^*/a_0\) (respectively \(b^*/b_0\)) for the diffusion case is contrary to the one of \(a^*/a_0\) (respectively \(b^*/b_0\)) for the advection-convection case (see Fig. 27, 28, 29 and 30 above). We can try to explain physically why those trend make sense. For diffusion for instance, it takes more time to reach the maximum temperature \(T_{\text{inj}}\) in a large cell (as there are in the coarse grid) than in small cells (as there are in the fine grid). Thus the temperature in the coarse grid is underestimated and then the viscosity is overestimated. That’s why \(b^*\) is low at the beginning (Fig. 28), in order to compensate that. It then increases as the temperature in the coarse grid and in the fine grid converge and we need less compensation. The exact opposite reasoning can be done for the advection-convection case. Here what take time is the transition between the cells, not the time of travel, more faster than for the diffusion case. So it takes less time to achieve \(T_{\text{inj}}\) in large cells compared too what we got for the equivalent length consisting of a large amount of small cells if we account the time of transmission between cells. Thus the temperature in the coarse grid will be overestimated, the viscosity underestimated and that’s why \(b^*\) is high at the beginning (Fig. 30) and then decreases as the temperature in the coarse grid and in the fine grid converge.

We can now come back to the real problem on STARS and apply, for every time step we have, the new viscosity-temperature correlation to the calculated temperature at this particular time step. If we consider for instance the Advection-Diffusion case, the coarse viscosity obtained (see Fig. 31 below) is then closer than before to the fine viscosity.
Conclusions

This paper shows that the upscaling of the viscosity cannot be forgotten in the case of thermal recovery. To address this issue, we can build very simple models on MATLAB to develop the tools needed to implement in more complex models, in STARS for instance. Results indicate that following the methodology provided can have a significant impact on the match between fine grid and coarse grid for the 1D case.

To improve the accuracy and the range of validity of this study, further work will be necessary. It would be worth doing the same process of sensitivity analysis for other parameters such as the grid block size, the heterogeneity… The next step would obviously be to consider what happens when we take the same problem in higher dimension. Vertically, the gravity forces will cause the development of segregation among the different fluids with the steam which has a tendency to move in the vicinity of the upper part of the layer and the condensed water which tends to sink down, all of that affecting vertical sweep efficiency. Horizontally, the sweep efficiency will depend on the ratio of the mobility of the displacing fluid to that of the displaced fluid, corrected by the ratio of the Darcy velocities in our case of displacement by steam (Harmsen, G.J.). In that case, we will need to know if our simplified MATLAB models will still be valid once upgraded to the proper dimension, or will need to be considerably modified and then lose their main advantage of being simple and convenient to develop solutions.

References


APPENDIX A – Critical Literature Review

<table>
<thead>
<tr>
<th>SPE Paper no</th>
<th>Year</th>
<th>Title</th>
<th>Authors</th>
<th>Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>2162</td>
<td>1937</td>
<td>“The Viscosity/Temperature Relationships of Hydrocarbons”</td>
<td>E. B. Evans</td>
<td>Development of viscosity-temperature tables for a wide range of hydrocarbon and test of existing viscosity-temperature correlation. Experimental and theoretical proof that three of them are of outstanding interest among every other else.</td>
</tr>
<tr>
<td>1266-G</td>
<td>1959</td>
<td>“Reservoir Heating by Hot Fluid Injection”</td>
<td>J.W. Marx R.H. Langenheim</td>
<td>Description of a method estimating thermal invasion rates, cumulative heated area, and theoretical economic limits for sustained hot-fluid injection at a constant rate into an idealized reservoir making full allowance for non-productive reservoir heat losses.</td>
</tr>
<tr>
<td>1537-G</td>
<td>1961</td>
<td>“Laboratory Studies of Oil Recovery by Steam Injection”</td>
<td>B.T. Willman V.V. Valleroy G.W. Runberg A.J. Cornelius L.W. Powers</td>
<td>This paper is a serious laboratory study into the use of steam as a recovery agent. It identifies the main mechanisms of recovery and their impact on different kind of oil as well as the reservoir parameters impacting it.</td>
</tr>
<tr>
<td>12247</td>
<td>1967</td>
<td>“A Current Review of Oil Recovery by Steam Injection”</td>
<td>H.J. Ramey Jr.</td>
<td>Review of field experience with cyclic steam injection in various part of the world. Discussion upon the importance of thermal recovery by steam injection as compared to thermal recovery by combustion.</td>
</tr>
<tr>
<td>1896</td>
<td>1967</td>
<td>“Heat and Mass Transport in Steam-Drive Processes”</td>
<td>G. Mandel C.W. Volek</td>
<td>Derivation of a critical time marking the transition from a conductive heat transfer to a convective heat transfer, in order to take into account the heat transport from the steam zone into the zone ahead, across the condensation front. Updated steam zone expansion and saturations are established and confronted to experimental data.</td>
</tr>
<tr>
<td>2917</td>
<td>1972</td>
<td>“A One-Dimensional Analytical Technique for Predicting Oil Recovery by Steamflooding”</td>
<td>N.D. Shutler T.C. Boberg</td>
<td>Development of a graphical method allowing calculating the oil recovery by steamflooding with accurate prediction of fluid saturation profiles and oil bank formation adapting the Buckley-Leverett method for isothermal, two-phase flow in porous media for a temperature depending on space and time.</td>
</tr>
</tbody>
</table>
SPE 2162 (1937)

The Viscosity-Temperature Relationships of Hydrocarbon

Authors:
Evans, E.B.

Contribution:
Development of viscosity-temperature tables for a wide range of hydrocarbon and test of existing viscosity-temperature correlation. Experimental and theoretical proof that three of them are of outstanding interest among every other else.

Objective of the paper:
This paper gathers a wide range of data on hydrocarbon viscosities. The material which had been previously published by other authors is critically examined and further measurements are carried out to broaden the accuracy and range of validity of this.

It then considers the applicability of viscosity-temperature correlations to the data summarized from new and already published work at the beginning. It discusses the theoretical basis of those correlations, as well as the experimental fit.

Methodology used:
- The hydrocarbons used in this work have been in most cases specially synthesised for the purpose and in all cases have been purified so that there is no doubt of their purity
- The measurements have been carried out using to glass viscometers of the B.S.I-Ostwald type, designed to avoid drainage and other errors
- 93 hydrocarbons viscosities are measured (cp) for 0°C, 20°C, 50°C, 80°C and 100°C.
- Each correlation and experimental data are plotted on the same graph and extrapolated in order to find the best match between the two for a wide range of temperature.

Conclusion reached:
Three equations stand out from the crowd, and are capable of relating experimental results with a considerable degree of accuracy. The first one is the ASTM equation which is empirical but only requires setting up two arbitrary constants while the others, named the modified Andrade-Silverman and Batschinski equations, appear to have sound theoretical justification but need three arbitrary constants to be fixed. The last one can be reduced to an approximate forth form with only two constants though.

Comments:
There is still no convenient and generally applicable means of expressing viscosity-temperature susceptibility. It has not been found possible to relate the three formulae given above.
SPE 1266-G (1959)

*Reservoir Heating by Hot Fluid Injection*

**Authors:**
Marx, J.W. and Langenheim, R. H.

**Contribution:**
Description of a method estimating thermal invasion rates, cumulative heated area, and theoretical economic limits for sustained hot-fluid injection at a constant rate into an idealized reservoir making full allowance for non-productive reservoir heat losses.

**Objective of the paper:**
This paper goal is to derive the equations describing thermal invasion rates, cumulative heated area, and theoretical economic limits when injecting hot fluid into the formation. Additionally, it provides numerical tables allowing the operator to find the solution of his problem in a convenient and straightforward manner as soon as the operating conditions are specified, without extensive mathematical manipulation.

**Methodology used:**
- It considers a radial flow system, concentric about the point of injection with the idealized temperature distribution being a step function
- The heat injection rate is assumed to be constant and heat losses are taken into account to the overburden and underburden
- It considers this problem to be analogous to the one describing fluid flow into a growing fracture bounded by permeable surfaces and therefore the derivation is the same, using the Laplace transformation.
- The *erfc* function appears and is tabulated in Appendix

**Conclusion reached:**
The theory of Marx and Langenheim can be used to determine steam zone growth if we neglect the heat transfer from the steam zone into the liquid zone ahead of the condensation zone separating the two latter.

**Comments:**
This simple approach is relatively accurate and convenient to use. However the obvious limitation results from its main assumption. The heat transfer between the steam zone and the liquid zone will affect the flow of water and oil and thus the growth of the steam zone.
SPE 1537-G (1961)

Laboratory Studies of Oil Recovery by Steam Injection

Authors:
Willman, B.T., Valleroy V.V., Runberg G.W., Cornelius A.J. and Powers L.W.

Contribution:
This paper is a serious laboratory study into the use of steam as a recovery agent. It identifies the main mechanisms of recovery and their impact on different kind of oil as well as the reservoir parameters impacting it.

Objective of the paper:
This paper aims to quantify the improvement in oil recovery that steam injection can bring compared to water injection. The effort is also put in investigating on the different recovery mechanisms occurring during cold water/hot water and steam injection. The authors report the results of the laboratory study and suggest an approach to assess the field behaviour of this process.

Methodology used:
- The system used consists of a core with temperature sensor, a feeding system to inject and produce steam or cold/hot water and a measuring machine for the output fluid. The core face allows measuring the heat losses in the adjacent formation. The composition of produced oil is obtained via a refractometer and the API gravity via an API hydrometer or a pycnometer.
- The steam injected is air-free and any condensate is removed continuously to guarantee a 100% quality steam all along. As for the hot water, an all liquid system is also assured.
- Several kinds of cores are used, varying in size, properties and mountings depending on the experience carried out.

Conclusion reached:
Laboratory tests show that steam injection recovers more oil than hot water injection, which recovers more than waterflood. The heat brought via the hot water triggers the mechanisms of thermal swelling and viscosity reduction of the oil. The presence of steam adds the effect of steam distillation, gas-drive and solvent extraction effect. High pressure steam is more effective but not so great percentagewise as is the extra heat required to heat the formation for that. The mass of water required to heat the reservoir at a particular temperature is far less with steam than with hot water.

Comments:
Even if heat requirements for exploiting a reservoir are independent of the amount of oil in place, good oil recovery per barrel of injected steam is depending upon a high Soi per unit of heated reservoir, a high percentage of net sand in the formation (interbedded shales absorb heat but do not contain oil), and high injection rate and/or small patterns into thick formation.
SPE 12247 (1967)

A current Review of Oil Recovery by Steam Injection

Authors:
Ramey Jr., H.J.

Contribution:
Review of field experience with cyclic steam injection in various part of the world. Discussion upon the importance of thermal recovery by steam injection as compared to thermal recovery by combustion.

Objective of the paper:
This paper aims to review current information regarding oil recovery by steam injection and transmit quantitative results to a broader audience. It must encourage engineers to share their experience on the subject currently affected by a paradox: most reports on continuous steam injection are theoretical while most papers about cyclic steam injection are field results only.

Methodology used:
- A history of the injection of hot fluid is provided to give us a reference on which to compare current practices.
- A presentation of existing theoretical studies is given regarding especially continuous injection, heat transmission within the medium and oil displacement by hot fluids.
- Field case histories are discussed and a link is made with economic considerations.

Conclusion reached:
The problem encounters is the most one considered in petroleum history and deals with non-isothermal transient flow of three immiscible fluids through porous media. The usual approach is to split heat and fluid flow as it has been done by Lauwerier, Marx-Langenheim and many others analytically.
Cyclic steam injection has been widely accepted because the increase in oil production happens immediately without a fillup period as it would have been the case with continuous injection.
At the time of the publication, most of the steam generators in California were barely economic and the process was regarded as being more a stimulation treatment in need of several technical problems to be solved.

Comments:
This paper gives a good overview of the different processes and their mechanisms; it was the state of the art back in the time. However, with the quick technology breakthrough of the last decades it is now a bit out-of-date.
SPE 1896 (1967)

*Heat and Mass Transport in Steam-Drive Processes*

Authors:
Mandel G. and Volek C.W.

Contribution:
Derivation of a critical time marking the transition from a conductive heat transfer to a convective heat transfer, in order to take into account the heat transport from the steam zone into the zone ahead, across the condensation front. Updated steam zone expansion and saturations are established and confronted to experimental data.

Objective of the paper:
This paper aims to describe the coupling mechanism of heat and mass transfer existing between the two regions on either sides of the condensation front. The critical time $t_c$ separating the two different heat transfer mode is derived and the steam zone expansion is determined depending on where we are compared to $t_c$.

Methodology used:
- Assuming the condensation front to be vertical, the paper considers a radial steam drive in a homogeneous and isotropic reservoir.
- A heat balance is derived for the moving condensation front.
- Steam zone growth equations of previous work without taking into account heat transport across the condensation front are plotted against experimental data and shown to be inconsistent at some point, called $t_c$.
- A new equation is derived when convective heat transfer becomes dominant and steam zone expansion is updated.
- This final result is confronted to experimental data.

Conclusion reached:
Experimental results show that this theory is accurate to describe the steam zone growth before the critical time. As for after the critical time, the steam zone is still slightly too high but far less than it was before without accounting for heat transport across the condensation front and surely accurate enough for practical purpose.

Comments:
This study does not take into account gravitational effects where the condensation front will be tilted and the critical time will vary along it. This can still be relevant in that case, if we can determine the distribution of steam and water fluxes along the condensation front.
SPE 2917 (1972)

A One-Dimensional Analytical Technique for Predicting Oil Recovery by Steamflooding

Authors:
Shutler, N.D. and Boberg, T. C.

Contribution:
Development of a graphical method allowing calculating the oil recovery by steamflooding with accurate prediction of fluid saturation profiles and oil bank formation adapting the Buckley-Leverett method for isothermal, two-phase flow in porous media for a temperature depending on space and time.

Objective of the paper:
To allow, in the case of partially depleted oil reservoirs, hand calculations of fluid saturations profiles and oil bank arrival delay. The effort is also put on the accuracy of the results compared to other simplified method already developed.

Methodology used:
- A 1D reservoir, bounded above and below by impermeable to flow layers is considered.
- The rate of steam front advance and the heat losses to adjacent strata are calculated by the theory of Marx-Langenheim. The isothermal, two-phase flow theory of Buckley-Leverett is adapted and applied to a series of n+1 moving isothermal zones separated by discontinuities of temperature modelling the global temperature distribution.
- The fact that steamflooding is three-phase problem and would imply the Buckley-Leverett theory to be banned is solved by a series of approximation, the simplest one being the assumption that only steam and oil flow within the steam front, Sw remaining equal to Swr.

Conclusion reached:
The graphical method developed in this study is more accurate than previous simplified methods thanks to the particular treatment made of the fluid flow happening during a steamflood. However the results are slightly optimistic except at steam breakthrough.

Comments:
The small difference remaining in ultimate oil recovery between this method and a numerical one lays on the approximation made to neglect oil distillation. We could however correct this by shifting the gas-oil fractional flow curve in order to get lower residual oil saturation. Furthermore, neglecting the effect of steam-oil displacement on downstream saturation profiles also leads to the optimistic recovery curve and must make us adopt some slight conservatism in the predictions. Finally, this study is not recommended for thick sands because the piston-like flow assumption would not be respected due to the gravity segregation occurring and reducing vertical sweep.
**APPENDIX B – Marx-Langenheim model**

Under the assumptions described in the main body of the thesis, the heat balance gives:

Heat injected into the pay zone = Heat loss into base/cap rock + Heat contained in the pay zone

And so we get:

\[
H_0 = 2 \int_0^t \left[ \frac{K \Delta T}{\sqrt{\pi D (t - \lambda)}} \right] \left( \frac{dA}{d\lambda} \right) d\lambda + M h \Delta T \frac{dA}{dt}
\]

The first term on the right stands for the nonproductive heat flux lost to the overburden and underburden formation and the second term represents the productive heat flow into the pay zone. This can be solved with a Laplace transformation, described by Carter R.D. (1957) and leads to:

\[
A = \left[ \frac{H_0 M h D}{4K^2 \Delta T} \right] \left[ \exp(t_D) \text{erfc} \left( \sqrt{t_D} \right) + 2 \frac{t_D}{\sqrt{\pi}} - 1 \right]
\]

With:

- \( H_0 \) = heat injection rate (Btu/hr)
- \( t \) = time since injection (hr)
- \( A(t) \) = cumulative heated area at time \( t \) (ft^2)
- \( K \) = overburden thermal conductivity (Btu/ft-hr-°F)
- \( D \) = overburden thermal diffusivity (ft^3/hr)
- \( H \) = pay thickness (ft)
- \( \Delta T \) = \( T_i - T_0 \)
- \( T_i \) = injection temperature (°F)
- \( T_0 \) = initial formation temperature (°F)
- \( \phi \) = porosity
- \( \rho_r, \rho_o, \rho_w \) = density of rock, oil and water (lb/ft^3)
- \( C_r, C_o, C_w \) = heat capacity of rock, oil and water (Btu/lbm-°F)
- \( S_{wi}, S_{oi} \) = initial saturation of water and oil
- \( S_{or} \) = irreducible oil saturation
- \( M \) = volumetric heat capacity of the solid matrix containing oil and water (Btu/ft^3-°F)

\[
M = (1 - \phi) \rho_r C_r + S_{wi} \phi_o C_w + S_{oi} \phi_o C_o
\]

\[
t_D = \left[ \frac{4K^2}{M^2 h^2 D} \right] t
\]

The differentiation of the function \( A(t) \) with respect to \( t \) gives the rate of expansion of the heated area and then the oil displacement rate (in Bbl per day):

\[
q_{oil} = 4.275 \left[ \frac{H_0 \phi(S_{oi} - S_{or})}{M \Delta T} \right] \left[ \exp(t_D) \text{erfc} \left( \sqrt{t_D} \right) \right]
\]

The implementation of this function on Excel allows us then to calculate the cumulative oil production in SC m^3. We obtain the following curve.
APPENDIX C – Derivation of the MATLAB Models

Heat Conduction.

\[ j(x, t) \rightarrow \frac{d}{dt}(\rho c S dx T(x,t)) \]

\[ \frac{d}{dt}(\rho c S dx T(x,t)) = \frac{d}{dx}(\rho c S dx T(x,t)) \]

\[ \frac{d}{dx}(\rho c S dx T(x,t)) = \rho c S dx T(x,t) \frac{d^2 T(x,t)}{dx^2} \]

\[ j = -\lambda \nabla T \]

Thus, we obtain,

\[ \frac{\partial T(x,t)}{\partial t} = \frac{\lambda}{\rho c} \frac{\partial^2 T(x,t)}{\partial x^2} \]

For a porous media, we have to consider separately the rocks and the fluid. If we take the subscripts s and f for solid and fluid respectively, and take \( \Phi \) as the porosity, \( cp \) as the specific heat at constant pressure of the fluid, we obtain two equations:
If we assume a local thermal equilibrium, we have \( T_s = T_f \). Adding the last two equations, we get:

\[
\frac{\partial T(x,t)}{\partial t} = D_m \frac{\partial^2 T(x,t)}{\partial x^2}
\]

With

\[
(\varphi)_m = (1-\varphi)(\varphi)_s + \varphi(\varphi)_f
\]

\[
\lambda_m = (1-\varphi)\lambda_s + \varphi\lambda_f
\]

\[
D_m = \frac{\lambda_m}{(\varphi)_m}
\]

Therefore, the system of equations with the Initial Condition (IC) and the Boundaries Conditions (BC) is the following:

\[
\frac{\partial T}{\partial t} = D_m \frac{\partial^2 T}{\partial x^2}
\quad \forall x \in [0,L],
\quad T(x,0) = T_0 \quad (IC)
\quad T(x=0,t) = T_{inj} \quad \text{and} \quad \frac{\partial T}{\partial x}(x=L,t) = 0 \quad (BC)
\]

We need to transform those equations in order to get dimensionless variables. So we consider:

\[
\theta_D = \frac{T - T_0}{T_{inj} - T_0}, \quad t_D = \frac{D_m L^2}{T_{inj} - T_0}, \quad x_D = \frac{x}{L}
\]

The system becomes:

\[
\frac{\partial \theta_D}{\partial t_D} = \frac{\partial^2 \theta_D}{\partial x_D^2}
\quad \forall x_D \in [0,1],
\quad \theta_D(x_D,0) = 0 \quad (IC)
\quad \theta_D(x_D=0,t_D) = 1 \quad \text{and} \quad \frac{\partial \theta_D}{\partial x_D}(x_D=1,t_D) = 0 \quad (BC)
\]

We need to develop this using a finite difference method. The Crank–Nicolson method will be used. The time and space domains are discretized and we get:

\[
\frac{\theta_{D,i}^{n+1} - \theta_{D,i}^n}{dt} = \frac{(\theta_{D,i+1}^{n+1} - 2\theta_{D,i}^{n+1} + \theta_{D,i-1}^{n+1}) + (\theta_{D,i+1}^n - 2\theta_{D,i}^n + \theta_{D,i-1}^n)}{2 dx^2}
\quad \forall i \in [0,N+1], \quad \theta_{D,0}^{(0)} = 0 \quad (IC)
\quad \forall n > 0, \quad \theta_{D,0}^{(n)} = 1 \quad \text{and} \quad \theta_{D,N}^{(n)} = \theta_{D,N+1}^{(n)} \quad (BC)
\]

The program in MATLAB was written with matrix. Below is how work the algorithm used.
The calculation of the viscosity is made with this formula:

\[ \eta = a \exp \left( \frac{b}{(T_{inj} - T_0) \theta_D + T_0} \right) \]

where

\[ a = 1.2215249 \text{ cp} \]
\[ b = 460.46 \ ^\circ\text{C} \]
\[ T_{inj} = 598.15 \ ^\circ\text{K} \]
\[ T_0 = 310.15 \ ^\circ\text{K} \]

**Advection-Diffusion with constant velocity**

The derivation is practically the same as the previous one but this time the fluid has a global movement with a velocity \( \vec{v} \) and so the local heat flux for the heat transfer within the fluid is given by

\[ \vec{j}_f = -\lambda_f \nabla T_f + (\rho c_p)_f \vec{v} T_f r, \] the one concerning the solid remaining unchanged.

We obtain those two equations:

\[ \Phi (\rho c_p)_f \frac{\partial T_f(x, t)}{\partial t} + (\rho c_p)_f \vec{v} \frac{\partial T_f(x, t)}{\partial x} = \Phi \lambda_f \frac{\partial^2 T_f(x, t)}{\partial x^2} \]
\[ (1 - \Phi)(\rho c_p)_s \frac{\partial T_s(x, t)}{\partial t} = (1 - \Phi) \lambda_s \frac{\partial^2 T_s(x, t)}{\partial x^2} \]

Again, if we assume a local thermal equilibrium, adding the last two equations gives us, with the same notation as before:

\[ (\rho c)_m \frac{\partial T_m(x, t)}{\partial t} + (\rho c)_f \vec{v} \frac{\partial T_m(x, t)}{\partial x} = \lambda_m \frac{\partial^2 T_m(x, t)}{\partial x^2} \]

Keeping the same initial and boundary conditions, and taking \( R_m = \frac{(\rho c)_f}{(\rho c)_m} \), we get:

\[
\left\{ \begin{array}{l}
\frac{\partial T}{\partial t} = D_m \frac{\partial^2 T}{\partial x^2} - R_m \vec{v} \frac{\partial T}{\partial x} \\
\forall x \in [0, L], \ T(x, t = 0) = T_0 \ (IC) \\
\forall t > 0, \ T(x = 0, t) = T_{inj} \ and \ \frac{\partial T}{\partial x}(x = L, t) = 0 \ (BC)
\end{array} \right.
\]

We transform again those equations to get dimensionless variables. This time we consider:

\[ \theta_D = \frac{T - T_0}{T_{inj} - T_0}, \ t_D = \frac{D_m}{\vec{v}^2} t, \ x_D = \frac{x}{L}, \ P_e = R_m \frac{\vec{v} \lambda_m}{D_m} \]

The system becomes:

\[
\left\{ \begin{array}{l}
\frac{\partial \theta_D}{\partial t_D} = \frac{\partial^2 \theta_D}{\partial x_D^2} - P_e \frac{\partial \theta_D}{\partial x_D} \\
\forall x_D \in [0, 1], \ \theta_D(x_D, t_D = 0) = 0 \ (IC) \\
\forall t_D > 0, \ \theta_D(x_D = 0, t_D) = 1 \ and \ \frac{\partial \theta_D}{\partial x_D}(x_D = 1, t_D) = 0 \ (BC)
\end{array} \right.
\]
We also develop this using a finite difference method. The Crank–Nicolson method will be used again. The time and space domains are discretized and we get:

\[
\frac{\theta_D^{n+1} - \theta_D^n}{\Delta t} = \frac{1}{2} \left[ \left( \frac{\partial \theta_D^{n+1}}{\partial x} + \frac{\partial \theta_D^{n+1}}{\partial x} \right) + \left( \frac{\partial \theta_D^n}{\partial x} - 2 \frac{\partial \theta_D^n}{\partial x} \right) \right] - \frac{\partial^2 \theta_D^n}{\partial x^2} - \frac{\partial \theta_D^n}{\partial x} \frac{\partial \theta_D^n}{\partial x} - \frac{\partial \theta_D^n}{\partial x} \frac{\partial \theta_D^n}{\partial x} - \frac{\partial \theta_D^n}{\partial x} \frac{\partial \theta_D^n}{\partial x} \right] \\
\forall i \in [0, N+1], \quad \theta_D^{(0)} = 0 \quad (IC) \\
\forall n > 0, \quad \theta_D^{(n)} = 1 \quad and \quad \theta_D^{(n)} = \theta_D^{(n)} \quad (BC)
\]

3. Advection-Diffusion with a Darcy’s velocity

This is the same problem as before but with \( v(x,t) = -\frac{k}{\rho \frac{\partial p}{\partial x}} \). It is quite common to assume the pressure gradient constant during the steam injection. For instance in Wingard (1994), they assume the displacement pressure to be constant. This is the hypothesis we have taken, so \( \frac{\partial \theta_D^n}{\partial x} = \frac{\partial \theta_D^n}{\partial x} = \text{constant} \). We have implemented a method which is of second order in space, first order in time, implicit in temperature and explicit in velocity. This time, instead of defining the matrix linking \( \theta_D^{n+1} \) and \( \theta_D^{n+1} \) at the beginning of the program with constant parameters, the matrix contain the changing velocity and must be recalculated for every step of time.

The equations are the same, including IC and BC ones but we add the equation linking velocity and temperature \( P_e(x,t) = \frac{R_m \alpha L}{b_m} v(x,t) \) which is represented numerically by:

\[
P_e^n = -\frac{R_m \alpha L}{b_m} \exp \left( \frac{k}{b} \left( T_{inj} - T_0 \right) \frac{\theta_D^n}{\theta_D^n + T_0} \right) \quad \text{II}
\]

We add also an initial condition on the velocity \( \forall x \in [0,L], \quad v(x,t = 0) = 0 \) which gives, with dimensionless parameters:

\[
\forall x \in [0,1], \quad P_e(x,t_D = 0) = 0 \quad . \quad \text{We eventually obtain:}
\]

\[
\begin{align*}
\frac{\partial \theta_D}{\partial t} &= \frac{\partial^2 \theta_D}{\partial x^2} - P_e \frac{\partial \theta_D}{\partial x} \\
P_e &= -\frac{R_m \alpha L}{b_m} \exp \left( \frac{k}{b} \left( T_{inj} - T_0 \right) \frac{\theta_D}{\theta_D + T_0} \right) \quad \text{II} \\
\forall x_D \in [0,1], \quad \theta_D(x_D, t_D = 0) = 0 \quad and \quad P_e(x_D, t_D = 0) = 0 \quad (IC) \\
\forall t_D > 0, \quad \theta_D(x_D = 0, t_D) = 1 \quad and \quad \frac{\partial \theta_D}{\partial x} (x_D = 1, t_D) = 0 \quad (BC)
\end{align*}
\]

APPENDIX D – Set up \( a_0 \) and \( b_0 \)

In order to use the viscosity-temperature correlation \( \eta = a_0 \exp \left( \frac{b_0}{T} \right) \), we need to set \( a_0 \) and \( b_0 \). The Table 4 of the STARS manual suggests values for \( a \) and \( b \) depending of the component. For instance, for C20H42, \( a_0 = 0.0095545 \) cp and \( b_0 = 1868.1 \) °K or °C. However, STARS does not provide values for component heavier than this one and unfortunately we are far from the values of viscosity we can meet in heavy oil reservoirs when we apply the range of temperature observed in the formation during a steam injection to the law above with those coefficients. Actually, beyond this point, we will have to provide a table of temperature/viscosity and it will extrapolate those values with the correlation \( \eta = a_0 \exp \left( \frac{b_0}{T} \right) \).
In order to set the values of a and b in MATLAB we have to do it ourselves. The table A.1 of Ashrafi et al (2011) provides a table of experimental measure of viscosity for different temperature, for the Athabasca heavy oil reservoir. By plotting \( \ln(\eta) \) function of \( \frac{1}{T} \), and doing a linear interpolation, we obtain the coefficient \( a_0 \) and \( b_0 \) wanted, the coefficient of determination being 0.9706:

\[
\begin{align*}
a_0 &= 1.2215249 \text{ cp} \\
b_0 &= 460.46 ^\circ \text{C}
\end{align*}
\]

![Figure 1 Athabasca Heavy Oil Viscosity](image)