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Failure of Anisotropic Shales under Triaxial Stress Conditions

by
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Declaration

I hereby declare that this thesis, entitled “Failure of Anisotropic Shales under Triaxial Stress Conditions”, is entirely my own work. All materials created by others that are used in the thesis have been given full acknowledgement.
Abstract

Shales are highly anisotropic in their mechanical behaviour. The strength of anisotropic shales depends not only on the magnitude of the principal stresses, but also on the bedding plane orientations relative to the principal stresses. In this study, the failure of shales are investigated using triaxial compression and extension tests, while the role of intermediate stress ($\sigma_2$) on the strength of anisotropic shale is evaluated using data from new triaxial extension tests, as well as data from the literature.

Triaxial compression and extension experiments were made on two organic-rich shales, at different confining stresses and bedding angles ($\beta$). Examination of post-failure computed tomography (CT) and thin section images for high strength anisotropy shale show that, for large and small values of $\beta$, the fracture plane follows the angle that is predicted by the Coulomb’s failure criterion for an isotropic material. In the range of angles of roughly $35^\circ<\beta<75^\circ$, failure occurs along the bedding plane. Both of these results are consistent with the assumptions of Jaeger’s plane of weakness (JPW) model. However, there exists a transition regime of loading angles lying between about $10^\circ$ and $35^\circ$, wherein the failure surface follows an irregular path that may jump between the bedding plane and the plane defined by the Coulomb criterion. In this regime, the strength of the rock is lower than the strength predicted by JPW model. For the shale with low strength anisotropy, the failure plane angles agree with the predictions of JPW model.

The triaxial compression experimental data on shales and several data sets from the literature were fit with both Pariseau’s continuum model for the failure of transversely isotropic materials and JPW model. Comparison of both models show that the Pariseau model provided a better fit for ten of the twelve rocks, whereas the JPW model provided a better fit only for two low strength anisotropy shales. It was noted that all the rocks with a strength anisotropy ratio (SAR) > 2 were fit more closely by the Pariseau model, whereas both shales that were a better fit with the JPW model had SAR < 2. Pariseau’s model is also more robust and accurate than Jaeger’s model when using a reduced numbers of data (i.e., data collected at fewer confining stresses and/or fewer angles).

Finally, both the JPW model and Pariseau’s model was applied in the true-triaxial stress regime, in which $\sigma_1 > \sigma_2 > \sigma_3$. When analysed with Mogi’s experimental data on Chichibu Schist, both models could predict failure under true-triaxial stress conditions. Mogi’s data and the triaxial extension experiments for the two shales shows that an increase in the intermediate stress $\sigma_2$ increases the intact rock strength, whereas weak plane failure depends not only on intermediate stress $\sigma_2$, but also on bedding plane angle $\beta$ and foliation direction ($\omega$).
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Contents

Chapter 1 Introduction
1.1 Importance of understanding the strength of shale formations................................. 8
1.2 Application of failure criteria in shale formations......................................................... 10

Chapter 2 Review of Rock Failure Criteria
2.1 Classification of failure criteria....................................................................................... 13
2.2 Review of failure criteria of anisotropic rocks................................................................. 15
2.3 Jaeger’s Plane of Weakness Model (JPW)..................................................................... 16
2.4 Jaeger’s continuously varying shear stress model......................................................... 17
2.5 McLamore and Gray model......................................................................................... 18
2.6 Walsh and Brace model............................................................................................... 19
2.7 Ramamurthy model..................................................................................................... 20
2.8 Pariseau’s model......................................................................................................... 21
2.9 Review of other anisotropic failure criteria................................................................. 22
2.10 Influence of intermediate stress in anisotropic rocks.................................................. 23

Chapter 3 Laboratory Experiments
3.1 Introduction..................................................................................................................... 26
3.2 Bossier shale description............................................................................................... 28
3.3 Vaca Muerta shale description..................................................................................... 29
3.4 Sample preparations and quality control...................................................................... 31
3.5 Triaxial compression and extension setup.................................................................... 32
3.6 Measurements and experimental results...................................................................... 33
    3.6.1 Strength of Bossier shale and Vaca Muerta shale................................................ 36
    3.6.2 Elastic moduli of Bossier shale and Vaca Muerta shale........................................ 42

Chapter 4 Fabric Analysis
4.1 Petrology......................................................................................................................... 47
4.2 CT Scan (Pre-test and Post-test) of Bossier shale and Vaca Muerta shale samples.... 49
4.3 Thin section analysis..................................................................................................... 58

Chapter 5 Jaeger Plane of Weakness (JPW) Model
5.1 Theory and data-fitting technique................................................................................ 62
5.2 Jaeger plane of weakness model applied to data......................................................... 70
    5.2.1 Bossier shale data analyzed using the JPW model.............................................. 75
    5.2.2 Vaca Muerta shale data analyzed using the JPW model.................................... 76
5.3 JPW Strength anisotropic ratio (SAR)......................................................................... 77
5.4 JPW model applied to triaxial extension..................................................................... 80
Chapter 6 Pariseau’s Model

6.1 Theory and data-fitting technique
6.2 Pariseau model applied to data
   6.2.1 Bossier shale data analyzed using Pariseau’s model
   6.2.2 Vaca Muerta shale data analyzed using Pariseau’s model
6.3 Pariseau Strength Anisotropy Ratio (SAR)
6.4 Pariseau’s model applied to triaxial extension
6.5 Pariseau’s model applied to data from literature
6.6 Pariseau model using reduced numbers of data sets
6.7 Pariseau model sensitivity analysis

Chapter 7 Failure of Anisotropic Rocks under True-Triaxial Conditions

7.0 Introduction
7.1 JPW model under true-triaxial conditions
7.2 Pariseau model under true-triaxial conditions
7.3 Mogi Chichibu Schist true-triaxial experiments
7.4 JPW model validation using true-triaxial Mogi Chichibu Schist data
7.5 Comparison of JPW model using conventional triaxial against true-triaxial data
7.6 Pariseau model validation using true-triaxial Mogi Chichibu Schist data
7.7 Bossier shale calibration of JPW and Pariseau models using compression and extension data
7.8 Vaca Muerta shale calibration of JPW and Pariseau models using compression and extension data

Chapter 8 Summary, Conclusions and Recommendations

8.1 Summary and conclusions
8.2 Recommendations

References

Appendix A Laboratory triaxial compression and extension data
   Appendix A1. Bossier shale experimental data
   Appendix A2. Vaca Muerta shale experimental data

Appendix B Matlab codes
   Appendix B1. JPW model objective function
   Appendix B2. JPW model main loop
   Appendix B3. Pariseau model objective function
1 Introduction

1.1 Importance of understanding the strength of shale formations

Jaeger (1962) described rock mechanics as a “new engineering subject” in his monograph “Elasticity, Flow and Fracture”. In the same year, Leopold Muller founded the ISRM with his motivation encapsulated by the comment “We don’t know the rock mass strength. That is why we need an International Society” (Hudson, 2008). During these times, rock mechanics was just starting to be recognized as a discipline worthy of a special course of lectures in an engineering program. Nowadays, rock mechanics is no longer described as a new subject, but the importance of predicting rock strength remains the same, but with more emphasis of increased appreciation of its anisotropic nature.

Isotropic intact rocks are relatively easy to test or interpret, and much is already understood regarding their behavior. However, predicting and modeling the strength of anisotropic rocks is one of the most important unsolved problems in rock mechanics (Hudson, 2008). Such solutions are valuable for finding answers for well construction or geomechanics applications, whereby failure of rock around the boundary of an excavation depends on the stress concentration around the opening and strength in different directions. Al-Ajmi (2006) and Zimmerman (2010) demonstrated the importance of incorporating the effect of the intermediate stress into wellbore stability analysis for isotropic rocks. However, the application of true-triaxial failure criteria for well construction was made only after many years of research on isotropic rocks. Now, with the present shift of the world energy market towards shale gas, it is natural that more studies are made to understand the anisotropic behavior of shales before applied for well construction in shale formations. This is important, because isotropic criteria should not be used for predicting the strength of anisotropic shales, as the strength of highly anisotropic shales can sometimes be ten times lower that its maximum strength, depending on the angle between the bedding plane and the direction of the maximum principal stress.

Tight shales are typically characterized by dominant matrix mineral composition, organic content and maturation, porosity and pore fluid saturation (Suarez-Rivera and Fjaer, 2012). However, for rock mechanics applications, it is more useful to think of shales in terms of their mineral and organic content. Goodman (1989) recognized the need for a behavior-based classification for rock mechanics analysis instead of the origin-based classification (i.e., metamorphic, sedimentary and igneous) and classified rocks as crystalline, clastic, fine-grained and organic. Within the fine-grained rocks, shales can be further sub-classified based on the degree of anisotropy and strength category. Organic-rich shales fall under the category of organic rocks. These classifications, although somewhat descriptive, are still too general and do not give a definition that can be standardized for organic-rich shales.
In this study, shale is described using the “mental picture” proposed in Figure 1.1. The mechanical behavior of shales is strongly influenced by three factors: its natural discontinuities, its texture, and its composition. The left-hand column of the mental picture in Figure 1.1 shows how the scale of the discontinuities, which represents the length, width and frequency of the discontinuities, relate to the overall mechanical behavior. When these discontinuities are weaker than the matrix rock, they are called “planes of weakness”, which result in strength anisotropy. Unlike for the case of isotropic rocks, anisotropic shales need to be analyzed down to finer \( \mu \text{m} \) scales, as shown by the thin section image in Figure 1.1. At this scale, lamination and bedding plane features that contribute to the mechanical behavior become relevant.

Figure 1.1. “Mental picture” of shales showing weak planes, texture and composition.

Shales are, strictly by definition, fissile rocks, but the industry usage of the term shale is broad and usually refers to the fine matrix rock. “Matrix rock” normally denotes rock having particles that are not distinguishable by the naked eye, while “sand” and “silt” generally refer to rock whose particles can be seen and are distinct. Using this broad definition, most shales can be categorized into calcite-rich, quartz-rich, or clay-rich. The texture of the shales will depend on the arrangement of mineral components within the matrix rock, resulting in
heterogeneity. An example of highly laminated shale is shown in the middle column of Figure 1.1, by a sample picture and a thin section image. The fine scale shown in the thin section image needs to be considered in order to distinguish different shale behaviors.

Taking a closer look at the shale fabric, the mechanical behavior of a shale can be described by its texture and composition. The shale diagram shown on the right hand column of Figure 1.1 describes this textural and compositional effect, categorized by its mineral composition. For the calcite rich shales, shale types with the same composition but with different textural effect (i.e., lumped vs. dispersed calcite), results in a wackestone or carbonate mudstone. Similarly, for the case of quartz shales, the arrangement of the quartz mineral within the shale matrix results in a silty mudstone or siliceous mudstone. Lastly, for clay-rich shales, high lamination or bioturbation would result in laminated claystone or bioturbiditic claystone. The presence of other material such as fossils, minerals, organics, and fluid within the shale matrix or laminations further alters and influences its mechanical behavior.

1.2 Application of failure criteria in shale formations

One of the most important applications of rock mechanics to petroleum engineering is the problem of wellbore stability (Zimmerman, 2010). Although the focus of this study is not on wellbore stability, there is an urgent need for an anisotropic failure criterion for the improved design of wellbores in shale formations. Apart from wellbore stability analysis, shale strength is also important in mining and in civil engineering (e.g., tunneling, bored piles, etc.) where shale strength anisotropy can have a large impact (Ewy et al., 2010). However, as the focus of this study is on reservoir shales, the discussion here is mostly aimed at application for wellbore stability analysis.

Wellbore stability analysis is usually conducted using isotropic models such as Mohr-Coulomb, Drucker-Prager, Von Mises, Modified Lade or Hoek-Brown (Han and Meng, 2014). Although isotropic models have been used for many years in the oil and gas industry, application of these models is severely limited in shale formations, as the strength of anisotropic shales are often overestimated and may lead to stability problems. To highlight the fact that shales are highly anisotropic, Ewy et al. (2010) tested six types of claystones, and found reduced strength along the weak plane by 10-70% for cohesion, and by 7-17% for friction angle.

To incorporate the effect of intermediate stress $\sigma_2$ on the strength of isotropic rocks, Al-Ajmi and Zimmerman (2005) developed the Mogi-Coulomb failure criterion that accounts for $\sigma_2$, and showed that this criterion was better at predicting the strength of a variety of isotropic rocks. Al-Ajmi and Zimmerman (2006), using data from the literature, applied their earlier findings on Mogi-Coulomb to wellbore stability analysis. Their main finding shows that the Drucker-Prager criterion overestimates wellbore strength, and the Mohr-Coulomb
criterion underestimates wellbore strength, whereas the Mogi-Coulomb criterion predicts wellbore strength reasonably well.

For anisotropic rocks, similar wellbore stability analyses were made for conventional triaxial stress conditions ($\sigma_1 > \sigma_2 = \sigma_3$) by various researchers. Wong et al. (1993) correlated shale strength and sonic measurements from the North Sea, and presented an approach to optimize shale formation drilling for high angle wells, suggesting practical methods to avoid stuck pipe situations. Santarelli et al. (1997) also suggested procedures to optimize drilling in shales and described that wellbore stability problems result in 10% to 15% of drilling cost, where a significant number of these issues are due to shale instability. Aadnoy et al. (1988) made one of the earliest evaluations of the plane of weakness criterion for wellbore stability analysis, and showed that ignoring strength anisotropy leads to erroneous wellbore strength predictions. Aadnoy et al. (2009) later applied the plane of weakness criterion for wellbores in Canada, which led to the successful completion of wells that previously had significant shales instability issues.

Wu and Tan (2010) used experimental data from Bohai Bay, China, and data sets from the literature, to evaluate the plane of weakness model for highly anisotropic shales, and applied this strength model for wellbore stability analysis. Their main finding suggests that shale strength anisotropy mainly affects high angle and horizontal wells. Narayanasamy et al. (2009) analyzed wells from the UK Continental Shelf that presented instabilities in the unstable Cretaceous mudstone. To overcome these shale instabilities, cores were taken from the shale formation and tested to determine its strength properties, and applying the plane of weakness model to design and complete the well successfully. Lee et al. (2012a) used the plane of weakness criterion to determine that the wellbore strength for anisotropic shales is significantly affected by the well orientation and in-situ stress field.

Past works on wellbore stability for anisotropic shales were mostly done using isotropic rock criteria, or the plane of weakness criterion. The former criteria should not be used for shales as it overestimates strength, whereas for the latter, no validation was made to determine how the plane of weakness criterion compares to other anisotropic rock models. Furthermore, there is also a lack of understanding of the role of the $\sigma_2$ effect for anisotropic rocks, which could significantly underestimate strength. Therefore, there is room for improving or validating existing anisotropic rock strength models, and testing their capabilities in the true-triaxial stress regime. In the following chapters, the experiments made on shales and the true-triaxial model described attempt to demonstrate a clear method and approach for using anisotropic models for predicting strength of anisotropic shales. The failure criteria used and validated in this study for anisotropic rocks is applicable for wellbore stability, or other civil engineering and construction applications.

The next chapter describes the various anisotropic rock failure criteria that are available in the literature. This is followed by details of the laboratory measurements on two types of shales made in this study. From the various anisotropic rock failure criteria, the Jaeger plane
of weakness (JPW) failure criterion and the Pariseau failure criterion will be described in detail. The JPW and Pariseau failure criteria will then be evaluated using data from the laboratory experiments and the literature. Lastly, the validity of these two criteria will be investigated under true-triaxial stress conditions using data from the literature and the experimental data from this study.


2 Review of Rock Failure Criteria

2.1 Classification of failure criteria

A “failure criterion” is an equation that defines, either implicitly or explicitly, the value of the maximum principal stress that will be necessary in order to cause the rock to “fail”, which in the case of brittle behavior can be interpreted as causing the rock to break along one or more “failure planes”. Rock failure criteria can be classified as isotropic or anisotropic, depending on whether or not they are intended to apply to rocks that exhibit anisotropic behavior. Pariseau (2012a, 2012b), Lade (1993) and Brady and Brown (1993) provided reviews of those failure criteria for isotropic rocks that are most commonly used in practice.

For anisotropic rocks, Duveau et al. (1998) classified failure criteria as either continuous or discontinuous, depending on whether or not they were expressed in terms of a single mathematical equation, or two or more equations that apply in different stress regimes. Within the continuous criteria, they were further categorized as mathematical, or empirical.

The Duveau et al. classification presented in Table 2.1 has been augmented with additional criteria uncovered during the present study.

An additional categorization can be made regarding whether or not the criterion accounts for the possibility that all three principal stresses may be unequal. Those criteria that do attempt to account for the influence of the intermediate principal stress, referred to hereinafter as being “true-triaxial criteria”, are identified with an asterisk.

Almost half of the criteria presented in Table 2.1 are “mathematical”, and in this approach, the rock is treated as a solid body with properties that vary continuously with direction. The usual features of these mathematical models are that the issues of orientation (bedding angle $\beta$, and foliation direction $\omega$) are accounted for, while parameters such as friction angle and cohesion are not explicitly required. Strictly speaking, these latter two parameters are necessary only in a Coulomb criterion. Most of these mathematical models have not yet been widely used in engineering practice – perhaps due to mathematical complexity, and perhaps also due to lack of experimental validation (e.g., Cazacu and Cristescu, 1999; Kusabuka et al., 1999; Lee and Pietruszczak, 2007; Mroz and Maciejewski, 2011). For anisotropic rocks, perhaps the most commonly used mathematical model is the Pariseau (1968) criterion.

The criteria that are classified as using the “empirical approach” are mainly extensions of the Coulomb or Von Mises isotropic criteria and do not use bedding angle or foliation direction. Instead, the various parameters are determined from fitting experimental data. Parameters determined from this approach are orientation specific, although orientation is not part of the equations defining the parameters. Sheorey (1997) conducted an extensive
review of those empirical rock failure criteria for isotropic and anisotropic rocks that are most common in the construction industry. This approach is easily developed or modified from existing failure criteria. Some of the latest empirical approaches proposed are extensions of the Hoek-Brown isotropic rock failure criterion adapted to true-triaxial conditions (e.g., Saroglou and Tsiambaos, 2007a; Zhang & Zhu 2007; Lee et al., 2012). This approach is not based on any physical or mathematical foundation, and been criticized for this reason (Duveau et al., 1998).

Table 2.1. Classification of anisotropic failure criteria; updated from Duveau et al. (1998).

<table>
<thead>
<tr>
<th>Mathematical approach</th>
<th>Empirical approach</th>
<th>Discontinuous criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Von Mises (1928)*</td>
<td>Casagrande and Carrillo (1944)</td>
<td>Jaeger (1960, 1964)*</td>
</tr>
<tr>
<td>Hill (1948)*</td>
<td>Jaeger variable shear (1960)</td>
<td>Walsh and Brace (1964)</td>
</tr>
<tr>
<td>Boehler and Raclin (1982)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Raclin (1984)</td>
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<td>Kaar et al. (1989)</td>
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<tr>
<td>Cazacu (1995)</td>
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<tr>
<td>Cazacu and Cristescu (1999)*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kusabuka, Takeda and Kojo (1999)*</td>
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<tr>
<td>Pietruszczak and Mroz (2001)*</td>
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<td>Lee and Pietruszczak (2007)*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mroz and Maciejewski (2011)*</td>
<td></td>
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</tr>
</tbody>
</table>

Bold – Criteria added since Duveau et al. (1998)
* – True-triaxial criteria

The discontinuous criteria are generally related to the Coulomb criterion (i.e., Jaeger, 1960). In these criteria, the failure mechanisms that occur along the weak planes or intact rock are distinguished. The assumption is that the rock fails either through shear fracture or sliding along weak planes. These two failure modes are used together to determine the actual failure criterion. Most of the criteria within this category are easily used for design applications, because they are based on the Coulomb strength parameters. However, some of the discontinuous criteria use an empirical approach (Hoek et al., 1992) or mathematical...
models (Pei, 2008) to account for failure along weak planes. Duveau and Henry (Duveau et al., 1998) combined Lade’s true-triaxial criteria for isotropic rocks with Barton’s criterion (Barton and Choubey, 1977) for anisotropic rocks. The relative ease of modifying and combining two criteria for isotropic and anisotropic rocks led to these various other combinations for the discontinuous model.

2.2 Review of failure criteria of anisotropic rocks

Although failure criteria for anisotropic rocks have not received nearly as much study as has the topic of failure criteria for isotropic rocks, nevertheless a literature review reveals several studies of the strength of anisotropic rocks. Among the pioneering and most well known approaches are Jaeger’s (Jaeger, 1960) “Plane of Weakness” model, denoted hereafter as JPW, and Jaeger’s variable shear strength model (Jaeger, 1964). To date, the JPW criterion seems to be the most widely used for predicting anisotropic rock strength.

Donath (1961) was the first to evaluate the applicability of Jaeger’s theory for Martinsburg slate. Chenevert and Gatlin (1964) also analyzed the Martinsburg slates to confirm Jaeger’s criterion, and observed that this rock’s elastic moduli are transversely isotropic. McLamore and Gray (1967) then extended Jaeger’s variable shear strength model by varying the friction angle, and applied this model to the Green River shale. In the same study, McLamore and Gray also presented evidence that the Walsh and Brace (1964) model is identical to the JPW because both models are derived from the same criteria.

Other notable early experimental works include the yielding of soft diatomite rocks under hydrostatic pressure (Allirot et al., 1977), revealing critical anisotropic properties supported by images of sample deformation. This study displayed evidence of a non-shear type failure for anisotropic rocks. This is uncommon, as most rocks fail in shear under compressive load. Another important research work was that of Attewell and Sandford (1974), who observe reduced anisotropy of the Penrhyn slate with increased stress. The phenomenon of reducing anisotropy was also reported by Ramamurthy et al. (1993), based on extensive experiments on the Himalayan schist, leading to development of an empirical failure criterion.

Since the role of $\sigma_2$ in the failure of isotropic rocks was recognized, various researchers extended this concept to anisotropic rocks, with limited success (e.g., Tiwari & Rao, 2007; Singh et al., 1998; Zhang and Zhu, 2007). While most research on strength anisotropy assumed conventional triaxial conditions, Jaeger (1964) and Pariseau (1968) extended strength anisotropy to account for the $\sigma_2$ effect. Jaeger introduced the first anisotropic rock failure criterion that includes $\sigma_2$, and discussed some of the results from his experimental work in his review of Donath’s work (Donath, 1964, p. 298).

Pariseau (1968) modified Hill’s theory for metal anisotropy (Hill, 1948), obtaining a failure criterion that incorporates the effect of $\sigma_2$. Although Duveau et al. (1998) validated
Pariseau’s model for the Angers schist, both Jaeger and Pariseau’s model were never validated using true-triaxial experimental data. The true-triaxial Pariseau model has similarities with the Drucker-Prager (1952) model for isotropic rocks, as several researchers have demonstrated (Tsai and Wu, 1971; Smith and Cheatham, 1980b; Kusabuka et al., 1999). Ong and Roegiers (1993), using Pariseau’s model, showed that high strength anisotropy significantly influences the stability of horizontal wells. A similar study of wellbore stability by Suarez-Rivera et al. (2009) showed that the Pariseau strength model, combined with an anisotropic elastic rock model, provides more conservative results than are obtained by using the JPW strength model in combination with an isotropic elastic model.

In the next sections, several failure criteria commonly known in the petroleum industry are described and presented in units of psi. Similarly, all data in this study will be presented in units of psi, to maintain consistency.

2.3 Jaeger’s Plane of Weakness Model (JPW)

The Coulomb, or the linear Mohr criterion, is sometimes referred to as the Mohr-Coulomb criterion. In 1773 Coulomb, based on experimental evidence, proposed that failure of geomaterials occurs along a plane when the shear stress $\tau$ acting along that plane reaches a critical value that is able to overcome a “frictional” type resistance force, plus an additional “cohesive” force (Jaeger et al., 2007). This condition for failure is

$$|\tau| = S + \sigma_n \tan \phi.$$  \hspace{1cm} (2.3.1)

The shear stress $\tau$ and normal stress $\sigma_n$, on a plane at angle $\beta$ to the direction of $\sigma_3$, defined as shown in Figure 2.1. The definition for angle $\beta$ described in Figure 2.1 is provided for clarity, whereas the remainder of this study uses the standard convention that is presented and used from Chapter 3 onwards.

![Figure 2.1. Triaxial compression test setup for sample angle $\beta$.](image_url)
For the angle $\beta$ defined in Figure 2.1, shear $\tau$ and normal stress $\sigma_n$ expressed in terms of maximum shear stress, $\tau_m = (\sigma_1 - \sigma_3)/2$, and mean normal stress, $\sigma_m = (\sigma_1 + \sigma_3)/2$, are given by

$$\sigma_n = \sigma_m - \tau_m \cos 2\beta,$$

$$\tau = -\tau_m \sin 2\beta,$$

where $\sigma_1$ and $\sigma_3$ are the major and minor principal stresses. Jaeger (1960) extended the Coulomb criterion to the idealized model of a rock that contains a preexisting plane that is weaker than the intact rock. This JPW criterion, expressed in terms $\tau_m$ and $\sigma_m$ (found by inserting Eqn. 2.3.2 and Eqn. 2.3.3 into Eqn. 2.3.1), is

$$\tau_m = (S_w \cos \phi_w + \sigma_m \sin \phi_w) / [\sin (2\beta + \phi_w)].$$

For intact rock failure with cohesion $S_o$ and friction angle $\phi_o$, there will be a critical plane at angle $\beta$ where the shear strength will be first reached as $\sigma_3$ is increased (Brady and Brown, 1993; p. 107; Parry, 2000). For intact rock this occurs at an angle of $\beta = 45 - \phi_o/2$ (Jaeger et al., 2007; p. 91) where $\beta$ is defined clockwise on the Mohr circle. Inserting the fracture angle $\beta = 45 - \phi_o/2$ into Eqn. 2.3.4 results in the isotropic rock Coulomb failure criterion:

$$\tau_m = S_o \cos \phi_o + \sigma_m \sin \phi_o.$$  (2.3.5)

The JPW criterion for failure along the weak plane is defined by Eqn. 2.3.4, and for failure of intact rock is defined by Eqn. 2.3.5. These two equations form the foundation of the various other criteria described in Table 2.1. Chapter 5 of this study gives a detailed explanation of the derivation and analysis of this criterion.

2.4 Jaeger’s continuously varying shear stress model

From the Coulomb criterion, Jaeger (1960) derived another approach based on the concept of “varying shear strength”. This method assumes that the cohesion of the rock, $S$, changes with bedding angle $\beta$, while the friction coefficient $\tan \phi$ is constant, where $S_1$ and $S_2$ are parameters, and the shear strength of the rock varies between $S_1 - S_2$ and $S_1 + S_2$:

$$S = S_1 - S_2 \cos 2(\alpha - \beta),$$

and where $\alpha$ is the critical plane at angle $\beta$ where the shear strength will be first reached as $\sigma_1$ is increased. For failure in a uniform medium with shear strength $S$ and friction coefficient $\tan \phi$, it is found from substituting Eqn. 2.3.2 and 2.3.3 into Eqn. 2.3.1 that $S$ has a maximum value when $\alpha = 45 - \phi/2$ (Jaeger, 1960). By assuming that $0 < \alpha < \pi/2$ and $0 < \beta < \pi/2$, and inserting Eqns. 2.3.2, 2.3.3 and 2.4.1 into the Coulomb criterion (Eqn. 2.3.1), the varying shear stress criterion can be written in terms of $\tau_m$ and $\sigma_m$ as
The parameters $S_1$ and $S_2$ are determined by fitting data to Eqn. 2.4.1, where cohesion $S$ is determined by the intercept on the Mohr circle. McLamore and Gray (1967) tested this approach, and concluded that Eqn. 2.4.1 does not describe the actual variation of $S$ over the entire range of angles $\beta$. To overcome this problem, they proposed an improvement, by assuming that the friction angle also varies with angle $\beta$, similar to Jaeger’s varying cohesion model.

2.5 McLamore and Gray model

McLamore and Gray (1967) modified Jaeger’s variable shear strength approach by proposing that cohesion $S$ and the angle of internal friction $\phi$ both vary with bedding angle $\beta$. McLamore and Gray’s varying $S$ and $\phi$ equations are

$$S = A + B[\cos 2(\alpha - \beta)]^n,$$

$$\tan \phi = C + D[\cos 2(\alpha - \beta)]^m,$$

where $A, B, C$ and $D$ in Eqn. 2.5.1 and Eqn. 2.5.2 are determined experimentally, and the “anisotropy type” factors $m$ and $n$ depend on rock type, with values ranging from 1.0 to 6.0. These varying strength parameters $S$ and $\phi$, when placed into the Coulomb equation (Eqn. 2.3.1), give a failure envelope that varies with angle $\beta$. This approach is used to determine the sets of parameters $A, B, C$ and $D$ at angles $0 \leq \beta \leq \alpha$ and $\alpha < \beta \leq 90$, where $\alpha$ is assumed to be the angle $\beta$ of lowest shear strength (i.e., $\alpha = 45 - \phi/2$).

To determine the validity of the approach, McLamore and Gray tested it on the Green River Shale-1 (GRS-1) and Green River Shale-2 (GRS-2) organic-rich mudstones. The Coulomb parameters $S$ and $\phi$ are determined by plotting a linear envelope using Mohr circles for each sample over different confining stress $\sigma_3$, and averaged for angle $\beta$. The intercept and slope of the linear fit on the Mohr circle gives the parameters $S$ and $\phi$, respectively. The rock parameter $S$ and $\phi$ are plotted against the angle $\beta$ to determine the best-fit equation to the parameters $A, B, C, D, m$ and $n$ (Figure 2.2).

By determining the fitting parameters from the plots in Figure 2.2, the varying $S$ and $\phi$ values determined by McLamore and Gray are, for GRS-1 and GRS-2 respectively:

$$S = 10,700 - 4,300[\cos 2(30^\circ - \beta)]^6; \quad \tan \phi = 0.589,$$

$$S = 6,640 - 2,440[\cos 2(30^\circ - \beta)]^6; \quad \tan \phi = 0.385 - 0.060[\cos 2(30^\circ - \beta)]^6,$$

$$S = 5,360 - 1,160[\cos 2(30^\circ - \beta)]^6; \quad \tan \phi = 0.378 - 0.053[\cos 2(30^\circ - \beta)]^6.$$
Figure 2.2. Fitting parameters for the McLamore and Gray (1967) model. (a) Varying cohesion $S$ ($\tau_0$ used in Figure 2.2), with respect to angle $\beta$ for Green River Shale 1. (b) Varying cohesion $S$ ($\tau_0$ used in Figure 2.2) and friction angle $\phi$ with respect to angle $\beta$ for Green River Shale 2.

Although the method proposed by McLamore and Gray seems to be straightforward and easy to apply, subsequent researchers have rarely attempted to use this approach. This is perhaps because the model does not vary significantly from Jaeger’s idea of varying shear strength, and also because the model provides no clear explanation of the failure mechanism.

2.6 Walsh and Brace model

The Walsh and Brace (1964) model is an extension of the McClintock and Walsh (1962) modification of Griffith’s tensile criterion (McLamore and Gray, 1967). This model assumes that anisotropic rocks contain long non-randomly oriented cracks that close under increased confining stress, superposed on an isotropic randomly distributed short crack.

Walsh and Brace assumed that fracture occurs through the long crack (subscript “L”) depending on angle $\beta$, or through the short crack (subscript “S”). The fracture stress for failure through randomly oriented short cracks as a function of confining stress $\sigma_3$ is

$$ (\sigma_1 - \sigma_3)_S = C_0 S + \left\{ 2 \mu_S \sigma_3 / \left[ (1 + \mu_S^2)^{1/2} - \mu_S \right] \right\}, \quad (2.6.1) $$
where \( C_{OS} \) is the unconfined compressive strength, and \( \mu_S \) is the coefficient of friction for the randomly oriented short crack material. This criterion for short random cracks is representative of the rock strength behavior for failure within the matrix.

For the case of material failure along the long oriented cracks, fracture stress as a function of confining stress is

\[
(\sigma_1 - \sigma_3)_L = \left[ C_{OL} \left( 1 + \frac{\mu_L^2}{2} \right)^{1/2} - \mu_L \right] + 2\mu_L\sigma_3 / \{2\sin\beta \cos\beta (1 - \mu_L \tan\beta) \}.
\] (2.6.2)

Equation 2.6.2 for long oriented cracks is representative of the rock strength for failure along a plane of weakness. The term \( C_{OL} \) is the unconfined compressive strength, and \( \mu_L \) represents the coefficient of friction of the long oriented crack.

The rock parameters \( C_{OS}, \mu_S, C_{OL} \) and \( \mu_L \) are determined from triaxial compression experiments for failure through the intact rock, and failure along the weak plane, respectively. Although it is not obvious from Eqns. 2.6.1 and 2.6.2, McLamore and Gray (1967) showed that this model is actually equivalent to the JPW model.

2.7 Ramamurthy model

Ramamurthy (1993) and his co-workers (Ramamurthy et al., 1993; Ramamurthy and Arora, 1994) proposed a failure criterion for rocks, which is similar to the Hoek-Brown failure criterion (Hoek and Brown, 1988). Ramamurthy and co-workers modified the Mohr-Coulomb criterion to account for nonlinear shear strength response of intact rock as

\[
(\sigma_1 - \sigma_3)/\sigma_3 = B_i (\sigma_c/\sigma_3)^{\alpha_i}.
\] (2.7.1)

Equation 2.7.1 represents the intact rock failure criterion, whereby the subscript “i” stands for intact rock, while \( \sigma_1 \) and \( \sigma_3 \) are major and minor principal stress, \( \sigma_c \) is the unconfined compressive strength, \( B_i \) is a rock-specific parameter, and \( \alpha_i \) is determined from fitting Eqn. 2.7.1. Reported values for the rock parameter \( B_i \) vary from 1.8 to 3.0 for argillaceous rocks (shales, slates, mudstones, etc.), sandstones, carbonates and igneous rocks.
Ramamurthy (1993) also recommended that the parameters $B_i$ and $\alpha_i$ be determined from two triaxial tests at values of $\sigma_3$ greater than 5% of $\sigma_c$.

For rocks with weak planes, Ramamurthy used the same equation for isotropic rocks, and extended it for anisotropic and jointed rocks by replacing the rock parameters. This failure criterion is

$$\frac{(\sigma_1 - \sigma_3)}{\sigma_3} = B_j \left( \frac{\sigma_{cj}}{\sigma_3} \right)^{\alpha_j}, \quad (2.7.2)$$

where the subscript “j” stands for anisotropic or jointed rocks, $\sigma_{cj}$ is the unconfined compressive strength at angle $0^\circ < \beta < 90^\circ$, and $B_j$ and $\alpha_j$ are the parameters at corresponding orientation. For anisotropic rocks, Ramamurthy (1993) suggested the following expressions to determine the coefficients $\alpha_j$ and $B_j$:

$$\frac{\alpha_j}{\alpha_{90}} = \left( \frac{\sigma_{cj}}{\sigma_{c90}} \right)^{1-\alpha_{90}}, \quad (2.7.3)$$

$$\frac{B_j}{B_{90}} = \left( \frac{\alpha_{90}}{\alpha_j} \right)^{0.5}, \quad (2.7.4)$$

where $\sigma_{c90}$ is the unconfined compressive strength at a bedding angle of 90°; and $\sigma_{90}$ and $B_{90}$ are determined from two or three triaxial tests at the angle 90° (Ramamurthy, 1993).

Ramamurthy and coworkers conducted many experiments on anisotropic Himalayan rocks (Ramamurthy, 1993; Ramamurthy et al., 1993; Ramamurthy and Arora, 1994; Ramamurthy, 2001; Nasseri et al., 2003) and were able to use large quantities of data to derive accurate rock parameters for various anisotropic rock types.

In Chapter 5 and 6 of the present study, data from Ramamurthy et al. (1993) on three rock types, namely Quartz Phyllite, Carbonate Phyllite and Micaceous Phyllite, are analyzed using the JPW and Pariseau’s model.

### 2.8 Pariseau’s model

Pariseau (1968) developed a failure theory for anisotropic rocks, modified from Hill’s theory for metal plasticity (Hill, 1948). Pariseau’s theory accounts for the yielding of geomaterials under hydrostatic stress and predicts a smooth, continuous variation of strength with bedding angle $\beta$. Although this mathematical model is able to account for the $\sigma_2$ effect, when applied to conventional triaxial data ($\sigma_2 = \sigma_3$) this model reduces to the following continuous failure criterion:

$$\frac{(\sigma_1 - \sigma_3)}{F \sin^4 \beta + G(\cos^4 \beta + \cos^2 2\beta) + 0.25M \sin^2 2\beta} = \frac{1 + \sigma_3(U + 2V)}{(U \cos^2 \beta + V \sin^2 \beta)^{0.5}}$$

$$\quad (2.8.1)$$
Pariseau’s criterion in Eqn. 2.8.1 is straightforward and easily applied to determine the rock strength as a function of confining stress $\sigma_3$ and angle $\beta$. However, the main issue in using this model is the difficulty of determining the rock parameters $F$, $G$, $U$, $V$ and $M$. Furthermore, the Pariseau model has never been validated for true-triaxial applications. In the present study, a detailed examination of this model will be described in Chapter 6 and 7, followed by evaluations using experimental data and data from the literature.

### 2.9 Review of other anisotropic failure criteria

Other works that are less known can also provide helpful ideas for understanding the different approaches to the problem of anisotropic rocks. Tiwari and Rao (2006, 2007) conducted true-triaxial experiments on reconstituted anisotropic rocks having various orientations and joint sets. From the experimental results on their reconstituted samples, they demonstrated that for each orientation, the strength of the rock is describable by the Von Mises criterion (Nadai, 1950). However, this is a direct interpolation of the isotropic criterion for anisotropic rocks, and uses varying parameters at different angles $\beta$.

Nova (1980) introduced a true-triaxial failure criterion that uses a friction angle and varying cohesion that were represented as tensors, showing that for transversely isotropic rock, cohesion and friction angle are sufficient to predict rock strength. However, this approach needs to be further tested.

The Hoek-Brown criterion has been widely used for intact and jointed rocks. Zhang and Zhu (2007) extended the conventional Hoek-Brown criterion to true-triaxial conditions, for isotropic and anisotropic jointed rocks. This true-triaxial equation reduces to the conventional Hoek-Brown criterion when a conventional triaxial stress condition exists. Developed for the coal industry, Ashour (1988) also proposed a true-triaxial approach that looks similar to the Pariseau (1968) model, but without simplifying the parameters for transverse isotropy. Yoshinaka and Yamabe (1981) proposed an interesting approach to show that normalized data that considers $\sigma_2$ lies on a straight line for all angles $\beta$ (where $\sigma_m = (\sigma_1+\sigma_2+\sigma_3)/3$; $\tau_m = (\sigma_1-\sigma_3)/2$; $\sigma_{mo}$ and $\tau_{mo}$ are the case when $\sigma_2 = \sigma_3 = 0$). This approach for the Penrhyn slate and Martinsburg slate shows plots using linear and log scales on Figure 2.4.

Based on extensive experience in tunneling, the Singh et al. (1998) criterion accounts for $\sigma_2$ for a single or dual joint set. Singh et al. modified the Coulomb criterion, and suggested that both $\sigma_2$ and $\sigma_3$ contribute to the normal stress acting on weak planes. This criterion suggests that the enhancement of strength in underground openings occurs because $\sigma_2$ along the tunnel cannot be ignored. However, the approach is semi-empirical, as the mean normal
stress acting on the plane includes $\sigma_2$ and $\sigma_3$, but does not address the angles $\beta$ and orientation $\omega$ - hence the difficulty in applying the approach to anisotropic rocks.

![Diagram](image)

**Figure 2.4.** Yoshinaka and Yamabe (1981) normalized linear plots (a) Penrhyn Slate normalized strength relation on linear scale; (b) Martinsburg Slate normalized strength relation on log scale.

### 2.10 Influence of intermediate stress in anisotropic rocks

There are many studies available on the subject of the influence of $\sigma_2$ for isotropic rocks (Handin et al., 1967; Mogi, 1967; Colmenares and Zoback, 2002; Al-Ajmi and Zimmerman, 2005, 2006; Haimson, 2009 and 2012). Through various experiments, Mogi (1967, 1973, and 1979) showed convincing evidence on the role of $\sigma_2$ on isotropic rock strength. The earliest works on true-triaxial experiments on anisotropic rocks were compiled by Kwaśniewski (1993) dating back to the late 1960s and 1970s. Unfortunately, these publications were available only in their native languages (Russian and Japanese). These earlier experimental works used biaxial compression machines ($\sigma_1 > \sigma_2 > \sigma_3 = 0$) for coal, slates, schist and limestone.

There are however, two important published researches on the role of $\sigma_2$ for anisotropic rocks. Jaeger (1964, p. 161) briefly described the true-triaxial JPW extended from Coulomb criterion. The same criterion was presented in Jaeger and Cook (1976), but remains untested due to the difficulty of conducting true-triaxial experiments for anisotropic rocks. Along with the true-triaxial JPW model, the Pariseau (1968) model also remains unverified for true-triaxial data, for the same reason.
The difficulty associated with preparing and testing natural anisotropic rocks led Reik and Zakas (1978) and Tiwari and Rao (2006, 2007) to conduct true-triaxial experiments using reconstituted anisotropic rocks. These samples, made of jointed blocks with specific bedding angle and direction, do not necessarily exhibit the response expected from natural rocks. The first published true-triaxial experiments on shales were for the Green River Shale (Smith and Cheatham, 1980a), but that study only considers bedding angle $\beta$ without input of foliation direction $\omega$, hence the limitations of these datasets.

Mogi (1979) conducted the most complete experiments on the true-triaxial response of anisotropic rocks for the Chichibu Schist. Mogi conducted forty-six true-triaxial and eighteen conventional triaxial experiments, with four sets of bedding angles $\beta$ and foliation directions $\omega$ (see Chapter 7, Figure 7.3 for further details on definition of $\beta$ and $\omega$). The Chichibu Schist is a macroscopically homogeneous green crystalline schist with dense foliation, originating from the Chichibu province, Honshu, Japan (Kwaśniewski and Mogi, 1990; Kwaśniewski, 2007). These experiments showed that the strength was influenced not only by the intermediate stress $\sigma_2$, the confining stress $\sigma_3$, and the bedding angle $\beta$, but also by the foliation direction, $\omega$. The definitions of angles $\beta$ and $\omega$ used by Mogi for the Chichibu Schist are as shown in Figure 2.5, wherein $\beta$ is the angle between the normal to the plane and the $\sigma_1$-direction and $\omega$ is the angle between the normal to the plane and the $\sigma_3$-direction. The foliation direction $\omega$ in this study will be referenced to $\sigma_2$ for ease of understanding and visualization.

![Figure 2.5. True-triaxial stress system with angle $\beta$ and $\omega$.](image)

For the Chichibu Schist, Mogi (2007; p. 173) used a simple model (Figure 2.6) to describe the effect of $\sigma_2$ on anisotropic rocks, based on the averaged response for an isotropic rock. To elaborate on this, Figure 2.6a shows the $\sigma_2$ effect strongly dependent on the orientation of the weak planes, whereas Figure 2.6b shows the $\sigma_2$ effect for isotropic rocks with intersecting small-scale oriented weak planes. Mogi explained that for isotropic rocks,
randomly distributed small-scale cracks or grain boundaries are present. The $\sigma_2$ effect in rocks containing many planes of weakness at some orientation may be represented by the average of the $\sigma_2$ effect of the three curves for anisotropic rock in Figure 2.6a. Figure 2.6b shows this average response for an isotropic rock with a $\sigma_2$ effect.

Although Figure 2.6 describes the influence of $\sigma_2$ on anisotropic rocks, Mogi did not suggest an analytical model to support this idea. This was probably because, a few years after experimenting on the Chichibu Schist, Mogi was assigned the task of leading the Japanese national earthquake prediction project from 1981 to his retirement in 2001, and was unable to continue his interest on true-triaxial failure criteria (Mogi, 2007; p. 186). Nevertheless, Mogi’s experimental results remain the most complete dataset available to help understand the influence of $\sigma_2$ on anisotropic rocks. This dataset is useful to validate true-triaxial models in the latter chapters of this study.

![Diagram](image)

Figure 2.6. Mogi (2007, p. 173) model describing the $\sigma_2$ effect for anisotropic and isotropic rocks. (a) Anisotropic rock with angle $\beta$ and orientations $\alpha_i$ (b) Isotropic rock with small-scale cracks or grain boundaries, randomly distributed at various orientations, that represent the averaged response.
3 Laboratory measurements

3.1 Introduction

Mechanical properties of natural rocks, and especially anisotropic rocks, are highly variable and not easily reproducible (Jaeger et al., 2007). Unlike manufactured materials, rocks vary significantly in texture and composition due to their mineralogy, geological history, and other natural processes. In the earlier stages of the development of rock mechanics, rock tests were conducted without sufficiently considering the complex behavior of the rock, hence leading to empirical formulations that lack physical or mathematical foundation, which should be avoided (Mogi, 2007). Therefore, to understand the mechanical properties of anisotropic rocks, careful laboratory measurements are necessary to evaluate this highly complex material.

To select suitable shales that would best represent the interest of this project, four organic-rich shales were tested from the Marcellus and Niobrara outcrops, and the Bossier and Vaca Muerta reservoir shales. Based on the interesting preliminary results from the latter reservoir shales, the Bossier and Vaca Muerta shales were then selected for further evaluation. The fact that the Bossier shale and Vaca Muerta shale were reservoir rocks, and not outcrops, also made these shales suitable for this study. A notable feature of these two shales is that the Bossier shale is highly laminated with organic-filled weak planes, whereas the Vaca Muerta shale has very high organic content that is dispersed throughout the shale matrix, but with poorer laminations. These different textural effects of the shale fabric led to the selection of these shales for further evaluation and experimentations.

In this chapter, the triaxial experimental procedure and measurements carried out for the Bossier shale and Vaca Muerta shale are described. To understand the mechanical properties of the shales tested, an introduction to the shales and their geology is provided. This is followed by descriptions of the sample preparation procedures and quality control measures that were used to ensure that the samples tested are of high quality and as uniform as possible, for the different test conditions applied. The triaxial compression and extension setup are also described with regards to equipment details and the stress-strain rates applied on the tested shale samples. Lastly, the main results presented in this study are the strength measurements for compression and extension tests at varying angles $\beta$ and confining stresses. To complement the strength data measurements, the elastic parameters from the triaxial experiments are also evaluated; this is done mainly to understand the deformation behavior and to determine if there is a relationship between strength and elastic parameters of anisotropic shales.

Traditionally, for the evaluation of anisotropic failure criterion, friction angle and cohesion are the most important mechanical properties obtained from laboratory experiments. These
strength parameters, however, may not be adequate to capture the heterogeneous nature of anisotropic rocks - particularly shales that are highly variable in composition and texture. For this reason, a complete range of measurements that includes axial and radial strain is carried out. The laboratory facility at TerraTek Schlumberger, Salt Lake City, USA, where these samples were tested, is also equipped with ultrasonic velocity (UV) measurement capabilities. However, although there are many empirical correlations relating UV derived mechanical properties for shales (Chang et al., 2006), the relationship between rock strength and elastic properties of shales still could not be properly established (Sone and Zoback, 2013b). As a result, it was decided that UV measurements would not be useful for the purpose of this study, and therefore were not included in these experiments.

In this study, two tight reservoir-quality shales from the Bossier and Vaca Muerta formations were tested at various orientations and confining stress levels using standard triaxial equipment. Most of the experiments were triaxial compression tests, although selected samples were also tested in triaxial extension. Despite the name “extension”, all test conditions are actually performed in compression. The traditional stress condition, $\sigma_1 > \sigma_2 = \sigma_3$, is applied for triaxial compression tests, whereas for the triaxial extension tests, the major principal stress is the confining pressure ($\sigma_2 = \sigma_3$) and the axial load is the minimum stress at failure ($\sigma_3$), i.e., $\sigma_1 = \sigma_2 > \sigma_3$. The purpose of the triaxial extension test is to understand the role of $\sigma_2$ on the strength of shales.

Previous experiments on anisotropic rocks were focused on extensive testing to understand the phenomenon of strength anisotropy (e.g., Donath, 1961; Chenever and Gatlin, 1965; Hoek, 1964; McLamore and Gray, 1967). The most important outcome of these experiments was the verification that isotropic models overestimate anisotropic rock strength by large amounts. Other more recent experiments focused on shales: Fjaer and Nas (2013), Ewy et al. (2010), and Islam et al. (2010) conducted tests on fewer samples, and verified the applicability of the JPW model. In most cases, for the past and recent experiments, reasonable agreements between models with experimental data are found. However, the latest trends in experiments for shales suggests that fewer samples are usually selected for testing, and this could be mainly due to sample availability or feasibility related issues. In the present study, to have a comprehensive evaluation of the true failure behavior of shales, optimum numbers of samples are selected that cover various orientation angles $\beta$ at multiple confining stresses. It is important that the experiments made are representative of different in-situ conditions, in order to yield a complete picture of the actual failure behavior.

Although various studies on anisotropic rocks are available from the literature, few of these studies focused on understanding the highly variable strength behavior of anisotropic shales. McLamore and Gray (1967) were the earliest researchers to make extensive compression tests, on Green River Shale, and also studied the shale failure modes. They concluded that the deformation structure types are controlled by the bedding angle $\beta$, weak
plane fabric, and confining pressure (Budd et al., 1967). However, these experiments were made over forty years ago, before high quality imaging of shales was possible. With currently available technology, the shale fabric mechanical behavior can be more accurately captured and evaluated, for the two different shale types, using CT (Computed Tomography) and thin section images.

Smith and Cheatham (1980a) made the first true-triaxial experiments on organic-rich shales. They conducted a series of true-triaxial experiments on the Green River Shales to assess the effect of $\sigma_2$ on the failure of anisotropic rocks, and evaluated the results by combining the JPW model for weak plane failure with a $J_2$-$I_1$ type relationship such as is usually used for isotropic rocks. Although the true-triaxial experimental results agree with the theoretical models, these experiments were not considered fully representative of true-triaxial conditions, as the sample orientation direction, $\omega$, was ignored. Mogi (1979) made a more representative true-triaxial experiment for anisotropic rock, considering $\omega$, these experiments are described in the later chapters of this study.

### 3.2 Bossier shale description

As described earlier, triaxial compression tests were conducted on two types of organic-rich shales, namely the Bossier shale and Vaca Muerta shale. The former is a North American shale, an argillaceous/calcareous organic-rich mudstone that lies above the Haynesville, in the Upper Jurassic and lower Cotton Valley formation, as shown in Figure 3.1 (Baker, 1995; Corley et al., 2011). This tight shale formation is a source rock with low permeability, and is rich with deposits of natural gas.

The Bossier shale (Mid Bossier argillaceous/calcareous facies) is described as highly layered and anisotropic (Figure 3.2 of the thin section image). The general composition of this argillaceous/calcareous mudstone is associated with a high authigenic calcite content of 20% by wt., with a clay-rich matrix that constitutes 40% by wt. The most obvious feature of the Bossier shale is its strong lamination, with preferential alignment of organics filling the bedding planes (i.e., planes of weakness), with carbonate cement dispersed throughout the rock matrix. The Bossier shale has moderate organic content, with reported total organic content (TOC) percentage by wt. of 1.62% (Suarez-Rivera and Fjaer, 2012). There were also moderate amounts of detrital quartz and feldspar grains, ranging from very fine silt to very fine sand present in the matrix. The clay-rich matrix consists of illite and mixed-layer illite-smectite, with varying amounts of chlorite.
### Figure 3.1. Louisiana Geological Society stratigraphic columns (from Corley et al., 2011).

![Stratigraphic columns](image)

### Figure 3.2. Thin section image of the Bossier shales.

![Thin section image](image)

#### 3.3 Vaca Muerta shale description

The second organic-rich shale tested in this study is the Vaca Muerta shale. The Vaca Muerta shale is a high-potential formation for shale oil and gas, located in the Neuquén basin, Argentina (Figure 3.3). The Tithonian Vaca Muerta shale (Figure 3.4 of the thin section image), which varies in thickness from 100 m to 450 m, holds significant reserves, and is potentially the largest oil shale field in the world (Monti et al., 2013).

This tight dark organic-rich source rock has an average TOC of 2.5-3.5% (Glorioso and Rattia, 2012), in some cases reaching up to 10-12% (Monreal et al., 2009). In comparison to the Bossier shale, the Vaca Muerta shale does not show obvious weak planes.
The Vaca Muerta shale shows poor lamination, due to bioturbation. This dark color shale matrix is predominantly calcareous, with moderate to high organic content. Thin section images show nondescript fecal pellets, charophyte spores and calcified algal material (Figure 3.4 of thin section image). This calcareous mudstone has low detrital silts or sand content within the matrix.
3.4 Sample preparations and quality control

The laboratory experiments in this study are designed to determine a suitable failure criterion that accounts for the angle $\beta$ between the planes of weakness and the maximum principal stress, and for the lateral confining stress. One important aspect of conducting experimental work is sample preparation and quality control. This is especially challenging for shales, which generally contain small-scale heterogeneity, whereas it is desired that each experimental sample is sufficiently homogeneous. Upon identifying a suitable core interval, sample preparation involves cutting plugs to the desired dimensions with a 2:1 ratio, at bedding angles (Figure 3.5) $\beta$ ranging from $0^\circ$ to $90^\circ$. Identifying the actual bedding angle $\beta$ is not straightforward, because natural sample bedding planes undulate, and a mean representative angle is verified using multiple measurements of physical samples and CT images.

Preliminary study and experiments are necessary in selecting high quality samples. Evaluations using scratch test and wireline logs give estimates of suitable shale zones to be tested. Triaxial compression tests of candidate samples tested at bedding angles of $\beta = 0^\circ$, $45^\circ$ and $90^\circ$ also provide basic mechanical properties to further scrutinize the suitability of the shale sample. These preliminary tests are useful to select suitable samples for overall triaxial experiments.

For the Bossier shale samples, a smaller sample of 0.75" (W) × 1.5" (H) was preferred, because of the highly laminated and fragile nature of this fissile mudstone. However, for the Vaca Muerta shale, the samples were much more competent, allowing for larger plugs of 1" (W) × 2" (H) to be prepared for the triaxial experiments. The plugs were then trimmed before the sample bulk density was measured. Samples whose densities were not within
±0.05 g/cc of the mean were considered outliers, and were removed from the batch. This ensures that the samples tested are reasonably representative, and of high quality.

During the sample preparation process, shale cores with significant microcracks are treated with low viscosity epoxy externally, vacuum suctioned to fill the external microcracks, and cured. TerraTek Schlumberger has done separate studies to verify that the treatment using epoxy does not alter the experiments, and is only used to ensure that the unstable shales are held together while being cut into plugs from the core.

3.5 Triaxial compression and extension setup

Triaxial compression and extension tests for the Bossier and Vaca Muerta shales were conducted at various bedding angles ($\beta$) and confining stress levels, to evaluate the applicability of the JPW and Pariseau models. Figure 3.5 shows the sample arrangement for the triaxial compression and extension experiments, wherein the angle $\beta$ is defined as the angle between the normal of the weak plane to the direction of the major principal stress, $\sigma_1$.

![Figure 3.5. Sample orientation and setup for triaxial compression and extension tests.](image)

The basic setup of the triaxial equipment is shown in Figure 3.6. The triaxial experiments procedure starts by filling the vessel with oil at a rate of 5 psi/sec, and increasing the cell pressure to the desired confinement. After reaching the confinement target, the sample is left for five minutes until all the pressure and strain measurements stabilize, and then axial loading is commenced. Axial load is applied at strain rate of $10^{-5}$/s, until sample failure occurs.
3.6 Measurements and experimental results

The triaxial experiments described above are further discussed below using example experimental data. From the triaxial experiments, the basic information obtained are stress and strain as a function of time. Using this information, stress-strain data plots are generated throughout the loading process, until the point where sample failure occurs. After sample failure, post-test residual strength data are of less interest in relation to peak strength behavior, and therefore not evaluated in this study.

An example of a typical experiment using the triaxial equipment is shown in Figure 3.7. Figure 3.7a shows that the sample is initially stressed hydrostatically to 6,000 psi. A pressure cycle from 6,000 psi to 9,000 psi is then applied, and the pressure is held at the final confining pressure before axial load is applied. At the end of the pressure cycle phase, the confining pressure is held constant for a few minutes, until all pressure and strain gauges stabilize, and then the axial load is applied. The axial load applied is increased at a steady strain rate of $1 \times 10^{-5}$ in/in per sec until the sample fails. The applied axial load is measured by the load cell located below the bottom end cap (Figure 3.6). Sample failure is followed by a sudden drop in axial load, resulting in a brittle abrupt failure, and is usually followed by a loud burst, pop or crack sound. This type II brittle rock failure (Fairhurst and Hudson, 1999) was evident for all the shales tested in this study. The only exceptions were for samples tested at high confining pressure, wherein ductile failure was observed, at higher strains compared to the samples tested at lower confining stresses.
The stress-strain response to axial loading is shown in Figure 3.7b. When axial load is applied, the sample strains axially, $\varepsilon_a$, causing the sample to shorten under axial compression. This axial stress-strain response to compressive loading is taken as positive, increasing almost linearly before the onset of nonlinear behaviour, which indicates pre-yielding, followed by erratic fluctuations in stress and strain, and finally yielding - also described as peak strength in this study. The failure of the sample occurs at peak stress, and the total axial stress acting on the sample at this point is referred to as the “rock strength”, $\sigma_1$. Although stress difference ($\sigma_2 - \sigma_3$) causes failure rather than just $\sigma_1$, in this study, rock strength is presented in terms of $\sigma_1$ rather than $\sigma_2 - \sigma_3$. For the triaxial extension test, similar peak strength behaviour is observed, but the increase in the major principal stress $\sigma_1$ that is applied laterally results in lengthening of the sample under applied confining pressure $\sigma_1$, before failure occurs at $\sigma_3$, which is the minor principal stress applied axially. The lengthening of the sample axially also results in reduced sample diameter.

Similar to the axial stress-strain response, the lateral strains due to axial loading are also measured using strain gauges. Four lateral strain gauges are used to measure the average strain parallel and perpendicular to the sample; see radial strain measurements on the graph in Figure 3.7b). The radial strain gauges ET1 and ET2 (shown in Figure 3.8) are placed perpendicular and parallel to the inclined sample. For a horizontal sample (Figure 3.8) the arrangement of the strain gauge is opposite to that of the inclined sample. For the triaxial compression tests, since axial strain response is taken as positive (shortening), the lateral strain response is taken as negative (fattening), resulting in radial stress-strain curves ET1 and ET2 increasing to the left of the plot. The volumetric stress-strain curve in Figure 3.7a shows the volumetric strain response, $\varepsilon_{vol}$ which is the sum of the axial and two radial strains ($\varepsilon_{vol} = \varepsilon_a + ET1 + ET2$). For horizontal samples the strain gauge ET1 and ET2 positions...
are opposite to the deviated samples, and this is an important point that needs to be considered for interpretation of experimental data. The reason for this swapped position is due to laboratory experimental procedures that are not covered in this study. The positions of the radial strain gauges ET1 and ET2 in relation to sample bedding orientation $\beta$ is shown in Figure 3.8 for an oriented sample at angle $0^\circ < \beta < 90^\circ$, and a horizontal sample at $\beta = 90^\circ$.

The stress-strain response curves described above are the direct measurements that are obtained from the triaxial experiments. To determine the apparent Young’s modulus of the sample, $E_z$, the linear elastic range of the axial stress-strain curve is determined at approximately 40-50% of the yield stress and within a 0.1% (0.001 strain window in Figure 3.7b) strain window. The ISRM Suggested Method (1981) proposes this linear elastic range to be approximately 50% of the ultimate yield stress, and the selection of this linear elastic range needs to be applied consistently. However, the linear elastic range may vary for unconfined tests, and different types of shales, and so identifying this elastic region is often challenging. The reason for this is that the linear elastic range occurs after the microcracks have closed (nonlinear early stage of loading), and only after that do shales become elastic. After further increased loading, the sample begins to yield, and ultimately failure occurs.

Although elastic parameters were acquired from the laboratory experiments, these parameters were not used for strength model evaluation. Therefore, for the Bossier shale and Vaca Muerta shale triaxial experiments, strength data are presented and discussed in detail, whereas the measured elastic moduli for both shales are evaluated as an experimental quality control measure, and also to determine if there are any possible correlation between strength and elastic moduli parameters.

![Figure 3.8. Strain gauge placements for oriented and horizontal sample.](image-url)
3.6.1 Strength of Bossier shale and Vaca Muerta shale

As described earlier, the strength measurements for the Bossier shale and Vaca Muerta shale are analyzed in detail in this study. For the Bossier shale samples, thirty-six triaxial compression tests were conducted for samples with bedding angles ranging from $\beta = 0^\circ$ to $90^\circ$, at confining stresses of 0 psi, 1,000 psi, 3,000 psi, 6,000 psi and 10,000 psi (Figure 3.9). Each data point represents a compression test and all samples showed increased strength at higher confining stress. The presented strength data were curve fit using polynomial least squares for each suites of $\sigma_3$ and not fit to any specific model. Overall, the compressive strength response shows a smooth change in strength as a function of $\beta$. The general strength response shows maximum strengths at angles of $\beta = 0^\circ$ and $90^\circ$, with the minimum strength occurring at about $60^\circ$. The only exception is for the case of zero confining stress (UCS). Shear strength under unconfined conditions showed a significant difference between $0^\circ$ and $90^\circ$, with the strength at $0^\circ$ being more than three times the strength at $90^\circ$. The reason for this is that, at $\beta = 0^\circ$, the sample failed by shear in the sample matrix, resulting in higher shear strength, whereas at $\beta = 90^\circ$ failure occurred due to tensile splitting. At these angles, at higher confining stresses of 1,000-10,000 psi, failure occurs predominantly by shear, and so the strength response under unconfined conditions does not have the same profile as for samples under non-zero confining stress.

For the Bossier shale samples tested at confining stresses of 1,000-10,000 psi, the strengths at angles of $0^\circ$ and $90^\circ$ are not equal, with slightly higher strengths observed at $\beta = 90^\circ$. This strength difference implies that the JPW model assumption of equal strengths at $\beta = 0^\circ$ and $90^\circ$ needs reevaluation. Further explanation for this could possibly be that the weak plane properties may affect the intact rock shear strength at $\beta = 0^\circ$ and $90^\circ$ in a way that is different to that assumed in the JPW model. The issue of strength prediction and failure modes needs further investigation using CT scans and thin section images; this work is described in Chapter 4 on fabric analysis. The complete data set for Bossier shale is contained in Appendix A1.

For the Vaca Muerta shale (Figure 3.10), twenty-one samples were tested at confining stresses of 0, 1000 psi, 2,500 psi, 5000 psi, and 20,000 psi. This data was also curve fit using polynomial least squares for each suites of $\sigma_3$ and not fit to any specific model. Most of the samples were tested at a confining stress of 2,500 psi to obtain the full failure curve as a function of $\beta$, whereas the three compressive strength tests conducted at 20,000 psi confining pressure were conducted to investigate the strength at very high confining stresses. The plots of the Vaca Muerta shear strengths $\sigma_1$ versus angle $\beta$ in Figure 3.10 show lower strength anisotropy than do the Bossier shale. All of the Vaca Muerta samples show increased strengths at higher confining stresses, with maximum strengths at $\beta = 0^\circ$ and $90^\circ$, whereas the lowest strength occurred at $\beta = 60^\circ$. The strength response for Vaca Muerta samples at angles $10^\circ < \beta < 40^\circ$ does not show a significant change in strength from the value at $\beta = 0^\circ$, which is different from the Bossier shale, which showed a sharper variation.
Figure 3.9. Bossier shale shear strength, $\sigma_1$ versus bedding angle $\beta$.

Figure 3.10. Vaca Muerta shale shear strength, $\sigma_1$ versus bedding angle $\beta$. 
in strength with angle $\beta$. One sample showed an unexpectedly higher strength at a confining stress of 5000 psi and an angle of $\beta = 80^\circ$; this was likely due to sample heterogeneity. The complete data set of Vaca Muerta shale is contained in Appendix A2. Comparing the strength response at $\beta = 0^\circ$ and $90^\circ$ shows that the shear strengths $\sigma_1$ at these angles are almost the same for the different confining stresses. There was no compression test conducted at angle $\beta = 90^\circ$ to verify if there are any tensile splitting issues, but the samples at 1,000 psi confining stress also does not show any tensile splitting, with similar values of $\sigma_1$ at $\beta = 0^\circ$ and $90^\circ$. Based on these results, the Vaca Muerta shale strength response is more consistent with the JPW model, and shows different strength behavior from the Bossier shale.

To understand the role of $\sigma_2$ for failure along weak planes, the Bossier shale triaxial extension ($\sigma_1 = \sigma_2 > \sigma_3$) strengths are compared to triaxial compression ($\sigma_1 > \sigma_2 = \sigma_3$) strengths. Five samples at angles $\beta = 30^\circ$, $45^\circ$ and $60^\circ$ are tested in extension, at two confining stress levels, whereby for triaxial extension tests the confining stress is the maximum principal stress $\sigma_1 = \sigma_2$ and the axial load is the minor principal stress $\sigma_3$ at failure.

For these extension tests, the maximum allowable cell pressure of 25,000 psi places a limit on the confining stress that can be applied. The lower limit cannot be too low, as the extension cell setup is designed for axial compressive load at failure, and is not designed for tensile failure. Five Bossier shale samples were tested in triaxial extension to failure. Extension tests for weak plane failure were made for samples with different angle $\beta$ and confining pressure $\sigma_1$. Figure 3.11a to 3.11c shows the stresses $\sigma_1$ versus $\sigma_3$ at sample failure with angles $\beta = 30^\circ$, $45^\circ$ and $60^\circ$.

Figure 3.11a shows the triaxial extension test made at angle $\beta = 30^\circ$ and confining pressure $\sigma_1 = 25,000$ psi, wherein the sample failure occurred at $\sigma_3 = 7,085$ psi. On the same plot, the compression test strengths $\sigma_1$ at angle $\beta = 30^\circ$ for varying confining pressure $\sigma_3$ show an almost linear strength behavior, increasing with confining pressure. The extension strength compared against the compression strengths plots very close to the linear compression line. This means that the triaxial extension test at this stress condition and angle $\beta$ is not significantly different from the strength of the sample in compression tests.

For the Bossier shale at $\beta = 45^\circ$, two triaxial extension tests were made (Figure 3.11b). The extension tests were made at high confining pressure $\sigma_1 = 25,000$ psi which failed at an axial load of $\sigma_3 = 7,661$ psi whereas the sample with low confining pressure $\sigma_1 = 8,020$ psi failed at an axial load of $\sigma_3 = 228$ psi. On the same graph, compression tests at angles $\beta = 45^\circ$ for confining pressures $\sigma_3 = 0$ to 10,000 psi show a linear relationship between strength $\sigma_1$ and confining pressure $\sigma_3$. Comparing the two triaxial extension test data points to the compression test, the extension test at $\sigma_3 = 7,661$ psi lies slightly below the compression line, whereas the test at $\sigma_3 = 228$ psi lies slightly above the compression line. Although the triaxial extension data points do not fall exactly on the compression line, the extension test results are only slightly different from the compression line. This shows that for the sample
Figure 3.11a. Bossier shale compression and extension plot ($\sigma_1$ vs. $\sigma_3$) at angle $\beta = 30^\circ$.

Figure 3.11b. Bossier shale compression and extension plots ($\sigma_1$ vs. $\sigma_3$) at angle $\beta = 45^\circ$.

Figure 3.11c. Bossier shale compression and extension plot ($\sigma_1$ vs. $\sigma_3$) at angle $\beta = 60^\circ$. 
with a bedding angle $\beta = 45^\circ$, triaxial extension and compression strengths are not significantly different.

Lastly, two Bossier shale extension tests were conducted for samples at an angle $\beta = 60^\circ$ shown in Figure 3.11c. The sample tested at higher confining pressure $\sigma_1$ of 25,000 psi failed at $\sigma_3 = 7,190$ psi, whereas the sample at a lower confining pressure of $\sigma_1 = 13,320$ psi failed at $\sigma_3 = 2,610$ psi. In the same Figure 3.11c, the triaxial compression tests conducted at confining pressures of $\sigma_3 = 0$ to 10,000 psi show a linear relationship between $\sigma_1$ and $\sigma_3$, similar to the earlier experiments at $\beta = 30^\circ$ and $45^\circ$. Comparing the extension test data points to the linear compression line, the extension result at $\sigma_3 = 7,190$ psi lies on the compression line, whereas for the case of $\sigma_3 = 2,610$, the data lies slightly above the compression line. The extension stresses at $\sigma_3 = 2,610$ psi also lie very close to the compression data point at $\sigma_3 = 3,000$ psi. In summary, the Bossier shale sample stresses at failure for compression and extension tests at $\beta = 60^\circ$ are essentially the same, without significant differences.

Overall, for the Bossier shale, comparing the triaxial extension strengths ($\sigma_1 = \sigma_2 > \sigma_3$) against triaxial compression strengths ($\sigma_1 > \sigma_2 = \sigma_3$), no significant difference in the stress plots are seen for the samples tested at $\beta = 30^\circ$, $45^\circ$ and $60^\circ$. This means that for the Bossier shale, for failure along the weak plane at angles $\beta = 30^\circ$, $45^\circ$ and $60^\circ$, the role of $\sigma_2$ is not significant. This result, however, cannot be used for the case of intact rock fracture (i.e., $\beta = 0^\circ$ and $90^\circ$). To determine the role of $\sigma_2$ for intact rock, extension tests for samples with intact rock failure such as at $\beta = 0^\circ$ need to be made separately.

For the Vaca Muerta shale, only two triaxial extension tests were conducted. The first Vaca Muerta shale extension test was carried out at $\beta = 0^\circ$ to determine if the intact rock strength under extension test conditions ($\sigma_1 = \sigma_2 > \sigma_3$) is different from intact rock strength under compression test conditions ($\sigma_1 > \sigma_2 = \sigma_3$). As the stresses required for intact rock failure is higher than for weak plane failure, the maximum cell pressure of 25,000 psi ($\sigma_1 = \sigma_2$) was applied, whereby the sample failed through the intact rock at $\sigma_3 = 1,161$ psi. Figure 3.12a shows the compression and extension test strengths for samples with intact rock failure. At first glance, the extension and compression strength may look close enough. However, a closer evaluation reveals that the extension stress ($\sigma_1 = \sigma_2$) is approximately 5,000 psi higher than the compressive strength $\sigma_1$ at confining stress of $\sigma_2 = \sigma_3 = 1,000$ psi. This means that for the Vaca Muerta shale with intact rock failure, the role of $\sigma_2$ cannot be ignored, and will be important in strength evaluation of shales.

The second Vaca Muerta extension test is made for failure along weak plane at $\beta = 60^\circ$ to verify the observations from the extension tests for the Bossier shale. Since the weak plane strength is expected to be less than the intact rock strength, a slightly lower confining pressure of $\sigma_1 = \sigma_2 = 22,600$ psi was applied, under which the sample failed along the weak
plane at $\sigma_3 = 3,109$ psi. The triaxial compression and extension strengths at failure are shown in Figure 3.12b, wherein the extension strength is slightly higher than the linear compressive strength line. The difference between the compression and extension strengths is, however, not as significant as that observed for the intact rock fracture shown in Figure 3.12a. The strength difference between compression and extension at $\beta = 60^\circ$ is comparable to that seen for the Bossier shale in Figure 3.11c, but less obvious, because only one extension test datum point is available. Based on the extension and compression data shown in Figure 3.12b, for the Vaca Muerta shale extension tests at $\beta = 60^\circ$, the role of $\sigma_2$ is less significant than that seen for intact rock fracture described earlier and shown in Figure 3.12a.

**Figure 3.12a.** Vaca Muerta shale compression and extension plot ($\sigma_1$ vs. $\sigma_3$) at $\beta = 0^\circ$.

**Figure 3.12b.** Vaca Muerta shale compression and extension plot ($\sigma_1$ vs. $\sigma_3$) at $\beta = 60^\circ$.
3.6.2 Elastic moduli of Bossier shale and Vaca Muerta shale

Intact anisotropic rock is stress dependent, with reduced anisotropy at increased confinement. As stresses increase, specifically the compression applied on the weak plane, discontinuities become stiffer due to the closing of microcracks along the weak planes, resulting in reduced anisotropy, nonlinear behavior and anisotropic pressure dependency (Amadei, 1996). For anisotropic rocks, Hooke’s law of elasticity for anisotropic media shows that the rock has at most twenty-one independent elastic components. As most anisotropic rocks are layered, the elastic parameters reduce to nine for orthotropic conditions, whereas for transverse isotropy the number of independent elastic parameters is only five.

The five independent parameters needed to describe the deformability of transversely isotropic rocks are the vertical Young’s Modulus ($E_v$), horizontal Young’s Modulus ($E_h$), vertical Poisson’s Ratio ($\nu_v$), horizontal Poisson’s Ratio ($\nu_h$) and vertical shear modulus ($G_v$). The theoretical elastic moduli equations mentioned above are described by Amadei (1983), Amadei (1996) and Choo et al. (2012).

Although the focus of this study is on strength behavior, the elastic parameters that were determined in this study serve several purposes. The first and most important purpose is as a measure of quality control of the experiments, which is used to verify that the shales are transversely isotropic, and that the measurements are consistent with theoretical predictions. The elastic parameters determined are also useful to distinguish the deformation behavior of two very different types of shales (highly anisotropic and low anisotropy) and to determine the level of elastic anisotropy compared to strength anisotropy. It is also interesting to determine if there are any usable correlations that could be established between strength and elastic parameters for transversely isotropic rocks, as such relationships are commonly used in the industry for isotropic rocks.

The compression test experiments on the Bossier and Vaca Muerta shales not only provide strength parameters, but also elastic moduli properties of the rock. Figure 3.13a shows the Young’s modulus, $E_z$, which is the slope of the measured axial stress versus axial strain (Figure 3.7b). Figure 3.13a shows that the Bossier shale $E_z$ data points for confining stresses from 0 to 10,000 psi generally increases from $\beta = 0^\circ$ to $90^\circ$ with an S-shape, consistent with the theoretical predictions. The theoretical curve $E_z$, calculated from the average elastic moduli parameters (values of average $E_v$, $E_h$, $\nu_v$, $\nu_h$ and $G_v$ for Bossier shale are listed in Appendix A1), also shows a smooth S-shape increasing from $\beta = 0^\circ$ to $90^\circ$. The $E_z$ data in Figure 3.13a show some scatter for the samples between $\beta = 10^\circ$ and $20^\circ$, which can be attributed to slight sample heterogeneity, as these samples were taken at lower depths of the core. This heterogeneity effect observed in $E_z$ is not seen in the strength behavior in Figure 3.9, because $E_z$ is more sensitive to changes in the rock fabric than is the strength; hence, strength at $\beta = 10^\circ$ and $20^\circ$ shows a smooth variation without any abrupt change.
The Bossier shale Young’s modulus $E_z$ values in Figure 3.13a show low values at $\beta = 0^\circ$ and the highest value at $\beta = 90^\circ$, with an $E_h/E_v$ ratio of approximately 2.9. The vertical Young’s modulus, $E_v$, at $\beta = 0^\circ$ shows values ranging from $E_v = 2.2 \times 10^6$ psi to $2.6 \times 10^6$ psi, whereas at $\beta = 90^\circ$ a larger scatter is observed. At UCS, $E_h$ is only about $4.3 \times 10^6$ psi, whereas at confining stresses of 1,000 psi to 10,000 psi, the $E_h$ values vary from $6.7 \times 10^6$ psi to $8.2 \times 10^6$ psi, generally showing increasing $E_h$ with confining pressure. However, the vertical Young’s modulus $E_v$, seems to be unaffected by this pressure effect. Excepting the measurements from $\beta = 10^\circ$ and $20^\circ$, $E_z$ seems to increase at higher confining pressure.

The Vaca Muerta compression data analysis with the apparent Young’s modulus $E_z$ is shown in Figure 3.13b. The $E_z$ data show less scatter, compared to the Bossier shale, with smoothly varying values with angle $\beta$. From the average elastic moduli parameters ($E_h$, $E_v$, $v_{hv}$, $v_v$ and $G_v$), the theoretical $E_z$ curve is calculated, and plotted in Figure 3.13b, showing a smooth S-shape that varies with angle $\beta$. Comparing the theoretical curve to the data, both the curve fit and data have the same S-shape. The theoretical curve at angles $\beta = 0^\circ$ and $90^\circ$ lies in the middle of the $E_z$ data, whereas at angles $0^\circ < \beta < 90^\circ$ the data are slightly lower than the theoretical curve. The $E_z$ data in Figure 3.13b indicate some dependence on confining pressure, as $E_z$ at 2,500 psi is lower than that at 5,000 psi or 20,000 psi. No clear trend is observed for the UCS data, because of limited data points, whereas $E_z$ at $\beta = 60^\circ$ at a confining stress of 20,000 psi was also lower than expected. Comparing $E_v$ at $\beta = 0^\circ$ and $E_h$ at $\beta = 90^\circ$, $E_v$ shows slight scatter with no confining pressure dependence, whereas $E_h$ shows a larger scatter with some confining pressure dependence. For $E_z$ at $\beta = 90^\circ$ ($E_h$), the highest value is $4.44 \times 10^6$ psi for 20,000 psi confining pressure, and the lowest is $3.52 \times 10^6$ psi at 5,000 psi confining pressure. The $E_z$ values at confining pressures 1,000 psi and 2,500 psi lie in between, but should theoretically be lower than $E_z$ at 5,000 psi. The average $E_h$ and $E_v$ values for the Vaca Muerta shale are $3.88 \times 10^6$ psi and $2.45 \times 10^6$ psi respectively, with an $E_h/E_v$ ratio of 1.58, which is lower than the $E_h/E_v$ ratio for Bossier shale.

Poisson’s ratio for shale can be determined from the axial and radial strain measurements from the triaxial compression experiments. The Poisson’s ratios in the Z-Y plane, $v_{zy}$, and Z-X plane, $v_{zx}$, are shown in Figures 3.14 and 3.15, for the Bossier shale and Vaca Muerta shale, respectively (Appendix A1 and A2 for sample reference coordinates of $X$, $Y$ and $Z$). For the Bossier shale, the $v_{zy}$ values in Figure 3.14a show higher scatter compared to the $v_{zx}$ values in Figure 3.15a, with the most scatter in $v_{zy}$ observed for the samples tested at 1,000 psi confining stress. This scatter may be due to low confinement, and for this reason, the $v_{zy}$ and $v_{zx}$ for UCS data are not presented in Figures 3.14a and 3.15a. The $v_{zy}$ and $v_{zx}$ values at confining stresses of 3,000 psi and 10,000 psi show a smooth change from bedding angle $\beta = 0^\circ$ to $90^\circ$. 

43
Figure 3.13. (a) Bossier shale Young’s Modulus, $E_z$; (b) Vaca Muerta shale Young’s Modulus, $E_z$.

Figure 3.14. (a) Bossier shale Poisson’s ratio, $v_{zy}$; (b) Vaca Muerta shale Poisson’s ratio, $v_{zy}$.

Figure 3.15. (a) Bossier shale Poisson’s ratio, $v_{zx}$; (b) Vaca Muerta shale Poisson’s ratio, $v_{zx}$. 
Comparing the Bossier shale $v_{xy}$ and $v_{xz}$ at $\beta = 0^\circ$, $v_{xy}$ shows higher values at 10,000 psi confining stress, whereas at all other confining stress, $v_{xy}$ and $v_{xz}$ does not show a clear increase with confining pressure. At $\beta = 90^\circ$, both $v_{xy}$ and $v_{xz}$ show increased values at higher confining stress, similar to $E_z$. In between these two angles at $0^\circ < \beta < 90^\circ$, $v_{xy}$ is higher than $v_{xz}$, and this is expected, as at these angles the rock strains along the weak planes while $v_{xz}$ is the secondary deformation that occurs perpendicular to $v_{xy}$. Comparing the $v_{xy}$ and $v_{xz}$ values at these angles, the general trend shows increasing values at higher angle $\beta$ with an S-shape trend. However, for $v_{xy}$, an initial increase occurs before slightly dipping when approaching $\beta = 90^\circ$. Similar observation was also made on data presented by Amadei (1983, Figure 2.4, p. 32) and Hakala et al. (2006) wherein the $v_{xy}$ values at $\beta = 90^\circ$ is lower than $\beta = 75^\circ$. Overall, the average $v_h/v_v$ ratio is approximately 2.3, and this value is close to the argillaceous shales presented by Suarez-Rivera et al. (2011).

The Vaca Muerta Poisson’s ratio plots shown in Figures 3.14b and 3.15b are not as straightforward to interpret as those for the Bossier shale. The Poisson’s ratio $v_{xy}$ (Figure 3.14b) and $v_{xz}$ (Figure 3.15b) shows some scatter in the data, similar to that seen for the Bossier shale. Comparing $v_{xy}$ and $v_{xz}$ at $\beta = 0^\circ$, the average vertical Poisson’s ratio $v_v$ shows only a slight variation at different confining stress, with an average value of 0.1748. At $\beta = 90^\circ$, $v_{xy}$ shows lesser scatter due to confining pressure, whereas $v_{xz}$ shows some scatter, whereby $v_{xz}$ decreases with confining pressure. At angles $0^\circ < \beta < 90^\circ$, $v_{xz}$ also show trends of decreasing value at higher confining stress, whereas $v_{xy}$ shows similar trends, but is less affected by confining pressure. The $v_{xy}$ and $v_{xz}$ values suggest that there is a general trend of reducing anisotropy with increased stress. Amadei (1996) explained that stress dependency on rock anisotropy should ideally reduce with increased stress, because pores and microcracks close with increased stress. However, the $v_{xy}$ and $v_{xz}$ plots in Figures 3.14b and 3.15b show some mixed results that are not easily attributable to being caused by confining pressure. Several issues may have contributed to this. Referring back at the strength data plot in Figure 3.10, from angles $0^\circ < \beta < 50^\circ$ and $80^\circ < \beta < 90^\circ$, the samples failed predominantly as an isotropic rock, and not along the weak planes. This means that, at these angles, the poorly laminated Vaca Muerta shales could have yielded “isotropically”, and not as an anisotropic material, with limited yielding along the weak planes. The competition between isotropic and anisotropic elastic behavior could have led to the scatter observed for $v_{xy}$ and $v_{xz}$.

The interpretation of $v_{xy}$ and $v_{xz}$ for Vaca Muerta shale is also made difficult by the fact that majority of the samples at angles $0^\circ < \beta < 50^\circ$ were tested at 2,500 psi confining stress, and these samples were taken at different sample depths, slightly deeper than the previous sample, in the later stages of the compression tests. This could have introduced slight sample heterogeneities, leading to scatter of the $v_{xy}$ and $v_{xz}$ values at these angles, hence affecting these extremely sensitive parameters. Excluding the 2,500 psi samples, there are still insufficient data points to see a clear trend for pressure effect on the $v_{xy}$ and $v_{xz}$ plots in
Figures 3.14b and 3.15b. If the data points at confining stress of 2,500 psi are excluded due to possible sample heterogeneity, the $v_{zy}$ and $v_{zx}$ values at 5,000 psi and 20,000 psi confining stress show some reasonable dependence on confining stress. The data at 1,000 psi confining stress could also be excluded, because microcracks may not have fully closed at this relatively low confining stress level. The Poisson’s ratios $v_{zy}$ and $v_{zx}$ show large scatter, due to some sample heterogeneities, and possible competition of isotropic-anisotropic elastic behavior. Overall, for the Vaca Muerta shale, Young’s Modulus $E_z$ is more consistent with the theoretical predictions, with $E_h/E_v = 1.58$ and $v_h/v_v = 1.38$, which are lower elastic anisotropy ratios compared to the Bossier shale.

In summary, the Bossier shale and Vaca Muerta shale elastic parameters $E_z$, $v_{zy}$ and $v_{zx}$ measured from laboratory experiments show reasonable agreement with theoretical predictions, displaying an S-shape trend when plotted versus the angle $\beta$. The Bossier shale shows higher elastic anisotropy ($E_h/E_v$ and $v_h/v_v$) compared to the Vaca Muerta shale. This trend is similar to the strength anisotropy behavior, whereby the Bossier shale showed higher strength anisotropy compared to the Vaca Muerta shale. Despite the consistent trends observed, attempts to find any immediate correlation between the strength anisotropy and elastic anisotropy parameters were unsuccessful. This could mean that the elastic behavior and strength of shales are independent and not directly related.


4 Fabric Analysis

4.1 Petrology

Earth scientists and seismologists are interested in the study of rock fabric to understand the importance of preferred orientation of crystals in the deformation of the Earth’s crust (Wenk and Houtte, 2004). The term “fabric” generally refers to the arrangement of the organic and mineral constituents of the rock matrix (Knopf, 1957). Brace (1965) provided another definition for rock fabric as being the geometrical arrangement and relative orientation of grains in a rock. However, for shales, Suarez-Rivera et al. (2013) provide a more specific definition by defining shale fabric as the presence of orientation and distribution of bed boundaries, lithological contacts, mineralized fractures, and any inherent weak planes, which create discontinuities that affect the physical behavior of the rock.

The study of rock fabric, also known as petrofabrics (Jaeger et al., 2007), can be carried out at micro and macroscales. Features of the rock fabric such as bedding planes, laminations, or preferential alignment of minerals due to depositional environment are important in describing the mechanical properties of anisotropic rocks. In the present study, analysis of the shale fabric, focusing on textural effects (i.e., weak planes) using computerized tomography (CT) and thin-section images, is compared for pre-test and post-test samples to understand the actual failure modes of shales at varying bedding angles $\beta$ and confining stresses. Further evaluations of the shale fabric were also made, whereby thin sections from post-test samples were analyzed under a microscope using a polarizing microscope equipped with a digital camera.

For isotropic rock, small scale mineral-filled layers may not influence the strength behavior of the rock. However, for anisotropic rocks, these layers are more significant and the minerals deposited along these layers alter the mechanical behavior of the rock. The effect of layers and their mineral composition are commonly also referred to as textural and compositional effects. For shales, the organization of minerals within the weak planes may not be uniform. This is because the formation of the minerals within the weak planes depends on the sedimentary processes. For shales, detrital minerals are more uniformly deposited in the weak planes compared to authigenic minerals that are formed later.

Two types of shales are evaluated in this study. The first is the Bossier shale, which is highly laminated, and the second is the Vaca Muerta shale, which has poor laminations. Before evaluating the shale fabric images, petrographic analysis of the mudstones provides a useful understanding of the shales tested. In Chapter 3, the general descriptions of the Bossier shale and Vaca Muerta shale was provided as an introduction of the shale samples tested. This general description of the shales is now supplemented with petrographic analysis, allowing for a more complete understanding of the shale fabric behavior. In general,
A petrographic study can be described as analysis of rock types based on pore-scale microscopic evaluation, distinguishing texture, composition, mineralogy, etc., from the micro images (Rushing et al., 2008). Table 4.1 shows the petrographic summary of both the Bossier and Vaca Muerta shales tested in this study. Mechanical properties such as elastic and strength anisotropy are also included in the table to relate the petrographic properties with engineering behavior of the shales.

Table 4.1. Summary of petrographic analysis for Bossier shale and Vaca Muerta shale.

<table>
<thead>
<tr>
<th></th>
<th>Bossier shale</th>
<th>Vaca Muerta shale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic anisotropy</td>
<td>$E_h/E_v = 2.9$</td>
<td>$E_h/E_v = 1.58$</td>
</tr>
<tr>
<td>Strength anisotropy</td>
<td>2.02 (&gt; 2.0)</td>
<td>1.87 (&lt; 2.0)</td>
</tr>
<tr>
<td></td>
<td>(See Eqn. 5.5.1 for definition)</td>
<td>(See Eqn. 5.5.1 for definition)</td>
</tr>
<tr>
<td>Lithology</td>
<td>Argillaceous Mudstone</td>
<td>Calcareous Mudstone</td>
</tr>
<tr>
<td>Clay minerals</td>
<td>Abundant Illite + Illite/Smectite and Sparse KA</td>
<td>Illite &gt; Illite/Smectite</td>
</tr>
<tr>
<td>Fossils</td>
<td>Moderate carbonate particles</td>
<td>Nondescript shell fragments, hash, echinoderm fragments, charophyte spores, phosphatic bone fragments and calcified algal remnants</td>
</tr>
<tr>
<td>Organic materials</td>
<td>Stringers, lenses with amorphous kerogen</td>
<td>Discrete particles</td>
</tr>
<tr>
<td>Authigenic minerals</td>
<td>Silica, calcite, pyrite</td>
<td>Calcite, dolomite, quartz</td>
</tr>
<tr>
<td>TOC (wt. %)</td>
<td>Low to Moderate (=1–2%)</td>
<td>Moderate to high (=2–8%)</td>
</tr>
<tr>
<td>Petrographic comments</td>
<td>Very well laminated with burrows and bioturbation</td>
<td>Poorly laminated; calcite lenses and fine shell with cement in matrix</td>
</tr>
</tbody>
</table>

The Bossier shale (Mid Bossier argillaceous/calcareous facies) has a lithotype of argillaceous mudstone, with the dominant matrix composition being argillaceous. The main clay minerals are illite and illite-smectite, with some presence of chlorite. Samples examined under a microscope did not show the presence of fossils. There is a significant presence of black or brown organic-filled weak planes throughout the rock matrix. The diagenetic minerals of the
Bossier shales are pyrite and calcite, with traces of dolomite and quartz. The most obvious petrographic feature of the Bossier shale is the strong lamination, with preferential alignment of organics parallel to the bedding planes, and carbonate cement dispersed throughout the rock matrix. The Bossier shale samples that were tested have low to moderate amounts of total organic content (TOC) of 1-2% by weight, where TOC is the amount of carbon bound in an organic compound, determined by subtracting the dissolved carbon (carbon dioxide and carbonic acid salts) from the total carbon measured.

The Vaca Muerta shale has a lithotype of calcareous/argillaceous mudstone, based on the concentration of fine-grained calcareous micrite contained within the matrix. The samples tested in this study are poorly laminated, with laminae composed of calcareous fecal pellets. The silt-sized sub angular detrital quartz and feldspar are evenly distributed, with some variation in the different samples evaluated. Notable authigenic minerals are pyrite, dolomite and small quantities of sphalarite and glaucomite. The Vaca Muerta is fossil rich with echinoderms, calcispheres and phosphatic bone fragments. The shale samples used in this study are organic-rich, with TOC ranging from 2-8% by weight.

4.2 CT Scan (Pre-test and Post-test) of Bossier shale and Vaca Muerta shale samples

X-Ray Computer Tomography (CT) scanning is a non-destructive imaging technique that is done externally to evaluate the inner fabric by determining the relative density of the rock (Spaw, 2012). The CT scanning technology was first developed by Hounsfield and Cormack in the early 1970s, and was originally used for medical investigations (Grace and Stewart, 1989). The application of CT imaging for the study of rocks started in the 1980s (Cook et al., 1993), and is now a standard tool used for petrographic evaluation worldwide.

The basic operation of the X-Ray CT scan was explained in detail by Wellington and Vinegar (1987), and is briefly described here. The X-Ray beam passes through the rock at various paths, and the attenuated beam is measured. Data from the transmitted and received beam are then used to determine the attenuation coefficient of the sample measured relative to the calibrated values for water and air. Based on this comparison of measured and calibrated values, density is computed for the rock, and displayed as pixel images in graphical form that is used to evaluate the internal structure of the rock fabric without damaging the rock. An example of the sample CT images is shown in Figure 4.1a, distinguishing the weak planes from the shale matrix for samples at various bedding angles and confining stresses, wherein the density contrast between the weak planes, shale matrix and other inclusions are clearly distinguishable.

The pre-test and post-test Bossier shale sample CT scans are shown in Figures 4.1a and 4.1b. The CT scan images are arranged in rows of increasing confining pressures for the triaxial compression tests indicated on the left-hand column, whereas the bottom row shows the
Figure 4.1a. Bossier shale pre-test CT scan images. The left hand column indicates the confining stress.
Figure 4.1b. Bossier shale post-test CT scan images, showing the failure planes.
angle $\beta$ of the sample’s bedding plane. These images show that the Bossier shale samples, at bedding angles of $\beta = 0^\circ$ to $10^\circ$, and $\beta = 90^\circ$, failed mostly along the planes whose orientations are consistent with the Coulomb criterion. There are some exceptions for the samples at $\beta = 90^\circ$ at low-confining stresses from 0–1,000 psi, where tensile splitting occurs, and the mixed failure mode of shear matrix and tensile splitting occurs at 1,000 psi. Tensile splitting occurs because, at low-confining stress, the sample at angle $\beta = 90^\circ$ is not constrained by any confining stress, causing the laminated sample to buckle under the axial load. Buckling of the sample causes an increase in tensile stresses normal to the weak planes, at the sample interior, resulting in splitting failure with lower strengths.

Tien et al. (2006) reported similar failure modes of tensile splitting and tensile fractures under unconfined conditions for argillaceous reconstituted samples. For samples at $\beta = 90^\circ$ at confining stress levels of 3,000–10,000 psi, the sample failure mode is predominantly that of shear matrix failure, with some post-failure tensile splitting. For the Bossier shale sample at bedding angles of $\beta = 45^\circ$–$75^\circ$, at confining stresses of 0–10,000 psi, all of the samples failed along a pre-existing plane of weakness. Sample failure along the weak plane occurs mainly along a single plane, but in some cases along multiple planes. Both the shear matrix and weak plane failure modes are consistent with the JPW model.

However, Figure 4.1b for Bossier shale shows a transition regime of loading angles lying between about $10^\circ$ and $35^\circ$, wherein the failure surface follows an irregular path that jumps between the weak plane and the matrix. In this irregular mixed-failure mode, denoted as the “transition zone”, the discontinuous weak planes intersect the shale matrix shear failure plane, providing an alternative path of least resistance that is lower than the shale matrix strength. This mixed-failure mode due to competition of shale matrix and weak planes causes an uneven distribution of high stresses at the transition zone. To further understand this phenomenon, thin section images will be evaluated in the following section.

For the Bossier shale samples, five triaxial extension experiments were made for samples at bedding angles $\beta = 30^\circ$, $45^\circ$ and $60^\circ$. Figures 4.2a and 4.2b show the five pre-test and post-test samples, with the bedding angles $\beta$ and confining stress $\sigma_1 = \sigma_2$ indicated below the samples images. The major principal stress $\sigma_1 = \sigma_2$ shown below sample images in Figure 4.2a are not applied while the CT images are taken, these are merely the prescribed stress levels for the samples. For Figure 4.2b, the $\sigma_3$ values shown below the samples are minor principal stress at sample failure. Comparison of the triaxial extension pre-test and post-test samples shows that the samples fail mainly along the weak plane. However, at least two of the samples did not fail exactly along the weak plane, and these are the samples at bedding angles $\beta = 30^\circ$ and $45^\circ$, with a high confining pressure of $\sigma_1 = \sigma_2 = 25,000$ psi. These two samples seems to have failed at $\beta = 60^\circ$, and not at $30^\circ$ or $45^\circ$. Despite these slight differences in failure angle $\beta$, strength values are unaffected (Figure 3.11a, b and c), whereby extension and compression strengths are the same for samples that failed along the weak plane.
<table>
<thead>
<tr>
<th>Orientation</th>
<th>Stress Level</th>
<th>Orientation</th>
<th>Stress Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>45deg @ $\sigma_1=\sigma_2=8,020$ psi</td>
<td>60deg @ $\sigma_1=\sigma_2=13,320$ psi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30deg @ $\sigma_1=\sigma_2=25$ kpsi</td>
<td>45deg @ $\sigma_3=229$ psi</td>
<td>60deg @ $\sigma_3=2,610$ psi</td>
<td></td>
</tr>
<tr>
<td>30deg @ $\sigma_3=7,085$ psi</td>
<td>45deg @ $\sigma_3=7,661$ psi</td>
<td>60deg @ $\sigma_3=7,190$ psi</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.2a. Bossier shale pre-test CT scans images (extension tests).

Figure 4.2b. Bossier shale post-test CT scans images (extension tests).
An observation is that both samples at $\beta = 30^\circ$ and $45^\circ$ at $\sigma_1 = \sigma_2 = 25,000$ psi failed close to the edge of the sample, which may have been due to slight end effects. However, the remaining samples at $\beta = 45^\circ$, for the case $\sigma_1 = \sigma_2 = 8,020$ psi, and both samples at $\beta = 60^\circ$, all failed along the weak plane and through the middle of the sample. For future applications, optimized sample height ratio and other methods to reduce end effects will be necessary to improve the current procedure for extension tests of anisotropic shales. In summary, the Bossier shale samples tested at angles $\beta = 30^\circ$, $45^\circ$ and $60^\circ$ under triaxial extension show failure occurring predominantly along the weak plane, similar to what was observed under triaxial compression.

Figures 4.3a and 4.3b show the pre-test and post-test Vaca Muerta shale samples. For samples at angles $0^\circ < \beta < 40^\circ$, the samples failed by shear failure through the matrix, whereas for samples at angles $50^\circ < \beta < 75^\circ$, the samples failed along the weak planes. The samples at angles $50^\circ$ and $60^\circ$ do not show this clearly, because in these cases, the weak-plane failure and shear-matrix failure occur at angles that are very close to each other. However, the sample with a confining stress of 2,500 psi at an angle of $\beta = 75^\circ$ clearly shows a weak-plane failure that is at a much higher angle than the shear matrix failure at angles $0^\circ < \beta < 40^\circ$. The Vaca Muerta post-test CT images in Figure 4.3b, with a confining stress of 20,000 psi and an angle of $\beta = 0^\circ$, also show a horizontal fracture along with a shear matrix failure plane. This horizontal fracture occurred post-failure, because the sample deformed and bulged to one side. There were other similar post-failure horizontal fractures at confining stresses of 5,000 psi at angles of $\beta = 0^\circ$ and $10^\circ$, and these features were also post-failure features that occurred as a result of the weak planes expanding when the loading was removed after shear matrix failure. The mixed mode of shear-matrix failure and tensile splitting at confining stress of 1,000 psi at $\beta = 90^\circ$ is predominantly post-failure, because the compressive strength of the sample with same confining stress at angle $\beta = 0^\circ$ has the same strength as the sample at $\beta = 90^\circ$, but without any tensile splitting.

For the Vaca Muerta extension tests, three samples were tested, at bedding angles $\beta = 0^\circ$, $60^\circ$ and $90^\circ$. The purpose of these extension tests is to evaluate the $\sigma_2$ effect on the intact rock using samples at $\beta = 0^\circ$ and $90^\circ$, whereas to assess the $\sigma_2$ effect along the weak plane, a sample with an angle of $\beta = 60^\circ$ is used. For the three Vaca Muerta samples tested, only two extension tests, at $\beta = 0^\circ$ and $60^\circ$, were successful. The sample at $\beta = 90^\circ$ at $\sigma_1 = \sigma_2 = 20,000$ psi failed at the end of the samples, by tensile splitting along the weak planes, showing a strong end effect. The reason for the end effect could be due to high friction at the contact point between the axial loading ram and the sample. For future applications, improvement in the design of the endcap (Figure 3.6) contact points needs to be considered.

Another possible approach to improve this experiment for future application is to use samples with a higher slenderness ratio of 3:1, as this may lead to improved smoothness of the stress acting at the center of the sample. To avoid tensile fractures initiating at the end
Figure 4.3a. Vaca Muerta shale pre-test CT scan images. The left hand column indicates the confining stress (Triaxial compression tests).
Figure 4.3b. Vaca Muerta shale post-test CT scan images, showing the failure planes (Triaxial compression tests).
Figure 4.4a. Vaca Muerta shale pre-test CT scans images (extension tests).

Figure 4.4b. Vaca Muerta shale post-test CT scans images (extension tests).
of the samples, epoxy could also be applied between the end caps and the sample edge (Mogi, 1967, Figure 1), arresting any tensile splitting that may form when the axial loading is reducing.

The Vaca Muerta sample at $\beta = 0^\circ$ failed at $\sigma_3 = 1,161$ psi, with a fracture through the intact rock, whereas the sample with a weak plane failure at a bedding angle $\beta = 60^\circ$ failed at $\sigma_3 = 3,109$ psi. Both samples showed some signs of slight influence of end effects, but not as severe as the sample at angle $\beta = 90^\circ$. In summary, the Vaca Muerta shale triaxial extension tests shows intact rock fracture for samples with loading angle acting perpendicular to the bedding plane at $\beta = 0^\circ$ and failure along the weak plane for sample at angle $\beta = 60^\circ$. Extension test for samples with loading angle parallel to major principal stress at $\beta = 90^\circ$ was unsuccessful, due to strong end effects. Due to the difficulties in equipment preparations for the extension tests, and the uncertainties with this experiment at that time, this unsuccessful test at $\beta = 90^\circ$ was excluded from further analysis, and no retest was made.

4.3 Thin section analysis

Selected post-test shale samples from the triaxial compression tests were also evaluated using thin sections under a microscope. The main purpose of examining thin section images in this study is to have a better understanding of the shale fabric interaction with the failure surface through the shale matrix or the weak plane. The samples selected for the Bossier shale had loading angles ranging from $\beta = 0^\circ$ to $45^\circ$, and for Vaca Muerta shale the angles ranged from $\beta = 0^\circ$ to $90^\circ$. To prepare these thin section slides, the samples are first impregnated with low-viscosity fluorescent red-dye epoxy resin under a vacuum, to highlight the rock fabric features when observed under the microscope. The impregnated samples are then surfaced into standard thin section slides. The thin sections from the Bossier shale and Vaca Muerta shale post-test samples are prepared on a 24 mm x 46 mm slide, and ground to a thickness of approximately 30 $\mu$m. The prepared thin sections are examined and digitally imaged under plane polarized, cross-polarized and reflected UV light. The Bossier and Vaca Muerta shale post-test thin sections are shown in Figures 4.5 and 4.6. The top row of these figures shows the scanned sample images, and the bottom row shows the failure plane micro-images.

The transition zone with mixed failure modes for the Bossier shale seems to be a new observation, and is shown in Figure 4.5. The sample failure plane occurs along the $Y$-axis (Figure 4.5), whereas the $X$-axis (Figure 4.5) is aligned along the bottom of the sample, perpendicular to the direction of axial loading. Thin-section images at angles $\beta = 0^\circ$ and $10^\circ$ show a single fracture plane angle, as predicted by the Coulomb criterion. The same thin-section images in the bottom row show an intact rock fracture through the rock matrix and across the dark organic-filled weak plane laminations. At this angle, the organic-filled weak-
plane laminations do not affect the matrix-failure plane, and only the shale matrix influences the shale shear failure.

In Figure 4.5, the Bossier shale thin-section images at angles $\beta = 15^\circ$ to $30^\circ$ show that the sample failure plane occurs through the intact rock or weak plane lamination, jumping between the two failure modes. Samples at angles $\beta = 15^\circ$ to $20^\circ$ show that the sample $X$-axis is not aligned to the figure, because the samples were significantly damaged during the compression tests. Nevertheless, these two samples capture the shale fabric failure plane interaction, showing mixed failure mode of intact rock and weak plane failure.

The mixed-failure modes in Figure 4.5, highlighted by the red circles at $\beta = 15^\circ$ to $20^\circ$, show a step-like pattern, wherein the weak planes interact with the matrix shear fractures. This result shows that the path of least resistance follows this step-like pattern through the shale matrix, intersected by discontinuous weak planes. The thin-section image at $\beta = 30^\circ$ shows an interesting change in failure plane angle, with slight mixed failure mode at a lower failure plane angle compared with samples at angles $\beta = 0^\circ$ and $10^\circ$. The reduced fracture angle in this transition zone is most likely also related to uneven stress distribution when the mixed failure mode occurs.

![Figure 4.5. Bossier shale post-test thin-section images; the top row shows the scanned sample images and the bottom row shows the failure plane micro-images.](image)

Lastly, the thin-section image at $\beta = 45^\circ$ shows the sample failure along the weak plane $Y$-axis (shown as yellow line) without any intact rock fracture interaction. A closer look at the thin-section image reveals weak plane failures that appear as small steps along the weak
plane. These observed mini-steps are weak plane crossings that occur due to intermittent weak planes. This means that the parallel weak planes are intermittent, and the failure plane jumps across the discontinuities at some angle, and then moves through to the next closest weak plane. The process of jumping from one weak plane to the next forms these mini steps. No significant increase in strength occurs when jumping across weak planes, as these discontinuities are formed by smaller weak planes having similar strength properties.

For the Vaca Muerta shale, the post-test thin section images are shown in Figure 4.6, with the bottom row showing the micro-image of the shale fabric. Unlike the Bossier shale, the Vaca Muerta shale shows poor laminations and is darker, as it has higher organic content than the Bossier shale. Because of this, the thin section images are not as easily evaluated as were those for the Bossier shale. The sample failure planes are also not easily distinguishable for most of the Vaca Muerta samples. However, it is interesting to note that the CT scans for Vaca Muerta provided very clear images of the sample failure planes, which were more easily evaluated to identify the failure planes, compared to the thin section images.

![Figure 4.6. Vaca Muerta shale post-test thin-section images; the top row shows the scanned sample images and the bottom row shows the failure plane micro-images.](image)

The thin section micro-images at the bottom row of Table 4.6 show some interesting details when compared with each other. The white spots on the bottom micro-images suggest that there is a significant presence of detrital material, composed mainly of fossil matter in a compact arrangement. As was discovered for the CT scan images, the thin section images do not show any clear weak plane laminations, even when observed under the microscope,
whereas the CT scan did show some visible bedding planes. This observation highlights the usefulness of the CT images as compared to thin section images, when used to distinguish weak plane textural effects for the Vaca Muerta shale.

All of the samples shown in Figure 4.6 have intact rock fracture, and none of these samples show failure along the weak plane. The reason for such sample selection for thin section images is that the thin section images for Vaca Muerta shale are more suitable to helping understand the interaction between the detrital components and the dark organic matter dispersed throughout the shale fabric.

In summary, the Vaca Muerta thin section images are not suitable for distinguishing failure planes of anisotropic shales. Instead, for poorly laminated shales, CT scans are more suitable for distinguishing failure planes and bedding angles. A high organic content in the Vaca Muerta shale matrix also makes the sample dark in color, hence more difficult to identify any failure features that may be of interest to understand the failure mode.
5 Jaeger’s Plane of Weakness Model

5.1 Theory and data-fitting technique

Jaeger (1960) introduced the Plane of Weakness model, hereinafter referred to as the “JPW model”, in an article entitled “Shear Failure of Anisotropic Rocks”. Jaeger envisioned “…an isotropic material whose behavior can be described in the ordinary way by a shear strength (or cohesion) $S_o$ and a coefficient of internal friction $\mu_o$, but which is supposed to have a plane (or parallel planes) of weakness with different values $S_w$ and $\mu_w$ of the shear strength and coefficient of internal friction.”

Although researchers have proposed various failure criteria for anisotropic rocks in the intervening fifty years, the JPW model remains the most commonly used. One obvious reason for this is that it is based on the widely accepted Mohr-Coulomb criterion for isotropic rocks, and therefore utilizes well-known parameters such as cohesion and angle of internal friction. These factors, and its simplicity, have made the JPW criterion the most preferred failure criterion for transversely isotropic rocks. However, the JPW criterion remains untested for true-triaxial applications, and in fact, even under traditional triaxial stress conditions, very few examples of its applicability can be found in the open literature.

Although the JPW criterion is well known, its origin and the complete construction of the model is not described in detail in the literature. In the following sections, the JPW criterion is described using the Coulomb criterion and the Mohr circle to explain the commonly used equations that are described in Jaeger et al. (2007).

According to Coulomb’s original failure criterion proposed in 1785 (Jaeger et al., 2007), the criterion for rock fracture is

$$|\tau| = S_o + \mu_o \sigma_n,$$  \hspace{1cm} (5.1.1)

where $\sigma_n$ is the normal traction and $\tau$ is the shear traction that act on the failure plane. The parameter $S_o$ is known as the cohesion, and $\mu_o = \tan \phi_o$ is the coefficient of internal friction. The shear $\tau$ and normal stress $\sigma_n$ acting on a plane inclined at angle $\beta$ to $\sigma_1$ are

$$\sigma_n = \sigma_m - \tau_m \cos 2\beta,$$  \hspace{1cm} (5.1.2)

$$\tau = -\tau_m \sin 2\beta,$$  \hspace{1cm} (5.1.3)

where $\tau_m = (\sigma_1 - \sigma_3)/2$ is the maximum shear stress acting on any plane, and $\sigma_m = (\sigma_1 + \sigma_3)/2$ is the mean normal stress at failure.
For failure in an isotropic medium with cohesion $S_o$ and coefficient of internal friction $\mu_o = \tan \phi_o$, inserting Eqn. 5.1.2 and 5.1.3 into 5.1.1, and recalling that $\phi = 90^\circ - 2\beta$ (Jaeger et al., 2007), Eqn. 5.1.1 gives

$$\tau_m = \sigma_m \sin \phi_o + S_o \cos \phi_o.$$  \hspace{1cm} (5.1.4)

Equation 5.1.4 is the Coulomb failure criterion for shear failure of intact isotropic rock.

Now consider a plane of weakness inclined at an angle $\beta$ to the direction of $\sigma_1$, with cohesion $S_w$ and coefficient of internal friction of $\mu_w = \tan \phi_w$. The Coulomb criterion for failure along the weak plane is

$$|\tau| = S_w + \mu_w \sigma_n.$$  \hspace{1cm} (5.1.5)

Inserting Eqn. 5.1.2 and Eqn. 5.1.3 into 5.1.5 gives

$$\tau_m \sin 2\beta = S_w + \tan \phi_w (\sigma_m - \tau_m \cos 2\beta),$$ \hspace{1cm} (5.1.6)

$$\tau_m (\sin 2\beta + \cos 2\beta \tan \phi_w) = S_w + \sigma_m \tan \phi_w.$$ \hspace{1cm} (5.1.7)

To simplify Eqn. 5.1.7 further, multiply by $\cos \phi_w$, resulting in the following expression:

$$\tau_m (\sin 2\beta \cos \phi_w + \cos 2\beta \sin \phi_w) = S_w \cos \phi_w + \sigma_m \sin \phi_w.$$ \hspace{1cm} (5.1.8)

Simplifying Eqn. 5.1.8 using the trigonometric relationship $\sin (2\beta + \phi_w) = \sin 2\beta \cos \phi_w + \cos 2\beta \sin \phi_w$ produces the following equation for failure along the weak plane:

$$\tau_m = (S_w \cos \phi_w + \sigma_m \sin \phi_w)/[\sin (2\beta + \phi_w)].$$ \hspace{1cm} (5.1.9)

This equation is for a plane inclined at angle $\beta$ to the direction of $\sigma_1$. This was the original definition of angle $\beta$ introduced by Jaeger (1960).

The current practice is to define the angle $\beta$ as the angle between the normal vector of the plane and $\sigma_1$, which results in Eqn. 5.1.9 with a denominator $\sin (2\beta - \phi_w)$. The definition of angle $\beta$ is shown in Figure 5.1, using the Mohr circle angle $\beta$ measured counterclockwise (Figure 5.2). The angle $\beta$ shown in Figure 5.1 is the definition that will be used throughout the remainder of this study.

However, both of the above definitions of angle $\beta$ are used in the literature, and this can sometimes cause confusion. To determine which definition of $\beta$ is used for a given data set, note that when the angle $\beta$ is defined as the angle between the normal to the plane and the direction of $\sigma_1$ (shown in Figure 5.1), the minimum of the strength failure envelope occurs at $\beta_{\min} = 45^\circ + \phi_w/2$, whereas, for the opposite definition (as defined for Eqn. 5.1.9), $\beta_{\min} = 45^\circ - \phi_w/2$. This can be used as a quick method to identify the definition of angle $\beta$ used by different researchers when reviewing data from the literature. On the other hand, when the
weak plane friction angle $\phi_w$ is very low (or for a frictionless weak plane having $\phi_w = 0^\circ$) the angle $\beta_{\text{min}}$ becomes closer to $45^\circ$, and the definition that has been used for angle $\beta$ becomes harder to ascertain from the experimental data.

![Figure 5.1. Triaxial compression test setup for sample angle $\beta$.](image1)

The problem of failure on a weak plane analyzed with the aid of the Mohr circle complements the Coulomb failure criterion. The condition for failure using the straight line PQR (Figure 5.2) is that failure will occur when point D lies within the arc QR (red arc on the Mohr circle). Hence, taking point D to coincide with point R, the angle $\angle RBC$ equals $2\beta$ and the angle $\angle PRB$ is given by the following relationship:

![Figure 5.2. JPW using Mohr circle; $\beta$ is positive in the clockwise direction.](image2)

$$180^\circ = (\phi_w + \angle PRB) + (180 - 2\beta), \quad (5.1.10)$$
Applying the law of sine for triangles, \( \angle PRB \) in Figure 5.2 relates to BR and PB as follows:

\[
|BR|/(\sin \phi_w) = |PB|/\sin(2\beta - \phi_w),
\]

where the terms BR and PB refer to the Mohr circle in Figure 5.2 as

\[
|BR| = \tau_m, \\
|PB| = |PO| + |OB| = S_w \cot \phi_w + \sigma_m.
\]

Rearranging Eqn. 5.1.12, the JPW criterion takes the form

\[
|BR| \sin(2\beta - \phi_w) = |PB| \sin(\phi_w).
\]

Inserting Eqn. 5.1.13 and 5.1.14 into Eqn. 5.1.15 yields

\[
\tau_m \sin(2\beta - \phi_w) = (S_w \cot \phi_w + \sigma_m) \sin \phi_w,
\]

\[
\tau_m = (S_w \cos \phi_w + \sigma_m \sin \phi_w) / \sin(2\beta - \phi_w).
\]

Equation 5.1.17 is the JPW criterion in Eqn. 5.1.9, but with the opposite reference angle \( \beta \).

To determine the angle \( \angle BQR \), subtract angle \( \angle BRQ \), and compute the difference of angles \( 2\beta_2 - 2\beta_1 \) from \( 180^\circ \), and input to Eqn. 5.1.18:

\[
\angle BQR = 180^\circ - \angle BRQ - (2\beta_2 - 2\beta_1),
\]

\[
\angle BQR = 180^\circ - (2\beta_2 - \phi_w) - (2\beta_2 - 2\beta_1) = 180^\circ - 2\beta_2 + \phi_w.
\]

Using the sine rule for triangles, and based on the angles \( \angle BRQ \) and \( \angle BQR \) in Eqn. 5.1.18 and 5.1.20, the relationship between angles \( \beta_1 \) and \( \beta_2 \) is found to be

\[
|BR|/(\sin \angle BQR) = |BQ|/(\sin \angle BRQ),
\]
Figure 5.3. JPW using Mohr circle; $2\beta_1$ and $2\beta_2$ are positive in the clockwise direction.

since $|BR| = |BQ|$ the angles $\angle BQR$ and $\angle BRQ$ are also equal. By equating Eqn. 5.1.18 and Eqn. 5.1.20, it follows that

$$2\beta_1 - \phi_w = 180^\circ - 2\beta_2 + \phi_w,$$
(5.1.22)

$$2\beta_1 + 2\beta_2 = 180^\circ + 2\phi_w,$$
(5.1.23)

$$\beta_1 + \beta_2 = 90^\circ + \phi_w,$$
(5.1.24)

To determine the angle $\beta_1$ as a function of $\{\tau_m, \phi_w, S_w, \sigma_m\}$, the angle $\beta_1$ can be substituted into Eqn. 5.1.16, after which rearrangement yields

$$\tau_m \sin(2\beta_1 - \phi_w) = (S_w \cot \phi_w + \sigma_m) \sin \phi_w,$$  
(5.1.25)

$$2\beta_1 = \phi_w + \sin^{-1}\{[(S_w \cot \phi_w + \sigma_m) \sin \phi_w]/\tau_m\}.$$  
(5.1.26)

To determine the angle $\beta_2$ as a function of $\{\tau_m, \phi_w, S_w, \sigma_m\}$, insert the angle $2\beta_1$ from Eqn. 5.1.23 into Eqn. 5.1.26 to derive the following:

$$2\beta_2 = 180^\circ + \phi_w - \sin^{-1}\{[(S_w \cot \phi_w + \sigma_m) \sin \phi_w]/\tau_m\}.$$  
(5.1.27)

Equations 5.1.26 and 5.1.27 (Jaeger et al., 2007, p. 75, Eqn. 3.34 & 3.35) shows the range of the angle $\beta$ for which sliding will occur on the weak plane within the limit angles $\beta_1 \leq \beta \leq \beta_2$. The corresponding angles $2\beta_1$ and $2\beta_2$ on the Mohr circle in Figure 5.3 show these limit angles for failure along the weak plane. The intercept point Q and R in Figure 5.3 represents these respective orientation angles $2\beta_1$ and $2\beta_2$ of the failure envelope in Figure 5.3.

The JPW failure envelope in Figure 5.3 has a minimum within the limits $\beta_1 \leq \beta \leq \beta_2$ at angle $\beta_{\min}$ (Figure 5.4). This point defines the inflexion point of the failure envelope defined by the JPW criterion. By differentiating Eqn. 5.1.17, the minimum value of $\sigma_1$ needed for failure is found to occur at the angle $\beta_{\min}$ that is related to $\phi_w$ (Jaeger et al., 2007, p. 74) as follows:
\[
\tan 2\beta_{\text{min}} = -\frac{1}{\tan \phi_w}. \quad (5.1.28)
\]

The method of differentiating the Coulomb criterion to find the angle of minimum \(\sigma_1\) or \(\tau_m\) is the same for the weak plane or intact rock fracture; an example is shown in Appendix C.

An easier method to determine the angle of minimum shear strength occurring at \(\beta_{\text{min}}\) on the failure envelope involves using the Mohr circle. From Figure 5.4, the failure envelope within the limits \(\beta_1\) and \(\beta_2\) is a semicircle, and its midpoint coincides at the lowest strength of the failure envelope at angle \(\beta_{\text{min}}\). The angle \(\beta_1\) and \(\beta_2\) can each be determined as a function of \(\phi_w\) (Eqn. 5.1.24 to Eqn. 5.1.27), and the average (midpoint) of the angle \(\beta_1\) and \(\beta_2\) is \(\beta_{\text{min}}\), which is

\[
\beta_{\text{min}} = (\beta_1 + \beta_2)/2 = 45^\circ + (\phi_w/2). \quad (5.1.29)
\]

![Figure 5.4. JPW model failure envelope.](image)

It is interesting to note that the form of Eqn. 5.1.29 for failure along the weak plane is similar to the angle \(\beta_o\), wherein intact rock fracture occurs as per the Coulomb criterion (Jaeger et al., 2007, p. 104, Eqn. 4.44 and Eqn. 4.46; Parry, 2000):

\[
\beta_o = 45^\circ + (\phi_w/2). \quad (5.1.30)
\]

Equation 5.1.30 has an interesting resemblance to Eqn. 5.1.29, and derives from Eqn. 5.1.28. To explore the significance of this relationship, consider that the two angles \(\beta_1\) and \(\beta_2\) in Eqn. 5.1.29 converge (i.e., \(\beta_1 = \beta_2 = \beta_{\text{min}}\)), in which case Eqn. 5.1.29 reduces to Eqn. 5.1.30 for isotropic rock. In other words, when \(\beta_1\), \(\beta_2\) and \(\beta_{\text{min}}\) coincide, line PQR in Figure 5.3 does not cut through the Mohr circle. Instead, it touches the Mohr circle, resulting in an isotropic failure condition (Eqn. 5.1.30). This means that, as the difference between the angles \(\beta_1\) and \(\beta_2\) becomes smaller, strength anisotropy is reduced. By equating Eqn. 5.1.26 and Eqn. 5.1.27 (i.e., \(\beta_1 = \beta_2\)), the resulting term is the isotropic failure criterion (Eqn. 5.1.4) with \(S_w\) and \(\phi_w\) terms. Consequently, when \(S_w = S_o\) and \(\phi_w = \phi_o\), strength anisotropy vanishes. These
relationships between the intact rock and weak plane criteria are possible because both
equations were derived from the Coulomb criterion.

Another demonstration that both Eqn. 5.1.17 and Eqn. 5.1.9 are the same, but with
different reference direction, can be carried out by using the Mohr circle diagram on Figure
5.2. The angle $\beta$ in Eqn. 5.1.17 is defined as increasing “counterclockwise” ($2\beta_1 = \text{angle } \angle \text{CBR}$). Using the same definition, the complementary angle to $2\beta_1$ is therefore ($\pi - 2\beta_1 = \text{angle } \angle \text{ABR}$). Replacing angle $2\beta_1$ with $\pi - 2\beta_1$ in Eqn. 5.1.17, the denominator becomes

$$\left[ \sin \left( (\pi - 2\beta_1) - \phi_w \right) \right] = \left[ \sin \left( \pi - (2\beta_1 + \phi_w) \right) \right]. \quad (5.1.31)$$

Using the trigonometric relationship, $\sin(A - B) = \sin A \cos B - \cos A \sin B$, and inserting
Eqn. 5.1.31 into the denominator of Eqn. 5.1.17, it becomes obvious that Eqn. 5.1.17 and
5.1.9 are the same, but with different reference directions for the angle $\beta$.

To determine the limits of angle $\beta$ for the JPW equation, manipulation of the equation for
the stress difference ($\sigma_1 - \sigma_3$) required to cause failure is necessary. The maximum shear
stress, $\tau_m$, for a plane of weakness gives the stress difference relationship as a function of $\beta$
and $\sigma_m$. Jaeger et al. (2007, p. 74, Eqn. 3.25) expressed Eqn. 5.1.7 with the angle $\beta_1$ (CBR in
Figure 5.2) as

$$\tau_m \left( \sin 2\beta - \cos 2\beta \tan \phi_w \right) = S_w + \sigma_m \tan \phi_w. \quad (5.1.32)$$

Multiplying Eqn. 5.1.32 with $\cot \phi_w$ results in

$$\tau_m \left( \sin 2\beta \cot \phi_w - \cos 2\beta \right) = S_w \cot \phi_w + \sigma_m. \quad (5.1.33)$$

Multiplying Eqn. 5.1.33 with $\sin \phi_w$ results in

$$\tau_m \left( \sin 2\beta \cos \phi_w - \cos 2\beta \sin \phi_w \right) = \left( S_w \cot \phi_w + \sigma_m \right) \sin \phi_w. \quad (5.1.34)$$

Using the trigonometric relationship $\sin(2\beta \pm \phi_w) = \sin 2\beta \cos \phi_w \pm \cos 2\beta \sin \phi_w$ into
Eqn. 5.1.34 gives

$$\tau_m \sin(2\beta - \phi_w) = \left( S_w \cot \phi_w + \sigma_m \right) \sin \phi_w, \quad (5.1.35)$$

$$\tau_m = \left( S_w \cot \phi_w + \sigma_m \right) \sin \phi_w \csc(2\beta - \phi_w). \quad (5.1.36)$$

Equation 5.1.36 (Jaeger et al., 2007, p. 74, Eqn. 3.26) expressed in terms of principal stresses are

$$(\sigma_1 - \sigma_3) = 2 \left( S_w \cot \phi_w + \left[ (\sigma_1 + \sigma_3)/2 \right] \right) \sin \phi_w \csc(2\beta - \phi_w), \quad (5.1.37)$$

$$(\sigma_1 - \sigma_3) = \left[ 2S_w \cot \phi_w \sin \phi_w \csc(2\beta - \phi_w) \right] + \left[ (\sigma_1 + \sigma_3) \sin \phi_w \csc(2\beta - \phi_w) \right].$$
\[(\sigma_1 - \sigma_3) - [(\sigma_1 + \sigma_3) \sin \phi_w \csc(2\beta - \phi_w)] = [2S_w \cos \phi_w \csc(2\beta - \phi_w)],\]
\[
\frac{\sigma_2 [1 - \sin \phi_w \csc(2\beta - \phi_w)] - \sigma_3 [1 + \sin \phi_w \csc(2\beta - \phi_w)]}{\csc(2\beta - \phi_w)} = 2S_w \cos \phi_w,
\]
\[
\sigma_1 \sin(2\beta - \phi_w) - \sin \phi_w - \sigma_3 \sin(2\beta - \phi_w) + \sin \phi_w = 2S_w \cos \phi_w.
\] (5.1.38)

Equation 5.1.38 (Jaeger et al., 2007, p. 74, Eqn. 3.27) further simplified gives
\[
[(\sigma_1 - \sigma_3) \sin(2\beta - \phi_w)] - \sigma_1 \sin \phi_w - \sigma_3 \sin \phi_w)/(\cos \phi_w) = 2S_w,
\] (5.1.39)
\[
[(\sigma_1 - \sigma_3) \sin 2\beta \cos \phi_w - \cos 2\beta \sin \phi_w] - \sigma_1 \sin \phi_w - \sigma_3 \sin \phi_w)/(\cos \phi_w) = 2S_w,
\]
\[
(\sigma_1 - \sigma_3) \sin 2\beta - \cos 2\beta \tan \phi_w - \sigma_1 \tan \phi_w - \sigma_3 \tan \phi_w = 2S_w.
\] (5.1.40)

Using the trigonometry identity \{\cos 2\beta = \sin 2\beta \cot \beta - 1\} in Eqn. 5.1.40 yields
\[
(\sigma_1 - \sigma_3) \sin 2\beta - (\sin 2\beta \cot \beta - 1) \tan \phi_w - \sigma_1 \tan \phi_w - \sigma_3 \tan \phi_w = 2S_w,
\]
\[
(\sigma_1 - \sigma_3) \sin 2\beta - \tan \phi_w \sin 2\beta \cot \beta + \tan \phi_w - \sigma_1 \tan \phi_w - \sigma_3 \tan \phi_w = 2S_w.
\] (5.1.41)

To simplify the above equation, replace Eqn. 5.1.41 with \(\mu_w = \tan \phi_w\)
\[
(\sigma_1 - \sigma_3) \sin 2\beta - \mu_w \sin 2\beta \cot \beta + \mu_w w - \mu_w \sigma_3 = 2S_w,
\]
\[
(\sigma_1 - \sigma_3) \sin 2\beta - \mu_w \sin 2\beta \cot \beta - 2\mu_w \sigma_3 = 2S_w,
\]
\[
(\sigma_1 - \sigma_3) \sin 2\beta (1 - \mu_w \cot \beta) = 2S_w + 2\mu_w \sigma_3,
\]
\[
(\sigma_1 - \sigma_3) = 2 \left( S_w + \mu_w \sigma_3 \right) / [\sin 2\beta (1 - \mu_w \cot \beta)].
\] (5.1.42)

Equation 5.1.42 (Jaeger et al., 2007, p. 74, Eqn. 3.28) shows the stress difference that is needed to cause failure as a function of \(\beta\), for a fixed value of \(\sigma_3\). From Eqn. 5.1.42, it can be seen that the stress difference \(\sigma_1 - \sigma_3\) approaches infinity or becomes negative when \(\beta \leq \phi_w\) and \(\beta = \pi/2\). This means that the failure along the weak plane will only occur for angles within the limits \(\phi_w < \beta < \pi/2\), as shown in Figure 5.5 (Jaeger et al., 2007, p. 104).

By inserting the principal stress ratio \(\sigma_3/\sigma_1 = k \leq 1\) into Eqn. 5.1.38, the stress difference term as a function of the stress ratio \(\sigma_3/\sigma_1\) (Jaeger et al., 2007, p. 75, Eqn. 3.2.9) can be expressed as
\[
\begin{align*}
\sigma_1 \sin(2\beta - \phi_w) - \sin \phi_w &- k \sigma_1 \sin(2\beta - \phi_w) + \sin \phi_w = 2S_w \cos \phi_w, \\
\sigma_2 \sin(2\beta - \phi_w) - \sin \phi_w - k \sin(2\beta - \phi_w) - k \sin \phi_w & = 2S_w \cos \phi_w, \\
\sigma_3 \sin(2\beta - \phi_w)(1 - k) - (1 + k) \sin \phi_w & = 2S_w \cos \phi_w, \\
\sigma_1 & = 2S_w \cos \phi_w / [\sin(2\beta - \phi_w)(1 - k) - (1 + k) \sin \phi_w].
\end{align*}
\] (5.1.43)

Figure 5.5. JPW model failure envelope for limit angle $\phi_w < \beta < \pi/2$.

Equation 5.1.43 (Jaeger et al., 2007, p. 74, Eqn. 3.29) allows a further generalization of the plane of weakness criterion to determine stress ratio $\sigma_1/\sigma_3$ in terms of $\beta$ and $\phi_w$, which is useful to determine practical values of $\sigma_1/\sigma_3$ that are required for failure to occur.

5.2 Jaeger plane of weakness model applied to data

Triaxial experiments conducted on the Bossier and Vaca Muerta shales provide the principal stresses $\sigma_1$, $\sigma_3$ and angle $\beta$ at failure. Using these strength data, anisotropic rock strength properties $\phi_0$, $S_0$, $P_w$ and $S_w$ are derived for JPW model and compared against experimental data and evaluated.

In the past, the most common method of determining the strength properties has been by using the Mohr circle construction (e.g., Donath, 1961; Hoek, 1964; Chenevert and Gatlin, 1964; McLamore and Gray, 1967; Handin, 1969 and Donath et al., 1972). The principal stresses at failure, $\sigma_1$ and $\sigma_3$, for different angles $\beta$ using graphical construction of the Mohr circles were used to determine the slope and intercept defining the strength properties $\phi_0$, $S_0$, $P_w$ and $S_w$. These properties are then used to define the failure envelope. The use of the JPW model provides a more accurate representation of the failure envelope, especially for materials with a high degree of anisotropy. The model is particularly useful in predicting the behavior of shales in the context of geomechanical and geotechnical applications.
So, \( S_w \), and \( S_{w'} \). Jaeger (1960, 1964, 1966 and 1971) however, was more inclined to use \( \tau_m \) versus \( \sigma_m \) plots. This is the same as using Mohr circles, but is easily implemented into simple programs to determine the strength properties \( \phi_b, S_o, \phi_w, \) and \( S_w \).

Both the methods of Mohr circles and using \( \tau_m \) versus \( \sigma_m \) can be implemented into a computer program to determine the strength properties of the rock and weak plane. A simple approach outlined here shows an example of using linear regression of best-fit equations using \( \tau_m \) versus \( \sigma_m \), which is easily evaluated using commonly available programs such as Excel.

The linear equation plots of maximum shear strength, \( \tau_m \) versus mean normal stress, \( \sigma_m \), in Figure 5.6 show the best-fit equations for angles \( \beta \) from \( 0^\circ \) to \( 90^\circ \). Figure 5.6a shows the plot for Bossier shale, and Figure 5.6b is for Vaca Muerta shale. The \( \tau_m \) versus \( \sigma_m \) plot at different angles \( \beta \) gives the slope and intercept that represents the linear Coulomb criterion.

For intact rock fracture, the Coulomb criterion is

\[
\tau_m = \sigma_m \sin \phi_o + S_o \cos \phi_o. \tag{5.2.1}
\]

From Eqn. 5.2.1, the slope of this equation is

\[
\text{slope, } m_o = \sin \phi_o, \tag{5.2.2}
\]

\[
\phi_o = \sin^{-1}(\text{slope, } m_o). \tag{5.2.3}
\]

The friction angle of the rock, \( \phi_o \), determined in Eqn. 5.2.3 is then used to evaluate the cohesion \( S_o \) from the intercept \( C_o \) of the linear Eqn. 5.2.1:

\[
C_o = S_o \cos \phi_o, \tag{5.2.4}
\]

\[
S_o = (C_o)/\cos \phi_o. \tag{5.2.5}
\]

For failure along a weak plane, the JPW criterion is:

\[
\tau_m = \left(S_w \cos \phi_w + \sigma_m \sin \phi \right)/\left[\sin(2\beta - \phi_w)\right]. \tag{5.2.6}
\]

To determine the strength properties of a weak plane, the angles where failure occurs only along the weak plane are used. This normally occurs around angles \( \beta \) of \( 45^\circ \) to \( 75^\circ \), but may vary depending on rock type. Distinguishing the samples with weak plane failure is important, because angles \( \beta \) lying outside this range may not be representative of the weak plane strength properties, \( \phi_w \) and \( S_w \). The slope \( m_w \) for \( \tau_m \) versus \( \sigma_m \) plot at angles \( 45^\circ \), \( 60^\circ \) and \( 75^\circ \) (Figure 5.6a) in Eqn. 5.2.6 is

\[
\text{slope, } m_w = \sin \phi_w / \sin(2\beta - \phi_w), \tag{5.2.7}
\]

\[
\sin(2\beta - \phi_w) = 2\beta \cos \phi_w - \cos(2\beta) \sin \phi_w, \tag{5.2.8}
\]
Figure 5.6a. Bossier shale \( \tau_m \) vs. \( \sigma_m \) plots at varying angles \( \beta \) to determine best-fit equation.

Figure 5.6b. Vaca Muerta shale \( \tau_m \) vs. \( \sigma_m \) plots at varying angles \( \beta \) to determine best-fit equation.
after which dividing Eqn. 5.2.9 by \( \sin \phi_w \) and rearranging, gives the weak plane friction angle \( \phi_w \) as

\[
\phi_w = \tan^{-1}\left\{ \frac{(m_w \sin 2\beta)}{(1 + m_w \cos 2\beta)} \right\}.
\] (5.2.10)

To determine the weak plane cohesion, \( S_w \), the intercept \( C_w \) from the linear Eqn. 5.2.6 is

\[
C_w = S_w \cos \phi_w / \sin(2\beta - \phi_w),
\] (5.2.11)

\[
S_w = C_w \sin(2\beta - \phi_w) / \cos \phi_w.
\] (5.2.12)

The approach described from Eqn. 5.2.1 to 5.2.12 gives the same results using the Mohr circle construction. This was verified separately in this study.

Despite the ease of using linear equations, this approach does not give strength parameters representative of the complete data set. The reason is that it is not clear from the strength data at which angles \( \beta \) the intact rock fracture or failure along weak planes occurs. The plots on Figure 5.6a,b show that the slopes and intercepts for intact rock fracture and failure along weak planes do not vary significantly. This causes difficulty in distinguishing representative failure modes and assigning the correct criterion at corresponding angles \( \beta \). This approach results in poor estimation of the strength parameters \( \phi_o, S_o, \phi_w, \) and \( S_w \) at angles where the failure mode is transitioning (Chapter 4 for definition of the transition zone) from intact rock fracture to sliding along the weak plane.

An alternative approach to determine the strength parameters representative of the complete data set is proposed in this study. The same basic input data are used, namely the strength data at failure, \( \sigma_1 \), the confining stress \( \sigma_3 \), and the corresponding angle \( \beta \). Using this input data, a Matlab code was written to determine the JPW strength parameters that are most representative of the complete data set (Appendix B1 and B2).

There are two parts to this Matlab code. The first part of the code (Appendix B1) contains the three input parameters (\( \sigma_3, \sigma_3 \) and angle \( \beta \)) and defines the objective function. The objective function in this case is the failure criterion used for intact rock fracture (Eqn. 5.2.1) and failure along weak planes (Eqn. 5.2.6). These two equations are applied with input data obtained experimentally (\( \sigma_3, \sigma_3 \) and angle \( \beta \)), for different values of strength parameters (\( \phi_o, S_o, \phi_w, S_w \)) described in the main loop code.

The main loop is the second part of the code (Appendix B2) which defines the range of strength parameters (\( \phi_o, S_o, \phi_w, S_w \)) iteratively. For the selected range of strength parameters, an output strength \( \sigma_{1\text{predict}} \) is calculated. The difference between the input strength \( \sigma_1 \) and output strength \( \sigma_{1\text{predict}} \) determines the deviation of the model. This deviation between
The above-described process is repeated iteratively until the optimal strength parameters of the rock are determined. For instance, the first range assigned for cohesion (\(S_o\) and \(S_w\)) is from 0 psi to 95,000 psi, while for the friction angle (\(\phi_o\) and \(\phi_w\)) the range is 0° to 95°. This broad range used as a first guess covers the strength properties that are applicable for most rock types, according to data available from the literature. In order to find the best fitting parameters to within, say, the nearest 1 psi and the nearest 0.1 degree, one could traverse the entire parameter space in increments of 1 psi and 0.1 degrees. However, this is not computationally feasible. Instead, the search is first done using increments of 1,000 psi, and 5 degrees. Once the best-fitting parameters are found on this grid, the grid is refined, and the search is repeated, but over a narrower range that is centered on the parameter values found in the previous iteration. In this way, the optimum parameters can be found after about three to five such iterations. The code used to determine the best-fit strength parameters for Bossier shale are given in Appendices B1 and B2.

A brief explanation on the operations of the Matlab code is described here. Appendix B1 is defined as the JPW model objective function, and in this code the experimental data (i.e., \(\sigma_1, \sigma_3\) and \(\beta\)) are input, followed by the equations that define the Coulomb criterion for intact rock fracture and JPW criterion for weak plane failure. At the end of the B1 code are the plot functions that are useful for plotting the fit data for the JPW model with least RMSE.

The code in Appendix B2 is used to input the Coulomb parameters (i.e., \(S_o, \phi_w, S_w, \phi_w\)) that are tested iteratively using the data set and equations defined in B1. The output Coulomb parameters with least RMSE are then fit to the complete dataset, and the process is repeated until the Coulomb parameters with lowest RMSE is determined for the dataset input into B1. The basic concept described here is also applicable for the Pariseau model Matlab codes in B3 and B4, wherein the Coulomb parameters are replaced by the Pariseau parameters.
5.2.1 Bossier shale data analyzed using the JPW model

The discussion in Section 5.2 shows how the JPW strength properties are determined. The failure criterion relationship of strength, \( \sigma_1 \) or \( \tau_m \), varies as a function of \( \sigma_3 \) and angle \( \beta \), whereby the JPW model strength is a maximum at angle \( \beta = 0^\circ \) or \( 90^\circ \) and a minimum at \( \beta = 45^\circ + \phi_m/2 \). In this section, the Bossier shale experimental data are analyzed using the JPW model. The JPW model analysis uses the Matlab code described in the earlier section (Appendices B1 and B2).

The JPW model failure envelope in Figure 5.7 shows the Bossier shale JPW model-predicted strength, \( \sigma_1 \) (on the y-axis), versus the angle \( \beta \) (on the x-axis) at different confining stresses \( \sigma_3 \), shown as solid lines. On the same plot, solid squares indicate the experimental data from triaxial compression tests that were used to determine the JPW model strength properties. The vertical red dotted line in Figure 5.7 is the theoretical minimum at the angle \( \beta = 45^\circ + \phi_m/2 \), while the strength properties for the Bossier shale are summarized in a box at the bottom right corner of Figure 5.7.

For the Bossier shale samples (Figure 5.7), thirty-six triaxial compression tests were conducted for samples with bedding angles ranging from \( \beta = 0^\circ \) to \( 90^\circ \), at confining stresses of 0 psi, 1,000 psi, 3,000 psi, 6,000 psi and 10,000 psi. All of the samples showed increased strengths at higher confining stresses. The general strength response shows maximum strengths at angles of \( \beta = 0^\circ \) and \( 90^\circ \), with a minimum strength occurring at approximately
The unconfined compressive strength (UCS) showed a significant difference between 0° and 90°, with the strength at 0° being more than three times the strength at 90°. The reason for this result is that at \( \beta = 0° \), the sample failed by shear in the sample matrix, whereas at \( \beta = 90° \), failure occurred due to tensile splitting (Figure 4.1b). At these angles with higher confining stresses of 1,000–10,000 psi, failure occurs predominantly by shear. Therefore, the strength response under unconfined conditions does not have the same profile as for samples under non-zero confining stress. For the samples tested at confining stresses of 1,000–10,000 psi, the strengths at 0° and 90° are not equal, with slightly higher strengths observed at \( \beta = 90° \). Overall, the compressive strength response shows a smooth change in strength as a function of \( \beta \) with an RMSE value of 4,171 psi.

However, Figure 5.7 for Bossier shale shows a transition zone between about 10° and 35°, wherein the JPW model overestimates the strength, resulting in an increased RMSE value for the model prediction. In this regime, the strength of the shale is lower than the strength of either the shale matrix, or the plane of weakness, which is not consistent with the JPW conceptual model. In this transition zone, mixed-failure mode occurs due to competition of shale matrix and weak planes causing an uneven distribution of high stresses, resulting in shale failure at lower strength than the JPW model prediction. Also, the mixed-failure mode causes the transition from shear fracture to sliding on a weak plane to occur over a range of angles \( \beta < \beta_1 \) and \( \beta > \beta_2 \), rather than exclusively between \( \beta_1 \) and \( \beta_2 \), as predicted by the JPW model; see Figure 5.5 for the definitions of \( \beta_1 \) and \( \beta_2 \).

In summary, the JPW model for the Bossier shale is mostly consistent with theoretical predictions. However, the JPW model does not correctly predict the strength in the transition zone. The JPW model also does not correctly represent the strength predictions at unconfined conditions. The difference in accuracy between the data and the model is mainly caused by the poor model predictions in the transition zone 10° < \( \beta \) < 35°, where the JPW model overestimates the strength.

### 5.2.2 Vaca Muerta shale data analyzed using the JPW model

Figure 5.8 shows the triaxial compression data for the Vaca Muerta shale, along with the strength properties determined using the JPW criterion. This plot shows the experimental data from the triaxial compressive strength \( \sigma_1 \) at varying angles \( \beta \), plotted as solid squares, while the JPW model is represented by the solid lines.

Twenty-one samples were tested at confining stresses of 0 psi, 1,000 psi, 2,500 psi, 5,000 psi, and 20,000 psi. Most of the samples were tested at a confining stress of 2,500 psi to obtain the full-failure curve as a function of \( \beta \), while the three compressive strength tests conducted at 20,000 psi confining pressure were conducted to investigate the strength at very high confining stresses. Compared to the Bossier shale, the Vaca Muerta shale fit using
JPW model has better fit with RMSE of 1,851 psi. All of the Vaca Muerta shale samples show increased strengths at higher confining stresses, with maximum strengths at $\beta = 0^\circ$ and $90^\circ$, while the lowest strength occurred at $\beta = 60^\circ$. The strength response for Vaca Muerta shale samples at angles $10^\circ < \beta < 40^\circ$ does not show a significant difference in strength from the value at $\beta = 0^\circ$, which is different from the Bossier shale, which showed a sharper variation in strength with angle $\beta$. One sample showed an unexpectedly higher strength at a confining stress of 5,000 psi and an angle of $\beta = 80^\circ$; this was likely due to sample heterogeneity.

Overall, the compressive strength response for the Vaca Muerta shale shows a sharper change in strength with angle $\beta$, as compared to the Bossier shale. The Vaca Muerta shale exhibited a nearly uniform strength in the range of angles $0^\circ < \beta < 40^\circ$, and did not show any transition zone; its behavior was therefore qualitatively consistent with the assumptions of the JPW model.

![Figure 5.8. Vaca Muerta shale fit with the JPW model using RMSE.](image)

### 5.3 JPW Strength anisotropic ratio (SAR)

Strength anisotropic ratio (SAR) is defined as the ratio of the average strength at the two angles of $\beta = 0^\circ$ and $90^\circ$, divided by the (minimum) strength at angle $\beta_{\text{min}} = 45 + \phi_0/2$. In this section, the SAR value using the JPW criterion (SAR-JPW) is compared against experimental data to predict strength anisotropy with confining stress $\sigma_3$ using strength parameters derived from the Coulomb criterion (i.e., $\phi_0, S_0, \phi_w, S_w$).
The ISRM (1981) defines SAR as the strength perpendicular to the planes of anisotropy (maximum strength) to the strength in the weakest direction (Saroglou and Tsambaraos, 2007b). Meanwhile, Ramamurthy (1993) only considered the unconfined (UCS) condition, and defined SAR as the maximum UCS strength over the minimum UCS strength. Neither of these approaches clearly defines SAR as a function of confining stress, \( \sigma_3 \).

Combining the ISRM definition with the JPW model, an equation to express SAR at varying stress levels is proposed in this study. The failure criteria for intact rock fracture, and failure along the weak plane, are

\[
\tau_m = \sigma_m \sin \phi_o + S_o \cos \phi_o, \quad \text{(5.3.1)}
\]

\[
\tau_m = (S_w \cos \phi_w + \sigma_m \sin \phi_w) / [\sin(2\beta - \phi_w)]. \quad \text{(5.3.2)}
\]

Figure 5.9 shows the intact rock fracture strength at \( a (\beta = 0^\circ) \) and \( b (\beta = 90^\circ) \) divided by the minimum strength for failure along the weak plane at \( c (\beta_{\text{min}} = 45 + \phi_w/2) \). Inserting the angle \( \beta_{\text{min}} \) into Eqn. 5.3.2 for the minimum strength, and using the Coulomb Eqn. 5.3.1 for intact rock fracture, the SAR-JPW is

\[
\text{SAR \text{-JPW}} = \left[ \sigma_m \sin \phi_o + S_o \cos \phi_o \right] / \left[ \sigma_m \sin \phi_w + S_w \cos \phi_w \right]. \quad \text{(5.3.3)}
\]

The denominator of Equation 5.3.3, which is the failure criterion for failure along the weak plane, resembles the Coulomb criterion for intact rock fracture, because when \( \beta_{\text{min}} = 45 + \phi_w/2 \) is inserted into Eqn. 5.3.2, \( \sin(2\beta - \phi_w) = 1 \), and the angle \( \beta \) disappears from the equation.

![Figure 5.9. SAR defined using the JPW model.](image)

The SAR-JPW in Eqn. 5.3.3 expressed in terms of \( \tau_m \) and \( \sigma_m \) does not clearly show how strength, \( \sigma_1 \), varies with confining stress, \( \sigma_3 \). By inserting \( \tau_m = (\sigma_1 - \sigma_3)/2 \) and \( \sigma_m = (\sigma_1 + \sigma_3)/2 \) into Eqn. 5.3.1 and 5.3.2, the SAR-JPW of strength \( \sigma_1 \) can be expressed as a function of confining stress \( \sigma_3 \) and strength parameters \( \phi_o, S_o, \phi_w, \) and \( S_w \) as
The Bossier shale SAR data compared to the SAR-JPW value in Figure 5.10 shows that SAR reduces with increasing confining stress, $\sigma_3$. The SAR-JPW value (solid red line) in Figure 5.10 shows a reducing SAR, but at a lesser rate compared to the actual SAR computed directly from the data (solid blue line with data points).

\[
\text{SAR JPW} = \left[ \frac{\sigma_3 (1+\sin \phi_0) + 2S_o \cos \phi_0}{\sigma_3 (1+\sin \phi_w) + 2S_w \cos \phi_w} \right] \times \frac{1-\sin \phi_w}{1-\sin \phi_0}.
\]

Figure 5.10. SAR-JPW plot for Bossier shale.

Figure 5.11. SAR-JPW plot for Vaca Muerta shale.
This difference between the SAR-JPW and the SAR computed directly from the experimental data is consistent with the strength predictions observed for the Bossier shale (Figure 5.7), whereby the model shows poor fit at unconfined (UCS) stress condition, gradually improving to a better fit at higher confining stresses. Since the SAR-JPW model is derived from the Coulomb criterion, the poor fit for the Bossier shale, similar to the JPW model, is expected. This also means that the SAR-JPW value is less reliable at unconfined conditions, but shows closer agreement to experimental data at higher confining stresses. This may be due to the fact that the Bossier shale has high strength anisotropy and is poorly represented by the JPW criterion, with significant error around the transition zone. The Bossier shale SAR data and SAR-JPW are also very different because SAR-JPW is derived using averaged Coulomb parameters over the different stress levels. These averaged parameters are unable to capture the actual change in high SAR data with varying stress. More experiments for different types of high SAR shales may be necessary to improve the understanding of this behavior.

For the Vaca Muerta shale, the SAR-JPW values, compared to data in Figure 5.11, show better agreement between model and experimental data, with lower SAR at higher confining stresses. The SAR-JPW (solid red line) shows a good fit to the experimental data (solid blue line with data points), displaying the reducing trend of SAR at increased confining stress. This good agreement with SAR-JPW is also observed for the strength predictions of the Vaca Muerta shale (Figure 5.8), which shows a good fit with the JPW strength model. In summary, for the Vaca Muerta shale, the SAR-JPW successfully predicts strength anisotropy from unconfined stress condition to higher confining stress up to $\sigma_3 = 20,000$ psi. This may be due to the fact that the Vaca Muerta shale has low strength anisotropy, and is represented reasonably well by the JPW criterion.

### 5.4 JPW model applied to triaxial extension

For many years, various researchers studied the roles of $\sigma_2$ for failure of isotropic rocks. Mogi (1967) demonstrated the effect of $\sigma_2$ for Dunham dolomite and Westerly Granite, observing that the fracture angle for triaxial compression and extension are notably different. The difference in fracture angle and higher strength for extension than compression suggests that the role of $\sigma_2$ cannot be ignored. In the same study, Mogi (1967) also studied the Solenhofen Limestone, observing that compression and extension failure strength are the same, suggesting that the role of $\sigma_2$ is not significant. However, the Solenhofen limestone triaxial extension results reported by Mogi had a lower fracture angle $\beta$ than in compression (Mogi, 2007). This inconsistency is not fully understood when only principal stresses are evaluated, without having a closer look at the fracture angle $\beta$.

Pariseau (2012a) explained the importance of a symmetry requirement for failure criteria and highlighted that since most criteria are plotted as $\sigma_1 = f(\sigma_3)$, this symmetry requirement
is often overlooked. The Coulomb criterion satisfies the symmetry requirement and does not have any deficiencies when the proper reference for failure angle $\beta$ is considered.

For the triaxial extension condition, it is obvious from the Mohr circle in Figure 5.12 that the difference between triaxial compression and extension is a shift by $\pi$. This shift by $\pi$ on the Mohr circle is represented by $\beta = \pi/2$ on a sample and the JPW failure criterion. Applying this shift of angle $\beta$ into the JPW failure criterion described in Eqn. 5.1.17 for triaxial compression, for triaxial extension condition the failure criterion shifted by $\beta + \pi/2$ is

$$\tau_m = (S \cos \phi + \sigma_m \sin \phi) / \sin[2(\beta + \pi/2) - \phi].$$  \hspace{1cm} (5.4.1)

Using the trigonometric relationship, $\sin(A + B) = \sin A \cos B + \cos A \sin B$ in Eqn. 5.4.1, the JPW criterion for triaxial extension is

$$\tau_m = -(S \cos \phi + \sigma_m \sin \phi) / \sin(2\beta - \phi).$$  \hspace{1cm} (5.4.2)

Equation 5.4.2 for triaxial extension has the same magnitude as Eqn. 5.1.17 for triaxial compression, but in opposite directions. This relationship for compression and extension is shown graphically on the Mohr circles in Figure 5.12. Theoretically, if the role of $\sigma_2$ is insignificant, the compression failure envelope on the Mohr circle forms a mirror image relationship with the extension failure envelope, along the $x$-axis.

![Figure 5.12. Mohr circle for triaxial compression and extension.](image)

To determine the role of $\sigma_2$ for anisotropic rocks with failure along the weak plane, triaxial compressions experiments were compared to triaxial extension for the Bossier shale and Vaca Muerta shale. The Bossier shale triaxial compression tests at angles $\beta = 30^\circ$, $45^\circ$ and
60° at different confining stresses $\sigma_3$ were compared to five triaxial extension samples. The triaxial compression experiments were conducted at the following confining stresses $\sigma_3$ of 0 psi (UCS), 1,000 psi, 3,000 psi, 6,000 psi, and 10,000 psi. Figures 5.13a, b and c show the Mohr circle plots for Bossier shale triaxial compression strengths compared against triaxial extension strengths at the corresponding angles $\beta$.

Figure 5.13a shows the triaxial extension strengths for $\sigma_1 = \sigma_2 = 25,000$ psi and $\sigma_3 = 7,085$ psi. Comparing extension data to the Mohr circle for triaxial compression at $\sigma_3 = 6,000$ psi, it is obvious that the Mohr circle diameters for compression and extension are approximately the same size. Although only one extension test was made at the angle $\beta = 30^\circ$, the angle $\beta_2$ can be estimated using Eqn. 5.1.24, shown as the failure envelope for the extension strength in Figure 5.13a. For the Bossier shale triaxial compression samples at angle $\beta = 30^\circ$, significant discrepancy was also observed from the JPW model, mainly due to the transition zone effect. However, this transition zone effect did not show any difference in strength, when compared to extension strength.

Two triaxial extension experiments were made for Bossier shale samples that fail along the weak plane at angle $\beta = 45^\circ$, as shown in Figure 5.13b. Comparison between the triaxial compression and extension at $\beta = 45^\circ$ shows that the Mohr circle diameters in compression and extension are similar.

Lastly, Figure 5.13c shows another two triaxial extension tests at angle $\beta = 60^\circ$, compared to triaxial compression strengths at the same angle. The extensional strength at the lower stress level shows the same Mohr circle diameter as does the compressive strength at the same stress level. For the extension test at higher stress levels, interpolating between the stress levels confirms that the diameter of the Mohr circle in compression and extension have nearly the same size. The slope and intercept of the samples strength in compression and extension also do not differ significantly.

For failure along the weak plane at angle $\beta = 30^\circ$, only one sample was tested under triaxial extension conditions. Comparison between compression and extension is also possible using a $\sigma_1$ versus $\sigma_3$ plot (described in Chapter 3). However, by comparing compression and extension using Mohr circles, the differences in strength are more easily distinguishable. In summary, using the Mohr circle to compare one triaxial extension sample at $\beta = 30^\circ$, and two triaxial extension samples each at $\beta = 45^\circ$ and $\beta = 60^\circ$, there is a clear trend indicating that the Bossier shale samples do not have significantly higher strengths in extension tests than in the compression tests. This means that for the Bossier shales, for samples that fail along the weak plane, the role of $\sigma_2$ is not significant.
**Figure 5.13a.** Bossier shale triaxial compression and extension at angle $\beta = 30^\circ$.

**Figure 5.13b.** Bossier shale triaxial compression and extension at angle $\beta = 45^\circ$. Only one extension data was available (@ $\sigma_1 = 25$ kpsi), while second point was obtained using the relationships:

\[
\frac{\beta_1 + \beta_2}{2} = \left(45 + \frac{w}{2}\right) \text{ and assuming } w \text{ is the same in compression and extension.}
\]
Figure 5.13c. Bossier shale triaxial compression and extension at angle $\beta = 60^\circ$.

For the Vaca Muerta shales (Figure 5.14a and b), three triaxial extension experiments were attempted, at angles $\beta = 0^\circ$, $90^\circ$ and $60^\circ$. From the two samples tested under triaxial extension conditions at angles $\beta = 0^\circ$ and $90^\circ$, the extension sample at angle $\beta = 90^\circ$ did not fail in shear, due to end effects (Figure 4.4b), resulting in lower shear strength failure by splitting. This sample at angle $\beta = 90^\circ$ was excluded from the extension result shown in Figure 5.14a, and is not included in the analyses described here. Only the sample at angle $\beta = 0^\circ$, which sheared sufficiently through the intact rock, shows significantly higher strength than did the triaxial compression sample. Figure 5.14a shows the Mohr circles for triaxial compression at angles $\beta = 0^\circ$ and $90^\circ$, compared to the triaxial extension at $\beta = 0^\circ$. The comparison between the triaxial compression and the triaxial extension at $\sigma_3 = 1,000$ psi shows that the diameter of the Mohr circle for triaxial extension is approximately 5,000 psi larger than that for triaxial compression. This means that the sample under triaxial extension is stronger than under triaxial compression, and hence underscores the importance of the role of $\sigma_2$ for intact rock failure for the Vaca Muerta shale.

Only one sample was selected for the Vaca Muerta shale triaxial extension test for failure along the weak plane at angle $\beta = 60^\circ$. This was mainly done to see if the result observed for the Bossier shale, indicating an insignificant role of $\sigma_2$, is also observed for Vaca Muerta. Figure 5.14b shows one triaxial extension sample with principal stresses $\sigma_1 = 22,619$ psi failed at $\sigma_3 = 3109$ psi, but there are no triaxial compression experiments for $\sigma_3$ close to 3,000 psi for direct comparison. However, the average of the triaxial compression Mohr circles for $\sigma_3 = 1,000$ psi and 5,000 psi shows that the diameters of the Mohr circles for triaxial compression and extension are of approximately the same size. This means that the
Figure 5.14a. Vaca Muerta shale triaxial compression and extension at angle $\beta = 0^\circ$.

Figure 5.14b. Vaca Muerta shale triaxial compression and extension at angle $\beta = 60^\circ$. 

$$y = 0.496x + 2888.9$$
$$R^2 = 0.9962$$
Vaca Muerta shale triaxial compression and extension strengths are not significantly different, which implies that the role of $\sigma_2$ for Vaca Muerta shale that fails along the weak plane can be ignored. This outcome for Vaca Muerta shale at $\beta = 60^\circ$ also verifies the results observed for the Bossier shales.

In summary, using Mohr circles to compare the triaxial compression and extension test data, it can be said that the strength for intact rock failure are much higher for extension tests. For weak plane failure, strength in compression and extension are not very different. This means that the role of $\sigma_2$ cannot be ignored for the intact rock failure, whereas for the weak plane failure, the role of $\sigma_2$ is not significant.

5.5 JPW model applied to data from literature

The Bossier shale evaluation with the JPW model in Section 5.2 shows poorer fits compared to those obtained for the Vaca Muerta shale. Evaluation of the CT scan and thin section images shows that the JPW model for Bossier shale had mixed failure modes with lower strength at the transition zones, while the Vaca Muerta shale showed a good fit using the JPW model. In this section, similar evaluations for various anisotropic rocks from the literature are made using the JPW model, but without access to any CT or thin section images.

The different rock types from the literature used for the JPW model analysis are the Angers schist (Duveau et al., 1998), Martinsberg slate (Donath, 1964), Austin slate, Green River Shale-1, Green River Shale-2 (McLamore and Gray, 1967), Quartz Phyllite, Carbona Phyllite, Micaceous Phyllite (Ramamurthy et al., 1993), Penrhyn slate (Attewell and Sandford, 1974) and Tournemire shale (Niandou, 1997). All data sets in this study are presented in units of psi, as this will facilitate comparing strength parameters for different rock types (Appendix D1).

Table 5.1 outlines the results of analysis for anisotropic rocks from experiments and the literature, using the JPW model. The first three columns show the source of the data, while the fourth column is the RMSE (defined in Eqn. 5.2.13) showing the amount of prediction error for different rock types. The next column is the measured strength divided by the model prediction ($\sigma_{\text{actual}}/\sigma_{\text{predict}}$), which provides a slightly different type of error measurement, indicating the average deviation of the data from the model. This gives an approximate error measurement in terms of percentage, when multiplied by 100%.
Table 5.1. Summary of various data analyzed using JPW model.

<table>
<thead>
<tr>
<th>No</th>
<th>Rock type</th>
<th>Reference</th>
<th>RMSE (psi)</th>
<th>$\sigma_{1,\text{actual}} / \sigma_{1,\text{predict}}$</th>
<th>$\phi_0$ (deg)</th>
<th>$S_o$ (psi)</th>
<th>$\phi_w$ (deg)</th>
<th>$S_w$ (psi)</th>
<th>SAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bossier shale</td>
<td>present work</td>
<td>4171</td>
<td>0.961</td>
<td>29.0</td>
<td>3750</td>
<td>24.0</td>
<td>2050</td>
<td>2.017</td>
</tr>
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<td>present work</td>
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<td>1.000</td>
<td>27.0</td>
<td>4850</td>
<td>26.0</td>
<td>2650</td>
<td>1.866</td>
</tr>
<tr>
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<td>Duveau et al., 1998</td>
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<td>41.2</td>
<td>5200</td>
<td>8.2</td>
<td>1600</td>
<td>6.206</td>
</tr>
<tr>
<td>4</td>
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<td>Donath, 1964</td>
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<td>30.3</td>
<td>7100</td>
<td>16.8</td>
<td>1900</td>
<td>4.836</td>
</tr>
<tr>
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<td>22.2</td>
<td>11750</td>
<td>13.6</td>
<td>6550</td>
<td>2.101</td>
</tr>
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<td>6350</td>
<td>18.7</td>
<td>4000</td>
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<td>Ramamurthy et al., 1993</td>
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<td>2250</td>
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</tr>
<tr>
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<td>Ramamurthy et al., 1993</td>
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<td>1800</td>
<td>1.932</td>
</tr>
<tr>
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<td>Micaceous Phyllite</td>
<td>Ramamurthy et al., 1993</td>
<td>5240</td>
<td>0.972</td>
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<td>3500</td>
<td>21.4</td>
<td>1500</td>
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<td>6018</td>
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<td>14.7</td>
<td>4970</td>
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<td>12</td>
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<td>Niandou et al., 1997</td>
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<td>20.2</td>
<td>1270</td>
<td>1.568</td>
</tr>
</tbody>
</table>

After the source information, RMSE, and the ratio of $\sigma_{1,\text{actual}} / \sigma_{1,\text{predict}}$, the fifth to the eight columns of Table 5.1 show the Coulomb strength parameters determined using the best-fit method by calculating the lowest RMSE for the combination of strength parameters for a specific anisotropic rock type. The details of this approach were described in Section 5.2, and the complete evaluation for different rock types with strength plots are provided in Appendix E. The intact rocks strength parameters evaluated in this study are $19.2^\circ < \phi_0 < 41.2^\circ$ and $1,850$ psi $< S_o < 11,750$ psi, while the weak plane strength parameters are $8.2^\circ < \phi_w < 30.5^\circ$ and $1,270$ psi $< S_w < 6,550$ psi. This broad range of anisotropic strength properties covers a wide range of rock types commonly encountered in the oil and gas industry.

Lastly, in the far right column, the SARs for different rock types are listed. To distinguish the different anisotropic rock types, the SAR derived from the JPW model at UCS is adopted. Inputting the $\sigma_3 = 0$ (UCS) condition into Eqn. 5.3.4, the SAR used to distinguish different rocks types becomes

$$\text{SAR} = \left( \frac{S_o \cos \phi_0}{S_w \cos \phi_w} \right) \frac{1-\sin \phi_w}{1-\sin \phi_0}. \quad (5.5.1)$$
Equation 5.5.1 is the SAR derived from the Coulomb criterion with four strength parameters, distinguishing different rock types. From this point onwards, SAR denotes the strength anisotropic ratio calculated from Coulomb strength parameters at the $\sigma_3 = 0$ (UCS) condition. For most rocks, the SAR defined in Eqn. 5.5.1 may not directly correlate with the strength parameters derived solely from unconfined experiments, because the strength parameters used here are averaged from experimental data at different confining stress levels.

Comparing the RMSE and $\sigma_{\text{actual}} / \sigma_{\text{predict}}$ of anisotropic rocks from experiments and the literature shows that the JPW model predicts rock strength reasonably well for most orientations, but with increased mean error at the transition zones (see Appendix E, Figure E1 for plot of $\sigma_{\text{actual}} / \sigma_{\text{predict}}$ vs. angle $\beta$). The transition zone is the angle $\beta$ where the failure mode changes from shear fracture to sliding failure along the weak plane. At these transition zones, the shear fracture changes from intact rock fracture to sliding along the weak plane. Between these two failure modes, the rock fails in mixed modes, and for some rocks this change is not adequately captured by the Coulomb criterion. The gradual change in strength at transition zone results in increased RMSE for the JPW model.

Most rocks from the literature shown in Table 5.1 show higher RMSE for rocks having higher SARs (*i.e.*, Bossier shale, Angers schist, Martinsberg slate, Austin slate, Micaceous Phyllite and Penrhyn slate). For these rocks with high strength anisotropy, the JPW model probably has increased RMSE at the transition zones, whereas for the rocks with low SAR (*i.e.*, SAR < 2.0, such as the Vaca Muerta, GRS-1, GRS-2, Quartz Phyllite, Carbona Phyllite and Tournemire shale), lower RMSE is obtained.

From the evaluation of the various rock types in this study, it is also possible to explore the possible shapes of the JPW model, which is a function of the four Coulomb parameters $S_o$, $\phi_o$, $S_w$ and $\phi_w$. The difference between $S_o$ and $S_w$, and between $\phi_o$ and $\phi_w$, determines the shape of the strength anisotropy as a function of confining stress $\sigma_3$. Increases in $S_w$ and $\phi_w$ generally shift the weak plane circle upwards and closer to the isotropic line. An increase in $S_w$ will only shift the circle upward, towards the isotropic line, moving along the centerline at $\beta = 45 + \phi_w/2$. An increase in $\phi_w$ only will shift the weak plane circle upward closer to the isotropic line, and sideways towards $\beta = 90^\circ$. Therefore, an increase in $S_w$ only shifts the circle upwards, while an increase in $\phi_w$ shifts the circle upwards and sideways. The minimum of the weak plane circle cannot be less than $45^\circ$ because $\beta_{\text{min}}$ occurs at $45 + \phi_w/2$, and for most rocks $\phi_w$ is not equal to zero. The angle $\beta_1$ also cannot be less than $\phi_w$, which was also shown earlier in this chapter. These are the basics of the JPW model shapes that are a function the four Coulomb parameters.

The more difficult aspect to understand is the shape of the failure curve for the extreme cases where there is a significant difference in $S_o$, $S_w$ and $\phi_o$, $\phi_w$. For most anisotropic rocks, where $S_o > S_w$ and $\phi_o > \phi_w$, the angles $\beta_1$ increases and $\beta_2$ decreases with increase in
confining stress $\sigma_3$. However, for extreme situations, $\beta_1$ and $\beta_2$ can also decrease and increase with increase in $\sigma_3$, and this was seen for the cases presented in Appendix E for the Angers Schist and Penrhyn Slate. These two anisotropic rocks had the highest difference in $\phi_b - \phi_w$ (i.e., Angers Schist $\phi_b - \phi_w = 33^\circ$, and Penrhyn Slate $\phi_b - \phi_w = 20.4^\circ$). These two cases are unique, showing a slight decrease in $\beta_1$, and an increase in $\beta_2$ at higher confining stresses. This is a rare scenario and could have been a consequence of the data-fitting technique; this issue needs to be further explored.

Figure 5.15 shows a plot of the RMSE using the JPW model, against SAR, for different rock types. In this plot, the different rock types are grouped into four categories: low SAR shales, Shales and Phyllite, Slates, and Schist. The organic-rich mudstones in this study show higher RMSE with SAR, while the Slates and Schist generally have higher RMSE. This plot gives a general idea of expected RMSE for different rock types, and is useful for comparison with other anisotropic rocks.

![Figure 5.15. Summary of JPW model RMSE vs. SAR.](image)

5.6 JPW model using reduced numbers of data sets

In this study, the JPW model applied to different rock types shows that the JPW model provides a good fit for low SAR rocks, but has a higher mean error for high SAR rocks. However, this JPW model evaluation for different anisotropic rocks uses significant amount of triaxial compression data at various confining stress and bedding angles $\beta$. These data provide the necessary information to understand the complete strength behavior and to
best calibrate the JPW model. In reality, such extensive experiments with so much data collected would not always be feasible for practical applications. Therefore, it is useful to identify the minimum possible number of data that would be necessary to implement the JPW model, without significantly affecting the accuracy of the fits.

To carry out the reduced data analysis for the JPW model, the data sets from experiments and the literature are analyzed under three different conditions. The RMSE for the complete data set at different confining stresses and angles $\beta$ is the base case scenario used for comparison. The same data set RMSE is then determined with the JPW model at two confining stress levels, covering the highest and lowest confining stresses. Lastly, the reduced data RMSE evaluation is made at two confining stress levels, at angles $\beta = 0^\circ$, $30^\circ$, $60^\circ$ and $90^\circ$. The selection of the eight data points based on earlier experimental evidence from the Bossier and Vaca Muerta shale shows that the main changes in failure modes occurs at these angles.

The reduced data analysis best-fit parameters are used to determine the JPW model’s Coulomb strength parameters $(\phi, S_0, \phi_0, S_0)$, and these parameters are then used to determine the RMSE against the complete data set available for the different rock type. Figure 5.16 shows the comparison of the JPW model RMSE using the complete data set, the data obtained at two stress levels at varying angles $\beta$, and lastly the data obtained at two stress levels at angles $\beta = 0^\circ$, $30^\circ$, $60^\circ$ and $90^\circ$.

![Figure 5.16. JPW model data using reduced data sets for various rock types.](image-url)
The JPW model analysis for the three scenarios of base case and reduced data set in Figure 5.16 shows a general trend of increasing mean errors when using reduced data sets. The base-case scenario using complete data sets represents the ideal condition and best-fit scenario for the JPW model at varying confining stress and bedding angles $\beta$. For the reduced data set, analysis at two confining stress levels at varying angles $\beta$ shows only a slight increase in mean error compared to the base case scenario that uses all the data. This means that although only two confining stress levels were used, the data in between these two stress levels do not significantly improve the overall model accuracy. However, for the case of using reduced data at two confining stress levels at angles $\beta = 0^\circ, 30^\circ, 60^\circ$ and $90^\circ$, rock types with high SAR showed higher increase in mean error. The Angers Schist, for example, shows more than a two-fold RMSE increase, while the Penrhyn Slate has an increase of about 25% in mean error, when analyzed using data at angles $\beta = 0^\circ, 30^\circ, 60^\circ$ and $90^\circ$. The main reason for this is that the increased RMSE at the transition zone is not fully captured by the data at $\beta = 30^\circ$. This suggests that for anisotropic rocks with high SAR, the JPW model analyses using reduced numbers of data can be done without significantly increasing RMSE at two confining stress levels, but needs data at various bedding angles such as $\beta = 0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ$, and $90^\circ$. For the case of low SAR rocks, no significant increase in RMSE was observed for both the reduced data set cases.

In summary, the JPW model analysis using reduced numbers of data shows that for most anisotropic rock types with low SAR, eight strength values obtained at two confining stress levels (i.e., four strength data at each stress level) at bedding angles of $\beta = 0^\circ, 30^\circ, 60^\circ$ and $90^\circ$, would be sufficient to obtain the failure envelope without causing significant increase in RMSE. On the other hand, for the case of rocks with high strength anisotropy, fourteen strength data obtained at two stress levels (i.e., seven strength data at each stress level) and bedding angles $\beta$ at intervals of $15^\circ$ (i.e., $\beta = 0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ$ and $90^\circ$) are required in order to predict the JPW model failure envelope without significantly increasing the RMSE.
6 Pariseau’s Model

6.1 Theory and data-fitting technique

Pariseau (1968) presented a failure model for anisotropic geological media at the 10th AIME Symposium on Rock Mechanics in a paper entitled “Plasticity Theory for Anisotropic Rocks and Soils”. Pariseau developed his theory by modifying Hill’s theory for metal plasticity (Hill, 1948). The main new contribution of Pariseau was the modification of Hill’s theory to account for yielding of geomaterials under hydrostatic stress. Pariseau’s continuum model is an extension of the Drucker-Prager model that satisfies the symmetry requirements for a transversely isotropic material. Unlike the plane of weakness model, this model predicts a smooth, continuous variation of strength with bedding angle $\beta$.

Although Pariseau’s criterion has been available for more than forty years, few researchers have attempted to apply it to laboratory data on reservoir rocks. Duveau et al. (1998) used the Pariseau criterion to assess the strength of the Angers Schist and concluded that this model is comparable in accuracy to the JPW criterion. Amadei (1983) showed that the five Pariseau parameters ($F, G, U, V$ and $M$) can be determined using simplified equations from unconfined compressive, tensile and torsional strengths, at $\beta = 0^\circ, 45^\circ$ and $90^\circ$. However, Amadei did not use experimental data from different rock types to verify the robustness of this approach at different confining stresses.

Although Duveau et al. (1988) validated Pariseau’s model for the Angers schist, this model was never validated for a range of anisotropic rocks. Pariseau’s model has similarities to the Drucker-Prager model for isotropic rocks (i.e., it uses the stress invariants $J_2$ and $I_1$), as several researchers have demonstrated (Tsai and Wu, 1971; Smith and Cheatham, 1980a; Kusabukua et al., 1999). Ong and Roegiers (1993), using Pariseau’s model, showed that high-strength anisotropic rocks significantly influence the stability of horizontal wells. A similar study of wellbore stability by Suarez-Rivera et al. (2009) showed that the Pariseau strength model, combined with an anisotropic elastic rock model, provides more conservative results than are obtained by using the JPW strength model in combination with an isotropic elastic model to compute the stresses.

One possible reason for the limited previous use of the Pariseau criterion is that the process required to determine the rock parameters is not straightforward. Sufficient data representing the failure envelope are necessary to determine the rock parameters used in this criterion, and this process involves finding solutions to a global minima problem.

Despite these problems, the Pariseau criterion offers the attractive possibility of being a continuous failure criterion, as opposed to the “discontinuous” JPW model. The most exciting prospect of this model is its inclusion of the role of $\sigma_2$. This chapter describes the
Pariseau criterion for the case of conventional triaxial experiments ($\sigma_2 = \sigma_3$ and $\beta = 0^\circ$ to 90$^\circ$) and the data-fitting technique shows the method to determine the parameters for strength prediction. The true-triaxial case ($\sigma_1 > \sigma_2 > \sigma_3$) is discussed separately in Chapter 7.

Pariseau (1968) classified failure criteria as those that either ignore the intermediate stress ($\sigma_1 > \sigma_2 = \sigma_3$) or those which include the effect of $\sigma_2$ ($\sigma_1 > \sigma_2 > \sigma_3$). For isotropic rocks, the failure criterion that ignores intermediate stress, $\sigma_2$, can generally be described as

$$\left[ \frac{1}{2} (\sigma_1 - \sigma_3) \right]^n = \frac{1}{2} (\sigma_1 + \sigma_3) A_2 + B_2,$$

$$|\tau_m|^n = \sigma_m A_2 + B_2. \quad (6.1.1)$$

The parameters $A_2$, $B_2$ and $n$ ($n \geq 1$) are material parameters, while $\sigma_1$ and $\sigma_3$ are the major and minor principal stresses. For the case of an isotropic rock that includes intermediate stress $\sigma_2$, the yield condition with parameters $A_3$ and $B_3$ are

$$|J_2|^n = A_3 I_1 + B_3. \quad (6.1.2)$$

For metal plasticity, Nadai (1950) proposed that the second invariant of the deviatoric stress $J_2$ is the term that drives failure, while the first invariant of stress $I_1$ is the term that resists failure (Jaeger et al., 2007). Nadai further proposed a more general strength relationship between the driving and resisting stress of $J_2$ and $I_1$, using the function $f$ as follows:

$$\sqrt{J_2} = f(I_1). \quad (6.1.3)$$

The invariant $J_2$ is often expressed in terms of the octahedral shear stress, $\tau_{\text{oct}}$, and this relationship between $\sqrt{J_2}$ (or $\tau_{\text{oct}}$) and $I_1$ was investigated by many researchers to describe the true-triaxial failure of isotropic rocks (i.e., Drucker and Prager, 1952; Mogi, 1967; Al-Ajmi and Zimmerman, 2005). The invariants $\sqrt{J_2}$ and $I_1$ are related to the three principal and shear stresses by the following equations:

$$\sqrt{J_2} = \sqrt[1/6]{[(\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2] + \tau_{23}^2 + \tau_{31}^2 + \tau_{12}^2}, \quad (6.1.4)$$

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3. \quad (6.1.5)$$

Pariseau’s criterion is an extension of Hill’s theory that includes the invariants $J_2$ and $I_1$. Hill’s criterion for metal anisotropy in a material coordinate system for an orthotropic material is

$$F(\sigma_2' - \sigma_3')^2 + G(\sigma_3' - \sigma_1')^2 + H(\sigma_1' - \sigma_2')^2 + L\tau_{23}'^2 + M\tau_{31}'^2 + N\tau_{12}'^2 = 1, \quad (6.1.6)$$

where the prime superscript indicates material coordinate system. The material parameters $F$, $G$, $H$, $L$, $M$ and $N$ are determined experimentally. The similarity between the $J_2$ invariant (Eqn. 6.1.4) and Hill’s criterion (Eqn. 6.1.6) is interesting. Pariseau utilized this relationship...
and proposed a criterion that has a similar form as the Hill’s criterion that includes the $I_1$ term with parameters $U$, $V$, and $W$ to account for the yielding of geomaterials under hydrostatic stress (Amadei, 1983):

$$1 = \left| F(\sigma_2' - \sigma_3')^2 + G(\sigma_3' - \sigma_1')^2 + H(\sigma_1' - \sigma_2')^2 + L\tau_{23}'^2 + M\tau_{31}'^2 + N\tau_{12}'^2 \right|^\frac{1}{2} - (U\sigma_1' + V\sigma_2' + W\sigma_3'). \quad (6.1.7)$$

To determine the parameters using experimental data, Eqn. 6.1.7 is changed into the principal coordinate system using the stress transformation equations. Figure 6.1 shows the stress system and orientation used in the stress transformation equations. For the triaxial stress conditions shown in Figure 6.1, the bedding angle $\beta$ is the normal vector of the plane to $\sigma_1$, while the foliation direction is $\omega = 0^\circ$ because the angle $\omega$ is parallel to $\sigma_2 = \sigma_3$ (see also Figure 7.3 for definition of $\beta$ and $\omega$ for true-triaxial stress system).

![Figure 6.1. Stress system in principal coordinate for triaxial compression.](image)

The direction cosines that are needed to transform Eqn. 6.1.7 are (Jaeger et al., 2007, p.34):

$$e_x' = (l_{11}, l_{12}, l_{13}) = (\cos \beta \cos \omega, \cos \beta \sin \omega, -\sin \beta),$$

$$e_y' = (l_{21}, l_{22}, l_{23}) = (-\sin \omega, \cos \omega, 0),$$

$$e_z' = (l_{31}, l_{32}, l_{33}) = (\sin \beta \cos \omega, \sin \beta \sin \omega, \cos \beta). \quad (6.1.8)$$

The principal and shear stresses in the material coordinate system are

$$\tau_{xx}' = \sigma_2'; \tau_{yy}' = \sigma_3'; \tau_{zz}' = \sigma_1'; \tau_{xy}' = \sigma_{23}'; \tau_{yx}' = \sigma_{31}'; \tau_{xz}' = \sigma_{21}'. \quad (6.1.9)$$

$$\tau_{xx}' = l_{11}^2 \tau_{xx} + l_{12}^2 \tau_{yy} + l_{13}^2 \tau_{zz},$$
\[
\dot{\sigma}_2 = (\cos^2 \beta \cos^2 \omega)\sigma_2 + (\cos^2 \beta \sin^2 \omega)\sigma_3 + \sin^2 \beta \sigma_1, \\
\]

For conventional triaxial stress conditions, \(\omega = 0\) and \(\sigma_2 = \sigma_3\):

\[
\dot{\sigma}_2 = (\cos^2 \beta)\sigma_3 + (\sin^2 \beta)\sigma_1. \\
\tau_{yy} = l_{21}^2 \tau_{xx} + l_{22}^2 \tau_{yy} + l_{23}^2 \tau_{zz}, \\
\dot{\sigma}_3 = (-\sin \omega)^2 \sigma_2 + (\cos \omega)^2 \sigma_3, \\
\dot{\sigma}_3 = (\sin \omega)^2 \sigma_2 + (\cos \omega)^2 \sigma_3.
\]

For conventional triaxial stress conditions, \(\omega = 0\) and \(\sigma_2 = \sigma_3\):

\[
\dot{\sigma}_3 = \sigma_3. \\
\tau_{zz} = l_{31}^2 \tau_{xx} + l_{32}^2 \tau_{yy} + l_{33}^2 \tau_{zz}, \\
\dot{\sigma}_1 = (\sin \beta \cos \omega)^2 \sigma_2 + (\sin \beta \sin \omega)^2 \sigma_3 + (\cos \beta)^2 \sigma_1.
\]

For conventional triaxial stress conditions, \(\omega = 0\) and \(\sigma_2 = \sigma_3\):

\[
\dot{\sigma}_1 = (\sin \beta)^2 \sigma_3 + (\cos \beta)^2 \sigma_1. \\
\tau_{xy} = l_{11}l_{21} \tau_{xx} + l_{12}l_{22} \tau_{yy} + l_{13}l_{23} \tau_{zz}, \\
\dot{\sigma}_{23} = (\cos \beta \cos \omega)(-\sin \omega)\sigma_2 + (\sin \beta \sin \omega)(\cos \omega)\sigma_3.
\]

For conventional triaxial stress conditions, \(\omega = 0\) and \(\sigma_2 = \sigma_3\):

\[
\dot{\sigma}_{23} = 0. \\
\tau_{yz} = l_{21}l_{31} \tau_{xx} + l_{22}l_{32} \tau_{yy} + l_{23}l_{33} \tau_{zz}, \\
\dot{\sigma}_{31} = (-\sin \omega)(\sin \beta \cos \omega)\sigma_2 + (\cos \omega)(\sin \beta \sin \omega)\sigma_3.
\]

For conventional triaxial stress conditions, \(\omega = 0\) and \(\sigma_2 = \sigma_3\):

\[
\dot{\sigma}_{31} = 0. \\
\tau_{xz} = l_{11}l_{31} \tau_{xx} + l_{12}l_{32} \tau_{yy} + l_{13}l_{33} \tau_{zz}, \\
\sigma_{21} = [\cos \beta \sin \beta (\cos^2 \omega)]\sigma_2 + [\cos \beta \sin \beta (\sin^2 \omega)]\sigma_3 - (\cos \beta \sin \beta)\sigma_1, \\
\dot{\sigma}_{21} = (\sin \beta \cos \omega)[\sigma_2 \cos^2 \omega + \sigma_3 \sin^2 \omega - \sigma_1],
\]

For conventional triaxial stress conditions, \(\omega = 0\) and \(\sigma_2 = \sigma_3\):

\[
\dot{\sigma}_{21} = \cos \beta \sin \beta [\sigma_3 - \sigma_1]. \\
\]

95
For transverse isotropy along the bedding plane:

\[ G = H, \quad M = N, \quad V = W, \quad L = 2G + 4F. \quad (6.1.10) \]

Inserting the parameters in Eqn. 6.1.10 into Pariseau’s equation Eqn. 6.1.7 for the case \( n = 1 \) yields

\[
1 = \left[ F\left(\sigma_2' - \sigma_3\right)^2 + G\left(\sigma_3' - \sigma_1\right)^2 + \left(\sigma_1' - \sigma_2\right)^2\right]^{0.5} + (2G + 4F)\sigma_{23}^2 + M\left(\sigma_{31}^2 + \sigma_{12}^2\right) - \left[U\sigma_1' + V(\sigma_2' + \sigma_3)\right]. \quad (6.1.11)
\]

Solving Eqn. 6.1.11 with inputs from Eqns. 6.1.9a-f, inserting the principal stresses, and rearranging the parameters \( F, G, U, V \) and \( M \), results in the Pariseau equation in principal coordinates (Pariseau, 1968, p. 281 Eqn. 26a):

\[
(\sigma_1 - \sigma_3) = \frac{1 + \sigma_3(U + 2V)}{(F \sin^4 \beta + G(\cos^4 \beta + \cos^2 2\beta) + 0.25M \sin^2 2\beta)^{0.5} - (U \cos^2 \beta + V \sin^2 \beta)}.
\]

(6.1.12)

### 6.2 Pariseau model applied to data

The Pariseau criterion (Eqn. 6.1.12) for conventional triaxial stress conditions \((\sigma_1 > \sigma_2 = \sigma_3)\) was described in the previous section, wherein \( F, \ G, \ U, \ V \) and \( M \) are rock constitutive parameters determined experimentally. These parameters may be determined by solving the global minimum problem iteratively, repeated until the parameters that yield the minimum mean error is determined. Appendices B3 and B4 show the Matlab code used for the Pariseau model.

In this iterative approach, the initial parameters estimated covers a large interval of possible solutions that estimates the range of possible values. Based on the estimated parameters, the RMSE (defined in Chapter 5) is computed, and a narrower interval is then assigned. This process is repeated until the lowest possible RMSE is obtained.

In the iterative process described above, the initial range of parameters \( F, G, U, V \) and \( M \) are estimated. The method described by Amadei (1983) using UCS data provides a reasonably good first estimate, but needs tensile strength data at angle \( \beta = 0^\circ \) and \( 90^\circ \). To determine the Pariseau rock parameters iteratively, it is easier if \( F, G, U, V \) and \( M \) are each expressed in terms of strength parameters that have units of stress. This avoids the need to use very small values of the Pariseau parameters in the calibration process. The parameters \( F, G, U, V \) and \( M \) expressed in terms of the strength data in psi are \( f, g, u, v \) and \( m \):

\[
F = \frac{1}{f^2}; \quad G = \frac{1}{g^2}; \quad M = \frac{1}{m^2}; \quad U = \frac{1}{u}; \quad V = \frac{1}{v}. \quad (6.1.13)
\]
An example of this iterative process is shown in Table G1 of Appendix G. The initial range for parameters $f$, $g$, $u$, $v$ and $m$ covers a wide range of possible values, and the combination of these parameters with lowest RMSE is determined as the new reference value for the next iteration.

For the Matlab code used in this study, one cycle of iteration takes around five minutes, and with further refinement (finer grid) to increase the accuracy of the model, the process could increase to 30 minutes or beyond. However, for the accuracy that is needed for the present purposes, a five-minute cycle was used to analyze all the data sets in this study, to an accuracy of within 100 psi for all five strength parameters. For all the anisotropic rocks tested in this study, when the selected grid is finer than 1,000 psi, very little change in RMSE is observed.

The parameters $f$, $g$, $u$, $v$ and $m$ determined from the iterative process when applied into equation 6.1.13 determine the rock parameters $F$, $G$, $U$, $V$ and $M$. The Pariseau rock parameters are then used in Eqn. 6.1.12 to calculate strength $\sigma_1$ as a function of confining stress $\sigma_3$ and angle $\beta$ at failure. The physical interpretation of the Pariseau rock parameters will also be explored in the following sections.

6.2.1 Bossier shale data analyzed using Pariseau’s model

In Chapter 5, the Bossier shale fit using the JPW model showed high RMSE, especially in the transition zones. In this chapter, the same Bossier shale data set is fit using the Pariseau model. The parameters $F$, $G$, $U$, $V$ and $M$ determined iteratively are:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>2.38E-09</td>
<td>9.07E-09</td>
<td>6.67E-05</td>
<td>4.17E-05</td>
<td>1.11E-07</td>
</tr>
</tbody>
</table>

Inserting these parameters into Eqn. 6.1.12 provides the continuous failure envelopes shown in Figure 6.2.

For the Bossier shale, the UCS failure envelope at different angles $\beta$ has a poor fit with experimental data, and is not adequately represented using the Pariseau model. This is because the sample at $\beta = 90^\circ$ fails in tensile splitting (see the CT scans in Chapter 4), whereas all the other samples used in this study failed in shear. The strength predicted for the Bossier shale at angle $\beta = 90^\circ$ is slightly higher than the strength predicted at angle $\beta = 0^\circ$. This was observed for samples with $\sigma_3$ of 1,000 psi and 10,000 psi, while at $\sigma_3$ of 3,000 psi and 6,000 psi the sample strength are almost the same at angles $\beta = 0^\circ$ and $90^\circ$. 
The Pariseau model for the Bossier shale provides a better fit than does the JPW model, with RMSE of 3,248 psi, which is lower than the RMSE of the JPW model, which was 4,171 psi. Unlike the JPW model, Pariseau’s model consists of a single equation over the entire range of angles, representing the complex transition of compressive strength from intact rock shear fracture, mixed mode, and sliding along weak planes reasonably well. Another attribute of the Pariseau’s model is that it is able to capture possible strength differences at $\beta = 0^\circ$ and $90^\circ$, while the JPW model assumes that strengths are equal at these angles. In this regard, it is worth noting that the JPW model contains four adjustable parameters, whereas Pariseau’s model contains five.

In summary, analysis of the Bossier shale using Pariseau’s model provides a better fit than does the JPW model. Pariseau’s model also gives a better representation of the compressive strengths at the transition zone. However, the same Pariseau model parameters determined at different stress levels are not applicable for unconfined condition at angle $\beta = 90^\circ$, because the failure mode is that of tensile splitting, resulting in lower strength.

Another observation from the analysis of the Bossier shale shows that the form of the failure envelope can be further explored to examine what controls the minimum $\beta$ (for Bossier shale $\beta_{\text{min}}$ this is close to $45^\circ$) and the slight strength difference observed at $\beta = 0^\circ$ and $90^\circ$. This will be explored further in the following sections.
6.2.2 Vaca Muerta shale data analyzed using Pariseau’s model

Compared to the Bossier shale, the Vaca Muerta shale is less argillaceous, and has lower strength anisotropy. In this section, the Vaca Muerta shale is fit using Pariseau’s model using the same approach described earlier. The parameters \( F, G, U, V \) and \( M \) determined for Vaca Muerta shale are:

Table 6.2. Summary of Pariseau Parameters for Vaca Muerta shale.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( F ) [psi(^2)]</th>
<th>( G ) [psi(^2)]</th>
<th>( U ) [psi(^{-1})]</th>
<th>( V ) [psi(^{-1})]</th>
<th>( M ) [psi(^2)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>6.40E-09</td>
<td>4.16E-09</td>
<td>2.99E-05</td>
<td>3.77E-05</td>
<td>4.00E-08</td>
</tr>
</tbody>
</table>

The Vaca Muerta shale data fit using Pariseau’s model gave an RMSE of 2,390 psi, which is higher than the RMSE obtained using the JPW model. Applying these parameters in Eqn. 6.1.12 provides the failure envelopes shown in Figure 6.3.

The Vaca Muerta shale was tested at confining stresses from 0 psi up to 20,000 psi, and Pariseau’s model follows the general trend of the failure envelope at all the tested stress levels. Looking in detail, the difference in RMSE between the Pariseau and JPW model for the Vaca Muerta shale is mainly due to the increased errors near \( \beta = 40^\circ \), where the smooth transition zone occurred for the Bossier shale. This transition zone does not occur for Vaca Muerta, resulting in higher RMSE for Pariseau’s model.

Figure 6.3. Vaca Muerta shale failure data fit using the Pariseau’s model.
In summary, the Vaca Muerta shale yields a higher RMSE when analyzed using Pariseau’s model than with the JPW model. This is despite the fact that Pariseau’s model contains one more adjustable parameter than does the JPW model. The Vaca Muerta shale has low strength anisotropy, and does not exhibit any significant transition zone, as had been observed for the Bossier shale. Based on this comparison, it can be concluded that Pariseau’s model gives a poorer fit than does the JPW model, for the Vaca Muerta shale.

6.3 Pariseau Strength Anisotropy Ratio (SAR)

The strength anisotropy ratio (SAR) for the JPW model was defined in Section 5.3, where it was discussed in light of the JPW model. Using the same approach, this section attempts to quantify SAR using the Pariseau model; the numerical value will be denoted by SAR-Pariseau. SAR-Pariseau is the ratio of maximum to minimum strength at a specified confining stress level, as determined by the equation that has been fitted to the laboratory failure data.

The SAR for Pariseau’s model is used to determine if the model is able to capture the reducing strength anisotropic behavior of shales at higher stress levels. It is straightforward to use Pariseau’s failure criterion in Eqn. 6.1.12 to define the SAR equation. According to Pariseau’s model, unlike the JPW, there can be differences in the maximum strength at angles $\beta = 0^\circ$ and $90^\circ$. From Eqns. 6.3.1 and 6.3.2, the ratio of maximum strength at $\beta = 0^\circ$ and $90^\circ$ to minimum strength at $\beta_{\text{min}}$ can be determined. For the case of $\beta = 0^\circ$ and $90^\circ$, the strength defined in Eqn. 6.1.12 reduces to the following expressions, respectively:

\[
(\sigma_1 - \sigma_3)_{\beta=0^\circ} = \frac{1+\sigma_3(U+2V)}{(2G)^{0.5}-(U)},
\]

\[
(\sigma_1 - \sigma_3)_{\beta=90^\circ} = \frac{1+\sigma_3(U+2V)}{(F+G)^{0.5}-(V)}.
\]

Using Pariseau’s equation, the minimum strength at $\beta_{\text{min}}$ can be determined by differentiating Eqn. 6.1.12, to find that its minimum occurs at an angle given by

\[
\beta_{\text{min}} = \tan^{-1}\left[\left(\frac{M-2F-4G}{M-6G}\right)^{0.5}\right].
\]

Similar to the approach used for the SAR-JPW model, using the results from Eqn. 6.3.1, Eqn. 6.3.2 and Eqn. 6.3.3, SAR-Pariseau can be expressed as

\[
0.5 * \left[\frac{1}{(2G)^{0.5}-(U)} + \frac{1}{(F+G)^{0.5}-(V)}\right] = \frac{1}{(F \sin^4 \beta_{\text{min}}+G\cos^4 \beta_{\text{min}}+\cos^2 2\beta_{\text{min}}+0.25M\sin^2 2\beta_{\text{min}})^{0.5}-(U\cos^2 \beta_{\text{min}}+V\sin^2 \beta_{\text{min}})}.
\]

\[
(6.3.4)
\]
Equation 6.3.4 cannot be further simplified, and remains a relatively complicated equation, containing all five Pariseau parameters, \( \{F, G, U, V, M\} \). Upon further examination, it becomes clear that the confining stress \( \sigma_3 \) cancels out from the SAR-Pariseau term in Eqn. 6.3.4. This means that the SAR-Pariseau in Eqn. 6.3.4 would not change as a function of confining stress \( \sigma_3 \). Instead, the SAR-Pariseau value is constant with confining stress. Based on this evidence, it is clear that the SAR-Pariseau parameter is not suitable for determining the change in strength anisotropy with confining stress. Various other attempts (not detailed here) to use the Pariseau parameters to determine SAR have also been unsuccessful. The SAR-Pariseau equation with \( \sigma_3 \) cancelling highlights the fact that this mathematical equation is a continuous criterion, whereas the JPW criterion is discontinuous.

Evaluation of Eqns. 6.3.1 to 6.3.3 provides some interesting information regarding the physical interpretation of the parameters \( F, G, U, V \) and \( M \). Equations 6.3.1 and 6.3.2 at \( \beta = 0^\circ \) and \( 90^\circ \) is comparable to the intact rock strength behavior, wherein at these angles the parameter \( M \) does not appear. However, at angles \( 0^\circ < \beta < 90^\circ \), parameter \( M \) is relevant with minimum strength defined at angle \( \beta \) described in Eqn. 6.3.3. This means that the parameter \( M \) is strongly related to the weak plane strength behavior.

Comparison of Eqn. 6.3.1 and 6.3.2 also shows that the intact rock strength is strongly influenced by the parameters \( F \) and \( G \) wherein the denominator \( 2G \) in Eqn. 6.3.1 for \( \beta = 0^\circ \) becomes \( F+G \) in Eqn. 6.3.2 at \( \beta = 90^\circ \). At these angles, the denominator \( U \) at \( \beta = 0^\circ \) becomes \( V \) at \( \beta = 90^\circ \), which means that the intact rock strength and strength difference observed for the Pariseau criterion at angles \( \beta = 0^\circ \) and \( 90^\circ \) is due to the parameters \( F, G \) and \( U, V \). These four parameters are therefore strongly related to the intact rock strength behavior and the strength difference observed at \( \beta = 0^\circ \) and \( 90^\circ \).

Parameters \( U \) and \( V \) that represent hydrostatic yielding are not present in Eqn. 6.3.3 where strength is lowest, but remain relevant terms for strength prediction at all angles \( \beta \). This means that the parameters \( U \) and \( V \) relate to both the intact rock and weak plane strength behavior. Some comparison of Eqn. 6.3.3 from Pariseau’s model and Eqn. 5.1.29 from the JPW model may also give an interesting relationship between \( \phi_w \) and parameters \( M, F, \) and \( G \). However, such evaluations for the available data set did not provide any useful relationship between these two models.

6.4 Pariseau’s model applied to triaxial extension

For the stress condition described in Figure 6.1, Pariseau’s model for triaxial compression (Eqns. 6.1.7 to 6.1.11) gives the failure criterion shown in Eqn. 6.1.12. For the case of triaxial extension, the stress condition is different, whereby the major principal stress \( \sigma_1 \) is the hydrostatic confinement, while the minor principal stress \( \sigma_3 \) is the axial stress at failure.
Figure 6.4 shows the stresses acting in the triaxial extension system. Introducing this stress condition into the transformation equation 6.1.9a to 6.1.9f, the principal stresses $\sigma_1$ and $\sigma_3$ are expressed for triaxial extension stress conditions as follows:

$$
\sigma_2' = (\cos^2 \beta)\sigma_1 + (\sin^2 \beta)\sigma_3, \quad (6.4.1a)
$$

$$
\sigma_3' = \sigma_1, \quad (6.4.1b)
$$

$$
\sigma_1' = (\sin \beta)^2\sigma_1 + (\cos \beta)^2\sigma_3, \quad (6.4.1c)
$$

$$
\sigma_{23}' = 0, \quad (6.4.1d)
$$

$$
\sigma_{31}' = 0, \quad (6.4.1e)
$$

$$
\sigma_{21}' = \cos \beta \sin \beta [\sigma_1 - \sigma_3]. \quad (6.4.1f)
$$

where again the superscript prime indicates material coordinate system.

Inserting equation 6.4.1a to 6.4.1f into Pariseau’s equation (Eqn. 6.1.7), the failure criterion for triaxial extension becomes

$$
\left(\sigma_1 - \sigma_3\right) = \frac{1 + \sigma_1(U + 2V)}{(F \sin^4 \beta + G(\cos^4 \beta + \cos^2 2\beta) + 0.25M \sin^2 2\beta)^{0.5} + (U \cos^2 \beta + V \sin^2 \beta)}.
$$

(6.4.2)

The extension criterion (Eqn. 6.4.2) is similar to the compression criterion (Eqn. 6.1.12), but for triaxial extension, $\sigma_3$ replaces $\sigma_1$, and the denominator term contains contributions from both the $U$ and $V$ parameters.

For the Bossier shale, triaxial extension results for five extension tests in Figure 6.5 show similar trends to that observed for triaxial compression, with a reasonably good fit for all five extension data. For the Vaca Muerta shale, only two triaxial extension experimental data are available, and there is insufficient data to fit using the Pariseau model. Therefore, for the case where few data is available, the JPW extension model is better suited to analyze
the strength properties. The Vaca Muerta extension data was presented earlier using the JPW model in Chapter 5.

Figure 6.5. Bossier shale triaxial extension data fit using Pariseau failure criterion.

6.5 Pariseau’s model applied to data from literature

The Bossier shale, Vaca Muerta shale and various data sets from literature are analyzed using Pariseau’s model, with the results summarized in Table 6.3. A summary of all the Pariseau parameters $F$, $G$, $U$, $V$ and $M$ applied into the failure criterion is shown in Table 6.4. An overall comparison of the RMSE of all the anisotropic rocks shows that the RMSE obtained from using Pariseau’s model is lower than the one obtained from the JPW model, for most of these rocks.

Detailed comparison of the RMSE in Table 6.3 for Pariseau’s model against Table 5.1 for the JPW model (Chapter 5) shows that the Pariseau model provides a better fit for ten of the twelve rocks, whereas the JPW model provide a better fit only for the Vaca Muerta shale (SAR = 1.866) and the Green River Shale-1 (SAR = 1.522). It is also noted that all rocks with SAR > 2 were fit more closely by the Pariseau model, whereas both shales that were better fit by the JPW model had SAR < 2. Nevertheless, three rocks with strength anisotropy ratios of less than 2 were fit more accurately by the Pariseau model. Therefore, the exact range of SAR for which the Pariseau model or JPW model is better cannot be determined definitively.

Referring to the individual failure surfaces (see Appendix F for analysis of each rock type using Pariseau’s model), it can be observed that Pariseau’s model seems to be able to capture more complex failure surfaces for anisotropic rocks, with strengths at an angle $\beta =$
0° that are different from the strength at angle $\beta = 90°$. This asymmetric strength behavior at angles $\beta = 0°$ and $90°$ is observed for the Angers schist, Martinsberg slate, Austin slate, Micaceous Phyllite and Penrhyn slate. The Pariseau failure envelopes are in general more representative of the smooth natural change from intact rock fracture to weak plane failure. The average ratio $\sigma_{\text{actual}}/\sigma_{\text{predict}}$ for all data sets using the Pariseau model is close to 1.0, which means that for all cases the strength prediction is reasonably good.

**Table 6.3. Summary of RMSE, SAR and related data using Pariseau’s model.**

<table>
<thead>
<tr>
<th>No.</th>
<th>Rock type</th>
<th>References</th>
<th>RMSE (psi)</th>
<th>$\sigma_{\text{actual}}/\sigma_{\text{predict}}$</th>
<th>$f$ (psi)</th>
<th>$g$ (psi)</th>
<th>$u$ (psi)</th>
<th>$v$ (psi)</th>
<th>$m$ (psi)</th>
<th>SAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bossier shale</td>
<td>present work</td>
<td>3248</td>
<td>0.951</td>
<td>20500</td>
<td>10500</td>
<td>15000</td>
<td>24000</td>
<td>3000</td>
<td>2.017</td>
</tr>
<tr>
<td>2</td>
<td>Vaca Muerta shale</td>
<td>present work</td>
<td>2390</td>
<td>0.981</td>
<td>12500</td>
<td>15500</td>
<td>33500</td>
<td>26500</td>
<td>5000</td>
<td>1.866</td>
</tr>
<tr>
<td>3</td>
<td>Angiers Schist</td>
<td>Duveau et al., 1998</td>
<td>2989</td>
<td>0.916</td>
<td>55000</td>
<td>23000</td>
<td>33500</td>
<td>65000</td>
<td>9500</td>
<td>6.206</td>
</tr>
<tr>
<td>4</td>
<td>Martinsburg Slate</td>
<td>Donath, 1964</td>
<td>5409</td>
<td>0.919</td>
<td>22000</td>
<td>15000</td>
<td>27000</td>
<td>84500</td>
<td>5500</td>
<td>4.836</td>
</tr>
<tr>
<td>5</td>
<td>Austin Slate</td>
<td>McLamore &amp; Gray, 1967</td>
<td>4453</td>
<td>0.994</td>
<td>270000</td>
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<td>10000</td>
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<td>1500</td>
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</tr>
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<td>57000</td>
<td>3500</td>
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<tr>
<td>7</td>
<td>GreenRiver Shale 2</td>
<td>McLamore &amp; Gray, 1967</td>
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<td>0.994</td>
<td>220000</td>
<td>29500</td>
<td>48000</td>
<td>119000</td>
<td>7500</td>
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<td>8</td>
<td>Quartz Phyllite</td>
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<td>2273</td>
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<td>8500</td>
<td>10500</td>
<td>21000</td>
<td>3000</td>
<td>1.746</td>
</tr>
<tr>
<td>9</td>
<td>Carbona Phyllite</td>
<td>Ramamurthy et al., 1993</td>
<td>2109</td>
<td>0.967</td>
<td>8500</td>
<td>10000</td>
<td>15000</td>
<td>14500</td>
<td>3000</td>
<td>1.932</td>
</tr>
<tr>
<td>10</td>
<td>Micaceous Phyllite</td>
<td>Ramamurthy et al., 1993</td>
<td>3284</td>
<td>1.003</td>
<td>26000</td>
<td>7500</td>
<td>9500</td>
<td>14000</td>
<td>1500</td>
<td>3.084</td>
</tr>
<tr>
<td>11</td>
<td>Penrhyn Slate</td>
<td>Attewell &amp; Sandford, 1974</td>
<td>3472</td>
<td>0.980</td>
<td>16000</td>
<td>28500</td>
<td>51500</td>
<td>27500</td>
<td>5000</td>
<td>2.095</td>
</tr>
<tr>
<td>12</td>
<td>Tournemire Shale</td>
<td>Niandou et al., 1997</td>
<td>1070</td>
<td>0.995</td>
<td>6000</td>
<td>5000</td>
<td>10000</td>
<td>11500</td>
<td>1500</td>
<td>1.568</td>
</tr>
</tbody>
</table>

Evaluation of various rock types categorized based on SAR in Figure 6.6 shows a lower RMSE using Pariseau’s model compared to the JPW model (Figure 5.15). The plot in Figure 6.6 using Pariseau’s model shows that this suite of anisotropic rocks have RMSE values that range from 1,070 psi to 5,409 psi, with the highest RMSE obtained for the Martinsberg slate. The Angers Schist, which has the highest SAR of 6.2, shows an average RMSE of approximately 2,989 psi. For the Pariseau model RMSE versus SAR plot, less scatter is observed, whereby the Shales, Phyllite and Slate can be grouped into the lower RMSE while Martinsberg slate and Angers schist fall in a different region with higher SAR. Compared to the JPW model (Figure 5.15), Figure 6.6 for the Pariseau model is not able to easily distinguish different rock types.

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bossier shale</td>
<td>present work</td>
<td>2.38E-09</td>
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<td>6.67E-05</td>
<td>4.17E-05</td>
<td>1.11E-07</td>
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<tr>
<td>2</td>
<td>Vaca Muerta shale</td>
<td>present work</td>
<td>6.40E-09</td>
<td>4.16E-09</td>
<td>2.99E-05</td>
<td>3.77E-05</td>
<td>4.00E-08</td>
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<tr>
<td>3</td>
<td>Angiers Schist</td>
<td>Duveau et al., 1998</td>
<td>3.31E-10</td>
<td>1.89E-09</td>
<td>2.99E-05</td>
<td>1.54E-05</td>
<td>1.11E-08</td>
</tr>
<tr>
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<td>Donath, 1964</td>
<td>2.07E-09</td>
<td>4.44E-09</td>
<td>3.70E-05</td>
<td>1.18E-05</td>
<td>3.31E-08</td>
</tr>
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<td>Austin Slate</td>
<td>McLamore &amp; Gray, 1967</td>
<td>1.37E-11</td>
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<td>1.00E-04</td>
<td>4.88E-05</td>
<td>4.44E-07</td>
</tr>
<tr>
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<td>7</td>
<td>GreenRiver Shale 2</td>
<td>McLamore &amp; Gray, 1967</td>
<td>2.07E-11</td>
<td>1.15E-09</td>
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<td>8.40E-06</td>
<td>1.78E-08</td>
</tr>
<tr>
<td>8</td>
<td>Quartz Phyllite</td>
<td>Ramamurthy et al., 1993</td>
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<td>9.52E-05</td>
<td>4.76E-05</td>
<td>1.11E-07</td>
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<td>9</td>
<td>Carbona Phyllite</td>
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<td>6.90E-05</td>
<td>1.11E-07</td>
</tr>
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<td>4.44E-07</td>
</tr>
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<td>Attewell &amp; Sandford, 1974</td>
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<tr>
<td>12</td>
<td>Tournemire Shale</td>
<td>Niandou et al., 1997</td>
<td>2.78E-08</td>
<td>4.00E-08</td>
<td>1.00E-04</td>
<td>8.70E-05</td>
<td>4.44E-07</td>
</tr>
</tbody>
</table>

Figure 6.6. Summary of Pariseau model RMSE vs. SAR.
6.6 Pariseau model using reduced numbers of data sets

The Pariseau model evaluated above provides a better fit than the JPW model, for most of the anisotropic rocks analyzed. However, the Pariseau model evaluation was made using significant amounts of data. In reality, for practical applications, fewer triaxial experiments are usually conducted to determine the rock strength parameters. It is therefore useful to identify the least number of data that would be needed in order to be able to use the Pariseau model, without significantly affecting its accuracy. For the reduced data analysis, the same anisotropic rock types from experiments and literature described in section 5.6 are used.

The three scenarios for the reduced data analysis are (a) the base case using all data sets, (b) data obtained at two confining stress levels at varying bedding angles $\beta$, and (c) data obtained at two confining stress levels at bedding angles $\beta = 0^\circ$, $30^\circ$, $60^\circ$ and $90^\circ$. Figure 6.7 shows the comparison of the Pariseau model RMSE using the complete data set, data at two confining stress levels at varying angles $\beta$, and for the case of two confining stress levels at angles $\beta = 0^\circ$, $30^\circ$, $60^\circ$ and $90^\circ$.

![Figure 6.7. Pariseau model data using reduced data sets for various rock types.](image)

In Figure 6.7, the Pariseau model using the complete data set provides the base case scenario have the lowest RMSE. For the case of reduced data at two confining stress levels at varying angles $\beta$, an increase of RMSE of around 5-20% is observed. The highest increase
in RMSE for the Martinsburg Slate is about 20%. However, this increase in RMSE must be viewed in light of the fact that only fourteen data points were used for the reduced data analysis, compared with the forty-two total data points used in the full analysis for the Martinsburg slate.

Lastly, for the reduced data analysis using two confining stress levels at bedding angles $\beta = 0^\circ, 30^\circ, 60^\circ$ and $90^\circ$, no significant increase in RMSE was observed, and in some cases this method of fitting gives a lower RMSE than was obtained using data at a larger number of bedding angles $\beta$. For low SAR shales such as Vaca Muerta and Green River Shale 2, the reduced data analysis and the base case scenario have almost the same RMSE, whereas for most high SAR rocks, the reduced data analysis using two confining stress at bedding angles $\beta = 0^\circ, 30^\circ, 60^\circ$ and $90^\circ$ does not show a significant increase in RMSE.

In summary, the Pariseau model reduced data analysis at two confining stress levels and bedding angles $\beta = 0^\circ, 30^\circ, 60^\circ$ and $90^\circ$ does not significantly increase the RMSE when compared to the base case scenario using the complete data set. The reduced data analysis shows approximately a less than 15% increase in RMSE, compared to the base case scenario. This reduced data analysis and associated increase in RMSE is especially useful for determining the minimum number of data required when using Pariseau’s model. This analysis shows that eight data points, obtained at two confining stress levels and at bedding angles $\beta = 0^\circ, 30^\circ, 60^\circ$ and $90^\circ$, is recommended as the minimum data needed for using Pariseau’s model.

### 6.7 Pariseau model sensitivity analysis

The analysis in section 6.5 shows that the Pariseau model is able to represent the failure behavior of various anisotropic rock types reasonably well. The main parameters used in this model are the Pariseau rock parameters $F$, $G$, $U$, $V$ and $M$ obtained from fitting the model to triaxial compression strength data sets. However, the sensitivity of the strength to the Pariseau rock parameters ($F$, $G$, $U$, $V$ and $M$) is not obvious. As this parameter may vary for different rock types, the focus in this section is limited to only organic-rich shales. To determine how these independent parameters affect the Pariseau model strength prediction, a sensitivity analysis approach using the Monte Carlo simulation technique is used.

The Monte Carlo simulation offers an alternative to analytical methods for understanding a sampling distribution and evaluating its behavior in random samples (Mooney, 1997). To do this, artificially generated data (in this case confining pressure and angle $\beta$) that resemble a possible actual distribution of parameters $F$, $G$, $U$, $V$ and $M$ are made. The effect that changing these parameters would have on the strength $\sigma_1$ is determined for each parameter, and the results are plotted on a tornado chart, as shown in Figure 6.8. The
Monte Carlo sensitivity analysis made in this study uses 100,000 randomly generated parameters computed with a statistical plug-in to Excel called @RISK.

The tornado charts for organic-rich shales in Figure 6.8 shows how strength $\sigma_1$ changes when the parameters $F$, $G$, $U$, $V$ and $M$, bedding angle $\beta$ and confining pressure $\sigma_3$ are assigned within a range of possible values. The rock parameters are assumed to have a uniform distribution with 10% standard deviation. The confining pressure is the range of confining pressures for the triaxial experiments made for the individual shale, whereas for angle $\beta$ the range is between $0^\circ$ and $90^\circ$. For these three randomly assigned sets of input parameters, the output strength $\sigma_1$ is measured with respect to the five Pariseau rock parameters.

**Figure 6.8.** Pariseau rock parameters ($F$, $G$, $U$, $V$ and $M$) vs. strength $\sigma_1$ sensitivity analysis.

The sensitivity analyses for the four organic-rich shales are shown in Figure 6.8. The Y-axis in Figure 6.8 shows the Pariseau rock parameters, while the X-axis gives the corresponding strengths, $\sigma_1$. For the randomly generated Pariseau rock parameters with a uniform distribution, confining pressure and angle $\beta$, the corresponding strengths $\sigma_1$ is recorded in the tornado chart. The mean that the strength values that the tornado chart shows, correspond to the mean Pariseau parameters, angle $\beta$ and confining pressure. On Figure 6.8, the left and right fluctuation around the mean of strength $\sigma_1$ corresponds to change in the Pariseau rock parameters $F$, $G$, $U$, $V$ and $M$. 

108
For the Bossier shale, GRS-1 (Green River Shale-1) and GRS-2 (Green River Shale-2), the parameter $U$ has the greatest effect on strength $\sigma_1$, whereas for Vaca Muerta shale, strength is most sensitive to the parameter $V$. Both $U$ and $V$ are hydrostatic yielding parameters introduced by Pariseau, and are now seen to be very important components for the Pariseau strength failure criterion.

The tornado charts also show that the least influential rock strength parameter is $F$, for all four shales presented here. The impact of parameter $F$ on these shales is almost negligible, and in fact, it could be set to zero without significantly degrading the model’s predictions. The other rock parameter $G$ seems to be important, and plays a significant role in strength prediction, although $G$ is not the most influential of the strength parameters. Both $F$ and $G$ are closely related to the intact rock strength as these two parameters are the main inputs at angle $\beta = 0^\circ$ and $90^\circ$ as shown in Eqn. 6.3.1 and 6.3.2.

Lastly, the shale rock strength is also reasonably sensitive to the rock parameter $M$, which is the fourth most influential parameter for three of the four shales presented here. The weak plane strength is related to the rock parameter $M$, and is significantly more important than the parameter $F$, but not as important as parameters $U$, $V$ or $G$. This relationship between weak plane strength and parameter $M$ is also seen by examining Eqn. 6.3.1 and 6.3.2.

In summary, the rock parameter $F$, $G$, $U$, $V$ and $M$ sensitivity analysis shows that the strength is most sensitive to the parameters $U$, $V$ and $G$, and least sensitive to the value of $F$. Other attempts to relate the Pariseau rock parameters to Coulomb strength parameters (i.e., $S_o$, $\phi_o$, $S_w$ and $\phi_w$) or to the elastic moduli ($E_h$, $E_v$, $v_h$, $v_v$, and $G_v$) of the shales tested in this study were unsuccessful. Therefore, although a method to determine the Pariseau rock parameters was developed in this study, further evaluations are necessary to understand the physical meaning of these parameters. This is important in order to be able to use the Pariseau criterion for practical applications.
7 Failure of Anisotropic Rocks under True-Triaxial Conditions

7.0 Introduction

Many researchers have done work to understand the role of the intermediate stress $\sigma_2$ on the strength of isotropic rocks (Drucker and Prager, 1952; Handin et al., 1967; Mogi, 1967, 1971, 1972, 1973 and 1979; Lade, 1993; Colmenares and Zoback, 2002; Al-Ajmi and Zimmerman, 2005 and 2006; Haimson, 2009; Ma and Haimson, 2012). The findings from these studies show that the effect of $\sigma_2$ on the strength of isotropic rocks is generally significant, and cannot be ignored.

Since the role of $\sigma_2$ in the failure of isotropic rocks was recognized, other researchers attempted to extend some of these isotropic criteria to anisotropic rocks, but with limited success (e.g., Tiwari & Rao, 2007; Singh et al., 1998; Zhang and Zhu, 2007; Pei, 2008). Smith and Cheatham (1980a), on the other hand, conducted true-triaxial experiments on the Green River Shale and used a $J_2$-$I_1$ type relationship to predict the strength behavior, but without considering the effect of orientation angle $\beta$ and direction $\omega$ on strength. Therefore, unlike for the case of isotropic rocks, additional studies need to be done to determine the role of $\sigma_2$ in the failure of anisotropic rocks.

Most research on strength anisotropy uses the JPW model (Jaeger, 1960) and assumes conventional triaxial conditions, $\sigma_2 = \sigma_3$. However, the JPW model can also be applied under true-triaxial stress conditions (Jaeger, 1964). Although JPW was derived using the Mohr-Coulomb criterion for intact rock, which itself has limitations under true-triaxial conditions, once the weak planes are taken into account, the JPW model does treat $\sigma_2$ and $\sigma_3$ differently, so it is worth investigating the applicability of JPW to true-triaxial data.

Pariseau (1968) modified Hill’s theory for metal anisotropy, obtaining a failure criterion for transversely isotropic rocks that incorporates the effect of $\sigma_2$. Pariseau’s model claims to be applicable under true-triaxial conditions because it intrinsically and separately accounts for the effect of both $\sigma_2$ and $\sigma_3$. However, the applicability of the Pariseau model under true-triaxial conditions is unknown, and to date untested. Therefore, this chapter aims to investigate whether or not the JPW and Pariseau models are indeed able to represent the true-triaxial strength of anisotropic rocks.

In this chapter, both the JPW and Pariseau models will be evaluated using the Chichibu Schist data (Mogi, 1979) obtained from the literature. Both true-triaxial models will be described in detail followed by evaluations using the Chichibu Schist data set.
7.1 JPW model under true-triaxial conditions

Jaeger (1960, 1964) explained that the conventional triaxial JPW model (when $\sigma_1 > \sigma_2 = \sigma_3$ or $\sigma_1 = \sigma_2 > \sigma_3$) can be also be applied under true-triaxial stress conditions ($\sigma_1 > \sigma_2 > \sigma_3$). Jaeger, however, was not able to validate the model under true-triaxial conditions, possibly due to the lack of good quality experimental data at that time. In the present study, the JPW model is tested using true-triaxial data from the Chichibu Schist (Mogi, 1979). In Chapter 5, the conventional triaxial Mohr circle was used to construct the JPW model. In this section, the JPW model is described using the true-triaxial Mohr circles. Figure 7.1a shows the stress vector unit sphere fixed by angles $\beta$ and $\omega$, whereas Figure 7.1b are the corresponding Mohr circle representation in three dimensions for determining the shear $\tau$ and normal stress $\sigma_n$ acting on a plane.

![Figure 7.1. True-triaxial Mohr circle (after Jaeger et al., 2007).](image)

Details of the Mohr circle representation for the true-triaxial stress states shown in Figure 7.1, is referred from Jaeger et al. (2007). Using this true-triaxial Mohr circle, relationships between the principal stresses and the weak plane orientations can be established to determine the shear $\tau$ and normal stress $\sigma_n$ acting on a plane at failure. For the orientations shown in Figure 7.1, the true-triaxial criterion for stresses acting on a plane can be determined for bedding angle $\beta$ ($\beta = \cos^{-1} l$) and direction $\omega$ ($\omega = \cos^{-1} n$). From the true-triaxial Mohr circles, the intercepts of the family of circles shown in Figure 7.1 determines the shear and normal stress acting on a plane. The family of circles DEF and GEH in Figure 7.1b are the $\tau$ and $\sigma_n$ at origins A and B, which vary with angles $\beta$ and $\omega$. The equation for the circles DEF and GEH shown to intersect at E in Figure 7.1b are
Simultaneously solving Eqns. 7.1.1 and 7.1.2 provides the intersection point $E$, which represents the shear $\tau$ and normal stress $\sigma_n$, acting on the plane. The normal stress acting on the plane, written as a function of the principal stresses, bedding angle $\beta$ and foliation direction $\omega$ is

$$\sigma_n = \sigma_2 - \cos^2 \omega (\sigma_2 - \sigma_3) - \cos^2 \beta (\sigma_2 - \sigma_1),$$ \hspace{1cm} (7.1.3)

while the shear stress acting on the plane is

$$\tau^2 = \cos^2 \omega (\sigma_1 - \sigma_3)(\sigma_2 - \sigma_3) - \left\{ [\sigma_1 - \sigma_2 + \cos^2 \omega (\sigma_2 - \sigma_3) + \cos^2 \beta (\sigma_2 - \sigma_1)] \times \right.$$

$$\left. \cos^2 \omega (\sigma_2 - \sigma_3) + \cos^2 \beta (\sigma_2 - \sigma_1) \right\}.$$ \hspace{1cm} (7.1.4)

Equations 7.1.3 and 7.1.4 can be simplified and presented as

$$\sigma_n = \sigma_2 - A,$$ \hspace{1cm} (7.1.5)

whereby,

$$A = \cos^2 \omega (\sigma_2 - \sigma_3) + \cos^2 \beta (\sigma_2 - \sigma_1).$$ \hspace{1cm} (7.1.6)

Inserting Eqn. 7.1.6 into Eqn. 7.1.4 simplifies the shear stress equation to

$$|\tau| = \sqrt{\cos^2 \omega (\sigma_1 - \sigma_3)(\sigma_2 - \sigma_3) - ((\sigma_1 - \sigma_2 + A) \times A)}.$$ \hspace{1cm} (7.1.7)

One quick method to check the validity of this solution is to use the graphical approach and compare the results against the numerical values of Eqns. 7.1.3 and 7.1.4 (e.g., use fictional numerical data $\sigma_1 = 100$, $\sigma_2 = 50$ and $\sigma_3 = 10$ at various angles $\beta$ and $\omega$ and compare the results against the graphical method).

Another way to verify the validity of Eqns. 7.1.3 and 7.1.4 is by applying the conventional triaxial stress condition of $\sigma_2 = \sigma_3$ in the true-triaxial equation. This leads the shear and normal stress in Eqns. 7.1.3 and 7.1.4 to become

$$\sigma_n = \frac{1}{2}(\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3)\cos 2\beta,$$ \hspace{1cm} (7.1.8)

$$\tau = -\tau_m |\sin 2\beta|.$$ \hspace{1cm} (7.1.9)

Equations 7.1.8 and 7.1.9 are the normal and shear stress acting at angle $\beta$ when $\sigma_2 = \sigma_3$ that was described for the JPW model earlier in Chapter 5. This verifies that the true-triaxial JPW criterion is the same as the JPW criterion used for conventional triaxial conditions. It is also interesting to note that when $\sigma_2 = \sigma_3$, the angle $\omega$ becomes irrelevant to the normal and shear stress terms (see Eqns. 7.1.3 and 7.1.4 for comparison).
Understanding the true-triaxial JPW criterion may also help to explain other phenomena that occur when studying the effect of \( \sigma_2 \) on strength. Some rocks are not significantly influenced by \( \sigma_2 \), and this has been demonstrated by Mogi (1967) for the Solenhofen limestone, where compression and extension strengths were almost the same. To understand how this could possibly occur for some rocks using the true-triaxial Mohr circle, a condition is examined by setting the parameter that causes the circles GEH and DEF (Figure 7.1b) to intersect the larger Mohr circle on the \( \sigma_3 - \sigma_1 \) line (Figure 7.2). Graphically, it can be shown that this intersection is achieved when \( \beta + \omega = 90^\circ \). As an example, and for ease of calculation, assume \( \beta = 60^\circ \) and \( \omega = 30^\circ \), and insert into Eqns. 7.1.3 and 7.1.4. The JPW equation for normal and shear stresses at these angles are

\[
\sigma_n = \frac{1}{4} (3\sigma_3 + \sigma_1), \quad \text{(7.1.10)}
\]

\[
|\tau| = \frac{\sqrt{3}}{4} (\sigma_1 - \sigma_3). \quad \text{(7.1.11)}
\]

![Figure 7.2. Mohr-circle intercept for \( \beta = 60^\circ \) and \( \omega = 30^\circ \) (adapted from Jaeger et al., 2007).](image)

The magnitude of the normal \( \sigma_n \) and shear stress \( \tau \) from Eqns. 7.1.10 and 7.1.11 that act on a plane is shown on Figure 7.2 in terms of principal stresses \( \sigma_1 \) and \( \sigma_3 \). Figure 7.2 shows that the normal and shear stress on the plane are represented by the intercept at the \( \sigma_1 - \sigma_3 \) circle, and from Eqns. 7.1.10 and 7.1.11, this means that shear and normal stress is independent of \( \sigma_2 \). Equations 7.1.10 and 7.1.11 show that the \( \sigma_2 \) term disappears from both the normal and shear stress equations. This example could possibly explain why some rocks show more pronounced \( \sigma_2 \) effect than others, as the shear and normal stress response is directly influenced by failure angle \( \beta \) and foliation direction \( \omega \). Therefore, when \( \beta + \omega = 90^\circ \),
an increase or decrease in the $\sigma_2$ strength effect under these conditions does not change $\sigma_n$ or $\tau$, as these terms are immune to $\sigma_2$, as seen from Eqns. 7.1.10 and 7.1.11. Hence, when the rock failure angles have the properties defined by $\beta + \omega = 90^\circ$, the role of $\sigma_2$ for the JPW criterion becomes insignificant. This $\sigma_2$ effect for different combinations of $\beta$ and $\omega$ was also verified numerically using Eqn. 7.1.3 and Eqn. 7.1.4.

### 7.2 Pariseau model under true-triaxial conditions

For conventional triaxial conditions (i.e., $\sigma_1 > \sigma_2 = \sigma_3$), Pariseau’s model was defined for the angles $\beta$ and foliation direction $\omega = 0^\circ$ (Eqn. 6.1.12). For true-triaxial stress conditions, where $\sigma_1 > \sigma_2 > \sigma_3$, the earlier assumption of $\omega = 0^\circ$ used for conventional triaxial conditions is not applicable. The following approach shows the method that is used to derive the true-triaxial Pariseau failure criterion. For an orthotropic material, the Pariseau criterion (1968) is

$$1 = \left| F(\sigma'_2 - \sigma'_3)^2 + G(\sigma'_3 - \sigma'_1)^2 + H(\sigma'_1 - \sigma'_2)^2 + L\tau_{23}^2 + M\tau_{31}^2 + N\tau_{12}^2 \right|^{\frac{1}{n}} - (U\sigma'_1 + V\sigma'_2 + W\sigma'_3). \quad (7.2.1)$$

The nine material parameters $F$, $G$, $H$, $L$, $M$, $N$, $U$, $V$ and $W$ reduce to five parameters because of symmetry along an axis (transverse isotropy) leading to the following simplification of $G = H$, $M = N$, $V = W$, $L = 2G + 4F$. For the true-triaxial Pariseau’s criterion, the same assumption of transverse isotropy applies, and for the purpose of this study, the material parameter $n = 1$ is assumed. For these conditions, Pariseau’s criterion is

$$1 = \left[ F(\sigma'_2 - \sigma'_3)^2 + G \left\{ (\sigma'_3 - \sigma'_1)^2 + (\sigma'_1 - \sigma'_2)^2 \right\} + (2G + 4F)\sigma_{23}^2 + M(\sigma_{31}^2 + \sigma_{12}^2) \right]^{0.5} - [U\sigma'_1 + V(\sigma'_2 + \sigma'_3)]. \quad (7.2.2)$$

Equation 7.2.2 is the same as that shown for conventional triaxial condition ($\sigma_2 = \sigma_3$) in Chapter 6 (Eqn. 6.1.11), wherein the principal axes of anisotropy are used as the Cartesian axes of reference. To transform Eqn. 7.2.2 from material coordinates to principal stress coordinates, the directional cosines are used, as described in Eqn. 6.1.8.

To describe the plane angle and orientation using zenith and longitudinal angles, the angle $\beta$ is the zenith angle and $\omega$ is the longitudinal angle. However, it is more convenient to specify the angles $\beta$ and $\omega$ in terms of directional cosines between the material plane and the principal stress coordinate system. Therefore, the angle $\beta$ is defined in reference to the normal vector of the plane to the direction of $\sigma_2$, whereas the direction $\omega$ is the angle of the foliation with respect to the direction of the intermediate principal stress, $\sigma_2$ (i.e., normal vector of the plane to $\sigma_3$). The definitions of angles $\beta$ and $\omega$ for a transversely isotropic material are shown in Figure 7.3. The principal and shear stresses in the material coordinate system uses the denotations described in Eqn. 6.1.9. Using the directional cosines in Eqn.
6.1.8, and the notation in Eqn. 6.1.9, the principal stress coordinate system and directional cosines input into the transformation equations are

\[ \tau_{xx} = l_{11}^2 \tau_{xx} + l_{12}^2 \tau_{yy} + l_{13}^2 \tau_{zz}, \]
\[ \sigma_2' = (\cos^2 \beta \cos^2 \omega) \sigma_2 + (\cos^2 \beta \sin^2 \omega) \sigma_3 + \sin^2 \beta \sigma_1. \]  
\[ \tau_{yy} = l_{21}^2 \tau_{xx} + l_{22}^2 \tau_{yy} + l_{23}^2 \tau_{zz}, \]

\[ \sigma_3' = (\sin \omega)^2 \sigma_2 + (\cos \omega)^2 \sigma_3. \]  
\[ \tau_{zz} = l_{31}^2 \tau_{xx} + l_{32}^2 \tau_{yy} + l_{33}^2 \tau_{zz}, \]
\[ \sigma_1' = (\sin \beta \cos \omega)^2 \sigma_2 + (\sin \beta \sin \omega)^2 \sigma_3 + (\cos \beta)^2 \sigma_1 \]
\[ \tau_{xy} = l_{11} l_{21} \tau_{xx} + l_{12} l_{22} \tau_{yy} + l_{13} l_{23} \tau_{zz}, \]
\[ \sigma_{23}' = (\cos \beta \cos \omega)(-\sin \omega) \sigma_2 + (\cos \beta \sin \omega)(\cos \omega) \sigma_3. \]  
\[ \tau_{yz} = l_{21} l_{31} \tau_{xx} + l_{22} l_{32} \tau_{yy} + l_{23} l_{33} \tau_{zz}, \]
\[ \sigma_{31}' = (-\sin \omega)(\sin \beta \cos \omega) \sigma_2 + (\cos \omega)(\sin \beta \sin \omega) \sigma_3. \]  
\[ \tau_{xz} = l_{11} l_{31} \tau_{xx} + l_{12} l_{32} \tau_{yy} + l_{13} l_{33} \tau_{zz}, \]
\[ \sigma_{21}' = (\cos \beta \sin \beta) \left[ \sigma_2 \cos^2 \omega + \sigma_3 \sin^2 \omega - \sigma_1 \right]. \]  

Figure 7.3. True-triaxial stress system and orientation in principal coordinates.
Equations 7.2.3a to 7.2.3f are similar to Eqns. 6.1.9a to 6.1.9f, except that now \( \omega \neq 0^\circ \), and \( \sigma_1 > \sigma_2 > \sigma_3 \). The failure criterion for transversely isotropic rocks in principal stress space represented by Eqn. 7.2.2 is shown in Figure 7.4.

The shape of this parabolic surface is useful when trying to understand a failure criterion plot (i.e., \( \sigma_1 \) versus \( \sigma_2 \)) on a plane of \( \sigma_3 \). A cross section of this parabolic surface would produce a plot with a curved shape as shown in Figure 7.5.

![Figure 7.4](image)

**Figure 7.4.** View of parabolic yield condition with transverse isotropy, plot on principal stress space (from Smith and Cheatham, 1980b).

![Figure 7.5](image)

**Figure 7.5.** Section view of parabolic yield condition with transverse isotropy, plot on principal stress space, \( \sigma_1 \) vs. \( \sigma_2 \) for fixed \( \sigma_3 \); (Refer section 7.6 for Mode I, II, III and IV).
7.3 Mogi’s Chichibu Schist true-triaxial experiments

This section describes the Chichibu Schist data set (Mogi, 1979) that will be used to validate the true-triaxial JPW and Pariseau models. The Chichibu Schist is the most complete true-triaxial experimental data set available on anisotropic rocks from the literature. The information on the Chichibu Schist used in this study was taken from Mogi (1979), Kwaśniewski and Mogi (1990) and Kwaśniewski (1993, 2007) and Mogi (2007). Although Mogi and coworkers worked mostly on isotropic rocks, their interest in the Chichibu Schist was also to understand the $\sigma_2$ strength effect for anisotropic rocks (Kwaśniewski, 2007).

Mogi (1979, 2007) reviewed the Martinsburg Slate experiments (Donath, 1964) and reported that the minimum strength angle $\beta_{\text{min}}$ (see Chapter 5 for the definition of $\beta_{\text{min}}$) for the Martinsburg Slate is not significantly different than the fracture angle of the intact rock, which was also approximately $\beta_{\text{min}} = 60^\circ$ for this rock. Although theoretically this is inaccurate, in some ways Mogi’s estimate was good enough as a first guess, because the difference between these two angles is not significant (i.e., for the Martinsburg Slate this difference is only approximately $7^\circ$). This is because intact rock fracture occurs at $\beta = 45 + \phi_u/2$, and the minimum strength for weak plane failure occurs at $\beta = 45 + \phi_w/2$, the difference between the two angles is only $(\phi_u - \phi_w)/2$.

The above explanation shaped Mogi’s idea to use samples at angles $\beta = 60^\circ$, but this does not describe how strength depends on the failure plane direction $\omega$. To understand the $\sigma_2$ effect for anisotropic rocks at different angles $\beta$ and $\omega$, Mogi designed four types of true-triaxial experiments called Mode I, II, III and IV shown in Figure 7.6. The weak plane strengths are represented by samples Mode I, II and III at angles $\beta = 60^\circ$ with $\omega = 0^\circ, 45^\circ$ and $90^\circ$, while the maximum strength properties was obtained for intact rock samples described as Mode IV.

![Figure 7.6. Chichibu Schist true-triaxial experiments with sample Mode I, II, III and IV (from Mogi, 2007).](image-url)
These samples with Mode I, II, III and IV represent the lower, mid-level and the upper strength responses of the anisotropic rocks. At the time that these true-triaxial experiments were conducted, the $\sigma_2$ effect for anisotropic rocks was relatively unknown. Therefore, these initial approximate estimates used by Mogi to design the experiments with four specific modes were innovative and creative.

To elaborate further on Mogi’s experiments and results, some details of the Chichibu Schist anisotropic rock are described here. The Chichibu Schist is highly anisotropic, with a macroscopically homogeneous green crystalline schist and dense foliation that originates from the Chichibu province, Honshu, Japan (Kwaśniewski and Mogi, 1990). This metamorphic rock with schistose structure has mineral composition (in volume percentage) of albite (19.4% to 26.4%), epidote (26.8% to 34.3%) calcite (5.1% to 10.1%), chlorite (3.0% to 5.7%), quartz (3.3% to 4.1%), pyroxene (0.9% to 1.4%) and iron oxides with trace minerals (2.7% to 3.7%). The Chichibu Schist samples were prepared from quarry outcrops with bulk density of 2.98 g/cc, and each specimen cut for the experiments was of rectangular shape of 15 mm (W) x 15 mm (L) x 30 mm (H) with an accuracy of ±2 μm. The samples were designed with these dimensions to achieve a slenderness ratio of 2. For the $\sigma_2 = \sigma_3$ experiments, the sample dimensions used were slightly different, with sample height of 40.0 mm and slenderness ratio of 2.67.

Mogi conducted forty-six true-triaxial compression tests ($\sigma_1 > \sigma_2 > \sigma_3$) and eighteen conventional triaxial stress experiments ($\sigma_1 > \sigma_2 = \sigma_3$) on the Chichibu Schist for samples under Mode I, II, III and IV. Prior to the true-triaxial experiments, Mogi carried out eighteen conventional triaxial stress experiments of Mode I, II and IV (Mogi, 2007; p. 167, Figure 3.105) to obtain the baseline intact rock and weak plane strength properties. The eighteen experiments of weak plane failures in Modes I and II for $\sigma_2 = \sigma_3$ conditions showed that the rock strengths were the same for both of these modes. This means that when $\sigma_2 = \sigma_3$, the difference in direction $\omega$ did not have any influence on strength. This point was also described in the discussion of the JPW model in section 7.1. Therefore, for conventional triaxial stress conditions both Mode I and II gave essentially the same strengths.

Following the conventional triaxial experiments, Mogi then made forty-six true-triaxial experiments in Mode I, II, III and IV and presented the results of the true-triaxial experiment as shown in Figure 7.7. This plot shows strength at failure $\sigma_1$ versus intermediate stress $\sigma_2$ for confining stress $\sigma_3 = 7,252$ psi. Sample with Mode I shows very little influence of $\sigma_2$ while Mode II shows an increased effect of $\sigma_2$ with higher scatter in the data. The sample Mode III that has $\sigma_2$ facing perpendicular to the weak plane shows a sharp increase in strength with increased $\sigma_2$, and has much higher strength than Mode I and II. For intact rock strength, sample Mode IV shows similar strength behavior to isotropic rocks, with strength increasing initially with $\sigma_2$, and then at a reduced rate until reaching the $\sigma_1 = \sigma_2$ extension line. A summary of the true-triaxial strength data shown in Figure 7.7 is available in Appendix D2. The solid lines in Figure 7.7 are fit arbitrarily, and do not represent any particular model.
An important outcome from the true-triaxial experiments for the Chichibu Schist is the fracture profiles or failure plane angles observed for the different Mode I, II, III and IV. This information is especially important for the JPW model, which needs the failure angles (i.e., $\beta$ and $\omega$) to define the strength properties. Mogi (1979 and 2007) explained that the samples Mode I, II and III failed along the weak planes, whereas Mode IV fractured through the intact rock. Kwaśniewski and Mogi (1990) provided further explanation to describe two samples for Mode III at high intermediate stress $\sigma_2$ that had mixed failure modes. These samples failed in two planes independent of loading path, resulting in fractures parallel and oblique to $\sigma_2$. These two samples are excluded from this study because of the mixed failure modes (see Appendix D2, Samples No. 50 and 51). These two outliers do not affect the analysis made in this study. Although the fracture profiles and failure planes were briefly described (Mogi, 2007; Kwaśniewski and Mogi, 1990), a complete side and top view picture of the fracture profiles, especially for Mode IV, would have been very useful to calibrate the JPW model of this rock. The fracture angles $\beta$ and $\omega$ necessary for the JPW model calibrations are described further in the following section.

![Figure 7.7. Chichibu Schist true-triaxial strength, $\sigma_1$ vs. $\sigma_2$ for $\sigma_3 = 7,252$ psi for sample Mode I, II, III and IV (from Mogi, 1979 and 2007).](image-url)
Despite extensive experiments on the Chichibu Schist, Mogi (2007) acknowledged that still much remains to be understood on the true-triaxial strength behavior of anisotropic rock. Nevertheless, the Chichibu Schist experiments provide sufficient evidence on the role of $\sigma_2$ on anisotropic rocks, and show that strength is influenced not only by intermediate stress $\sigma_2$, confining stress $\sigma_3$, and bedding angle $\beta$, but also by the foliation direction $\omega$. The Chichibu Schist data set will be applied in the following sections to test and validate the JPW and Pariseau models.

### 7.4 JPW model validation using true-triaxial Mogi Chichibu Schist data

The true-triaxial Mohr circle described earlier in this chapter shows how the shear $\tau$ and normal stress $\sigma_n$ acting on a plane is determined by the angle $\beta$, the foliation direction $\omega$, and the principal stresses $\sigma_1$, $\sigma_2$ and $\sigma_3$. For intact rock fracture under conventional triaxial $\sigma_2 = \sigma_3$ stress conditions, there is a critical plane at angle $\beta = 45 + \phi_w/2$, at which shear strength at failure will be first reached as $\sigma_1$ is increased (Brady and Brown, 1993; p. 107). Alternatively, this critical rock fracture angle, $\beta$, can also be measured from the failed sample and input into the Coulomb criterion to determine the $\tau$ and $\sigma_n$ acting on the failure plane. For rock failure under true-triaxial stress condition, a similar approach is used and is described below.

For the JPW criterion under true-triaxial stress conditions, it is not possible to determine the critical angle $\beta$ and direction $\omega$ where shear failure occurs by solving $d\sigma_1/d\beta = 0$, due to the complicated terms for $\tau$ and $\sigma_n$ in Eqn. 7.1.4 and 7.1.8. Attempts to solve this equation were unsuccessful, producing long terms that are not useful for practical applications. Therefore, for intact rock fracture under true-triaxial stress conditions, laboratory measurements of fracture angle $\beta$ and direction $\omega$ are necessary to determine the intact rock strength properties for the JPW model. There are two parts to solving this true-triaxial stress problem, which is firstly for the case of failure along the weak plane, and the second is for intact rock fracture.

First, for the case of failure along the weak plane under true-triaxial stress conditions, the JPW criterion described by Eqn. 7.1.3 and 7.1.4 is used, wherein the rock failure plane ($i.e.$, $\tau$ and $\sigma_n$ acting on the weak plane at failure) is predefined by angle $\beta$ and direction $\omega$, and the strength properties of this weak plane can be determined using the true-triaxial Mohr circles according to the Coulomb criterion (Jaeger, 1964, p. 161) shown in Figure 7.8. The Coulomb plot of $\tau$ vs. $\sigma_n$ from Eqns. 7.1.3 and 7.1.4 for $\sigma_1$, $\sigma_2$ and $\sigma_3$, $\beta$ and $\omega$ gives a linear relation, shown as PA in Figure 7.8. This plot is then fit to determine the slope that defines the weak plane friction angle, $\phi_w$, and intercept represents the weak plane cohesion, $S_w$. 


Each point along the line PA gives values of the angles $\beta$ and $\omega$ described in section 7.1. In other words, the $\tau$ and $\sigma_n$ acting on a plane for a true-triaxial stress system is determined from the intercept at point Q, which plots in a straight line PA. For the Chichibu Schist anisotropic rock, samples of Mode I, II and III failed along the weak planes at angle $\beta$ / direction $\omega = 60^\circ$ / $0^\circ$ (Mode I), $60^\circ$ / $45^\circ$ (Mode II) and $60^\circ$ / $90^\circ$ (Mode III). From these three Modes, samples with Mode I and II were the most representative of weak plane failure, while Mode III was not used to determine the plane property, because at this stress level the failure modes are transitioning and for some cases approaching isotropic strengths.

The $\tau$ versus $\sigma_n$ plot for Mode I and II representing the weak plane strength properties for the Chichibu Schist is shown in Figure 7.9.

![Figure 7.8. JPW criterion under true-triaxial stress conditions (from Jaeger, 1964).](image)

![Figure 7.9. Chichibu Schist true-triaxial data Mode I and II plot of $\tau$ vs. $\sigma_n$ to determine strength properties of weak plane ($S_w = 4,031.3$ psi; $\phi_w = 30.5^\circ$).](image)
The \( \tau \) vs. \( \sigma_n \) plot shows best-fit parameters with intercept \( S_w = 4031.3 \) psi and slope angle, \( \phi_w = \tan^{-1}(0.5899) = 30.5^\circ \). The second part of the data analysis is to determine the strength properties of the intact rock fracture plane. For true-triaxial stress conditions, the intact rock fracture plane angle \( \beta \) and direction \( \omega \) needs to be determined experimentally. This, however, is not possible using the available data from literature because although some of the fracture angles \( \beta \) are available, the fracture directions \( \omega \) are not available. Therefore, for the Chichibu Schist intact rock fracture angles, the assumption of \( \beta = 45 + \phi_b/2 \) and \( \omega = \phi_b \) was used as an estimate for the Mode IV fracture angles.

For the Chichibu Schist, using the data from Mode IV true-triaxial experiments (\( \sigma_1, \sigma_2, \sigma_3 \) and angles \( \beta = 45 + \phi_b/2 \) and direction \( \omega = \phi_b \)), the best-fit \( \tau \) vs. \( \sigma_n \) plot was computed as shown in Figure 7.10. The intercept of the plot shows that the cohesion of the intact rock \( S_o = 7,309.3 \) psi and slope angle is \( \phi_b = \tan^{-1}(0.8139) = 39.1^\circ \).

\[
\begin{align*}
\text{Figure 7.10. Chichibu Schist true-triaxial data Mode IV plot of } &\tau \text{ vs. } \sigma_n \text{ to determine strength properties of intact rock (} S_o = 7,309.3 \text{ psi; } \phi_b = 39.1^\circ). \\
&y = 0.8139x + 7309.3 \\
&R^2 = 0.922 \text{ (Mode IV)}
\end{align*}
\]

Using the strength properties of the weak plane and the intact rock, the Coulomb criterion can be applied to predict strength \( \sigma_5 \) as a function of the other two principal stresses \( \sigma_2, \sigma_3 \) and plane orientation defined by angles \( \beta \) and \( \omega \).

For failure along the weak plane, the Coulomb criterion is

\[
|\tau| = S_w + \sigma_n \tan \phi_w, \tag{7.4.1}
\]
while for intact rock failure the Coulomb criterion is

\[ |\tau| = S_o + \sigma_n \tan \phi_o. \]  

(7.4.2)

Solving Eqns. 7.1.3 and 7.1.7 to determine strength \( \sigma_1 \) is not straightforward because of the complicated terms in \( \tau \) and \( \sigma_n \). Therefore, a Matlab code with symbolic math function was used (Appendix B8) to find the strength \( \sigma_1 \) as a function of the input parameters

\[ \sigma_1 = f(\sigma_2, \sigma_3, \beta, \omega). \]  

(7.4.3)

From the Chichibu Schist true-triaxial experiments, the strength parameters \( S_w, \phi_w, S_o \) and \( \phi_o \) of the intact rock and weak plane applied to Eqns. 7.4.1 and 7.4.2 respectively, gives \( \sigma_1 \) as described in Eqn.7.4.3, and presented in terms of \( \sigma_1 \) vs. \( \sigma_2 \) for a fixed value of \( \sigma_3 \) as shown in Figure 7.11.

Figure 7.11. JPW model fit for Chichibu Schist true-triaxial data (Mogi, 1979). Plot shows \( \sigma_1 \) vs. \( \sigma_2 \) for \( \sigma_3 = 7,252 \) psi for samples Mode I, II, III and IV.

Figure 7.11 shows the JPW model applied in the true-triaxial stress regime. For samples that failed along the weak plane, Mode I and II predictions of the plane strength properties were reasonably good. The JPW model shows an interesting result for Mode I, wherein the strength \( \sigma_1 \) reduces with increased stress \( \sigma_2 \), and with further increase in \( \sigma_2 \), strength \( \sigma_1 \) increases again. Experimental results also show that \( \sigma_1 \) reduces initially before increasing,
which is an interesting and unexpected behavior for the \( \sigma_2 \) effect. This reduced strength in \( \sigma_1 \) with increase in \( \sigma_2 \) is a new response discovered in this study, and the JPW model captures this behavior. Referring back to Figure 7.2, this weakening effect with increased \( \sigma_2 \) occurs when \( \beta + \omega < 90^\circ \). For Mode I, \( \beta = 60^\circ \) and \( \omega = 0^\circ \), hence \( \beta + \omega < 90^\circ \) with strength profile as shown for Mode I.

One of the most difficult and confusing phenomenon to understand in this study is the Mode I behavior that shows a “U-shape” strength \( \sigma_1 \) response with increase of \( \sigma_2 \). This behavior is difficult to understand because, for most isotropic rocks, an increase in \( \sigma_2 \) usually causes an increase in \( \sigma_1 \). This U-shape strength behavior means that the commonly understood analogy of driving and resisting failure is not directly applicable. Other approaches to describe this phenomenon using Mohr circles are also not straightforward because the principal stresses \( \sigma_1, \sigma_2 \) and angle \( \beta \) using the Mohr circle change accordingly, and the center of the \( \sigma_1-\sigma_2 \) circle is not known \textit{a priori} to predict the \( \sigma_1 \) strength behavior. What is certain is that for Mode I, \( \beta = 60^\circ \) and \( \omega = 0^\circ \), which means that the \( \tau \) and \( \sigma_n \) intersects the \( \sigma_1, \sigma_3 \) circle, which was described in Section 7.1. Therefore, for the Mode I case, this phenomenon of U-shape strength \( \sigma_1 \) response with increase in \( \sigma_2 \) cannot be determined directly, even using the graphical method, because \( \sigma_1 \) is not known \textit{a priori}. To understand the Mode I behavior, a rigorous mathematical approach using Matlab codes, such as that used in this study, is necessary.

For the Mode II strength behavior, the sample shows some increase in strength \( \sigma_1 \) at higher intermediate stress \( \sigma_2 \), and as \( \sigma_2 \) approaches the extension line where \( \sigma_1 = \sigma_2 \), strength \( \sigma_1 \) reduces slightly. It is interesting to note that the properties of the plane for Mode I and II are the same, but the only difference is the direction \( \omega \) which results in significantly higher strength \( \sigma_1 \) at around \( \sigma_2 = 30,000 \) psi. This strengthening effect can be explained using the JPW model as strength \( \sigma_1 \) increases with increased intermediate stress \( \sigma_2 \). This strengthening \( \sigma_2 \) effect occurs because \( \beta + \omega > 90^\circ \). For Mode II, \( \beta = 60^\circ \), \( \omega = 45^\circ \) and \( \beta + \omega > 90^\circ \).

For the sample in Mode III, the rock fails along the weak plane aligned perpendicular to \( \sigma_2 \) (i.e., weak plane is facing \( \sigma_2 \); hence, it fails against intermediate stress \( \sigma_2 \)). This causes the rock strength \( \sigma_1 \) to increase rapidly with \( \sigma_2 \) until reaching the isotropic failure envelope. When the \( \sigma_2 \) reaches 20,000 psi, the strength predicted increases rapidly and reaches the isotropic rock failure line, and from this point onwards, the intact rock fracture model supersedes the weak plane model. An interesting point is that the Mode III weak plane property in Figure 7.11 was derived from samples in Mode I and II, because Mode III samples transitions from weak plane failure to intact rock fracture when approaching \( \sigma_2 = 20,000 \) psi. This change in failure Mode affects the evaluation of \( S_w \) and \( \phi_w \) of the Mode III sample. Figure 7.11 also shows that the plane property from Mode I and II was able to predict the strength behavior of Mode III reasonable well, without using any of the Mode III
data to determine $S_w$ and $\phi_w$. This is a good validation of the JPW model for failure along weak plane. For Mode III, the strengthening effect occurs rapidly with increase in $\sigma_2$ because $\beta + \omega > 90^\circ$ and is higher than the Mode II case. For Mode III, $\beta = 60^\circ$, $\omega = 90^\circ$ and $\beta + \omega > 90^\circ$.

Lastly, for the samples in Mode IV, the intact rock strength properties $S_o$ and $\phi_o$ for Chichibu Schist true-triaxial experimental data was determined using Eqn. 7.4.2 (Figure 7.10). Using the intact rock strength properties, strength $\sigma_1$ is predicted for intact rock fracture under true-triaxial stress conditions. The Mode IV plot of $\sigma_1$ vs. $\sigma_2$ for intact rock fracture (Figure 7.11) shows increasing strength $\sigma_1$ with increased $\sigma_2$. The Mode IV JPW model shows rock strength $\sigma_1$ increasing from $\sigma_2 = \sigma_3 = 7,252$ psi, almost parallels the extension line ($\sigma_2 = \sigma_2$), and later shows strength reduction at further increased $\sigma_2$ until reaching the extension line. This is a typical strength response for intact rock fracture of isotropic rocks. The Mode IV strength is higher than Mode I, II and III because of the higher intact rock strength properties (i.e., $S_o$ and $\phi_o$). The strengthening effect of Mode IV (slope of $\sigma_2$ vs. $\sigma_2$) is lower than the rate of increase seen for Mode III, and almost similar to that of Mode II. This is because the angle $\beta + \omega$ for Mode IV is closer to the case of Mode II. For Mode IV, the fracture angle $\beta$ and $\omega$ are estimated to be $\beta = 45 + (39.1/2)^\circ$, $\omega = 39.1^\circ$ and $\beta + \omega > 90^\circ$.

Evaluation of the Chichibu Schist Mode I, II, III and IV true-triaxial experimental data using the JPW model shows that the mean error of the model calculated using RMSE (Eqn. 5.2.13) is 5,416 psi. This represents about a 10% mean error when compared to the mean strengths $\sigma_1$ of the experimental data shown in Figure 7.11.

### 7.5 Comparison of JPW model using conventional triaxial against true-triaxial data

From the Chichibu Schist experiments by Mogi (1979), eighteen of the samples were conventional triaxial experiments ($\sigma_2 = \sigma_3$). These experiments were used to understand the strength properties of the weak plane and intact rock under conventional triaxial stress conditions. The outcome from these experiments was also useful to design the forty-six samples of Mode I, II, III and IV true-triaxial experiments. For the conventional stress condition ($\sigma_2 = \sigma_3$) samples, the JPW model as described in Chapter 5 applies, whereas for the true-triaxial stress conditions ($\sigma_1 > \sigma_2 = \sigma_3$), the JPW model described in this chapter applies. These JPW models however are essentially the same, as both models use the strength properties of the intact rock ($S_o$ and $\phi_o$) and the weak plane ($S_w$ and $\phi_w$). Therefore, in this section, the strength properties derived from the $\sigma_2 = \sigma_3$ conditions (eighteen samples) of Mode I, II and IV are compared to the strength properties from the (forty-six samples) true-triaxial experiments.

The eighteen conventional triaxial experiments of Mode I, II and IV (Appendix D2 for raw data) plot using maximum shear stress $\tau_m$ versus mean normal stress $\sigma_m$ is shown in Figure
7.12. The two linear plots represent the strength properties of the weak plane (Mode I and II) and intact rock (Mode IV). Comparison of the intact rock and weak plane strength properties under true-triaxial and conventional triaxial stress conditions are summarized in Table 7.1.

**Figure 7.12.** Chichibu Schist conventional triaxial data Mode I, II and IV. Plot of $\tau_m$ vs. $\sigma_m$ for intact rock (Mode IV) and weak plane (Mode I and II). Intact rock properties $S_o = 5,147.3$ psi, $\phi_o = 40.5^\circ$; Weak plane strength properties $S_w = 2,608.1$ psi, $\phi_w = 33.7^\circ$.

The strength parameter of the intact rock in Table 7.1 shows that the friction angle of $\phi_o = 40.5^\circ$ for conventional triaxial conditions is slightly higher than the friction angle for the true-triaxial experiments, which is $\phi_o = 39.1^\circ$. For cohesion, the opposite is the case: cohesion for true-triaxial experiments of $S_o = 7,309.3$ psi is higher than cohesion for conventional triaxial experiments of $S_o = 5,147.3$ psi. The same trend is observed for the weak plane strength parameters, where the conventional triaxial friction angle and cohesion are $\phi_w = 33.7^\circ$ and $S_w = 2,608.1$ psi, while for the true-triaxial condition they are $\phi_w = 30.5^\circ$ and $S_w = 4,031.3$ psi, respectively.

**Table 7.1.** Strength parameters derived from true-triaxial and conventional triaxial tests on Chichibu Schist.

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<thead>
<tr>
<th>Strength parameters</th>
<th>True-triaxial</th>
<th>Conventional triaxial</th>
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<tr>
<td>Friction angle intact rock, $\phi_o$</td>
<td>39.1°</td>
<td>40.5°</td>
</tr>
<tr>
<td>Cohesion intact rock, $S_o$</td>
<td>7309.3 psi</td>
<td>5147.3 psi</td>
</tr>
<tr>
<td>Friction angle weak plane, $\phi_w$</td>
<td>30.5°</td>
<td>33.7°</td>
</tr>
<tr>
<td>Cohesion weak plane, $S_w$</td>
<td>4031.3 psi</td>
<td>2608.1 psi</td>
</tr>
</tbody>
</table>
From the strength parameters listed in Table 7.1, it is not directly obvious how the changes in strength parameters would compare when used to derive a true-triaxial model, because the friction angle and cohesion seem to vary differently under conventional triaxial and true-triaxial stress condition. To evaluate these strength parameters, the conventional triaxial strength parameters in Table 7.1 were used to generate a true-triaxial model, similar to the method described earlier, for Mode I, II, III and IV. For Mode IV, the same assumption for fracture plane angle is used to defined the orientation angles $\beta = 45 + \phi_0/2$ and $\omega = \phi_0$, while for Mode I, II and III angle $\beta$ and direction $\omega$ defines the weak plane orientations.

Figure 7.13 shows the JPW model derived using conventional triaxial strength parameters compared to true-triaxial strength parameters. The solid lines are the JPW model derived using true-triaxial strength parameters, while the dashed lines are the JPW model derived using conventional strength parameters. The two JPW model plots in Figure 7.13 do not show much difference. The Mode II and IV solid line (JPW from true-triaxial data) shows a slight difference from the dashed line (JPW from conventional triaxial) as intermediate stress $\sigma_2$ approaches the $\sigma_2 = \sigma_2$ extension line.

![Figure 7.13](image)

**Figure 7.13.** JPW model fit for Chichibu Schist using conventional triaxial strength parameters (dashed line); compared to JPW model using true-triaxial strength parameters (solid line).
Comparing the JPW model in Modes I, II and III for only failure along weak planes, the parameters derived from conventional triaxial data and true-triaxial data do not show a significant difference. For the Mode II data fit to the JPW model, the conventional triaxial strength parameter (dashed line) overestimates the true-triaxial strength by about 10% when approaching the $\sigma_1 = \sigma_2$ line. For the case of intact rock failure, Mode IV conventional triaxial strength model (dashed line) seems to predict the true-triaxial strength reasonably well. However, Mode IV conventional triaxial strength (dashed line) shows around 10% difference compared to the true-triaxial strength (solid line) at low and high levels of $\sigma_2$. Comparing Mode II and IV data fits shows over-predictions occur close to the triaxial extension line near $\sigma_1 = \sigma_2$. Overall, this analysis shows that for the Chichibu Schist, the JPW parameters derived from conventional and true-triaxial experiments have almost similar strengths. Based on these results, for the Chichibu Schist, it is possible to use conventional triaxial strength parameters to predict strength in the true-triaxial stress regime.

7.6 Pariseau model validation using true-triaxial Mogi Chichibu Schist data

The Pariseau continuum model was described in section 7.2 under true-triaxial stress conditions. When Eqn. 7.2.2 is transformed to the principal stress coordinate system using Eqns. 7.2.3a-f, a failure criterion to predict strength $\sigma_1$ as described in Eqn. 7.4.3 is produced. For the Chichibu Schist data analysis using the Pariseau model, the failure envelope will be described using a $\sigma_1$ vs. $\sigma_2$ plot for fixed $\sigma_3$, as shown in Figure 7.5.

The Pariseau model, unlike the JPW model, is not related in any explicit way to traditional rock properties such as friction angle and cohesion. Instead, the Pariseau model uses the parameters $F$, $G$, $U$, $V$ and $M$ to represent the rock strength properties. To determine these parameters, data from Chichibu Schist conventional triaxial and true-triaxial experiments will be used to determine the parameters $F$, $G$, $U$, $V$ and $M$ (Appendix D2). The Pariseau parameters are determined iteratively, similar to the approach described in section 6.2, whereby the rock parameters are determined so as to yield the least possible mean error (i.e., smallest RMSE). Shown in Appendix B5 and B6 is the Matlab code used to determine the Pariseau parameters iteratively in the true-triaxial stress regime. Once the Pariseau parameters $F$, $G$, $U$, $V$ and $M$ are determined, the true-triaxial failure envelope of $\sigma_1$ as a function of $\sigma_2$, $\sigma_3$, $\beta$ and $\omega$ can be determined by inserting the Pariseau parameters into Eqn. 7.2.2 and Eqns. 7.2.3a-f, using the Matlab code shown in Appendix B7. The Pariseau model parameters calibrated for the Chichibu Schist are:

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<tbody>
<tr>
<td>Values</td>
<td>4.72E-10</td>
<td>4.34E-11</td>
<td>1.25E-05</td>
<td>7.41E-06</td>
<td>1.23E-08</td>
</tr>
</tbody>
</table>
For Mode I, II, III and IV true-triaxial data, the Pariseau model failure surface is shown in Figure 7.14. The Pariseau model shows similar trends for the Chichibu Schist as those seen using the JPW model. However, the most obvious difference in the Pariseau model in Figure 7.14, compared to JPW model in Figure 7.11, is the Mode I failure surface. The Mode I failure surface from Pariseau’s model predicts a slightly increasing strength $\sigma_1$ with increased intermediate stress $\sigma_2$, while experimental results shows trends that are more consistent with the prediction using the JPW model. This may be because the Pariseau model is defined mainly by the overall data set trends in this case, and there are more data for Mode II and III than Mode I. Therefore, for the Mode I failure envelope, when using Pariseau’s criterion, a more conservative estimate is necessary to avoid overestimating strength in the true-triaxial stress regime. Overall, Mode I is better predicted using the JPW model than the Pariseau model.

For Mode II failure, the Pariseau model predicts the Chichibu Schist strength reasonably well, comparable to the JPW model. Whereas for the case of Mode III, the Pariseau model slightly underpredicts strength, and provides a better prediction of this transition zone than the JPW model. Overall, for Mode III, the Pariseau model performs slightly better than the JPW model.

Figure 7.14. Pariseau model fit for Chichibu Schist true-triaxial data (Mogi, 1979). Plot shows $\sigma_1$ vs. $\sigma_2$ for $\sigma_3 = 7,252$ psi for samples Mode I, II, III and IV.
For the case of intact rock failure in Mode IV, the Pariseau model prediction is only slightly higher than the strengths predicted by the JPW model. Referring to Figure 7.14, the strength prediction in the $\sigma_1 > 90,000$ psi stress regime indicates a reduced $\sigma_2$ strength effect. Due to lack of experimental data at the higher strength $\sigma_1$ stress regime, the $\sigma_2$ effect at this higher stress regime is derived based on the trends of $\sigma_1 < 90,000$ psi. To constrain the failure envelope dependence on $\sigma_2$ at the higher stress level, an extension experiment where $\sigma_1 = \sigma_2$ for Mode IV would have been useful. This could provide better information regarding the $\sigma_2$ dependence at higher levels of $\sigma_2$. For the Chichibu Schist, most of the data are at stress levels of $\sigma_2 < 40,000$ psi. Therefore, rock strength behavior at the lower $\sigma_2$ stress regime is being used to predict the strength response at the higher $\sigma_2$ stress regime. This may not be fully representative of the rock strength behavior at the higher $\sigma_2$ stress regimes. Instead, the Pariseau model prediction could be improved if triaxial extension experiments were available to constrain the $\sigma_2$ effect in the higher $\sigma_2$ stress regime.

The Pariseau model mean errors measured using the RMSE (see section 6.2 for the definition of RMSE) for Chichibu Schist is 5,389 psi, which is slightly lower than that determined for the JPW model, which gives an RMSE of 5,416 psi. Overall, it can be concluded that both the Pariseau and JPW models predict the true-triaxial strength behavior of the Chichibu Schist equally well.

### 7.7 Bossier shale calibration of JPW and Pariseau models using compression and extension data

In this chapter, the JPW and Pariseau models were validated using the Chichibu Schist true-triaxial experimental data. Both models were shown to be able to represent the strength behavior in the true-triaxial stress regime equally well. However, the true-triaxial experiments used for the Chichibu Schist are still not a common practice for industry applications, and are largely used only for research purposes. Apart from that, cost and equipment availability are the other issues that makes it impractical to make true-triaxial experiments. For this reason, it would be very useful if JPW and Pariseau models calibrations were made using only conventional triaxial experimental data.

Conventional triaxial equipment, which is standard rock testing equipment, can be used to test the strength behavior under compressive loading conditions $\sigma_1 > \sigma_2 = \sigma_3$, and extension loading conditions $\sigma_1 = \sigma_2 > \sigma_3$. Using the rock strength behavior at these stress conditions may be sufficient to calibrate the models in the true-triaxial stress regime $\sigma_1 > \sigma_2 > \sigma_3$. In this section, the compression and extension data for the shale samples will be used to determine if these data are sufficient to calibrate both JPW and Pariseau models in the true-triaxial stress regime.
For the Bossier shale, thirty-six triaxial compression tests were made under various orientations and confining stresses for failure along the weak plane and intact rock. However, for the Bossier shale triaxial extension tests, only five experiments were made for failure along the weak plane. Therefore, for the Bossier shale samples in this study, the conventional triaxial compression data were used to obtain the intact rock and weak plane strength properties (i.e., $\phi_v$, $S_o$, $\phi_w$ and $S_w$). These strength properties were then applied in the true-triaxial stress regime to predict the triaxial extension strengths, and compare them to experimental results at extensional stress conditions $\sigma_1 = \sigma_2 = 25,000$ psi and $\sigma_3 = 7,190$ psi (section 5.4 for Bossier shale sample shows extension tests at $\beta = 60^\circ$).

Figure 7.15a shows the JPW model for Mode I, II, III and IV in the true-triaxial stress regime, as predicted using triaxial compression strength properties. Mode IV for intact rock strength shows that triaxial compression and extension strengths are almost the same, with a slight $\sigma_2$ strength effect at intermediate stress levels. The JPW model Mode II and III predictions shows a more pronounced $\sigma_2$ effect. However, there are no extension data available at these Modes, and therefore this effect cannot be verified with experimental data.

For the Mode I condition, extension test data was compared with the JPW model that was calibrated using conventional triaxial experimental data. The purpose of this exercise is to evaluate whether or not the extension strength could be predicted using the JPW model calibrated using strength properties from compression data. Figure 7.15a shows that the JPW model Mode I prediction is slightly higher than the triaxial extension strength obtained from experimental data at $\beta = 60^\circ$; $\sigma_1 = \sigma_2 = 25,000$ psi and $\sigma_3 = 7,190$ psi.

For the Pariseau model, triaxial compression and extension data were needed to calibrate the model. These parameters were then applied in the true-triaxial stress regime to predict the triaxial extension strength of the sample at $\beta = 60^\circ$, $\sigma_1 = \sigma_2 = 25,000$ psi and $\sigma_3 = 7,190$ psi. Figure 7.15b shows the failure modes I, II, III and IV predicted using Pariseau's model. The Pariseau model Mode IV shows that true-triaxial failure surface shows a significant $\sigma_2$ effect. This is because there are no extension data available for the intact rock failure under Mode IV to sufficiently constrain the $\sigma_2$ effect. Alternatively, by introducing the intact rock strength data in the triaxial extension stress regime, the $\sigma_2$ effect could be reduced, resulting in a more constrained failure envelope, as that seen in Mode IV of Figure 7.15a for the JPW model.

The plots in Figure 7.15a and 7.15b for Bossier shale in the true-triaxial stress regime show that using triaxial compression data to predict extension strength could serve as an approximate approach to determine the true-triaxial behavior of the Bossier shale. However, due to lack of extension data for intact Mode IV rock failure, the true-triaxial strength behavior for intact rock shows a low $\sigma_2$ effect for the JPW model, and a high $\sigma_2$ effect for the Pariseau model. Therefore, extension data of the intact rock are necessary in order to calibrate the JPW and Pariseau models in the true-triaxial stress regime.
Figure 7.15a. JPW model fit for Bossier shale derived using strength parameters from triaxial compression data. Plot shows $\sigma_1$ vs. $\sigma_2$ for $\sigma_3 = 7,190$ psi for samples Mode I, II, III and IV.

Figure 7.15b. Pariseau model fit for Bossier shale derived using strength parameters from triaxial compression and extension data. Plot shows $\sigma_1$ vs. $\sigma_2$ for $\sigma_3 = 7,190$ psi for samples Mode I, II, III and IV.
7.8 Vaca Muerta shale calibration of JPW and Pariseau models using compression and extension data

For the Vaca Muerta shale, twenty-one triaxial compression tests were conducted at different orientations and confining stresses. For the triaxial extension tests, only two experiments were successfully made for the intact rock fracture and failure along the weak plane. For the Vaca Muerta triaxial extension tests of intact rock failure (section 5.4), comparison of the compression and extension strengths show that the extension test for intact rock strength is higher than the triaxial compressive strength. Using the Vaca Muerta intact rock extension test data, the extension strength properties is estimated to be $\phi_b = 37.6^\circ$ and $S_o = 3,805.1$ psi, which are higher than the triaxial compression strength parameters. This means that there is a significant $\sigma_2$ effect for the Vaca Muerta intact rock strength. On the other hand, for failure along the weak plane, the Vaca Muerta Mohr circles for triaxial compression and extension (section 5.4) show almost the same diameter, indicating that the strength in compression and extension are the same. Based on this observation, the strength parameter for failure along the weak plane are assumed to be $\phi_w = 26.0^\circ$ and $S_w = 2650$ psi.

The JPW model for intact rock and weak plane failure applied in the true-triaxial stress regime is shown in Figure 7.16a. The JPW model Mode IV intact rock failure envelope predicts the strength at $\sigma_1 = \sigma_2 = 25,000$ psi and $\sigma_3 = 1,150$ psi, showing a significant $\sigma_2$ effect. The $\sigma_1$ strength under triaxial extension conditions is approximately 5,000 psi higher than in triaxial compression. For modes I, II, and III, the failure envelopes are derived using the weak plane strength properties. For failure along the weak plane, the Mode I failure envelope shows an initially reducing $\sigma_2$ effect, which increases again when approaching $\sigma_1 = \sigma_2$, before asymptoting along the extension line. The JPW Mode II and Mode III failure envelope shows higher $\sigma_2$ effects with increased direction $\omega$.

For the Pariseau model, all the Vaca Muerta shale triaxial compression and extension data were used to calibrate the model in the true-triaxial stress regime shown in Figure 7.16b. Comparing the Pariseau model failure envelope to extension data shows that the Mode IV failure envelope slightly overestimates $\sigma_1$ strength for triaxial extension strength by 2,000 psi. This is reasonable, considering that only one triaxial extension data is available for intact rock failure. For failure along the weak plane, however, the Mode I failure envelope shows an insignificant role of $\sigma_2$, while Mode II and III failure envelope shows stronger $\sigma_2$ effect. For the Pariseau Mode IV failure envelope, the model prediction could be improved by applying further constrains in calibrating the parameters in the true-triaxial stress regime.
Figure 7.16a. JPW model for Vaca Muerta shale using strength parameters from triaxial compression and extension data. Plot shows $\sigma_1$ vs. $\sigma_2$ for $\sigma_3 = 1,150$ psi for Mode I, II, III and IV.

Figure 7.16b. Pariseau model for Vaca Muerta shale using strength parameters from triaxial compression and extension data. Plot shows $\sigma_1$ vs. $\sigma_2$ for $\sigma_3 = 1,150$ psi for Mode I, II, III and IV.
In summary, comparing Figures 7.15a,b and 7.16a,b, the JPW and Pariseau models were able to use compression and extension data to predict the true-triaxial strength behavior. However, additional triaxial extension data (Mode I and IV) would significantly improve the model calibration. In addition, using only compression and extension test data, the intermediate stress levels in between compression and extension may not be properly captured by the models, leading to failure envelopes that do not correctly represent the actual true-triaxial failure behavior of the rock. Therefore, for JPW and Pariseau models, true-triaxial experiments are still recommended, while more studies need to be done on using compression and extension data for calibrating true-triaxial failure models.
8 Summary, Conclusions and Recommendations

8.1 Summary and conclusions

In this chapter, the summary and conclusions of this thesis are described based on the materials presented from the literature review, the experiments, the JPW and Pariseau models, and finally the true-triaxial failure criteria. Before summarizing the overall thesis, it is probably best to start by emphasizing the importance of understanding that shales are complex natural materials. Organic-rich shales are even more complex because of the presence of organic matter in the shale fabric, with planes of weakness that significantly alters its mechanical behavior. Therefore, it is only correct to say that the focus of this study, although providing a glimpse into the role of organic matter on strength behavior of shales, does not give a complete picture of the various different textural or compositional effects that may influence shale strength. To use the JPW or Pariseau models for organic-rich shales, the user must be able to understand the different failure modes that may exist for different types of shales.

In the introductory chapter, a mental picture of shales was presented to distinguish shales by the scale of their discontinuities, texture and composition. The rock matrix that is composed of discontinuities filled by minerals and sediments forms shales with planes of weakness. Two key aspects that control the mechanical behavior of shales are texture and composition. The effect of texture and composition were highlighted using three types of minerals found predominantly in shales, namely calcite, quartz and clay. Here, shales with similar composition but different texture result in different shale types. Therefore, it is important to highlight that discontinuities, texture and composition play a key role in understanding the mechanical response of shales.

A literature review uncovered many anisotropic rock failure criteria, which is categorized according to the Duveau et al. (1998) classification as mathematical, empirical or discontinuous (Table 2.1). Within these three categories of failure criteria, the mathematical models are not widely used in engineering practice due to their complexity and lack of experimental validation. As for the empirical criteria, these criteria are mainly extensions of the isotropic criteria that use various parameters from fitting experimental data, and are easily modified from existing failure criteria. Lastly, for the discontinuous criteria, most of these criteria are related to the Coulomb criterion, which combines the failure mechanisms along the weak plane and intact rock. The relative ease of modifying the Coulomb criterion for weak plane and intact rock with other empirical or mathematical criterion led various other combinations for the discontinuous models. Among these many criteria, to date the...
JPW criterion (discontinuous) is still the industry-preferred model that is commonly used for anisotropic rocks. This is probably because the JPW criterion originates from the well-known Coulomb criterion, which is widely used in the industry for isotropic rocks. In this study, the JPW criterion was presented in detail and tested using data from laboratory and the literature, which are presented in Chapter 5. As for the Pariseau criterion (mathematical), although this approach has been available for more than forty years, few researchers have attempted to apply it to laboratory data. Therefore, the Pariseau criterion was tested extensively in this study, and the results were presented in Chapter 6.

The laboratory measurements carried out in this study provide a unique understanding of the strength behavior of the highly anisotropic Bossier shale and poorly laminated Vaca Muerta Shale. The extensive experiments made on these organic-rich shales were described in as much detail as possible, so that future studies could benefit from the lessons learned in this study. An equally important finding from the experiments is that the quality control of the experiments is paramount for ensuring good results. This is especially important, since many experimental data are tested and analyzed, and all the data interpreted must be consistent and as accurate as possible. Any gap or error in the processes used from sample selection and interpretation would be difficult to correct because retesting or getting a replacement sample is usually costly or in many cases not possible. Once all the quality control measures from sample preparations up to data interpretation are followed, the analysis is straightforward, with fewer uncertainties.

The Bossier shale and Vaca Muerta shale compressive strength tests show that strength increases at higher confining stress. The compression tests also show that strength is maximum at bedding angles $\beta = 0^\circ$ and $90^\circ$, while the lowest strength was observed for the samples tested at angle $\beta = 60^\circ$. The highly laminated Bossier shale showed higher strength anisotropy compared to the poorly laminated Vaca Muerta shale. Another important observation from the experimental results is that the UCS strength at angle $\beta = 90^\circ$ is much lower than at $\beta = 0^\circ$, which means that UCS values are not representative of the shale strength behavior under non-zero confining stress. This also means that the Bossier shale UCS data should not be used for design of wellbore stability analysis. The same condition could not be verified for the Vaca Muerta shale, due to lack of data at $\beta = 90^\circ$, but the issue of unconfined strength is suspected to occur only for highly anisotropic shales such as the Bossier shale, due to tensile splitting at low confining stresses. Low strength anisotropy shale such as the Vaca Muerta did not show any tensile splitting behavior at angle $\beta = 90^\circ$.

Five extension tests were performed for the Bossier shale, while for Vaca Muerta shale only two extension tests were conducted. All of the extension tests were for failure along the weak plane, except for one Vaca Muerta sample that was tested for intact rock failure. Results show that for weak plane failure, there is no significant difference in strength between the triaxial compression and triaxial extension tests. However, a significant difference was found between the Vaca Muerta compression and extension tests for intact
rock failure. This means that the $\sigma_2$ effect is not significant for failure along the weak plane, whereas for the intact rock failure the $\sigma_2$ effect is important and cannot be ignored. However, these experiments do not address the complete true-triaxial stress condition, which also includes the direction, $\omega$.

Although the main focus of this study was on strength behavior, the elastic behavior of the anisotropic shales was very useful as a measure of quality control. For the highly anisotropic Bossier shale, the Young’s modulus and Poisson’s ratio data showed S-shaped behavior consistent with the theoretical predictions. However, for the Vaca Muerta shale, although the theoretical curve shows a similar S-shape, the experimental data shows significant scatter due to possible sample heterogeneity and competition of isotropic-anisotropic elastic behavior. This means that some of the Vaca Muerta samples tested behaved in a transition manner and showed more elastic isotropy and less elastic anisotropic behavior. Another observation for both the shales tested was that confining pressure affected the elastic moduli. The Bossier shale elastic moduli increased at higher confining pressures, whereas for Vaca Muerta shale, this effect of pressure on the elastic moduli was not detectable, due to significant scatter in the data.

Using CT scans and thin section images, two types of shale fabric were evaluated in this study. The first is the Bossier shale, which is highly laminated with discontinuous weak planes, wherein the planes are filled with organic material. The second shale evaluated was the Vaca Muerta shale, which is slightly different from the Bossier shale, wherein the fabric arrangement shows poor lamination, and the organic matter is dispersed throughout the shale matrix. Evaluations of the CT and thin-section images show that the fabric of the organic matter affects the failure modes and strength behavior of shales. For the Bossier shale, both CT and thin section images were necessary to evaluate the shale fabric. However, for the Vaca Muerta shale, the CT scan images provided better information than the thin section images regarding the failure modes.

Comparison of the Bossier shale pre-test and post-test thin section images shows that for samples with bedding angles $\beta = 0^\circ$ to $10^\circ$ and $\beta = 90^\circ$, failure occurs at a fracture plane angle consistent with the Coulomb criterion. At angles between $45^\circ < \beta < 75^\circ$, the samples failed predominantly along the weak planes, also consistent with the Coulomb criterion. However, there is a transition regime of angles $\beta$ lying in the range of about $10^\circ < \beta < 35^\circ$, wherein the failure surface follows an irregular path, and shows a mixed failure mode of intact rock fracture and weak plane failure. The transition zone with mixed failure modes for the Bossier shale is a new observation, and was further examined using thin section images. Thin section images at angles $\beta = 15^\circ$ to $30^\circ$ show that the sample failure plane occurs through the intact rock or weak plane lamination, jumping between the two failure modes with a step-like pattern, wherein the weak planes interact with the matrix shear fractures, following the path of least resistance. One possible explanation for this behavior is that at the transition zone, the discontinuous weak planes intersect the shale matrix failure plane,
providing an alternative fracture path, resulting in shale strength that is lower than the matrix rock strength. This mixed-failure mode due to competition of shale matrix and weak planes is caused by uneven distribution of high stresses at the transition zone, resulting in shale failure at lower strength than the JPW model prediction. The mixed-failure mode also causes the transition from shear fracture to sliding on a weak plane to occur over a range of angles $\beta < \beta_1$ and $\beta > \beta_2$, rather than exclusively at $\beta_1$ and $\beta_2$, as predicted by the JPW model.

For the Vaca Muerta shale, a comparison of the pre-test and post-test CT scan images does not show the presence of a transition zone or mixed failure modes, as was observed for the Bossier shale. All of the compressive test samples failed in shear through the matrix, or along a weak plane, in agreement with the JPW model predictions. The failure surfaces and weak plane laminations were also more easily observed using the CT scan images than in the thin section images.

Although the JPW model is already well known in the industry, it is difficult to find in the literature a clear explanation of how this model is derived from the Coulomb criterion. This was described in detail by deriving the JPW model using the Coulomb criterion equations and Mohr’s circle. The definition of angle $\beta$ was also discussed in detail, to show that the same Coulomb criterion is applicable in compression and extension conditions.

The Pariseau continuum model was the other criterion that was described in detail. Both the JPW and Pariseau models for triaxial compression and extension condition were then fit for the Bossier shale and Vaca Muerta shales tested in this study. Comparing both models, Pariseau’s model gives a better fit to the Bossier shale data than does the JPW model, with a RMSE of 3,248 psi, as compared with 4,171 psi for the JPW model. The difference in accuracy between the two models is mainly caused by the different model predictions in the transition zone $10^\circ < \beta < 35^\circ$, where the JPW model overestimates strength. For the Vaca Muerta shale, the Pariseau model provided a poorer fit than did the JPW model, with an RMSE of 2,390 psi, compared with the JPW model’s RMSE of 1,851 psi. The Vaca Muerta shale exhibited a nearly uniform strength in the range of angles $0^\circ < \beta < 40^\circ$, and did not show any transition zone; its behavior was therefore more consistent with the assumptions of the JPW model. In summary, both the JPW and Pariseau models show a reasonable ability to fit strength data on the Bossier and Vaca Muerta shales. The Bossier shale was a better fit with the Pariseau model, having an RMSE error smaller by 28%, whereas the JPW model gave a better fit to the Vaca Muerta shale, with an RMSE error smaller by 29%.

To distinguish the different anisotropic rock types, a method to determine the strength anisotropy ratio (SAR) was proposed, which can be expressed in terms of the fitting parameters of the JPW model as follows:

$$\text{SAR} = \frac{S_0 \cos \phi_0}{S_w \cos \phi_w} \left( \frac{1 - \sin \phi_w}{1 - \sin \phi_0} \right)$$

(8.1)
The JPW and Pariseau models were then applied to the ten data sets from the literature, and the two shales tested in this study. The best-fit model with the least RMSE for each anisotropic rock type was calculated, and is displayed in a bar chart shown in Figure 8.1. Each column shows the RMSE for both the Pariseau and JPW model, along with the SAR value derived from the JPW fit.

**Figure 8.1. Comparison of JPW and Pariseau models for various anisotropic rocks.**

*denotes data from the present study.

Comparison of the RMSE for all the rock types shows that the Pariseau model provides a better fit for ten of the twelve rocks, whereas the JPW model provided a better fit only for the Vaca Muerta shale (SAR = 1.87) and the Green River Shale-1 (SAR = 1.52). It can be noted that all rocks with SAR > 2 fit more closely by the Pariseau model, whereas both shales that were a better fit with the JPW model had SAR < 2. Nevertheless, three rocks with SAR < 2 fit more accurately by the Pariseau model; therefore, it seems that no definitive conclusions can be reached in this regard.

Using the available data from experiments and the literature, the JPW and Pariseau models were also evaluated using reduced data to determine the minimum number of triaxial experiments necessary to determine the strength parameters without significantly lowering the model accuracy. Evaluations of both models show that the Pariseau model gives a lower
RMSE than the JPW model, when evaluated using reduced number of data. The Pariseau model analysis shows that eight data points, obtained at two confining stress levels and at bedding angles $\beta = 0^\circ, 30^\circ, 60^\circ$ and $90^\circ$, is the recommended minimum data needed for using Pariseau's model. This recommendation of minimum numbers of data is also applicable using JPW models for rocks with low SAR. For the analysis of anisotropic rocks with high SAR, the JPW model analysis using minimum number of data set will need at least fourteen triaxial compression data at bedding angles of $\beta = 0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ 75^\circ$ and $90^\circ$ at two confining stress levels.

A good understanding of the JPW and Pariseau models was gained from the experimental work for the Bossier shale and Vaca Muerta shale, and applying these two models for various anisotropic rocks gave further insight into both models. The experience from using the JPW and Pariseau models in Chapter 5 and 6, under conventional triaxial stress conditions, provided the right understanding for applying the same models for true-triaxial stress conditions. The JPW and Pariseau failure criteria for true-triaxial stress conditions were described in Chapter 7, and validated using the Chichibu Schist data set published by Mogi (1979). The Chichibu Schist true-triaxial data fit using the JPW model gave an RMSE of 5,416 psi, whereas the Pariseau model produced an RMSE of 5,389 psi. Overall, it can be concluded that both Pariseau and JPW models predict the true-triaxial strength behavior of the Chichibu Schist equally well. An interesting observation from the Mode I true-triaxial experiments show that $\sigma_2$ decreases initially before increasing, which is an interesting and unexpected phenomenon for the $\sigma_2$ effect. This reduced strength $\sigma_1$ with increased $\sigma_2$ is a new response discovered in this study, and the true-triaxial JPW model captures this behavior. All the Chichibu Schist samples tested under true-triaxial stress conditions with Mode I, II, III and IV show that strength $\sigma_1$ depends not only on confining pressure $\sigma_3$, intermediate stress $\sigma_2$ and angle $\beta$, but also is significantly influenced by the direction $\omega$ (direction $\omega$ is the angle of the foliation with respect to the direction of the intermediate principal stress, $\sigma_2$).

8.2 Recommendations

Two types of organic-rich shales, one highly laminated and one poorly laminated, were tested in this study. The fabric of the organic matter plays a key role in the failure mode of both shales, as was discovered in this study. The highly laminated Bossier shale with organic-filled weak planes shows a mixed failure mode, while the dispersed organic matter in the Vaca Muerta shale with poor laminations showed smaller discrepancies with the predictions of the JPW model. Further experiments for different types of organic-rich shales are therefore recommended to uncover other potential failure modes and improve the model predictions.
The Bossier shale and Vaca Muerta shale were tested under conventional triaxial compression or extension conditions. However, the actual strength behavior of shales under true-triaxial stress conditions remains untested. Although the JPW and Pariseau true-triaxial models were validated in this study for the Chichibu Schist, failure behavior of organic-rich shales under true-triaxial stress condition could be different due to the presence of organic matter. The influence of the transition zone, organic-filled weak planes and fracture direction $\omega$ and angle $\beta$ for true-triaxial experiments needs to be further evaluated. Other failure modes under true-triaxial stress condition, similar to the transition zone for the Bossier shales, remain to be discovered.

During the triaxial extension experiments, some end effects were observed on the Bossier shale and Vaca Muerta shales samples, which causes problems in interpreting the results. For future experiments on organic-rich shales, further studies need to be made to determine if a different sample dimensions are necessary (i.e., height ratio, smaller sample, larger samples, etc.) to reduce possible end effect such as that seen for the Vaca Muerta sample at $\beta = 90^\circ$. Further evaluations also need to be made to determine if the extension tests could be affected due to high friction at the contact point between the axial loading ram and the sample. Improvement in the design of the endcap contact points between sample and axial loading ram should also be considered.

In view of possible applications on the findings from this study, the JPW and Pariseau models are recommended to be applied to wellbore stability analysis using actual field data. To determine the stresses around the wellbore, the anisotropic rock elastic model should be used to have a better estimate of the near wellbore stresses. Obtaining such field data to be tested is a time consuming process and needs careful planning and investment. A single case study may not give clear results, and tests should be done for several wells to have meaningful results.

In the study of shale fabric analysis, one of the most pertinent questions in the industry is the relationship, if any, between texture and composition and mechanical behavior. Textural and compositional information used in the industry quantitatively define shale petrology for geological characterization. Quantitative geology (i.e., TOC, porosity, density, volume, etc.) is useful for identification and classification purposes, but has limited use for determining mechanical behavior. Sone and Zoback (2013a, b) provided equations relating shale properties such as clay % (vol.), carbonate % (vol.), kerogen % (vol.) and bulk density, to mechanical properties such as creep, strength and elastic moduli. However, the amounts of scatter in these relationships are significant and in some cases, there are no clear trends observed.

For estimating strength properties from the shale fabric, one area that could be explored is the use of X-Ray technology such as CT (Computed Tomography) scans. In recent years, CT scans have become standard equipment used in the industry to characterize the internal features of the rock. Although CT scans provide bulk density quantitatively, most of the
textural features are determined qualitatively. Therefore, the use of texture and composition to determine rock strength is an issue that may not be solved using existing CT technology. Further development is necessary to measure the shale fabric quantitatively, and applying additional analysis for estimating strength behavior.

Improvement in CT technology to capture the textural features of the shale fabric needs to be complemented by further development of digital analysis and numerical methods. Textural features of the shale fabric captures the actual discontinuities and flaws that exist in the rock, which is an improvement from the idealized analytical models used in this study. The advantage of numerical methods is the ability to model complex features in the rock, which is a limitation for analytical models described in this study. Capturing the actual textural features in shale fabric and analyzing the rock behavior using numerical simulation still needs to be supplemented with experimental data that accounts for both textural and compositional effects. Numerical methods using CT technology and experimental data can then be used to determine the mechanical behavior of very complex shales, which could then lead to complete analysis under true-triaxial stress conditions for highly anisotropic shales. This is an exciting area of research with tremendous potential for future applications.
References


Suarez-Rivera, R., Deenadayalu, C. and Yang, Y.K., 2009. Unlocking the unconventional oil and gas reservoir: The effect of lamination heterogeneity in wellbore stability and


**Bossier Shale – Summary of experimental results**

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<th>( \epsilon_a )</th>
<th>( \epsilon_{ET1} )</th>
<th>( \epsilon_{ET2} )</th>
<th>evol</th>
<th>( v_{zx} )</th>
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**Average E moduli**

- \( E = 2.31E+06 \)
- \( E' = 6.78E+06 \)
- \( E'' = 0.31E+06 \)
- \( G_E = 1.41E+06 \)

**Deformed Samples**

![Deformed Samples Diagram](image)

**Oriented Samples**

![Oriented Samples Diagram](image)
Bossier Shale – Compression test

\[ \sigma_1 - \sigma_3 = 16,466 \]
\[ \sigma_3 = 0 \]
\[ E = 2.34 \times 10^6 \]
\[ \varepsilon_a = 0.011082 \]
\[ \varepsilon_{ET1} = -0.004768 \]
\[ \varepsilon_{ET2} = -0.018114 \]
\[ \varepsilon_{vol} = -0.0118 \]
\[ v_{zx} = 0.0983 \]
\[ v_{zy} = \ldots \]

Note: Elastic parameter window is >0.3% (0.6-0.7%) strain as material is linear elastic only at higher stress.
Bossier Shale – Compression test

Axial Stress vs Strain
Triaxial compression JBC 1-2 (β = 30°), σ₃ = UCS

Poissons Ratio, (nₓᵧ, nᵧₓ)
Triaxial compression JBC 1-2 (β = 30°), σ₁ = UCS

Note: Elastic parameter window is >0.3% (0.0-0.4%) strain as material is linear elastic only at higher stress.
Bossier Shale – Compression test

Axial Stress vs Strain
Triaxial compression JBC 1-3 ($\beta = 45^0$), $\sigma_3 = UCS$

- Axial Stress
- Radial Strain ET1
- Radial Strain ET2
- Volumetric Strain
- $E$ modulus @ 45-65% of UCS

Note: Elastic parameter window is < 0.1% (0.20 - 0.26%) because non-linear PR out of this strain window.

$\sigma_1 - \sigma_3 = 5,653$
$\sigma_3 = 0$
$E = 2.12E+06$
$\varepsilon_a = 0.003785$
$\varepsilon_{ET1} = -0.000369$
$\varepsilon_{ET2} = -0.000273$
$\varepsilon_{vol} = 0.003143$
$v_{zx} = 0.1285$
$v_{zy} = ...$

Poissons Ratio, $(v_{zy}, v_{zx})$
Triaxial compression JBC 1-3 ($\beta = 45^0$), $\sigma_3 = UCS$

$y = 2.12E+06x - 1.82E+03$
$R^2 = 0.9772$

$y = 0.1285x + 0.0005$
$R^2 = 0.9833$

$y = -0.1579x + 0.0005$
$R^2 = 0.9870$
Bossier Shale – Compression test

Axial Stress vs Strain
Triaxial compression JBC 1-8 ($\beta = 45^\circ$), $\sigma_3 = UCS$

Slope: Radial Strain ET1 vs Axial Strain
Slope: Radial Strain ET2 vs Axial Strain

Poisson's Ratio, ($v_{zy}, v_{zx}$)
Triaxial compression JBC 1-8 ($\beta = 45^\circ$), $\sigma_3 = UCS$

Note: Elastic parameter window is $>0.35 - 0.3\%$ as material is linear elastic at higher stress
Bossier Shale – Compression test

Axial Stress vs Strain
Triaxial compression JBC 1-4 (β = 60°), σ₁ = UCS

Note:
* Elastic parameter range is > 50% (60-70% of yield) because of low strain values @ 50% yield (< 0.1% strain window selected from 0.08 to 0.09%).

Axial Strain vs Radial Strain 2
Axial Strain vs Radial Strain 1
Series 4
Series 5

Poisson’s Ratio, (νzx, νzy)
Triaxial compression JBC 1-4 (β = 60°), σ₁ = UCS

Note:
* ET1 > ET2

Axial Strain vs Strain difference, σ₁ - σ₃

- Strain εa, εET1, εET2, εvol
- Axial Stress difference, σ₁ - σ₃
- E modulus @ 60-70% of UCS
- Peak Strength

- E = 1.94E+06
- ϵ1 = 0.001533
- ϵET1 = -0.000786
- ϵET2 = -0.000221
- ϵvol = 0.000526
- νzx = 0.2977
- νzy = 0.0307

- σ1 - σ3 = 1,811
- σ3 = 0
- E = 1.94E+06
- ϵ1 = 0.001533
- ϵET1 = -0.000786
- ϵET2 = -0.000221
- ϵvol = 0.000526
- νzx = 0.2977
- νzy = 0.0307
Bossier Shale – Compression test

\[ \sigma_1 - \sigma_3 = 6,140 \]
\[ \sigma_3 = 0 \]
\[ E = 4.32 \times 10^6 \]
\[ \varepsilon_a = 0.001147 \]
\[ \varepsilon_{ET1} = -0.00255 \]
\[ \varepsilon_{ET2} = -0.00166 \]
\[ \varepsilon_{vol} = -0.003063 \]
\[ v_{zx} = 0.3964 \]
\[ v_{zy} = ... \]

Note: \( \varepsilon_{vol} \) shows higher strain laterally than axially. This is comparable to the CT data indicating the material failed by forming parallel lines vertically, hence failure by crushing and not in Mohr-Coulomb criterion.
Bossier Shale – Compression test

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Note:
* Elastic parameter range >0.3% strain as linear behavior was achieved at 0.38 – 0.48% axial strain.
Bossier Shale – Compression test

Axial Stress vs Strain
Triaxial compression JBC 4-6 (β = 30°), σ_3 = 1,000psi

Poissons Ratio, (ν_zx, ν_zy)
Triaxial compression JBC 4-6 (β = 30°), σ_3 = 1,000psi
Bossier Shale – Compression test

Axial Stress vs Strain
Triaxial compression JBC 4-3 ($\beta = 45^\circ$), $\sigma_3 = 1,000$psi

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<td>2.53E+06</td>
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<tr>
<td>$\epsilon_a$</td>
<td>0.00296</td>
</tr>
<tr>
<td>$\epsilon_{ET1}$</td>
<td>-0.000496</td>
</tr>
<tr>
<td>$\epsilon_{ET2}$</td>
<td>-0.000579</td>
</tr>
<tr>
<td>$\epsilon_{vol}$</td>
<td>0.001885</td>
</tr>
<tr>
<td>$v_{zx}$</td>
<td>0.1966</td>
</tr>
<tr>
<td>$v_{zy}$</td>
<td>0.1842</td>
</tr>
</tbody>
</table>

Poisson’s Ratio, $(v_{zx}, v_{zy})$

Triaxial compression JBC 4-3 ($\beta = 45^\circ$), $\sigma_3 = 1,000$psi

Axial Strain vs Radial Strain

Slope: Radial Strain ET1 vs Axial Strain
Slope: Radial Strain ET2 vs Axial Strain

Poisson’s Ratio $(v_{zx}, v_{zy})$

Axial Strain vs Radial Strain

$y = 2.53E+06x - 5.43E+01$
$R^2 = 1.00E+00$

$y = 0.1966x + 0.0001$
$R^2 = 0.9985$

$y = 0.1842x - 2E-05$
$R^2 = 0.9989$
Bossier Shale – Compression test

Axial Stress vs Strain
Triaxial compression JBC 4-4 (β = 60°), σ3 = 1,000psi

Axial Stress
Radial Stress ET1
Radial Stress ET2
Volumetric Stress
E modulus @ 50-60% of Yield

Note:
* Elastic parameter range is > 50% (60-70% of yield) because of low strain values @ 50% yield (≤ 1.5% strain window selected from 0.08 to 0.1%).
* ET1 > ET2.
* Similar in behavior to JBC 1-4 for Elastic range and ET1 > ET2, and both samples at 60deg.

Poissons Ratio, (νzy, νzx)
Triaxial compression JBC 4-5 (β = 60°), σ3 = 1,000psi

Axial strain vs Radial Strain 1
Axial strain vs Radial Strain 2
Series4
Series5

y = 0.1114x + 0.0005  
R² = 0.9931

y = 0.2056x + 0.0000  
R² = 0.9990

y = 0.2056x + 0.0000  
R² = 0.9990

y = 2,962.44x + 2,054.02  
R² = 0.9991
Bossier Shale – Compression test

Axial Stress vs Strain
Triaxial compression JBC 4-1 ($\beta = 90^\circ$), $\sigma_3 = 1,000$

- $\sigma_1 - \sigma_3 = 20,371$
- $\sigma_3 = 1,000$
- $E = 7.16 \times 10^6$
- $\varepsilon_a = 0.002987$
- $\varepsilon_{ET1} = -0.000722$
- $\varepsilon_{ET2} = -0.000901$
- $\varepsilon_{vol} = 0.001364$
- $v = 0.3241$
- $v = 0.2445$

Radial Strain $\varepsilon_{ET1}$ vs Axial Strain

Poissons Ratio, $(v_y, v_x)$
Triaxial compression JBC 4-1 ($\beta = 90^\circ$), $\sigma_3 = 1,000$

- $v_y = 0.3241$
- $v_x = 0.2445$

Axial Strain
Radial Strain $\varepsilon_{ET1}$
Radial Strain $\varepsilon_{ET2}$
Volumetric Strain
E modulus @ 46-75% of Yield
Bossier Shale – Compression test

### Stress vs Strain

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_1 - \sigma_3$</td>
<td>27,060</td>
</tr>
<tr>
<td>$\sigma_3$</td>
<td>3,000</td>
</tr>
<tr>
<td>$E$</td>
<td>$2.25E+06$</td>
</tr>
<tr>
<td>$\epsilon_a$</td>
<td>0.012693</td>
</tr>
<tr>
<td>$\epsilon_{ET1}$</td>
<td>-0.002143</td>
</tr>
<tr>
<td>$\epsilon_{ET2}$</td>
<td>-0.002531</td>
</tr>
<tr>
<td>$\epsilon_{vol}$</td>
<td>0.008019</td>
</tr>
</tbody>
</table>

- Axial Stress Difference, $\sigma_1 - \sigma_3$ vs Strain
- Radial Strain $\epsilon_{ET1}$ vs Axial Strain
- Radial Strain $\epsilon_{ET2}$ vs Axial Strain
- Volumetric Strain $\epsilon_{vol}$
- E modulus 14-22% of Yield
- Peak strength

### Poissons Ratio

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_{zx}$</td>
<td>0.1115</td>
</tr>
<tr>
<td>$v_{zy}$</td>
<td>0.1468</td>
</tr>
</tbody>
</table>

- Axial Strain vs Radial Strain $\epsilon_{a}$ $\epsilon_{ET1}$
- Axial Strain vs Radial Strain $\epsilon_{a}$ $\epsilon_{ET2}$
- Slope: Radial Strain $\epsilon_{ET1}$ vs Axial Strain
- Slope: Radial Strain $\epsilon_{ET2}$ vs Axial Strain

### Triaxial Compression

- JBC 2-5 ($\beta = 0^\circ$)
- $\sigma_3 = 3,000$ psi

---

165
Bossier Shale – Compression test

**Stress vs Strain**

Triaxial compression JBC 2-7 ($\beta = 15^\circ$), $\sigma_3 = 3,000$psi

- $\sigma_1 - \sigma_3 = 14,103$
- $\sigma_3 = 3,000$
- $E = 2.45 \times 10^6$
- $\varepsilon_a = 0.006381$
- $\varepsilon_{ET1} = -0.001082$
- $\varepsilon_{ET2} = -0.00112$
- $\varepsilon_{vol} = 0.004179$
- $v_{zx} = 0.1740$
- $v_{zy} = 0.1783$

**Poisson’s Ratio**

- $v_{zx}$
- $v_{zy}$

**Triaxial compression JBC 2-7 ($\beta = 15^\circ$), $\sigma_3 = 3,000$psi**

Axial Strain vs Radial Strain 1
Axial Strain vs Radial Strain 2
Slope: Radial Strain ET1 vs Axial Strain
Slope: Radial Strain ET2 vs Axial Strain

**Poissons Ratio**

- $(v_{zx}, v_{zy})$
Bossier Shale – Compression test

\[ \sigma_1 - \sigma_3 = 17,956 \]
\[ \sigma_3 = 3,000 \]
\[ E = 4.27 \times 10^6 \]
\[ \varepsilon_a = 0.005072 \]
\[ \varepsilon_{ET1} = -0.001266 \]
\[ \varepsilon_{ET2} = -0.000755 \]
\[ \varepsilon_{vol} = 0.003051 \]
\[ v_{zx} = 0.2247 \]
\[ v_{zy} = 0.2057 \]
\[ y = 4.27 \times 10^6 x + 1.26 \times 10^2 \]
\[ R^2 = 1.00 \]

\[ y = 0.2247x + 0.0000 \]
\[ R^2 = 0.9997 \]

\[ y = 0.2057x + 2 \times 10^{-5} \]
\[ R^2 = 0.9995 \]
Bossier Shale – Compression test

**Stress vs Strain**

Triaxial compression JBC 2-2 ($\beta = 30^\circ$), $\sigma_3 = 3,000$psi

- $\sigma_1 - \sigma_3 = 16,367$
- $\sigma_3 = 3,000$
- $E = 2.29 \times 10^6$
- $\varepsilon_a = 0.008162$
- $\varepsilon_{ET1} = -0.001549$
- $\varepsilon_{ET2} = -0.001835$
- $\varepsilon_{vol} = 0.004778$
- $v_{zx} = \ldots$
- $v_{zy} = \ldots$

**Poissons Ratio, $(v_{zy}, v_{zx})$**

Triaxial compression JBC 2-2 ($\beta = 30^\circ$), $\sigma_3 = 3,000$psi

- $y = 0.2046x + 0.0001$ $R^2 = 0.9888$
- $y = 0.184x + 0.0001$ $R^2 = 0.9898$

**Graphs**

- Stress vs Strain
- Poissons Ratio
Bossier Shale – Compression test

Stress vs Strain
Triaxial compression JBC 2-9 (β = 30°), σ3 = 3,000psi

- Axial Strain
- Radial Strain ET1
- Radial Strain ET2
- Volumetric Strain
- E modulus 33-50% of Yield
- Peak strength

\[ \sigma_1 - \sigma_3 = 14,500 \]
\[ \sigma_3 = 3,000 \]
\[ E = 2.68 \times 10^6 \]
\[ \varepsilon_a = 0.005726 \]
\[ \varepsilon_{ET1} = -0.001229 \]
\[ \varepsilon_{ET2} = -0.001354 \]
\[ \varepsilon_{vol} = 0.003143 \]
\[ v_{zx} = 0.2085 \]
\[ v_{zy} = 0.2226 \]

\[ y = 2.68E+06x + 2.80E+02 \]
\[ R^2 = 0.9995 \]

Poissons Ratio, \((v_{zy}, v_{zx})\)
Triaxial compression JBC 2-9 (β = 30°), σ3 = 3,000psi

\[ y = -0.2086x + 4E-05 \]
\[ R^2 = 0.9995 \]
Bossier Shale – Compression test

\[ \sigma_1 - \sigma_3 = 12,678 \]  
\[ \sigma_3 = 3,000 \]  
\[ E = 3.13 \times 10^6 \]  
\[ \varepsilon_a = 0.004893 \]  
\[ \varepsilon_{ET1} = -0.001332 \]  
\[ \varepsilon_{ET2} = -0.001016 \]  
\[ \varepsilon_{vol} = 0.002545 \]  
\[ v_{zx} = 0.1739 \]  
\[ v_{zy} = 0.2534 \]  

\[ y = 3.13 \times 10^6 x + 2.99 \times 10^1 \]  
\[ R^2 = 1.00 \]  

\[ y = -0.1739x + 0.0001 \]  
\[ R^2 = 0.9977 \]  

\[ y = -0.2534x - 6 \times 10^{-5} \]  
\[ R^2 = 0.9994 \]  

Poisson's Ratio, \((v_{zy}, v_{zx})\)

Triaxial compression JBC 2-3 (β = 45°), \(\sigma_3 = 3,000\) psi

Slope: Radial Strain ET1 vs Axial Strain
Slope: Radial Strain ET2 vs Axial Strain
Bossier Shale – Compression test

Stress vs Strain
Triaxial compression JBC 2-4 ($\beta = 60^\circ$), $\sigma_3 = 3,000$ psi

Poissons Ratio, ($v_{zy}, v_{zx}$)
Triaxial compression JBC 2-4 ($\beta = 60^\circ$), $\sigma_3 = 3,000$ psi

\[
\begin{align*}
\sigma_1 - \sigma_3 & = 10,751 \\
\sigma_3 & = 3,000 \\
E & = 3.05 \times 10^6 \\
\varepsilon_a & = 0.004438 \\
\varepsilon_{ET1} & = -0.001324 \\
\varepsilon_{ET2} & = -0.002141 \\
\varepsilon_{vol} & = 0.000973 \\
v_{zx} & = 0.1748 \\
v_{zy} & = 0.2424
\end{align*}
\]
Bossier Shale – Compression test

\[ \sigma_1 - \sigma_3 = 16,834 \]
\[ \sigma_3 = 3,000 \]
\[ E = 6.31 \times 10^6 \]
\[ \varepsilon_a = 0.003222 \]
\[ \varepsilon_{ET1} = -0.001047 \]
\[ \varepsilon_{ET2} = -0.000575 \]
\[ \varepsilon_{vol} = 0.0016 \]
\[ v_{zx} = 0.2103 \]
\[ v_{zy} = 0.2028 \]

\[ y = 6.310 \times 10^6 x - 6.620 \times 10^3 \]
\[ R^2 = 0.9941 \]

\[ y = -0.2103x + 2 \times 10^{-5} \]
\[ R^2 = 0.9993 \]
Bossier Shale – Compression test

\[
\sigma_1 - \sigma_3 = 25,988 \\
\sigma_3 = 3,000 \\
E = 6.71 \times 10^6 \\
\epsilon_a = 0.005232 \\
\epsilon_{ET1} = 0.000243 \\
\epsilon_{ET2} = -0.004581 \\
\epsilon_{vol} = 0.000894 \\
v_{zx} = 0.2483 \\
v_{zy} = 0.1910
\]

\[y = 6.715 \times 10^6 x - 1.069 \times 10^6 \]
\[R^2 = 1.00\]

\[y = 0.715x + 1.66 \times 10^2 \]
\[R^2 = 0.9976\]
Bossier Shale – Compression test

Stress vs Strain
Triaxial compression JBC 7-5 ($\beta = 0^\circ$), $\sigma_3 = 6,000$ psi

$$\sigma_1 - \sigma_3 = 30,951$$
$$\sigma_3 = 6,000$$
$$E = 2.28 \times 10^6$$
$$\varepsilon_a = 0.01551$$
$$\varepsilon_{ET1} = -0.003041$$
$$\varepsilon_{ET2} = -0.003556$$
$$\varepsilon_{vol} = 0.008913$$
$$v_{zx} = 0.1190$$
$$v_{zy} = 0.1549$$

$$y = 2.28 \times 10^6 x - 2.99 \times 10^2$$
$$R^2 = 0.9999$$

$$y = -0.1190 x + 0.0000$$
$$R^2 = 0.9987$$

Poissons Ratio, $(v_{zy}, v_{zx})$

Triaxial compression JBC 7-5 ($\beta = 0^\circ$), $\sigma_3 = 6,000$ psi

Slope: Radial Strain ET1 vs Axial Strain
Slope: Radial Strain ET2 vs Axial Strain
Bossier Shale – Compression test

\[ \sigma_1 - \sigma_3 = 24,920 \]
\[ \sigma_3 = 6,000 \]
\[ E = 4.28 \times 10^6 \]
\[ \varepsilon_a = 0.006849 \]
\[ \varepsilon_{ET1} = -0.002001 \]
\[ \varepsilon_{ET2} = -0.001581 \]
\[ \varepsilon_{vol} = 0.003267 \]
\[ v_{zx} = 0.2345 \]
\[ v_{zy} = 0.2372 \]

\[ y = 4.28 \times 10^6 x + 3.50 \times 10^2 \]
\[ R^2 = 1.00 \]

Stress vs Strain
Triaxial compression JBC 7-6 (\( \beta = 20^\circ \)), \( \sigma_3 = 6,000 \text{psi} \)

Poissons Ratio, \((v_{zx}, v_{zy})\)
Triaxial compression JBC 7-6 (\( \beta = 20^\circ \)), \( \sigma_3 = 6,000 \text{psi} \)
Bossier Shale – Compression test

\[
\begin{align*}
\sigma_1 - \sigma_3 &= 19,984 \\
\sigma_3 &= 6,000 \\
E &= 2.51 \times 10^6 \\
\varepsilon_a &= 0.00852 \\
\varepsilon_{ET1} &= -0.002958 \\
\varepsilon_{ET2} &= -0.001642 \\
\varepsilon_{vol} &= 0.00392 \\
v_{zx} &= 0.2018 \\
v_{zy} &= 0.1988 \\
y &= 2.51 \times 10^6 x + 1.31 \times 10^3 \\
R^2 &= 1.00 \times 10^0
\end{align*}
\]
Bossier Shale – Compression test

\[
\begin{align*}
\sigma_1 - \sigma_3 &= 17,468 \\
\sigma_3 &= 6,000 \\
E &= 2.67 \times 10^6 \\
\varepsilon_a &= 0.008164 \\
\varepsilon_{ET1} &= -0.001306 \\
\varepsilon_{ET2} &= -0.002828 \\
\varepsilon_{vol} &= 0.00403 \\
v_{zx} &= 0.2064 \\
v_{zy} &= 0.2392 \\
y &= 2.67 \times 10^6 x + 4.87 \times 10^2 \\
R^2 &= 1.00 \times 10^0
\end{align*}
\]
Bossier Shale – Compression test

\[ \sigma_1 - \sigma_3 = 18,026 \]
\[ \sigma_3 = 6,000 \]
\[ E = 2.74 \times 10^6 \]
\[ \varepsilon_a = 0.00733 \]
\[ \varepsilon_{ET1} = -0.001886 \]
\[ \varepsilon_{ET2} = -0.003266 \]
\[ \varepsilon_{vol} = 0.002178 \]
\[ v_{zx} = 0.2190 \]
\[ v_{zy} = 0.2608 \]

\[ y = 2.74E+06x + 7.00E+02 \]
\[ R^2 = 1.00E+00 \]

\[ y = -0.2190x + 0.0000 \]
\[ R^2 = 0.9995 \]

\[ y = -0.2608x + 2E-05 \]
\[ R^2 = 0.9996 \]

Stress vs Strain
Triaxial compression JBC 7-8 (\( \beta = 30^\circ \)), \( \sigma_3 = 6,000 \text{psi} \)

Poissons Ratio, \((v_{zx}, v_{zy})\)
Triaxial compression JBC 7-8 (\( \beta = 30^\circ \)), \( \sigma_3 = 6,000 \text{psi} \)
**Bossier Shale – Compression test**

### Stress vs Strain

**Triaxial compression JBC 7-3 (β = 45°), σ_3 = 6,000 psi**

- Axial Stress
- Radial Stress ET1
- Radial Stress ET2
- Volumetric Stress
- E modulus 24-50% of Yield
- Peak strength

- Axial strain vs Radial strain 1
- Axial strain vs Radial strain 2
- Slope: Radial strain ET1 vs Axial strain
- Slope: Radial strain ET2 vs Axial strain

### Poisson's Ratio, (ν_zy, ν_zx)

**Triaxial compression JBC 7-3 (β = 45°), σ_3 = 6,000 psi**

- Axial strain vs Radial strain 1
- Axial strain vs Radial strain 2
- Slope: Radial strain ET1 vs Axial strain
- Slope: Radial strain ET2 vs Axial strain

---

<table>
<thead>
<tr>
<th>β</th>
<th>E</th>
<th>ν</th>
<th>ε_a</th>
<th>ε_ET1</th>
<th>ε_ET2</th>
<th>ε_vol</th>
<th>( \sigma_1 - \sigma_3 )</th>
<th>( \sigma_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>2.62E+06</td>
<td>0.2169</td>
<td>0.007064</td>
<td>-0.001607</td>
<td>-0.001788</td>
<td>0.003669</td>
<td>16,722</td>
<td>6,000</td>
</tr>
</tbody>
</table>

\( R^2 = 1.00E+00 \)

---

\( y = 2.62E+06x + 6.00E+02 \)

\( R^2 = 0.9997 \)

\( y = -0.2169x + 0.0001 \)

\( R^2 = 0.9996 \)

---

179
Bossier Shale – Compression test

Stress vs Strain
Triaxial compression JBC 7-4 (β = 60°), σ3 = 6,000 psi

- Axial Strain
- Radial Strain ET1
- Radial Strain ET2
- Volumetric Strain
- E modulus, 25-50% of yield
- Peak strength

Poissons Ratio, (νzx, νzy)
Triaxial compression JBC 7-4 (β = 60°), σ3 = 6,000 psi

Axial strain vs Radial Strain 1
Axial strain vs Radial Strain 2
Slope: Radial Strain ET1 vs Axial Strain
Slope: Radial Strain ET2 vs Axial Strain

Axial stress difference, σ1 - σ3 = 14,330
σ3 = 6,000
E = 3.78E+06
εa = 0.004151
εET1 = -0.000944
εET2 = -0.001443
εvol = 0.001764
νzx = 0.1938
νzy = 0.2502

y = 3.78E+06x + 5.63E+02
R² = 0.9987

y = -0.1938x + 0.0001
R² = 0.9987

y = -0.2502x - 0.0001
R² = 0.9987

y = 0.1938x + 0.0001
R² = 0.9987

y = -0.2502x - 0.0001
R² = 0.9987
Bossier Shale – Compression test

**Stress vs Strain**
Triaxial compression JBC 7-1 (β = 90°), σ₃ = 6,000psi

- **Axial Strain**
- **Radial Strain ET1**
- **Radial Strain ET2**
- **Volumetric Strain**
- **E modulus: 26-50% of Yield**
- **Peak strength**

**Load-Deflection Curves**

**Poissons Ratio, (νₓᵧ, νᵢᵢ) vs Axial Strain**
Triaxial compression JBC 7-1 (β = 90°), σ₃ = 6,000psi

- **Axial strain vs Radial Strain 1**
- **Axial strain vs Radial Strain 2**
- **Slope: Radial Strain ET1 vs Axial Strain**
- **Slope: Radial Strain ET2 vs Axial Strain**
### Bossier Shale – Compression test

#### Stress vs Strain

Triaxial compression JBC 5-5 ($\beta = 0^\circ$), $\sigma_3 = 10,000$ psi

- $\sigma_1 - \sigma_3 = 32,614$ psi
- $\sigma_3 = 10,000$ psi
- $E = 2.61 \times 10^6$ psi
- $\epsilon_a = 0.014577$
- $\epsilon_{ET1} = -0.003235$
- $\epsilon_{ET2} = -0.004098$
- $\epsilon_{vol} = 0.007244$
- $v_{zx} = 0.1456$
- $v_{zy} = 0.2024$

#### Poisson's Ratio ($v_{ET1}, v_{ET2}$)

Triaxial compression JBC 5-5 ($\beta = 0^\circ$), $\sigma_3 = 10,000$ psi

- $y = 2.615E-06x + 1.245E-03$, $R^2 = 1.00E+00$
- $y = 0.1456x - 0.0000$, $R^2 = 0.9996$
- $y = 0.2024x - 3E-05$, $R^2 = 0.9996$

---

182
Bossier Shale – Compression test

Stress vs Strain
Triaxial compression JBC 5-8 (β = 10^0), σ_3 = 10,000psi

\[ \sigma_1 - \sigma_3 = 28,263 \]
\[ \sigma_3 = 10,000 \]
\[ E = 4.21 \times 10^6 \]
\[ \varepsilon_a = 0.007673 \]
\[ \varepsilon_{ET1} = -0.001908 \]
\[ \varepsilon_{ET2} = -0.001719 \]
\[ \varepsilon_{vol} = 0.004046 \]
\[ v_{zx} = 0.2388 \]
\[ v_{zy} = 0.2164 \]

y = 4.21E+06x + 4.74E+02
R² = 1.00E+00

\[ y = 0.2388x + 0.0000 \]
R² = 0.9997

\[ y = 0.2164x - 3E-05 \]
R² = 0.9996

Axial stress, \( \sigma \)
Strain, \( \varepsilon \)
Axial strain
Radial Strain ET1, \( \varepsilon_{ET1} \)
Radial Strain ET2, \( \varepsilon_{ET2} \)
Volumetric strain
\( E \) modulus 31-46% of yield
Peak strength

Poissons Ratio, \( (v_{zy}, v_{zx}) \)
Triaxial compression JBC 5-8 (β = 10^0), σ_3 = 10,000psi
Bossier Shale – Compression test

Stress vs Strain
Triaxial compression JBC 5-9 (β = 15°), σ3 = 10,000psi

-0.006 -0.004 -0.002 0 0.002 0.004 0.006 0.008 0.01 0.012
Axial stress, ε

-0.006 -0.004 -0.002 0 0.002 0.004 0.006 0.008 0.01 0.012
Strain εa, εET1, εET2, εvol

Poissons Ratio, (νzy, νzx)
Triaxial compression JBC 5-9 (β = 15°), σ3 = 10,000psi

Axial strain vs Radial strain 1
Axial strain vs Radial strain 2
Slope: Radial strain ET1 vs Axial strain
Slope: Radial strain ET2 vs Axial strain

Symbols and Values:
- ε1 - ε3 = 32,278
- ε3 = 10,000
- E = 4.26E+06
- εa = 0.01024
- εET1 = -0.002801
- εET2 = -0.002447
- εvol = 0.004992
- νzx = 0.2333
- νzy = 0.2157

Equations:
y = 0.2333x + 0.0001
R² = 0.9998

y = 0.2157x + 4E-05
R² = 0.9997

y = 6.29E+04x + 4.38E+01
R² = 1.0000
Bossier Shale – Compression test

\[ \sigma_1 - \sigma_3 = 26,198 \]
\[ \sigma_3 = 10,000 \]
\[ E = 2.70 \times 10^6 \]
\[ \varepsilon_a = 0.012005 \]
\[ \varepsilon_{ET1} = -0.003732 \]
\[ \varepsilon_{ET2} = -0.003479 \]
\[ \varepsilon_{vol} = 0.004794 \]
\[ v_{zx} = 0.2127 \]
\[ v_{zy} = 0.2448 \]

\[ y = 2.70 \times 10^6 x - 6.97 \times 10^3 \]
\[ R^2 = 0.9995 \]

\[ y = -0.2127x + 0.0001 \]
\[ R^2 = 0.9993 \]

\[ y = -0.2448x + 8 \times 10^{-5} \]
\[ R^2 = 0.9993 \]
Bossier Shale – Compression test

Stress vs Strain
Triaxial compression JBC 5-3 (β = 45°), $\sigma_3 = 10,000$ psi

- $\sigma_1 - \sigma_3 = 22,556$
- $\sigma_3 = 10,000$
- $E = 2.94 \times 10^6$
- $\varepsilon_a = 0.00931$
- $\varepsilon_{ET1} = -0.003458$
- $\varepsilon_{ET2} = -0.00294$
- $\varepsilon_{vol} = 0.002912$
- $\nu_{zx} = 0.2792$
- $\nu_{zy} = 0.2964$

Poissons Ratio, $(\nu_{zy}, \nu_{zx})$
Triaxial compression JBC 5-3 (β = 45°), $\sigma_3 = 10,000$ psi
Bossier Shale – Compression test

**Stress vs Strain**

Triaxial compression JBC 5-4 ($\beta = 60^\circ$), $\sigma_3 = 10,000$psi

- $\sigma_1 - \sigma_3 = 23,197$
- $\sigma_3 = 10,000$
- $E = 4.15 \times 10^6$
- $\varepsilon_a = 0.007123$
- $\varepsilon_{ET1} = -0.002771$
- $\varepsilon_{ET2} = -0.002826$
- $\varepsilon_{vol} = 0.001526$
- $v_{zx} = 0.2611$
- $v_{zy} = 0.3067$

**Poissons Ratio ($v_{xy}, v_{yx}$)**

Triaxial compression JBC 5-4 ($\beta = 60^\circ$), $\sigma_3 = 10,000$psi

- $y = 0.2611x + 0.0001$
  $R^2 = 0.9994$
- $y = -0.3067x$
  $R^2 = 0.9997$

---

**Graphs**

- Stress vs Strain
- Poisson's Ratio vs Axial Strain

---

187
Bossier Shale – Compression test

Stress vs Strain
Triaxial compression JBC 5-6 (β = 75°), $\sigma_3 = 10,000$psi

- Axial strain vs Radial Strain 1
- Axial strain vs Radial Strain 2
- Axial strain vs Volumetric strain
- Elastic modulus 20-45% of Yield
- Peak strength

Poisson's Ratio, ($\nu_{xy}$, $\nu_{yz}$)
Triaxial compression JBC 5-6 (β = 75°), $\sigma_3 = 10,000$psi

- Axial strain vs Radial Strain 1
- Axial strain vs Radial Strain 2
- Axial strain vs Volumetric strain
- Slope: Radial Strain ET1 vs Axial Strain
- GeoRes

\[ \sigma_1 - \sigma_3 = 29,812 \]
\[ \sigma_3 = 10,000 \]
\[ E = 7.51 \times 10^6 \]
\[ \varepsilon_a = 0.005216 \]
\[ \varepsilon_{ET1} = -0.001655 \]
\[ \varepsilon_{ET2} = -0.002187 \]
\[ \varepsilon_{vol} = 0.001374 \]
\[ v_{zx} = 0.2664 \]
\[ v_{zy} = 0.3659 \]

Graphical representation of stress-strain relationships and poisson's ratio with respective equations and values.
**Bossier Shale – Compression test**

**Stress vs Strain**
Triaxial compression JBC 5-1 ($\beta = 90^\circ$), $\sigma_3 = 10,000$ psi

-  $\sigma_1 - \sigma_3 = 37,097$
-  $\sigma_3 = 10,000$
-  $E = 8.19 \times 10^6$
-  $\varepsilon_a = 0.004846$
-  $\varepsilon_{ET1} = -0.001513$
-  $\varepsilon_{ET2} = -0.001999$
-  $\varepsilon_{vol} = 0.001334$
-  $v_{zx} = 0.3594$
-  $v_{zy} = 0.3038$

**Axial strain vs Radial Strain**
- $y = 8.19 \times 10^6 x + 2.41 \times 10^3$
- $R^2 = 9.99 \times 10^{-01}$

**Poissons Ratio, ($v_{zy}, v_{zx}$)**
Triaxial compression JBC 5-1 ($\beta = 90^\circ$), $\sigma_3 = 10,000$ psi

- $y = -0.3038x - 0.0000$
- $R^2 = 0.9990$

- $y = -0.3594x - 8 \times 10^{-05}$
- $R^2 = 0.9990$
Bossier Shale – Extension test

\[ \sigma_1 - \sigma_3 = 7,796 \]
\[ \sigma_3 = 228 \]
\[ E = 3.67 \times 10^6 \]
\[ \varepsilon_a = -0.00325114 \]
\[ \varepsilon_{ET1} = -0.00018799 \]
\[ \varepsilon_{ET2} = 0.002717798 \]
\[ \varepsilon_{vol} = -0.00072133 \]
\[ v_{zx} = 0.1057 \]
\[ v_{zy} = 0.2820 \]

\[ y = -3.67 \times 10^6 x + 5.42 \times 10^2 \]

Axial Strain vs Radial Strain 1
Axial Strain vs Radial Strain 2
Slope: Radial Strain ET2 vs Axial Strain

Poissons Ratio, (\(v_{zy}, v_{zx}\))
Triaxial compression MUE 9-1 (\(\beta = 45^\circ\)), \(\sigma_1 = 8,024\)psi
**Bossier Shale – Extension test**

**Stress vs Strain**

Triaxial extension JBC 3-1 (β = 60°), \( \sigma_1 = 13,334 \text{psi} \)

- \( \sigma_1 - \sigma_3 = 10,709 \)
- \( \sigma_3 = 2,610 \)
- \( E = 3.95 \times 10^6 \)
- \( \varepsilon_a = -0.00342954 \)
- \( \varepsilon_{ET1} = -0.00016641 \)
- \( \varepsilon_{ET2} = 0.000776718 \)
- \( \varepsilon_{vol} = -0.00281923 \)
- \( v_{zx} = 0.1198 \)
- \( v_{zy} = 0.1808 \)

**Poisson's Ratio, \((v_{zy}, v_{zx})\)**

Triaxial compression MUE 3-1 (\( \beta = 60° \)), \( \sigma_1 = 13,334 \text{psi} \)

- \( y = 0.1198x + 0.0000 \)
- \( R^2 = 0.8759 \)
- \( y = -0.1808x - 0.0001 \)
- \( R^2 = 0.8046 \)
Bossier Shale – Extension test

\[ \sigma_1 - \sigma_3 = 17,923 \]
\[ \sigma_3 = 7,085 \]
\[ E = 5.62 \times 10^6 \]
\[ \varepsilon_a = -0.00420461 \]
\[ \varepsilon_{ET1} = 0.000767473 \]
\[ \varepsilon_{ET2} = 0.002395817 \]
\[ \varepsilon_{vol} = -0.00104132 \]
\[ v_{zx} = 0.2409 \]
\[ v_{zy} = 0.4097 \]

\[ y = -5.624549x + 3.08 \times 10^{-2} \]
\[ R^2 = 0.9591 \]

\[ y = -0.4097x - 5 \times 10^{-5} \]
\[ R^2 = 0.972 \]
Bossier Shale – Extension test

\[ \sigma_1 - \sigma_3 = 17,331 \]
\[ \sigma_3 = 7,661 \]
\[ E = 5.19 \times 10^6 \]
\[ \varepsilon_a = -0.00513524 \]
\[ \varepsilon_{ET1} = 0.000597042 \]
\[ \varepsilon_{ET2} = 0.003024213 \]
\[ \varepsilon_{vol} = -0.00151398 \]
\[ v_{zx} = 0.125 \]
\[ v_{zy} = 0.3866 \]

\[ y = -5.19 \times 10^6 x + 2.75 \times 10^1 \]
\[ R^2 = 9.78 \times 10^{-01} \]

\[ y = -0.3866 x - 0.0000 \]
\[ R^2 = 0.9126 \]

\[ y = -0.1250 x - 0.0000 \]
\[ R^2 = 0.5396 \]
Bossier Shale – Extension test

Stress vs Strain
Triaxial extension JBC 6-4 (β = 60°), σ1 = 25,007psi

\[
\begin{align*}
\sigma_1 - \sigma_3 &= 17,817 \\
\sigma_3 &= 7,190 \\
E &= 4.48 \times 10^6 \\
\varepsilon_a &= -0.00511752 \\
\varepsilon_{ET1} &= 0.000840627 \\
\varepsilon_{ET2} &= 0.002503805 \\
\varepsilon_{vol} &= -0.00177309 \\
v_{zx} &= 0.1597 \\
v_{zy} &= 0.4056
\end{align*}
\]

\[y = -0.6460x + 7.43 \times 10^2 \quad R^2 = 9.97 \times 10^{-01}\]

\[y = -0.4056x + 4 \times 10^{-5} \quad R^2 = 0.9869\]

\[
\begin{align*}
\text{Axial Strain vs Radial Strain ET1} \\
\text{Axial Strain vs Radial Strain ET2} \\
\text{Volumetric Strain vs Axial Strain} \\
\text{E modulus vs Axial Strain} \\
\text{Peak strength vs Axial Strain}
\end{align*}
\]

Poissons Ratio, \((v_{zy}, v_{zx})\)
Triaxial compression MUE 6-4 (β = 60°), σ1 = 25,007psi

\[
\begin{align*}
y &= -0.8056x + 0.0625 \quad R^2 = 0.9890 \\
y &= -0.3758x + 0.0000 \quad R^2 = 0.0026
\end{align*}
\]

\[
\begin{align*}
\text{Slope: Radial Strain ET1 vs Axial Strain} \\
\text{Slope: Radial Strain ET2 vs Axial Strain}
\end{align*}
\]
Vaca Muerta Shale – Summary of experimental results

<table>
<thead>
<tr>
<th>Triaxial experiment type</th>
<th>Sample ID</th>
<th></th>
<th>Bulk Density</th>
<th>&lt;sub&gt;1&lt;/sub&gt;−&lt;sub&gt;3&lt;/sub&gt;</th>
<th>&lt;sub&gt;3&lt;/sub&gt;</th>
<th>Ez</th>
<th>g&lt;sub&gt;a&lt;/sub&gt;</th>
<th>g&lt;sub&gt;ET1&lt;/sub&gt;</th>
<th>g&lt;sub&gt;ET2&lt;/sub&gt;</th>
<th>g&lt;sub&gt;vol&lt;/sub&gt;</th>
<th>v&lt;sub&gt;xz&lt;/sub&gt;</th>
<th>v&lt;sub&gt;yz&lt;/sub&gt;</th>
<th>strain in (in/in) at failure</th>
</tr>
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<tbody>
<tr>
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<td>MUE 1-1</td>
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<td>0.000</td>
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<td>2.12±0.06</td>
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<tr>
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<td>2.366</td>
<td>18,035</td>
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<td>23,802</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>2.15±0.06</td>
</tr>
</tbody>
</table>

Average E result

-  = 2.454±0.06
- Eh = 3.888±0.06
- v<sub>zz</sub> = 0.24±0.06
- v<sub>xz</sub> = 0.24±0.06
- v<sub>yz</sub> = 0.24±0.06
- Ez = 0.24±0.06
Vaca Muerta Shale – Compression test

\[ \sigma_1 - \sigma_3 = 13,730 \]
\[ \sigma_3 = 0 \]
\[ E = 2.67 \times 10^6 \]
\[ \epsilon_a = 0.006367 \]
\[ \epsilon_{ET1} = -0.000961 \]
\[ \epsilon_{ET2} = -0.001001 \]
\[ \epsilon_{vol} = 0.004405 \]
\[ v_{zx} = 0.1586 \]
\[ v_{zy} = 0.1690 \]

\[ y = 2.67 \times 10^6 x - 1.30 \times 10^3 \]
\[ R^2 = 1.00 \]

\[ y = \frac{1}{1.00} x + 0.0001 \]
\[ R^2 = 0.9993 \]

\[ y = \frac{1}{1.00} x + 0.0002 \]
\[ R^2 = 0.9992 \]
Vaca Muerta Shale – Compression test

\[ \sigma_1 - \sigma_3 = 7,874 \]
\[ \sigma_3 = 0 \]
\[ E = 3.03 \times 10^6 \]
\[ \varepsilon_a = 0.0029 \]
\[ \varepsilon_{ET1} = -0.003215 \]
\[ \varepsilon_{ET2} = -0.000991 \]
\[ \varepsilon_{vol} = -0.001306 \]
\[ v_{zx} = 0.2207 \]
\[ v_{zy} = \ldots \]

\[ y = 3.03 \times 10^6 x \]
\[ R^2 = 1.00 \times 10^0 \]

\[ y = -0.2207x + 0.0001 \]
\[ R^2 = 0.9836 \]

\[ y = -0.0965x - 9 \times 10^{-06} \]
\[ R^2 = 0.9685 \]

\[ \beta = 60 \]
\[ \sigma_3 = UCS \]

Axial Strain
Radial Strain ET1
Radial Strain ET2
Volumetric Strain
E modulus 40-50% of Yield
Peak strength

Poissons Ratio, \( (v_{zy}, v_{zx}) \)

Axial strain vs Radial Strain 1
Axial strain vs Radial Strain 2
Slope: Radial Strain ET1 vs Axial Strain
Slope: Radial Strain ET2 vs Axial Strain

Elastic modulus 40-50% of Yield
Peak strength
Vaca Muerta Shale – Compression test

Stress vs Strain
Triaxial compression MUE 1-4 (β = 0°), σ3 = 1,000psi

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>σ1-σ3</td>
<td>19,118</td>
</tr>
<tr>
<td>σ3</td>
<td>1,000</td>
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<tr>
<td>E</td>
<td>2.65E+06</td>
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<tr>
<td>εa</td>
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<td>νzx</td>
<td>0.1850</td>
</tr>
<tr>
<td>νzy</td>
<td>0.1877</td>
</tr>
</tbody>
</table>

y = 2.65E+06x + 1.27E+03
R² = 1.00E+00

Axial Strain
Radial Strain ET1
Radial Strain ET2
Volumetric Strain

E modulus 40-50% of Yield
Peak strength

Poissons Ratio, (νzy, νzx)
Triaxial compression MUE 1-4 (β = 0°), σ3 = 1,000psi

y = -0.1850x + 0.0000
R² = 0.9992

y = -0.1877x + 5E-05
R² = 0.9996
Vaca Muerta Shale – Compression test

\[ \sigma_1 - \sigma_3 = 11,337 \]
\[ \sigma_3 = 1,000 \]
\[ E = 3.77 \times 10^6 \]
\[ \varepsilon_a = 0.003259 \]
\[ \varepsilon_{ET1} = -0.000857 \]
\[ \varepsilon_{ET2} = -0.001049 \]
\[ \varepsilon_{vol} = 0.001353 \]
\[ v_{zx} = 0.2568 \]
\[ v_{zy} = 0.2864 \]

\[ y = 3.77 \times 10^6 x + 0.77 \times 10^{-2} \]
\[ R^2 = 1.00 \]

\[ y = -0.2568 x + 0.0000 \]
\[ R^2 = 0.9983 \]

\[ y = -0.2864 x + 3 \times 10^{-5} \]
\[ R^2 = 0.9979 \]

Axial stress vs Strain

Triaxial compression MUE 1-9 (\( \beta = 60^\circ \)), \( \sigma_3 = 1,000 \) psi

Poissons Ratio, \((v_{zy}, v_{zx})\)

Triaxial compression MUE 1-9 (\( \beta = 60^\circ \)), \( \sigma_3 = 1,000 \) psi
Vaca Muerta Shale – Compression test

**Stress vs Strain**
Triaxial compression MUE 1-16 ($\beta = 90^\circ$), $\sigma_3 = 1,000$psi

- $\sigma_1 - \sigma_3 = 18,073$
- $\sigma_3 = 1,000$
- $E = 3.80 \times 10^6$
- $\varepsilon_a = 0.00536$
- $\varepsilon_{ET1} = -0.005983$
- $\varepsilon_{ET2} = -0.000887$
- $\varepsilon_{vol} = -0.00151$
- $v_{zx} = 0.2793$
- $v_{zy} = 0.4195$

**Poissons Ratio** ($v_{xy}, v_{yx}$)
Triaxial compression MUE 1-16 ($\beta = 90^\circ$), $\sigma_3 = 1,000$psi

- $v_{xy} = 0.2793 \times 10^{-3}$ $R^2 = 0.9989$
- $v_{yx} = 0.4195$
Vaca Muerta Shale – Compression test

Stress vs Strain
Triaxial compression MUE 1-33 ($\beta = 15^\circ$), $\sigma_3 = 2,500$psi

- $\sigma_1 - \sigma_3 = 17,604$
- $\sigma_3 = 2,500$
- $E = 1.68E+06$
- $\varepsilon_a = 0.014801$
- $\varepsilon_{ET1} = -0.004027$
- $\varepsilon_{ET2} = -0.003228$
- $\varepsilon_{vol} = 0.007546$
- $v_{zx} = 0.2164$
- $v_{zy} = 0.1346$

$y = 1.68E+06x + 1.50E+03$
$R^2 = 1.00E+00$

Axial stress diff, ($\sigma_1 - \sigma_3$)

Strain $\varepsilon_a$, $\varepsilon_{ET1}$, $\varepsilon_{ET2}$, $\varepsilon_{vol}$

Poissons Ratio, ($v_{zy}$, $v_{zx}$)

Axial Strain vs Radial Strain 1
Axial Strain vs Radial Strain 2
Radial Strain ET1 vs Axial Strain
Radial Strain ET2 vs Axial Strain

Slope: Radial Strain ET1 vs Axial Strain
Slope: Radial Strain ET2 vs Axial Strain

Graphs showing the relationship between axial strain and radial strain for different stress conditions.
Vaca Muerta Shale – Compression test

Stress vs Strain
Triaxial compression MUE 1-22 ($\beta = 20^\circ$), $\sigma_3 = 2,500$psi

- $\sigma_{1-3} = 19,451$
- $\sigma_3 = 2,500$
- $E = 2.30 \times 10^6$
- $\varepsilon_a = 0.012081$
- $\varepsilon_{ET1} = -0.006045$
- $\varepsilon_{ET2} = -0.005598$
- $\varepsilon_{vol} = 0.000438$
- $v_{zx} = 0.2617$
- $v_{zy} = 0.2186$

Axial stress diff. ($\sigma_1 - \sigma_3$)

Axial Strain
Radial Strain ET1
Radial Strain ET2
Volumetric Strain
$E$ modulus: 40-50% of Yield
Peak strength

y = $2.30 \times 10^6 x + 1.81 \times 10^3$
$R^2 = 1.00 \times 10^0$

Poissons Ratio, ($v_{zy}, v_{zx}$)
Triaxial compression MUE 1-22 ($\beta = 20^\circ$), $\sigma_3 = 2,500$psi

Radial Strain ET1 vs Axial Strain
Radial Strain ET2 vs Axial Strain

Slope: Radial Strain ET1 vs Axial Strain
Slope: Radial Strain ET2 vs Axial Strain

Axial strain vs Radial Strain 1
Axial strain vs Radial Strain 2
Axial strain vs Radial Strain 3
Axial strain vs Radial Strain 4
Axial strain vs Radial Strain 5
Axial strain vs Radial Strain 6
Axial strain vs Radial Strain 7
Axial strain vs Radial Strain 8
Axial strain vs Radial Strain 9
Axial strain vs Radial Strain 10
Axial strain vs Radial Strain 11
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Axial strain vs Radial Strain 98
Axial strain vs Radial Strain 99
Axial strain vs Radial Strain 100
Vaca Muerta Shale – Compression test

Stress vs Strain
Triaxial compression MUE 1-32 \((\beta = 30^\circ)\), \(\sigma_3 = 2,500\text{psi}\)

- Axial Strain
- Radial Strain ET1
- Radial Strain ET2
- Volumetric Strain
- Stress vs Strain
- Peak Strength
- E modulus 40-50% of Yield

Axial stress diff \(\sigma_1 - \sigma_3 = 18,036\)
\(\sigma_3 = 2,500\)
\(E = 2.19 \times 10^6\)
\(\epsilon_a = 0.011023\)
\(\epsilon_{ET1} = -0.003189\)
\(\epsilon_{ET2} = -0.003514\)
\(\epsilon_{vol} = 0.00432\)
\(v_{zx} = 0.2104\)
\(v_{zy} = 0.1996\)

\[ y = 2.39 \times 10^6 x + 1.78 \times 10^3 \]
\[ R^2 = 1.00 \times 10^0 \]

Poissons Ratio, \((v_{zy}, v_{zx})\)
Triaxial compression MUE 1-32 \((\beta = 30^\circ)\), \(\sigma_3 = 2,500\text{psi}\)

- Axial strain vs Radial strain 1
- Axial strain vs Radial strain 2
- Slope: Radial strain ET1 vs Axial strain
- Slope: Radial strain ET2 vs Axial strain

\[ y = -0.2104 x + 0.0001 \]
\[ R^2 = 1.00 \times 10^{-05} \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_1 - \sigma_3)</td>
<td>18,036</td>
</tr>
<tr>
<td>(\sigma_3)</td>
<td>2,500</td>
</tr>
<tr>
<td>(E)</td>
<td>2.19 \times 10^6</td>
</tr>
<tr>
<td>(\epsilon_a)</td>
<td>0.011023</td>
</tr>
<tr>
<td>(\epsilon_{ET1})</td>
<td>-0.003189</td>
</tr>
<tr>
<td>(\epsilon_{ET2})</td>
<td>-0.003514</td>
</tr>
<tr>
<td>(\epsilon_{vol})</td>
<td>0.00432</td>
</tr>
<tr>
<td>(v_{zx})</td>
<td>0.2104</td>
</tr>
<tr>
<td>(v_{zy})</td>
<td>0.1996</td>
</tr>
</tbody>
</table>
Vaca Muerta Shale – Compression test

Stress vs Strain
Triaxial compression MUE 1-27 ($\beta = 40^\circ$), $\sigma_3 = 2,500$psi

- $\sigma_1 - \sigma_3 = 15,507$
- $\sigma_3 = 2,500$
- $E = 2.20 \times 10^6$
- $\varepsilon_a = 0.009666$
- $\varepsilon_{ET1} = -0.003237$
- $\varepsilon_{ET2} = -0.002728$
- $\varepsilon_{vol} = 0.003701$
- $v_{zx} = 0.2324$
- $v_{zy} = 0.1664$

Axial stress diff ($\sigma_1 - \sigma_3$)
Strain $\varepsilon_a$, $\varepsilon_{ET1}$, $\varepsilon_{ET2}$, $\varepsilon_{vol}$
E modulus 40-50% of Yield
Peak Strength

Poissons Ratio, $(v_{zy}, v_{zx})$

Axial strain vs Radial strain 1
Axial strain vs Radial strain 2
Slope: Radial strain ET1 vs Axial strain
Slope: Radial strain ET2 vs Axial strain
Vaca Muerta Shale – Compression test

\[
\begin{align*}
\sigma_1 - \sigma_3 &= 14,512 \\
\sigma_3 &= 2,500 \\
E &= 2.66 \times 10^6 \\
\varepsilon_a &= 0.007998 \\
\varepsilon_{ET1} &= -0.003035 \\
\varepsilon_{ET2} &= -0.001751 \\
\varepsilon_{vol} &= 0.003212 \\
v_{zx} &= 0.2781 \\
v_{zy} &= 0.1666 \\
y &= 2.66 \times 10^6 x + 4.02 \times 10^2 \\
R^2 &= 1.00 \\
\end{align*}
\]
Vaca Muerta Shale – Compression test

\[
\begin{align*}
\sigma_1 - \sigma_3 &= 17,433 \\
\sigma_3 &= 2,500 \\
E &= 3.00 \times 10^6 \\
\varepsilon_a &= 0.008272 \\
\varepsilon_{ET1} &= -0.002348 \\
\varepsilon_{ET2} &= -0.003285 \\
\varepsilon_{vol} &= 0.002639 \\
v_{zx} &= 0.2015 \\
v_{zy} &= 0.3286 \\
y &= 3.00 \times 10^6 x + 1.61 \times 10^3 \\
R^2 &= 1.00 \times 10^0 \\
\end{align*}
\]
Vaca Muerta Shale – Compression test

\[ \sigma_1 - \sigma_3 = 19,347 \]
\[ \sigma_3 = 2,500 \]
\[ E = 3.76 \times 10^6 \]
\[ \varepsilon_a = 0.005929 \]
\[ \varepsilon_{ET1} = -0.001725 \]
\[ \varepsilon_{ET2} = -0.001769 \]
\[ \varepsilon_{vol} = 2.44 \times 10^{-3} \]
\[ v_{zx} = 0.2599 \]
\[ v_{zy} = 0.2485 \]

\[ y = 3.7646x + 1.7948 \]
\[ R^2 = 1.00 \times 10^0 \]

\[ y = 0.2485x - 0.0000 \]
\[ R^2 = 0.9998 \]

\[ y = 0.2599x - 0.0000 \]
\[ R^2 = 0.9998 \]

Poissons Ratio, \((v_{zy}, v_{zx})\)
Vaca Muerta Shale – Compression test

**Stress vs Strain**
Triaxial compression MUE 1-2 ($\beta = 0^\circ$), $\sigma_3 = 5,000$psi

- $\sigma_1 - \sigma_3 = 25,744$
- $\sigma_3 = 5,000$
- $E = 2.12 \times 10^6$
- $\varepsilon_a = 0.017101$
- $\varepsilon_{ET1} = -0.004515$
- $\varepsilon_{ET2} = -0.004311$
- $\varepsilon_{vol} = 8.28 \times 10^{-3}$
- $v_{zx} = 0.1998$
- $v_{zy} = 0.1900$

**Poissons Ratio** ($v_{zx}, v_{zy}$)
Triaxial compression MUE 1-2 ($\beta = 0^\circ$), $\sigma_3 = 5,000$psi

- $v_{zx} = 0.1986$
- $v_{zy} = 0.1986$

**Graphs**
- Stress vs Strain
- Poisson's Ratio ($v_{zx}, v_{zy}$)
Vaca Muerta Shale – Compression test

Stress vs Strain
Triaxial compression MUE 1-5 (β = 100), σ3 = 5,000psi

Poison's Ratio, (nzy, nzx)
Triaxial compression MUE 1-5 (β = 100), σ3 = 5,000psi
Vaca Muerta Shale – Compression test

### Stress vs Strain

**Triaxial compression MUE 1-21 (β = 40°), σ₃ = 5,000psi**

- **Axial Strain (εₐ)**
- **Radial Strain ET1 (εₑ₁)**
- **Radial Strain ET2 (εₑ₂)**
- **Volumetric Strain (εᵥ)**
- **E modulus 40-50% of Yield**
- **Peak strength**

### Poisson's Ratio (νₓᵧ, νᵧₓ)

**Triaxial compression MUE 1-21 (β = 40°), σ₃ = 5,000psi**

- **Axial strain vs Radial Strain 1**
- **Axial strain vs Radial Strain 2**
- **Slope: Radial Strain ET1 vs Axial Strain**
- **Slope: Radial Strain ET2 vs Axial Strain**

### Equations

- $\sigma_1 - \sigma_3 = 24,355$
- $\sigma_3 = 5,000$
- $E = 3.01 \times 10^6$
- $\varepsilon_a = 0.012017$
- $\varepsilon_{ET1} = -0.004343$
- $\varepsilon_{ET2} = -0.004881$
- $\varepsilon_{vol} = 2.79 \times 10^{-3}$
- $v_{zx} = 0.2137$
- $v_{zy} = 0.2071$

**Graphical Data**

[Stress vs Strain Graph]

[Stress vs Strain Graph]

[Poison's Ratio Graph]
**Vaca Muerta Shale – Compression test**

### Stress vs Strain

**Triaxial compression MUE 1-10 (β = 60°), σ₃ = 5,000psi**

- $\sigma_1 - \sigma_3 = 18,913$
- $\sigma_3 = 5,000$
- $E = 4.10E+06$
- $\varepsilon_a = 0.005754$
- $\varepsilon_{ET1} = -0.00171$
- $\varepsilon_{ET2} = -0.000904$
- $\varepsilon_{vol} = 3.14E-03$
- $v_{zx} = 0.2408$
- $v_{zy} = 0.2006$

### Poisson's Ratio, $(v_{zy}, v_{zx})$

**Triaxial compression MUE 1-10 (β = 60°), σ₁ = 5,000psi**

- $y = 0.2408x + 0.0000$
  $R^2 = 0.9995$
- $y = 0.2006x + 0.0000$
  $R^2 = 0.9998$

---

**Graphs:**
- Stress vs Strain graph showing $\sigma_1 - \sigma_3$ and corresponding strains.
- Poisson's Ratio graph showing $v_{zy}$ vs $\varepsilon_a$ and $v_{zx}$ vs $\varepsilon_a$. 

---

**Summary:**
- The stress-strain behavior of Vaca Muerta Shale under triaxial compression at $\beta = 60°$ with $\sigma_3 = 5,000psi$ is analyzed, providing key material properties like modulus and Poisson's ratios. The graphs illustrate the relationship between axial, radial, and volumetric strains. 

---

**Equations:**
- $\varepsilon_a = \frac{y - 5000}{E}$
- $v_{zx} = \frac{\varepsilon_{ET1} - \varepsilon_a}{\varepsilon_a}$
- $v_{zy} = \frac{\varepsilon_{ET2} - \varepsilon_a}{\varepsilon_a}$
Vaca Muerta Shale – Compression test

\[
\begin{align*}
\sigma_1 - \sigma_3 &= 28,639 \\
\sigma_3 &= 5,000 \\
E &= 4.66 \times 10^6 \\
\varepsilon_a &= 0.007758 \\
\varepsilon_{ET1} &= -0.002374 \\
\varepsilon_{ET2} &= -0.00199 \\
\varepsilon_{vol} &= 3.39 \times 10^{-3} \\
v_{zx} &= 0.2551 \\
v_{zy} &= 0.2282
\end{align*}
\]

\[y = 4.66 \times 10^6 x + 1.29 \times 10^3 \quad R^2 = 1.00 \]

\[y = -0.2551 x + 0.0000 \quad R^2 = 0.9998\]

\[y = -0.2282 x + 0.0000 \quad R^2 = 0.9996\]
Vaca Muerta Shale – Compression test

\[
\sigma_1 - \sigma_3 = 22,673 \\
\sigma_3 = 5,000 \\
E = 3.52 \times 10^6 \\
\varepsilon_a = 0.008917 \\
\varepsilon_{ET1} = -0.00215 \\
\varepsilon_{ET2} = -0.002433 \\
\varepsilon_{vol} = 4.33 \times 10^{-3} \\
v_{zx} = 0.2256 \\
v_{zy} = 0.2249
\]
Vaca Muerta Shale – Compression test

\[ \sigma_1 - \sigma_3 = 49,963 \]
\[ \sigma_3 = 20,000 \]
\[ E = 2.35 \times 10^6 \]
\[ \varepsilon_a = 0.067036 \]
\[ \varepsilon_{ET1} = -0.022181 \]
\[ \varepsilon_{ET2} = -0.022008 \]
\[ \varepsilon_{vol} = 2.28 \times 10^{-2} \]
\[ v_{zx} = 0.1605 \]
\[ v_{zy} = 0.1474 \]

\[ y = 2.35 \times 10^6 x + 2.34 \times 10^3 \]
\[ R^2 = 0.9981 \]

\[ y = -0.1605x - 0.0000 \]
\[ R^2 = 0.9993 \]

\[ y = -0.1474x - 0.0000 \]
\[ R^2 = 0.9994 \]
Vaca Muerta Shale – Compression test

\[ \sigma_1 - \sigma_3 = 41,119 \]
\[ \sigma_3 = 20,000 \]
\[ E = 3.01 \times 10^6 \]
\[ \varepsilon_a = 0.058103 \]
\[ \varepsilon_{ET1} = -0.021896 \]
\[ \varepsilon_{ET2} = -0.037697 \]
\[ \varepsilon_{vol} = -1.49 \times 10^{-3} \]
\[ v_{zx} = 0.1909 \]
\[ v_{zy} = 0.1849 \]

\[ y = 3.015 \times 10^6 x + 1.15 \]
\[ R^2 = 0.9986 \]

\[ y = -0.037697 x \]
\[ R^2 = 0.9997 \]

\[ y = -0.0005 x \]
\[ R^2 = 0.9986 \]
Vaca Muerta Shale – Compression test

\[ \sigma_{1} - \sigma_{3} = 47,894 \]
\[ \sigma_{3} = 20,000 \]
\[ E = 4.44 \times 10^{6} \]
\[ \varepsilon_{a} = 0.081562 \]
\[ \varepsilon_{ET1} = -0.059534 \]
\[ \varepsilon_{ET2} = -0.042261 \]
\[ \varepsilon_{vol} = -2.02 \times 10^{-2} \]
\[ v_{zx} = 0.2030 \]
\[ v_{zy} = 0.2499 \]

\[ y = 4.44 \times 10^{6}x + 5.90 \times 10^{2} \]
\[ R^{2} = 0.9962 \]

\[ y = -0.2499x - 0.0000 \]
\[ R^{2} = 0.9933 \]
Vaca Muerta Shale – Extension test

Stress vs Strain
Triaxial compression MUE 1-24 (β = 0°), σ2 = 24,963psi

\[ \sigma_1 - \sigma_3 = 23,802 \]
\[ \sigma_3 = 1,161 \]
\[ E = 4.74 \times 10^6 \]
\[ \varepsilon_a = -0.00775522 \]
\[ \varepsilon_{ET1} = 0.003801683 \]
\[ \varepsilon_{ET2} = 0.001848205 \]
\[ \varepsilon_{vol} = -0.00210533 \]
\[ v_{zx} = 0.2666 \]
\[ v_{zy} = 0.4501 \]

\[ y = -4.74 \times 10^6 x + 1.89 \times 10^3 \]
\[ R^2 = 0.9921 \]

Poisson’s Ratio, (ν_{zy}, ν_{zx})

Triaxial compression MUE 1-24 (β = 0°), σ2 = 24,963psi

\[ v_{xy} = 0.001093629 \]
\[ v_{yx} = 0.00091065 \]
\[ v_{xy} = 0.2506 \]
\[ v_{yx} = 0.4901 \]
Vaca Muerta Shale – Extension test

### Stress vs Strain

**Triaxial compression MUE 1-25 (β = 60°), σ₁ = 22,619psi**

![Stress vs Strain Graph](image)

- **Axial Stress Diff:** 
  \[ \sigma_1 - \sigma_3 = 19,510 \]
  \[ \sigma_3 = 3,109 \]
  \[ E = 3.25 \times 10^6 \]
  \[ \varepsilon_a = -0.00820747 \]
  \[ \varepsilon_{ET1} = 0.002294112 \]
  \[ \varepsilon_{ET2} = 0.004119127 \]
  \[ \varepsilon_{vol} = -0.00179423 \]

- **Axial Strain:** \[ y = -3.25 \times 10^6 x + 2.04 \times 10^3 \]
  \[ R^2 = 0.9492 \]

- **Radial Strain ET1:** \[ y = -0.4239 x - 7 \times 10^{-5} \]
  \[ R^2 = 0.9837 \]

### Poisson's Ratio, (νzx, νzy)

**Triaxial compression MUE 1-25 (β = 60°), σ₁ = 22,619psi**

![Poisson's Ratio Graph](image)

- **Radial Strain ET1 vs Axial Strain:** \[ y = 0.4239 x - 7 \times 10^{-5} \]
  \[ R^2 = 0.9837 \]

- **Radial Strain ET2 vs Axial Strain:** \[ y = -0.2289 x - 0.0001 \]
  \[ R^2 = 0.9827 \]

### Diagrams

1. **Stress vs Strain Graph**
   - Axial Strain
   - Radial Strain ET1
   - Radial Strain ET2
   - Volumetric Strain
   - E modulus 40-50% of Yield
   - Peak strength

2. **Poisson's Ratio Graph**
   - Axial strain vs Radial Strain ET1
   - Axial strain vs Radial Strain ET2
   - Slope: Radial Strain ET1 vs Axial Strain
   - Slope: Radial Strain ET2 vs Axial Strain

3. **Axial Strain vs Radial Strain**
   - ![Axial Strain vs Radial Strain](image)

4. **Radial Strain vs Axial Strain**
   - ![Radial Strain vs Axial Strain](image)
B1  JPW model objective function

% Matlab code - Objective function (Jaeger plane of weakness for $\sigma_1 > \sigma_2 = \sigma_3$)
% This Matlab code is used to input strength data, and model RMSE is calculated for the
% combination of strength parameters taken from the main loop (B2). The strength
% parameters with least RMSE is determined from the main Loop (B2) and used to
calculate % strength $\sigma_1$ and plotted against data sets for comparison.

function Error=Objectivefun(x,options);
format compact
Data = ...
[ ... 
  0  90  6140
  0  60  1811
  0  45  6874
  0  45  5653
  0  30  6577
  0   0  16466
1000 90  21371
1000 60  6672
1000 45  8021
1000 30  13321
1000  0  18106
3000 90  28988
3000 75  19834
3000 60  13751
3000 45  15678
3000 30  19367
3000 30  17500
3000 15  17103
3000 15  20956
3000  0  30060
6000 90  37437
6000 60  20330
6000 45  22722
6000 30  25984
6000 30  24026
6000 20  23468
6000 20  30920
6000  0  36951
10000 90  47097
10000 75  39812
10000 60  33197
10000 45  32556
10000 30  36198
10000 15  42278
10000 10  38263
10000  0  42614
];
t = Data(:,2);
sig1 = Data(:,3);
sig3 = Data(:,1);

So=x(1);
fo=x(2);
Sw=x(3);
fw=x(4);

sig1T= 2*So*((cos(fo*pi/180))./(1- (sin(fo*pi/180)))).+sig3*((1+(sin(fo*pi/180)))./(1-(sin(fo*pi/180)))).+0*t;

sig1W=sig3+(2*(Sw+sig3*(tan(fw*pi/180))))./((1- ((tan(fw*pi/180))./(tan(t*pi/180))).*(sin((2*t)*pi/180));
for i=1:length(sig1W)
   if (sig1W(i)<0)
      sig1W(i)=1.0E50;
   end
end

sig1T=min(sig1T,sig1W);
Error=((1/36)*sum((sig1-sig1T).^2)).^0.5
if (options(1)==1)
   %Start plot for sig3=10kpsi
   tt=0:1:90;
sig1_10k= 2*So*((cos(fo*pi/180))./(1-
   (sin(fo*pi/180)))).+10000*((1+(sin(fo*pi/180)))./(1-(sin(fo*pi/180)))).+0*tt;
sig1W_10k=10000+(2*(Sw+10000*(tan(fw*pi/180))))./((1-
   ((tan(fw*pi/180))./(tan(tt*pi/180))).*(sin((2*tt)*pi/180));
   for i=1:length(sig1W_10k);
      if (sig1W_10k(i)<0);
         sig1W_10k(i)=1.0E50;
      end
   end
   sig1_10k=min(sig1_10k,sig1W_10k);

   %Start plot for sig3=6kpsi
   sig1_6k= 2*So*((cos(fo*pi/180))./(1-
   (sin(fo*pi/180)))).+6000*((1+(sin(fo*pi/180)))./(1-(sin(fo*pi/180)))).+0*tt;
sig1W_6k=6000+(2*(Sw+6000*(tan(fw*pi/180))))./((1-
   ((tan(fw*pi/180))./(tan(tt*pi/180))).*(sin((2*tt)*pi/180));
   for i=1:length(sig1W_6k);
      if (sig1W_6k(i)<0);
         sig1W_6k(i)=1.0E50;
      end
   end
   sig1_6k=min(sig1_6k,sig1W_6k);

   %Start plot for sig3=3kpsi
   sig1_3k= 2*So*((cos(fo*pi/180))./(1-
   (sin(fo*pi/180)))).+3000*((1+(sin(fo*pi/180)))./(1-(sin(fo*pi/180)))).+0*tt;
sig1W_3k=3000+(2*(Sw+3000*(tan(fw*pi/180))))./((1-
   ((tan(fw*pi/180))./(tan(tt*pi/180))).*(sin((2*tt)*pi/180));
   for i=1:length(sig1W_3k);
      if (sig1W_3k(i)<0);
         sig1W_3k(i)=1.0E50;
      end
   end
   sig1_3k=min(sig1_3k,sig1W_3k);

   %Start plot for sig3=1kpsi
sig1_1k = 2*So*((cos(fo*pi/180))./(1-(sin(fo*pi/180))))+1000*((1+(sin(fo*pi/180)))./(1-(sin(fo*pi/180))))+0*tt; 
sig1W_1k=1000+(2*(Sw+1000*(tan(fw*pi/180))))./((1-(tan(fw*pi/180))./(tan(tt*pi/180)))).*(sin((2*tt)*pi/180));
for i=1:length(sig1W_1k);
   if (sig1W_1k(i)<0);
      sig1W_1k(i)=1.0E50;
   end
end
sig1_1k=min(sig1_1k,sig1W_1k);

%Start plot for sig3=UCS
sig1_UCS= 2*So* ((cos(fo*pi/180))./(1-(sin(fo*pi/180))))+0*((1+(sin(fo*pi/180)))./(1-(sin(fo*pi/180))))+0*tt;
sig1W_UCS=0+(2*(Sw+0*(tan(fw*pi/180))))./((1-(tan(fw*pi/180))./(tan(tt*pi/180)))).*(sin((2*tt)*pi/180));
for i=1:length(sig1W_UCS);
   if (sig1W_UCS(i)<0);
      sig1W_UCS(i)=1.0E50;
   end
end
sig1_UCS=min(sig1_UCS,sig1W_UCS);

plot(t(1:6),sig1(1:6),'r*');
plot(t(7:11),sig1(7:11),'co');
plot(t(12:20),sig1(12:20),'go');
plot(t(21:28),sig1(21:28),'bo');
plot(t(29:36),sig1(29:36),'ro');

hold on;
plot (tt,sig1_10k,'-r');
plot (tt,sig1_6k,'-b');
plot (tt,sig1_3k,'-g');
plot (tt,sig1_1k,'-c');
plot (tt,sig1_UCS,'-r');
end
end
B2 JPW model main loop

% Matlab code - MainLoop (Jaeger plane of weakness for $\sigma_1 > \sigma_2 = \sigma_3$)
% This Matlab code is where strength parameters are input iteratively to be used in B1
clc
tic

nN=20;
Y(1:nN*nN*nN*nN*nN*nN+10)=10^20;
for iSo=1:1:nN
  for ifo=1:1:nN
    for iSw=1:1:nN
      for ifw=1:1:nN
        So=50*(iSo)+3300; % 3.3k<So<4.3k ; 0.05kpsi No.n
        fo=0.1*(ifo)+28; % 28.1deg<fo<30deg ; 0.1deg
        Sw=50*(iSw)+2000; % 2.02k<Sw<3k ; 0.05kpsi
        fw=0.1*(ifw)+23.5; % 23.6deg-fw<24.5deg ; 0.1deg

        % Different intervals applied here
        % So=5000*(iSo)-5000; % 0k<So<95k ; 5kpsi No.1
        % fo=5*(ifo)-5; % 0deg<fo<95deg ; 5deg
        % Sw=5000*(iSw)-5000; % 0k<Sw<95k ; 5kpsi
        % fw=5*(ifw)-5; % 0deg<fw<95deg ; 5deg

        xxxx=[So fo Sw fw];
        initialStepSize=0.1*abs(xxxx)+100; % changed the step size

        index=iSo*nN*nN+ifo*nN+iSw*nN+ifw;
        X6Dim(index,1:4)=xxxx;
        Ytmp=Objectivefun(xxxx,[0,0]);
        Y(index)=Ytmp;
      end
    end
  end
end
Ymin=min(Y)
indexminAll=find(Y==Ymin, 1 );
indexmin=min(find(Y==Ymin));
Objectivefun(X6Dim(indexmin,1:4),[1,1]);

So=X6Dim(indexmin,1)
fo=X6Dim(indexmin,2)
Sw=X6Dim(indexmin,3)
wfw=X6Dim(indexmin,4)

xxxx=[So fo Sw fw];
Objectivefun(xxxx,[1,1]);
toc
Pariseau model objective function

% Matlab code – Objective function (Pariseau model for $\sigma_1 > \sigma_2 = \sigma_3$)
% This Matlab code is used to input strength data, and Pariseau model RMSE is calculated.
% The strength parameters $F$, $G$, $U$, $V$ and $M$ determined with least RMSE from the main
Loop % (B4) is then used to calculate strength $\sigma_1$ and this is plotted against data sets for
% comparison

function Error=Objectivefun(x,options);
format compact
Data = ... 
[ ... 
10000 90 32614 
10000 80 28263 
10000 75 32278 
10000 60 26198 
10000 45 22556 
10000 30 23197 
10000 15 29812 
10000 0 37097 
6000 90 30951 
6000 70 24920 
6000 60 19984 
6000 60 17468 
6000 60 18026 
6000 45 16722 
6000 30 14330 
6000 0 31437 
3000 90 27060 
3000 75 14103 
3000 75 17956 
3000 60 16367 
3000 60 14500 
3000 45 12678 
3000 30 10751 
3000 15 16834 
3000 0 25988 
1000 90 17106 
1000 60 12321 
1000 45 7021 
1000 30 5672 
1000 0 20371 
0 90 16466 
0 60 6577 
0 45 5653 
0 45 6874 
0 30 1811 
0 0 6140];
t = Data(:,2);
sig13 = Data(:,3);
sig3 = Data(:,1);

F=1.0/(x(1)*(x(1)));
G=1.0/(x(2)*(x(2)));  
U=1.0/x(3);  
V=1.0/x(4);  
M=1.0/(x(5)*(x(5)));  

xPos=((F*(sin(t*pi/180)).^4)+G*((cos(t*pi/180)).^4+((cos(2*t*pi/180)).^2))+0.25*M*((sin(2*t*pi/180))).^2);  

Error=((1/36)*sum((sig13-((1+(U+2*V)*sig3))/((((F*(sin(t*pi/180)).^4)+G*((cos(t*pi/180)).^4+((cos(2*t*pi/180)).^2))+0.25*M*((sin(2*t*pi/180))).^2)).^0.5-((U*(cos(t*pi/180)).^2+V*(sin(t*pi/180)).^2))).^0.5))^0.5;  

if (options(1)>0 & xPos>0)  
tt=0:1:90;  

sig13_10k=((1+(U+2*V)*10000)./(((F*(sin(tt*pi/180)).^4)+G*((cos(tt*pi/180)).^4+((cos(2*tt*pi/180)).^2))+0.25*M*((sin(2*tt*pi/180))).^2)).^0.5-((U*(cos(tt*pi/180)).^2+V*(sin(tt*pi/180)).^2))).^0.5;  

sig13_6k=((1+(U+2*V)*6000)./(((F*(sin(tt*pi/180)).^4)+G*((cos(tt*pi/180)).^4+((cos(2*tt*pi/180)).^2))+0.25*M*((sin(2*tt*pi/180))).^2)).^0.5-((U*(cos(tt*pi/180)).^2+V*(sin(tt*pi/180)).^2))).^0.5;  

sig13_3k=((1+(U+2*V)*3000)./(((F*(sin(tt*pi/180)).^4)+G*((cos(tt*pi/180)).^4+((cos(2*tt*pi/180)).^2))+0.25*M*((sin(2*tt*pi/180))).^2)).^0.5-((U*(cos(tt*pi/180)).^2+V*(sin(tt*pi/180)).^2))).^0.5;  

sig13_1k=((1+(U+2*V)*1000)./(((F*(sin(tt*pi/180)).^4)+G*((cos(tt*pi/180)).^4+((cos(2*tt*pi/180)).^2))+0.25*M*((sin(2*tt*pi/180))).^2)).^0.5-((U*(cos(tt*pi/180)).^2+V*(sin(tt*pi/180)).^2))).^0.5;  

sig13_UCS=((1+(U+2*V)*0)./(((F*(sin(tt*pi/180)).^4)+G*((cos(tt*pi/180)).^4+((cos(2*tt*pi/180)).^2))+0.25*M*((sin(2*tt*pi/180))).^2)).^0.5-((U*(cos(tt*pi/180)).^2+V*(sin(tt*pi/180)).^2))).^0.5;  

plot(t,sig13,'bo');  
    hold on;  
plot (tt,sig13_10k,'rx');  
plot (tt,sig13_6k,'g*');  
plot (tt,sig13_3k,'bx');  
plot (tt,sig13_1k,'go');  
plot (tt,sig13_UCS,'r*');  
end  
end
% Matlab code - MainLoop (Pariseau model for $\sigma_1 > \sigma_2 = \sigma_3$)
% This Matlab code is where parameters $F, G, U, V$ and $M$ are input iteratively to use in B3

clear
clc
tic

nN=12;
Y(1:nN*nN*nN*nN*nN*nN+10)=10^12;
for iF=1:1:nN
  for iG=1:1:nN
    for iU=1:1:nN
      for iV=1:1:nN
        for iM=1:1:nN
          FinvSqrt=100*(iF)+19900; % 20.0k<F<21.1k ; 0.1kpsi interval
          GinvSqrt=100*(iG)+8900; % 9.0k<G<10.1k ; 0.1kpsi interval
          Uinv=100*(iU)+12400; % 12.5k<U<13.6k ; 0.1kpsi interval
          Vinv=100*(iV)+20900; % 21.0k<V<22.1k ; 0.1kpsi interval
          MinvSqrt=100*(iM)+1900; % 2.0<M<3.1k ; 0.1kpsi interval

          xxxx=[FinvSqrt GinvSqrt Uinv Vinv MinvSqrt];
          initialStepSize=0.1*abs(xxxx)+100; % changed the step size
          index=iF*nN*nN*nN*nN+iG*nN*nN*nN+iU*nN*nN+iV*nN+iM;
          X6Dim(index,1:5)=xxxx;
          Ytmp=Objectivefun(xxxx,[0,0]);
          Y(index)=Ytmp;
        end
      end
    end
  end
end

Ymin=min(Y)
indexminAll=find(Y==Ymin, 1)
indexmin=min(find(Y==Ymin));
Objectivefun(X6Dim(indexmin,1:5),[1,1])

FinvSqrt=X6Dim(indexmin,1)
GinvSqrt=X6Dim(indexmin,2)
Uinv=X6Dim(indexmin,3)
Vinv=X6Dim(indexmin,4)
MinvSqrt=X6Dim(indexmin,5)
xxxx=[FinvSqrt GinvSqrt Uinv Vinv MinvSqrt];
Objectivefun(xxxx,[1,1])

Toc
True-triaxial Pariseau model objective function

% Matlab code – Objective function (Pariseau model for $\sigma_1 > \sigma_2 > \sigma_3$)
% This Matlab code is used to input strength data, and Pariseau model RMSE is calculated.
% The strength parameters $F$, $G$, $U$, $V$ and $M$ determined from the main Loop (B6) with least
% RMSE is then used to calculate strength $\sigma_1$ and plotted against data sets for comparison

function Error=Objectivefun(x,options);
format compact
Data = ...
[ ... 
0   0   60  8557    0 
0   0   60  10298   0 
0   3626  60  22336   3626 
0   7252  60  35389   7252 
0  10878  60  47572  10878 
45  0   60  9572    0 
45  0   60  9863    0 
45  3626  60  24801   3626 
45  7252  60  33794   7252 
45  7252  60  35244   7252 
45  7252  60  35389   7252 
45  10878  60  47137  10878 
45  10878  60  48733  10878 
0   0   0  21321    0 
0   0   0  21611    0 
0   3626  0  42641   3626 
0   7252  0  55840   7252 
0  10878  0  72664  10878 
0   7252  60  35389   7252 
0   7252  60  32633   12328 
0   7252  60  34809   17550 
0   7252  60  32633   24076 
0   7252  60  33794   24076 
45  7252  60  35824   7252 
45  7252  60  35824   7252 
45  7252  60  39595   8122 
45  7252  60  42061  10153 
45  7252  60  42641  12473 
45  7252  60  43511  14069 
45  7252  60  43221  14069 
45  7252  60  50183  17550 
45  7252  60  48588  17550 
45  7252  60  46122  20885 
45  7252  60  46267  24221 
45  7252  60  50183  24366 
45  7252  60  44091  26107]
L = Data(:,1);
sig3 = Data(:,2);
t = Data(:,3);
sig1 = Data(:,4);
sig2 = Data(:,5);

F=1.0/(x(1)*(x(1)))
G=1.0/(x(2)*(x(2)))
U=1.0/x(3);
V=1.0/x(4);
M=1.0/(x(5)*(x(5)))

xPos=(((F*(sin(t*pi/180)).^4)+G*((cos(t*pi/180)).^4+((cos(2*t*pi/180)).^2))+
0.25*M*((sin(2*t*pi/180)).^2);
sig22=(cos(t*pi/180)).^2.*((sig2.*(cos(L*pi/180)).^2)+sig3.*(sin(L*pi/180)).^2)+sig1.*(sin(t*pi/180)).^2;
sig33 = (((sig2.*(sin(L*pi/180)).^2) + sig3.*((cos(L*pi/180)).^2)));
sig11= (sin(t*pi/180)).^2.*((sig2.*(cos(L*pi/180)).^2)+sig3.*(sin(L*pi/180)).^2)+sig1.*(cos(t*pi/180)).^2;
sig23 = (cos(t*pi/180)).*(cos(L*pi/180)).*(sin(L*pi/180)).*(sig3-sig2);
sig31 = (sin(t*pi/180)).*(cos(L*pi/180)).*(sin(L*pi/180)).*(sig3-sig2);
sig12 = (cos(t*pi/180)).*(sin(t*pi/180)).*(cos(L*pi/180)).*(sig2.*(cos(L*pi/180)).^2)+sig3.*(sin(L*pi/180)).^2-sig1);

Error= (1/62)*sum((1-((F*(sig22-sig33)).^2+G*((sig33-sig11).^2)+((sig11-sig22).^2)+(2*G+4*F)*sig23.^2+M*(sig31.^2+sig12.^2)).^0.5-
(U*sig11+V*(sig22+sig33))).^2).^0.5
if (options(1)>0 & xPos>0)

    tt=0:1:90;

    sig10_8k=((1+(U+2*V)*10878)./(((F*(sin(tt*pi/180)).^4)+G*((cos(tt*pi/180)).^4+((cos(2*tt*pi/180)).^2))+0.25*M*((sin(2*tt*pi/180))).^2)).^0.5... - (U*(cos(tt*pi/180)).^2+V*(sin(tt*pi/180)).^2)))+10878;

    sig7_2k=((1+(U+2*V)*7252)./(((F*(sin(tt*pi/180)).^4)+G*((cos(tt*pi/180)).^4+((cos(2*tt*pi/180)).^2))+0.25*M*((sin(2*tt*pi/180))).^2)).^0.5... - (U*(cos(tt*pi/180)).^2+V*(sin(tt*pi/180)).^2)))+7252;

    sig3_6k=((1+(U+2*V)*3626)./(((F*(sin(tt*pi/180)).^4)+G*((cos(tt*pi/180)).^4+((cos(2*tt*pi/180)).^2))+0.25*M*((sin(2*tt*pi/180))).^2)).^0.5... - (U*(cos(tt*pi/180)).^2+V*(sin(tt*pi/180)).^2)))+3626;

    sig1_UCS=((1+(U+2*V)*0)./(((F*(sin(tt*pi/180)).^4)+G*((cos(tt*pi/180)).^4+((cos(2*tt*pi/180)).^2))+0.25*M*((sin(2*tt*pi/180))).^2)).^0.5... - (U*(cos(tt*pi/180)).^2+V*(sin(tt*pi/180)).^2)));

plot(t, sig1, 'bo');
    hold on;
    plot (tt, sig10_8k, 'rx');
    plot (tt, sig7_2k, 'g*');
    plot (tt, sig3_6k, 'bx');
    % plot (tt, sig1k, 'go');
    plot (tt, sig1_UCS, 'r*');

end

end
% Matlab code - MainLoop (Pariseau model for $\sigma_1 > \sigma_2 > \sigma_3$)
% This Matlab code is where parameters $F$, $G$, $U$, $V$ and $M$ are input iteratively to use in B5

clear
clc
tic

nN=12;
Y(1:nN*nN*nN*nN*nN*nN+10)=10^12;
for iF=1:nN
for iG=1:nN
for iU=1:nN
for iV=1:nN
for iM=1:nN
    FinvSqrt=-1000*(iF)-34000;  % -46k<F<-57k ; 1kpsi interval
    GinvSqrt=2000*(iG)+50000;  % 52<G<74k ; 2kpsi interval
    Uinv=5000*(iU)+75000;  % 80k<U<135k ; 5kpsi interval
    Vinv=5000*(iV)+75000;  % 80k<V<135k ; 5kpsi interval
    MinvSqrt=500*(iM)+8000;  % 8.5<M<14k ; 0.5kpsi interval
    % FinvSqrt=-1000*(iF)-45000;  % -46k<F<-57k ; 1kpsi interval  No. 30
    % GinvSqrt=2000*(iG)+50000;  % 52<G<74k ; 2kpsi interval
    % Uinv=5000*(iU)+75000;  % 80k<U<135k ; 5kpsi interval
    % Vinv=5000*(iV)+75000;  % 80k<V<135k ; 5kpsi interval
    % MinvSqrt=1000*(iM)+9000;  % 10<M<22k ; 1kpsi interval
    % ...  iteration...  No. 2 to 29
    % ...  
    % FinvSqrt=20000*(iF-nN/2);  % -100k<F<120k ; 20kpsi interval  No. 2
    % GinvSqrt=10000*(iG)-10000;  % 0<G<110k ; 10kpsi interval
    % Uinv=20000*(iU)-20000;  % 0<k<u<220k ; 20kpsi interval
    % Vinv=20000*(iV)-20000;  % 0<k<V<220k ; 20kpsi interval
    % MinvSqrt=5000*(iM)-10000;  % 0<M<60k ; 5kpsi interval
    % FinvSqrt=20000*(iF-nN/2);  % -100k<F<120k ; 20kpsi interval  No. 1
    % GinvSqrt=10000*(iG)-10000;  % 0<G<110k ; 10kpsi interval
    % Uinv=10000*(iU)-10000;  % 0<k<u<110k ; 5kpsi interval
    % Vinv=10000*(iV)-10000;  % 0<k<V<110k ; 5kpsi interval
    % MinvSqrt=10000*(iM)-10000;  % 0<M<100k ; 10kpsi interval

    x=FinvSqrt GinvSqrt Uinv Vinv MinvSqrt;
    initialStepSize=0.1*abs(x)+100;  % changed the step size
    index=iF*nN*nN*nN*nN+iG*nN*nN+iU*nN+iV*nN+iM;
    X6Dim(index,1:5)=x;
    Ytmp=Objectivefun(x,[0,0]);
    Y(index)=Ytmp;
end
Ymin=min(Y)

indexminAll=find(Y==Ymin, 1 )
indexmin=min(find(Y==Ymin));
Objectivefun(X6Dim(indexmin,1:5),[1,1])

FinvSqrt=x6Dim(indexmin,1)
GinvSqrt=x6Dim(indexmin,2)
Uinv=x6Dim(indexmin,3)
Vinv=x6Dim(indexmin,4)
MinvSqrt=x6Dim(indexmin,5)
xxxx=[FinvSqrt GinvSqrt Uinv Vinv MinvSqrt];
Objectivefun(xxxx,[1,1])

toc
B7 True-triaxial Pariseau model strength
$\sigma_1$ prediction

% The Objective of this Matlab code is to determine strength $\sigma_1$ using the parameters $F$, $G$, $U$, $V$ and $M$ determined for least RMSE from B5 and B6. The Pariseau model strength $\sigma_1$ is
% determined as a function of $\sigma_2$, $\sigma_3$, $\beta$ and $\omega$ for defined parameters $F$, $G$, $U$, $V$ and $M$. The
% final output is a plot of $\sigma_1$ vs. $\sigma_2$ for defined $\sigma_3$, $\beta$ and $\omega$

clc;
clear all;
tic

F=4.7259E-10;
G=4.34028E-10;
U=0.0000125;
V=7.40741E-06;
M=1.23457E-08;
firstPass = 1;

for sig2=7.251e3:2e3:150e3

beta=60;  % Input angle beta $\beta$ here
t=90-beta;

omega=0;  % Input angle omega $\omega$ here
L=90-omega;

sig3=7252;

sig1 = sym('sig1','positive');

sig22=(cos(t*pi/180)).^2.*((sig2.*(cos(L*pi/180)).^2)+sig3.*(sin(L*pi/180)).^2)+sig1.*(sin(t*pi/180)).^2;

sig33 = (((sig2*(sin(L*pi/180)).^2) + sig3*((cos(L*pi/180)).^2));

sig11=(sin(t*pi/180)).^2.*((sig2*(cos(L*pi/180)).^2)+sig3*(sin(L*pi/180)).^2)+sig1*(cos(t*pi/180)).^2;

sig23 =(cos(t*pi/180)).*(cos(L*pi/180)).*(sin(L*pi/180))*(sig3-sig2);

sig31 =(sin(t*pi/180)).*(cos(L*pi/180)).*(sin(L*pi/180))*(sig3-sig2);

% End of code
\[ \text{sig12} = (\cos(t \pi/180)) \cdot (\sin(t \pi/180)) \cdot ((\text{sig2} \cdot (\cos(L \pi/180))^2 + \text{sig3} \cdot (\sin(L \pi/180))^2)^2 - \text{sig1}); \]

\[
\text{sig1\_calc} = \text{solve}(((F \cdot (\text{sig2}^2 - \text{sig3}^2) + G \cdot ((\text{sig3}^2 - \text{sig1}^2) + (\text{sig1}^2 - \text{sig2}^2)^2) + (2G+4F) \cdot \text{sig23}^2 + M \cdot (\text{sig31}^2 + \text{sig12}^2))^{0.5} - (U \cdot \text{sig11} + V \cdot (\text{sig22} + \text{sig33}) - 1), \text{sig1});
\]

\[ \text{sig1\_calc} = \text{max(double(sig1\_calc))} \]

\[
\text{sig2} \quad \text{if} \quad (\text{firstPass} == 1)
\]
\[
\quad \text{sig2\_calc} = \text{sig2};
\quad \text{sig1\_calc\_toPlot} = \text{sig1\_calc};
\quad \text{firstPass} = 0;
\]

\[
\text{else}
\quad \text{sig2\_calc} = [\text{sig2\_calc}, \text{sig2}];
\quad \text{sig1\_calc\_toPlot} = [\text{sig1\_calc\_toPlot}, \text{sig1\_calc}];
\text{end}
\]

\[
\text{plot} \quad (\text{sig2\_calc}, \text{sig1\_calc\_toPlot}, 'bx');
\]

\[
\text{title}('\text{sig1 vs sig2}');
\text{xlabel}('\text{sig2}');
\text{ylabel}('\text{sig1}');
\text{axis}([0 140e3 0 140e3]);
\]

\[
\text{end}
\]

toc
B8 True-triaxial JPW model strength $\sigma_1$ prediction

% Matlab code – True-triaxial JPW model
% This Matlab code is used to compute strength $\sigma_1$ using Coulomb strength parameters $S_o$, $\phi_o$, $S_w$ and $\phi_w$, for defined $\sigma_3$, $\beta$ and $\omega$

clc
tic
beta= 61.4 ; % % Input angle beta $\beta$ here
omega=32.8 ; % % Input angle omega $\omega$ here
n=(cos(omega*pi/180)); % n=(cos(Lambda*pi/180)) ; n=>omega=w=direction
l=(cos(beta*pi/180)); % l=(cos(Lambda*pi/180)) ; l=>bedding angle

S=4101.2 "%Sw=1694.8"; "So=4101.2";
phi=32.8 "%fw=25.8" ; "fo=32.8";
u=(tan(phi*pi/180)); %tan phi

firstPass = 1;

for sig2=7212:0.5e3:120e3;
sig3=7312;
sig1 = sym('sig1', 'positive');
sig=sig2-(sig2-sig3)*n^2-(sig2-sig1)*l^2;
Tau=(n^2*(sig1-sig3)*(sig2-sig3)-(sig1-sig)*(sig2-sig))^0.5;
sig1_calc = solve((Tau-S-u*sig), sig1);
sig1_calc = max(double(sig1_calc))
sig2
if( firstPass == 1)
sig2_calc = sig2;
sig1_calc_toPlot = sig1_calc;
firstPass = 0;
else
sig2_calc=[sig2_calc,sig2] ;
sig1_calc_toPlot = [sig1_calc_toPlot,sig1_calc];
end

plot (sig2_calc,sig1_calc_toPlot,'bx');
title('sig1 vs sig2');
xlabel('sig2');
ylabel('sig1');
axis([0 120e3 0 120e3]);

end
toc
Differentiate Coulomb criterion for $\beta_{\text{min}}$

The Coulomb criterion for shear stress $\tau$ acting along the plane is a function of cohesion $S$, friction angle $\phi$ and normal stress $\sigma_n$. For shear fracture of rock to occur, the Coulomb criterion needs to be satisfied, and the mathematical representation of the Coulomb criterion is

$$|\tau| = S + \mu \sigma_n. \quad \text{C.1.1}$$

The shear and normal stress can be determined using the Mohr circle and expressed as a function of principal stresses using maximum shear stress $\tau_m$ and mean normal stress $\sigma_n$ as follows

$$\tau = |\tau_m \sin 2\beta|, \quad \text{C.1.2}$$
$$\sigma_n = \sigma_m - \tau_m \cos 2\beta. \quad \text{C.1.3}$$

Replacing the shear $\tau$ and normal stress $\sigma_n$ into the Coulomb criterion gives the general Coulomb criterion for shear failure at an arbitrary angle $\beta$ as

$$\tau_m = (S_w \cos \phi_w + \sigma_m \sin \phi_w)/[\sin(2\beta - \phi_w)]. \quad \text{C.1.4}$$

The above eqn. C.1.4 is also the plane of weakness criterion derived by Jaeger (1960) using the Coulomb criterion. To determine the Coulomb criterion (i.e., angle $\beta$ where minimum shear strength occurs), Coulomb differentiated the general solution in Eqn. C.1.4 with respect to angle $\beta$ as follows

$$\frac{d\tau_m}{d\beta} = \frac{d}{d\beta} \left\{\frac{(S_w \cos \phi_w + \sigma_m \sin \phi_w)}{[\sin(2\beta - \phi_w)]}\right\}. \quad \text{C.1.5}$$

Since the numerator is a constant, replace this with a new term $k$:

$$\frac{d\tau_m}{d\beta} = \frac{d}{d\beta} \left\{\left(\frac{k}{[\sin(2\beta - \phi_w)]}\right)\right\}. \quad \text{C.1.6}$$
$$\frac{d\tau_m}{d\beta} = (-2k) \cdot \cos(\phi - 2\beta) / \sin(\phi - 2\beta)^2. \quad \text{C.1.7}$$

At minimum shear, the slope of the function $\tau_m$ is zero, hence $\frac{d\tau_m}{d\beta} = 0$

$$\frac{d\tau_m}{d\beta} = (-2k) \cdot \cos(\phi - 2\beta) / \sin(\phi - 2\beta)^2 = 0. \quad \text{C.1.8}$$

By equating $\mp \cos(\phi - 2\beta) = 0$, the solution for minimum shear occurs at $\beta = \mp 45 \pm \phi/2$

Alternatively, using trigonometry and rearranging Eqn. C.1.8

$$\frac{d\tau_m}{d\beta} = (-2k) / [\tan(\phi - 2\beta) \cdot \sin(\phi - 2\beta)] = 0, \quad \text{C.1.9}$$

$$(-2k) / [\tan(\phi - 2\beta) \cdot \sin(\phi - 2\beta)] = 0, \quad \text{C.1.10}$$
\[ \frac{(-2k) \ast \left( \frac{\tan \phi - \tan 2\beta}{1 + \tan \phi \tan 2\beta} \right) \ast \sin(\phi - 2\beta)}{\sin(\phi - 2\beta)} = 0, \quad \text{C.1.11} \]

\[ (-2k) \ast (1 + \tan \phi \tan 2\beta) / [(\tan \phi - \tan 2\beta) \ast \sin(\phi - 2\beta)] = 0, \quad \text{C.1.12} \]

\[ (2k) \ast (-1 - \tan \phi \tan 2\beta) / [(\tan \phi - \tan 2\beta) \ast \sin(\phi - 2\beta)] = 0. \quad \text{C.1.13} \]

Equation C.1.13 solved for the prescribed condition

\[ (-1 - \tan \phi \tan 2\beta) = 0, \quad \text{C.1.14} \]

\[ \tan 2\beta = -\left(\frac{1}{\tan \phi}\right). \quad \text{C.1.15} \]

Equation C.1.15 is the same equation shown in Jaeger et al. (2007, pg. 104). Using shifts or periodicity eqn. C.1.15 is

\[ \tan 2\beta = \left(\frac{1}{\cot(\phi + 90)}\right), \quad \text{C.1.16} \]

\[ \tan 2\beta = \tan(\phi + 90), \quad \text{C.1.17} \]

\[ \beta = 45 + \phi/2. \quad \text{C.1.18} \]

The above Coulomb solution is the angle where shear failure of intact rock occurs, and depending on orientation and periodicity, Eqn. C.1.18 is also sometimes written as

\[ \beta = \mp 45 \pm \phi/2. \quad \text{C.1.19} \]
### Data from literature - various anisotropic rocks

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<th>Authors</th>
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Note: σ₁ and σ₃ are in units of psi; angle β is as defined for triaxial compression tests in chapter 5

236
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Note: σ_1 and σ_3 are in units of psi; angle β is as defined for triaxial compression tests in chapter 5.
### D2 Data from literature – Chichibu Schist
(Mogi, 1979, 2007)

Chichibu Schist true-triaxial experimental

<table>
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<th>Conventional triaxial $\sigma_2 = \sigma_3$</th>
<th>True-triaxial data $\sigma_2 &gt; \sigma_3$</th>
<th>Mode IV, $\beta=0^\circ$, $\omega=\infty$</th>
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<td>$\sigma_1$ (psi)</td>
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<td>No.4</td>
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<td>7,251.9</td>
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<tr>
<td>No.5</td>
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| Mode IV, $\beta=45^\circ$, $\omega=\infty$ (psi) | No. | $\sigma_1$ (psi) | $\sigma_2$ (psi) | $\sigma_3$ (psi) |
|------------------------------------------|------------------------------------------|------------------------------------------|
| No.15 | 21,320.5 | 0.0 | 0.0 |
| No.16 | 21,610.6 | 0.0 | 0.0 |
| No.17 | 42,641.1 | 3,625.9 | 3,625.9 |
| No.18 | 55,839.5 | 7,251.9 | 7,251.9 |
| No.19 | 72,663.9 | 10,877.8 | 10,877.8 |
| Mode III, $\beta=60^\circ$, $\omega=90^\circ$ (psi) | No.43 | 31,763.3 | 7,251.9 | 7,251.9 |
| No.44 | 35,389.2 | 7,251.9 | 7,251.9 |
| No.45 | 45,106.7 | 12,473.2 | 7,251.9 |
| No.46 | 46,122.0 | 14,068.7 | 7,251.9 |
| No.47 | 57,580.0 | 17,549.6 | 7,251.9 |
| No.48 | 64,541.8 | 22,480.8 | 7,251.9 |
| No.49 | 70,778.4 | 22,625.9 | 7,251.9 |
| No.50 | 62,076.1 | 27,847.2 | 7,251.9 |
| Excluded from study due to mixed failure mode |
| No.51 | 62,656.3 | 27,992.3 | 7,251.9 |
| Excluded from study due to mixed failure mode |
JPW model analysis for various anisotropic rocks

Figure E.1. Plot of the $\sigma_1$ actual / $\sigma_1$ predicted for JPW model
### Table 1: Angle vs. Predicted Stress

<table>
<thead>
<tr>
<th>Angle</th>
<th>$\beta$</th>
<th>$\sigma_1$</th>
<th>Actual</th>
<th>$\sigma_3$</th>
<th>Predicted</th>
<th>(Actual - Predicted)</th>
<th>Ratio</th>
<th>(Actual/Predicted)</th>
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</thead>
<tbody>
<tr>
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<td>15,615</td>
<td>33,136,907</td>
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### Table 2: Angle vs. Predicted Stress

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<th>Angle</th>
<th>$\beta$</th>
<th>$\sigma_1$</th>
<th>Actual</th>
<th>$\sigma_3$</th>
<th>Predicted</th>
<th>(Actual - Predicted)</th>
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### Table 3: Angle vs. Predicted Stress

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### Figure

- **Jaeger Plane of Weakness - Bossier shale**
- $\phi_1 = 29.0 \text{ deg}$
- $S_o = 3750 \text{ psi}$
- $\phi_w = 24.0 \text{ deg}$
- $S_w = 2050 \text{ psi}$

### Graphical Representation

- Plot of angle $\beta$ vs. predicted stress $\sigma_1$ with actual stress $\sigma_3$ values.

### Calculation

- $MSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (Data - Predicted)^2}$
- $MSE = 4.171$
## Angle Calculation

### Units in psi

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<th>$\sigma_1$</th>
<th>$\sigma_1$ Predict</th>
<th>$(\text{Actual} - \text{Predicted})^2$</th>
<th>Ratio $(\text{Actual}/\text{Predicted})$</th>
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### MSE Calculation

$$\text{MSE} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (\text{Data} - \text{Predicted})^2}$$

$$\text{MSE} = 1,851$$

### Plot Details

- $\phi_s = 27.0$ deg
- $S_o = 4850.0$ psi
- $\phi_w = 26.0$ deg
- $S_w = 2650.0$ psi

### Jaeger Plane of Weakness - Vaca Muerta

- $\sigma_3 = 0$
- $\sigma_3 = 1,000$
- $\sigma_3 = 2,500$
- $\sigma_3 = 5,000$
- $\sigma_3 = 20,000$

- $\beta_{\text{min}} = 58.0$ deg
- $S_o = 4850.0$ psi
- $\sigma_3 = 0$
- $\sigma_3 = 1,000$
- $\sigma_3 = 2,500$
- $\sigma_3 = 5,000$
- $\sigma_3 = 20,000$

- $\beta_{\text{min}} = 27.0$ deg
- $S_o = 4850.0$ psi
- $\sigma_3 = 0$
- $\sigma_3 = 1,000$
- $\sigma_3 = 2,500$
- $\sigma_3 = 5,000$
- $\sigma_3 = 20,000$
\[ \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (Data - Predicted)^2} \]
### Table

<table>
<thead>
<tr>
<th>Angle (°)</th>
<th>Actual (σ3)</th>
<th>Predicted (σ3)</th>
<th>Ratio (Actual/Predicted)</th>
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### Diagram

- **Jaeger Plane of weakness - Austin Slate (McLamore & Gray 1967)**

- **JPW**
  - σ3 = 5,000 psi
  - σ3 = 10,000 psi
  - σ3 = 20,000 psi
  - σ3 = 30,000 psi
  - σ3 = 40,000 psi

- **βmin**
  - 0° to 90°
  - \( \phi_0 = 22.2 \text{deg} \)
  - \( S_0 = 11,750.0 \text{ psi} \)
  - \( \phi_w = 13.6 \text{deg} \)
  - \( S_w = 6550.0 \text{ psi} \)

### Equation

\[
\text{MSE} = \sqrt{\frac{1}{m} \sum_{i=1}^{m} (y_i - \hat{y}_i)^2}
\]
Jaeger Plane of weakness - GreenRiver1 (McLamore & Gray1967)

Angle θ Actual σ3 Actual - Predicted² Dev in Ratio
90 36,553 1,000 35,239 1,725,483 1.037
75 30,060 1,000 32,771 7,349,399 0.917
70 28,165 1,000 27,492 452,847 1.024
60 23,207 1,000 24,409 1,445,075 0.951
45 31,649 1,000 33,554 3,626,979 0.943
30 32,174 1,000 35,239 9,396,208 0.913
15 31,439 1,000 35,239 14,442,458 0.892
0 33,224 1,000 35,239 4,063,544 0.943

β Actual σ1 Predict (Actual - Predicted)² Dev in Ratio
90 50,090 5,000 46,231 14,889,938 1.083
75 46,566 5,000 46,231 111,922 1.007
70 42,692 5,000 40,824 3,489,099 1.046
60 37,733 5,000 36,654 1,164,520 1.029
45 43,387 5,000 46,231 8,090,906 0.938
30 44,092 5,000 46,231 4,577,682 0.954
15 44,347 5,000 46,231 3,553,043 0.959
0 46,671 5,000 46,231 193,203 1.010

β Actual σ1 Predict (Actual - Predicted)² Dev in Ratio
90 63,638 10,000 59,972 13,441,666 1.061
75 61,103 10,000 59,972 1,280,943 1.019
70 58,399 10,000 57,489 828,336 1.016
60 53,710 10,000 51,961 3,059,638 1.034
45 58,824 10,000 59,972 1,317,014 0.981
30 61,328 10,000 59,972 1,840,059 1.023
15 60,953 10,000 59,972 963,711 1.016
0 61,928 10,000 59,972 3,826,670 1.033

β Actual σ1 Predict (Actual - Predicted)² Dev in Ratio
90 103,921 25,000 101,192 7,444,201 1.027
75 99,137 25,000 101,192 4,223,410 0.980
70 98,232 25,000 101,192 8,760,379 0.971
60 96,333 25,000 97,882 2,398,965 0.984
45 96,768 25,000 101,192 19,576,145 0.956
30 99,362 25,000 101,192 3,350,341 0.982
15 100,516 25,000 101,192 456,697 0.993
0 100,591 25,000 101,192 360,953 0.994

MSE = > 2,165 0.997

\[ \sqrt{\frac{1}{n} \sum_{i=1}^{n} (Data - Predicted)^2} \]
### Angle Plane of Weakness - GreenRiver2 (Mclamore & Gray1967)

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\[ \sqrt{\frac{\sum_{i=1}^{N} (\text{Data} - \text{Predicted})^2}{N}} \]

\[ \text{MSE} = 1.811 \]
MSE => 2.692 0.991

\[
\sqrt{\frac{1}{n} \sum_{i=1}^{n} (Data - Predicted)^2}
\]
### Data Table

<table>
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<tr>
<th>Angle (°)</th>
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### Diagram

- **Jaeger Plane of Weakness - Carbona Phyllite (Ramamurthy et al., 1993)**

- $\phi_c = 32.8$ deg
- $S_c = 3200.0$ psi
- $\phi_w = 28.7$ deg
- $S_w = 1800.0$ psi

### Units
- $\sigma_1$, $\sigma_3$ in psi
- $\beta_{min}$ = 59.4°
<table>
<thead>
<tr>
<th>Angle (°)</th>
<th>Actual (σ1)</th>
<th>Predicted (σ1)</th>
<th>(Actual - Predicted)$^2$</th>
<th>Dev in Ratio</th>
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<tbody>
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<table>
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<th>Angle (°)</th>
<th>Actual (σ1)</th>
<th>Predicted (σ1)</th>
<th>(Actual - Predicted)$^2$</th>
<th>Dev in Ratio</th>
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<td>1.119</td>
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<tr>
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<th>Predicted (σ1)</th>
<th>(Actual - Predicted)$^2$</th>
<th>Dev in Ratio</th>
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<tr>
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<td>31,652</td>
<td>0</td>
<td>29,895</td>
<td>1.059</td>
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<table>
<thead>
<tr>
<th>Angle (°)</th>
<th>Actual (σ1)</th>
<th>Predicted (σ1)</th>
<th>(Actual - Predicted)$^2$</th>
<th>Dev in Ratio</th>
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<td>1.059</td>
</tr>
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MSE => 5,240 0.972
\[ \frac{1}{N} \sum_{i=1}^{N} (Data - Predicted)^2 \]
Jaeger Plane of weakness - Penrhyn Slate (Attenwell & Sandford, 1974)

\[ \sigma_3 = 0 \text{ psi} \]

\[ \sigma_3 = 1,000 \text{ psi} \]

\[ \sigma_3 = 2,000 \text{ psi} \]

\[ \sigma_3 = 6,000 \text{ psi} \]

\[ \sigma_3 = 8,000 \text{ psi} \]

\[ \sigma_3 = 10,000 \text{ psi} \]

\[ \phi_o = 35.1 \text{ deg} \]

\[ \sigma_w = 7010.0 \text{ psi} \]

\[ \phi_w = 14.7 \text{ deg} \]

\[ \sigma_w = 4970.0 \text{ psi} \]
Jaeger Plane of weakness - Tournemire Shale (Niandou, 1997)

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<tr>
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<tr>
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<td>6,825</td>
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<tr>
<td>0</td>
<td>6,825</td>
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</table>

MSE: $1,447.7$ 0.982

$\text{MSE} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (Data - Predicted)^2}$

$\sigma_3 = 145$ psi
$\sigma_3 = 725$ psi
$\sigma_3 = 2,901$ psi
$\sigma_3 = 5,802$ psi
$\sigma_3 = 7,252$ psi

$\beta_{min} = 55.1$ deg
$S_o = 1850.0$ psi
$\phi_o = 24.1$ deg
$S_w = 1270.0$ psi
$\phi_w = 20.2$ deg
F Pariseau model analysis for various anisotropic rocks

Figure F.1. Plot of the $\sigma_1$ actual / $\sigma_1$ predicted for Pariseau model
\[ \begin{align*}
\sigma_3 & \quad \beta \quad \sigma_1 \quad \text{Actual} \quad \text{Predicted} \quad \text{(Actual - Predicted)}^2 \quad \text{Ratio (Actual/Predicted)} \\
0 & \quad 90 & \quad 6,140 & \quad 15,305 & \quad 84,001,130 & \quad 0.401 \\
0 & \quad 60 & \quad 1,811 & \quad 9,073 & \quad 52,766,672 & \quad 0.200 \\
0 & \quad 45 & \quad 6,874 & \quad 8,273 & \quad 6,843,782 & \quad 0.831 \\
0 & \quad 30 & \quad 6,577 & \quad 8,262 & \quad 7,313,067 & \quad 0.710 \\
0 & \quad 0 & \quad 16,466 & \quad 14,701 & \quad 3,113,513 & \quad 1.120 \\
1,000 & \quad 90 & \quad 21,371 & \quad 18,601 & \quad 7,672,928 & \quad 1.149 \\
1,000 & \quad 60 & \quad 6,672 & \quad 11,434 & \quad 22,676,189 & \quad 0.584 \\
1,000 & \quad 45 & \quad 8,021 & \quad 10,514 & \quad 6,214,124 & \quad 0.763 \\
1,000 & \quad 30 & \quad 13,321 & \quad 11,652 & \quad 2,786,195 & \quad 1.143 \\
1,000 & \quad 0 & \quad 18,106 & \quad 17,907 & \quad 39,717 & \quad 1.011 \\
3,000 & \quad 90 & \quad 28,988 & \quad 25,193 & \quad 14,405,373 & \quad 1.151 \\
3,000 & \quad 75 & \quad 19,834 & \quad 20,556 & \quad 521,453 & \quad 0.965 \\
3,000 & \quad 60 & \quad 13,751 & \quad 16,156 & \quad 5,783,317 & \quad 0.851 \\
3,000 & \quad 45 & \quad 15,678 & \quad 14,996 & \quad 465,562 & \quad 1.046 \\
3,000 & \quad 30 & \quad 18,106 & \quad 17,907 & \quad 39,717 & \quad 1.011 \\
3,000 & \quad 0 & \quad 30,060 & \quad 24,317 & \quad 8,622,779 & \quad 1.179 \\
6,000 & \quad 90 & \quad 37,437 & \quad 35,080 & \quad 5,555,898 & \quad 1.067 \\
6,000 & \quad 60 & \quad 20,330 & \quad 23,239 & \quad 8,460,557 & \quad 0.875 \\
6,000 & \quad 45 & \quad 22,722 & \quad 21,718 & \quad 1,007,080 & \quad 1.046 \\
6,000 & \quad 30 & \quad 25,984 & \quad 23,599 & \quad 5,689,929 & \quad 1.046 \\
6,000 & \quad 0 & \quad 36,951 & \quad 26,909 & \quad 16,086,779 & \quad 1.149 \\
10,000 & \quad 90 & \quad 47,097 & \quad 48,263 & \quad 1,359,632 & \quad 0.976 \\
10,000 & \quad 75 & \quad 39,812 & \quad 40,269 & \quad 209,002 & \quad 0.961 \\
10,000 & \quad 60 & \quad 33,197 & \quad 32,683 & \quad 264,705 & \quad 1.016 \\
10,000 & \quad 45 & \quad 32,556 & \quad 30,682 & \quad 3,511,106 & \quad 1.061 \\
10,000 & \quad 30 & \quad 36,198 & \quad 33,156 & \quad 5,246,258 & \quad 1.046 \\
10,000 & \quad 15 & \quad 42,278 & \quad 40,429 & \quad 3,420,638 & \quad 1.046 \\
10,000 & \quad 10 & \quad 38,263 & \quad 43,435 & \quad 26,747,121 & \quad 0.881 \\
10,000 & \quad 0 & \quad 42,614 & \quad 46,754 & \quad 17,137,220 & \quad 0.911 \\
\end{align*}\]
\[ \sigma_3 = \beta \sigma_1 \]

Actual vs. Predicted

\[
\frac{(\text{Actual} - \text{Predicted})^2}{\text{Ratio (Actual/Predicted)}}
\]

<table>
<thead>
<tr>
<th>(\sigma_1) (psi)</th>
<th>(\beta)</th>
<th>(\sigma_3) Actual</th>
<th>(\sigma_3) Predicted</th>
<th>(\text{Ratio (Actual/Predicted)}^2)</th>
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</thead>
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<td>21,924</td>
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</table>

Mean Error = \[ \sqrt{\frac{1}{N} \sum_{i=1}^{N} (\text{Data} - \text{Predicted})^2} \] = 2.390 psi
Pariseau Model (Martinsburg Slate, Donath 1964)

<table>
<thead>
<tr>
<th>$\sigma_3$</th>
<th>$\beta$</th>
<th>$\sigma_1$ Actual</th>
<th>$\sigma_1$ Predicted</th>
<th>$(\sigma_1 - \text{Predicted})^2$</th>
<th>Ratio (Actual/Predicted)</th>
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</thead>
<tbody>
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<td>53,905</td>
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<td>0.997</td>
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</table>

Mean error = $\sqrt{\frac{1}{n} \sum (\text{Data} - \text{Predicted})^2}$

- Mean Error = 5,409 psi
### Table 1: Pariseau Model (Green River Shale 1, McLamore & Gray 1967)

<table>
<thead>
<tr>
<th>$\sigma_3$ (psi)</th>
<th>$\beta$ (%)</th>
<th>$\sigma_1$ Actual</th>
<th>$\sigma_1$ Predicted</th>
<th>$(\sigma_1 - \text{Predicted})^2$ Ratio (Actual/Predicted)</th>
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<tbody>
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### Table 2: Mean Error = 2.818 psi

### Graph: Pariseau Model (Green River Shale 1, McLamore & Gray 1967)

- $F = 1.31E-10$ ps$^{-2}$
- $G = 1.89E-09$ ps$^{-2}$
- $U = 2.99E-05$ ps$^{-1}$
- $V = 5.54E-05$ ps$^{-1}$
- $M = 1.11E-08$ ps$^{-2}$

Mean Error = 2.818 psi
### Pariseau Model (Green River Shale 2, McLamore & Gray 1967)

<table>
<thead>
<tr>
<th>$\sigma_3$ (psi)</th>
<th>$\beta$</th>
<th>$\sigma_1$ (Actual)</th>
<th>$\sigma_1$ (Predicted)</th>
<th>$(\text{Actual} - \text{Predicted})^2$</th>
<th>Ratio $\sigma_1$ (Actual)/$\sigma_1$ (Predicted)</th>
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<tr>
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<td>64,181</td>
<td>64,762</td>
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<td>0.991</td>
</tr>
<tr>
<td>25,000</td>
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<td>58,620</td>
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<td>5,076,565</td>
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<tr>
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<td>68,981</td>
<td>2,583,057</td>
<td>0.977</td>
</tr>
</tbody>
</table>

Mean Error = 1,633 psi 0.994
<table>
<thead>
<tr>
<th>$\sigma_3$</th>
<th>$\beta$</th>
<th>$\sigma_1$ Actual</th>
<th>$\sigma_1$ Predicted</th>
<th>$(\sigma_1 - \text{Predicted})^2$</th>
<th>Ratio $(\text{Actual} / \text{Predicted})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,176</td>
<td>90</td>
<td>20,564</td>
<td>19,456</td>
<td>819,169</td>
<td>1.047</td>
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<tr>
<td>2,176</td>
<td>60</td>
<td>14,719</td>
<td>15,751</td>
<td>1,063,716</td>
<td>0.935</td>
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<tr>
<td>2,176</td>
<td>30</td>
<td>18,117</td>
<td>16,866</td>
<td>1,564,096</td>
<td>1.074</td>
</tr>
<tr>
<td>2,176</td>
<td>20</td>
<td>20,730</td>
<td>18,866</td>
<td>1,564,096</td>
<td>1.074</td>
</tr>
<tr>
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<td>25,382</td>
<td>22,057</td>
<td>11,050,261</td>
<td>1.099</td>
</tr>
</tbody>
</table>

Pariseau Model (QuartzPhyllite, Ramamurthy et al. 1993)

<table>
<thead>
<tr>
<th>$\sigma_3$</th>
<th>$\beta$</th>
<th>$\sigma_1$ Actual</th>
<th>$\sigma_1$ Predicted</th>
<th>$(\sigma_1 - \text{Predicted})^2$</th>
<th>Ratio $(\text{Actual} / \text{Predicted})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4,351</td>
<td>90</td>
<td>28,550</td>
<td>26,694</td>
<td>3,446,278</td>
<td>1.070</td>
</tr>
<tr>
<td>4,351</td>
<td>50</td>
<td>19,822</td>
<td>21,285</td>
<td>2,140,784</td>
<td>0.931</td>
</tr>
<tr>
<td>4,351</td>
<td>40</td>
<td>23,742</td>
<td>21,759</td>
<td>3,930,853</td>
<td>1.091</td>
</tr>
<tr>
<td>4,351</td>
<td>30</td>
<td>25,623</td>
<td>23,345</td>
<td>5,185,975</td>
<td>1.091</td>
</tr>
<tr>
<td>4,351</td>
<td>20</td>
<td>29,073</td>
<td>25,923</td>
<td>9,923,734</td>
<td>1.122</td>
</tr>
<tr>
<td>4,351</td>
<td>0</td>
<td>32,627</td>
<td>30,058</td>
<td>6,599,466</td>
<td>1.085</td>
</tr>
</tbody>
</table>

Mean error = $\frac{1}{N} \sum_{i=1}^{N} (\text{Data} - \text{Predicted})^2$ $= 2.778$ psi

Mean Error = 2.273 0.975
### Table 1: Actual vs. Predicted Values

<table>
<thead>
<tr>
<th>ε0</th>
<th>β</th>
<th>(σ1) Actual</th>
<th>(σ1) Predicted</th>
<th>(Actual - Predicted)²</th>
<th>Ratio (Actual/Predicted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>90</td>
<td>9,446</td>
<td>11,704</td>
<td>5,099,896</td>
<td>0.807</td>
</tr>
<tr>
<td>0</td>
<td>65</td>
<td>5,584</td>
<td>9,532</td>
<td>15,586,978</td>
<td>0.586</td>
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<tr>
<td>0</td>
<td>60</td>
<td>5,427</td>
<td>9,109</td>
<td>13,556,160</td>
<td>0.596</td>
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<tr>
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<td>30</td>
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<td>9,483</td>
<td>5,941,798</td>
<td>0.743</td>
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<tr>
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<td>0</td>
<td>11,533</td>
<td>13,377</td>
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### Table 2: Additional Actual vs. Predicted Values

<table>
<thead>
<tr>
<th>ε0</th>
<th>β</th>
<th>(σ1) Actual</th>
<th>(σ1) Predicted</th>
<th>(Actual - Predicted)²</th>
<th>Ratio (Actual/Predicted)</th>
</tr>
</thead>
<tbody>
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<td>3,063,082</td>
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<tr>
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<td>65</td>
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<td>11,671</td>
<td>8,462,008</td>
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<tr>
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<td>30</td>
<td>11,267</td>
<td>11,615</td>
<td>120,722</td>
<td>0.970</td>
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<tr>
<td>0</td>
<td>0</td>
<td>16,747</td>
<td>16,086</td>
<td>436,158</td>
<td>1.041</td>
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</table>

### Table 3: Additional Actual vs. Predicted Values

<table>
<thead>
<tr>
<th>ε0</th>
<th>β</th>
<th>(σ1) Actual</th>
<th>(σ1) Predicted</th>
<th>(Actual - Predicted)²</th>
<th>Ratio (Actual/Predicted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>90</td>
<td>25,852</td>
<td>26,475</td>
<td>387,421</td>
<td>0.976</td>
</tr>
<tr>
<td>0</td>
<td>60</td>
<td>29,088</td>
<td>26,475</td>
<td>6,828,890</td>
<td>1.099</td>
</tr>
<tr>
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<td>55</td>
<td>28,462</td>
<td>25,458</td>
<td>9,025,005</td>
<td>1.118</td>
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<tr>
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<td>25</td>
<td>27,001</td>
<td>23,422</td>
<td>12,808,116</td>
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<tr>
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<td>0</td>
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<td>28,013</td>
<td>844,299</td>
<td>1.033</td>
</tr>
</tbody>
</table>

### Table 4: Additional Actual vs. Predicted Values

<table>
<thead>
<tr>
<th>ε0</th>
<th>β</th>
<th>(σ1) Actual</th>
<th>(σ1) Predicted</th>
<th>(Actual - Predicted)²</th>
<th>Ratio (Actual/Predicted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>36,216</td>
<td>36,322</td>
<td>11,230</td>
<td>0.997</td>
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<tr>
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<td>60</td>
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<td>29,877</td>
<td>362,412</td>
<td>0.980</td>
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<tr>
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<tr>
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<td>0</td>
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<td>32,310</td>
<td>449,108</td>
<td>1.021</td>
</tr>
</tbody>
</table>

### Table 5: Additional Actual vs. Predicted Values

<table>
<thead>
<tr>
<th>ε0</th>
<th>β</th>
<th>(σ1) Actual</th>
<th>(σ1) Predicted</th>
<th>(Actual - Predicted)²</th>
<th>Ratio (Actual/Predicted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>46,169</td>
<td>64,511</td>
<td>1.006</td>
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<tr>
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<td>55</td>
<td>37,447</td>
<td>37,268</td>
<td>31,784</td>
<td>1.005</td>
</tr>
<tr>
<td>0</td>
<td>30</td>
<td>39,847</td>
<td>39,333</td>
<td>264,117</td>
<td>1.013</td>
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<tr>
<td>0</td>
<td>0</td>
<td>46,893</td>
<td>50,592</td>
<td>13,688,139</td>
<td>0.927</td>
</tr>
</tbody>
</table>

### Mean Error

\[ \text{Mean Error} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (\text{Data} - \text{Predicted})^2} \]

\[ \text{Mean Error} = 2.109 \text{ psi} \]
Pariseau Model (Micaceous Phyllite, Ramamurthy et al. 1993)

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\sigma_3$ Actual</th>
<th>$\sigma_3$ Predicted</th>
<th>$(\sigma_1 - \sigma_3)^2$</th>
<th>Ratio Actual/Predicted</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.660</td>
</tr>
<tr>
<td>0</td>
<td>75</td>
<td>7,552</td>
<td>6,607,356</td>
<td>0.660</td>
</tr>
<tr>
<td>0</td>
<td>60</td>
<td>4,553</td>
<td>4,327,380</td>
<td>0.543</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>12,005</td>
<td>4,865,094</td>
<td>1.184</td>
</tr>
</tbody>
</table>

$\sigma_3$ Actual = $\sigma_1$ @ $\sigma_3=

\begin{align*}
F &= 1.48E-09 \text{ psi}^{-2} \\
G &= 1.78E-08 \text{ psi}^{-2} \\
U &= 1.05E-04 \text{ psi}^{-2} \\
V &= 7.14E-05 \text{ psi}^{-2} \\
M &= 4.44E-07 \text{ psi}^{-2}
\end{align*}

Mean error = $\sqrt{\frac{1}{n} \sum_{i=1}^{n} (Data - Predicted)^2}$ = 3,283 psi

$\frac{1}{n} \sum_{i=1}^{n} (Data - Predicted)^2 = 3,283 \text{ psi}$

$\frac{1}{n} \sum_{i=1}^{n} \frac{\sigma_1}{\sigma_3} = 3,283 \text{ psi}$

Mean Error = 3,283 psi
### Pariseau Model (Penrhyn Slate, Attewell & Sandford 1974)

#### σ<sub>3</sub> vs. β

<table>
<thead>
<tr>
<th>β</th>
<th>σ&lt;sub&gt;1&lt;/sub&gt; Actual</th>
<th>σ&lt;sub&gt;1&lt;/sub&gt; Predicted</th>
<th>Ratio Actual/Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>27,167</td>
<td>28,319</td>
<td>0.959</td>
</tr>
<tr>
<td>90</td>
<td>27,167</td>
<td>28,319</td>
<td>0.959</td>
</tr>
</tbody>
</table>

#### Mean Error = 3,472 psi

---

### Pariseau Model (Penrhyn Slate, Attewell & Sandford 1974)

#### σ<sub>3</sub> vs. β

<table>
<thead>
<tr>
<th>β</th>
<th>σ&lt;sub&gt;1&lt;/sub&gt; Actual</th>
<th>σ&lt;sub&gt;1&lt;/sub&gt; Predicted</th>
<th>Ratio Actual/Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>27,167</td>
<td>28,319</td>
<td>0.959</td>
</tr>
<tr>
<td>90</td>
<td>27,167</td>
<td>28,319</td>
<td>0.959</td>
</tr>
</tbody>
</table>

#### Mean Error = 3,472 psi

---

### Pariseau Model (Penrhyn Slate, Attewell & Sandford 1974)

#### σ<sub>3</sub> vs. β

<table>
<thead>
<tr>
<th>β</th>
<th>σ&lt;sub&gt;1&lt;/sub&gt; Actual</th>
<th>σ&lt;sub&gt;1&lt;/sub&gt; Predicted</th>
<th>Ratio Actual/Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>27,167</td>
<td>28,319</td>
<td>0.959</td>
</tr>
<tr>
<td>90</td>
<td>27,167</td>
<td>28,319</td>
<td>0.959</td>
</tr>
</tbody>
</table>

#### Mean Error = 3,472 psi

---
**Pariseau Model (Tournemire Shale, Niandou 1997)**

### Table 1: Actual vs Predicted Values

<table>
<thead>
<tr>
<th>$\sigma_3$ (psi)</th>
<th>$\beta$</th>
<th>Actual $\sigma_1$</th>
<th>Predicted $\sigma_1$</th>
<th>$(\text{Actual} - \text{Predicted})^2$</th>
<th>Ratio $(\text{Actual}/\text{Predicted})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>145.0</td>
<td>90</td>
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<td>6,142</td>
<td>958,355</td>
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<td>5,000</td>
<td>421,692</td>
<td>0.870</td>
</tr>
<tr>
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<td>45</td>
<td>3,669</td>
<td>4,269</td>
<td>448,901</td>
<td>0.847</td>
</tr>
<tr>
<td>145.0</td>
<td>30</td>
<td>2,926</td>
<td>4,413</td>
<td>2,211,319</td>
<td>0.663</td>
</tr>
<tr>
<td>145.0</td>
<td>15</td>
<td>4,587</td>
<td>5,208</td>
<td>384,453</td>
<td>0.881</td>
</tr>
</tbody>
</table>

### Table 2: Actual vs Predicted Values

<table>
<thead>
<tr>
<th>$\sigma_3$ (psi)</th>
<th>$\beta$</th>
<th>Actual $\sigma_1$</th>
<th>Predicted $\sigma_1$</th>
<th>$(\text{Actual} - \text{Predicted})^2$</th>
<th>Ratio $(\text{Actual}/\text{Predicted})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>725.2</td>
<td>90</td>
<td>8,722</td>
<td>7,638</td>
<td>1,174,060</td>
<td>1.142</td>
</tr>
<tr>
<td>725.2</td>
<td>45</td>
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<td>4,878</td>
<td>227,838</td>
<td>0.915</td>
</tr>
<tr>
<td>725.2</td>
<td>30</td>
<td>5,506</td>
<td>5,595</td>
<td>494,284</td>
<td>1.134</td>
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<tr>
<td>725.2</td>
<td>0</td>
<td>8,503</td>
<td>5,281</td>
<td>1,042,838</td>
<td>1.168</td>
</tr>
</tbody>
</table>

### Table 3: Actual vs Predicted Values

<table>
<thead>
<tr>
<th>$\sigma_3$ (psi)</th>
<th>$\beta$</th>
<th>Actual $\sigma_1$</th>
<th>Predicted $\sigma_1$</th>
<th>$(\text{Actual} - \text{Predicted})^2$</th>
<th>Ratio $(\text{Actual}/\text{Predicted})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2901.5</td>
<td>90</td>
<td>15,256</td>
<td>13,515</td>
<td>4,022,369</td>
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<td>2,611,881</td>
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<td>14,591</td>
<td>12,725</td>
<td>4,822,362</td>
<td>1.179</td>
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</tbody>
</table>

### Table 4: Actual vs Predicted Values

<table>
<thead>
<tr>
<th>$\sigma_3$ (psi)</th>
<th>$\beta$</th>
<th>Actual $\sigma_1$</th>
<th>Predicted $\sigma_1$</th>
<th>$(\text{Actual} - \text{Predicted})^2$</th>
<th>Ratio $(\text{Actual}/\text{Predicted})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5801.5</td>
<td>90</td>
<td>20,886</td>
<td>20,734</td>
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<td>16,428</td>
<td>1,436</td>
<td>0.998</td>
</tr>
<tr>
<td>5801.5</td>
<td>30</td>
<td>15,753</td>
<td>15,505</td>
<td>322,088</td>
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</tr>
<tr>
<td>5801.5</td>
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<td>19,788</td>
<td>19,962</td>
<td>30,211</td>
<td>0.995</td>
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</tbody>
</table>

**Mean Error** = $\sqrt{\frac{1}{N} \sum (\text{Data} - \text{Predicted})^2} = 1.070$ psi

**Mean Error** = 1.070 0.995
## Example of Pariseau model iteration

### Table G1. Example of Iterative process for Pariseau model

<table>
<thead>
<tr>
<th>Parameters in psi</th>
<th>Interval (Steps)</th>
<th>Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>$g$</td>
<td>$u$</td>
</tr>
<tr>
<td>-20,000</td>
<td>20,000</td>
<td>110,000</td>
</tr>
<tr>
<td></td>
<td></td>
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</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-10,000</td>
<td>20,000</td>
<td>80,000</td>
</tr>
<tr>
<td></td>
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</tr>
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</tr>
<tr>
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<td></td>
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<td>82,000</td>
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