
Structural behaviour and Design criteria of Under-deck cable-stayed bridges and Combined cable-stayed bridges.

Single-span bridges.

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Abstract

This paper examines two new types of bridges, namely under-deck cable-stayed bridges and combined cable-stayed bridges, for prestressed concrete road bridges with single-spans of medium length. Both bridge types offer many advantages over conventional schemes in several aspects, such as structural efficiency, enhanced construction possibilities, and both economic and aesthetical considerations. These are very slender structural types with a very high structural efficiency, for which the materials used in the deck are reduced to one third of that in conventional bridges without stay cables. In this paper, the most important aspects of the structural behaviour of these bridge types are set out through the description of a careful selection of an extensive collection of bridges designed and analysed by the authors in a previous research project. In addition, a detailed set of design criteria for these bridge types is presented, based on the results of the extensive parametric study undertaken in the aforementioned research project.

Key words:

under-deck cable-stayed bridge; combined cable-stayed bridge; cable-stayed bridges; prestressed concrete; bridges
1. Introduction

Over the past thirty years, outstanding engineers (including Leonhardt, Schlaich, Menn, Virlogeux and Cremer, among others) have designed and built over twenty cable-stayed bridges with highly innovative morphologies, namely under-deck cable-stayed bridges and combined cable-stayed bridges. In these new types of bridges, the stay cables are placed below the deck, or above-and-below the deck, respectively, rather than in the conventional position above the deck. The state-of-the-art of these new types of bridges (Ruiz-Teran 2005; Ruiz-Teran and Aparicio 2007a) as well as the parameters that govern their structural response (Ruiz-Teran and Aparicio 2007b) has already been analysed by the authors.

In this paper, the most relevant aspects of the structural behaviour of single-span bridges with under-deck cable-stayed systems and combined cable-stayed systems are set out through the analysis of a carefully chosen and limited selection of an extensive bridge collection designed and studied by the authors in a previous research project. A detailed set of design criteria are also established. Although many of the conclusions drawn in this paper are of a general nature, the study focuses on the application of under-deck cable-staying systems and combined cable-staying systems to road bridges with prestressed concrete decks having medium range spans (80 metres). A companion paper concerning multi-span bridges has also been written by the authors (Ruiz Teran and Aparicio 2007c).

2. Under-deck cable-stayed bridges

In under-deck cable-stayed bridges, the stay cables have a polygonal layout under the intrados of the deck, they are self-anchored in the deck in the sections which are supported over the abutments, and they are deflected by struts which, working under compression, introduce the cable upward deviation forces into the deck (Figure 1).
2.1 Under-deck cable-stayed bridges with 2 struts

In this section, a meticulous description is made of one of the bridges analysed with this morphology (under-deck cable-stayed bridges with two struts), in order to give a detailed picture of the structural response of this structural type (under-deck cable-stayed bridges). Once this general behaviour is ascertained, many alternative schemes with different morphologies will be compared and contrasted by considering certain key aspects.

2.1.1 Detailed description of one selected bridge of this morphology

The selected bridge has a span of 80 metres (Figure 2). The deck is a voided slab of prestressed concrete with a depth of one metre (Figure 3), giving a depth/span ratio of 1/80. The characteristic strength of the concrete is 40 MPa.

The under-deck cable-staying system, made up of 258 strands, each with a cross-section of 140 mm$^2$ and an ultimate stress of 1860 MPa, is self-anchored to the deck and deflected by two struts. The struts support the deck at third-span sections, where diaphragms are positioned, and are connected to it by means of pins. The struts are oriented along the bisector of the angle formed by the stay cables so that the stay cables work under the same tension over their full length. The under-deck cable-staying system has an eccentricity of 8 metres (1/10 of the span) at midspan. The greater the eccentricity of the stay cables, the higher the efficiency of the cable-staying system under traffic live load, however, the eccentricity should be limited on the basis of aesthetic considerations (Ruiz-Teran and Aparicio 2007b).

2.1.2 Response to permanent state

During permanent state, three actions exist: the dead load (self-weight of the structural elements: $g_1=188.25$ kN/m), the superimposed dead load (self-weight of the non structural
elements: \( g_2 = 43.10 \text{kN/m} \) and the prestressing. The stay cables are prestressed to compensate 100\% of the permanent load \((g_1 + g_2)\), so that the vertical component of the load introduced by the struts into the deck is equal to the vertical reaction that would be found in a continuous beam with supports at the points where the deck lays over the struts. As a result of both the prestressing and the appropriate layout of the struts, the span is effectively subdivided and the bending of the deck is reduced to the local bending between the struts (Figure 4a). The external isostatic moment due to the permanent load \((g_1 + g_2)^2/8 = 185.08 \text{ MN.m}\) is resisted through two mechanisms of response: 20.12 MN.m (11\%) by means of the bending of the deck at midspan (flexural response) and the rest (89\%) by the under-deck cable-stayed system by means of a couple that compresses the deck and tenses the stay cables with a lever arm of 8 metres (axial response). In addition, the axial response is enhanced due to the prestressing of the stay cables. Consequently, the bending moments in the deck in the permanent state are much smaller than those that would be found in a bridge without stay cables. In this case, the ‘efficiency of the under-deck cable-staying system’ \( \xi \), under the permanent load \((g_1 + g_2)\), which is the portion of the external isostatic moment resisted by the cable-staying system (Ruiz-Teran and Aparicio 2007b), is very high \((\xi = 0.89)\).

In the permanent state, the stay cables are stressed to 22.62 MN. Because of the high efficiency of the cable-staying system, 91\% of this axial load is due to the response of the structure to the permanent load and only the remaining 9\% is due to prestressing action. In addition, the response of the structure in permanent state is very stable over time (Figures 4a and 4b). Time-dependent effects (shrinkage and creep of the concrete and relaxation of the internal prestressing) produce losses of only 2.5\% because of two facts: (1) the delayed vertical movements of the points where the deck lays over the struts are very small —as 100\% of the permanent load has been compensated for and the upwards and downwards deviation
forces of the internal prestressing in the deck are strongly balanced—; and (2) the delayed shortening of the deck causes a very minor redistribution of forces due to the high flexibility of the deck.

2.1.3 Static response to traffic live load

The bending moment envelopes of the deck due to traffic live loads (q=52.8 kN/m and Q=600 kN in the Spanish Code (IAP 1998)) (Figure 5) have three major differences in comparison to those obtained for a single-span bridge without stay cables: (1) the maximum sagging bending moment does not occur at midspan; (2) there are hogging bending moments in the sections where the deck lays over the struts that are of the same order of magnitude as the maximum sagging bending moments; and (3) the bending moment due to traffic live load at midspan is much smaller — only 19% of that found in a bridge without stay cables. An eccentricity factor equal to 2 has been used in order to take into consideration the non uniform transversal distribution of the longitudinal bending moments due to the 600-kN vehicle. The eccentricity factor, also called the distribution coefficient (Cusens and Parma, 1975), is the ratio between the maximum and the mean longitudinal bending moment per unit of transversal width due to an eccentric point load. The value of 2 is conservative and has been selected on the basis of a detailed model of the bridge.

2.1.4 Dynamic response to traffic live load

The dynamic response of the structure under two 400 kN vehicles crossing the bridge at speeds of 60 km/h —which is the dynamic test recommended in Spain (Fomento 1999)— has been analysed by means of a step-by-step time integration (with increments of 0.001 seconds). A damping factor equal to 2% has been adopted —similar values have been measured in Glacis Bridge (Schlaich and Werwigk 2001) and Takehana Bridge (Nakagawa et
al. 2001), both of which are under-deck cable-stayed bridges. The maximum vertical acceleration registered, 0.41 m/s², occurs at around the quarter-span section (Figure 6). It is important to stress that the vertical accelerations are fourteen times greater than those found in a conventional prestressed concrete bridge (without stay cables) with the same span.

2.1.5 Verification of Service Limit States (SLS)

The internal prestressing in the deck, made up of only 190 strands, each of 140 mm², and stressed to 1450 MPa, satisfies the required limit states for concrete structures (EHE 1999; Eurocode-2-1-1 2004): (1) the decompression SLS under quasi-permanent action combination (permanent actions + 20% live load); (2) the controlled cracking SLS under the frequent action combination (permanent actions + 50% live load); and (3) the stress limitation SLS under the rare action combination (permanent actions + 100% live load). It is interesting to stress that the internal prestressing has a layout that is similar to that of a beam with supports at the sections where the deck lays over the struts (Figure 7) as well as the fact that the amount of prestressing is much smaller than in conventional bridges without stay cables.

The vibration SLS is just satisfied, since the maximum acceleration practically coincides with the allowed maximum (0.45 m/s²). Vertical accelerations are limited to $0.5\sqrt{f_0}$, where $f_0$ (in this case $f_0 = 0.80$ Hz) is the fundamental frequency of vibration of the structure (BS 5400-2 1978; Eurocode 2-2 2001).

2.1.6 Verification of Ultimate Limit States (ULS)

The ULS of fatigue determines the cross-sectional area of the stay cables. External prestressing anchorages, that allow frequent stress changes of 80 MPa and maximum tensile stresses equal to 65% of the tensile strength of the stay cables, have been used. The stress changes due to frequent live load are 80 MPa, while maximum stress reaches 42% of the
tensile strength of the stay cables. The tension ULS in the stay cables as well as both the bending ULS and the shear ULS in the deck are amply satisfied.

2.1.7 Geometric and mechanical non-linearity

The effects of considering the geometric and mechanical non-linearity of the structure have been analysed. The redistribution of internal forces in the deck and in the stay cables is negligible. Only the increase in compression of the struts (10%) is significant.

2.1.8 Aspects relating to construction

In the event of on-site construction, it will be feasible to use lighter falsework due to the substantial reduction of the self-weight. It is important to stress that, for stressing the stay cables, the struts must be immobilised by means of temporary props that connect the lower points of the struts to the deck.

Given the layout of the internal prestressing in the deck, a construction method using three longitudinal precast elements —with straight bonded pretensioned reinforcement— assembled on-site precisely in the sections where the deck lays over the struts would be more than suitable, since only two temporary support towers would be required.

2.2 Under-deck cable-stayed bridges with multiple struts

2.2.1 Description of one selected bridge of this morphology

An under-deck cable-stayed bridge with multiple struts has been designed (Figure 8), with the same cross-section as in Figure 3 (depth/span = 1/80), and with a 35 MPa concrete.

The under-deck cable-staying system, made up of 264 strands, each with a cross-section of 140 mm², and an ultimate stress of 1860 MPa, is self-anchored in the deck and deflected by
multiple struts. The geometry of the under-deck cable-staying system is defined by the following conditions: (1) the eccentricity at midspan is 1/10 of the span; (2) the struts are connected to the deck at equidistant sections; (3) the struts are oriented along the bisector of the angle formed by the stay cables; and (4) the upward vertical components of the loads introduced by the struts into the deck are constant over the full length of the deck. Observance of these conditions allows us to achieve several objectives: (1) the tension in the stay cable is uniform, which is profitable from the viewpoint of both cost (optimum use of the cross-section of the stay cable) and construction (simplification of the stressing process); (2) the struts work under compression and do not introduce any bending moment into such a slender deck; and (3) the permanent load can be compensated by means of the prestressing of the stay cables.

**2.2.2 Response to permanent state**

Due to the prestressing and the greater number of struts placed, the bending moments of the deck in permanent state are minimal (Figure 9). The efficiency of the cable-staying system under the permanent load \((g_1+g_2)\) is very high \((\xi=0.92)\), and therefore the main contribution in permanent state is due to the response to permanent load (91%), rather than the prestressing of the stay cables (9%). Losses of tension in the stay cables due to time-dependent effects are only 1.7%.

**2.2.3 Static response to traffic live load**

The shapes of the bending moment envelopes of the deck due to traffic live load (Figure 10) are very similar to those that occur in a bowstring. The maximum bending moment at midspan is only 17% of the moment that would occur in a conventional bridge without stay cables.
2.2.4 Dynamic response to traffic live load

The maximum vertical acceleration registered is 0.41 m/s$^2$ and the distribution of accelerations is similar to that for 2-strut bridges (Figure 6). Hence the number of struts placed has not changed the maximum dynamic response to traffic live load.

2.2.5 Verification of SLSs

Given the substantial reduction of the flexural response in permanent state, it is sufficient to use internal prestressing in the deck made up of 60 strands of 140 mm$^2$ to satisfy the SLSs. Prestressing with a parabolic profile has been used, although straight prestressing would also have been suitable.

The vibration SLS is again just satisfied, since the maximum acceleration practically coincides with the allowed maximum (0.45 m/sec$^2$, given that $f_0 = 0.80$ Hz).

2.2.6 Verification of ULSs

The ULS of fatigue determines the cross-sectional area of the stay cables. They are strictly dimensioned using external prestressing anchorages. Stress variations due to frequent live load are 80 MPa, while the maximum stress reaches 42% of the stay cables tensile strength.

The ULS of tension in the stay cables as well as both bending and shear ULSs in the deck are amply met.

2.2.7 Aspects relating to construction

In this case it might be more appropriate to stress the stay cables using some other procedure, since a large number of temporary props would be required. The stressing of the stay cables would be carried out from both abutments, immobilising only the strut located at midspan,
attaching the stay cables to the deviators by means of clamps, and allowing the struts to turn on the pins that join them to the deck until they reach their final position when the stay cables have been stressed.

2.3 Comparative analysis depending on the number of struts placed

Table 1 shows a conventional scheme without stay cables and three others with under-deck cable-staying systems using one, two and multiple (fifteen) struts. The last two cases correspond to the examples described above. The comparison of these structural systems brings to light several key points. Due to the greater subdivision of the span given by the increase in the number of struts and to the prestressing of the stay cables, bending of the deck in permanent state is reduced to the local bending between struts. Therefore, by increasing the number of struts, the flexural response (bending of the deck) is reduced and the axial response (tension of the stay cables and compression of the struts and deck) is promoted in permanent state. Consequently, the design bending moments are reduced, allowing the reduction of the depth of the deck that in turn produces a further reduction of the bending moments in the deck both in permanent state —because of the reduction of the self-weight of the deck— and under traffic live load —because of the increased efficiency of the cable-staying system. The reduction of the design bending moments in the deck allows a reduction of the amount of internal prestressing and a reduction of the characteristic strength of the concrete of the deck.

The results shown in Table 1 confirm that the efficiency of cable-staying systems under traffic live load increases with the reduction of the depth and with the increase in the number of struts, as the authors have demonstrated analytically (Ruiz-Teran and Aparicio 2007b). In addition, Table 1 shows that the influence of the deck depth is very significant (the efficiency doubles when the depth is reduced from 1/34 to 1/80 of the span) whereas the influence of the number of struts is minor (the efficiency increases by less than 3% when the number of struts
In the two examples shown (with 2 and with 15 struts), the efficiency of the cable-staying systems is so great that the depth of the deck is not limited by the bending ULS, but rather by the vibration SLS. Similar bridges to the ones described above but with a depth of only 0.70 m (1/115 of span) could have been designed. They would have satisfied all the limit states, except the vibration SLS, since the maximum accelerations would have increased by 80%. The maximum acceleration is therefore very sensitive to variations of the deck depth.

In summary, it is possible to design single-span bridges with under-deck cable-staying systems that have a high structural efficiency. To achieve this, it is necessary to subdivide the span —adding an appropriate number of struts— and to prestress the stay cables so that the permanent loads are compensated, along with using the minimum depth that satisfies both the vibration SLS and the bending ULS. In addition to its great structural efficiency, this structural type is attractive from an aesthetical point of view; and even from an economical point of view, since it allows a very substantial reduction in the amount of materials used (reducing the amounts of concrete and active steel to one third of that used in conventional schemes without stay cables).

3. Combined cable-stayed bridges

In combined cable-stayed bridges the stay cables are located above the extrados and below the intrados of the deck (Figure 11). Where the stay cables are above the extrados, they are deflected by the pylons, which take the cable downward deviation forces directly to the supports; and where the stay cables are below the intrados of the deck, they have a polygonal layout and are deflected by the struts that, under compression, introduce the cable upward deviation forces into the deck.
First of all, it is necessary to expand and apply the concept of efficiency of under-deck cable-staying systems to cable-stayed bridges, and particularly to combined cable-stayed bridges. Conventional cable-stayed bridges, extradosed bridges, under-deck cable-stayed bridges and combined cable-stayed bridges have two mechanisms for response to vertical forces, namely axial response —tension of stay cables and compression of struts, pylons and deck— and flexural response —bending of the deck— (Ruiz-Teran and Aparicio 2007a). It is best to design bridges in which the contribution of the axial response is promoted, since it is the most efficient response from a structural point of view and makes it possible to build lighter and less expensive structures (Schlaich and Bergermann 2004). Therefore, it is important to establish parameters for knowing and measuring to what extent the axial response is promoted. The authors have presented the ‘efficiency of the cable-staying system under live load’ $\xi$ (Ruiz-Teran and Aparicio 2007b), as the parameter that allows this to be assessed in under-deck cable-stayed bridges. However, this definition is so powerful that it can be extended to any type of cable-stayed bridge.

The ‘efficiency of the cable-staying system under live load’ $\xi$, is the fraction of the external isostatic moment of live load ($qL^2/8$ for a uniform load $q$) resisted by means of the cable-staying system, i.e.

$$
\xi = \frac{M_{\text{CABLE\_STAYED\_SYSTEM}}}{M_{\text{ISOSTATIC}}} = \frac{M_{\text{CABLE\_STAYED\_SYSTEM}}}{M_{\text{CABLE\_STAYED\_SYSTEM}} + M_{\text{DECK}}}
$$

The portion of the isostatic moment resisted by means of the flexural response of the deck is given by:
where $M_{DECK \_SUPPORT \_SECTION \_1}$, $M_{DECK \_SUPPORT \_SECTION \_2}$ and $M_{DECK \_MID\_SPAN \_SECTION}$ are the bending moments in the deck in the end sections of the main span (sections 1 and 2) and at midspan section, respectively.

The portion of the isostatic moment resisted by means of the axial response of the whole cable-staying system is given by:

\[
M_{CABLE\_STAYED\_SYSTEM} = -\frac{M_{CABLE\_STAYED\_SYSTEM \_SUPPORT \_SECTION \_1}}{2} + \frac{M_{CABLE\_STAYED\_SYSTEM \_SUPPORT \_SECTION \_2}}{2} + M_{CABLE\_STAYED\_SYSTEM \_MID\_SPAN \_SECTION}
\]

where $M_{CABLE\_STAYED\_SYSTEM \_SUPPORT \_SECTION \_1}$, $M_{CABLE\_STAYED\_SYSTEM \_SUPPORT \_SECTION \_2}$ and $M_{CABLE\_STAYED\_SYSTEM \_MID\_SPAN \_SECTION}$ are the moments contributed by the cable-staying system at the end sections of the main span and at the midspan section, respectively. The bending moment contributed by the cable-staying system at any section is given by:

\[
M_{CABLE\_STAYED\_SYSTEM \_SECTION \_i} = \sum_{j=1}^{n_i} e_{j,i} N_{j,i} \cos \alpha_{j,i}
\]

where $e_{j,i}$ is the eccentricity of the stay cable $j$ at the section $i$ (it will be positive for intradosed stay cables and, negative for extradosed stay cables); $N_{j,i}$ is the axial force in the stay cable $j$ in the section $i$, $\alpha_{j,i}$ is the angle between the stay cable $j$ and the deck at the section $i$, and $n_i$ is the number of stay cables in the section $i$.

### 3.2 Comparison of structural response to live load in under-deck cable-stayed bridges and combined cable-stayed bridges.

In under-deck cable-stayed bridges, stay cables are eccentric in the midspan section, but not in
the support sections. Therefore, the components contributed by the cable-staying system in the support sections are zero (Equation 4 is zero in the support sections). Consequently, all of the isostatic moment resisted by means of the axial response of the cable-staying system (Equation 3) is due solely to the component contributed in the midspan section. However, in a combined cable-stayed bridge (Figure 11), the stay cables are also eccentric in the sections over supports (Equation 4 is not zero in the support sections), meaning that the cable-staying system also helps to resist the isostatic moment in the support sections (Equation 3). In an under-deck cable-stayed bridge, the under-deck cable-staying system works with an eccentricity that is precisely the eccentricity of the stay cables at midspan. However, in a combined cable-stayed bridge, the cable-staying system works with a much greater eccentricity, which is close to the sum of the average eccentricity in the support sections and the eccentricity at midspan section. The main consequence is that, for the same efficiency of the cable-staying system, the axial forces in the stay cables will be much smaller in a combined cable-stayed bridge, and this fact will lead to a reduction in the cross-sectional area of the stay cables in comparison with under-deck cable-stayed bridges.

3.3 Comparative analysis of combined cable-stay bridges

Figure 11 shows three different combined cable-stayed bridges with 80-metre spans — with the same previous cross-section (Figure 3) —: a) the stay cables have no connection with the deck in the sections where they cross from the extrados to the intrados of the deck — they are displaced inside guide pipes where they pass through the deck cross-section —; b) the stay cables are anchored in the deck where they cross through it; and c) the stay cables have no connection with the deck in the sections where they cross from the extrados to the intrados of the deck — they are displaced inside guide pipes where they pass through the deck cross-section —and the pylons are inclined with the aim of recovering the compression lost in the
deck in the preceding cases—a) and b)—as a result of the elimination of the self-anchoring of the stay cables in the deck. Table 2 shows the main features of these three bridges and compares them to that of the under-deck cable-stayed bridge with two struts presented above.

It is shown that the combined cable-stayed bridges maintain the high efficiency of the cable-staying system at around 90% ($\xi=0.90$). As it was foreseen in Section 3.2, the cross-sectional area of the stay cables is almost halved. Nevertheless, this reduction in the cross-sectional area of the stay cables does not give rise to a proportional reduction of the amount of active steel for stay cables, due to the fact that, in combined cable-stay bridges, back stay cables are required—that have to be anchored to large counterweights whose construction considerably increases the cost of the structure.

Comparing the combined cable-stayed bridge with stay cables passing through the deck (case a from above) to the under-deck cable-stayed bridge, the loss of compression in the deck as a result of eliminating the self-anchorage of the stay cables in the deck must be compensated by an increase in the amount of internal prestressing. Therefore, combined cable-stayed bridges have a smaller amount of active steel in stay cables, whereas under-deck cable-stayed bridges have a smaller amount of active steel in internal prestressing, a fact that leads to a very similar total amount of active steel in both structural types.

Anchoring the through-passing stay cables to the deck (case b from above) allows a greater subdivision of the span—in five sections—but introduces an axial tension into the three middle sections of the deck between these anchorages. The reduction of the amount of internal prestressing is small, since the decrease that is achieved through reduction of the bending moments—due to the greater subdivision—is partially lost with the required compensation for the introduced tension. In addition, the necessity of having to anchor all the extradosed and intradosed stay cables in the same section results in a more difficult design
and construction. This extra intricacy is not offset by a significant reduction in the amounts of materials used, as is shown in Table 2. Therefore, in combined cable-stayed bridges with a single span, it is not worthwhile anchoring the stay cables in the deck.

In case c from above, the pylons have been inclined with the aim of recovering the compression lost in the deck in the preceding cases—as a result of the elimination of the self-anchoring of the stay cables in the deck—and, consequently, reducing the amount of internal prestressing. However, the reduction in the total amount of active steel is minimal due to the increase in the amount of active steel required for the stay cables—due to the loss of eccentricity of the extradosed stay cables in the support sections as a result of having inclined the pylons and having maintained their maximum height. Therefore, the total amount of active steel is very similar to that in the combined cable-stayed bridge with passing-through-the-deck stay cables and vertical pylons (case a). Inclining the pylons and placing the back stay cables in vertical position has other implications: (1) the requirement of much heavier counterweights; (2) the possibility of designing a unique single element to perform the functions of the abutment and counterweight—using the self-weight of the abutment as the anchorage counterweight of the back stay cables—, due to their close proximity; and (3) the possibility for greater expression from an architectural point of view. Given the above considerations, factors such as the construction cost, the space available and the aesthetical significance will determine which of these schemes—a) or c)—is the most appropriate in any particular case, since both of them are suitable from a structural point of view.

In all of the schemes studied in this section, the depth is limited to 1/80 of the span in order to satisfy the vibration SLS.

4. Comparative analysis of the schemes using under-deck cable-staying systems or combined cable-staying systems for single-span bridges.
The examples set out above show that both schemes using under-deck cable-staying systems and schemes using combined cable-staying systems are suitable from a structural point of view for the design of single-span bridges with medium spans (80 metres). From an economical point of view, schemes using under-deck cable-staying systems are more attractive, since they do not require the extra-costs associated with the construction of counterweights and pylons. From an aesthetic and architectural point of view, schemes using combined cable-staying systems can be justified in certain locations, since the fact that the stay cables are located above the deck, framing the space of passage over the structure, could have an extra value for the road users.

5. Design criteria

The analysis of the results and conclusions of the parametric study developed has allowed us to formulate the design guidelines set out below.

5.1 Morphology

1) **Structural elements.** Decks of under-deck cable-stayed bridges require struts and under-deck stay cables. Decks of combined cable-stayed bridges require struts, combined stay cables, pylons, back stay cables and counterweights.

2) **Connection of the stay cables to the deck.** In under-deck cable-stayed bridges, the stay cables are anchored in the end sections of the deck. In combined cable-stayed bridges, there should not be a direct connection between these two structural elements and, therefore, guide pipes should be employed where the stay cables pass through the deck cross-section.

3) **Number of struts.** The span is subdivided through the use of struts and by prestressing the stay cables, compensating for 100% of the permanent load (dead load and superimposed dead
The larger the number of struts, the smaller the bending moments — and the shear forces — in the deck, the larger the reduction in the amount of internal prestressing, the smaller the required characteristic strength required for the concrete of the deck and the smaller the depth — provided that the vibration SLS is not reached. However, the larger the number of struts, the higher their cost and the more complicated the stressing process becomes. Therefore, the number of struts used should be a compromise between structural, aesthetic, economic and construction considerations.

4) **Depth of the deck.** The smaller the depth, the higher the efficiency of the cable-staying system. Consequently, the smallest depth that satisfies both the bending ultimate and the vibration SLSs should be used. In bridges with 80-metre spans, the depth should be around 1/80 of the span, being limited by the vibration SLS.

5) **Connection between the deck and the struts.** The struts should be connected to the deck by means of pins in order to avoid the introduction of concentrated bending moments into the slender deck and struts as well as to strengthen the axial response. Diaphragms are positioned in these connection sections. This configuration greatly influences the procedure for stressing the stay cables, that can be performed by two methods: (1) preventing the struts from moving during the stressing process by using temporary props, or (2) allowing all the struts, except for the central strut, to move during the stressing process so that at the end of the process they will have taken up their required positions. This second stressing procedure, that is more complicated and requires the use of clamps on all the deviators, is only advisable when a large number of struts is used.

6) **Location of the connection points of the cable-staying system to the deck.** These points are uniformly distributed along the span. Shorter subsections can be placed close to the supports, although this is not a determinant factor as in conventional bridges.
7) **Orientation of the struts.** The struts are placed along the bisector of the angle formed by the stay cables in order to ensure that the stress in the stay cable remains constant along its length.

8) **Orientation of the pylons.** In combined cable-stayed bridges, the pylons can be placed either vertically or inclined—in order to introduce compression into the deck.

9) **Eccentricity of the cable-staying systems.** Eccentricities in the critical sections of midspan and supports on the order of 1/10 of the span are appropriate. Although greater eccentricities will increase structural efficiency, they are usually not considered suitable from an aesthetic point of view (Ruiz-Teran and Aparicio 2007b). The eccentricity used should be a compromise between structural, clearance and aesthetic considerations.

10) **Layout of the cable-staying systems.** Once both the points where the deck lays over struts and the eccentricity of the cable-staying system in critical sections have been established, the layout of the stay cables is determined by three conditions: (1) the eccentricity in the critical sections must be as established; (2) all the struts must be placed along the bisector of the angle formed by the stay cables; and (3) the vertical component of the deviation force introduced through the struts into the deck must be uniform, in order to compensate the permanent load.

11) **Layout of the internal prestressing in the deck.** A layout similar to that of a bridge with supports at the points where the deck lays over the struts is appropriate. If the number of struts is increased, it is more suitable to choose a centred prestressing formed by two eccentric families that guarantee a high contribution in the ultimate moment resistance of the sections.

12) **Cross-sections of the deck.** Voided slabs are suitable, since the efficiency of the cable-staying system increases with the reduction in the flexural rigidity of the deck and in the
radius of gyration (Ruiz-Teran and Aparicio 2007b). The separation between voids will be conditioned by the placement of the internal prestressing and by construction requirements, but it will not be conditioned by the shear ULS.

5.2 Limit states that govern the dimensions of the different structural elements

1) Cross-section of the stay cables. The cross-section of the stay cables is determined by the ULS of fatigue that is linked to the anchorage technology used: stay-cable anchorages ($\sigma_{\text{max}} \leq 0.45f_{\text{pu}}$ and $\Delta \sigma_{\text{max}} \leq 200$ MPa) or conventional external-prestressing anchorages ($\sigma_{\text{max}} \leq 0.65f_{\text{pu}}$ and $\Delta \sigma_{\text{max}} \leq 80$ MPa). This limit state requires a double verification: (1) the maximum stress must be less than $\sigma_{\text{max}}$ and (2) the variation in stress due to frequent live load must be less than $\Delta \sigma_{\text{max}}$. Stress changes due to rotation of the anchorages of the stay cables as well as the wind effects should be considered if they are not negligible.

2) Depth of the deck. For medium spans (80 metres), the depth of the deck is governed by the vibration SLS.

3) Characteristic strength of the concrete of the deck. It is advisable to use the minimum strength that ensures both the verification of the stress limitation SLS and the durability of the structure. Values between 35 and 40 MPa are appropriate.

4) Amount of internal prestressing in the deck. In contrast with conventional bridges, the amount of internal prestressing is determined by the controlled cracking SLS and even by the bending ULS. Attempting to satisfy these limit states through the increase of the passive reinforcement (using only the internal prestressing required to satisfy the decompression SLS) gives rise to sections that are so highly reinforced that they are not recommended from both construction and durability points of view.
5) **Amount of internal reinforcement.** Because of the huge reduction of the flexural response of the deck and in order to avoid a brittle failure, it is necessary to verify the bending ULS taking into consideration at least the hogging and sagging cracking bending moments in the case they are larger than the design bending moments.

6) **Cross-section of struts.** Struts must satisfy the compression ULS. Design forces must be obtained on the basis of a non-linear analysis.

### 6. Conclusions

1) A complete and systematic parametric study of under-deck cable-stayed bridges and combined cable-stayed bridges with single spans of medium length (80 m) has been conducted. The study has been focused on road bridges with prestressed concrete decks, and the full design of several different schemes has been addressed, thereby allowing the analysis of their structural response and the definition of design criteria.

2) In permanent state, as a result of the prestressing of the stay cables (compensating for 100% of dead load and superimposed dead load) and the use of intermediate struts, a span subdivision is achieved: the bending of the deck is reduced to the local bending between struts, strengthening the axial response (tension of the stay cables and compression of the deck, struts and pylons). In addition, the loss of tension in the stay cables due to time-dependent effects is very small (less than 2%), and the span subdivision attained after stressing the stay cables is maintained over time.

3) In the static response of these structural types to traffic live load, the axial response is also strengthened in relation to the flexural response. Efficiencies of 90% are attained for cable-staying systems in single-span bridges with medium-length spans. This high efficiency results in the ULS of fatigue being the critical limit state for determining the
4) The total eccentricity of a combined cable-staying system is approximately double that of an under-deck cable-staying system. Therefore, for the same efficiency of the cable-staying systems, the cross-sectional area of the stay cables in combined cable-stayed bridges is approximately half that of the stay cables in under-deck cable-stayed bridges. Nevertheless, this reduction in the cross-sectional area does not give rise to a proportional reduction in the amount of active steel for stay cables since back stay cables are required. Furthermore, the construction of the corresponding counterweights to which the back stay cables are anchored considerably increases the cost of the structure.

5) The strengthening of the axial response and consequently the reduction of the flexural response makes it possible to design extremely slender structures that make optimal structural use of the materials disposed. There is a very substantial reduction in the depth of the deck. However, the reduction in the depth of the deck involves a significant increase in the vertical accelerations associated with the dynamic response of the structure to traffic live load.

6) For reference proposes, in single-span bridges with a medium-length span (80 metres), a conventional bridge without stay cables would have a depth of 1/15 of the span, whereas an under-deck cable-stayed bridge or a combined cable-stayed bridge will have a depth of 1/80 of the span. The huge reduction of the depth (by ~80%) leads to the vibration SLS being the critical limit state for determining the depth of the deck in these types of bridges.

7) In single-span bridges with medium-length spans (80 metres), as a result of the high efficiency of these structural types, substantial reductions are achieved in the amounts of cross-sectional area of the stay cables.
materials used in comparison with conventional schemes: the amounts of concrete and active steel are reduced to one third. This result is achieved using conventional concrete (35 or 40 MPa) and stay cable anchorages with an anchorage technology of external prestressing.

8) Design criteria for these types of bridges have been established in Section 5 of this paper.

9) These structural types offer a wide range of possibilities from a construction point of view. The significant reduction in the depth of the deck and the shape of the required prestressing layouts leads us to consider the possibility of building these types of bridges using longitudinal precast elements assembled on site. This would allow a substantial extension of the span range for which longitudinal precast elements could be used in bridge-building.

10) Such a substantial reduction in the amounts of materials used for the decks and also of the design actions on other structural elements (bearings, piers, abutments and foundations), as well as the wide range of possibilities from a construction point of view, lead us to believe that these schemes could be much more economical than conventional schemes for medium spans, as long as there is enough vertical clearance to set up the stay cables.
References


EHE 1999. Structural Concrete Code. Ministerio de Fomento, Spain (In Spanish)


Table 1: Comparison of under-deck cable-stayed bridges

<table>
<thead>
<tr>
<th></th>
<th>Without stay cables</th>
<th>With under-deck stay cables</th>
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<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Number of struts</td>
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<tr>
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<td>1/34</td>
</tr>
<tr>
<td>Section type</td>
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<td>box girder</td>
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<td>Self-weight (kN/m)</td>
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<td>202</td>
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<td>Number of stay cable strands</td>
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<tr>
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<td>staying system (ξ)</td>
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<td>vehicles (m/s²)</td>
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*a Amounts of active steel given in kg/m² / span(m)
Table 2: Comparison of combined cable-stayed bridges

<table>
<thead>
<tr>
<th></th>
<th>Under-deck cable-stayed bridge</th>
<th>Stay cables without deck connection (inside guide pipes)</th>
<th>Stay cables anchored to the deck</th>
<th>Stay cables without deck connection (inside guide pipes) and inclined pylons</th>
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<td>1/80</td>
<td>1/80</td>
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<td>Number of stay cable strands</td>
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<td>142</td>
<td>184/107^b</td>
<td>161</td>
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<tr>
<td>Total amount of active-steel^a</td>
<td>0.48</td>
<td>0.49</td>
<td>0.46</td>
<td>0.47</td>
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<td>Amount of internal prestressing in the deck^a</td>
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<td>0.27</td>
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<td>Amount of stay cables^a</td>
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<td>fck (MPa)</td>
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<td>35</td>
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<td>Counterweights</td>
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<td>Weight (kN)</td>
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<td>100</td>
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<td>Bending moment in permanent state</td>
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<td>Maximum (MN.m)</td>
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<td>-17.73</td>
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<td>Bending moment due to traffic live load</td>
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<td>0.91</td>
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<td>Acceleration due to heavy vehicles (m/s^2)</td>
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<td>0.41</td>
<td>0.39</td>
<td>0.29</td>
<td>0.46</td>
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^a Amounts of active steel given in kg/m^2 / span(m)

^b Extradosed stay cables / intradosed stay cables
Figure captions

Figure 1: Tobu recreation resort footbridge, Japan (courtesy of Meguru Tsunomoto, Oriental Construction Co.).

Figure 2: Under-deck cable-stayed bridge with two struts. Elevation.
Figure 3: Cross-sections: a) calculation cross-section, b) real cross-section and c) section showing the connection with a strut

Figure 4: Bending moment diagrams in permanent: a) due to g₁ (dead load) + g₂ (superimposed dead load) + prestressing of the stay cables; b) due to g₁ + g₂ + prestressing of the stay cables + shrinkage + creep + relaxation (internal prestressing).

Figure 5: Bending moment envelopes due to traffic live load: a) q=52.8 kN/m (4 kN/m²); b) Q=600kN (M_Q TOTAL = 2M_Q GRAPH)

Figure 6: Envelope of vertical accelerations due to passage of two vehicles of 400 kN at 60 km/h (moving from left to right)
Figure 7: Layout of internal prestressing.

Figure 8: Under-deck cable-stayed bridge with multiple struts. Elevation

Figure 9: Bending moment diagram in permanent state: a) due to g1+g2 + prestressing of stay cables; b) due to g1+g2+ prestressing of stay cables + Shrinkage + Creep + Relaxation (internal prestressing)

Figure 10: Bending moment envelopes due to traffic live load: a) q=52.8 kN/m (4 kN/m²); b) Q=600kN ($M_Q\ TOTAL = 2M_Q\ GRAPH$)
Figure 11: Combined cable-stayed bridges: a) with stay-cables passing through the deck; b) with stay cables anchored in the deck; c) with passing-through-the-deck stay cables and inclined pylons