Photon Coupling Effects and Advanced Characterisations of Multiple-Quantum-Well Multi-Junction Solar Cells

Kan-Hua Lee

June 9, 2014
Statement of Originality

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## List of Symbols

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Abstract

Achieving optimal band-gap combinations of multi-junction solar cells at production level is the most difficult challenge in concentrator photovoltaics. To improve the state-of-the-art InGaP/InGaAs/Ge triple-junction cells, it requires that the band gaps of the top and middle junction to be lower or an additional 1 eV junction. This involves lattice-mismatch growth or introducing dilute nitrides materials, which makes it difficult to scale up to production at low cost. Strain-balanced multiple quantum wells (MQWs) in the middle junction has been very well-studied as a means to adjust the absorption edges of the middle junction in multi-junction solar cells. To fully optimise the efficiency of solar cells with MQW GaAs subcell, an InGaP top cell with MQWs also has to be introduced to achieve current-matching. The aim of this thesis is to address the issues of production multi-junction cell with MQWs. We studied the material properties of MQW InGaP subcells and demonstrated its strong photon coupling effects in multi-junction devices. Several characterisation techniques were developed to acquire deeper understanding of the material qualities and sheet resistance of MQW solar cells.
Chapter 1

Introduction

1.1 III-V solar cells

III-V compound semiconductor solar cells have held the record efficiency of solar cells for very long time [5]. Currently, the most efficient III-V solar cell has the conversion efficiency of more than 44% at high sun concentration. However, the material cost of III-V solar cells are also very high compared to other solar cell materials. Therefore, III-V solar cells are only commercially viable for the applications for which customers are willing to pay a premium in return for better efficiency.

One of such area where III-V solar cells have succeeded is photovoltaic (PV) systems in space, and it is still the biggest market for III-V solar cells. The cost for launching satellites makes the cost of the solar cells relatively marginal. Apart from the efficiencies, harsh environmental conditions, such as very high energy radiation and dramatic temperature differences need to be considered for space PV applications. III-V solar cells have demonstrated to be very radiation hard and their performance is insensitive to environment temperature. III-V solar cells are also very lightweight. The weight of state-of-the-art III-V solar cells is mainly influenced by the substrate, and this can be further reduced when substrate removal technology becomes more mature [6].
Concentrator photovoltaics (CPV) offers a way for III-V solar cells to be employed in terrestrial applications. Concentrator photovoltaics focuses light onto the solar cells using concentrating optics. In this way, the cost of the solar cell material can be replaced by much cheaper materials for the optics, such as plastics or metal reflectors. Typically, the cost of solar cells in a CPV system is only 15% to 20%, which makes the high areal manufacturing cost less significant.

Concentrated sunlight brings additional benefits to the performance of solar cells. First of all, high injection of photons can saturate the defect states in the solar cell so that the efficiency can be increased. Also, from the fundamental thermodynamic point of view, the concentrated light can reduce the increase of entropy during the photovoltaic process [7], and therefore increase the conversion efficiency.

However, many challenges remain for III-V solar cells to make CPV more cost competitive. The first challenge is efficiency. Currently, the world-record efficiency of triple-junction solar cell is 44.4% [5], and the goal is to achieve more than 50%, so that the cost per watt can be further reduced [8]. Another challenge is raising the operating concentration of the solar cells. Operating the solar cell at very high current also comes with significant power loss due to parasitic resistance of the solar cell. The mainstream of the concentration ratio on the market is 500 to 1000 suns. Raising the concentration ratio further, up to 1500 suns to 2000 suns, is necessary to make the price of CPV competitive.

1.2 Basics of III-V solar cells

Multi-junction solar cells comprise junctions with different band gaps to increase the harvest of photon energies [9]. A two-terminal, series-connected, monolithic multi-junction solar cell is a very common design for III-V solar cells. Triple-junction solar cell with InGaP (1.82 eV), GaAs (1.42 eV) and Ge (0.7eV) as subcells is currently the most popular design of III-V multi-junction solar cells. The layer structure of a typical triple-junction
solar cell is drawn in Figure (1.2) [1].

A typical III-V solar cell is made of layers as follows:

**Anti-reflection coating**  Anti-reflection (AR) coatings are deposited on the surface of solar cells to lower the reflection caused by the refractive index difference between the semiconductor layer and air. Silicon nitrides, silicon oxides, and titanium oxides are commonly used as AR coatings for III-V solar cells.

**Electrodes and capping layers**  Metals are deposited on both sides of the solar cells to form the ohmic contacts. Silver or gold is usually used for ohmic contacts due to their high conductivities. Below the ohmic contacts is the capping layer. The capping layer is a very highly doped semiconductor layer which serves two purposes: (1) to protect the solar cell from oxidisation before its processing, and (2) to help forming the ohmic contacts and
Figure 1.2: The left hand figure shows layer structure of a typical two-terminal triple-junction solar cell, and the table lists the band gaps and thicknesses of each layer. Both are excerpted from Ref. [1].
increase the conductivity for collecting current.

**Window layer and back-surface field layer**  Window layer is for reducing the surface combination of photogenerated carriers. The dangling bonds on the surface of materials form the recombination centres for carriers. The purpose of window layer is stopping the minority carriers reaching the surface of the solar cells. Similar to the window layer, the back-surface field layer is for reducing the minority carrier flowing to the substrate or the tunnel junction.

**Active layer**  The active layer generates photocurrent for the solar cell devices. The typical structures for the active layers of semiconductor solar cells are p-n or p-i-n structures. The p-n junction in a solar cell usually has one side heavily doped, known as the emitter, and the lightly doped layer, known as the base. A commercial III-V solar cell generally uses a n-type emitter and p-type base.

**Tunnel junction**  The interconnections of different p-n junctions in a multi-junction solar cell are implemented by tunnel junction diodes. Ideal tunnel diodes should have very low resistance, high peak current and very low absorption.

### 1.3 Searching for the best band gap combination

Commercial InGaP/GaAs/Ge triple-junction solar cells can achieve efficiency at about 40%. To achieve even higher efficiencies of multi-junction solar cells, the band gaps combinations need to be further optimised. Under the standard sun spectrum, the photons absorbed in the Ge junction are much more than the photons absorbed in the top two junctions. In principle, this can be balanced by decreasing both the band gaps of the top and middle cell or adding another junction with band gap ~1 eV between the middle cell and the Ge junction. However, neither of these is easy to implement. Figure (1.3)
shows the band gaps against the lattice constants of some semiconductors. The coloured line in the figure marks lattice constants of GaAs. It clearly shows the absence of the materials required to lower the band gaps of the top two junctions, or the material as a 1 eV junction between the GaAs and Ge. Several approaches have been attempted to solve this problem:

**Upright metamorphic growth** Materials with different lattice parameters are monolithically grown by growing buffer layers to transfer the lattice parameters for one material to another [10], as illustrated in Figure (1.4) (a). The advantage of this method is that it does not add too much additional cost compared to standard triple-junction, but maintaining the quality of the subcells grown above the buffer layer is a challenge.

**Inverted metamorphic growth** Unlike upright metamorphic growth, the materials are grown from the top cell, followed by a buffer layer to transfer the lattice parameter. After that, the bottom cell is grown on the top of the buffer layer, and the substrate is removed from the top cell [11]. This process is illustrated in Figure (1.4)(b). In this way, the material quality of the top two junctions can be maintained because they are not grown on a buffer layer. However, substrate removal causes additional labour and material cost.

**Lattice-matched materials** Adding dilute nitrides into GaAs or InGaAs can lower the band gaps of these materials to achieve 1 eV junction, while keeping the lattice constant matched to GaAs. Diffusion lengths of minority carriers in dilute nitrides are very short, making it very challenging to apply it in high efficiency solar cells [12]. Although it has been demonstrated that triple junction solar cells with dilute nitrides bottom cell grown by molecular beam epitaxy (MBE) can achieve a 43.5% efficiency [13], growing dilute nitrides using metal organic chemical vapour deposition (MOCVD) is still known to be very difficult.
Figure 1.3: Energy gaps versus lattice constants of some common compound semiconductors. The coloured line indicates the lattice constants close to GaAs. This figure is excerpted from Ref. [2].

**Low-dimensional nanostructures** Employing quantum wells or quantum dots in the solar cells can tune the absorption edges while keeping the lattice parameters nearly constant throughout the growth. This technique has been widely used in light emitting diodes or semiconductor lasers. This method also allows the absorption edges to be highly customised towards the local illumination spectrum of a PV site. Growing large stacks of quantum wells or quantum dots with very high quality to achieve high absorption is also challenging.

### 1.4 Quantum well solar cells

The concept of quantum well solar cells was proposed by Barnham and Duggan [14], and was further developed by Quantumphotovoltaics group at Imperial College London. In 2009, QuantaSol Ltd., a spin-out company of this research group, demonstrated a
Figure 1.4: An illustration of (a) upright metamorphic growth and (b) inverted metamorphic growth. In (a), the device is grown from the bottom cell, with a buffer layer grown between the bottom cell and the middle cell to transfer the lattice constant. In (b), the device is grown in the reverse order, and the substrate is removed after the epitaxy.

A single-junction GaAs solar cell with shallow multiple quantum wells (MQWs) achieved 28% efficiency and was the world record. In 2011, QuantaSol demonstrated triple-junction solar cells with MQWs in the GaAs middle cell with efficiencies more than 39%. After that, the technology and the proprietary intellectual properties were transferred to JDS Uniphase (JDSU). In 2013, JDSU demonstrated that the median efficiencies of triple-junction solar cells with MQWs in both top and middle cells could achieve 41.5% at 500 suns [15].

Figure (1.5) illustrates the working principles of quantum well solar cells. MQWs are used in the i-region of a p-i-n cell structure. When the solar cell is illuminated, the photons excite electron and hole pairs inside the quantum wells. The photogenerated electron-hole pairs that thermally escape from the well and reach the n or p-layers can then be collected as photocurrent. State-of-the-art GaAs MQW solar cells use GaAsP as the barrier and InGaAs as the well materials. The compressive strain caused by GaAsP is balanced by the tensile strain, which is in turn caused by InGaAs, using a so-called strain-balanced
Figure 1.5: An illustration of quantum well solar cell. Electron-hole pairs can be excited by photons with energy that is lower than the band gap. The generated carrier can escape from the quantum well thermally, but some of these carriers will be re-trapped into the quantum well. Some of the photogenerated carriers in the quantum wells can eventually reach the p or n-type material, and be collected by the electrodes.

technique [16]. The confinement potentials of the quantum wells are adjusted by altering the compositions of GaAsP and InGaAs. The absorption edges of the MQW GaAs solar cell can be extended to around 1.2 eV.

1.5 Outline of this thesis

This thesis is aimed at addressing the issues of commercial MQW multi-junction solar cells. The first part of this thesis will present the study of a novel InGaP/InGaAsP quantum wells for top cells and its photon coupling effects in a multi-junction device. Although MQWs for extending the band edges of GaAs cell has been well studied, many material properties of MQWs for InGaP top cells remain unknown. In Chapter 3, material properties studied via several characterisation techniques will be presented. As the photon-coupling effect in multi-junction solar cells is recognised as a means to improve
the energy harvest, demonstrations of photon coupling in the MQW solar cells will be reported in Chapter 4. The second part of this thesis presents two new techniques for characterising III-V solar cells, including a very low-cost flash system for high concentration I-V test, and a technique that uses electroluminescence to determine the local sheet resistivity of a solar cell in Chapter 5 and Chapter 6, respectively.
Chapter 2

Theory and Methods

The theoretical backgrounds and experimental methods are outlined in this chapter. We start from the detailed balance model, which is widely used to predict the efficiency limit of solar cells. After that, the semiconductor statistics and several of its approximations will be introduced. Then, the carrier generation and recombination models commonly used in III-V solar cells will be explained, followed by a short introduction of p-n junction analysis. In the second part of this chapter, a number of experimental methods for studying the material properties of the semiconductor solar cells will be explained.

2.1 Predicting the efficiency limit of solar cells using detailed balance model

The efficiency limit of converting solar energy to work has been analysed using the framework of thermodynamics. Landsberg [17] uses a purely thermodynamic framework to predict the efficiency of solar energy converter, which analysed the incoming energy and entropy of photons with the emitted photon energy and entropy without considering the nature of photovoltaic materials. This calculation gives the limiting efficiency of solar energy conversion material to be 95%.
Shockley and Queisser [18] analysed the efficiency limit of semiconductor solar cells by applying detailed balance, or so called Roosbroeck-Shockley principle [19]. Their predicted efficiency limit, known as Shockley-Queisser limit, detailed balance limit or radiative limit, assumes the following criteria for an ideal solar cell:

- Every incoming photon with energy larger than the band gap generates an electron-hole pair. The generated electron-hole pair can only be collected as photocurrents or recombine radiatively in the material.

- The solar cell is in thermal equilibrium and its radiative emission is described by generalised Planck distribution.

This model is very successful and widely used in studying solar cell physics for many reasons. First of all, nearly all of the photovoltaic material are based on similar principle: one electron-hole pair or exciton are generated by absorbing one photon, thus the first assumption in Shockley-Queisser works for nearly all types of solar cells. Hence this model provides a more practical efficiency limit for solar cells. Moreover, since thermodynamics coordinates such as entropy in a solar cell are more difficult to quantify, detailed balance model offers a way to assess the model based on electrical and optical parameters.

**Solar cell at detailed balance limit**

Detailed balance principle states that the absorbed photons should equal to the sum of the electrons collected by the external circuit and the emitted photons of the material, which can be written as

\[
\phi_{abs} = I_{ext}/q + \phi_{ext}
\]  

(2.1)

where \(\phi_{abs}\) is the absorption rate of photon flux, \(I_{ext}\) is the current collected by the external circuit, and \(\phi_{em}\) is the rate of photon flux flowing out of the material. At open-circuit,
Eq. (2.1) becomes

\[ \phi_{\text{abs}} = \phi_{\text{ext}}, \]  \hspace{1cm} (2.2)

implying that the photon absorption rate equals to the photon emission rate at open circuit. This process is illustrated in Figure (2.1). Detailed balance model assumes the emissivity \( \phi_e(E) \) between energy \( E \) and \( E + dE \) can be described by Shockley-van Roosbroeck’s law [19]:

\[ \phi_e(E)dE = a(E)\phi_{BB}(E)dE \]  \hspace{1cm} (2.3)

where \( a(E) \) is the absorptivity, and \( \phi_{BB}(E) \) is generalised Planck distribution:

\[ \phi_{BB}(E) = \frac{2n_d^2}{h^3c^2} \frac{E^2}{\exp\left(\frac{E-\mu}{kT}\right) - 1} \]  \hspace{1cm} (2.4)

where \( \mu \) is the chemical potential of the photons, and \( n_d \) is refractive index of the material. The total photon emission rate of the material can then be calculated by integrating the outgoing photon flux over the surface:

\[ \phi_{\text{ext}} = \int_S \int_\Omega \int_E a(E)\phi_{BB}(E)d\Omega dSdE \]  \hspace{1cm} (2.5)

where \( \Omega \) is the external solid angle, \( E \) is the photon energy, and \( S \) is the surface normal vector. Considering the limit \( \exp((E - \mu)/kT) \gg 1 \), \( \phi_{\text{ext}} \) becomes:

\[ \phi_{\text{ext}} = \frac{2n_d^2}{h^3c^2} \int_S \int_\Omega \int_E a(E)E^2d\Omega dSdE \exp\left(\frac{\mu}{kT}\right) \]  \hspace{1cm} (2.6)

As will be discussed in Section 2.3, the chemical potential \( \mu \) is equal to the electrical potential, namely \( \mu = qV \). With short-circuit current \( I_{sc} = q\phi_{abs} \), Eq. (2.1) can rewritten as the current-voltage relation:

\[ I_{\text{ext}}(V) = -I_{sc} + q\phi_{\text{ext}}(V) = -I_{sc} + I_{\text{rad}}(V) \]  \hspace{1cm} (2.7)

where \( I_{\text{rad}} \equiv q\phi_{\text{ext}}(V) \) has the opposite direction of \( I_{sc} \) due to the radiative recombination photon flux \( \phi_{\text{ext}} \).
Non-ideal solar cells

Eq. (2.1) can be generalised to describe a more realistic solar cells by adding an extra term $\phi_{nr}$ that accounts for the photon flux not emitted out of the material owing to the internal photon loss or non-radiative recombination:

$$\phi_{abs} = \frac{I_{ext}}{q} + \phi_{ext} + \phi_{nr} \quad (2.8)$$

By defining a term $\eta_{ext}$ to be

$$\eta_{ext} \equiv \frac{\phi_{ext}}{\phi_{ext} + \phi_{nr}} \quad (2.9)$$

Eq. (2.8) can be rewritten as:

$$\phi_{abs} = \frac{I_{ext}}{q} + \left( \frac{1}{\eta_{ext}} \right) \phi_{ext} \quad (2.10)$$

$\eta_{ext}$ is often called external radiative efficiency or external luminescence efficiency, which is used as a parameter for describing the extent of photon loss against detailed balance limit. External radiative efficiency of state-of-the-art solar cells are summarised in Ref. [20], which shows that the external radiative efficiencies of standard single-junction GaAs is around 1.26%, but it can be raised to 22.2% by substrate removal and optimising its device structure to reduce the optical loss.

The photon absorption and emission in a solar cell can be described a two-step process. First, a photon is absorbed and re-radiate another photon with the probability referred to as internal luminescence efficiency or internal radiative efficiency, $\eta_{int}$. Then, the re-radiated photons will escape out of the material with optical efficiency $\eta_{opt}$, which is the probability that the reradiated photon can escape out of the cell. As a result, $\eta_{int}$ merely depends on material quality, but $\eta_{ext}$ is a function of both the material quality and the optical design of the cell. More details of external and internal radiative efficiencies will be discussed in Ref. [21,22].
Figure 2.1: An illustration of detailed balance model. At detailed balance limit, the incident photons ($\phi_{\text{abs}}$) can only either emit out of the solar cell or be converted to external current $I_{\text{ext}}$. This is described by Eq. (2.1).

External radiative efficiency can be rewritten into other forms. According to Eq. (2.10), $\eta_{\text{ext}}$ is the ratio of emitted photon and absorbed photons at open-circuit ($I_{\text{ext}} = 0$),

$$\eta_{\text{ext}} = \frac{\phi_{\text{ext}}(V_{oc})}{\phi_{\text{abs}}} = \frac{q\phi_{\text{ext}}(V_{oc})}{I_{sc}} \quad (2.11)$$

When the cell is forward-biased at a voltage $V$ in the dark, which is equivalent to replacing the photon influx term $\phi_{\text{abs}}$ by an external carrier injection term $I_{\text{inj}}(V)/q$ in Eq. (2.10). This makes the external radiative efficiency become

$$\eta_{\text{ext}} = \frac{\phi_{\text{ext}}(V)/q}{I_{\text{inj}}(V)} = \frac{I_{\text{rad}}}{I_{\text{inj}}(V)} \quad (2.12)$$

An application of using the equivalence of Eq. (2.11) and Eq. (2.12) will be presented in Chapter 4.
2.2 Semiconductor statistics

At thermal equilibrium, the concentration of free electrons in an undoped semiconductor is given by

\[ n = \int_{E_C}^{\infty} N(E)F(E)dE \]  \hfill (2.13)

\( N(E) \) is the density of states and \( F(E) \) is the Fermi-Dirac distribution function:

\[ F(E) = \frac{g}{\exp\left(\frac{E-\mu}{kT}\right) + 1} \]  \hfill (2.14)

where \( g \) is the degeneracy and \( \mu \) is the chemical potential. In semiconductor physics, we usually use Fermi energy level \( E_F \) to represent the chemical potential \( \mu \). Fermi energy level is the maximum occupancy energy when \( T \) approaches zero. In the limit of \( \exp\left(\frac{E-\mu}{kT}\right) \approx 1 \), \( F(E) \) approximates to Maxwell-Boltzmann distribution \( \exp\left(-\frac{E-\mu}{kT}\right) \), thus Eq. (2.13) can be reduced to

\[ n = N_C \exp\left(-\frac{E_C - E_F}{kT}\right) \]  \hfill (2.15)

for electrons, and

\[ p = N_V \exp\left(-\frac{E_F - E_V}{kT}\right) \]  \hfill (2.16)

for holes. \( N_C \) and \( N_V \) are effective density of states of electrons and holes. The above equations are also known as Boltzmann approximation, which will be used for calculating the analytical solution of I-V characteristics in the later sections.

2.3 Chemical potential of semiconductors

Chemical potential is a thermodynamic coordinate that determines the flow of the particles. Typically, chemical potential is determined by the population of the particles.
However, since carriers in semiconductors are charged particles, the electrical potential should also be included in chemical potential, that is,

$$\mu \equiv \mu_N + qV$$  \hspace{1cm} (2.17)

where $\mu_N$ is the chemical potential of the carriers if they are uncharged.

Chemical potential not only determines how the carriers will flow but also the reaction direction of carrier recombination and generation:

$$e^- + h^+ \rightleftharpoons \nu$$  \hspace{1cm} (2.18)

At equilibrium, the chemical potential of electron and holes $\mu_{e^-h}$ should be equal to the chemical potential of the photons $\mu_\nu$. Therefore, the chemical potential of photons is equal to the Fermi-level split of electron and holes:

$$\mu_\nu = E_{F_n} - E_{F_p}$$  \hspace{1cm} (2.19)

### 2.4 Optical absorption in semiconductors

In semiconductor solar cells, the carrier generation rate $g(E, x)$ of photons with energy $E$ at depth $x$ is usually assumed to be the product of the photon flux and the absorption rate

$$g(E, x) = \phi(E, x)\alpha(E, x)$$  \hspace{1cm} (2.20)

where $\phi(E, x)$ and $\alpha(E, x)$ are the photon flux and the absorption rate of photon energy $E$ at $x$, respectively. To calculate $\phi(E, x)$, we need to know the attenuation of the material, which can be described by

$$\frac{d\phi}{dx} = -\alpha(E, x)\phi$$  \hspace{1cm} (2.21)
Hence the photon flux at depth $x$ is

$$\phi(E, x) = \phi_0(E)(1 - r(E)) \exp \left( \int_0^x \alpha(E, x') \, dx' \right) \quad (2.22)$$

where $\phi_0(E)$ is the incoming photon flux and $r(E)$ is the reflectivity at the surface.

The absorption coefficient $\alpha$ is related to the imaginary part of refractive index $\xi$ by

$$\alpha(\lambda) = 4\pi\xi/\lambda \quad (2.23)$$

The dominant absorption mechanism of semiconductor solar cell is valence band to conduction band transition. The probability of band-to-band transition can be predicted by Fermi-Golden rule.

### 2.5 Carrier recombination in semiconductors

#### 2.5.1 Radiative recombination

The radiative recombination rate $U_R$ can be described by a recombination coefficient multiplying by the product of free electron and hole concentration:

$$U_R = Bnp \quad (2.24)$$

At thermal equilibrium, the radiative recombination rates balances the carrier generation rates and the law of mass action should be satisfied, which is

$$np = n_i^2 \quad (2.25)$$

When excess carriers with densities $\Delta n$ and $\Delta p$ are injected, the net recombination rate becomes [23]

$$U'_R = \frac{d[np]}{dt} - \frac{d[n_i^2]}{dt} = B \left( (n_0 + \Delta n)(p_0 + \Delta p) - n_i^2 \right) = B \left( (n_0 + \Delta n)(p_0 + \Delta p) - n_0p_0 \right) \quad (2.26)$$
In a n-type semiconductor with doping density \( N_D \), Eq. (2.26) becomes

\[
U'_R = B (\Delta n p_0 + \Delta p N_D + \Delta n \Delta p)
\]

\[
\simeq B (\Delta p(N_D + \Delta p))
\]

(2.27)

since \( N_D \gg p_0 \) and \( \Delta n = \Delta p \). At low excess carrier density \( N_D \gg \Delta p \), the net recombination rate \( U'_R \) reduces to

\[
U'_R = B \Delta p N_D
\]

(2.28)

and is thus linear to the excess minority carrier density \( \Delta p \). At higher injection, the net radiative recombination rate will also have the contribution of higher order term \( (\Delta p)^2 \), as shown in Eq. (2.27).

### 2.5.2 Recombination involving traps

Defects in semiconductors result in energy states between the conduction band and valence band, which is usually called trap states. The electron capturing rate of a trap state can be described by

\[
\frac{dn}{dt} = -c_n n = -v_{th} N_T \sigma_n n
\]

(2.29)

where \( c_n \) is capture rate per electron, \( v_{th} \) is thermal velocity of carriers, \( N_T \) is trap density and \( \sigma_n \) is the capture cross section of the electron traps. Similarly, the capture rate for holes of this trap state can be written as

\[
\frac{dp}{dt} = -c_p p = -v_{th} N_T \sigma_p p
\]

(2.30)

where \( \sigma_p \) is the capture cross section for hole traps. The thermal emission rates of the electrons and holes from the trap states are given by the form of Arrhenius equation:

\[
\frac{dn_T}{dt} = c_n n_T \exp \left( \frac{-(E_C - E_T)}{kT} \right)
\]

\[
\frac{dp_T}{dt} = c_p p_T \exp \left( \frac{-(E_T - E_V)}{kT} \right)
\]

(2.31)
If a trap level cannot capture another electron and hole before the captured carrier re-emit, this type of trap will not cause any carrier loss. It only affects the carrier transport. However, if a trap level can capture an electron and a hole before either one is released, this trap becomes a recombination centre. This process is called Shockley-Read-Hall (SRH) recombination. The net recombination rate can be described by [24]

\[
U_{SRH} = \frac{\sigma_n \sigma_p v_{th} N_T (np - n_i^2)}{\sigma_n (n + n_T) + \sigma_p (p + p_T)}
\]  
(2.32)

where \(n_T\) and \(p_T\) are the electrons and hole densities above \(E_T\), which can be described by Boltzmann approximation as

\[
n_T = n_i \exp((E_T - E_i)/kT)
\]

\[
p_T = n_i \exp((E_i - E_T)/kT)
\]  
(2.33)

The SRH recombination rate reaches maximum when \(E_T = E_i\). Assuming \(E_T \simeq E_i\), Eq. (2.32) reduces to

\[
U_{SRH} = \frac{\sigma_n \sigma_p v_{th} N_T (np - n_i^2)}{\sigma_n (n + n_i) + \sigma_p (p + p_i)}
\]  
(2.34)

At low injection level in a n-type semiconductor, Eq. (2.34) can be reduced to

\[
U_{SRH} \simeq \sigma_p v_{th} N_T \Delta p
\]  
(2.35)

by using the result of Eq. (2.27). It clearly shows that SRH recombination rate in a doped semiconductor is linear to the excess minority carrier density.

2.6 Analysis of p-n junction

2.6.1 Depletion approximation

The p-n junction that we considered in this section is illustrated in Figure (2.2). The device is symmetrical in the in-plane direction. The light comes from the side of p-type
Figure 2.2: An illustration of p-n junction. The junction interface is chosen as zero point. The coordinates of the surface, the interface between the neutral p-region and the depletion region, and the interface between the neutral n region and the depletion region are \(-x_p, -w_p\) and \(w_n\), respectively. The coordinates are chosen to follow the convention in Ref. [3].

A depletion region is formed near the interface of the p-type and n-type material. In the depletion region, the majority carriers diffuse to the other side of the junction, leaving the impurity atoms become charged. The interfaces between the neutral regions and the depletion region are assumed to be abrupt. The zero point of the coordinate is set to be the interface between p and n layer. The depletion width of p and n-layer are \(w_p\) and \(w_n\) respectively. The coordinate of the front surface is set to be \(x_p\). This follows the convention in Ref. [3].

The carrier and current densities in a semiconductor device are described by several
sets of equations [3,24]. The first one is the continuity equations:

\[
\frac{\partial n}{\partial t} = \frac{1}{q} \nabla \cdot J_n + G_n - U_n
\]

\[
\frac{\partial p}{\partial t} = \frac{1}{q} \nabla \cdot J_p + G_p - U_p
\]

(2.36)

The second set of equation is the drift-diffusion equations:

\[
J_n(r) = qD_n \nabla n + q\mu_n F_n
\]

\[
J_p(r) = qD_p \nabla p + q\mu_p F_p
\]

(2.37)

The third one is Poisson’s equation:

\[
\nabla^2 \psi = -\frac{\rho(r)}{\epsilon}
\]

(2.38)

where \(\psi\) is the electric potential, \(\rho(r)\) is the local charge density at \(r\), and \(\epsilon\) is the permittivity of the material.

The problem here is to solve these equations to get the I-V characteristics of the p-n junction structure. In general, the coupled differential equations Eq. (2.36), Eq. (2.37) and (2.38) can only be solved numerically. However, with some approximations, it is possible to obtain analytical solutions for the I-V characteristics. The approximations can be summarised as follows:

1. In the depletion region, the atoms of donors and acceptors are fully ionised, leaving the total charge densities in the depletion region equals to the doping density.

2. The quasi-Fermi level separation is constant throughout the p-n region.

3. No electric fields exist in the neutral regions.

4. Boltzmann approximation is valid in all regions.

To simplify Eq. (2.36), it is also assumed that the recombination rates \(U_n\) and \(U_p\) in neutral regions are linear to the minority carrier densities, as indicated by Eq. (2.28)
and Eq. (2.35). Moreover, the majority carrier concentrations in the neutral regions are assumed to be constant. The current in the neutral regions is only caused by the diffusion current of the minority carriers.

These assumptions dramatically simplify the coupled differential equations. With assumption 3, the neutral region can be completely decoupled from the depletion region and the other neutral region. Therefore, the n-type and p-type neutral regions and the depletion region can be treated separately to obtain the I-V characteristics:

**Neutral regions** For p-neutral region, substituting Eq. (2.37) into Eq. (2.36) and set \( F_n = 0 \) yields the differential equation for free electrons in p-region:

\[
D_n \frac{d^2 n}{dx^2} - U(n - n_0) + g(E, x) = 0 \tag{2.39}
\]

where \( g(E, x) \) is the carrier generation rate due to photons with energy \( E \), \( n_0 \) is the background concentration of the free electrons, and \( U(n - n_0) \) is the total recombination rate as the function of excess carrier density \( \Delta n = (n - n_0) \). Using Eq. (2.28) and Eq. (2.35), the total recombination rate \( U \) can be written as

\[
U(n - n_0) = \frac{(n - n_0)}{\tau_n} \tag{2.40}
\]

where \( \tau_n = 1/(B \Delta p N_D) + 1/(\sigma_n v_{th} N_t) \). Using the minority carrier diffusion length \( L_n = D_n \tau_n \), Eq. (2.39) can be rewritten as

\[
\frac{d^2 n}{dx^2} - \frac{(n - n_0)}{L_n} + \frac{g(E, x)}{D_n} = 0 \tag{2.41}
\]

With the boundary conditions at \(-w_p\),

\[
n(-w_p) - n_0 = \frac{n_i^2}{N_A} (\exp(qV/kT) - 1) \tag{2.42}
\]

and at \(-x_p\)

\[
D_n \frac{dn}{dx} = S_n(n(-x_p) - n_0) \tag{2.43}
\]
$g(E, x)$ is the photogeneration rate given in Eq. (2.20) and Eq. (2.22).

The concentration of free electrons $n(E, x)$ in Eq. (2.41) can be solved analytically. Substituting the solved $n(E, x)$ back into the drift-diffusion equation Eq. (2.37), the current density can be calculated by

$$j_n(E, x) = qD_n \frac{d^2 n(E, x)}{dx^2}$$

The hole current in n-type neutral region can be calculated in a similar way. Full solution these equations are given in Ref. [3].

**Depletion region** In some literature, the recombination current generated in depletion region is ignored. However, since solar cell generally has wide depletion region, carrier generation and recombination in the depletion region become very important. Also, the free carrier densities $n$ and $p$ are closer in depletion region, making SRH recombination become even more significant.

The current in depletion region can be derived by integrating the recombination rates within this region

$$J_{SCR} = q \int_{w_p}^{w_n} (U - g(E, x)) \, dx$$

where $U$ is the SRH recombination rate, and $g(E, x)$ is the carrier generation rate described in Eq. (2.20) and Eq. (2.22).

The carrier densities in the depletion region can be calculated using Boltzmann approximation, i.e.,

$$n(x) = n_i \exp \left( \frac{E_{F_n} - E_i(x)}{kT} \right)$$

$$p(x) = n_i \exp \left( \frac{E_{F_p} - E_i(x)}{kT} \right)$$

where $E_{F_n}$ and $E_{F_p}$ are quasi-Fermi level of electrons and holes in the depletion region respectively.
In summary, these assumptions work well when the carrier injection density is much lower than the doping density. As a result, when the solar cell is illuminated at intensities more than hundreds of suns or forward biased at high voltages, these assumptions would breakdown. Firstly, majority carrier flow in the neutral region can no longer be ignored. Moreover, the diffusion lengths will no longer be constant but will depends on the carrier concentrations. These results make the short circuit current of the solar cells at high injection be coupled with the recombination currents.

2.6.2 Approximations for p-i-n structure

A very lowly or unintentionally doped layer can be added between the p-type and n-type semiconductor to form a p-i-n structure. The i-region can be approximated as an extended depletion region and all the depletion approximations for the p-n junction can still be applied to p-i-n structures.

p-i-n structure is commonly used in solar cells because the carriers generated in the i-region has much higher probabilities to be collected as photocurrent due to the coverage of electric fields in this region. Therefore, materials with very short diffusion length, such as amorphous silicon or dilute nitrides, are usually fabricated as p-i-n structure to increase the photocurrent. However, since the SRH recombination is most active in the i-region, this structure also increases the recombination loss. State-of-the-art quantum well solar cells are usually fabricated as p-i-n diodes with quantum wells in the i-region to get more efficient carrier collection.

2.6.3 Modelling EQE with the results on depletion approximation

An important application is to use this result to analyse the external quantum efficiencies (EQE). EQE of solar cells are typically measured with low illumination intensity and low
bias voltages, making it suitable for applying the analysis provided in the previous section. Figure (2.3) presents the experimental and modelled EQE of a GaAs solar cell with 50 MQWs using the solutions of the differential equations discussed above. This calculation is performed by Dr. James Connolly using the software package Sol [4]. The parameters \( L_n, L_p, D_n, D_p, S_n \) and \( S_p \) are obtained by fitting the experimental EQE.

The absorption coefficient of GaAs for this fitting can be obtained from published data, but the absorption coefficients of MQWs are estimated using the experiment EQE result. Since Eq. (2.45) assumes that all the absorbed photons will eventually become photocurrent when \( U = 0 \). For quantum well solar cells, photons absorbed in the i-region has very small absorptivity in neutral region, thus Eq. (2.45) becomes

\[
J_{\text{SCR}}(E) = q \int_{-w_p}^{w_n} -g(E, x)dx \\
= \phi_0(E)(1 - r(E)) \exp(-\alpha_{\text{MQW}}(E)(x_p - w_p))(1 - \exp(-\alpha_{\text{MQW}}(E)(w_p + w_n)))
\]

(2.47)

Because \( EQE(E) = J_{\text{SCR}}(E)/[\phi_0(E)(1 - r(E))] \), which can be further rewritten as

\[
EQE(E) = 1 - \exp(-\alpha_{\text{MQW}}(E)(w_p + w_n))
\]

(2.48)

This results state that the quantum efficiency of photon energy \( E \) equals to the absorptivity of the MQW region. Therefore, the absorption coefficients of MQWs can be estimated using this empirical method.

Figure (2.3) shows that the modelled QE fits with the experiment data very well. The contributions of the EQE of each layer are also plotted in the figure. It clearly shows that most of the photocurrent are generated in emitter and the i-region. The base only contributes the photocurrent mostly in the wavelength region near the absorption edge of the band gap.
Figure 2.3: Experimental and modelled EQE of a p-i-n GaAs solar cell with 50 quantum wells in the i-regions. This graph is reproduced from Figure 1 of Ref. [4]. Black dots indicate the measured EQE of a p-i-n GaAs solar cell. The red line shows the modelled EQE calculated by solving Eq. (2.41). Full analytical solution of these differential equations are listed in Section 6.5 of Ref. [3]. Blue, green and black dashed lines are the modelled EQE of neutral p-region, depletion region and neutral n-region, respectively.
2.6.4 Dark I-V characteristics

An important special case of the Eq. (2.41) and Eq. (2.45) is \( g(E, x) = 0 \), in which case the solved I-V only has the contribution of recombination current. The solution for the diffusion current generated in neutral regions is

\[
J_{\text{diff}}(V) = \left( \frac{qD_p n_i^2}{L_p N_D} + \frac{qD_n n_i^2}{L_n N_A} \right) \exp \left( \frac{qV}{kT} \right) \tag{2.49}
\]

which is also known as Shockley injection current.

Recombination current in the depletion region can be calculated from Eq. (2.45) and Eq. (2.46), which can be simplified by assuming \( E_T = E_i \) and \( \sigma_n = \sigma_p = \sigma \). Eq. (2.45) then becomes

\[
J_{\text{SCR}}(V) = q \int_{-w_p}^{w_n} U(x) \, dx
\]

\[
= q \int_{-w_p}^{w_n} \sigma v_{th} N_T (np - n_i^2) \, dx
\]

\[
= q \int_{-w_p}^{w_n} \sigma v_{th} N_T n_i^2 (\exp(qV/kT) - 1) + \exp(E_{Fn} - E_i/kT) + \exp(E_{Fp} - E_i/kT) + 2 \, dx \tag{2.50}
\]

The integrand reaches maximum when \( E_i = (E_{Fn} - E_{Fp})/2 = qV/2 \). Assuming \( qV \gg kT \), the maximum of \( U(x) \) can be reduces to

\[
U_{\text{max}}(x) = \sigma v_{th} N_T n_i \left( \exp \left( \frac{qV}{2kT} \right) \right) \tag{2.51}
\]

Assuming that half of the depletion width satisfies \( E_i \simeq qV/2 \), the integral Eq. (2.50) can be estimated as

\[
J_{\text{SCR}}(V) \simeq \frac{1}{2} (w_n + w_p) v_{th} N_T n_i \left( \exp \left( \frac{qV}{2kT} \right) \right) \tag{2.52}
\]

Therefore, the total recombination current in a p-n structure has the form:

\[
J_{\text{rec}}(V) = J_{\text{diff},0} \exp(qV/kT) + J_{\text{scr},0} \exp(qV/2kT) \tag{2.53}
\]
or in a more compact form:

\[ J_{\text{rec}}(V) = \sum_i J_{0i} \exp(qV/\eta_i kT) \]  \hspace{1cm} (2.54)

\( \eta_i \) is usually called ideality factor.

Figure (2.4) shows the experiment and modelled dark I-V characteristics of the same device plotted in Figure (2.3). The SRH recombination current is obtained by fitting the \( n = 2 \) part of the experiment dark I-V. The Shockley injection current is calculated using the \( L_n, L_p, D_n \) and \( D_p \).

Dark-IV characteristics can be used to study the external radiative efficiency of a solar cell, which is done by separating the radiative recombination current from the total recombination current. In some texts, the term with \( \eta_i = 1 \) is attributed to radiative recombination term and \( \eta_i = 2 \) to non-radiative recombination term, but this is not precise. As discussed when deriving Eq. (2.41), the recombination term can have contributions from both radiative and non-radiative recombination. As a result, to calculate modelled external radiative efficiency, it is necessary to separate the radiative recombination current from the total Shockley injection current.

In Figure (2.4), the radiative recombination current is calculated by Eq. (2.6). Note that the levels of the non-radiative Shockley injection current and the radiative recombination current are close, so they almost overlap in the figure. The external radiative efficiency, defined as the ratio of the radiative current and total current, is plotted using the second y-axis.

### 2.7 Equivalent circuit model for solar cells

Using the results in the previous section, a solar cell can be modelled by a equivalent circuit model illustrated in Figure (2.5). Apart from the equivalent diodes and the current source, the series resistance \( R_s \) and shunt resistance \( R_{sh} \) are also included. The current-voltage...
Figure 2.4: Experiment and modelled dark-IV of the same sample as plotted in Figure (2.3). The graph is reproduced from Figure 3 in Ref. [4]. The purple dashed line plots the radiative current calculated by Eq. (2.6). The blue line plots the Shockley-Read-Hall recombination current calculated by Eq. (2.52). The green broken line indicates the calculated the ideal Shockley injection current using Eq. (2.49) subtracts the calculated radiative current using Eq. (2.6). The red line plots the total recombination current, which is the sum of different recombination current components plotted in broken lines. The black dotted line is the experimental dark I-V characteristics.
Figure 2.5: The equivalent circuit model of a solar cell. This model includes two diodes that models the Shockley and SRH recombination currents, a current source, a lumped series resistor and a shunt resistor.

Relation is given by this implicit equation:

\[
J(V) = J_{01} \left( \exp \left( \frac{q(V - JR_s)}{kT} \right) - 1 \right) + J_{02} \left( \left( \exp \frac{q(V - JR_s)}{2kT} \right) - 1 \right) - J_{sc} + \frac{V - JR_s}{R_{sh}} \tag{2.55}
\]

Monolithic multi-junction solar cell can be modelled by connecting this equivalent circuit model in series. A couple of examples of using this model to solve multi-junction I-Vs will be presented in Section 2.15.

2.8 Fabrication of solar cells

The samples studied in this thesis were all grown by metalorganic chemical vapour deposition (MOCVD), also known as organometalllic vapour phase epitaxy (OMVPE) or
metalorganic vapour phase epitaxy (MOVPE) [25]. MOCVD is currently the standard of growing III-V compound semiconductors in the industry. MOCVD uses the surface reactions of chemical compounds containing the required elements, known as precursors, to deposit the epitaxy layer of desired composition and doping. The major advantage of this method over other epitaxy approaches is that it has high growth rate and the low maintenance cost, making it very suitable for mass production.

The front contacts and anti-reflection coatings are patterned using ultra-violet photolithography and etching [26]. The width of metal grids on the solar cell is usually around tens of microns and the pitches between the metal grids are usually around a couple of microns. The anti-reflection coating are deposited using plasma-enhanced chemical vapour deposition (PECVD).

The samples presented in this thesis were fabricated by EPSRC National Center for III-V Technologies in Sheffield, and QuantaSol Limited.

### 2.9 X-ray diffraction

X-Ray diffraction (XRD) is a very powerful method for investigating the crystal structure. Since the intensity and direction of the X-ray is sensitive to the crystal structure and the species of atoms, it is a very important tool for calibrating the thicknesses and compositions of epitaxial layers.

The principle of XRD measurement is illustrated in Figure (2.6). $k_{in}$ is the wavevector of the incoming X-ray, $k_{out}$ is the wavevector of diffracted x-ray beam. Constructive interference of diffracted x-ray beam forms when the phase-matching condition is met:

$$k_{in} + \Delta k = k_{out} \quad (2.56)$$

where $\Delta k$ is equals to the reciprocal lattice vector of the crystal. Taking the square on the both sides of Eq. (2.56) and using $|k_{in}|^2 = |k_{out}|^2$, Eq. (2.56) can then be reduced to

$$2k_{in} \cdot \Delta k + (\Delta k)^2 = 0 \quad (2.57)$$
Figure 2.6: An illustration of XRD measurement. $k_{\text{in}}$ is the wavevector of incoming probe beam, and $k_{\text{out}}$ is the diffracted beam. $Q_x$ and $Q_z$ are the basis vectors in the reciprocal space.

Since $k_{\text{in}} = 2n_d\pi/\lambda$ and $\Delta k = 2\pi/d$, where $\lambda$ is the wavelength of X-ray, $d$ is the lattice period, and $n$ is the refractive index, the above equation then can be reduced to Bragg’s condition:

$$2d\sin \theta = n_d\lambda$$

(2.58)

where $2\theta$ is the angle between $k_{\text{in}}$ and $k_{\text{out}}$, as illustrated in Figure (2.6).

For convenience, the basis vectors of a two-dimensional reciprocal space can be defined as

$$Q_x = \frac{2\pi}{\lambda} [\cos(\omega) - \cos(2\theta - \omega)]$$

$$Q_z = \frac{2\pi}{\lambda} [\sin(\omega) + \sin(2\theta - \omega)]$$

(2.59)

Note that the basis vector $Q_z$ is chosen to be parallel to the reciprocal lattice vector $\Delta k$.

Two types of XRD measurement are performed in this thesis. The first one is measuring the intensity XRD over a wide range of both $\omega$ and $\theta$ to construct a two-dimensional
map, which is called reciprocal space map (RSM). Another measurement is a one-dimensional scan by keeping \( \delta 2\theta = \delta \omega \), which is usually called XRD rocking curves. This one-dimensional scan keeps \( Q_x \) in Eq. (2.59) constant in order to study the XRD intensity as the function of only \( Q_z \).

2.10 Steady-state photoluminescence

Steady-state photoluminescence (PL) experiment measures the emission of a sample illuminated by continuous light. The test sample can then be considered to be in thermal equilibrium. As discussed in section 2.1, the PL of a semiconductor sample \( \phi_{ext}(E) \) can be described by detailed balance model, i.e.,

\[
\phi_{ext}(E) \propto \frac{a(E)\eta_{ext}(E)E^2}{\exp\left(\frac{E}{kT}\right) - 1}
\] (2.60)

Temperature dependent PL can be used to assess the internal radiative efficiency of a material. At very low temperature, the non-radiative recombination of carriers becomes less active due to smaller thermal velocity \( v_{th} \) and carrier capture rate in Eq. (2.29) and Eq. (2.30). Assuming that the internal radiative efficiency is 100% at very low temperature and the optical loss of the PL emissions in the sample are the same at all temperatures, the internal quantum efficiency of the test sample at temperature \( T \) is given by

\[
\eta_{int}(T) = \frac{\phi_{ext}(T)}{\phi_{ext}(T_L)}
\] (2.61)

where \( T_L \) is a temperature that sample has \( \eta_{int} = 1 \).

Another application of temperature dependent PL is analysing thermal activation energy of the carrier loss mechanism in quantum wells [27, 28]. Energy states in quantum wells can be understood as a two-level model illustrated in Figure (2.7). The ground state of the barrier in a quantum well can be considered as the higher energy state \( E_C \), and the ground state in the well can be considered as the lower energy state \( E_T \). The carriers are injected into the state \( E_C \) with the pumping rate \( P \). Carriers at the state \( E_C \)
Figure 2.7: A diagram that illustrates the capture, emission and recombination processes between the quantum well state $E_t$ and the barrier state $E_c$. At equilibrium, the net increase of carrier in energy state $E_T$ or $E_C$ due to the pumping rate $P$, recombination rate $R$ and $B$, capture and re-emission rate $c_n$ and $c_n \exp(-E_A/kT)$, is zero.

Can either be captured by the quantum well state $E_T$ or recombine with the rate $R'$. $R'$ can be radiative or non-radiative. The carriers captured by the state $E_T$ are assumed to either radiately recombine at a rate $B$, or re-emit back to the state $E_C$. When this system reaches equilibrium, we have $\frac{dn}{dt} = 0$ and $\frac{dn_t}{dt} = 0$. Using capture and emission rate given in Eq. (2.29) and Eq. (2.31), $\frac{dn}{dt}$ and $\frac{dn_t}{dt}$ are given by

$$\frac{dn}{dt} = c_n n_T \exp(-E_A/kT) - c_n n - R'n + P = 0$$

$$\frac{dn_t}{dt} = c_n n - c_n n_T \exp(-E_A/kT) - B n_T = 0$$

(2.62)

where $E_A = E_C - E_T$. Solving the above equations for $Bn_t$ yields the PL intensity [27]

$$I(T) = Bn_t = P \left[ 1 + \exp\left( -\frac{E_A}{kT} \right) \frac{c_n R'}{B c_n + \frac{R'}{B}} \right]^{-1}$$

(2.63)

As a result, temperature dependent PL intensity can be used to analyse the confinement potential carrier potential $E_A$ if non-radiative recombination in the quantum well is negligible.
2.11 Electroluminescence

Electroluminescence (EL) is generated by injecting the carriers into the sample electrically. Similar to PL, the EL intensity is given by

\[
\Phi_{EL}(V,E) \propto \frac{a(E)\eta_{ext}(E)E^2}{\exp\left(\frac{E-qV}{kT}\right) - 1}dE
\]

(2.64)

The spatial distribution of EL of solar cells can be used as a quick test to inspect the uniformity of materials. In solar cells, the EL intensity distribution are related to the sheet resistance of the emitter layer. More details of this technique will be discussed in Chapter 6.

2.12 Time-resolved photoluminescence decay

Time-resolved photoluminescence (TRPL) decay measures the decay of PL versus time after the carrier is injected into the sample in a very short duration, which is reused to analyse the recombination rate of minority carriers in a material.

TRPL decay measures the photons emitted from the sample \( \Phi(t) \):

\[
\Phi(t) = \int_A \phi(r,t)dr
\]

(2.65)

The sheet density of photons \( \phi(r,t) \) is can be related to the local excess carrier density \( n(r,t) \) by

\[
\phi(r,t) = B(n(r,t)(N_D + n(r,t))
\]

(2.66)

In the limit of \( N_D \gg n(r,t) \), substituting Eq. (2.66) into Eq. (2.65) gives

\[
\Phi(t) = \int_A BN_Dn(r,t)dr
\]

(2.67)

The excess carrier density \( n(r,t) \) can be described by the rate equation

\[
\frac{dn(r,t)}{dt} = -An(r,t) - Bn^2(r,t)
\]

(2.68)
With $A \gg Bn(r, t)$, the nonlinear terms in Eq. (2.66) and Eq. (2.68) can be ignored, $\Phi(t)$ then becomes

$$\Phi(t) = \int_A^B B D n(r, t) \, dr$$

$$= \int_A^B B D n_0(r) \exp(-At) \, dr$$

(2.69)

where $n_0(r)$ is the initial excess carrier density at $r$. As a result, the measured decay rate of $\Phi(t)$ is the same as the excess carrier decay rate if the linear term in Eq. (2.68) dominates the rate equation.

However, if the nonlinear term in Eq. (2.68) dominates, the decay rate of $\Phi(t)$ is no longer equal to the excess carrier decay rate, because the excess carrier decay rate $A'$ is then a function of carrier density $n(r, t)$. Eq. (2.69) can then be expressed as

$$\Phi(t) = \int_r B D n_0(r) \exp(-A'(n(r, t))t) \, dr$$

(2.70)

Therefore, the excess carrier injection profile must be taken into account when retrieving the recombination coefficients $A$ and $B$ from $\Phi(t)$.

The TRPL decay measurement is implemented by time-correlated single-photon counting technique. The sample is injected by a very short laser pulse with pulse width around $\sim 200$ ps. The emission from the sample is kept at an intensity level that the probability of receiving one photon per laser pulse is less than unity. The measurement electronics of the system measures the elapsed time $t$ between the laser pulse and the detector receives a photon. This process is repeated many times to build a histogram $N(t)$, where $N(t)$ is the number of photons received at elapsed time $t$.

2.13 Measuring I-V characteristics and efficiency of solar cells

I-V characteristics is the most fundamental measurement for solar cells. The I-V characteristics is usually measured by commercial current-voltage sourcemeter, e.g. Keithley
The sourcemeter applies the designated current or voltage to the test device, and measures the responded voltage or current of the device. I-V measurements can be performed when the cell is either in the dark or illuminated, which are usually called dark I-V and light I-V respectively.

The conversion efficiency of a solar cell is determined by measuring its light-IV under standard illumination spectrum. ASTM G173-03 is commonly used for testing concentrator solar cells [29]. The efficiency is defined by the ratio of maximum electrical power output and the input power of light illuminated on the test device, namely,

\[
\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{\text{max}(I \cdot V)}{\phi \cdot A}
\]

\( P_{\text{out}} \) is the maximum power point of the I-V of the test device, given by the maximum of \( I \cdot V \). \( \phi \) is the illumination intensity and \( A \) is the illumination area of the device, which usually refers to the total mesa area of the device, including the bus bars and metal fingers covered on the device. An example of light I-V and its power-voltage curve is plotted in Figure (2.8).

To test the concentrator solar cells, the illumination intensity needs to be raised to a few hundred suns to one thousand suns. This is usually achieved by using concentrating optics or flash lamps. More details of this measurement will be explained in Chapter 5.

Another useful figure of merit for solar cells is fill factor. Fill factor is defined by the ratio of maximum power point and the product of open circuit voltage \( V_{oc} \) and short-circuit current \( I_{sc} \):

\[
\text{FF} = \frac{\text{max}(I \cdot V)}{I_{sc}V_{oc}}
\]

Fill factor is significantly affected by the parasitic resistances \( R_s \) and \( R_{sh} \) in the device. In multi-junction devices, fill factor is also affected by the extent of current mismatch between the subcells.
2.14 Transmission line measurement

Transmission line measurement (TLM) is a standard method for measuring the sheet resistance and contact resistivity. Contact resistivity is the resistance of the ohmic contact formed between the semiconductor and the metal, which is usually defined as [30]

$$\rho = R \frac{Wd}{L} \quad (\Omega \cdot \text{[Length]})$$

(2.73)

where $R$ is the total resistance, $L$ is the length of the material, and the product of $W$ and $d$ is the total cross-section area of the current flow. This is depicted in Figure (2.9).

The resistance of the emitter layer is usually expressed as sheet resistance, which is defined as [30]

$$R_{\square} = R \frac{W}{L} \quad (\Omega/\square)$$

(2.74)

As illustrated in Figure (2.10), parallel metal grids are fabricated on the test material. The resistance of the material between the neighbouring metal grids are obtained by
probing the pairing metal grids and measuring its I-V. By Ohm’s law, the measured I-V on the i-th pair metal contacts can be written as

\[ V^i = I^i (R_c + R_i^i) \]  \hspace{1cm} (2.75)

where \( R_c \) is the total contact resistance and \( R_i^i \) is the total resistance of pitch width \( L_i \).

By Eq. (2.74), this is

\[ R_{i}^i = R_{i}^{\text{res}} \frac{L_i}{W} \]  \hspace{1cm} (2.76)

The measured total resistance \((R_c + R_i^i)\) can be calculated from each I-V measurements and be plotted against \(L_i\) as shown in Figure (2.10). Assuming that the contact resistances seen by each pair of metal contacts are identical and using Eq. (2.76), it clearly shows that the intercept of this plot is \(R_c\) and the slope is \(R_{i}^{\text{res}}/W\).

### 2.15 Spectral response measurement

A spectral response measurement setup can measure the physical responses of a sample against the illumination at a specific wavelength, including external quantum efficiency, reflectance, and transmission. The test sample is illuminated by a monochromatic beam, which comes out from a white light source and filtered by a monochromator. The electrical or optical response of the sample is then measured by an appropriate technique depending
Figure 2.10: An illustration of the working principles of TLM. The resistances of L1, L2 and L3 are measured by the I-V characteristics, and then the measured resistances are plotted again the widths of L1, L2 and L3. The contact and sheet resistances can then be obtained from the slope and intercept of the fitted line.

on the level of signal. Since the power of the monochromatic beam is typically within the range between nanowatts to microwatts, lock-in amplification thus becomes the standard of measuring spectral responses.

Lock-in amplification is essentially an amplitude modulation method to improve the signal-to-noise ratio of the measurement. The monochromatic beam for illuminating the sample is modulated by a chopper wheel, and the measured signal is demodulated by a lock-in amplifier.

The schematic diagram of this setup is illustrated in Figure (2.11). The light module has two light sources, an xenon light bulb and a halogen light bulb. The wavelengths of the light are selected by a Bentham monochromator. The beam coming out of the monochromator is relayed to the sample by a pair of lenses or off-axis parabolic mirrors. The photocurrent from the test sample or a photodetector is measured through a Signal Recovery #7230 lock-in amplifier. The entire measurement is automated using the programming interface provided by the manufacturers of the lock-in amplifier and the
monochromator. A windows software for driving this experiment was implemented using C# programming language. A couple of screenshots of this software are provided in Figure (2.12).

2.15.1 Measuring external quantum efficiency of single junction devices

External quantum efficiency measures the photocurrent per illuminated photons. This measurement has two parts. The first part is measuring the photocurrent of the device-under-test (DUT) $I_{DUT}(\lambda)$ against different wavelengths $\lambda$. The second part is measuring the photocurrent $I_{REF}(\lambda)$ of a calibrated photodetector under the same illumination. With the known external quantum efficiency of the photodetector $\eta_{REF}(\lambda)$. By assuming the external quantum efficiencies of the DUT and the calibrated photodetector are independent of illumination intensity, the external quantum efficiency $\eta_{DUT}(\lambda)$ of the DUT is given by

$$EQE_{DUT}(\lambda) = \frac{I_{DUT}(\lambda)}{I_{REF}(\lambda)} EQE_{REF}(\lambda)$$  \hspace{1cm} (2.77)

2.15.2 Measuring quantum efficiency of multi-junction devices

Measuring the external quantum efficiency of a subcell in a multi-junction device accurately can be very difficult, because more than one cell are connected in series in a device
Figure 2.12: Screenshots of the control software that automates the spectral response measurement.
but only two contacts are available for measuring a subcell. However, if the subcells behave as an ideal diode, measuring the external quantum efficiencies of individual diodes is possible. Principles of measuring the EQE of a subcell in a multi-junction device will be explained in this section and more detailed discussions on this topic can be found in Ref. [31,32].

**EQE without shunted subcells**

Here we consider a multi-junction device in which all the subcells behave as ideal diodes. From Eq. (2.54), the dark-IV of the subcells can be written as

\[
J_{tot}^i(V) = \sum_m J_{0m}^i \exp \left( \frac{qV}{\eta_m kT} \right)
\]

(2.78)

where \( J_{tot}^i \) is the current of the i-th junction in the device, and \( \eta \) is the ideality factor. When this multi-junction device is illuminated with a modulated monochromatic light, ideally only the \( k \)-th subcell will have photocurrent \( j_{sc}^k \). The I-V of the \( k \)-th subcell is given by

\[
J_{tot}^k(V) = \sum_m J_{0m}^k \exp \left( \frac{qV}{\eta_m kT} \right) - j_{sc}^k
\]

(2.79)

Let us denote \( J_{tot}^{all} \) as the current of this multi-junction device. To be able to measure \( j_{sc}^k \), we need to find the operating voltages for each subcell such that

\[
J_{tot}^{all} = J_{sc}^i (V^i) = j_{sc}^k
\]

(2.80)

for each subcell \( i \). However, \( J_{tot}^i \) can only be positive in the dark because it is a sum of exponential functions. Therefore, \( j_{sc}^k \) cannot be measured if every subcell other than subcell \( k \) is kept in the dark.

To resolve this, each subcell except subcell \( k \) is illuminated with a stronger continuous light in order to get a photocurrent term \( J_{sc}^i \) such that \( J_{sc}^i \gg j_{sc}^k \). The I-V of the illuminated subcells become

\[
J_{tot}^i = \sum_m J_{0m}^i \exp \left( \frac{qV^i}{\eta_m kT} \right) - J_{sc}^i
\]

(2.81)
It is now possible to reach a solution for Eq. (2.80) because the subcell $k$ is the current-limiting junction in this device.

A simulation and an illustration of this process is presented in Figure (2.13). This simulation considers a dual-junction device and uses Eq. (2.78) to model each subcell. Figure (2.13)(a) simulates the case that only subcell 1 is illuminated with a weak monochromatic beam which generates $J_{sc}^1 = 0.1 \text{A/cm}^2$. The I-V of the whole device is plotted as the red line in Figure (2.13) (a), showing that the photocurrent is nearly zero for this dual-junction device. Figure (2.13)(b) simulates the case that subcell 2 illuminated with larger intensity of light that gives $J_{sc}^2 = 5 \text{A/cm}^2$, and subcell 1 still has the photocurrent of 0.1 A/cm$^2$. The I-V of the dual-junction devices shows that the photocurrent generated in subcell 2 can now be measured.

**EQE with shunted subcells**

Measuring the quantum efficiency of a subcell in a multi-junction device becomes more complicated when one of the subcells has low shunt resistance. In this case, an additional shunt resistance term is added into the diode equation, making Eq. (2.78) becomes

$$J_{tot}^i(V^i) = \sum_m J_{0m}^i \exp \left( \frac{qV^i}{\eta m kT} \right) + \frac{V^i}{R_p^i}$$  \hspace{1cm} (2.82)

Eq. (2.82) shows that it is possible to get negative values of $J_{tot}^i$ without having a photocurrent term. This couples measured photocurrent of a subcell with the spectral response of other subcells. Figure (2.15)(a) illustrates an example of this measurement artefact. In this example, the goal is to measure the EQE of subcell 2 in a dual-junction device. Subcell 2 is a junction with low shunt resistance. When this dual-junction device is illuminated by photons with wavelengths that can generate photocurrent in subcell 1 but not in subcell 2, this photocurrent will be blocked by subcell 2 if it has high shunt resistance. However, with low shunt resistance in subcell 2, the photocurrent generated by other subcells will be measured, which causes measurement artefacts. Figure (2.16) shows some...
Figure 2.13: Simulated I-V characteristics of a dual-junction device and its individual subcells with (a) only subcell 1 illuminated with weak light, and (b) subcell 1 and 2 illuminated. In (a), since subcell #2 blocks the photocurrent generated in subcell #1, the total current of the dual-junction device is negligible. In (b), subcell #2 is illuminated by a much stronger light bias, the total short-circuit current of the dual-junction device is equal to the short-circuit current of subcell #1, since subcell #1 is the current-limiting junction.
examples of this measurement artefacts. It shows that the shunt resistance in a subcell can lower the measured EQE in the wavelength region that the subcell is supposed to have response and increase the measured EQE that the subcell is not supposed to have response.

This measurement artefacts can be suppressed by applying a forward bias to the multi-junction device. As shown in Figure (2.15)(b), the applied forward bias can offset the voltage across subcell 2 so that the photocurrent generated in subcell 1 cannot flow through subcell 2.

This idea can be illustrated by a simulation of a dual-junction device that includes the shunt resistance in subcell 2. This simulation compares two sets of parameters. One with shunted subcell 2 and the other has no shunted subcells. Both simulation has subcell 1 illuminated and subcell 2 in the dark. The goal is to measure the EQE of subcell 2, and thus the photocurrent generated by subcell 1 should not be collected. Details of all the parameters are listed in Table 2.1 and Table 2.2.

Figure (2.14) (a) and (b) shows the calculated I-Vs of each subcell and the whole device. To compare the differences of these two graphs, the I-V of the dual-junction device between -0.5 V to 0.5 V are plotted in Figure (2.14) (c) and (d). It clearly shows that $J_{all}^{sc} \sim 0.1J_{sc}^1$ if subcell 2 is shunted, while $J_{all}^{sc}$ is negligible when subcell 2 is not shunted.

Figure (2.14) (e) and (f) shows the difference of $J_{tot}^{all}$ between the monochromatic probe beam turning on and off, namely $J_{tot}^{all}(\text{probe on}) - J_{tot}^{all}(\text{probe off})$. Since the monochromatic beam is modulated in the experiment, this calculation simulates the result that is measured by the lock-in amplifier versus a range of device bias. It shows that almost no current can be measured between -0.5 V to 0.5 V if subcell 2 is not shunted. With shunted subcell 2, significant photocurrent can be measured be this can be minimised by tuning forward bias to around 0.7 V, which is the voltage required to stop the photocurrent flowing through subcell 2.
Figure 2.14: The simulated I-Vs of dual-junction devices. The left column are the results using the parameters in Table 2.1 and the right column are the results using the parameters in Table 2.2. (a) and (b) are the I-Vs of simulated I-Vs of subcell 1 (J1), subcell 2 (J2) and the dual-junction device (2J). (c) and (d) shows the I-V of 2J between -0.5 V to 0.5 V. (e) and (f) shows the difference of I-V between the illumination on and off.
Figure 2.15: An illustration of how the external bias can suppress the measurement artefacts caused by the shunt resistance of the a subcell. When subcell #1 is illuminated and subcell #2 is kept in the dark, the current can leak through the parasitic resistance of subcell #2, as depicted in (a). An additional bias can be added to offset the voltage across the resistance in order to suppress the leakage current, which is illustrated in (b).
Table 2.1: Details of simulation parameters (no shunt)

<table>
<thead>
<tr>
<th>subcell</th>
<th>( J^i_{01} )</th>
<th>( J^i_{02} )</th>
<th>( R^i_p )</th>
<th>( J^i_{ph} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>subcell #1 (i=1)</td>
<td>( 2.2 \times 10^{-18} )</td>
<td>( 1 \times 10^{-10} )</td>
<td>( 1 \times 10^6 )</td>
<td>0.2</td>
</tr>
<tr>
<td>subcell #2 (i=2)</td>
<td>( 9.6 \times 10^{-6} )</td>
<td>( 7.9 \times 10^{-5} )</td>
<td>( 1 \times 10^6 )</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2.2: Details of simulation parameters (shunted)

<table>
<thead>
<tr>
<th>subcell</th>
<th>( J^i_{01} )</th>
<th>( J^i_{02} )</th>
<th>( R^i_p )</th>
<th>( J^i_{ph} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>subcell #1 (i=1)</td>
<td>( 2.2 \times 10^{-18} )</td>
<td>( 1 \times 10^{-10} )</td>
<td>( 1 \times 10^6 )</td>
<td>0.2</td>
</tr>
<tr>
<td>subcell #2 (i=2)</td>
<td>( 9.6 \times 10^{-6} )</td>
<td>( 7.9 \times 10^{-5} )</td>
<td>( 500 )</td>
<td>0</td>
</tr>
</tbody>
</table>

2.15.3 Experimental results of triple-junction solar cells

As discussed in the previous section, the sample needs to be light biased and electrical biased to measure the EQEs of the subcells in a multi-junction device. In this spectral response measurement setup, the light bias is provided by LEDs with three different wavelengths for light biasing each junction of a standard InGaP/InGaAs/Ge triple-junction device. The wavelengths of these LEDs are 650 nm, 850 nm, and 1.3 um.

A electrical circuit box was designed and implemented to use the analogue voltage outputs to control the device bias and the light intensities of the LEDs. Since the voltages of the analogue output ports of the lock-in amplifier can also be programmed by the software, the parameters for controlling device bias and light bias can all be controlled through the software. The circuitry for controlling the device bias is depicted in Figure (B.2) in the Appendix.

To measure the EQE of a subcell in a triple-junction device, it is required to search the best parameter set of the device and light bias that minimise the photocurrent in the wavelength region not expected to have response and maximize the photocurrent in the region that should have response. The optimised device bias is usually the near the sum...
of open-circuit voltages of the other two subcells, and the light bias should be as high as possible.

Figure (2.16) presents the measured EQE of a triple-junction InGaP/InGaAs/Ge device (2404-5-trip-x11y10) at various device bias and light bias conditions. The numbers in the legends labels the bias conditions listed in Table 2.3 and Table 2.4. These table lists the applied electrical bias and the applied current of the LEDs for relative comparisons of light-biasing intensity. These results show that the measurement artefact can be suppressed by selecting appropriate device bias and light bias. Although the EQE of the triple-junction device is dependent on the bias conditions, there is usually a small region in the parameter space that has minimised measurement artefact and very close EQE levels. As shown in the table and the EQE plots, the different sets of parameters result in minimised measurement artefacts and overlapped EQEs. This property makes the random error cause by different operators or the selection of parameter very small.
Figure 2.16: Measured external quantum efficiencies of (a) top cell, (b) middle cell and (c) bottom cell in the triple-junction sample 2404-5-trip-x11y10. The numbers in the legend correspond to the number that identifies the conditions in Table 2.3 2.4, and 2.5.
Table 2.3: Bias conditions of measured top cell EQE in Figure (2.16) (a).

<table>
<thead>
<tr>
<th>condition</th>
<th>device bias (V)</th>
<th>LED (650 nm)</th>
<th>LED (850 nm)</th>
<th>LED (1.3 µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>200 mA</td>
<td>200 mA</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
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</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0</td>
<td>50 mA</td>
<td>200 mA</td>
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</tbody>
</table>

Table 2.4: Bias conditions of measured middle cell EQE in Figure (2.16) (b)

<table>
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<th>condition</th>
<th>device bias (V)</th>
<th>LED (650 nm)</th>
<th>LED (850 nm)</th>
<th>LED (1.3 µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>200 mA</td>
<td>0</td>
<td>10 mA</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>200 mA</td>
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<td>200 mA</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>200 mA</td>
<td>0</td>
<td>100 mA</td>
</tr>
</tbody>
</table>

Table 2.5: Bias conditions of measured bottom cell EQE in Figure (2.16) (c).

<table>
<thead>
<tr>
<th>condition</th>
<th>device bias (V)</th>
<th>LED (650 nm)</th>
<th>LED (850 nm)</th>
<th>LED (1.3 µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5</td>
<td>200 mA</td>
<td>200 mA</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>50 mA</td>
<td>50 mA</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1.8</td>
<td>200 mA</td>
<td>200 mA</td>
<td>0</td>
</tr>
</tbody>
</table>
Chapter 3

InGaP/InGaAsP Multiple Quantum Wells

The efficiency of state-of-the-art InGaP/GaAs/Ge triple junction solar cells can be further improved by lowering the band gaps of the top two junction because the Ge junction absorbs much more photons than the top two junctions. A novel InGaP/InGaAsP quantum well for tuning the absorption edge of the top cell has been patented and implemented in commercial triple-junction solar cells [33]. JDSU has demonstrated that triple junction solar cells with dual-MQWs can achieve over 41% efficiency at 500 suns [15]. Studies of this MQW structure were also reported in several publications [34] [35] [36], mainly through EQE, I-V characteristics and detailed balance calculations. Other material properties such as crystal structures or carrier recombination dynamics have not been reported yet.

In this chapter, several material properties of InGaP/InGaAsP are presented for the first time. RSMs and XRD rocking curves measured on this MQW structure will be reported in the first part of this chapter, followed by the carrier recombination dynamics studied via analysing temperature dependent PL and TRPL decays. After that, the EQE and I-V result of a dual-junction dual-MQW device will be presented.
3.1 InGaAsP as the well material for MQWs

InGaAsP lattice-matched to GaAs has been used in semiconductor lasers as an aluminium-free option to generate laser at wavelengths from 730 nm to 808 nm [37–40] as the active layer. The main advantage of $\text{In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$ as the well material for top cell is that one can change both the compositions $x$ and $y$ to tune the band gap while maintaining a fixed lattice constant.

Figure (3.1) (a) and (b) shows band gaps and lattice constants of $\text{In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$ as the function of Ga and As compositions respectively. The graphs are approximated by interpolating [41] the known parameters of GaAs, InP, InAs, and GaP using

$$Q(x, y) = (1 - x)yB_{\text{InAs}} + xyB_{\text{GaAs}} + (1 - x)(1 - y)B_{\text{InP}} + x(1 - y)B_{\text{GaP}}$$  \hspace{1cm} (3.1)

where $B_{\text{InAs}}$, $B_{\text{GaAs}}$, $B_{\text{InP}}$ and $B_{\text{GaP}}$ are the parameters of InAs, GaAs, InP, and GaP listed in Table 3.1. The dashed line in Figure (3.1) (a) and (c) represent the combination of $x$ and $y$ that keeps its lattice constant equal to GaAs (5.6 Å). This line in Figure (3.1) (a) shows that the band gap of $\text{In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y}$ can be varied from 1.45 eV to 1.9 eV while keeping the lattice constant equal to GaAs. Figure (3.1) shows that this line almost overlaps with the contour of 4.1 eV electron affinity, which is very close to the electron affinity of GaAs and InGaP. This makes the well of the electrons in InGaP/InGaAsP QW very shallow.

As discussed in Section 1.3, the detailed balance model predicts that the best band gap combination of top and middle cells for triple junction solar cells with Ge as the bottom cell is 1.7 eV and 1.2 eV. The dashed green line on Figure (3.1) shows that the desired As composition falls in the range between 0 to 0.6. However, it was reported both theoretically and experimentally that the InGaAsP composition is thermodynamically unstable and easily separates to a GaP-phase and InAs-phase, which makes it difficult to grow this material in bulk form [42]. However, since epitaxial growth takes place far from
Table 3.1: The material parameters of the binary materials for estimating the band gaps, electron affinities and lattice constants of InGaAsP

<table>
<thead>
<tr>
<th>Material</th>
<th>Band Gap (eV)</th>
<th>Lattice Constants (Å)</th>
<th>Electron Affinity (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GaP</td>
<td>2.776</td>
<td>5.4512</td>
<td>3.8</td>
</tr>
<tr>
<td>GaAs</td>
<td>1.415</td>
<td>5.6533</td>
<td>4.07</td>
</tr>
<tr>
<td>InP</td>
<td>1.35</td>
<td>5.8688</td>
<td>4.38</td>
</tr>
<tr>
<td>InAs</td>
<td>0.36</td>
<td>6.0584</td>
<td>4.9</td>
</tr>
</tbody>
</table>

equilibrium, it was found that this material can be grown as very thin layers, making it possible to grow this material as the quantum well.

3.2 Growth of InGaP/InGaAsP MQWs

3.2.1 Top cell heterostructure sample

Two heterostructure samples, TS1111 and TS1114, were fabricated to study the material properties of InGaP/InGaAsP quantum wells. The nominal sample structures are listed in Table 3.3. TS1111 has 20 repeats of InGaP/InGaAsP quantum wells as the active layer, which has 20 repeats of 4.4-nm InGaAsP wells separated by 13.1-nm InGaP barriers. The control sample, TS1114, has bulk InGaP with the same active layer thickness of TS1111. Both samples were grown by John Roberts at the University of Sheffield on GaAs substrates 10 degrees misorientated towards the (001) plane to form disordered InGaP by MOCVD. The compositions of InGaAsP and InGaP are designed to be lattice-matched to GaAs. The background doping concentrations of both samples are lower than $2 \times 10^{15}$ cm$^{-3}$. This quantum well structure supports a fundamental interband transition at 1.69 eV.
Figure 3.1: The calculated (a) band gap, (b) lattice constant and (c) electron affinity contours of In$_{1-x}$Ga$_x$As$_y$P$_{1-y}$ as a function of $x$ and $y$ using Eq. (3.1) and the parameters in Table3.1. Green dashed lines in (a) and (c) represent the In$_{1-x}$Ga$_x$As$_y$P$_{1-y}$ lattice-matched to GaAs.
3.2.2 X-ray diffraction study

X-ray diffraction (XRD) was performed on the sample TS1111 and TS1114 to investigate the crystal structure of InGaP/InGaAsP MQWs. The X-ray diffraction of the samples are measured by a Philips X’PERT-MPD diffractometer system. The X-ray source generates the probe photon beam with wavelength at 1.54 Å.

Figure (3.2) and 3.3 (a) shows measured reciprocal space maps (RSM) of TS1111 and TS1114. The definitions of reciprocal space vectors $Q_x$ and $Q_z$ in the plots can be referred to section 2.9. The dashed lines in Figure (3.2)(a) and Figure (3.3)(a) indicate the [004] direction of GaAs substrate. For a fully strained sample without any substrate misorientation, it is expected that all the peaks in the map lie on the [004] lines drawn on the RSMs. However, the lines aligned by the peak intensities are tilted from the [004] line, which can be the result of substrate misorientation or strain relaxation in the epitaxial layers. To further clarify this, Figure (3.2)(b) and Figure (3.3)(b) are plotted as the functions of $\omega$ and $2\theta$. The fitted lines of the high intensity points in the graphs are plotted as the blue lines, and the fitted parameters are shown in the legends of the plots. It shows that both fitted lines have an offset at around $8^\circ$, which corresponds to the misorientation of the substrate. The slopes for those fit are expected to be unity if a sample is perfectly strained, but the fitted slopes for both samples are around 0.7.

In principle, one can make a one-dimensional XRD scan along the [004] axis of the epitaxial layer by providing an offset in $\omega$. In this case, it is an XRD scan along the line of $\omega = \theta + 8^\circ$, but this line has no intersections with the high intensity points in Figure (3.2)(b) and Figure (3.3)(b) due to the mismatch of the fitted slope. A compromise is to do a one-dimensional scan along the line $\theta = \omega$, which corresponds to the [004] direction of the substrate. Although the $\theta = \omega$ line does not completely overlap with the high intensity points in Figure (3.2)(b) and Figure (3.3)(b), the XRD rocking curves can still be measured because the detection angle of the detector is wide enough to collect the
signals that are slightly off from the centre. Therefore, the measured XRD rocking curves can be regarded as the projection of the peaks in RSMs to [004] axis of the substrate. The results are plotted in Figure (3.4).

Figure (3.4) also shows the best fit of the measured XRD rocking curves. The fitting was performed by X’Pert Epitaxy software package. The fitted layer thicknesses and compositions are listed in Table 3.3. This XRD fitting assumes that the experiment XRD curves were taken along [004] direction of the epitaxial layers, leading to some discrepancies between the nominal and fitted parameters of the layer structure.

Although an unique set of layer thicknesses and alloy composition cannot be obtained through fitting, some qualitative features of the samples can be inferred from these results. Firstly, both the XRD data of TS1111 and TS1114 have a broad peak R1 on the right hand side of the substrate peak, suggesting that some layers in both samples cause relaxed tensile strain. Since TS1114 does not have layers other than AlInP and InGaP, the XRD fitting of TS1114 suggests that the peaks R1 come from the AlInP layer if we assume that the Ga composition of InGaP layer does not exceed 52%. Also, the XRD plot and the fitting show that TS1111 has uniform repeats of MQW layers, otherwise the satellite peaks of the measured XRD will not be able to match the fitted peaks well by assuming each repeat of MQW layers is identical. In addition, in the XRD plot of TS1111, the peaks L1 to L4 are much stronger than R1 to R3, indicating that the MQWs are compressively-strained.

### 3.3 Carrier dynamics study of InGaP/InGaAsP via photoluminescence

The carrier recombination dynamics of InGaP/InGaAsP MQWs through steady-state PL and TRPL decay, as described in section 2.11 and 2.12. In addition to TS1111 and TS1114, we add a AlGaAs/GaAs MQW heterostructure sample, KLB494, as another
Figure 3.2: Reciprocal space mapping of TS1111 plotted in (a) reciprocal space vector $\mathbf{Q}$ and (b) $2\theta - \omega$. The definitions of vector components $Q_x$ and $Q_z$ can be referred to Eq. (2.59) and Figure (2.6).
Figure 3.3: Reciprocal space mapping of TS1114 plotted in (a) reciprocal space vector $\mathbf{Q}$ and (b) $2\theta - \omega$. The definitions of vector components $Q_x$ and $Q_z$ can be referred to Eq. (2.59) and Figure (2.6).
Figure 3.4: The measured and simulated XRD rocking curves of (a) TS1111 and (b) TS1114 at the [004] direction of the substrate.
### Table 3.2: Layer structure of TS1111

<table>
<thead>
<tr>
<th>layer name</th>
<th>nominal</th>
<th>nominal thickness (nm)</th>
<th>fitted composition</th>
<th>fitted thickness (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>cap</td>
<td>( \text{In}<em>{0.52}\text{Ga}</em>{0.48}\text{P} )</td>
<td>10</td>
<td>( \text{In}<em>{0.52}\text{Ga}</em>{0.48}\text{P} )</td>
<td>5.8</td>
</tr>
<tr>
<td>cladding</td>
<td>( \text{Al}<em>{0.53}\text{In}</em>{0.47}\text{P} )</td>
<td>60</td>
<td>( \text{Al}<em>{0.59}\text{In}</em>{0.41}\text{P} )</td>
<td>46.1</td>
</tr>
<tr>
<td>barrier x20</td>
<td>( \text{In}<em>{0.52}\text{Ga}</em>{0.48}\text{P} )</td>
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<td>( \text{In}<em>{0.52}\text{Ga}</em>{0.48}\text{P} )</td>
<td>27.8</td>
</tr>
<tr>
<td>well x20</td>
<td>( \text{Ga}<em>{0.62}\text{In}</em>{0.38}\text{As}<em>{0.34}\text{P}</em>{0.66} )</td>
<td>13.1</td>
<td>( \text{Ga}<em>{0.62}\text{In}</em>{0.38}\text{As}<em>{0.34}\text{P}</em>{0.66} )</td>
<td>2.9</td>
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<td>( \text{Al}<em>{0.59}\text{In}</em>{0.41}\text{P} )</td>
<td>45</td>
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<tr>
<td>substrate</td>
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### Table 3.3: Layer structure of TS1114

<table>
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<th>fitted composition</th>
<th>fitted thickness (nm)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>10</td>
<td>( \text{In}<em>{0.52}\text{Ga}</em>{0.48}\text{P} )</td>
<td>5.8</td>
</tr>
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<td>( \text{Al}<em>{0.59}\text{In}</em>{0.41}\text{P} )</td>
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<td></td>
<td></td>
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reference sample. The layer structure of this sample is provided in Table A.1. This sample is provided by Professor Philips Dawson from University of Manchester. The peak PL emission wavelength of KLB 494 is close to TS1111 and known to be very radiatively efficient, making it suitable for benchmarking the measurement of TS1111 and TS1114.

### 3.3.1 Steady-state PL spectra

The PL spectra of TS1111, TS1114 and KLB494 measured at different temperatures are plotted in Figure (3.5). The samples were excited by a PicoQuant diode pulsed laser with emission wavelength at 485 nm and a repetition rate of 80 MHz, and a pulse duration of a few hundreds picoseconds. The average power of the laser beam was 4 mW, and the beam focused onto the test sample was close to a Gaussian profile with $1/e^2$ radius of 49 µm. This gives the injection density at the Gaussian peak to be $1.27 \times 10^{12}$ cm$^{-2}$ sec$^{-1}$. The peak injection density is equivalent to the order of $10^5$ suns. The PL spectrum was resolved by a Princeton Instrument SP2500 monochromator, and the intensities of the monochromatic light were measured by a silicon photodetector through a Stanford Research SR530 lock-in amplifier. The measured PL of TS1111, TS1114 and KLB494 are normalised to the same scale, with the response of detector and grating calibrated. The peak PL photon energy of TS1111 is 1.69 eV at room temperature, which matches the absorption edges of EQE reported in Ref. [43,44]. The peak PL photon energy of TS1114 is around 1.9 eV at room temperature, which is higher than the PL peak of typical InGaP cell ($\sim 1.82$ eV) because it is highly disordered [45–47].

Figure (3.6) (a) shows the spectrally integrated PL in photon numbers normalised to the same scale. The spectrally integrated PL of TS1111 saturates at temperatures below 200 K, suggesting that radiative recombination process play a dominant role at these temperatures. Also, a relative comparison of the three samples shows that their integrated PL intensities are similar at 30 K. Thus we can infer that the recombination of both samples are likely to be highly radiative at this temperature. The radiative efficiency
at room temperature, which is defined as the ratio between the emitted photons and the recombined electron-hole pairs, can be estimated as the ratio of the spectrally integrated PL at room temperature and low temperature. This gives the radiative efficiency of TS1111 to be 84%, while the radiative efficiency of TS1114 and KLB494 are less than 20%. The peak PL intensities against different temperatures plotted in Fig. 3.6 (b) shows very similar trend. The peak PL intensity of TS1111 at room temperature is around 60% of its PL intensity at low temperature, but the peak PL intensities of the other two samples at room temperature are one or two orders of magnitude lower than their peak PL intensities at the lowest temperatures.

The radiative efficiency of TS1111 calculated from spectrally integrated PL is comparable to the values reported for the radiatively coupled junctions described in Section 4.3 and Ref. [43] via measuring the photon coupling efficiencies of dual-MQW dual-junction devices, which results in a external radiative efficiency of 68%.

Figure (3.6)(d) presents a plot of the PL full width at half maximum height, showing that the emission of TS1111 is very broad, which indicates that there is disorder at the MQW interface. The broad PL emission suggests a slow onset to the density of states in the QW and is consistent with modest increase in absorption at the band-edge evident from the external EQE of this MQW top cell [43, 44].

The Arrhenius plot of spectrally integrated PL is plotted in Figure (3.7). Fittings to the spectrally integrated PL using Eq. (2.63) are also presented in this figure. It shows that TS1111 has a relatively large fitted activation energy 0.16 eV compared to the other two samples. Since the internal radiative efficiency loss is a combination of the thermal escape to the barriers and the non-radiative recombination in the well, the fitted activation energy can be considered as the lower limit of the confinement potential of the quantum well.

The difference between the PL peak photon energy and the band gap of InGaP is $1.9 - 1.69 = 0.21$ eV, and the fitted activation energy of TS1111 suggests that either
the confinement potentials of either electrons or holes is no less than 0.16 eV, showing that either electron or hole well is very shallow. From the electron affinities of InGaAsP predicted in Figure (3.1), it is likely that the confinement potential for holes is larger than for electrons.

3.3.2 Time-resolved photoluminescence decay

Time-resolved photoluminescence (TRPL) decay curves at the PL peak wavelengths were measured by time-correlated single photon counting to further study the recombination mechanism of these samples. TRPL decay of these samples were illuminated at the same pulse power and beam size described in the previous section, but the repetition rate is reduced to a few MHz in order to measure longer decay times. The measured PL decays are plotted in Figure (3.8).

Carrier recombination model for describing TRPL decays

The TRPL decay is essentially measuring the instantaneous excess carrier density, which can be described by the rate equation:

\[
\frac{\partial n}{\partial t} = -(An + Bn^2 + Cn^3) + D\nabla^2 n
\]  

(3.2)

The linear term coefficient \(A\) is the function of Shockley-Read-Hall recombination, surface recombination and the linear radiative recombination in Eq. (2.27), \(B\) is usually attributed to the second order term of radiative recombination rate in Eq. (2.27), \(C\) is the coefficient that relates to Auger recombination and \(D\) is the diffusion coefficient of the free carriers [23]. Since the carrier generation term is not considered in Eq. (3.2), the coefficients \(A\), \(B\) and \(C\) will be lower than than the real recombination rates if photon reabsorption is significant. Although separating the contributions of the real recombination coefficients and photon reabsorption factors is difficult, it is the effective recombination rates, \(A\), \(B\) and \(C\), that are of the most interest for solar cells since they are directly linked to the
Figure 3.5: Steady-state temperature dependent PL spectra of (a) TS1111, (b) TS1114 and (c) KLB494. The legends labels the temperatures in Kelvins. All PL spectra are normalised to the peak intensity of the PL spectrum of TS1111 at 30 K.
Figure 3.6: (a) Spectrally integrated PL, (b) peak PL intensities, (c) peak PL photon energies and (d) FWHM of PL of all the test samples against various temperatures.
Figure 3.7: The Arrhenius plot of the spectrally integrated PL shown in Figure (3.5)(a). The solid lines are the fitted PL intensity using Eq. (2.63).
Figure 3.8: The measured and fitted TRPL decay curves of (a) TS1111, (b) TS1114 and (c) KLB494 at their PL peak wavelengths. Fitted TRPL decay curves are presented in black solid lines.
diffusion length of the carriers. The recombination coefficients $A$, $B$ and $C$ of the samples can be retrieved by solving the rate equation to fit the TRPL decay data. Recall that the excess carrier density profile needs to be taken into account due to the nonlinear terms in the rate equation, as mentioned in Section 2.12.

To simplify the calculation, we assume that the recombination coefficients $A$, $B$ and $C$ are constant throughout the PL decay. Also, by assuming that the excited excess carriers are distributed uniformly within an effective layer of thickness $w$, Eq. (3.2) can be reduced to

$$\frac{\partial \sigma_n}{\partial t} = - A_s \sigma_n - B_s \sigma_n^2 - C_s \sigma_n^3 + D \nabla^2 \sigma_n$$

(3.3)

$\sigma_n$ is the excess sheet carrier density and $D$ is the diffusion coefficient. This is the equation that is solved to fit $A_s$, $B_s$ and $C_s$. The diffusion coefficient of lowly-doped InP/InGaAsP MQWs is reported to be of the order of few cm$^2$sec$^{-1}$ at room temperatures [48, 49]. The effect of diffusion at this level is found to be negligible in the decay time scale that we considered here. The difference of fitted $A_s$ due to varying $D$ from zero to 10 cm$^2$sec$^{-1}$ is around 10$^5$ sec$^{-1}$, and the difference of fitted $B_s$ caused by varying $D$ in the same range is negligible.

In most cases, setting the coefficient of the third-order term $C$ to be zero yields reasonable fit. However, at the temperatures below 70 K, better fit of TS1114 and KLB494 require negative values of $C_s$ at around $1 \times 10^{23}$ cm$^6$/sec, suggesting that the carrier recombinations at this temperature range cannot be fully described by free-carrier recombination model, since disorder in the QWs would lead to localised trapped charge at low temperatures.

The effective layer of thickness $w$ of MQWs and bulk materials need to be considered respectively due to different carrier redistribution mechanisms in each type of material. Since both InGaP and InGaAsP have the absorption coefficients around $1.5 \times 10^5$ cm$^{-1}$ at the laser wavelength [50,51], 64%($\sim 1 - 1/e$) of the carriers are absorbed in the first
75 nm of the active layer of both samples, so this value can be considered as a lower limit of \( w \) for both samples. For TS1111, most of the carriers are captured and recombined in the quantum well, showing that the carriers are transferred from one well to another by thermal escape or photon reabsorption. Therefore, \( w \) of TS1111 has a range from few wells to the total thickness of 20 wells depending on how uniform that the carriers are distributed. For TS1114, the excess carriers are redistributed by diffusion. The diffusion length of the carriers is \( \sim 700 \) nm even if the diffusion coefficient is just \( 1 \) cm\(^2\)sec\(^{-1}\) and the carrier lifetime is just \( 5 \) ns, so we can assume that the excess carriers in TS1114 are redistributed within the whole bulk InGaP layer. As a result, \( w \) of TS1114 can be estimated as the whole active layer thickness.

With the approximations and assumptions mentioned above, Eq. (3.3) can be solved using the laser profile as the initial condition of \( \sigma_n(r) \). The total PL decay can then be calculated from the solved \( \sigma_n(r; t) \) using the relation [23]

\[
\phi \propto B_s \int_A \sigma_n(r)[(\sigma_n(r) + \sigma_D)]dA
\]

where \( \sigma_D \) is the sheet background doping density of the samples. The background doping concentration \( N_D \) is of the order of \( 10^{15} \) to \( 10^{16} \) cm\(^{-3}\). The sheet background doping density can be written as \( \sigma_D = N_D w \), which gives a sheet background doping density \( \sigma_D \) an upper bound \( \sim 10^9 \) cm\(^{-2}\) by taking \( w \) as the thickness of the whole active layers. Since the excess density range covered by this TRPL decay measurement is from \( 10^9 \) to \( 10^{12} \) cm\(^{-2}\), we have \( \sigma_n(r) \gg \sigma_D \). Eq. (3.4) thus becomes

\[
\phi \propto B_s \int_A \sigma_n^2(r)dA
\]

Therefore, the fitted radiative recombination \( A_s \) and \( B_s \) are not affected by the background doping in this case.
Interpretation of the TRPL fitting

The fitted TRPL decay curves and the fitted recombination coefficients are plotted in Figure (3.8) and Figure (3.9). This model fits the PL decay very well across most of the curves. However, the tails of the experimental PL decay are not fitted very well, which may be due to the occupancies of long-lived trap states in the samples that slower the non-radiative recombination rates at late stage of the decay.

The fitted $B_s$ of TS1111 is of the order of $10^{-4}$ cm$^2$/sec, thus the radiative recombination coefficient $B$ is between $10^{-11}$ to $10^{-10}$ cm$^3$/s, depending on the value of $w$. The range of $B$ is comparable to the radiative recombination coefficients of InP/InGaAsP MQWs reported in Ref. [49,52,53]. Also, the fitted $B_s$ of both TS1111 and TS1114 drop as the temperature increases, because spontaneous radiative recombination rates becomes higher and $w$ becomes lower as the temperature decreases.

The fitted $A_s$ of TS1111 and TS1114 have different trends. Below 100 K, the fitted $A_s$ of TS1114 increases with temperature, and then stays at the same level above 100 K. This result suggests that $A_s$ of TS1114 is dominated by non-radiative recombination, which generally increases with temperature. However, the fitted $A_s$ of TS1111 decreases with temperature, which suggests that radiative recombination is still dominant at high temperatures. This matches the qualitative behaviours of the TRPL decays shown in Figure (3.8). When the carrier injection density $n$ is low or the $A$ coefficient is high ($A \gg Bn$), the PL decay is dominated by the term $An$ and therefore is a straight line in the logarithmic plot. In contrast, if the carrier density $n$ is high or $A$ is low ($A \ll Bn$), the decay will not be single-exponential and the instantaneous decay rate will change with the remaining excess carrier densities. Therefore, at high injection regime, the initial decay of the TRPL is dominated by radiative recombination and dependent on $B$ and $n$, and the decay rate in the late stage is determined by $A$. At low temperature, all the three samples have non-single-exponential PL decays. However, at high temperatures, TS1114 and KLB494 are dominated by single-exponential decay, but TS1111 is still non-single-
exponential because that TS1111 has much lower $A$. The $A_s$ of TS1111 is around $10^6$ sec$^{-1}$, which is one order of magnitude lower than TS1114 at all temperatures. Since $A_s$ is the upper limit of the non-radiative recombination rates, this supports the observation of PL that the barriers dominates the radiative efficiency loss in the MQWs.

In Ref. [49], the $A_s$ of undoped InP/InGaAsP MQWs at room temperature is reported to be $2 \times 10^5$ sec$^{-1}$, which is one order of magnitude lower than the fitted $A_s$ of TS1111. This may be due to that the qualities of InGaP/InGaAsP MQWs are not as optimised as InP/InGaAsP MQWs.

Comparing the fitted recombination coefficient $B_s$ of these three samples gives some indication of the carrier recombination dynamics of the MQWs in TS1111. At the lowest temperature, the $B_s$ of TS1114 and KLB494 are very close, but the $B_s$ of TS1111 is much lower. From spectrally integrated PL we know that the internal radiative efficiency of all these samples are nearly 100% at this temperature, so we can ensure that $B_s$ nearly depends on the radiative transitions and not coupled by effects from non-radiative recombination mechanisms. As a result, lower $B_s$ shows that the radiative transitions of TS1111 is less efficient than the other two samples. This observation supports the conclusion from spectrally integrated PL that the barrier of the electrons are low that causes low joint density of states.

### 3.4 Solar cell devices with InGaP/InGaAsP top cells

A single junction InGaP solar cell, TS462, with the same InGaP/InGaAsP MQWs as TS1111 was fabricated to explore the performance of this MQWs. The layer structure of this sample is listed in Table A.4.

Figure (3.10) presents the experimental and simulated dark IV characteristics of the sample. The fraction of the recombination current that is radiative, also referred to as external radiative efficiency, is also shown on the second y-axis. The method for extracting
the radiative current and external radiative efficiency is described in Section 2.6.4 and Ref. [54, 55]. The modelled and experimental EQE are plotted in Figure (3.11). This dark IV and EQE analysis are both performed by Dr. James Connolly [43]. The measured radiative efficiency of TS1111 is higher than the values measured in Ref. [43] because the instantaneous injection intensity is higher and there are no emitter or base layers which cause additional recombination losses, i.e. here we measure the internal quantum efficiency of the MQWs only, not a complete device.

The characterisation results of dual-junction devices with the InGaP/InGaAsP MQWs are presented in this section. Figure (3.12) and (3.15) show the EQE and the conversion efficiencies of one of the best performed dual-MQW dual-junction device, 416-2698-h17. The layer structure of this sample is listed in Table A.2.

This EQE shows that the absorption edge of the top cell is at 740 nm and the bottom cell is at 1026 nm, which is close to the optimal band gap combination (1.7/1.4 eV) of a dual or triple junction device. Under standard one-sun spectrum ASTM G173-03, the bottom cell is the current-limiting cell in this dual-junction device, which can potentially
Figure 3.10: Measured and simulated recombination current of a single junction MQW top cell TS462. The blue line plots the radiative current calculated by Eq. (2.6). The green line plots the Shockley-Read-Hall recombination current calculated by Eq. (2.52). The red line indicates the calculated the ideal Shockley injection current using Eq. (2.49) subtracts the calculated radiative current using Eq. (2.6). The red line plots the total recombination current, which is the sum of all the different recombination current components. Note that the non-radiative Shockley injection current and radiative current overlaps each other because their values are very close. The fraction of the recombination current that is radiative, referred to as external radiative efficiency, is also shown on the second y-axis. This analysis is performed by Dr. James Connolly.
Figure 3.11: Measured and simulated quantum efficiencies of a single-junction top cell (device #1 in Table 4.2). This analysis is performed by Dr. James Connolly, using the methods described in Section 2.6
be improved by increasing the number of quantum wells in the bottom cell.

Recall that the absorption coefficient $\alpha(E)$ of the i-region can be estimated using Eq. (2.48), that is,

$$\alpha(E) = -\log(1 - \text{EQE}(E))/L$$  \hspace{1cm} (3.6)

where $L$ is total thickness of the MQWs. This calculation can be verified by substituting the calculated absorptivity $a(E)$ into Eq. (2.60) to obtain the PL spectrum and comparing it to the measured PL.

The modelled and measured PL spectra are plotted in Figure (3.13) for comparison. It shows that the modelled PL agrees with the measured PL spectrum very well, which confirms the reciprocity of absorption and emission of the MQWs.

The calculated absorption coefficients of the these two subcells are shown in Figure (3.14). It clearly shows that the bottom cell has an exciton absorption peak near its absorption edge, but no significant exciton absorption is observed in the top cell’s absorption spectrum. Also, the absorption edge of the bottom cell steep, close to what
Figure 3.13: The modelled and experiment steady-state PL of sample TS1111 and 416-2698. The modelled PL were calculated by substituting the absorptivity into the generalised Planck’s equation Eq. (2.60). The absorptivity is extracted from the measured EQE using Eq. (3.6).
we expect from a type I quantum well, but the absorption of the top cell MQWs does not have this steep edges. These observations suggest that the distribution of the energy states in the top cell is very broad, which may be due to the interdiffusion of the interfaces in the MQWs.

Figure (3.15) (a) shows the measured efficiencies and fill factors versus concentration of this dual-junction device characterized by Fhg-ISE. The maximum efficiency of this device is 28.5% at 100 suns. Other devices on the same wafer have peak efficiencies between 25% and 28%. The fill factors start to decrease at low concentrations, indicating that the peak cell efficiency is mainly limited by the series resistance. This is supported by the linearity of the measured open-circuit voltage versus logarithms of concentrations in Figure (3.15) (b).
Figure 3.15: (a) Efficiencies and fill factors and (b) open circuit voltages of the sample 416-2698-h17 versus concentrations.

3.5 Conclusions

InGaP/InGaAsP MQWs can extend the absorption edge of an InGaP solar cell to reduce the current mismatch of state-of-the-art triple junction MQW solar cells. Although this quantum well material has been implemented in commercial triple junction solar cells, only its device I-V characteristics and EQEs have been reported. Many other material properties have not been studied yet.

The material properties of InGaP/InGaAsP MQWs were studied in a number of ways for the first time in this chapter. Crystal structures of the InGaP/InGaAsP MQWs were investigated by XRD rocking curves and RSMs. The analysis of RSMs confirmed that the misorientation of the substrate is nearly 10°. Large misorientation of the substrates makes it difficult to accurately determine the exact layer thicknesses and material compositions of the MQWs by fitting the XRD rocking curves, but the fitting still suggests that this sample has uniform periodic layer structures in the MQWs.

Carrier recombination dynamics were studied via PL and TRPL decay. The spectrally
integrated PL shows that the InGaP/InGaAsP MQWs has an internal radiative efficiency of 84% at high carrier injection regime. The activation energy extracted from spectrally integrated PL confirms the shallow well depth for the electrons and deep well depth for holes, as predicted by the interpolated electron affinity of InGaAsP. Deep hole well depth also contributes to high internal radiative efficiency because it reduces the thermal loss of carriers.

The PL spectrum of the InGaP/InGaAsP MQWs were modelled by detailed balance model using the absorption coefficient extracted from the measured EQE of the MQW devices. The modelled PL shows good agreement with the experimental PL spectrum, indicating the broad PL of the MQWs is consistent with the modest increase in the absorption at the band-edge of the MQW. This result suggest a slow onset to the density of states in the QW.

TRPL decays were studied in detail by separating the contribution of different recombination components. The result shows that InGaP/InGaAsP MQWs have very slow non-radiative recombination rate, which is consistent with high internal radiative efficiency observed in steady-state PL. However, low radiative recombination rates at low temperatures of InGaP/InGaAsP MQW sample suggest the absorption is less efficient than the other two reference samples. A comparison of the absorption coefficients between InGaP/InGaAsP and GaAsP/InGaAs MQWs also supports this observation.

The EQE and I-V measurements of a dual-MQW dual-junction device were presented. It is demonstrated that dual-MQW dual-junction devices can reach a maximum efficiency of 28.5% at 100 suns, the trend of fill factors and the open circuit voltages against concentrations show that the peak conversion efficiency is limited by the series resistance in the device instead of the material quality of the MQWs.
Chapter 4

Demonstration of Photon Coupling in Dual-MQW Dual-Junction Solar Cells

Multi-junction devices operate most efficiently when the sub-cells are current matched under a specific spectrum, i.e., each of the subcells are operated at its maximum power points. In space, this is straightforward to achieve since the extraterrestrial AM0 spectrum is constant. However, for terrestrial applications, atmospheric conditions cause variations in the illumination spectrum. Current-mismatch in a multi-junction device raised the voltage of one or more of the subcells requiring the device to emit radiative photons because only part of the absorbed photon can be collected by the external circuit. The radiative photons can be absorbed by the subsequent junction to increase its photocurrent. This process is is called photon coupling; also sometimes known as radiative coupling or luminescent coupling. Photon coupling can help reduce the current-mismatch in the multi-junction device and make the multi-junction device less sensitive to the variations of the illumination spectrum, which could potentially increase the energy yields of a multi-junction device.
Photon coupling was considered by Marti and Araujo [9] to determine the limiting efficiencies of multi-junction solar cells. It was later elaborated by Brown and Green [56] who highlighted the role radiative coupling can mitigate the power loss due to spectral mismatch. Yoon et al. has made one of the first, practical demonstrations of photon coupling in multi-junction devices [57] although the overall coupling efficiency was found to be low.

In recent years, photon coupling receives more attention because of the improvement of the material qualities, leading to much more significant photon coupling effects in multi-junction devices. Photon coupling was demonstrated and quantified by various of methods. For instance, it can be demonstrated by illuminating the multi-junction device with a spectrum that creates strong current-mismatch, and the photon coupling efficiencies were quantified by some analysis, such as in Ref. [58–62]. Another way to induce photon coupling is electrically biasing the top cell in a three-terminal device [57]. EQE measurement can also be used to measure photon coupling. However, photon coupling signal needs to be separated from the measurement artefacts due to a shunted subcell as they have the same signatures. This can be resolved by using a very non-radiative device [57,63] as a reference or perform a very rigorous analysis of measured EQE [32,64,65].

There are only certain spectral conditions when photon coupling actually improves the overall energy yield of a multi-junction device, namely when there is a blue rich spectrum resulting in a cascade of power from high gap solar cells to low gap. In conventional solar bulk solar cells and depending upon the cell design, the improvement in power output can reach 9% [66].

MQWs in solar cells tend to become dominated by radiative recombination in the QW layers that lend to photon coupling. If the QW is compressively strained, the direction of the emission can be engineered to propagate perpendicular to the plane of the QW, which will promote photon coupling [67]. An energy yield model of multi-junction quantum well solar cell has been presented in Refs. [34,35,43]. The results of these calculations
indicated that photon coupling can raise the efficiencies of quantum well solar cells and potentially reduce the spectral sensitivity of power output.

Irrespective of the practical application, photon coupling is still of great interest because it is a direct measurement of the lower bound of external radiative efficiency, which is very critical to the performance of the solar cell as discussed in Section 2.1.

Photon coupling in MQW solar cells are demonstrated for the first time in this chapter. In the first part of this chapter, the definition of photon coupling efficiency and its the relevant mathematical formalisms will explained in detail. Then, the experimental results of photon coupling by electrically and optically injecting the top cells will be reported. The comparison of modelled and experiment photon coupling efficiencies will also be presented.

4.1 Photon coupling efficiency

The photon coupling efficiency in a dual-junction device can be defined as the fraction of the total recombination current of the top cell (\(J_1\)) that becomes the photocurrent of the bottom cell (\(J_2\)), which can then be expressed as

\[
\eta_{\text{coup}} = \frac{J_2^{2'} - J_2^2}{J_1^{1\text{sc}}(V)}
\]  
(4.1)

where \(J_1^{1\text{rec}}(V)\) is the recombination current of \(J_1\) at its operating voltage \(V\), \(J_2^{2'}\) and \(J_2^2\) are the short-circuit current of \(J_2\) with and without the presence of photon coupling, respectively. Thus \(J_2^{2'} - J_2^2\) is the short-circuit current increase due to photon coupling.

Recall that J-V characteristics of each subcell at illumination \(\phi\) is

\[
J_{i\text{tot}}(V, \phi) = -J_{i\text{sc}}(\phi) + J_{i\text{rec}}(V^i), \ i = 1, 2
\]  
(4.2)

Substituting Eq. (4.2) into Eq. (4.1) yields

\[
\eta_{\text{coup}} = \frac{J_2^{2'} - J_2^2}{J_{i\text{tot}}^1(V, \phi) + J_{i\text{sc}}^1(\phi)} = \frac{J_2^{2'} - J_2^2}{J_{i\text{sc}}^1(\phi) - |J_{i\text{tot}}^1(V^i, \phi)|}
\]  
(4.3)
Note that we only consider that regime that \( J_{i_{ed}}(V^i, \phi) < 0 \).

One special case is that both the numerator and denominator of Eq. (4.3) under a illumination \( \phi \) are maximised, i.e., \( J_{i_{ed}}(V_{oc}, \phi) = J_{sc}^2 = 0 \). Since \( J_{sc}^2 \) is the short-circuit current of J2 without photon the presence of photon coupling, \( J_{sc}^2 = 0 \) means that all the incident photons are absorbed by J1. Eq. (4.3) can then be reduced to:

\[
\eta_{coup} = \frac{J_{sc}^2}{J_{sc}^1(\phi)} \tag{4.4}
\]
or

\[
\eta_{coup} = \frac{J_{sc}^2}{J_{rec}^1(V_{oc}^1(\phi))} \tag{4.5}
\]

where \( V_{oc}^1(\phi) \) denotes the open-circuit voltage of J1 at illumination \( \phi \). Eq. (4.4) and Eq. (4.5) are important because they can be easily related to the external radiative efficiency of J1. Since \( J_{sc}^2 \) is only contributed by the radiative photons emitted from J1, \( J_{sc}^2 \) can be calculated using detailed balance model

\[
J_{sc}^2 = q \int_E \gamma(E)\phi_{ext}^1(V_{oc}^1(\phi), E)dE \tag{4.6}
\]

where \( \gamma \) is the probability of a radiative photon from J1 that can be absorbed and becomes photocurrent in J2. \( \gamma \) can be estimated by the product the escape cone loss \( \beta(E) \), transmission of layers between J1 and J2 \( t(E) \), and the EQE of the bottom cell, that is,

\[
\gamma(E) = (1 - \beta(E))t(E)EQE(E) \tag{4.7}
\]

The escape loss \( \beta(E) \) is \( \sim 1/4n_d^2 \). Since the spectral width of \( \phi_{ext} \) is usually very narrow, the coefficients \( \beta(E) \), \( t(E) \) and \( EQE(E) \) can all assumed to be constants. Hence Eq. (4.6) can be reduced to

\[
J_{sc}^2 = q\gamma\phi_{ext}^1(V_{oc}^1(\phi)) = \gamma J_{rad}^1(V_{oc}(\phi)) \tag{4.8}
\]

Substituting \( J_{sc}^2 \) back to Eq. (4.5) yields

\[
\eta_{coup} = \frac{\gamma J_{rad}^1(V_{oc}(\phi))}{J_{rec}^1(V_{oc}^1(\phi))} = \gamma \eta_{ext}^1(V_{oc}^1(\phi)) \tag{4.9}
\]
where $\eta_{\text{ext}}(V_{oc}(\phi))$ is the external radiative efficiency $J_1$ at its open circuit voltage. Eq. (4.9) is an important result because it links the photon coupling efficiency and the external radiative efficiency in a linear relation. It shows that photon coupling is significant only when the top subcell has high external radiative efficiency. Also, photon coupling provides a means to measure external radiative efficiency if $\gamma$ is known.

However, using Eq. (4.4) or Eq. (4.5) to measure photon coupling requires that the bias of each subcell can be controlled independently. Here we will show that the photon coupling efficiency measurement can be also be performed on a standard dual-junction device, in which the subcells are connected in series with only two electrodes.

Consider that $J_1$ of a dual-junction device is illuminated with $\phi$ and $J_2$ is kept in the dark, we have $J_{2\text{sc}} = 0$ but the radiative photon in $J_1$ induces a current $J_{2\text{cr}}$ because of photon coupling. In this case, when is dual-junction device is kept at short-circuit, $J_1$ is the current-limiting cell and is operated at a voltage $V^1$ that is very close to its open-circuit voltage. In other words, at short-circuit of the dual-junction device, we have $J_{2\text{sc}} = J_{1\text{tot}}(V^1) = J_{2\text{tot}}(-V^1)$. Assuming that $J_2$ is not shunted so that $J_{2\text{sc}} = J_{2\text{tot}}(-V^1) = J_{2\text{sc}} = J_{1\text{tot}}(V^1)$, Eq. (4.3) then becomes

$$\eta_{\text{coup}} = \frac{J_{2\text{sc}}}{J_{1\text{sc}}(\phi) - J_{2\text{sc}}}$$

With this result, $\eta_{\text{coup}}$ can be measured on a two-terminal dual-junction device as long as its subcell has very high shunt resistance.

### 4.2 Measuring photon coupling using EL

In this section, the photon coupling of a dual-junction device is demonstrated by using EL of the top cell to generate photocurrent in the bottom cell, which is essentially using the bottom cell as a photodetector to measure the EL intensity of the top cell. The photon coupling efficiency can thus be measured using Eq. (4.5).
Figure 4.1: The device layer structure of the three-terminal device TS462γ. The polarity of the top cell is inverted and the subcells are interconnected by a highly doped lateral conduction layer instead of a tunnel junction.

Since this measurement requires the voltages of J1 and J2 to be individually controlled, a three-terminal dual-junction device TS462γ was fabricated at the University of Sheffield for this purpose; illustrated in Figure (4.1). This device has an InGaP MQW top cell and GaAs MQW bottom cell. To reduce the complexities of fabrication, in particular the fabrication of a tunnel junction, the polarity of one junction is inverted and only a highly doped n-type layer is grown between the top and bottom junction. As a result, this device cannot be operated as a two-terminal dual-junction device, but requires independent current extraction from each sub-cell via the three electrical connections. A detailed layer structure of this device is presented in Table A.3.

These devices were processed into 1 mm² mesa structures with a typical solar cell de-
sign on the front. The dark current and quantum efficiencies of the subcells are presented in Figure (4.2) and Figure (4.3). As shown in Figure (4.3), both the top and bottom cells have reasonable spectral response. However, the dark current measurement shows both devices has very high series resistance since the curve becomes flat at high bias region in the logarithm plot.

Since the current level of $J_{sc}^{\gamma}$ is very small at low $J_{rec}^{1}$, lock-in amplification technique is thus needed to acquire reasonable signal to noise ratio. The experimental setup is illustrated in Figure (4.4). The forward bias of the top cell was modulated by a function generator and an emitter follower, which is essentially a current amplifier but keeps the output voltage same as input. The modulated forward bias was a 188 Hz square wave with 50% duty cycle. The high level of the square wave was the demanded forward bias voltage and the low level of the square wave was set to be zero. The bottom cell was connected to a transimpedance amplifier, which is a current-to-voltage converter and maintains the voltage of the bottom cell to be zero. The output of the transimpedance amplifier was
Figure 4.3: Measured quantum efficiency of the three terminal device TS462γ.

then demodulated by a lock-in amplifier.

To ensure that the measured photocurrent from the bottom cell does not come from the cross-talk between the two subcells, a null test was performed by forward biasing the bottom cell and then measuring the response of top cell, as illustrated in Figure (4.5). Since the radiative photons emitted from bottom cell cannot be absorbed by the top cell, any current measured in the top cell circuit loop can only result from the leakage into the bottom cell circuit loop.

When the bottom cell is forward biased at 1.0 V with injection current of 2.4 mA, the measured current across the top cell is less than $2 \times 10^{-5}$ mA. However, when the top cell is forward biased at 1.4 V with the injection current of only 0.05 mA, the measured current across the bottom cell is $8 \times 10^{-3}$ mA. Therefore, we can conclude that the electrical cross-talk of these two devices are negligible compared to photon coupling.

The measured bottom cell photocurrent $I_{\text{bot}}$ against top cell bias $V$ is plotted in Figure (4.6). The measured photon coupling efficiencies $\eta_{\text{coup}}$ using Eq. (4.1) are also plotted
Figure 4.4: The circuit diagram for the EL photon coupling measurement. The top cell is biased by a modulated voltage signal, and the response of the bottom cell is demodulated by a lock-in amplifier.

Figure 4.5: The circuit diagram of the null test for ruling out electrical coupling between subcells. The aim of this setup is to measure the photocurrent of the top cell when the bottom cell is forward biased. Since the photons generated in the bottom cell cannot induce current in the top cell, the current measured from the top cell can be regarded as the noise due to the electrical cross-talk in the circuits.
$J_{\text{rec}}^1$ is the photocurrent density of the top cell at the bias voltage $V^1$, and $J_{\text{sc}}^2$ is the measured photocurrent of the bottom cell when the top cell is biased at the voltage $V^1$. $J_{\text{sc}}^2/J_{\text{rec}}^1$ is the photon coupling efficiency defined in Eq. (4.1).

in the second axis of Figure (4.6). It clearly shows that coupling efficiency rises as the injected current of top cell increase, and the trend is very similar to the radiative efficiency shown in Figure (3.10).

### 4.3 Measuring photon coupling efficiency via PL

In the previous section photon coupling of a dual-junction device was demonstrated via EL, but series resistance in the lateral conduction layer limits the current that can be injected into the top cell. This can be overcome by optically generating electron-hole pairs in the top cell and operating it at open-circuit where the maximum radiative recombination will take place. This is essentially using the bottom cell as a photodetector to measure PL of the top cell. Moreover, since the introduction of carriers is optical, this
experiment can be extended to two terminal dual-junction samples without the need for a third contact by using the result of Eq. (4.10).

For three-terminal devices described in the previous section, $\eta_{coup}$ can be measured using Eq. (4.4) with two steps. First, the top cell is illuminated with an intensity $\phi$, the short-circuit current $J_{sc}^1(\phi)$ can then be measured. After that, the top cell is operated at $V_{oc}$ at the same illumination and the short-circuit current of the bottom cell $J_{sc}^2$ can be measured.

For standard dual-junction devices without a third terminal, the device is still illuminated by an intensity $\phi$ but only the short-circuit current of this dual-junction device $J_{sc}^{2J}$ can be measured. The other parameter, $J_{sc}^1$, can only be obtained by indirect measurements, such as measuring the short-circuit current of single-junction devices with identical structure to the top cell or estimating it from the EQE and the intensity $\phi$.

In this experiment, the top cell was illuminated with a 532 nm laser beam from a Spectra Physics Millennia V neodymium vanadate laser capable of delivering up to 5.5 W. The extinction of the 532 nm beam in the top cell was almost complete, with an intensity only $10^{-11}$ of its original intensity arriving at the bottom cell. $J_{sc}^1$, $J_{sc}^2$ or $J_{sc}^{2J}$ were measured over a range of laser intensities.

Since Eq. (4.10) assumes $J_{sc}^2 = J_{tot}^2(-V) = J_{sc}^{2J}$, meaning that the bottom cell passes the same photocurrent at reverse bias as at short-circuit current. In other words, the bottom cell cannot be seriously shunted or have low reverse breakdown voltage. This was checked by varying the bias of the dual-junction device under illumination. If the measured current was not constant when the bias is shifted a few hundred millivolts from the zero point into either forward or reverse bias, then the cell was not suitable for this test.

Three dual-junction devices, labelled as device #1, #2 and #3, were measured in this experiment. Device #1 is a three-terminal device, same as the one measured in section 4.2. Device #2 and device #3 are standard two-terminal dual-junction devices.
All these three samples have MQWs in their top and bottom cells. The details of the three test samples are listed in Table 4.1 and 4.2.

The top cell short-circuit current $J_{sc}^1$ under the same illumination intensity $\phi$ was estimated by different methods. Device #1 is a three-terminal device, so its $J_{sc}^1$ was simply measured by keeping its top cell at short-circuit. $J_{sc}^1$ of device #2 is measured from a single junction top cell with the same layer structure as the dual-junction top cell. The difference between measured EQE of the top cell of device #2 and the top cell on its own is within 2% across all top cell wavelengths. $J_{sc}^1$ of device #3 is estimated from its measured top cell EQE and the known laser intensity, since the single-junction top cell with an identical layer structure to the top cell of device #3 is not available. This gives around 10% of error in estimating $J_{sc}^1$ of device #3, mainly coming from the linearity between the photocurrent and the intensity of incoming light. This results in around 10% relative error in its photon coupling efficiency.

Figure (4.7) shows the photon coupling efficiencies $\eta_{\text{coup}}$ against $J_{sc}^1$ for device #1 and $J_{sc}^1 - J_{sc}^{2J}$ for device #2 and #3. As discussed in section 4.1, both $J_{sc}^1$ and $J_{sc}^1 - J_{sc}^{2J}$ represent the magnitudes of the induced recombination current in the top cell of these three devices. The axis for the estimated concentrations is calculated by taking the ratio of the short-circuit current density of the top cell of device #1 and its one-sun current density listed in Table 4.2. The one-sun current densities are calculated from multiplying the measured spectral response by ASTMg173-03 spectrum normalized to 1 kW/m².

The errors caused by indirect methods of extracting $IJ_{sc}^1$ are plotted in these figures. The measurement error is less than the size of the data points in the graphs, so no error bars are plotted with device #1’s data. The broken line in Figure (4.7) is the modelled radiative efficiency shown in Figure (3.10). The device in Figure (3.10) has the same nominal layer structures with device #1. The modelled photon coupling efficiency is calculated by using Eq. (4.7), that is,

$$\eta_{\text{coup}} = \gamma \eta_{\text{ext}} = (1 - \beta(E)) \tau(E) EQE(E) \eta_{\text{ext}}$$

(4.11)
Since $\beta(E) = 1/4n_d^2$, $1-\beta(E) = 98\%$ by assuming $n_d = 3.5$. The measured EQE of bottom cell near the top cell emission wavelength (730 nm) is 81%, as shown in Figure (4.3). The total transmission $t(E)$ of the Al$_{0.4}$Ga$_{0.6}$As LCL and the highly doped GaAs layer can be estimated by interpolating the data in [68], which is around 95%. By substituting these values into Eq. (4.11), we get modelled coupling efficiency:

$$\eta_{\text{coupl}} \approx 0.81 \times 0.98 \times 0.95\eta_{\text{ext}} = 0.754\eta_{\text{ext}}$$

(4.12)

For device#1, Figure (4.7) shows reasonable agreement between the predictions obtained from the radiative efficiency determined by the fit to the top cell dark current and the measurement of the photon coupling efficiency by PL. Also, the coupling efficiencies of device#1, #2 and #3 have very similar trends in the radiative efficiencies due to similar top cell layer structure designs as listed in Table 4.2.

### 4.4 Conclusion

The formalism of photon coupling depends critically on the radiative efficiency of the semiconductor junction and it is shown that the external radiative efficiency of the top cell is linear to the photon coupling efficiency of a dual-junction device.

Photon coupling in dual-MQW dual-junction solar cells has been demonstrated using both electrical and optical methods for inducing the radiative process in the top cell for the first time. This amounts using the bottom cell as a photodetector to measure the EL and PL of the top cell.

Photon coupling measured by electrically biasing the top cell was performed on a three-terminal device. Since the series resistance in the three-terminal device limited the level of current that can be injected into the top cell, an electronic circuitry was developed to measure the photon coupling efficiency at injection current density ranging from 10 to $10^3$ A/m$^2$. The measured photon coupling efficiency at this regime is less than 10%.
Figure 4.7: Photon coupling efficiency of device#1 and photon coupling efficiencies of device#2 and #3, with modelled radiative and photon coupling efficiencies of device#1.

Optically biasing the top cell to measure photon coupling avoided the series resistance problem because the subcells were operated at either short-circuit or open-circuit where series resistance is negligible. Using this technique, the injection current density can be raised to more than $5 \times 10^4 \, \text{A/m}^2$, which is equivalent to around 400 suns. Also, photon coupling efficiency of conventional, two-terminal dual-junction solar cell can be evaluated with this technique. More than 50\% photon coupling efficiency was measured at an equivalent optical injection intensity of 350 suns.

The photon coupling efficiency was modelled by extracting the external radiative efficiencies from dark I-V characteristics and calculating the optical loss caused by the escape cone, transmission of the layers between the subcells, and the EQE of the bottom cell. The measured photon coupling efficiencies are found to be consistent with the calculated values.
Table 4.1: Details of the measured dual-junction devices

<table>
<thead>
<tr>
<th>device</th>
<th>interconnect</th>
<th>terminals</th>
<th>method of extracting $I_{sc}^{top}$</th>
<th>grower</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1(TS462γ)</td>
<td>LCL</td>
<td>3</td>
<td>directly on the subcell</td>
<td>National centre for III-V technologies</td>
</tr>
<tr>
<td>#2(416-2698)</td>
<td>tunnel junction</td>
<td>2</td>
<td>from an single junction top cell with identical layer structure</td>
<td>QuantaSol</td>
</tr>
<tr>
<td>#3(TS287)</td>
<td>tunnel junction</td>
<td>2</td>
<td>measure $I_{sc}^{top}$ of a reference single junction top cell, and then multiply this reference $I_{sc}^{top}$ by the EQE ratio between the test device and the reference device.</td>
<td>National centre for III-V technologies</td>
</tr>
</tbody>
</table>
Table 4.2: Details of the top cells in the dual-junction devices

<table>
<thead>
<tr>
<th>Device</th>
<th>MQW numbers</th>
<th>Emitter thickness (nm)</th>
<th>Emitter doping (cm(^{-3}))</th>
<th>Base thickness (nm)</th>
<th>Base doping (cm(^{-3}))</th>
<th>1-Sun current density (A/m(^2))</th>
<th>QE absorption edge (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1(TS462γ)</td>
<td>20</td>
<td>100</td>
<td>(1 \times 10^{18})</td>
<td>570</td>
<td>(1 \times 10^{17})</td>
<td>123</td>
<td>740</td>
</tr>
<tr>
<td>#2(416-2698)</td>
<td>40</td>
<td>50</td>
<td>undoped</td>
<td>570</td>
<td>(1 \times 10^{17})</td>
<td>149</td>
<td>740</td>
</tr>
<tr>
<td>#3(TS287)</td>
<td>37</td>
<td>50</td>
<td>undoped</td>
<td>570</td>
<td>(1 \times 10^{17})</td>
<td>126</td>
<td>738</td>
</tr>
</tbody>
</table>
Chapter 5

A Flash IV Test System

Measuring solar cell efficiencies at very high concentration is challenging. A simple way to achieve it is using lenses or other concentrating optics to raise the illumination intensity of a continuous light source; conditions that are very close to the cell’s operating condition in a practical system. Also, the I-V acquisition can be easily implemented by using commercial source-meter units. However, concentrator optics causes problems in several aspects. First of all, it is difficult to alter the solar concentration over a wide range without changing the illumination spatial profile. Although the concentration in a solar simulator can be altered simply by moving the test plane closer or farther from the light source, to achieve high concentration of several hundred suns, additional optics are required and these can introduce aberrations and non-uniform illumination of the solar cell. Since there are distributed resistances across the surface of the solar cell [69–72] non-uniform illumination leads to a loss in efficiency and hence an inaccurate IV measurement. Therefore, commercial concentrator solar cell testing systems that use continuous light source can only illuminate the test cells at some fixed intensity, because every concentration requires its own concentrator optics to maintain the spectral and illumination uniformity. These problems become increasingly serious at very high concentration (>100X). In addition, temperatures of the test cell need to be very well controlled because a continuous,
concentrated sunlight generates large amount of heat which must be removed from the device. These features make the continuous light source suitable for testing long-term performance of concentrator solar cells at a specific concentration but not for daily cell measurement that requires variable illumination intensities.

An alternative approach is to use a flash lamp as the light source. Here electrical power is accumulated in a capacitor and then discharged over a very short time through an arc lamp to achieve very high instantaneous light intensity. Flash light sources have many advantages over continuous light source. Firstly, it can reach high concentration without using concentrator optics. The concentrations can be altered by simply changing the distance between the lamp and the sample, so it is easier to keep the illumination spectrum constant against concentrations. The distance between the flash lamp and the sample is usually between a few tens of centimetres to a few meters even for high concentrations, making the illumination profile much more uniform than using concentrator optics with continuous light source. Also, it consumes low power and thus transfers less heat to the test device. These advantages make it become the industry standard for measuring the efficiencies of concentrator solar cells.

However, using flash lamp to characterise I-V of solar cells brings its own challenges. The pulse duration of the flash lamp typically ranges from a few hundreds of microseconds to a few hundreds of milliseconds and data must be acquired during this relatively short burst of illumination. The full I-V measurement can be either completed within one flash pulse or over several pulses to build up an I-V measurement. Either way the measurement requires high-speed power electronics, and commercial source meters usually do not meet these requirements. Fast I-V sweeping also causes transient errors, which refer to the current or voltage shifts due to the parasitic capacitance or inductance in the device. Large current and voltage gradients during a fast I-V sweep can make the transient error significant, especially in silicon solar cells [73, 74] where the carrier lifetime extends from microseconds to milliseconds. Also, measuring the illumination spectrum of a single flash
is more difficult to characterise compared to continuous light sources, since the spectrum is broad band and short lived.

Commercial flash testing system are therefore expensive because they aim to provide high spatial uniformity and spectral matching to standard reference spectrum to reach high accuracy [75]. However, the research interest of most academic research groups is developing new materials instead of fine-tuning the state-of-the-art high efficiency solar cells. For these research activities, the main purpose of measuring devices at high intensity illumination is to explore the properties of the material at high carrier injection regime, where precise efficiency measurements are not essential. As a result, a more cost-effective solution to the commercial offerings are required.

One of the implementations of low-cost flash systems has been reported in [76]. This system holds the test device at constant voltage during the light pulse, so multiple pulses are required to complete one I-V acquisition of a concentration. This constant-voltage approach can reduce the error due to the diffusion capacitance effect in the test cells, which is significant in long minority carrier lifetime materials [77]. This approach was further improved and patented by Sinton et al. [78], but it still requires several pulses to complete one I-V acquisition. This increases the time needed for a measurement and decreases the lifetime of the light bulbs.

III-V materials generally have shorter carrier lifetime compared to silicon, typically a few tens of nanoseconds. This allows a high-speed I-V sweep within short light pulse duration without introducing much transient error. As a result, the trend of high-concentration solar cell industry is moving towards single-pulse approach rather than multi-pulse [75]. However, publications on the details of the implementation and performance of the single-pulsed flash I-V system are limited. Hu et al.’s work [79] provides the most comprehensive details of implementing a single-pulsed flash I-V system to the best of our knowledge. However, the flash lamp in their implementation has a pulse duration more than 10 ms which is unusually long. Typical flash lamps that are commercially available at low cost
have stable flash intensities for 1 ms or less, so Hu et al.’s measurement electronics are not fast enough for acquiring data with a low cost light source, which requires less than 200 – µs measurement window.

In this chapter, we report an implementation of a very cost-effective flash system aiming at addressing the requirement of research groups that wish to test solar cells at high concentration but willing to sacrifice some measurement accuracy for cost. This flash tester adopts single-pulse method using homemade electronics and software to synchronise and measure the I-V characteristics. These details including the light source and measurement electronics are described in the first part of this chapter, followed by measurement results and a discussion of measurement errors and suggestions for further improvement.

5.1 Photographic flash unit as a solar simulator

Xenon arc lamps are commonly used in a solar simulator because of their close match to the solar spectrum, which is also the reason the xenon lamp is used in photographic flash to generate white light. As a result, photographic flash can be a very cost effective option for solar simulators. A Bowen Gemini 500R photographic flash head was chosen as the light source in this instrument and was sent to the National Renewable Energy Laboratory (NREL), USA to characterise its spectrum. Figure (5.1) shows the spectrum of this flash lamp over a 160-µs measurement window. The measured spectrum is plotted with standard reference spectrum ASTM G173-03 [29]. Both spectra are normalised to 1 kW/m². It shows that the both spectra match closely, apart from the characteristics emission peaks of xenon in near infra-red region.
Figure 5.1: The comparison of the flash spectrum measured by NREL and the ASTM G173-03 spectrum. Both spectra are normalised to 1000 W/m².

5.2 Time-resolved spectrum of the flash lamp

Time-resolved emission spectrum of the flash lamp was later characterised at Imperial College using an Andor DH340T-25U-03 spectrograph and an Andor iStar intensified CCD camera. The image intensifier in this camera can be gated and operated as a fast optical shutter to select the measurement window with a resolution of 1.2 ns. Owing to the grating used in the spectrograph, the camera could only capture a limited range of wavelengths at a time. Two measurement regimes were used, from 400 nm to 670 nm and from 690 nm to 960 nm. Time-resolved spectra were obtained by taking multiple measurements with different trigger times of the intensifier. The integration time for all the measurements was 10 µs.

Since the measurement of the time-resolved spectrum requires a number of flash shots, it is important to know the shot-to-shot variation of the flash lamp so the time-resolved spectrum can be interpreted correctly. The shot-to-shot variations of the flash was ob-
tained by repeating the measurement of the flash spectrum at the same trigger time many times. Figure (5.2)(a) and (b) show the averaged spectrum of 20 flash shots at the same shutter trigger time, with a repetition rate of 0.2 Hz. The reflectivity of the grating and the quantum efficiency of the CCD camera are not calibrated, hence we can only make relative comparisons of spectral intensity in time domain. Figure (5.2)(c) and (d) presents the standard deviation of each wavelength of the 20 shots normalised to the average intensity, which is also known as coefficient of variation:

$$c_v(\lambda) = \frac{\sigma(\lambda)}{\mu(\lambda)}$$ (5.1)

where $\sigma(\lambda)$ denotes standard deviation in this chapter, and $\mu(\lambda)$ is the average intensity at $\lambda$ of these 20 consecutive flash shots. The results in Figure (5.2)(c) and (d) show that $c_v(\lambda)$ are generally lower than 3% except some spikes at some wavelengths.

The time-resolved flash spectrum measurement were performed at two different time ranges. Figure (5.3)(a) and (b) shows the measured flash spectrum at a time range of 3 ms with a time step of 100 $\mu$s. Figure (5.5)(a) shows the temporal profile of the flash at several wavelengths that presented in Figure (5.3). It clearly shows that the temporal profile of these wavelengths generally follow the same trend. The intensity profiles rises sharply at the beginning, followed by an exponential-like decay with lifetimes of a few milliseconds.

Figure (5.4)(a) and (b) shows the measured flash spectrum at a time range of 1.2 ms with a time step of 10 $\mu$s, and Figure (5.5) (b) shows the temporal profile or several selected wavelengths presented in Figure (5.4). The analysis used in Figure (5.2) is applied to the measured spectral intensity between 200 $\mu$s and 400 $\mu$s, which is the target window for I-V measurements. The results are plotted in Figure (5.6). It shows that the $c_v$ within this window is slightly higher than the $c_v(\lambda)$ in Figure (5.2), suggesting that random error due to shot-to-shot variation dominates the $c_v(\lambda)$ in the 200 $\mu$s to 400 $\mu$s window.
Figure 5.2: (a) and (b): The averaged spectrum of the 20 consecutive flash shots measured at the same trigger time between (a) 400 nm to 670 nm and (b) 690 nm to 960 nm. (c) and (d): The coefficient of variations of the 20 consecutive flash shots measured at the same trigger time between (c) 400 nm to 670 nm and (d) 690 nm to 960 nm.
Figure 5.3: The measured time-resolved flash spectrum between (a) 400 nm to 670 nm and (b) 690 nm to 960 nm. The time step is 100-µs and the measured time range is 3 ms. Temporal intensity profiles of selected wavelengths are plotted in Figure (5.5)(a).
Figure 5.4: The measured time-resolved flash spectrum between (a) 400 nm to 670 nm and (b) 690 nm to 960 nm. The time step is 10-µs and the measured time range is 1.25 ms. Temporal intensity profiles of selected wavelengths are plotted in Figure (5.5)(b).
Figure 5.5: The time-resolved intensity profiles of the flash lamp at various wavelength with (a) 100-µs time step and (b) 10-µs time step. Both graphs share the same legend in (a). Full time-resolved profiles of all wavelengths measured with 100-µs and 10-µs are plotted in Figure (5.3) and Figure (5.4) respectively.
Figure 5.6: (a) and (b): The averaged spectrum within the 200-µs to 400-µs range between (a) 400 nm to 670 nm and (b) 690 nm to 960 nm. (c) and (d): The coefficient of variations within the 200-µs to 400-µs range between (c) 400 nm to 670 nm and (d) 690 nm to 960 nm.
5.3 The measurement electronics

To obtain the IV characteristics of a solar cell within the stable duration of the flash, some bespoke electronics were developed. The main circuit of this flash tester is illustrated in Figure (5.7). The device under test (DUT), current-measuring resistor, a voltage source and a field-effect transistor (FET) form a closed loop. The main concept behind the design is to use the FET as a variable resistor. The I-V of the DUT is swept by controlling the equivalent resistance of the FET, similar to the circuit reported in Ref. [80]. The current of the DUT is measured from the voltage across the current-measuring resistor. The voltages of DUT and the current-measuring resistor are measured via the instrumentation amplifier and recorded by a data acquisition (DAQ) unit. The equivalent resistance between the drain and source of the FET is demanded by the voltage of the gate-source voltage $V_{GS}$. When the FET is operated in linear regime, this relation can be approximated as [81]

$$R_{DS} = \frac{1}{2k(V_{GS} - V_T)}$$  \hspace{1cm} (5.2)

where $R_{DS}$ is the effective resistor across drain and source, $k$ is the scale factors, and $V_T$ is the gate threshold voltage. As a result, to sweep the DUT from short-circuit to open-circuit, $V_{GS}(t)$ should rise as a linear ramp. This can be achieved using an arbitrary function generator or a LM556 timer chip. The system constructed used the latter approach and a detailed circuit diagram is drawn in Figure (B.3) in the Appendix.

The I-V measurement is initiated by a 5 V transistor-transistor logic (TTL) pulse generated by a TTL output port of the data acquisition card. This TTL pulse triggers the flash and a delay gate generator. The delay gate generator generates a ramp signal as $V_{GS}$ to start the I-V measurement when the flash intensity starts to reach its peak. When the delay gate generator starts ramping $V_{GS}$, it sends out another TTL to the DAQ card to record the measured voltages from the DUT and the current-measuring resistor. This process is illustrated in Figure (5.8). Computer software was written to automate the entire synchronisation and data logging processes of the I-V measurement. When
the user triggers the I-V measurement from the software interface, the rest of the I-V measurement processes is completed by the software within one second.

The main advantage of this implementation is its simplicity. It does not require high power electronics to supply the current or voltage for measuring the I-V at very high concentration. Also, the simplicity of this circuit allows the circuit to be operated at high speed at large current. This implementation also has some drawbacks. Without the presence of power electronics, the lowest voltage that the DUT can reach is $IR_m$, where $I$ is the DUT current and $R_m$ is the resistance of the current-measuring resistor. For multi-junction cells, it is desirable to keep $IR_m$ below 0.5 V to retain the shape of an I-V measurement. This implementation also limits the maximum current that can be measured. For device current over several amps, the cables and wires in the circuit can create significant voltage offset, making the lowest available voltage of DUT shift further away from short-circuit.

The cost of the components in this system are listed in Table 5.1. It shows that the
Figure 5.8: The illustration of the triggers and measurement window of a flash I-V measurement. The flash lamp unit starts to discharge after it receives the initial trigger. The I-V measurement starts after a certain amount of delay of the initial trigger signal. The coloured area indicates the 160-us measurement window.

Table 5.1: The cost sheet of the flash I-V system

<table>
<thead>
<tr>
<th>Item</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bowen Gemini 500R flash head</td>
<td>$690</td>
</tr>
<tr>
<td>Picoscope data acquisition card</td>
<td>$2000</td>
</tr>
<tr>
<td>Instrumentational amplifiers and other electronics consumables</td>
<td>$500</td>
</tr>
<tr>
<td><strong>total</strong></td>
<td><strong>$3190</strong></td>
</tr>
</tbody>
</table>

total cost of this system can be less than USD 4,000, making it a very cost-effective option.

5.4 Measurement results on multi-junction solar cells

Concentrator solar cells usually need to be tested over a range of different concentrations. The intensity of illumination is altered by varying the distance between the DUT and the flash lamp unit. The highest concentration that this system can achieve is around 1000 suns.
A reference device is placed next to the DUT to calibrate the concentration of illumination. The short-circuit current of the reference device is measured simultaneously with the DUT. The one-sun current of the reference \( I_{1-SUN}^R \) was calibrated by the Fraunhofer Institute for Solar Energy Systems, Germany (Fhg-ISE). The concentrations of the flash is then estimated by taking the ratio of the measured short-circuit current of the device \( I_{SC}^R \) and \( I_{1-SUN}^R \). The efficiency can then be calculated using

\[
\eta = \frac{P_{max}}{P_{in}} = \frac{\max(IV)}{(I_{SC}^R/I_{1-SUN}^R) \cdot (\text{one-sun power}) \cdot (\text{device area})}
\] (5.3)

For each illumination intensity, ten repeated I-Vs are taken to reduce the random errors due to the fluctuation of the flash intensity. The efficiencies and fill factors are calculated by averaging the efficiencies and fill factors of each individual I-V.

Figure (5.9) shows the ten measurements of the dual-junction sample 416-2698-h17 at around 100 suns. The sample is the same as the dual-junction device presented in Section 3.4. As discussed in the previous section, the measured I-V cannot reach short circuit due to the voltage offset of the current-measuring resistor. The short-circuit current is then estimated by fitting the linear parts of the I-V curves. The coefficient of variation \( c_v \) of these I-Vs is around 3%, which matches the variation of the flash spectrum presented in the previous section.

The measured efficiencies and fill factors of the device are presented in Figure (5.10) with the results calibrated by Fhg-ISE on the same device. The difference of efficiencies between these two measurements is around 1.5% (absolute).

### 5.5 Discussions on measurement errors

**Spectrum mismatch against standard test spectrum** The mismatch of the flash spectrum against standard test spectrum causes two different errors. One is the estimation of concentrations, and the other is current-matching condition of the subcells.
Figure 5.9: (a) Ten measured I-Vs of sample 416-2698-h17 at near 100 suns using the flash I-V system. (b) The I-Vs of (a) near the maximum power points.

As mentioned in the previous section, the concentration is estimated by the ratio of the measured short-circuit current and the known one-sun current. The mismatch of flash spectrum and the standard test spectrum can thus cause errors in estimating concentration. Table 5.2 compares the one-sun current of each subcell in the sample 2698 under the flash spectrum and standard test spectrum normalised to 1 kW/m$^2$. The one-sun current is calculated by measured EQE of each subcell, that is,

$$J_{1\text{-sun}} = \int_0^\infty EQE(E)\phi_t(E)dE$$

where $J_{1\text{-sun}}$ is the one-sun current density and $\phi_t(E)$ is the illumination spectrum. Under the illumination of flash, the current is limited by the top cell, however, the current is limited by the the bottom cell under the standard test spectrum. For this device, the illumination concentration is overestimated due to the mismatch of two spectra. For example, when the device is illuminated by the flash with intensity 1 kW/m$^2$, it generates the current density of 119.8 A/m$^2$, but using 128.4 A/m$^2$ as the one-sun current reports only 0.93 suns, leading to an overestimation of efficiencies.

As shown in Table 5.2, the spectral mismatch changes the current-limiting subcell and
Figure 5.10: The comparisons of (a) efficiencies and (b) fill factors of sample 416-2698-h17 against concentrations measured by this flash I-V system and Fhg-ISE’s concentrator testing system.

Table 5.2: Predicted one-sun current density of the subcells in sample 416-2698-h17 using measured EQE as shown in Figure (3.12). The absorption edge of the top cell is around 740 nm and the bottom cell is 1050 nm.

<table>
<thead>
<tr>
<th></th>
<th>top cell (A/m²)</th>
<th>bottom cell (A/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>measured flash spectrum</td>
<td>119.8</td>
<td>166.1</td>
</tr>
<tr>
<td>ASTM G173-03</td>
<td>133.0</td>
<td>128.4</td>
</tr>
</tbody>
</table>
its short-circuit current. In this case, the red-rich flash spectrum creates larger current-mismatch, leading to higher fill factors than under standard spectrum [82].

**Transient effects of the device and the circuit** Transient effects can cause errors in measuring the current of DUT. When the device is illuminated and operated at high voltage, the injected free carriers increase the capacitance of the device. The capacitance cause current drift when large \( \frac{dV}{dt} \) is applied to the cell during the measurement, which is given by

\[
I_D = C_d \frac{dV}{dt} \tag{5.5}
\]

where \( C_d \) is the capacitance caused by the carriers. When the concentration of illumination is below one thousand suns, we can assume that the minority carrier diffusion current still dominates the recombination current in the device. The capacitance causes by the minority carriers in neutral regions can be described by [77]

\[
C_d = \frac{q}{kT} \tau_n I_{rec} \tag{5.6}
\]

where \( \tau_n \) is the minority carrier lifetime and \( I_{rec} \) is the recombination current. In this setup, \( \frac{dV}{dt} \) is around \( 2 \times 10^{-4} \) V/sec, and \( \tau_n \) is around 10 ns for a GaAs solar cell. This gives \( I_D \sim 0.008I_{SC} \), and it will be even smaller in lower bias. As a result, the transient effects in III-V solar cells are very low in the regime that minority carrier diffusion current is dominant, which is mainly due to the short carrier lifetime \( \tau_n \) in III-V compound semiconductor.

However, the transient effects arising from other circuit components can be significant when the device is operated at high current. It was observed that using very long cables can distort the I-V curves at high current regime, suggesting that the inductances of the circuit components becomes very significant due to large current gradient \( \frac{dI}{dt} \).
5.6 Suggestions for improving the flash system

A flash lamp with longer pulse duration will help improve the measurement accuracy. A longer pulse duration provides longer time window for I-V measurement, which reduces the transient error in the DUT and other circuit components. Moreover, a longer I-V measurement window makes it possible to have more precise control and design options of the measurement electronics. For example, with longer I-V sweeping time, we can add a feedback control circuit to offset the voltage across current-measuring resistor, making it possible to start the I-V sweeping from negative voltage. With the capabilities of offsetting the voltages of the current-measuring resistor, higher resistances can be chosen for the current-measuring resistor to raise the signal to noise of the current measurement. Cost effective flash lamps with longer pulse durations may not be commercially available but can possible be implemented by referring to Ref. [83].

Measurement errors due to the light sources can also be improved. Spectral mismatch can be further calibrated by using the time-resolved spectrometer, with the reflectivity of grating and spectral response of the CCD camera calibrated. Spatial uniformity becomes crucial for measuring very large-area devices, which can be improved by implementing an enclosure based on a number of publications [79,84].

5.7 Conclusions

A very cost-effective concentrator testing system was implemented using a commercial photographic flash unit as the light source. An electronic circuit and software for synchronisation were developed to measure the entire I-V characteristics of a solar cell within a 160 µs window at the flash pulse peak. The circuit is implemented by controlling the effective resistance of a FET to sweep the DUT from open-circuit to nearly short circuit. Another circuit and software were developed to synchronise the measurement electronics
and the flash pulse. This system can measure the I-V of a concentrator solar cell to near 1000 suns, and the total cost of the system is less than USD 4000, making it a very cost-effective option for the purposes that high measurement accuracy is not essential.

The time-resolved spectrum of the light source in visible wavelengths was characterised by using a gated intensified CCD camera capable of operating with a high-speed (ns) optical shutter. The time-resolved flash spectrum showed that the intensity variations across almost all wavelengths are less than 4%, and the shot-to-shot variation are less than 3% across all measured wavelengths.

The main systematic errors come from the spectral mismatch of the light source against the standard test spectrum. The illumination spectrum of the flash unit make the GaAs subcell overproduces current in a standard dual (InGaP/GaAs) or triple junction (InGaP/GaAs/Ge) solar cells. Since the concentrations are estimated by the short-circuit current of the current-limiting subcell, the spectral mismatch causes errors in estimating concentrations. Also, the extent of current mismatch also affects the measured fill factors.

Another sources of systematic errors come from the transient errors of the DUT and other components of the circuits. Transient errors are mainly caused by the parasitic inductance and capacitance of the DUT and other circuit components. For III-V solar cells, the errors of measured short-circuit current due to the its diffusion capacitance is less than 1%, but it was found that other components such as cables contribute to the transient errors.

Despite these sources of errors, this system shows reasonable agreement with the measurement made by Fhg-ISE. Longer pulse duration of the flash lamp would further improve the measurement since it enables more design options for the measurement electronics and helps reduce the transient errors of the DUT and other circuit components. Spectral mismatch can be corrected using a calibrated time-resolved spectrometer and the spatial uniformity can be improved by designing a suitable enclosure.
Chapter 6

Measuring Sheet Resistance Values of Single Junction Solar Cells by Electroluminescence Imaging

6.1 Series resistance in solar cells

In general, a highly efficient multi-junction solar cell will yield the lowest cost of electricity when operated in a concentrator system at 1000X solar concentration. Most commercial solar cells have a peak efficiency at a few hundred suns due to resistive losses in the solar cell. These losses significantly degrade the fill factor and efficiency of the solar cell. In this chapter, an electroluminescent technique is developed for determining the sheet resistivity of the solar cell emitter.

Series resistance arises from several components in a solar cell: the sheet resistance of the emitter, tunnel junction, the contact resistance at metal-semiconductor interface, and the resistance of the metal grids. However, the trade-offs between the series resistance and other solar cell parameters need to be considered. For instance, sheet resistance of the emitter layer can be reduced by either decreasing the width between front metal
contact grid or increasing the emitter doping. However, decreasing the pitch size means increase the coverage of the metal on the device, and increasing the doping will also increase the non-radiative recombination in the emitter layer. Moreover, although the resistance of metal grids can be reduced by increasing their heights or widths, higher metal grids increase shading and wider widths reduces the photon active area of the solar cell. As result, to optimise the series resistance, it is essential know the values of sheet resistance and contact resistances. Likewise, an increased doping will yield lower emitter sheet resistance but will compromise the minority carrier diffusion length in the emitter and lower the quantum efficiency of the solar cell.

A lumped value of series resistance can be approximated by fitting the dark I-V or light I-V characteristics based on Eq. (2.55) [85–88], but this method is accurate only when contact resistance dominates the series resistance. As illustrated in Figure (6.1), when emitter sheet resistance is negligible, the circuit network drawn in Figure (6.1) can be reduced to Figure (2.5). A more accurate method is using the full distributed circuit network model to fit the I-V characteristics of the device [89–91], which can separate the contributions of sheet resistance and contact resistance. However, fitting a single I-V curve using so many parameters can sometimes yield many different parameter sets that give good fit, which reduces the accuracy of this method.

The Transmission Line Measurement (TLM) is an accurate method for determining the sheet and contact resistance of the material, but it has several disadvantages that limit its applicability of studying resistance in solar cells. First of all, it requires the fabrication of special structures on the candidate material. Second, the method assumes that the piece of material for TLM is identical to the solar cell, which is not always a valid assumption, especially when the area of the solar cell is large. As a result, a technique capable of determining the sheet resistance and contact resistance of an actual solar cell without any additional fabrication is desirable.

Since the intensity of electroluminescence (EL) depends on the local quasi-Fermi level
Figure 6.1: An illustration of EL and the equivalent circuit model.

separation, EL can be used to determine a voltage profile over the surface of the solar cell, and can then be related to the sheet and metal contact resistance. Figure (6.1) illustrates a circuit model that describes the relation between EL and resistance. Assuming that the material of the solar cell is very uniform, i.e., every diode and resistor in the figure are equivalent, the EL emission near the metal front grid will be larger than the emission at the centre because of the power loss caused by the resistance in the emitter.

Figure (6.2) demonstrates the relation between the solar cell efficiency and EL distribution. The main figure shows the efficiency and fill factors of a single junction solar cell as the function of short circuit current. The inset figures are the EL images operated at the voltage that generate the same recombination current as the short circuit current indicated by the dashed line. At low short-circuit current, the EL intensity is uniform and the fill factors increase with short circuit current. When the short-circuit current reaches $10^{-2}$ A, both efficiencies and fill factors start to drop. Also, the EL intensity becomes non-uniform because the sheet resistance starts to dominate the IV characteristics of the cell.

Although EL imaging is widely used in the photovoltaic industry to make a qualitative
Figure 6.2: The efficiencies and fill factors of a single junction solar cell at different short-circuit currents with its EL images. The inset figures are the EL images operated at the voltages that generate the same recombination currents as the short circuit currents indicated by the dashed line.
estimate of series resistance, methods to extract the exact local sheet resistance have not been fully developed. Reyna et al. uses EL to separate the contributions of contact and sheet resistances [92]. Ramspeck et al. combines EL and lock-in thermography [93,94] to measure the local series resistance [95]. Hinken et al. [96] demonstrated an elegant method to retrieve local resistance from EL images and the result agreed with Ref. [95] very well. However, their model can only be applied to situations where there is either low current regime or very wide pitch for the metal grids, making it unsuitable for concentrator solar cells. Xiong et al. [97] used a different approach to analyse the EL images of InGaP solar cells and matched their result with simulation, but made no comparison against other experimental methods.

In this chapter, a new method based on a circuit network model is developed to quantify the sheet resistance from EL imaging. The local sheet resistance of a single-junction GaAs solar cell is determined and for the first time, compared with the result measured from the same sample by TLM.

### 6.2 Modelling EL images

The EL image can be modelled by an equivalent electrical circuit model by extending the diode model. Every pixel on the EL image is considered as a radiative diode. The EL intensity $\phi_k$ of the diode on pixel $k$ can be assumed to be proportional to the current $J_k$ flowing through this diode, which can be written as

$$\phi_k(V) = \beta J_k(V)$$  \hspace{1cm} (6.1)

where $\beta$ is a coefficient related to the emission angle of EL and optical loss of the EL imaging setup. Each radiative diode on $k$-th pixel is assumed to be an ideal diode:

$$J_k(V) = J_{01} \exp\left(\frac{qV}{kT}\right)$$  \hspace{1cm} (6.2)
The circuit network can be constructed as illustrated in Figure (6.3) to simulate the spatial EL of a solar cell. Figure (6.4) (a) shows the simulated voltage map of a circuit network simulation. The circuit network contains 75 × 75 pixels. The voltage sources are connected to the pixels at \( x = 0 \). It clearly shows that the voltages near the voltage sources are higher than the voltages on the other end. Figure (6.4) (b) is the voltage map profile along x-direction in (a). Note that the voltage maps can be considered as EL images plotted in log scale because of Eq. (6.2).

### 6.3 EL imaging experiment

The experimental setup for the EL imaging experiment is illustrated in Figure (6.5). A fixed voltage is applied by a Keithley 2430 1-kW pulse mode sourcemeter. The 1 mm × 1
Figure 6.4: (a) The simulated voltage maps of a circuit network with $75 \times 75$ pixels using the equivalent circuit model drawn in Figure (6.3). This calculation assumes that the front contact lies on the $x=0$ axis. (b) The voltage profile of (a) in $x$-direction.
Figure 6.5: Schematic drawing of the EL imaging setup. The sample is imaged by an microscope object onto the CCD camera and forward biased by a sourcemeter. The CCD camera and the sourcemeter are synchronised by a homemade control software.

mm concentrator solar cell is imaged onto a $8978.4 \, \mu \text{m} \times 6708 \, \mu \text{m}$ charge-coupled device (CCD) chip through a microscope. A QImaging QiClick 12-bit digital CCD camera with resolution $1392 \times 1040$ is used to capture the EL images. To reduce the heat dissipated to the test cell, the forward bias is applied momentarily and synchronized with exposure time of the camera.

The test sample is a p-i-n single junction device 414-3874D with 50 stacks of GaAsP/InGaAs quantum wells and GaAs p and n regions, which gives EL emission at around 930 nm. The layer structure of this sample is listed in Table A.6. The EL image of this sample at low current appears to be very uniform, so we can assume the equivalent diode of every pixel is almost identical in this sample.

To cover the large range of the measured EL intensity at different forward biases, it is necessary to change the exposure time of the CCD camera to optimise the contrast of the image. It is therefore important to confirm that the exposure time has a linear relationship with the EL intensity, enabling the data to be placed onto the same scale.

To test the linearity of the exposure time, a sample was set at a forward bias that
Figure 6.6: The CCD camera read counts of the EL image against the exposure times of the CCD camera. The y-axis is the averaged read count of bright areas in the EL image. The solid line is the linear fitting of the averaged CCD counts and the exposure times.

gives uniform EL images. After that, the EL image is taken with different exposure times. The averaged CCD read count against the exposure times are plotted in Figure (6.6), confirming the linearity of the exposure time over the practical test range.

### 6.4 Fitting the EL images with circuit network model

Figure (6.9) shows the measured EL images and the line profiles for the analysis. The EL intensity is plotted along the red line of the inset EL image. Since the front metal grid of the device is either parallel or intersected by 90°, this symmetry allows the two-dimensional circuit network to be reduced to one-dimensional in order to simplify the computation. The intensity profile along the red line is chosen to represent the full EL image, because it meets the criteria that the EL intensity gradient along y-directions is nearly zero, showing that the intensity profile is not coupled with the resistances in y-directions. Hence a 1-D analysis can be applied on this line profile.
Figure 6.7: The 1-D equivalent circuit model for fitting the EL profile. The diodes of each pixel are labelled from $D_0$ to $D_n$ and the sheet resistors are labelled from $R_0$ to $R_n$. The amount of current that flows through the resistor $R_k$ is labelled as $j_k$. $R_c$ is the contact resistance.

In this 1-D model, the pixels from the metal grid to the middle between the two grids can be modelled by the circuit illustrated in Figure (6.7). The cell is divided into $n+1$ pixels from the position of a front contact to the middle between the front contacts. Each diode $D_k$ is connected to a sheet resistor $R_k$, except that $D_n$ is connected to the external circuit through a contact resistor $R_c$. Recall that each diode is described by

$$J_k(V) = J_{01} \exp \left(\frac{qV}{kT}\right) \quad (6.3)$$

The relation of voltage and current for each diode $D_k$ and $R_k$ can be calculated by using Kirchhoff’s law, that is,

$$j_k R_k + V_k = V_{k+1} \quad (6.4)$$

$$j_k = J_k(V_k) + j_{k-1} \quad (6.5)$$

where $j_k$ is the current that flows through the sheet resistance $R_k$. This set of equations are calculated numerically. Once the voltage of the diode $D_0$ is set, the voltage and
Table 6.1: Fitted parameters from EL images

<table>
<thead>
<tr>
<th>applied voltage (V)</th>
<th>1.25</th>
<th>1.31</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_{01}$</td>
<td>$6 \times 10^{-28}$</td>
<td>$6 \times 10^{-28}$</td>
</tr>
<tr>
<td>$V_0$</td>
<td>1.242</td>
<td>1.17</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$4 \times 10^6$</td>
<td>$4 \times 10^6$</td>
</tr>
<tr>
<td>$R_k(\Omega/\Box)$</td>
<td>330</td>
<td>390</td>
</tr>
</tbody>
</table>

current flowing through each diode $D_k$ and resistor $R_k$ can be calculated subsequently by iterating Eq. (6.4) and Eq. (6.5).

The unknown parameters for calculating Eq. (6.5) are $R_k$, $V_0$, $\beta$ and $J_{01}$. The range of the parameter $J_{01}$ can be estimated by fitting the dark IV characteristics of the sample. In this case, the dark IV was consistent with $J_{01}$ of around $10^{-16}$ A/m$^2$, which is equivalent to $\sim 10^{-28}$ A/pixel. The fitted $J_{01}$ from EL is not exactly the same as the value fitted from dark IV, since the measured dark IV characteristics may be distorted by contribution of series resistance. $\beta$ is the calibration factor which should stay constant throughout the measurement. $R_k$ is the sheet resistance that we are trying to extract.

The fitted parameters for EL images measured at two different forward biases are listed in Table 6.1 and the experimental and modelled EL line profiles are plotted in Figure (6.9). The fitted parameters meet the requirements stated in the last paragraph and fit the EL profile very well.

The sheet resistance values extracted for EL fitting were verified by TLM. A device for TLM is fabricated near the test sample, as shown in Figure (6.8). The results of TLM is plotted in Figure (6.10). Figure (6.10)(a) shows the individual IV characteristics of each pair of neighbouring metal grids. The resistance yielded from each IV characteristics is plotted as the function of neighbouring grid distances in Figure (6.10)(b). The slope of the fitted line in Figure (6.10)(b) is 0.72 $\Omega/\mu$m. The sheet resistance is derived by multiplying this value by the width of the metal grid (500 $\mu$m). This gives the sheet
resistance value of $360 \, \Omega/\square$, which is very close to the sheet resistance extracted by EL fitting.

## 6.5 Conclusion

Achieving concentrations more than 1000X of CPV cells is necessary to drive the cost of electricity down, however, the series resistance in CPV solar cells starts to dominate the power loss at high concentrations. Although series resistance can be decreased by adding more front grid coverage or raising the doping densities of the emitter, these approaches accompany with lower photoactive areas of the solar and higher carrier recombination rates, making it a difficult optimisation problem of designing CPV. Therefore, knowing the sources of resistance in a CPV cell is crucial for optimising the performance of a concentrator cell. Although using TLM is an accurate method for determining sheet and contact resistances, the requirement of additional device fabrication limits its applicability.

A two dimensional circuit network model was developed to simulate the EL image of a solar cell. This uses one diode and several resistors to simulate a very small area of a solar cell, which can be used to represent a pixel in an EL image. This model provides a tool to optimise the design of the solar cell or extract the sheet resistance of the solar
Figure 6.9: (a) The experimental and fitted EL profiles at two forward biases 1.3 V and 1.25 V along the red line of the EL image. (b) The experiment and fitted EL profile between pixel 250 and pixel 350 in (a).
Figure 6.10: TLM results of a device near the test sample for EL. (a) shows the individual I-Vs of each pair of neighbouring grids, and (b) is the resistance extracted from the I-Vs in (a) versus grid distances. The sheet resistance and contact resistance can then be extracted from fitted slopes and interception in (b), as described in Section 2.14.
cell from the measured EL images. An EL experiment was performed on a single junction device to extract its sheet resistance by fitting the EL images with the circuit network model. To obtain accurate EL imaging results for quantitative analysis, the linearity of the CCD camera are carefully calibrated. Also, a software was written to synchronise the sourcemeter and the image acquisition to keep the cell temperature constant. The sheet resistance of the device was extracted by fitting the circuit network model with some other known parameters.

To validate this model, the sheet resistance also was measured by TLM method on the same semiconductor wafer. The result matches the sheet resistance by this new EL imaging method. This confirms the validity of the circuit network model, which can be used as a design tool for more complex front metal grid designs.
Chapter 7

Summary and Further Work

The first part of this thesis investigates the material properties of a novel InGaP/InGaAsP MQW structure, which has been demonstrated as a means to adjust the absorption edge of an InGaP top cell in a multi-junction solar cell.

In Chapter 3, InGaP/InGaAsP MQWs are studied by a number of characterisation techniques. XRD rocking curves and RSMs are studied on these MQW samples. The result shows that the quantum wells are compressively strained. The layer structures and compositions are extracted by fitting the XRD rocking curves, however, large misorientation between the substrate and the epitaxial layer make the fitting less accurate.

The carrier dynamics of InGaP/InGaAsP MQWs are studied by PL and TRPL decay experiment. The results of temperature dependent PL shows that the InGaP/InGaAsP MQWs has an internal radiative efficiency of 82%. The activation energy analysis of PL shows that this MQW structure has an activation energy of 0.16 eV, which is mainly due to confinement potential of holes according to the estimation of electron affinities.

TRPL decays of the InGaP/InGaAsP MQWs are also studied. The measured TRPL decays were fitted using a free-carrier recombination model to extract the recombination coefficients. This analysis shows that the InGaP/InGaAsP MQWs has very slow non-radiative recombination rate, which explains the high internal radiative efficiency.
measured by PL.

Dual-junction dual-MQW devices were also characterised. The absorption coefficients of the MQWs were extracted from the measured EQE. With the extracted absorption coefficient of MQW, the PL spectrum were modelled using detailed balance model. The modelled and experimental PL show good agreement.

In Chapter 4, the formalisms of photon coupling and its relation to radiative efficiencies are discussed. Photon coupling of dual-MQW dual-junction devices were measured by both EL and PL. It was demonstrated that the photon coupling efficiencies dual-MQW dual-junction devices can reach over 40%. The measured photon coupling efficiency also matches the modelled photon coupling efficiency very well.

To further optimise InGaP/InGaAsP MQWs for top cells, the crystal structure of the MQWs needs to be studied in more details. Wider range of RSMs or transmission electron microscopy can provide more information of the crystal structure, which helps determine the layer thicknesses and compositions with much better accuracies. These information can also provide more understandings of the growth mechanisms of this MQW structure, which is essential for growing large stack of this structure with high qualities.

The second part of this thesis reports the development of several advanced characterisation techniques of concentrator solar cells.

The development of a low-cost flash I-V measuring system is reported in Chapter 5. Time-resolved spectrum of the flash lamp in visible wavelengths was characterised and analysed. The normalised standard deviation of shot-to-shot flash intensities are less than 3% in across almost all the visible wavelengths. The normalised standard deviation within the 200 $\mu$s I-V measurement window is less than 4%. The results measured from this flash I-V shows good agreement with the results characterised by Fhg-ISE. The main error comes from the spectral mismatch between the flash spectrum and the standard test spectrum. The spectral mismatch results in the inaccuracies of estimating the input power of the solar cells and the fill factors. Transient effects due to the high-speed I-V
measurement also causes errors.

The spectral mismatch of the flash should be further characterised and reduced to raise the accuracies of this system. Moreover, employing a flash lamp with longer peak pulse duration can help improve the accuracies in many ways. First of all, longer I-V measurement window allows more feasible options of electronics design. Also, the transient effects in the circuit components and DUT can be reduced.

A new approach for assessing the local sheet resistivity using spatial EL imaging is presented in Chapter 6. This approach is based on fitting the modelled EL profile to the experimental EL profile. The modelled EL profile is calculated by a circuit network model. The fitted sheet resistivity agrees with the result measured by TLM very well. However, this method needs to be validated by measuring more different samples. Moreover, this work can be extended to multi-junction solar cells.
References


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Appendix A

Layer Structure of Samples

Table A.1: Layer structure of sample KLB494

<table>
<thead>
<tr>
<th>layer name</th>
<th>material</th>
<th>doping</th>
<th>thickness (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>cap</td>
<td>Al(0.33)Ga(0.67)</td>
<td>UD</td>
<td>0.14</td>
</tr>
<tr>
<td>wellx60</td>
<td>GaAs</td>
<td>UD</td>
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<td></td>
<td>Al(0.33)Ga(0.67)As</td>
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<td>0.5</td>
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<td>substrate</td>
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<td>layer name</td>
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<td>doping</td>
<td>thickness (µm)</td>
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<td>----------------------</td>
<td>-------------------</td>
<td>-----------------</td>
<td>----------------</td>
</tr>
<tr>
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<td></td>
</tr>
<tr>
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<td>0.01</td>
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<td>N:Si 2.00e17</td>
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<td></td>
<td>GaAs</td>
<td>N:Te 1.00e19</td>
<td>0.200</td>
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<td>GaAs</td>
<td>N:Te 1.0e19</td>
<td>0.100</td>
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<td>Al(0.11)GaAs</td>
<td>N:Si 2.0e18</td>
<td>0.0641</td>
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<td>Al(0.47)GaAs</td>
<td>N:Si 2.0e18</td>
<td>0.0163</td>
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<td>Al(0.80)GaAs</td>
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<td>Al(0.47)GaAs</td>
<td>N:Si 2.0e18</td>
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<td>N:si 2.0e18</td>
<td>0.0641</td>
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<td>GaAs</td>
<td>N:Si 2.0e18</td>
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Table A.3: Layer structure of TS462γ

<table>
<thead>
<tr>
<th>layer name</th>
<th>material</th>
<th>doping</th>
<th>thickness (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>cap</td>
<td>GaAs</td>
<td>P:C &gt; 1e20</td>
<td>0.150</td>
</tr>
<tr>
<td>window layer</td>
<td>Al(0.8)GaAs</td>
<td>P:C &gt; 1e20</td>
<td>0.030</td>
</tr>
<tr>
<td>emitter</td>
<td>GaInP</td>
<td>P:Zn 3e18</td>
<td>0.040</td>
</tr>
<tr>
<td></td>
<td>GaInP</td>
<td>UD</td>
<td>0.060</td>
</tr>
<tr>
<td></td>
<td>GaInP</td>
<td>UD</td>
<td>0.1</td>
</tr>
<tr>
<td>well</td>
<td>InGaAsP x20</td>
<td>UD</td>
<td>0.0046</td>
</tr>
<tr>
<td>barrier</td>
<td>InGaP x20</td>
<td>UD</td>
<td>0.0131</td>
</tr>
<tr>
<td>base</td>
<td>InGaP</td>
<td>N:Si 1e17</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>Al(0.8)Ga(0.2)As</td>
<td>N:Si 1e18</td>
<td>0.03</td>
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<tr>
<td></td>
<td>GaAs</td>
<td>N:Si &gt; 1e20</td>
<td>0.030</td>
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<tr>
<td>lateral conducting layer</td>
<td>Al(0.4)Ga(0.6)As</td>
<td>N:Si &gt; 1e20</td>
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<tr>
<td></td>
<td>Al(0.8)Ga(0.2)As</td>
<td>N:Si 1e18</td>
<td>0.03</td>
</tr>
<tr>
<td>emitter</td>
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<td>N:Si 6e18</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>GaAs</td>
<td>N:Si 2e18</td>
<td>0.03</td>
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<tr>
<td></td>
<td>GaAs</td>
<td>UD</td>
<td>0.01</td>
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<tr>
<td>barrier</td>
<td>GaAs(0.14)P(0.86) x40</td>
<td>UD</td>
<td>0.0095</td>
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<tr>
<td>well</td>
<td>Ga(0.89)In(0.11)As x40</td>
<td>UD</td>
<td>0.0077</td>
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<tr>
<td>barrier</td>
<td>GaAs(0.14)P(0.86) x40</td>
<td>UD</td>
<td>0.0095</td>
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<tr>
<td>base</td>
<td>GaAs</td>
<td>P:C 2e17</td>
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GaAs substrate
Table A.4: Layer structure of TS462

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<th>layer name</th>
<th>material</th>
<th>doping</th>
<th>thickness (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>cap</td>
<td>GaAs</td>
<td>P:C &gt; 1e20</td>
<td>0.150</td>
</tr>
<tr>
<td>window layer</td>
<td>Al(0.8)GaAs</td>
<td>P:C &gt; 1e20</td>
<td>0.030</td>
</tr>
<tr>
<td>emitter</td>
<td>GaInP</td>
<td>Zn: 3e18</td>
<td>0.040</td>
</tr>
<tr>
<td></td>
<td>GaInP</td>
<td>UD</td>
<td>0.060</td>
</tr>
<tr>
<td></td>
<td>GaInP</td>
<td>UD</td>
<td>0.1</td>
</tr>
<tr>
<td>well</td>
<td>InGaAsP x20</td>
<td>UD</td>
<td>0.0046</td>
</tr>
<tr>
<td>barrier</td>
<td>InGaP x20</td>
<td>UD</td>
<td>0.0131</td>
</tr>
<tr>
<td>base</td>
<td>InGaP</td>
<td>N:Si 1e17</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>Al(0.8)Ga(0.2)As</td>
<td>N:Si 1e18</td>
<td>0.03</td>
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<tr>
<td></td>
<td>GaAs</td>
<td>N:Si &gt; 1e20</td>
<td>0.030</td>
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Table A.5: Layer structure of TS287

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<th>doping</th>
<th>thickness (µm)</th>
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<tr>
<td>cap</td>
<td>GaAs</td>
<td>P:C 8e19</td>
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</tr>
<tr>
<td>window layer</td>
<td>AlInP</td>
<td>P:Zn &gt; 1e20</td>
<td>0.020</td>
</tr>
<tr>
<td>emitter</td>
<td>Al(0.8)Ga(0.2)As</td>
<td>P:C 1.1e20</td>
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</tr>
<tr>
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<td>GaInP</td>
<td>UD</td>
<td>0.05</td>
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<tr>
<td>well</td>
<td>InGaAsP x37</td>
<td>UD</td>
<td>0.0037</td>
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<tr>
<td>barrier</td>
<td>GaInP x37</td>
<td>UD</td>
<td>0.015</td>
</tr>
<tr>
<td>base</td>
<td>GaInP</td>
<td>N:Si 1e17</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>AllInP</td>
<td>N+:Si</td>
<td>0.03</td>
</tr>
<tr>
<td>tunnel junction</td>
<td>GaAs</td>
<td>N++: Si</td>
<td>0.015</td>
</tr>
<tr>
<td>tunnel junction</td>
<td>Al(0.45)GaAs</td>
<td>P++ C</td>
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<td>Al(0.8)GaAs</td>
<td>P+:C</td>
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<td>emitter</td>
<td>GaAs</td>
<td>P:C 2E18</td>
<td>0.4</td>
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<td></td>
<td>GaAs</td>
<td>UD</td>
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<td>In(0.23)Ga(0.77)As</td>
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Table A.6: Layer structure of 414-3874D

<table>
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<th>doping</th>
<th>thickness (µm)</th>
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<td>GaAs</td>
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<tr>
<td>window layer</td>
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<td>GaAs</td>
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<td>0.07</td>
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<td>emitter</td>
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<td>P:C 2.0e18</td>
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<td>GaAs</td>
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<td>barrier</td>
<td>GaAs(0.90)P</td>
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<td>GaIn(0.109)As x50</td>
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<td>barrier</td>
<td>GaAs(0.90)P x50</td>
<td>UD</td>
<td>0.0158</td>
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<td>GaAs</td>
<td>UD</td>
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<td>base</td>
<td>GaAs</td>
<td>N:Si 2.0e17</td>
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<td></td>
<td>GaAs</td>
<td>N:Si 1e18</td>
<td>0.1</td>
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GaAs substrate
Appendix B

Circuit Diagrams
Figure B.1: The circuit for controlling the light-biasing LEDs in the spectral response measurement setup.
Figure B.2: Transimpedance amplifier for controlling the device bias in spectral response measurement setup.
Figure B.3: Delay and ramp function generator for flash I-V.
Publications

Journal Articles


Conference Presentations

- Oral presentation, InGaP/InGaAsP quantum well top cells and the pathways to high-efficiency multi-junction quantum well solar cells, 23rd Photovoltaics and Engineering Conference, October, 2013

- Oral presentation, Dual-junction solar cells with multiple-quantum-well top cell, 9th International Conference on Concentrator Photovoltaic Systems, April, 2013, DOI: 10.1063/1.4822196


- Oral presentation, Measuring sheet resistance values of single junction solar cells by electroluminescence imaging, 21st International Photovoltaic Science and Engineering Conference, Fukuoka, Japan, November 2011

- Invited oral presentation, Multiple-quantum-well top cells for multijunction concentrator solar cells, 37th IEEE Photovoltaic Specialists Conference, Seattle, USA, June 2011