Development of a particle image velocimetry technique for three-dimensional flame structure analysis

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Thesis submitted for the degree of
Doctor of Philosophy
of
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Abstract

A technique for the measurement of three-dimensional quantities in turbulent premixed flames was developed. The need for this information arises when a deeper understanding of the flame-flow interactions is sought. As yet, information was mostly obtained in a two-dimensional manner using planar laser light sheet based measurement techniques. Although they are well established, the data gained is only a projection of the reality into a two-dimensional plane.

In an effort to gather truly three-dimensional quantities, four laser light sheets have been crossed in a single line and particle image velocimetry (PIV) has been performed in each of them. By using the vaporisation of seeded silicon oil droplets at the flame front, the flame structure can be extracted as additional information. Combining the information about velocity and flame structure, flame displacement speeds were deduced.

For the separation of the four laser light sheets, different wavelengths and polarisations were used. The readily available frequency doubled output of a Nd:YAG laser at 532 nm was utilised to illuminate two of the light sheets, separating them by polarisation. A third light sheet was produced with the frequency-tripled output of another Nd:YAG laser at 355 nm. To create the fourth light sheet, a solid state external Raman laser with barium nitrate as the active material was set up.

This quad-crossed plane PIV experiment was applied in a model flame stabilised in a diffuser type combustor, which involves decelerating the premixed methane/air flow to a point where the flow velocity matches the turbulent burning velocity and therefore results in a rather flat reaction zone. The diffuser was made from quartz glass to allow optical access.

The information gained was compared to established theories and numerical simulation results. Furthermore, a comparison of three-dimensional and two-dimensional data was performed to critically analyse the significance of two-dimensional measurements.
Acknowledgements

First of all, I want to thank my supervisor Frank Beyrau, who gave me the opportunity to do a PhD and also offered me the position in London. I also want to thank Peter Lindstedt who helped me having a good start from the administrative side.

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<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_t$</td>
<td>Tangential strain rate</td>
</tr>
<tr>
<td>$c$</td>
<td>Reaction progress variable</td>
</tr>
<tr>
<td>$c$</td>
<td>Speed of light</td>
</tr>
<tr>
<td>$c_P$</td>
<td>Heat capacity</td>
</tr>
<tr>
<td>$g$</td>
<td>Diffusion function</td>
</tr>
<tr>
<td>$g_R$</td>
<td>Gain coefficient</td>
</tr>
<tr>
<td>$h$</td>
<td>Mathematical abbreviation</td>
</tr>
<tr>
<td>$k$</td>
<td>Wavenumber</td>
</tr>
<tr>
<td>$l$</td>
<td>Longitudinal correlation</td>
</tr>
<tr>
<td>$m$</td>
<td>Positive integer in the diffusion function</td>
</tr>
<tr>
<td>$m$</td>
<td>Reduced mass of the oscillating molecules</td>
</tr>
<tr>
<td>$n$</td>
<td>Counter variable</td>
</tr>
<tr>
<td>$n_i$</td>
<td>Flame surface normal component in direction $i$</td>
</tr>
<tr>
<td>$n_i$</td>
<td>Eigenvector component in direction $i$</td>
</tr>
<tr>
<td>$p_i$</td>
<td>Line point coordinate component in direction $i$</td>
</tr>
<tr>
<td>$p,q,r,s,t$</td>
<td>Mathematical abbreviation</td>
</tr>
<tr>
<td>$q$</td>
<td>Transverse correlation</td>
</tr>
<tr>
<td>$r_{ij}$</td>
<td>Partial two dimensional derivative in $i$ and $j$</td>
</tr>
<tr>
<td>$\dot{s}$</td>
<td>Strain rate</td>
</tr>
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<td>Counter variable</td>
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<td>$s_D$</td>
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<td>$s_h$</td>
<td>Shape factor</td>
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<td>$s_L$</td>
<td>Laminar flame speed</td>
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<tr>
<td>$s_n$</td>
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<tr>
<td>$s_r$</td>
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<td>-------------</td>
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<tr>
<td>$t$</td>
<td>Second directional derivative along $(n_x, n_y)$ (Chapter 4.2.5)</td>
</tr>
<tr>
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<td>time</td>
</tr>
<tr>
<td>$u$</td>
<td>Velocity</td>
</tr>
<tr>
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<td>Two dimensional image intensity data</td>
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<tr>
<td>$u_k$</td>
<td>Kolmogorov velocity</td>
</tr>
<tr>
<td>$\dot{\omega}$</td>
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**Latin Symbols, upper case**

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<thead>
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<tr>
<td>$A$</td>
<td>Area</td>
</tr>
<tr>
<td>$C_m$</td>
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</tr>
<tr>
<td>$D$</td>
<td>Molecular diffusivity</td>
</tr>
<tr>
<td>$E$</td>
<td>Electric field</td>
</tr>
<tr>
<td>$G_\sigma$</td>
<td>Gaussian convolution kernel of width $\sigma$</td>
</tr>
<tr>
<td>$H$</td>
<td>Hessian matrix</td>
</tr>
<tr>
<td>$I$</td>
<td>Intensity information of the image</td>
</tr>
<tr>
<td>$I_P$</td>
<td>Intensity of the pump laser beam</td>
</tr>
<tr>
<td>$I_S$</td>
<td>Intensity of the Stokes beam</td>
</tr>
<tr>
<td>$L$</td>
<td>Characteristic length</td>
</tr>
<tr>
<td>$L$</td>
<td>Interaction pathway</td>
</tr>
<tr>
<td>$L_x$</td>
<td>Integral length scale</td>
</tr>
<tr>
<td>$M$</td>
<td>Magnification factor</td>
</tr>
<tr>
<td>$\mathbf{N}$</td>
<td>Flame normal vector</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of Raman active molecules</td>
</tr>
<tr>
<td>$P$</td>
<td>Polarisation</td>
</tr>
<tr>
<td>$R_{ij}$</td>
<td>Correlation coefficient</td>
</tr>
<tr>
<td>$T_P$</td>
<td>Temperature of the products</td>
</tr>
<tr>
<td>$T_R$</td>
<td>Temperature of the reactants</td>
</tr>
<tr>
<td>$X$</td>
<td>Sparse matrix for the numerical implementation of the diffusion function</td>
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Nomenclature

<table>
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<th>Definition</th>
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<td>( Y )</td>
<td>Sparse matrix for the numerical implementation of the diffusion function</td>
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<td>( Y_P )</td>
<td>Mass fraction of the products</td>
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Greek Symbols

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<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>( \alpha_0 )</td>
<td>Thermal diffusivity of nitrogen at 300 K</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Thermal diffusivity</td>
</tr>
<tr>
<td>( \delta_{ij} )</td>
<td>Delta function</td>
</tr>
<tr>
<td>( \delta_L )</td>
<td>Laminar flame thickness</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>Rate of dissipation</td>
</tr>
<tr>
<td>( \varepsilon_0 )</td>
<td>Dielectric constant</td>
</tr>
<tr>
<td>( \phi )</td>
<td>Equivalence ratio</td>
</tr>
<tr>
<td>( \phi )</td>
<td>Arbitrary variable</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Conversion efficiency</td>
</tr>
<tr>
<td>( \eta_k )</td>
<td>Kolmogorov length scale</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Curvature</td>
</tr>
<tr>
<td>( \kappa_m )</td>
<td>Mean curvature</td>
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<tr>
<td>( \lambda )</td>
<td>Positive integer in the diffusion function</td>
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<tr>
<td>( \lambda )</td>
<td>Thermal conductivity</td>
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<tr>
<td>( \lambda_T )</td>
<td>Taylor length scale</td>
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<tr>
<td>( \mu_p )</td>
<td>Refractive index for the pump laser frequency</td>
</tr>
<tr>
<td>( \mu_s )</td>
<td>Refractive index for the Stokes frequency</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Kinematic viscosity</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Density</td>
</tr>
<tr>
<td>( \tau_F )</td>
<td>Flame time</td>
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<td>( \tau_M )</td>
<td>Molecular diffusion time</td>
</tr>
<tr>
<td>( \tau_R )</td>
<td>Reaction time</td>
</tr>
<tr>
<td>( \chi^i )</td>
<td>Optical susceptibility of order ( i )</td>
</tr>
<tr>
<td>( \omega_{AS} )</td>
<td>Raman anti-Stokes frequency</td>
</tr>
</tbody>
</table>
Nomenclature

\( \omega_p \)  Pump laser frequency
\( \omega_R \)  Characteristic Raman shift
\( \omega_S \)  Raman Stokes frequency

Chemical Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>Ba(NO(_3)_2)</td>
<td>Barium nitrate</td>
</tr>
<tr>
<td>CO(_2)</td>
<td>Carbon dioxide</td>
</tr>
<tr>
<td>OH</td>
<td>Hydroxyl</td>
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Dimensionless numbers

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<td>Da</td>
<td>Damkoehler number</td>
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<tr>
<td>Ka</td>
<td>Karlovitz number</td>
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<tr>
<td>Le</td>
<td>Lewis number</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>Re(_t)</td>
<td>Turbulent Reynolds number</td>
</tr>
<tr>
<td>Sc</td>
<td>Schmidt number</td>
</tr>
<tr>
<td>Tu</td>
<td>Turbulence intensity</td>
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Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>BBO</td>
<td>Beta Barium Borate</td>
</tr>
<tr>
<td>CCD</td>
<td>Charge-Coupled Device</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>LIF</td>
<td>Laser Induced Fluorescence</td>
</tr>
<tr>
<td>LHS</td>
<td>Left Hand Side</td>
</tr>
<tr>
<td>pdf</td>
<td>Probability density function</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>-------------</td>
</tr>
<tr>
<td>Nd:YAG</td>
<td>Neodymium Yttrium Aluminum Garnet</td>
</tr>
<tr>
<td>PIPM</td>
<td>Particle Image Pattern Matching</td>
</tr>
<tr>
<td>(C)PIV</td>
<td>(Conditioned) Particle Image Velocimetry</td>
</tr>
<tr>
<td>RHS</td>
<td>Right Hand Side</td>
</tr>
<tr>
<td>ROI</td>
<td>Region of interest</td>
</tr>
<tr>
<td>SRS</td>
<td>Stimulated Raman scattering</td>
</tr>
</tbody>
</table>
1 Introduction

The world is facing an increasing demand for energy. Projections assume an increase of 35% over the next 30 years [1]. The electricity production will be the highest contributor to cover this, with a share of 41%. While coal will still cover 32% of the demand for electricity (down from 46% in 2010), natural gas will become increasingly important as it will account for 29% of the electricity production, which is an increase of 85% from 2010. The better part of the electricity will therefore still be produced from combustion of fossil fuels [2, 3]. If an environmentally friendly solution for covering the increasing demand for energy is to be found, the efficiency of existing combustion devices needs to be increased in order to reduce emissions and the cost of after-treatment for e.g. CO$_2$ separation and capture [4].

This can only be accomplished if a deep understanding of the interactions of the two main elements in a reactive flow, the flow-field and the reaction itself, is gained. The focus of the current work lies on the analysis of turbulent premixed combustion. High turbulence levels enhance the energy density in technical systems. Furthermore, premixing of fuel and oxidiser leads to well defined combustion conditions and additionally helps to reduce pollutant output such as carbon monoxide or nitrogen oxides.

Well-designed laser-based techniques applied to laboratory scale combustors can help to provide an insight into the fundamental processes of combustion. They offer non-intrusive measurements of flow properties such as velocities and strain rates, as well as chemistry-related properties such as the flame speed. However, the applied laser techniques are typically only two-dimensional and hence can capture only an incomplete picture of the real, three-dimensional nature of the flame. Recently, direct numerical simulations have shown that such planar measurements provide only limited access to the true three-dimensional properties sought after [5], unless assumptions are made which have only limited applicability [6, 7].
In this thesis, the challenge of measuring three-dimensional properties of lean premixed flames was approached by expanding the existing planar measurement technique of Particle Image Velocimetry (PIV) to four light sheets that cross in a common intersection line. The motion of seeded silicon oil droplets was measured to yield the velocity field in each of the investigated planes using a conventional PIV approach. Additionally, the evaporation of the seeded droplets at around 600 K can be exploited in premixed flames within the thin flame and corrugated regime to identify the location of the flame front and thus derive other quantities such as the local flame curvature and shape. When the time separation between two consecutive particle raw images was chosen to be sufficiently long, the displacement of the flame front between the two exposures could be determined, allowing simultaneous measurement of the flame displacement speed. The data from each of the planes was then combined to yield several three-dimensional parameters of the flame near the intersection line.

In the following section the fundamentals of turbulent premixed combustion will be explained before giving an overview of the recent studies involving laser-based measurement techniques. This is followed by the explanation of the theory behind the four crossed plane technique. After that, an excursion into Stimulated Raman Scattering and its application in an external Raman laser will be shown. This was a crucial step in the development of the measurement system. The processing of the raw data gained by the experimental set-up is shown thereafter. Lastly, the application of the developed technique to a diffuser burner and results of the experiments are presented, before summarising the work performed.
2 Fundamentals of turbulent premixed combustion

Before the details about turbulent combustion are described, an introduction is given to explain the fundamental properties and dimensionless numbers defined in turbulent flows in general and reactive flows in particular.

2.1 Fluid dynamics of turbulent flows

In technical combustion processes turbulent flows are almost exclusively used to improve the mixing of fuel and air before combustion and also increase the energy density in the flame by mixing the products and reactants. This on one hand keeps combustion chambers small in size while on the other hand helps reducing pollutants due to short residence times.

Turbulent flows are characterised by the temporally and spatially random movement of the fluid. A range of eddies is responsible for this behaviour which leads to a rapid transfer of heat, mass and momentum. Due to the strong non-linear effects in these kinds of flows a little variation can have a large impact on the flow behaviour. The system is not in thermodynamical equilibrium and therefore has to be supplied with energy to maintain its state.

To characterise the flow condition as laminar or turbulent, the Reynolds number is defined [8] as the ratio between the destabilising forces of inertia and the dampening viscosity.

\[
Re = \frac{\bar{u}L}{\nu}
\]  

(2.1)

Where \(\bar{u}\) signifies the mean flow velocity, \(L\) a characteristic length of the flow and \(\nu\) the kinematic viscosity of the fluid. The Reynolds number can be used to determine the transition point between laminar and turbulent regime.
As turbulent flows consist of random motion, a statistical approach offers a good way of describing the system. Reynolds therefore suggested a separation of a variable $\varphi(x, t)$ into its temporally independent mean $\bar{\varphi}(x)$ and the temporally dependent fluctuations $\varphi'(x, t)$ [9].

\[
\varphi(x, t) = \bar{\varphi}(x) + \varphi'(x, t)
\]  (2.2)

The fluctuations are considered when calculating the root-mean-square value of the variable. This builds the second momentum of the variable and is defined as

\[
\varphi'_{RMS} = \sqrt{\bar{\varphi}'^2}
\]  (2.3)

In many cases $\varphi'_{RMS}$ is simplified as $\varphi'$. If the flow velocity $u$ as a variable is considered, the mean flow velocity $\bar{u}$ and the root-mean-square value $u'$ can be combined to characterise the turbulence intensity $Tu$ of the flow as

\[
Tu = \frac{u'}{\bar{u}}
\]  (2.4)

$Tu$ gives a measure of the fluctuations in the flow normalised by the mean flow velocity. Another method to analyse turbulent flows is to describe them by means of turbulent length scales. Here, the characteristic length $L$ used in equation (2.1) is the largest possible spatial scale of turbulence and refers to a geometrical parameter.

Other length scales are often defined by means of temporal or spatial correlations between the flow fluctuations. The definition of the resulting correlation coefficient $R_{ij}$ is

\[
R_{ij}(\vec{r}, t, \tau) = \frac{u_i'(\vec{x}, t)u_j'(\vec{x} + \vec{r}, t + \tau)}{}
\]  (2.5)

The correlation is given along a vector $r$ – describing the distance from the starting point – in direction $i$. Depending on which correlation is chosen, the functions $l(r)$ and $q(r)$ result,
describing longitudinal and transverse correlations, respectively. For a longitudinal correlation, the vector components in the same direction as the correlation are chosen.

\[
l(r) = \frac{R_{ij}(r)}{u_i r^2}, \quad q(r) = \frac{R_{jj}(r)}{u_j r^2}
\]  

(2.6)

With these correlations the integral length of the flow field, which describes the size of the largest eddies in the flow, can be determined by integration of the correlation coefficient.

\[
L_{x,t} = \int_{r=0}^{\infty} l(r) \, dr, \quad L_{x,q} = \int_{r=0}^{\infty} q(r) \, dr
\]  

(2.7)

In the case of an isotropic turbulence, the longitudinal length scale is half of the transverse length scale, hence \( L_{x,t} = \frac{1}{2} L_{x,q} \) [9].

Another length scale that can be derived from the described correlation coefficients is the Taylor microscale [9, 10]. In isotropic turbulence, it can be calculated from either longitudinal or transverse correlations as given in equation (2.8).

\[
\lambda_T = \sqrt{- \left( \frac{\partial^2 l}{\partial r^2} \right)^{-1}_{r=0}} = \sqrt{- \left( \frac{\partial^2 q}{\partial r^2} \right)^{-1}_{r=0}}
\]  

(2.8)

It is an intermediate scale between the largest eddies of the integral length scale and the smallest eddies of the Kolmogorov length scale and can be considered as the length scale at which the geometry of the system is not affecting the turbulence anymore [10].

The Kolmogorov length scale gives the size of the smallest eddies in the flow before they dissipate. In sufficiently turbulent flows, Kolmogorov’s first hypothesis [11-13] stated that the statistics of small scale motion can be defined uniquely by the kinematic viscosity \( \nu \) and the rate of dissipation \( \varepsilon \). Thus the Kolmogorov length scale can be defined as given in equation (2.9).
The introduced length scales can be represented in an energy spectrum, in which the turbulent kinetic energy per unit wavenumber \( k \) is plotted in a log-log fashion versus the wavenumber itself. The wavenumber is defined as the reciprocal of the corresponding length scale.

The behaviour of the turbulent kinetic energy spectrum for the entire wavenumber range is shown schematically in Figure 2.1.

In the large scale regions, corresponding to low wavenumbers, the energy increases with a power law. This range is dependent on large scale fluctuations, which in terms are different for different boundary conditions of the flow. At a wavenumber that corresponds to the integral length scale, the spectrum has a maximum, indicating that most of the energy in the flow is present in eddies of this size. In the following inertial subrange, the energy decays following a \( k^{-5/3} \) law. Finally there is a cut-off at the Kolmogorov length scale in the viscous subrange, in which the energy drops exponentially due to dissipation effects.
2.2 Turbulent premixed combustion

An overview of turbulent premixed combustion can be found in [15]. In this kind of combustion, the fuel and oxidiser are present in a homogeneous mixture before ignition which allows for a good control of the combustion process by the introduction of another degree of freedom in the stoichiometry. This concept is for example applied in spark ignition engines and power generation gas turbines. One of the most important properties for premixed combustion is the laminar flame speed \( s_L \). It is influenced by the thermal diffusivity of the fluid \( \alpha \) and a characteristic chemical reaction time \( \tau_R \), which is dependent on pressure, temperature and composition of the mixture.

\[
\frac{L}{s} = \sqrt{\frac{\alpha}{\tau_R}} \quad (2.10)
\]

In a laminar flame, the flame front is moving with the laminar flame speed towards the reactants, consuming them while advancing. If the flame is burning in a turbulent flow, the flame is stretched or compressed by its interaction with it, increasing or reducing the active flame area [16]. Flame stretch rate \( \dot{s} \) can be defined as the time rate of change of flame area \( dA/dt \) per unit area \( A \) [17].

\[
\dot{s} = \frac{1}{A} \frac{dA}{dt} \quad (2.11)
\]

\( \dot{s} \) can be related to the strain rate acting on the flame due to the fluid motion and the contribution of the reaction to the stretch, expressed in terms of curvature and flame displacement speed [17, 18].

\[
\dot{s} = a_t + 2s_D \kappa_m \quad (2.12)
\]
\( \kappa_m \) being the mean of the principal curvatures of the flame and \( s_D \) the local flame displacement speed, which is defined later on. \( a_t \) is described as the surface divergence due to the velocity gradients tangent to the flame front. It is defined by [18, 19] as

\[
a_t = (\delta_{ij} - n_i n_j) \frac{\partial u_i}{\partial x_j}
\]

(2.13)

\( u_i \) and \( n_i \) are the velocity and flame normal vector component, respectively. \( \delta_{ij} \) denotes the delta function.

The flame displacement speed which influences the flame stretch – as can be seen in equation (2.12) – is defined as the movement of the flame perpendicular to its surface within a certain time relative to the unburnt gas and can be measured by its relation to the apparent flame displacement and the convective movement. It is an important property for numerical simulations using level-set [14] and flame surface density methodologies [20]. It may be analysed in terms of the components due to the reaction, due to normal diffusion, and due to tangential diffusion [5, 6, 19, 21, 22]:

\[
s_D = (s_r + s_n) + s_t
\]

(2.14)

With the reaction component

\[
s_r = \frac{\dot{w}}{\rho|\nabla c|}_{c^*}
\]

(2.15)

\( \dot{w} \) denotes the reaction rate, \( \rho \) is the density and \( c \) is the reaction progress variable which can be defined as a function of the product mass fraction or the temperature:

\[
c = \frac{Y_p - Y_p^0}{Y_{p\infty} - Y_p^0} = \frac{T - T_{min}}{T_{max} - T_{min}}
\]

(2.16)
\( Y_P \) is the mass fraction of the product and subscripts \( \theta \) and \( \infty \) represent the unburnt reactants and the fully burned products, respectively. \( T \) is the temperature, and \( T_{\text{min}} \) and \( T_{\text{max}} \) denote the minimum and maximum temperature in the system, respectively.

The normal diffusion component is given as

\[
 s_n = \frac{\vec{N} \cdot \nabla (\rho D \vec{N} \cdot \nabla c)}{\rho |\nabla c|} \bigg|_{c=c^*}
\]  

(2.17)

With \( \vec{N} \) representing the flame normal vector and \( D \) the molecular diffusivity. Finally, the tangential-diffusion component can be expressed by

\[
 s_t = -2D \kappa_m
\]  

(2.18)

With the mean curvature

\[
 \kappa_m = \frac{1}{2} \frac{\partial N_i}{\partial x_i} \bigg|_{c=c^*}
\]  

(2.19)

From equation (2.18) it can be seen that the tangential-diffusion component is indirectly proportional to the mean curvature. This decomposition is often used in modelling approaches [5].

The response of the flame to stretch can be analysed using the dimensionless Lewis number \( Le \), which is defined as the ratio of thermal to molecular diffusivity [10]:

\[
 Le = \frac{\alpha}{D} = \frac{\lambda}{\rho c_p D}
\]  

(2.20)

\( \alpha \) is the thermal diffusivity and \( D \) the molecular diffusivity of the deficient reactant, which is the fuel in lean flames. \( \lambda \) denotes the thermal conductivity and \( c_p \) the heat capacity of the fluid, whereas \( \rho \) is the density. For mixtures with Lewis numbers close to unity mass and heat
transfer effects are balanced. For $Le \neq 1$ flames will curve towards the products for a Lewis number lower than one, leading to convex shapes. This is due to the fact that mass diffusion outweighs thermal conduction. On the other hand, if the Lewis number is greater than one, thermal conduction outweighs the mass diffusion. This leads to concave shapes of the flame and a loss of heat towards the reactants which will in turn accelerate the flame due to preheating effects [10].

The Damköehler number $Da$ describes the ratio of molecular diffusion time $\tau_M$ associated with mixing to the reaction time $\tau_R$.

$$Da = \frac{\tau_M}{\tau_R} \tag{2.21}$$

For a Damköehler number much smaller than unity the reaction takes longer than the molecular diffusion, which leads to a strong interaction of eddies with the flame structure. Contrary, for a Damköehler number far greater than unity, the reaction occurs fast in comparison to the molecular diffusion and combustion is therefore characterised by fast chemistry [23].

The Karlovitz number $Ka$ is defined as

$$Ka = \tau_F \cdot \dot{s} \tag{2.22}$$

With the flame time $\tau_F$

$$\tau_F = \frac{\alpha}{s_{L0}^2} \tag{2.23}$$

Peters [14] shows that the flame stretch term equals the Kolmogorov time scale
The thermal diffusivity $\alpha$ is

$$\alpha = \alpha_0 \left[ \frac{(T_R + T_P)}{2} / 300K \right]^{1.5}$$  \hspace{1cm} (2.27)

where $\alpha_0$ is the diffusivity of nitrogen at 300 K, which is 0.2 cm$^2$/s [24] and $T_R$ and $T_P$ are the temperatures of the reactants and the products, respectively. The Karlovitz number $Ka$ can therefore be calculated by [15]

$$Ka = \left( \frac{u'}{s_{L0}} \right)^{3/2} \left( \frac{s_{L0} L_x}{\alpha_0} \right)^{-1/2} \left( \frac{(T_R + T_P)/2}{300K} \right)^{3/4}$$  \hspace{1cm} (2.28)

The Karlovitz number is a measure for the transition of the combustion taking place only in a laminar-like layer to a broader, thickened combustion region.

In order to visualise the different states of turbulent premixed combustion, a regime diagram was proposed by Borghi and Peters [14] (see Figure 2.2). In this diagram, regions in which different combustion characteristics occur are visualised.
The three-dimensional quantities of strain rate, curvature and flame displacement speed are of high importance in turbulent premixed combustion. Their investigation is the goal in most combustion research measurements.
3 State of the technology

For flow investigations, the technique of Particle Image Velocimetry is widely used. It is applied in the fundamental research of flow turbulence [25-28] or even in tornado research [29] and has reached a state of commercial availability. Recently, it was applied in tissue deformation analysis, too [30]. A good overview over the technique is given by Raffel et. al. [31] and Kompenhans et. al. [32]. Developments and investigations involving the technique itself were performed by Kaene et. al. [33], Westerweel et. al. [34], Nogueira et. al. [35], Kähler et. al. [36] or Mitchell et. al. [37], among others. The developments go as far as tomographic or holographic PIV, which enables the analysis of the three-dimensional velocity field in a three-dimensional space [38, 39]. These techniques, however, are restricted to isothermal measurements.

In combustion research, PIV is only applied in a plane and usually combined with other techniques to gain simultaneous two-dimensional information on flow fields and reaction parameters. Hartung et. al. [6] for example uses the simultaneous laser induced fluorescence of the hydroxyl radical to gain information on the reaction while PIV allows the detection of the flow field. This enables the investigation of the flame displacement. Three-dimensional effects, however, could only be studied with Monte Carlo simulations.

The technique of extracting the flame front position from PIV raw particle images was used in several studies [40-47]. With this development it is, under certain conditions, not necessary to do simultaneous measurements together with the PIV. This can simplify experimental set-ups. It also gives room for the development of more sophisticated techniques as open resources can be put to use elsewhere [43, 44].

Pfadler et. al. [43] used a dual-plane set-up to investigate the three-dimensional stress and strain in a V-shaped flame. Their emphasis lays on the comparison of calculated model terms
to measured data. Although highly valuable data had been gained, it was not possible to derive information on the three dimensional structure of the flame from these measurements.

Chen et al. [48] presented a method to derive the three-dimensional normal of the flame surface and the curvature in premixed flames by means of crossed-plane tomography. Two crossed light-sheets with equal wavelengths illuminate the tracer particles with a short time delay in a turbulent premixed stagnation point flame and the intensity of the scattered signal is detected by one CCD camera for each. Although data on the three-dimensional flame surface structure could be gained, no information on the velocity field could be extracted.

Karpetis [49] faced similar challenges when applying the crossed-plane set-up for an OH-LIF experiment to resolve three-dimensional scalar gradients. For the investigation of combustion processes, a T-shaped arrangement of the light sheets has been used by Steinberg and Driscoll [42], where one light sheet is placed horizontally to gain inflow conditions. Utilising Taylor’s hypothesis, velocity gradients in a Bunsen flame could be measured.

In an effort by Shimura et. al. [50] to investigate the three-dimensional flame structure and velocity field simultaneously, two different laser induced fluorescence techniques in three investigation planes and two PIV planes had to be used in order to yield the intended information. However, flame displacement speeds could not be measured.

Long et. al. [51] measured the local flame displacement speed by means of asynchronous Particle Image Velocimetry. The interaction of a single axisymmetrical vortex (quasi two-dimensional) with a laminar flame was investigated under ambient conditions in a methane/air flame. In the processing of the data, a method was used to first determine the flame displacement purely by the flow, creating a virtual flame front. Subsequently, this was compared to the flame front of the second image taken after several microseconds to determine the flame displacement by the reaction.
Klein et. al. [22] did correlations of flame kernel radius and flame displacement speed for different flame kernel radii. Although all these measurements of flame displacement speeds exist, there is only one three dimensional measurement of it in the literature. Trunk et. al. [52] recently described an effort to gain this information in unstabilised, freely propagating flames. Their set-up involves two laser induced fluorescence planes and a single PIV plane. However, information on the curvature of the flame could not be gained with the described technique.

Chakraborty et. al. [5] performed an investigation of a turbulent premixed flat flame from direct numerical simulations to relate three-dimensional flame information to usually measured two-dimensional data. This paper showed on the one hand, that two-dimensional information is in most cases not sufficient to describe the inherently three-dimensional processes in turbulent premixed flames. On the other hand, it presented correlations of flame curvature, flame displacement speed and flow field, which have never been simultaneously measured in a three-dimensional fashion. This gave the motivation for the current work and the necessary foundation to relate the newly gained data to already published ones.
4 Development of quad-plane Particle Image Velocimetry

In the literature there has not been a description of a successful attempt to gain full access to the crucial three-dimensional flow and reaction information. In this chapter, a new approach will be introduced, yielding three-dimensional flow field information, as well as flame curvature and flame displacement speed. The experimental set-up, as far as the detection of the information is concerned, is explained in the following sections. Thereafter, the processing of the detected signals is illustrated and the gathering of the final information is explained. In chapter 5, the application of the technique to a diffuser type burner system and results are shown.

4.1 Experimental set-up of the measurement technique

The experimental approach involves the creation of four light sheets with 45° angles between them, intersecting in one line, at which all the information is gained. This will overcome the lack of three-dimensional data to access the quantities of interest. The optical set-up is introduced in this section, whereas the burner system and operating conditions for a specific application are shown in chapter 5.

4.1.1 Laser diagnostics

In Figure 4.1 a schematic of the optical setup for the quad-plane experiment is shown. A photograph of the set-up can be found in the Appendix (figure A2). The four light sheets (LS) intersect at a common line with an angle of 45° between each of them. To create the light sheets, three of the four lines of a Thales multi-channel Nd:YAG laser system are utilised in double-pulse operation with a time separation of 150 microseconds. The laser system produces double-pulses by pumping one cavity twice, which restricts the minimum and maximum delay. An advantage of the system is the inherent overlap of the two laser pulses, which is important for Particle Image Velocimetry measurements. Light sheet forming optics (LSFO) were installed to create the light sheets from the laser beams.
Figure 4.1. Experimental setup of the quad-plane PIV experiment. (Cam: Camera, LS: light sheet, LSFO: light sheet forming optics, BPx: band pass filter for wavelength x, PF: polarisation filter, UG11: Schott UG11 filter). Scheimpflug mounts and angles for all cameras not depicted

Light sheet 1 is created from the frequency tripled emission of one line of the multi-Nd:YAG system at a wavelength of 355 nm. The respective camera is equipped with a 105 mm f/2.8 UV camera lens to capture the signal and a Schott UG11 filter to block interfering signal from the other three light sheets. Cameras 2 and 3 are equipped with Nikon 105 mm f/2.0 and Sigma 105 mm f/2.8 camera lenses, respectively, and band-pass filters for 532 nm to collect the scattered light from the corresponding light sheets. The signals are separated through different polarisations using half-wave plates for the two frequency doubled lines of the multi Nd:YAG system. Finally, the last light sheet has an emission wavelength of 563 nm and is produced by an external Raman laser [53] pumped with one of the frequency-doubled lines and then separated from the pump beam. This set-up is explained in more detail in section 4.1.2. The produced measurement signal is captured by camera 4, which is equipped with a band-pass filter for 560 nm.

All cameras were equipped with Scheimpflug mounts to facilitate focusing the cameras when looking from a narrow angle onto the light sheets. This is not depicted in Figure 4.1 for clarity reasons. The Scheimpflug criterion enables the focussing of a camera even though it is in an angle to the imaging plane. It is fulfilled if the imaging plane, the plane of the camera
lens and the plane of the camera chip all intersect in the same line. A schematic of this is shown in Figure 4.2.

![Schematic showing the principle of the Scheimpflug criterion](image)

**Figure 4.2. Schematic showing the principle of the Scheimpflug criterion**

In practice, the angle between camera lens and chip is changed and the focus of the camera onto the imaging plane checked interactively, until the focus in all regions of the image is the same. The cameras were arranged in a way that the laser beams were not directed towards them to avoid possible damage.

To identify the position of intersection in each of the camera images, the four laser light sheets were aligned to a thin filament of about 300 µm hanging freely. This procedure was done before each series of measurements. Then, the measurements were taken with the cameras being focussed to their respective light sheet. After the last measurement of the day, the filament was replaced into the burner and the alignment checked again. With the laser turned off, images of the filament were taken with each of the cameras and an image correction target consisting of a dot-pattern was imaged onto the cameras to allow for de-warping. The measured data and images of the filament were afterwards treated with the same de-warping algorithms to guarantee the precise determination of the position of intersection. This was a very sensitive part of the experiment as a slight misalignment can lead to the data not being correlated, which has a huge negative influence on all three-
dimensional measurements, making them become meaningless. It is therefore of paramount importance to ensure the alignment of the four light sheets.

4.1.2 An external Raman laser for Crossed plane PIV

As mentioned in section 4.1.1, one of the light sheets was created by the use of an external Raman laser. In this section, the fundamentals and set-up of the external Raman laser are explained.

4.1.2.a Set-up of the laser

*Stimulated Raman scattering*

Before the Stimulated Raman Scattering (SRS) is explained, spontaneous Raman scattering will be introduced first [54]. This inelastic scattering process is often used for laser diagnostics of several different species [55]. It is related to the elastic Rayleigh scattering which is also shortly explained here. A schematic of the quantum processes in molecules when illuminated with high intensity light is shown in Figure 4.3.

![Figure 4.3. Principle of Rayleigh and Raman scattering](image)

Figure 4.3. Principle of Rayleigh and Raman scattering

When a molecule is excited by a photon with a pump frequency \( \omega_P \) too low to lift it to the next electronic state \( (x) \), but too high to lift it to a vibrational or rotational energy state, it goes to a virtual energy level \( (n) \) with a lifetime of less than one picosecond and finally falls
back to its ground state (a), emitting light of the same wavelength and polarisation. This effect is called Rayleigh scattering (i). In very few cases, the molecule does not fall back to its ground state and spontaneous Raman scattering occurs. There are two different ways of Raman scattering. In one case, the molecule stays at an elevated energy level (b). The emitted photon has then less energy and a lower frequency $\omega_S$ than the absorbed photon. The energy difference here is molecule-specific and is called the Raman shift $\omega_R$:

$$\omega_S = \omega_p - \omega_R$$  \hspace{1cm} (4.1)

The scattered light is called the Stokes line (ii). If the molecule is already in an elevated energy state (b) and then falls back to the ground level (a), it emits a photon of more energy and therefore higher frequency $\omega_{AS}$ than the incident photon.

$$\omega_{AS} = \omega_p + \omega_R$$  \hspace{1cm} (4.2)

This is called the anti-Stokes line (iii). The Raman shift in this case is identical for the same molecule. At room temperature, the anti-Stokes line is negligible compared to the Stokes line because of the Boltzmann distribution of the molecules in different energy levels. For the frequency conversion in crystals, molecular vibrational changes are the ones most relevant. That means that the elevated level (b) corresponds to a different vibrational energy of the molecule.

If the intensity of the pump laser beam becomes high enough, the macroscopic polarisation $P$ induced in the material is not only linearly dependent on the electric field $E$, but also nonlinear terms become significant:

$$P = \varepsilon_0 \chi_1 E + \varepsilon_0 \chi_2 |E|^2 + \varepsilon_0 \chi_3 |E|^3 + \cdots$$  \hspace{1cm} (4.3)

Where $\varepsilon_0$ is the dielectric constant in vacuum and $\chi_1$ is the linear optical susceptibility. For the process of Stimulated Raman Scattering, the term including the nonlinear optical susceptibility $\chi_3$ is responsible. This process can be understood as a continuous population
inversion similar to a flash lamp-pumped laser. The laser beam is created by the stimulated emission of light at the Stokes frequency. A complete description of the underlying processes is given in [56, 57]. For the understanding of the phenomena regarding this thesis the most important property is the intensity of the Stokes beam $I_S$, to which the laser light is converted via SRS. It is determined by the pump laser energy $I_P$, the length of the Raman active material $L$, and the so called gain coefficient $g_R$ in the following relationship:

$$I_S \propto e^{(g_R I_P L)} \tag{4.4}$$

Where the gain coefficient is a material and wavelength dependent quantity:

$$g_R = \frac{4\pi^2 \omega_S N}{\mu_p \mu_S c^2 m_\omega R \Delta \nu} \left( \frac{\partial \alpha}{\partial q} \right)^2 \tag{4.5}$$

$N$ is the number of Raman-active molecules in the material, $\mu_p$ and $\mu_S$ are the refractive indices of the material for the pump and Stokes frequencies, respectively. The speed of light is denoted by $c$ and the reduced mass for the oscillating molecules by $m$. Furthermore, $\left( \frac{\partial \alpha}{\partial q} \right)^2$ is proportional to the imaginary part of the nonlinear optical susceptibility $\chi_3$ and $\Delta \nu$ is the full-width at half maximum of the Raman spectral line.

From equation (4.5) it can be seen that for a given Raman-active material, the gain coefficient is only dependent on the wavelength of the produced Stokes beam. Therefore, this coefficient is given as a performance criterion for Raman-active materials.

One other parameter that is important for an experimental set-up – not only for the material itself – is the conversion efficiency. This additionally takes into account the geometry-based quantities given in equation (4.4). It is defined as the ratio of the Stokes laser energy and the pump laser energy:

$$\eta = \frac{I_S}{I_P} \tag{4.6}$$
As the gain coefficient is only material and wavelength specific, the achievable conversion efficiency can be increased in two ways. One is to increase the pump laser energy, which increases the Stokes energy exponentially while it diminishes the denominator in the definition of the conversion efficiency only linearly. The other way is to increase the Stokes energy by elongating the interaction pathway $L$. This can be done either by an increase in the length of the Raman-active material, which goes along with a higher cost, or by reflecting parts of the pump beam back into the crystal, which effectively elongates the path without having to physically change the material dimensions.

**Choice of crystal**

An overview of commercially available Raman-active materials can be found in [58, 59] and Appendix A1. From these, Ba(NO$_3$)$_2$ was chosen since several of its properties make it attractive for laser diagnostics. It has a high damage threshold for nanosecond pulses of 0.4 GW/cm$^2$ and a high gain coefficient of 47 cm/GW at 532 nm [59, 60]. One particular interesting property of this crystal is its Raman shift $\omega_R$ of 1047 cm$^{-1}$ [61] resulting in a first Stokes component at 563.5 nm which yields 281.7 nm after frequency doubling. This wavelength is suitable to excite the OH radical which is a good marker for burnt gases in flames. Moreover, Ba(NO$_3$)$_2$ shows a remarkably small line broadening of only 0.4 cm$^{-1}$ [58]. This enables rather efficient excitation of the narrowband OH transitions. However, this feature has not been used for the quad-plane application.

**The cavity**

For our investigations, the Ba(NO$_3$)$_2$ crystal was placed in a plano-parallel cavity, pumped by a dual-cavity Nd:YAG laser. A schematic of the set-up is shown in Figure 4.4. A photograph of the cavity can be found in the Appendix (figure A1).
A telescope (T1) was used to collimate the beam and to reduce its diameter to 6 mm, in order to fit into the aperture of the Raman-active crystal which had a 7x7 mm\(^2\) cross section and a length of 50 mm. The reflectivities of the cavity mirrors were chosen to get a good conversion ratio to the 1st Stokes transition [61, 62]. The rear cavity mirror (M) is highly transmittive for 532 nm and highly reflective for the first Stokes component at 563.5 nm, while the output coupler (OC) is 30 % reflective throughout the visible. The end faces of the crystal were anti-reflective coated. After the cavity, a dichroic (DC) was used to separate the remaining pump beam from the generated Stokes beam [63, 64]. To demonstrate another possible application of the Raman laser in combustion research, the set-up was expanded as shown and explained in Appendix A3.

4.1.2.b Characterisation

\textit{Conversion efficiency}

The first investigated parameter is the conversion efficiency which is shown in Figure 4.5. It was calculated using equation (4.6) with the measured data of the intensity of the Stokes beam and the pump beam.
Figure 4.5. Conversion efficiency of the Ba(NO_3)_2 crystal versus the pump laser fluence. Efficiencies of 30 % can be achieved with fluences well below the damage threshold of the crystal.

Here, the conversion efficiency is plotted with respect to the pump laser fluence. Clearly, efficiencies of up to 30 % can be easily achieved, which are comparable to or even better than those of commonly used dye lasers.

**Saturation effects**

For applications such as PIV or dual-pulse LIF for flame front tracking [6], successive pulses with very short time separation of the order of microseconds are required. Since the Raman laser is a solid state laser type, saturation effects occurring in dye laser systems when pumped in rapid succession are not expected. To prove this point, two successive laser pulses with variable inter-pulse timing were used to pump the Raman laser and pulse energy readings were taken in front of the Raman laser cavity for the pump and after the beam splitter for the Stokes beam. Figure 4.6 shows the overall conversion efficiency of the two Stokes pulses versus the temporal separation of the two pump pulses.
Clearly, no saturation effects can be seen which makes the crystal ideal for applications such as dual-plane PIV [42, 44]. For such applications it is also beneficial that the green laser beam can still be utilised unlike in a dye laser where the beam is absorbed by a beam dump within the laser. When the Stokes component is separated from the fundamental, the two beams at 532 nm and 563.5 nm can be used to form the two light sheets. Thus, it is possible to set up a complete dual-plane PIV experiment with only one dual-cavity laser and the external Raman laser. Another practical benefit is that the beams are inherently synchronised.

During the course of measurements for this thesis, an unexpected temperature effect of the Raman crystal was observed. When measurements were taken at the specified measurement conditions the test cell would heat up and the second pulse of Raman-shifted light would disappear gradually. Despite shielding the crystal from direct radiation of the burner and optical table and having a fan in place to increase the convectional cooling of the crystal, the problem could not be solved.

Therefore, for the set of measurements presented in this thesis, three sub-sets had to be taken. Between the measurements of the sub-sets, which took around 20 minutes each, about 30 minutes time was allowed for the crystal to cool down. The alignment was carefully checked after the sub-sets were taken to be sure that conditions had not changed during the measurements and cool down times.
4.2 Data processing

The data gained in each of the measurement planes is evaluated to gather information on the flow field and flame characteristics in the respective plane and then combine the data to get three-dimensional quantities. The procedures are explained in the following section.

4.2.1 Particle Image Velocimetry

To measure velocities non-intrusively and in a two-dimensional domain, Particle Image Velocimetry (PIV) is usually used. To apply the technique, the flow to be analysed needs to be seeded with a tracer such as gas bubbles, particles or droplets if they are not already naturally present in the fluid. The movement of the tracer boundary layer ensembles within a certain time determines the velocity the volume element has that surrounds the ensemble.

The minimal equipment that is needed consists of a light source and light sheet forming optics to illuminate the measurement plane twice in rapid succession and a CCD-camera to detect the light scattered from the tracer in the fluid [31]. There are two measurement strategies to calculate the velocity. One is to expose one image with the two successive illuminations and apply an auto-correlation algorithm. In this case, information on the direction of the flow needs to be known as the result of the auto-correlation algorithm is bidirectional. Therefore, the application is restricted to directional flows. This option allows the use of a relatively cheap camera as the exposure time doesn’t need to be controlled precisely.

The second strategy involves the use of a more expensive interline-transfer-CCD camera, which allows the separation of the two illuminations in two different images, called frames, and a subsequent cross-correlation. The advantage is a generally higher signal-to-noise ratio – the magnitude of the effect depending on the application – and additional information on the flow direction. The latter method is preferred in most applications. It is also applied in this thesis due to its experimental advantages over the auto-correlation method.
In both methods the image is divided into regions of interest (ROIs) and the following correlation is performed for the ensemble in the respective region. The raw information gained is the motion of the tracer ensemble $\dot{x}$ within the time interval of the illuminations $\Delta t$. The velocity $\vec{u}(\vec{x}, t)$ for each ROI can therefore be calculated as

$$\vec{u}(\vec{x}, t) = \frac{\dot{x}(t + \Delta t) - \dot{x}(t)}{M \cdot \Delta t}$$

(4.7)

Here, $M$ denotes the magnification factor determined by a calibration of the system. The correlation can either be done directly in the spatial domain via a Particle Image Pattern Matching (PIPM) algorithm or can be more efficiently realised by doing a Fast Fourier Transform (FFT) of the information and correlating it in the frequency domain [31].

In this work, silicon oil droplets are seeded to the flow (Dow Corning 200/50cs, 1 $\mu$m Diameter). They evaporate at 600 K, yielding velocity information only in the unburnt part of the flame, but additionally giving the opportunity to extract the flame front from the raw particle images. The procedure to be followed to achieve this is shown in the next chapter.

### 4.2.2 Filter procedure for two-dimensional flame front reconstruction

In the present work, a combination of filtering algorithms and contour extraction is used to determine the flame front position and characteristics with sub-pixel accuracy. The filter procedures applied to the instantaneous raw particle images are shown for an example image in Figure 4.7 to Figure 4.14.

In Figure 4.7 the raw particle image is shown in a three-dimensional representation on the LHS and a two-dimensional plot on the RHS.
The blue line represents the final result of the contour finding algorithm described in section 4.2.2.e. A very good agreement with vaporisation line of the silicon oil droplets at 318 °C can be seen which represents the location of the flame front. Furthermore, an intensity extract of an arbitrary vertical and horizontal line are shown next to the two-dimensional image to illustrate the signal behaviour with the applied filter procedures (red lines). It can be seen that the distinction between the reactants present in the lower part of the image and the products in the upper part is rather made in terms of “signal-noise” due to the character of the elastically scattered light from the droplets than in terms of intensity, as in the upper part of the image the intensities in the products due to flame luminescence are similar to the ones in the reactants.

4.2.2.a Local standard deviation filter

To overcome this issue with flame luminescence, the first filter applied to the raw image is a local standard deviation filter which is calculating the standard deviation in a neighbourhood of 3x3 Pixels in order to get the noise extracted from the raw image. As said
earlier, the signal is noisy in the reactants due to the droplets being present there, whereas it is more homogenous in the products. The result of this filter procedure is shown in Figure 4.8.

Figure 4.8. Image processing series: local standard deviation filter with a 3x3 filter width

It can be seen that the filter suppressed the signal in the reactants leading to a better separation of the signals in the reactant and the products. This is especially important for cameras 1 and 4, for which the optical filters have a large bandwidth. As the exposure times of the respective second frames are long for interline-transfer CCD-cameras it is not possible to suppress the flame luminescence to a higher degree so that this filter procedure is necessary for the evaluation of these frames. To emphasise this, the example image was chosen to be the second frame of an image of camera 4. For consistency reasons, the filter is applied to every evaluated image.

4.2.2.b Removing high intensity peaks

In a second processing step, peak maxima are equalised. This is accomplished by an analysis of the image’s histogram showing the number of pixels with a specific intensity value. The intensity value, at which the histogram reaches 1% of its maximum magnitude, is chosen as the cut-off intensity, as the value is high enough to substantially reduce the peak
intensities but still is not influencing the main contour of the flame front. To illustrate this, a schematic is shown in Figure 4.9.

![Figure 4.9. Schematic of the intensity distribution in an image illustrating the procedure to find the threshold intensity.](image)

Every pixel with a value higher than the cut-off intensity is set to this value to accomplish a homogenisation of the high intensity part of the image. The result of this processing step is shown in Figure 4.10.
Figure 4.10. Image processing series: high intensity peaks cut.

It can be seen that high intensity peaks are cut to avoid artefacts in the following processing step, which is the application of a smoothing filter. Nonetheless, the number of pixels processed in this way is only in the order of 2 % of the total pixels in the image.

4.2.2.c Non-linear filtering, background subtraction and normalisation

The next step consists of a non-linear sliding average filtering of the image to smooth the flame structure, avoiding the detection of artificial wrinkles produced by the particle pattern. This is done by applying the following filter function four times, going from left to right, right to left, top to bottom and from bottom to the top of the image [65]:

\[ I_{avg}(i) = \frac{(n - 1)}{n} \cdot I_{avg}(i - 1) + \frac{1}{n} \cdot I(i); \quad I_{avg}(0) = I(0) \] (4.8)

With \( I(i) \) being the intensity of a pixel \( i \) and \( n \) being the filter length. The reason for applying this filter four times is to rule out bias errors which result if the operation is only done in one direction. To prepare the image for further processing, a background subtraction and normalisation is applied again using histogram analysis. This time, the start and end of
the histogram are determined and the dynamics of these values are normalised by 1000 (see Figure 4.9). The result is shown in Figure 4.11.

![Image](image.png)

Figure 4.11. Image processing series: non-linear sliding average filter with filter width 9, image background subtraction and normalisation by 1000 (after this, the image was 2x2 binned, which is not shown here because the resolution of this print is too coarse for the effect to be seen)

Here, a clear step in intensity can be seen that can also be picked up by the contour finding algorithm due to its large gradient. But before this the image needs to be smoothed in regions with no flame front while still maintaining the high gradient at the flame front position to avoid the detection of false contour lines.

4.2.2.d Non-linear diffusion filter

This was achieved by applying a non-linear diffusion filter algorithm published by Malm et al. [66] using a diffusivity function originally proposed by Weickert [67]. As this step is very processing expensive, the image was binned before applying the filter function. This filter provides a way of smoothing noisy images without blurring or dislocating contours. Hence, the edges are enhanced in relation to the noise.
The principle equation of the filter is similar to a linear diffusion function given in equation (4.9) for an image $u$.

$$
\partial_t u = \text{div}(d \nabla u) 
$$

(4.9)

Where $d$ is a scalar constant. Mathematically it is exactly the same as convolving the image $u$ with a Gaussian kernel $G_\sigma$ of width $\sigma = \sqrt{2t}$, described by:

$$
G_\sigma(x) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{x^2}{2\sigma^2}\right)
$$

(4.10)

As this operation signifies a linear diffusion process, the contours can be blurred or dislocated by it. Perona and Malik [68] therefore proposed an exchange of the scalar constant $d$ for a function $g$ of the gradient $\nabla u$, which results in the following diffusion equation:

$$
\partial_t u = \text{div}[g(|\nabla u|)\nabla u]
$$

(4.11)

It has been empirically shown that the length of the gradient $|\nabla u|$ is a good measure of the edge strength of the current location. However, this dependence of the diffusion function on the edge strength makes the diffusion nonlinear. The function $g$ is required to be smooth and nonincreasing with $g(0) = 1$ and $g(s) \to 0$ for $s \to \infty$. In this work, the diffusivity function originally proposed by Weickert [67] in a similar setting was implemented. This reads

$$
g(s) = 1 - \exp\left(-\frac{C_m}{(s/\lambda)^m}\right)
$$

(4.12)

$m$ is a positive integer, $\lambda > 0$ and $C_m > 0$. $C_m$ is chosen in a way that the flux function $\Phi(\nabla u) = g(|\nabla u|)\nabla u$ is increasing for $s<\lambda$ and decreasing for $s>\lambda$. That way, a backward diffusion is produced when $|\nabla u| > \lambda$ and a forward diffusion when $|\nabla u| < \lambda$, which increases the slopes at the edges. Here, it is chosen to be $\ln(100)$, so that the maximum flow
occurs exactly at \(s=\lambda\). The parameter \(\lambda\) can be adjusted to define the separation of regions where forward diffusion should take place and where backward diffusion occurs. In this work, it was set to be 1.2 times the standard deviation of the noise gradient distribution of each image (see [66]). \(m\) was chosen to be 8, in accordance to the proposed function by Weickert [67].

As equation (4.11) can show unstable behaviour [66] and due to the backward diffusion properties sensitive to image noise, Catté et al [69] proposed to replace \(g(|\nabla u|)\) with \(g(|\nabla(G_\sigma \ast u)|)\). In this way, the gradient inside the diffusivity function is smoothed by a Gaussian kernel of width \(\sigma\) (see equation (4.10)). With this change, the diffusion equation is:

\[
\frac{\partial_t u}{u} = \text{div}[g(|\nabla(G_\sigma \ast u)|)\nabla u] \tag{4.13}
\]

This equation is insensitive to noise at a level defined by \(\sigma\) and therefore suppresses noise while the apparent edges are enhanced, which makes it much more stable than equation (4.11).

To implement the diffusion equation numerically, a discretisation is used based on central finite difference approximations of the spatial derivatives. The step sizes in the \(x\) and \(y\)-direction \(\Delta x\) and \(\Delta y\) are set to be 1. Therefore, the spatial derivatives read:

\[
\partial_x u_{i,j} = \frac{u_{i+1,j} - u_{i-1,j}}{2} \tag{4.14}
\]

and

\[
\partial_y u_{i,j} = \frac{u_{i,j+1} - u_{i,j-1}}{2} \tag{4.15}
\]

respectively. Sparse matrices are introduced and the multiplication with these replaces difference calculations, leading to improved computational time. With \(\partial_x u = uX\) and
\[ \partial_y u = Y u \]

the sparse matrices of the following form are used, where \( N \) and \( M \) are the numbers of discrete pixels along the \( x \) and \( y \) axis of \( u \), respectively.

\[
X = \frac{1}{2} \begin{bmatrix}
0 & -1 & 0 & \cdots & 0 & 0 \\
1 & 0 & -1 & \cdots & 0 & 0 \\
0 & 1 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 0 & -1 \\
0 & 0 & 0 & \cdots & 1 & 0 \\
\end{bmatrix}_{N \times N} 
\]  

(4.16)

\[
Y = \frac{1}{2} \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 & 0 \\
-1 & 0 & 1 & \cdots & 0 & 0 \\
0 & -1 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 0 & 1 \\
0 & 0 & 0 & \cdots & -1 & 0 \\
\end{bmatrix}_{M \times M} 
\]  

(4.17)

To iterate, first \( G_{\sigma} \ast u \) is calculated followed by a differentiation of the result, giving \( \nabla(G_{\sigma} \ast u) \). With this term, the function \( g(\nabla(G_{\sigma} \ast u)) \) is evaluated at all pixels. Then, \( u \) is updated by

\[
u^n = u^{n-1} + \Delta t \left[ (g \cdot \partial_x u^{n-1})X + (g \cdot \partial_y u^{n-1})Y \right] 
\]  

(4.18)

Here, \( g \) is the abbreviated form of \( g(\nabla(G_{\sigma} \ast u)) \) and indicates an element-wise multiplication with the respective gradient \( \partial_x u^{n-1} \) and \( \partial_y u^{n-1} \), respectively. The time step was chosen to be 1 to improve processing time and still have the advantage of the non-linear diffusion filtering with only 300 iterations. The number was determined by a sensitivity study of the number of iterations.

The filter leads to a step function at the flame front position that can be clearly seen in Figure 4.12.
Development of quad-plane Particle Image Velocimetry

Figure 4.12. Image processing series: non-linear diffusion filter according to Malm [66] (step size of 1, 300 iterations, cut off at 1.2 times the standard deviation of the image)

However, there are still artefacts of strong peak values near the flame front which are in a last step smoothed by another application of the non-linear sliding average filter. The final processed image fed into the contour finding algorithm can be seen in Figure 4.13.

Figure 4.13. Image processing series: non-linear sliding average filter with a filter width of 3
To illustrate the behaviour of the signal along the horizontal line during the whole procedure a comparison at the different stages of processing is shown in Figure 4.14.

![Figure 4.14](image)

Figure 4.14. Image processing series: comparison of the different stages of processing (raw: raw image, std: image after applying the standard deviation filter, dot: image after the removal of high intensity peaks, back: image after background subtraction, bin: binned image, malm: image after the application of the non-linear smoothing filter suggested by Malm et.al. [66], avg: image after the final averaging step)

The vertical black line represents the position of the detected flame front. It can be seen that in each processing step the position of the flame front is not influenced by the applied filter algorithms whereas the rest of the image is progressively smoothed to allow for an optimal detection of the main gradient.

The whole sequence of processing shown in Figure 4.14 is again presented on a step by step basis in Figure 4.15 for the same horizontal line.
Application of a local standard deviation filter with filter width 3.
(see section 4.2.2.a)

Removing high intensity peaks
(see section 4.2.2.b)

Application of non-linear sliding average filter with filter width 9, background subtraction and normalisation (see section 4.2.2.c)

Figure 4.15. Sequence of processing (black: initial situation, red: processed)
Binning of the image (2x2) (see section 4.2.2.c)

Application of non-linear diffusion filter with step size 1, 300 iterations and a cut off at 1.2 times the standard deviation of the image (see section 4.2.2.d)

Application of non-linear sliding average filter with filter width 3 (see section 4.2.2.d)

Figure 4.15 (cont). Sequence of processing (black: initial situation, red: processed)
**4.2.2.e Contour finding algorithm according to Steger**

A Gaussian convolution of the image was used to extract the gradient map. The equation used is the first derivative of equation (4.10):

\[
G'_\sigma(x) = \frac{-x}{\sqrt{2\pi}\sigma^3} \exp \left( -\frac{x^2}{2\sigma^2} \right) \tag{4.19}
\]

The resulting image was fed into an algorithm proposed by Steger [70] for the sub-pixel accurate detection of curvilinear structures. This algorithm has been used in combustion diagnostics by Sweeney et al. [71] to improve the results from a Canny algorithm [72] but without utilising the advantage of sub-pixel accurate line detection. The calculation is performed by the use of a Hessian matrix given in equation (4.20). With this, eigenvalues and eigenvectors can be determined at each point, giving the strength of a line by means of its curvature perpendicular to it (second derivative) and its direction. Lines in the gradient map correspond to the gradient in the flame image, giving flame contour information.

\[
H(x, y) = \begin{pmatrix} r_{xx} & r_{xy} \\ r_{yx} & r_{yy} \end{pmatrix} \tag{4.20}
\]

\(r_{xx}, r_{xy} = r_{yx} \) and \(r_{yy}\) are the partial derivatives of the image which are calculated by two-dimensional Gaussian convolution. This matrix is calculated for each pixel of the image and therefore, a map of the magnitude of the gradients can be produced. The eigenvector corresponding to the maximum absolute eigenvalue is given by \((n_x, n_y)\) with \(\| (n_x, n_y) \| = 1\). This vector is pointing perpendicular to the direction of the line. A quadratic polynomial is used to determine if the investigated pixel can indeed be considered a line pixel. It is specified to be a line pixel if the directional derivative along \((n_x, n_y)\) vanishes within the pixel. This is calculated by inserting \((tn_x, tn_y)\) into a Taylor polynomial, setting the derivatives along \(t\) to zero. The point can then be calculated by
\[
(p_x, p_y) = (tn_x, tn_y)
\]  
(4.21)

with

\[
t = -\frac{r_x n_x + r_y n_y}{r_{xx} n_x^2 + 2r_{xy} n_x n_y + r_{yy} n_y^2}
\]  
(4.22)

For a point to be declared a line point, \[(p_x, p_y) \in \left[-\frac{1}{2}, \frac{1}{2}\right] \times \left[-\frac{1}{2}, \frac{1}{2}\right]\] needs to be fulfilled. With the points being given in sub-pixel range, the results are not discrete anymore in contrast to the Canny algorithm and therefore do not produce high curvature values artificially that need to be dealt with (see [73]). The second directional derivative along \((n_x, n_y)\) can be used to find the prominent line from the maximum eigenvalue.

After the line points are found, they need to be linked by an algorithm. This is described in detail in [70]. The principle is to find a prominent line point, the one with the highest eigenvalue in the image. From this point, a search algorithm looks for line points in the perpendicular direction of the eigenvector, which is the direction of the line. Here, an interval of \([-22.5^\circ, 22.5^\circ]\) is covered in order to be able to detect curvatures in the lines. From this point the algorithm searches along the line. This procedure is then repeated in the opposite direction, starting from the original point, and the resulting points are sorted accordingly. As there is only one continuous line representing the flame front in each image, an additional step to find junctions is not required in contrast to the original algorithm given in [70]. The result is parameterised contour information for each of the two frames of the four camera images.

### 4.2.3 Accessible two-dimensional data

In each light sheet, curvature and flame displacement speed can be evaluated to compare them with the three-dimensional measurements. For the evaluation of these, the same routines
as for the three-dimensional data are used. Only the respective out-of-plane components and
derivatives are neglected. Therefore, the reader is referred to the following section for a
detailed description of the equations used.

4.2.4 Reconstruction of three-dimensional information

The three-dimensional mean curvature $\kappa_m$ is defined as the mean of the two principal
curvatures $\kappa_1$ and $\kappa_2$ of the given surface. Equation (4.23) shows this relation.

$$\kappa_m = \frac{(\kappa_1 + \kappa_2)}{2} \quad (4.23)$$

$\kappa_1$ and $\kappa_2$ are determined by calculating their reciprocals, the principal radii $R_1 = \frac{1}{\kappa_1}$ and
$R_2 = \frac{1}{\kappa_2}$, by solving equation (4.24) [74].

$$(rt - s^2)R^2 + h[2pq - (1 - p^2)t - (1 - q^2)r]R + h^4 = 0 \quad (4.24)$$

With

$$p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}, r = \frac{\partial^2 z}{\partial x^2}, s = \frac{\partial^2 z}{\partial x \partial y}, t = \frac{\partial^2 z}{\partial y^2} \quad (4.25)$$

and

$$h = \sqrt{1 + p^2 + q^2} \quad (4.26)$$

The resulting equation for the mean curvature expressed as a function $f(x,y,z)$ with $z$
denoting the flame height information and $x$ and $y$ being the axes of the planes perpendicular
to the intersection line is given as

$$\kappa_m = \frac{(\kappa_1 + \kappa_2)}{2} = \frac{r(1 + q^2) - 2pq + t(1 + p^2)}{2(1 + p^2 + q^2)^{\frac{3}{2}}} \quad (4.27)$$
The mean curvature is defined as the mean of the curvatures of the two principal directions of a two-dimensional surface. It describes what is conventionally known as the three-dimensional curvature.

To analyse the flame shape, a shape factor can be defined [75, 76]:

$$s_h = \frac{\kappa_1}{\kappa_2}, \text{with } |\kappa_1| < |\kappa_2|$$

(4.28)

This value gives information on the three-dimensional structure of the surface, being greater than 0 for elliptic, “sphere-like” and smaller than 0 for hyperbolic, “saddle-like” structures. For a value of 0, the flame is cylindrically shaped (parabolic). To illustrate these different flame shapes, schematics are given in Figure 4.16. This is a crucial distinction as it can give a hint if a two-dimensional projection can be viable for the flame in question. For mainly hyperbolic structures, a projection will probably not capture the real three-dimensional structure of the flame surface.

Figure 4.16. Schematics of different possible flame shapes. From left to right: hyperbolic ($s_h<0$), parabolic ($s_h=0$), elliptic ($s_h>0$)

Knowing the flow field, the three-dimensional curvatures and the respective normal vector to the flame surface at the intersection, the next possible step is to determine the three-dimensional flame displacement speed. This is done by translating the intersection point of the flame front in the first frame by the velocity measured 0.4 mm upstream in the unburnt, as it is not possible to evaluate the velocity exactly at the intersection point due to a lack of
seeding droplets in the burnt region. A side effect of this choice of location is that the flame displacement speed does not need to be corrected for reduced gas densities, which would be the case if the velocities were taken at a location closer to the flame front or even in the burnt region [5]. Additionally, thermophoretic effects negatively influence the accuracy of PIV near the flame front [77]. Following the flame surface normal from this advected virtual point, one can calculate the distance to the flame surface of the second frame by applying a cubic interpolation spline to it and solving the set of linear equations leading to the distance. The procedure is explained for a two-dimensional measurement in [51].

The accuracy of determining the flame front position is assumed to be 0.25 pixels due to the sub-pixel nature of the information gained. It is furthermore assumed to be the biggest factor of uncertainty. Thus, the error in determining the flame displacement speed is estimated to be around 10%.

The flame stretch rate is defined as the time rate of change of flame area per unit area [17]

$$\dot{s} = \frac{1}{A} \frac{dA}{dt}$$

(4.29)

It can be written in terms of strain, local flame displacement speed and curvature [17, 18]

$$\dot{s} = a_t + 2s_\rho \kappa_m$$

(4.30)

\( \kappa_m \) being the mean of the principal curvatures of the flame and \( s_\rho \) the local flame displacement speed. \( a_t \) is described as the surface divergence due to the velocity gradients tangent to the flame front. It is defined by [18, 19] as

$$a_t = (\delta_{ij} - n_in_j) \frac{\partial u_i}{\partial x_j}$$

(4.31)
Here $u_i$ and $x_i$ are the velocity and scales component, respectively. $\delta_{ij}$ denotes the delta function, and $n_i$ and $n_j$ are the unit normal vector components of the flame surface. This information was extracted from the vector fields of all four planes. The flame stretch rate is therefore a combination of flow field and reaction information which could only be obtained by simultaneously measuring flame structure and flow field in a three-dimensional manner.

In the graphs shown in chapter 5, these quantities are sometimes normalised. This is done for the curvature by multiplication with the laminar flame thickness $\delta_L$ calculated as the ratio of the thermal diffusivity $\alpha$ evaluated at the mean flame temperature [15] and laminar burning velocity $s_L$ [78]. For the operating condition here, these values were $s_L=0.35$ m/s, $\alpha=1.7*10^{-4}$ m$^2$/s, and thus $\delta_L = 0.4$ mm. Stretch rates are normalised by multiplying by a timescale, $\delta_L/s_L=1.1$ ms, to form one possible definition of the Karlovitz number:

$$K_a = \frac{s L}{\delta_L} = \frac{s L}{\delta_L^2}$$

The flame displacement speeds are normalised by the laminar burning velocity. They may be analysed in terms of the components due to the reaction, due to normal diffusion, and due to tangential diffusion [5]:

$$s_D = s_r + s_n + s_t$$

Where $s_t = -2D\kappa_m$. Thus, $s_r + s_n$ can also be analysed by a subtraction of $s_t$ from $s_D$. The normalised expression for equation (4.33) reads

$$\frac{s_D}{s_L} = \frac{s_r + s_n}{s_L} - 2\delta_l\kappa_m$$

All the three-dimensional quantities are now defined and can be calculated. These can be compared to their two-dimensional counterparts to analyse the reliability of planar
measurements. To do this, the newly developed measurement technique is applied to a model burner and results are shown in the following chapter.
5 Measurement of three dimensional quantities in a diffuser based burner

The technique of quad-plane PIV to extract various flame and flow quantities was applied to a diffuser type burner, which is described in section 5.1. Results from these measurements are then presented in order to show the achievable information of e.g. flame curvature, strain rates and flame displacement speeds, among others. Major parts of this chapter have been published in [79].

5.1 Experimental set-up of the burner

The experimental set-up for the laser diagnostics and data acquisition is described in section 4.1. This section focusses on the burner geometry and the operating conditions for this specific application.

5.1.1 Burner geometry

The investigated diffuser burner has been previously characterised using planar techniques [80, 81]. A schematic of which can be seen in Figure 5.1.
The diffuser itself was made of quartz glass and has a diffuser half-angle of 12.7° from an inlet diameter of 58 mm. It was chosen for this study because of the perpendicularity of the flame surface to the mean flow direction which can be achieved if a high momentum annular wall jet is used to avoid separation. Aligning the four light sheets perpendicular to the flame surface yields highly accurate flame displacement measurements. Furthermore this arrangement allows long interframing times, while it is not experiencing too many out-of-plane losses of seeding particles.

Because of the decrease of the mean flow velocity in the quartz glass diffuser, the flame is stabilised at a specific height due to the instantaneous equilibrium of the turbulent flame speed and the mean flow velocity. For turbulence generation, a perforated plate with a
blockage ratio of 37% (triangular pattern of holes with a diameter of 2.5 mm and a distance of 0.5 mm) was installed 25 mm upstream of the burner nozzle.

Upstream of the turbulence generating grid, a honeycomb flow straightener of 100 mm length was installed which was fed from a 90 mm section filled with glass beads (diameter of 3 mm) to get a uniform velocity distribution. This in turn was preceded by a 12 mm diameter flexible inlet hose.

The burner was fed by Bronkhorst mass flow controllers F-203AV-RAD-55-V for air, which was connected to a 12 mm hose, and F-202AC-RAD-22-V for methane with a 6 mm hose. A part of the airflow was separated and directed through a PALAS Aerosol Generator Type AGF 10.0, which seeded 1 µm silicon oil droplets (Dow Corning 200/50cs) to the flow. The three flows (seeded air, pure air and methane) were mixed 1 m before entering the burner to ensure a well-mixed state of the air and fuel composition. The air Co-Flow was fed by a third Bronkhorst mass flow controller F-203AV-RAD-55-V through a 6 mm flexible hose.

5.1.2 Operating conditions

The isothermal measurements were performed with an inlet bulk velocity of 4 m/s and a wall jet velocity of 6 m/s, resulting in a momentum ratio of 0.167. These flow conditions were matched to the reactive measurements, in which a planar premixed methane air flame was stabilised within the diffuser burner; they can be seen in sections 5.2.2-5.2.4. An equivalence ratio of $\phi=0.95$ with a bulk velocity of 4 m/s at the inlet was chosen as operating conditions. In the investigated flame stabilisation region at a height of around 63 mm above the burner exit turbulence intensities of around 20% were achieved.

5.2 Measurement results

The results of the measurements taken in the diffuser burner at the centreline are described in this section. First, the isothermal data are presented in order to show the characteristics of
the flow field. After that, the reactive measurements provide an overview of the possible results from the measurement technique.

### 5.2.1 Isothermal measurements

Figure 5.2 shows the axial velocity, root-mean-square fluctuations and the resulting turbulence intensity along the centreline for all four light sheets. A very good agreement of the axial velocity component in all four light sheets can be seen. Thus, light sheet calibration procedure is working properly. Due to the area expansion of the glass diffuser the mean axial flow rate decreases downstream as expected [82]. The area increase of 23% between 53 mm and 73 mm accounts almost exactly for the axial velocity decrease in Figure 5.2.

![Figure 5.2. Axial velocity (o), fluctuations (x) and turbulence intensity (□) along the centreline in all four light sheets](image)

With decreasing velocities, the fluctuations in the flow are slightly increasing. This results in a linear rise in turbulence intensity within the measured region from a value of 0.14 to 0.24. At the mean flame height, a turbulence intensity of 0.18 is reached. This development of turbulence intensities is typical for a conical diffuser [82].
In Figure 5.3 a comparison of the axial velocity component in radial direction at the lower end of the measured region – 53 mm downstream of the burner exit – is presented. Here, a slight inhomogeneity of the flow can be seen. However, deviations are only about 10% of the mean values.

These deviations are not relevant due to the fact that data are only gained close to or directly at the intersection position. Although the mean velocity distributions show inhomogeneity, the turbulence distribution is rather homogeneous. This can be seen from the root-mean-square fluctuations of the axial velocity component which are similar in all four light sheets.

Figure 5.4 shows the radial mean velocities along the centreline for all four light sheets and their respective root-mean-squared fluctuations. The fluctuations of the radial velocity component are only half as large as the fluctuations of the axial component. This is an indicator for an anisotropic turbulence behaviour of the flow. However, this was expected in a diffuser decelerated configuration [82], which is similar to the turbulence profile in a pipe flow.
Measurement of three dimensional quantities in a diffuser based burner

The radial components of the mean velocity are not equal on the axis. This was expected as the radial components are projections of the real flow velocity into the respective investigated plane. The magnitude of the radial velocity is only around 3 % of the axial component and only a slight asymmetry is needed to account for this. This asymmetry is the reason the radial components are not precisely zero. The root-mean-square fluctuations show a good agreement for all four investigated light sheets. However, the values for camera 4 are less reliable in the lower region of the investigated area due to a lower signal-to-noise ratio of the camera system.

5.2.2 Three-dimensional curvatures

Figure 5.5 shows a comparison of the three-dimensional curvatures calculated from frame 1 and frame 2, respectively, in the form of a scatter plot. As expected from two non-independent images, there is a strong correlation between the values. Still scatter occurs due to the time delay between the two shots. As a strong deviation of the two values has no physical explanation, the relation of the curvatures was also used as an exclusion criterion for measurement errors. Only images for which these two values were closer than 0.1 mm\(^1\) were
taken into account in the evaluation. With this criterion in place, the number of recordings was 1104 for the following statistical examination.

![Graph showing three-dimensional curvature of frame 1 and frame 2.](image)

Figure 5.5. Three-dimensional curvature of frame 1 and frame 2.

It can also be seen that mainly moderate positive values of curvature are detected, with very few extreme values either positive or negative.

This can be further corroborated by plotting the probability density function, shown in Figure 5.6. Along with the three-dimensional curvature, the probability density function (pdf) of two-dimensional curvatures extracted from light sheet 3 are shown (see Figure 4.1).
Both pdfs show a peak at slightly positive values, but the three-dimensional curvature peak is narrower, while the two-dimensional curvature forms a broader probability density function. The mean of the pdfs is around zero with a skew to positive values. To further illustrate the dependence of two- and three-dimensional curvature values, a scatter plot is presented in Figure 5.7. Differently coloured points indicate a difference in the shape of the three-dimensional flame surface, where blue depicts an elliptic (sphere-like) and red a hyperbolic (saddle-like) shape of the flame front. Two- and three-dimensional curvatures clearly correlate, but the scatter is significant. This indicates that the planar measurement cannot capture the 3D curvature properly. It is also notable that for elliptic shapes values occur mainly close to the bisecting line, indicating a better correlation, whereas for hyperbolic shapes values mainly occur off the bisecting line. This behaviour can be explained with the definition of the mean three-dimensional curvature and the shape factor. For elliptic shapes the two principal curvatures have the same sign. They are by definition the highest and lowest curvatures of the structure. So every two-dimensional curvature chosen from a random plane through the same point needs to have the sign of the principal curvatures. This
leads to a rather good positive correlation. On the other hand hyperbolic shapes have the principal curvatures of different sign. This leads to a non-predictable sign of two-dimensional curvatures chosen from a random plane through the same point. They can have either positive or negative sign, which leads to a negative correlation or no correlation at all.

![Figure 5.7](image-url)

Figure 5.7. Normalised three-dimensional curvature calculated from four light sheets in comparison to the normalised two-dimensional curvature in light sheet 3.

As the correlations of two- and three-dimensional values are not satisfying, a crossed plane configuration involving two planes is investigated for its potential to improve curvature measurements. The curvature values gained are quasi-three-dimensional as they are calculated by the mean of the curvatures in the two perpendicular planes and not with equation (4.27), which is the exact description of three-dimensional curvature. A comparison of this quasi-three-dimensional curvature with the true three-dimensional curvature from four planes is presented in Figure 5.8. The information for the two perpendicular planes was taken from light sheets 1 and 3 (see Figure 4.1).
Figure 5.8. Comparison of normalised three-dimensional curvature calculated from four planes versus the normalised mean of curvatures from only two perpendicular planes (light sheets 1 and 3).

The correlation of the compared values is very good, resulting in limited scatter around the bisecting line. Thus, a less complex set-up of only two crossed light sheets already yields a good estimate of the three-dimensional curvature in this configuration. Furthermore, it can be seen that hyperbolic shapes mainly occur for curvatures around zero, whereas elliptic shapes are seen more often for higher values of curvature. The trend to positive curvature values can also be observed here.

A method to investigate the flame surface further is to consider the shape factor of curvatures. Thus, the pdf of the shape factor is presented in Figure 5.9. Negative values correspond to hyperbolic saddle structures, whereas positive values indicate spherically-shaped elliptic structures. For cylindrical structures, values of zero are calculated.
A general preference for spherically shaped flame structures can be clearly seen for the burner investigated. A possible explanation for this distribution is the nature of the burner system, which favours elliptic shapes, because of the tendency to mean curvature of the flame. This conclusion is reinforced by a comparison with the results of DNS calculations [5], where the peak of the pdf was at zero indicating cylindrical structures.

In Figure 5.10 the mean curvature is compared with the shape factor by means of a scatter plot. A trend to spherically-shaped curvatures can be observed there as well. Elliptic shapes are present throughout the spectrum of mean curvatures, whereas hyperbolically-shaped flame fronts are scarce for high absolute mean curvatures and mainly occur from -0.25 mm$^{-1}$ to 0.25 mm$^{-1}$.
This was already observed in previous plots and can be explained by the definition of both shape factor and mean curvature. For a hyperbolic structure, the shape factor is smaller than zero and the principal curvatures therefore have different signs, resulting in low mean curvatures. To achieve high mean curvatures on the other hand, the principal curvatures need to be high and of the same order of magnitude, which leads to a positive shape factor representing elliptic shapes.

Here, a simplified experimental set-up of two perpendicular crossed planes can be investigated again. The shape factor derived from the two crossed planes is compared with the “true” shape factor from the four plane measurements. For the two-plane case, the shape factor is not calculated by the principal curvatures, but by the two curvatures measured in perpendicular planes. Here, light sheets 1 and 3 are chosen again. For light sheets 2 and 4, very similar results were observed, but are not shown here. A comparison to the real shape factor is presented in Figure 5.11.
Figure 5.11. Comparison of the shape factor calculated from four planes and the shape factor determined from only two perpendicular planes (light sheets 1 and 3)

Although a clear correlation of the two values is evident, it can be seen that the scatter increases for high absolute values of shape factor. Analysing the data conditioned by the mean curvatures two main observations can be made. One was already seen in Figure 5.10, i.e. that mainly elliptic shapes are present for high mean curvatures and for negative shape factors, mainly small mean curvatures around zero exist. For the small mean curvatures the scatter in the shape factor plot is the highest. This can be explained by the vicinity of the two-dimensional curvatures to zero. A slight change in orientation of the corresponding coordinate system can lead to a change in sign for one of the curvatures, which results in an anti-correlating behaviour of the shape factor. Bearing this in mind, the absence of strongly anti-correlating points for high positive values of shape factor can also be explained. As both principal curvatures are of comparable magnitude, a change in orientation does not result in a change in sign so that the shape factor is more or less correlated. This suggests that, especially for low absolute mean curvatures, the use of a set-up with only two crossed planes will lead to unreliable information on the shape of the flame. Keeping in mind that the
curvature distribution indicates that a majority of the curvatures have low values around zero, the simplified experimental set-up is clearly not favourable.

Another interesting structural feature is the orientation of the flame surface. As the flame is propagating perpendicular to its surface, the orientation of the surface determines the main propagation direction. To investigate this, the unit flame normal vector is split up into its components $N_1$, $N_2$ and $N_3$. $N_1$ is the component parallel to the burner axis and $N_2$ the component aligned with the x-axis, which is found in light sheet 1 (see Figure 4.1). $N_3$ corresponds to the component aligned with the y-axis and therefore is within light sheet 3.

In Figure 5.12 the probability density functions of $(N_1^2+N_2^2)$ and $(N_1^2+N_3^2)$ are presented. Both distributions involving $N_1$ show a high probability around 1, indicating a predominant propagation of the flame front in the mean flow direction. This also means that it is well aligned with the intersection line of the four light sheets leading to the maximum possible sensitivity for flame displacement measurements, which are evaluated in section 5.2.3.

The relations between the orientations $N_1$, $N_2$ and $N_3$ and thus the propagation direction of the flame can be further investigated by creating scatter plots. These can be seen in Figure 5.13. The predominant propagation direction of the flame front is here also along the $N_1$ direction opposite to the flow, with some arbitrary scatter in $N_2$ and $N_3$. The scatter cloud

![Figure 5.12. Probability density function of $(N_1^2+N_2^2)$ and $(N_1^2+N_3^2)$ showing an orientation of the flame along the centreline](image-url)
for $N_2$ and $N_3$ is not completely homogeneous, which indicates that the flame surface orientation was not perfectly parallel to the intersection line. This behaviour was also observed in DNS simulations of statistically planar flames [5].

Figure 5.13. Scatter plots of the relations between N1, N2, and N3 showing a predominant propagation of the flame against the flow.

Although the main propagation direction of the flame is aligned with the axis of the burner, for the comparison of two-dimensional and three-dimensional flame displacement speeds, one other parameter needs to be found to describe whether the flame is mainly propagating within the investigated light sheet or whether the out-of-plane component is
significant. Therefore, a probability density function of $\alpha$ – defined as the angle between the two-dimensional plane investigated, which is light sheet 3 in this case, and the flame normal vector – is shown in Figure 5.14. It can be seen that the propagation of the flame does align well with the detection plane, which is in good agreement with the data found in [5] for statistically planar flames. For the orthogonal plane a plot of the angle shows a similar trend but is not depicted here.

![Figure 5.14](image)

Figure 5.14. Probability density function of the angle $\alpha$ between the two-dimensional plane of light sheet 3 and the flame surface normal

Although the main propagation direction is aligned with the axis of the burner, the movement of the flame is significant and therefore a projection of three-dimensional displacement speed into one plane is expected to give higher values of displacement speed. This will be further analysed in the next section.

### 5.2.3 Determining the flame displacement speed

A comparison of the three-dimensional flame displacement speed with the two-dimensional one calculated from light sheet 3 – both normalised by the laminar flame speed – is presented in Figure 5.15 in the form of a scatter plot. Although a positive correlation can be
observed, there is a large scatter. This indicates that a reconstruction of three-dimensional flame displacement speeds from two-dimensional data is unreliable.

Figure 5.15. Normalised three-dimensional flame displacement speed calculated from four planes in comparison to the normalised two-dimensional flame displacement speed evaluated from light sheet 3

This point can be further corroborated by the comparison of probability density functions from three- and two-dimensional flame displacement speeds, as shown in Figure 5.16. The distribution for the three-dimensional data is narrower whereas the probability of finding high values is larger in the two-dimensional case.
The mean value of the normalised three-dimensional flame displacement speed is 1.1 and therefore the mean displacement speed is approximately equal to the laminar flame speed. However, there is a very broad distribution with a standard deviation of 0.71 in $s_D/s_L$. This deviation is very similar to those in the simulations of Chakraborty et. al.[5]. The figure constitutes the major result of this investigation, to be discussed in section 5.2.4.

The two-dimensional mean value, however, shows a shift to higher values of flame displacement speed. Bearing in mind the findings from Figure 5.14 for the angle of flame propagation it can be explained by the misalignment of the flame propagation direction with the investigated light sheet. This misalignment can be corrected by using equation (5.1) [6] if the angle $\alpha$ and the out-of-plane component $u_3$ of the velocity are available. This information can be gained in the orthogonal plane, so a simplified experimental set-up is investigated here again.
The resulting probability density function can be seen in Figure 5.17.

![Figure 5.17. Probability density function of the normalised three-dimensional flame displacement speed calculated from four planes compared to the angle corrected normalised two-dimensional flame displacement speed evaluated with information from light sheets 1 and 3.](image)

With the correction the statistics of two-dimensional and three-dimensional flame displacement speeds overlap almost perfectly. This suggests that even with two orthogonal planes, the statistics of three-dimensional flame displacement speed can be derived by the two-dimensional flame displacement speed determined from one plane and the angle $\alpha$ and the out-of-plane component of the velocity determined in the orthogonal plane.

With a scatter plot of the three-dimensional flame displacement speed and the corrected two-dimensional one, the point-by-point correlation can be investigated. This is shown in Figure 5.18.
There is a clear correlation near the bisecting line, but still a lot of scatter, although the statistics of three-dimensional flame displacement speed can be deduced from the two-dimensional data. Thus, in order to yield data of significance, the measurement of fully three-dimensional information is still essential.

In Figure 5.19 and Figure 5.20, the pdfs of the tangential-diffusion component and the combined normal-diffusion and reaction component of the flame displacement speed is shown, respectively. This decomposition was introduced in equation (4.34) and has been shown to be important in the context of level-set modelling [20, 83].
5 Measurement of three dimensional quantities in a diffuser based burner

Figure 5.19. Probability density function of the tangential diffusion component of the normalised three-dimensional flame displacement speed calculated from four planes compared to the tangential diffusion component of the normalised two-dimensional flame displacement speed evaluated from light sheet 3.

Predominantly negative values can be found in the plot of the tangential-diffusion component. This can be explained by the definition of the component and the fact that the mean curvature of the investigated flame is positive. The distribution is therefore more or less a mirrored graph of the mean curvature histogram in Figure 5.6. Thus, the two-dimensional distribution is here also broader than the three-dimensional one, but the mean values of both are similar, with $s_v/s_L = -0.1$. This component accounts for backwards diffusion effects occurring in the flame.
The pdf of the combined normal-diffusion and reaction components shows a similar trend to the original flame displacement speed pdf, but there is a smaller chance of finding values in the negative part of the histogram. This leads to the conclusion that the tangential-diffusion component is mainly responsible for displacement speeds in the same direction as the mean flow, whereas the combined normal-diffusion and reaction components tend to move the flame against the flow direction. Similarly to Figure 5.16, the two-dimensional distribution is overestimating the combined normal-diffusion and reaction components of the flame displacement speed.

### 5.2.4 Flame stretch relations

It is of interest to investigate whether these deviations in the flame displacement speed from the unstretched laminar burning velocity can be related to the flame stretch. To analyse this, the relations of the quantities contributing to the flame stretch – they can be found in equation (4.34) – have been plotted against the flame stretch. A scatter plot of the normalised
flame stretch, the Karlovitz number, versus the normalised mean curvature of the flame is presented in Figure 5.21.

![Figure 5.21. Normalised flame stretch versus normalised mean curvature, both calculated from four planes.](image)

A positive correlation can clearly be seen for both hyperbolic and elliptic flame structures, but it is in combination with the flame displacement speed that the contribution to flame stretch is made.

The contribution of the tangential component of the strain rate can be seen in Figure 5.22. Here, a positive correlation is also present. Furthermore, negative values for tangential strain rate are relatively scarce. The strain rate was calculated 0.4 mm upstream of the flame position. The distance is about a fifth of the smallest radius of flame curvature, but about the same as the laminar flame thickness. This was chosen to be close enough to the flame to be of relevance but far enough to have sufficient velocity data in the vicinity.
In summary, Figures 5.21 and 5.22 show that the flame stretch rate is almost always positive for the measured condition which indicates that the flame is constantly stretched, not only by the convective contribution of the flow field but also by the chemical reaction. From the slope of the positive correlation with strain rate, it can be concluded that about 80% of the flame stretch is due to the motion of the fluid.

In Figure 5.23 the flame displacement speed is displayed as a function of the flame stretch, but no substantial correlation can be seen.
Even more surprisingly in view of the DNS results [5], there was no correlation between the flame displacement speed and the three dimensional curvature or the strain rate when these contributions to flame stretch were isolated and examined independently. A possible explanation is that the DNS results were produced in a way higher turbulent flame with values of $u'/s_L$ of 2.37 to 7.44, which were shown to give a better correlation of curvature and flame displacement speed with increasing turbulence. The flame measured in this thesis, on the other hand, had a value of $u'/s_L$ of around 1 near the measurement region. Therefore, the correlation of the two values may be too weak to be detected within the accuracy of the measurement system.

Therefore, the newly developed technique is capable of producing data of significance for three-dimensional analysis of flame curvature and shape as well as for flame displacement speeds. To investigate correlations with flame stretch further, the set-up needs to be improved and applied to flames of a higher turbulence level.
6 Conclusions and Summary

A quad plane Particle Image Velocimetry technique for the three dimensional investigation of turbulent premixed flame structures was developed. The described technique gives access to crucial data of the flow field and flame geometry and therefore can help to gain a better understanding of the processes in turbulent premixed flames and the interaction of flow and reaction. This knowledge can be used for the design of more efficient, less polluting burner systems in industrial applications.

For the set-up, an external Raman-laser was investigated and applied for the first time in combustion research. This gave a reliable source of light of a different wavelength. Additionally, the external Raman laser can also be applied for laser induced fluorescence measurements in other experimental set-ups [47]. The high efficiencies and lack of saturation effects observed also make it a good choice for high speed measurements.

The described technique uses the principle of Particle Image Velocimetry and expands it to gain access to flow characteristics, such as the strain rate, and the flame displacement speed. A combination of these can yield the flame stretch rate or a decomposition of the flame displacement speed into its tangential-diffusion and combined normal-diffusion and reaction components. These very relevant quantities can all be gathered in a three dimensional fashion, and not merely in a two-dimensional light sheet, as in many earlier studies.

Thus, a comparison of two- and three-dimensional data is possible and shows that for certain flame geometries, two-dimensional data may be sufficient to represent the processes occurring. Especially low turbulent flames with symmetric flame structures would fall into this category. But in most real flames, a two-dimensional measurement cannot reproduce the three-dimensional information necessary for model validation or a deeper understanding of the underlying processes. However, a simplification of the measurement technique using only
two crossed planes was explored and found to deliver sufficiently accurate data in comparison to the fully three-dimensional ones. This would be applicable to flames which are mainly parabolically shaped or nearly flat, as the orientation of the perpendicular measurement planes would then have little effect on the outcome. Contrary, if a flame has a highly hyperbolic structure, the measurement in only two planes will not give reliable data. To explore the flame structure, a set of measurements can be taken with a crossed-plane arrangement. Based on the result it can be decided if an expansion to four planes is necessary.

To get additional information on the applicability and limitations of the technique, measurements of additional data sets or the application of the technique to different burner systems should be taken. However, when applied to a different burner geometry, the main flow direction, flame orientation and the time separation between laser pulses need to be considered beforehand to avoid out-of-plane losses of particles and retain a high sensitivity for flame displacement measurement.

If a simplified version of the set-up proves to be sufficiently accurate in different burner configurations and operating conditions, more data sets can be gained with even less effort. This makes the build-up of a data base for deeper analysis a relatively easy task. It could even be used for a comparison to numerical simulations due to the high resolution of the data.

The developed set-up gives access to a vast number of quantities in three dimensions and allows for a further investigation of premixed turbulent flames than any previous technique. The foundation was laid in this thesis, but further development needs to be done in order to use the full potential of this measurement technique.
List of References


[84] C. Endres, Set up and characterisation of an external Raman laser for use in PIV applications, in Department of Mechanical Engineering. 2009, Imperial College London: London.


## Appendix

### A1 Raman-active materials and their properties [84]

Parameters of selected Raman crystals which have been used for frequency conversion. Shortcuts to read the table: VIS = visible regime of light, IR = infrared regime of light, s = steady state, t = transient, Ref. = references

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<th>Material</th>
<th>Frequency shift [cm(^{-1})]</th>
<th>Raman linewidth [cm(^{-1})]</th>
<th>Integral cross section [%]</th>
<th>Peak intensity [%]</th>
<th>Gain coefficient (VIS) [GW/cm(^2)]</th>
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<td>11 (s), 1.1 (t)</td>
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<td>8.5 (s), 3.8 (t)</td>
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<td>LiIO(_3)</td>
<td>821.6</td>
<td>5.0</td>
<td>54</td>
<td>25</td>
<td>–</td>
<td>4.8</td>
<td>[56, 85]</td>
</tr>
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<td>LiO(_2)</td>
<td>770</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>[56]</td>
</tr>
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<td>872</td>
<td>21.4</td>
<td>44</td>
<td>5</td>
<td>–</td>
<td>–</td>
<td>[85]</td>
</tr>
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<td>LiNbO(_3)</td>
<td>632</td>
<td>27</td>
<td>166</td>
<td>18</td>
<td>–</td>
<td>–</td>
<td>[85]</td>
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<td>LiNbO(_3)</td>
<td>250</td>
<td>28</td>
<td>–</td>
<td>22</td>
<td>–</td>
<td>–</td>
<td>[85]</td>
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<tr>
<td>NaBi(WO(_4))(_2)</td>
<td>910</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>8</td>
<td>[56]</td>
</tr>
<tr>
<td>NaBi(MoO(_4))(_2)</td>
<td>877</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>9</td>
<td>[56]</td>
</tr>
<tr>
<td>NaBrO(_3)</td>
<td>795</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>[56]</td>
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<tr>
<td>NaN(_3)</td>
<td>1069</td>
<td>1.0</td>
<td>23</td>
<td>44</td>
<td>–</td>
<td>–</td>
<td>[85]</td>
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<td>NaY(WO(_4))(_2)</td>
<td>914</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.8</td>
<td>–</td>
<td>[56]</td>
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<tr>
<td>PbWO(_4)</td>
<td>901</td>
<td>4.2</td>
<td>–</td>
<td>8.4</td>
<td>3.1</td>
<td>–</td>
<td>[56, 86]</td>
</tr>
<tr>
<td>PbWO(_4)</td>
<td>323</td>
<td>7.5</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>[86]</td>
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<td>SiO(_2)</td>
<td>464</td>
<td>7</td>
<td>2.2</td>
<td>1.2</td>
<td>–</td>
<td>–</td>
<td>[85]</td>
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<tr>
<td>Sr(_3)(PO(_4))(_3)F</td>
<td>950.3</td>
<td>2.8</td>
<td>3.4</td>
<td>3.8</td>
<td>–</td>
<td>–</td>
<td>[85]</td>
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<td>SrMoO(_4)</td>
<td>887.7</td>
<td>2.8</td>
<td>55</td>
<td>51</td>
<td>–</td>
<td>–</td>
<td>[85]</td>
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<td>SrWO(_4)</td>
<td>921.5</td>
<td>2.74</td>
<td>50</td>
<td>3.8 (s)</td>
<td>–</td>
<td>–</td>
<td>[85]</td>
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<td>TiO(_2)</td>
<td>612</td>
<td>48</td>
<td>159</td>
<td>9.8</td>
<td>–</td>
<td>–</td>
<td>[85]</td>
</tr>
<tr>
<td>Y(_2) (AlO(_3))(_2)</td>
<td>379</td>
<td>4.0</td>
<td>3.3</td>
<td>2.16</td>
<td>–</td>
<td>–</td>
<td>[85]</td>
</tr>
<tr>
<td>Y(_2) (AlO(_3))(_2)</td>
<td>783</td>
<td>8</td>
<td>3</td>
<td>1.0</td>
<td>–</td>
<td>–</td>
<td>[85]</td>
</tr>
<tr>
<td>Y(_2) (AlO(_3))(_2)</td>
<td>370</td>
<td>5.7</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.1 ± 0.05</td>
<td>[86]</td>
</tr>
<tr>
<td>YVO(_4)</td>
<td>890</td>
<td>3</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>&gt; 4.5</td>
<td>[56, 86]</td>
</tr>
<tr>
<td>ZnO</td>
<td>438</td>
<td>6.0</td>
<td>1.14</td>
<td>2.7</td>
<td>–</td>
<td>–</td>
<td>[85]</td>
</tr>
<tr>
<td>ZnWO(_4)</td>
<td>907</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>[56]</td>
</tr>
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</table>
Appendix

A2 Images

Figure A1. The external Raman laser cavity

Figure A2. Quad plane PIV experiment
A3 Simultaneous PIV/OH-LIF measurements using a Raman-laser

For simultaneous PIV/LIF measurements, the Raman shifted beam was collimated and reduced in diameter with another telescope (T2) to be frequency doubled in a beta barium borate (BBO) crystal, see Figure A3.

Another dichroic mirror was used to separate the fundamental frequency component from the frequency doubled light which was then formed into a lightsheet and spatially overlapped with the lightsheet formed from the remaining 532 nm beam.

An interline-transfer CCD camera (LaVision Imager Intense) equipped with a 532 nm bandpass filter was used to acquire the PIV images, and the fluorescence signal was captured by a combination of an image intensifier with a fast P46 phosphor (LaVision IRO 9, decay time of 400 ns) and another interline-transfer CCD camera (LaVision Imager Intense). A combination of Schott UG11 and WG295 filters was used in order to block chemiluminescence from the flame and scattered laser light from the particles, respectively. \( R_2(5) \) line has only half the absorption cross section of the usually used \( Q_1(8) \) line. However, the absorbed energy is still sufficient to do laser induced fluorescence, as the example images show. The fluorescence of the OH radical is collected from the \((1,1)\) and \((0,0)\) bands between 305 to 320 nm. The absorption spectrum of the OH radical can be seen in Figure A4.
Appendix

Figure A4. Absorption spectrum for the OH radical at 2200K from 30000 to 40000 wavenumbers

The Raman shifted light is thus frequency-doubled for OH excitation and simultaneous PIV/OH-LIF measurements can be performed using the remaining 532 nm beam for PIV. An example set of images is depicted in Figure A5.

Figure A5. Set of simultaneously taken particle images (a,b) and LIF images (c,d)
This shows an application of the device to another measurement technique widely used in combustion research, i.e. the laser induced fluorescence of the hydroxyl radical. For the current work, the external Raman laser was not used for excitation of the OH radical.
Statement of Originality

I declare that I wrote the present work myself and only used the sources referenced therein. All sources that I used literally are labelled as such.