Modelling and optimisation of Electro-Active Polymer (EAP) devices

by

Florence Rosenblatt-Weinberg

Department of Aeronautics
Imperial College of Science, Technology and Medicine
Prince Consort Road
London SW7 2AZ

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I confirm that this thesis is my own work, and I have referenced appropriately when the work is not mine.

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Abstract

The control of boundary layers either for skin-friction reduction or for flight control can be achieved by their manipulation using deformable surfaces. In the case of the former, it is known that the manipulation of coherent structures in the turbulent boundary layer can lead to significant drag reductions. However, the challenge is to find actuators and sensors that are functional at these spatial scales (10 μm to 0.1 mm) and the associated temporal scale (100 kHz). Electro-Active Polymers (EAPs) provide excellent performance, are light weight, flexible, and low cost. Therefore EAPs, and in particular Dielectric Elastomer Actuators (DEAs), provide many potential applications as micro-actuators and micro-sensors.

Modelling DEA devices is a cost-effective way of providing a better understanding of the devices and optimising their designs. Acquiring a model for the EAP material itself is the first essential step in DEA modelling. A modelling technique taking into account the material non-linearities and its behaviour at large deformations (‘hyperelasticity’) is presented in the third chapter of this thesis. The main challenge in modelling DEA devices is the modelling of their electro-mechanical coupling. Commercially available electro-mechanical modelling does not apply to non-linear materials such as EAPs. The ANSYS Finite Element (FE) software is the tool used in this work to develop a novel model presented in the fourth chapter. Various means of optimising the design of DEA devices are suggested in the sixth chapter using the developed DEA model. A novel design of an EAP-based pressure sensor is suggested in the seventh chapter; FE modelling is used to study the abilities and performance of such a device. To complete the model, its time-dependent properties are examined by a modal analysis examined in an eighth chapter. The thesis is completed by examining the potential for DEA in providing a ‘smart’ surface for distributed aerodynamic control.
To my husband Alexander, children
and parents Mr and Dr. J. Rosenblatt
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<tr>
<td>A</td>
<td>Electrode area</td>
</tr>
<tr>
<td>a</td>
<td>Dimple radius</td>
</tr>
<tr>
<td>[B]</td>
<td>Strain displacement matrix</td>
</tr>
<tr>
<td>C</td>
<td>Capacitance</td>
</tr>
<tr>
<td>[C]</td>
<td>Element structural damping matrix</td>
</tr>
<tr>
<td>C\eq</td>
<td>Element structural damping matrix for system with one degree of freedom</td>
</tr>
<tr>
<td>C_{10}</td>
<td>First Mooney-Rivlin constant</td>
</tr>
<tr>
<td>C_{01}</td>
<td>Second Mooney-Rivlin constant</td>
</tr>
<tr>
<td>C_{20}</td>
<td>Third Mooney-Rivlin constant</td>
</tr>
<tr>
<td>C_{11}</td>
<td>Fourth Mooney-Rivlin constant</td>
</tr>
<tr>
<td>C_{02}</td>
<td>Fifth Mooney-Rivlin constant</td>
</tr>
<tr>
<td>C_{eq}</td>
<td>Equivalent capacitance</td>
</tr>
<tr>
<td>c_p</td>
<td>Specific heat capacity of the material</td>
</tr>
<tr>
<td>D</td>
<td>Dimple diameter</td>
</tr>
<tr>
<td>D</td>
<td>Membrane Stiffness</td>
</tr>
<tr>
<td>{D}</td>
<td>Electric flux density vector</td>
</tr>
<tr>
<td>d</td>
<td>Sheet thickness</td>
</tr>
<tr>
<td>dt</td>
<td>Time step</td>
</tr>
<tr>
<td>E</td>
<td>Young’s modulus</td>
</tr>
<tr>
<td>E(t)</td>
<td>Relaxation modulus function</td>
</tr>
<tr>
<td>{E}</td>
<td>Electric field intensity vector</td>
</tr>
</tbody>
</table>
\{F\}  Vector of nodal and surface forces
\[ F(t) \]  Load vector
\{F_e\}  Vector of nodal electrostatic forces
\[ F_{el} \]  Electrostatic forces
\[ F_{ext} \]  External forces
\[ F_0 \]  Forcing input intensity
\{F^a\}  Total applied vector
\{F^r\}  Reaction load vector
\( f \)  Forcing frequency of the dimple
\( g \)  Linear acceleration
\[ H(t) \]  Heaviside function
\( h \)  Height of gap
\( h_{air} \)  Thickness of the air layer of the FE model
\{I\}  Identity matrix
\( I_1 \)  Strain invariant in the 1 direction
\( I_2 \)  Strain invariant in the 2 direction
\( I_3 \)  Strain invariant in the 3 direction
\[ J(t) \]  Creep function
\( J_1(u) \)  Bessel function of first order of the first kind
\[ [K] \]  Stiffness matrix
\( K \)  Element structural matrix for system with one degree of freedom
\[ [K_d] \]  Element dielectric permittivity coefficient matrix
\[ [K_e] \]  Element stiffness matrix
\[ [K^{VS}] \]  Dielectric permittivity matrix
\( kg \)  Kilograms
\( kV \)  Kilovolts
\[ [L] \]  Vector of nodal, surface and body charges
\( L^a_e \)  Nodal charge vector
\[ [M] \]  Element mass matrix
\( M \)  Element mass matrix for system with one degree of freedom
\( m \)  Mass of the material
N  Number of elements
N  Element shape function
Pa  Pascal
Pst  Prestretch
$P_{el}$  Electrostatic pressure
Q  Shearing force per unit of length
q  Force density
R  Resistance
R  Radius
r  Distance of any point measured from the centre of the plate
r  Location on the circular membrane
$[S]$  Stiffness matrix
$St$  Strouhal number
T  Material temperature
$T_i$  Engineering stress in the $i$ direction
t  Membrane thickness
t$_{elastomer}$  Thickness of the elastomer
U  Voltage across electrodes
$U_{∞}$  Free stream velocity
$\{u\}$  Nodal displacement vector in the $x$ direction
$\ddot{u}$  Nodal acceleration vector in the $x$ direction
$\dot{u}$  Nodal velocity vector in the $x$ direction
V  Volts
V  Scalar potential
$V_{cr}$  Buckling voltage
$V_e$  Nodal electric scalar potential
$\ddot{v}$  Nodal acceleration vector in the $y$ direction
$\dot{v}$  Nodal velocity vector in the $y$ direction
$\{v\}$  Nodal displacement vector in the $y$ direction
W  Strain energy function
$w(r)$  Membrane deflection at radial location $r$
\( w_n \)  
Natural frequency

\( w_0 \)  
Maximum deflection at the centre of a circular membrane

\( X \)  
Oscillation amplitude

\( x \)  
Horizontal direction

\( Y_1(u) \)  
Bessel function of first order of the second kind

\( y \)  
Vertical direction

\( \alpha_x \)  
Thermal expansion constant in the x direction

\( \alpha_y \)  
Thermal expansion constant in the y direction

\( \beta_i \)  
Material damping

\( \Delta T \)  
Temperature difference

\( \delta \)  
Dimple out-of-plane displacement

\( \epsilon \)  
Material strain

\( [\epsilon] \)  
Permittivity matrix

\( \epsilon_a \)  
Air permittivity

\( \epsilon_e \)  
Relative permittivity of the elastomer

\( \epsilon_{xx} \)  
Permittivity in the x direction

\( \epsilon_{yy} \)  
Permittivity in the x direction

\( \epsilon_{zz} \)  
Permittivity in the x direction

\( \epsilon_0 \)  
Free space permittivity

\( \eta \)  
Material viscosity

\( \lambda_i \)  
Eigenvalue

\( \lambda_1 \)  
Stretch ratio in the 1 direction

\( \lambda_2 \)  
Stretch ratio in the 2 direction

\( \lambda_3 \)  
Stretch ratio in the 3 direction

\( \nu \)  
Poisson ratio

\( \xi \)  
Damping ratio

\( \rho \)  
Electric-charge density or material density

\( \rho_s \)  
Surface charge density vector

\( \sigma \)  
Material stress

\( \sigma_{cr} \)  
Buckling stress

\( \sigma_e \)  
Maxwell stress
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_f$</td>
<td>Radial thermal stress</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>Stress in the $i$ direction</td>
</tr>
<tr>
<td>$\sigma_{ii}$</td>
<td>Stress in the $i$ direction</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>Residual stress</td>
</tr>
<tr>
<td>$[\sigma^M]$</td>
<td>Maxwell stress tensor</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Phase displacement of the oscillation with respect to the exciting force</td>
</tr>
<tr>
<td>$\phi_i$</td>
<td>Eigenvector</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Oscillation frequency</td>
</tr>
<tr>
<td>$1D$</td>
<td>One dimensional</td>
</tr>
<tr>
<td>$2D$</td>
<td>Two-dimensional</td>
</tr>
<tr>
<td>$3D$</td>
<td>Three-dimensional</td>
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</table>
## List of acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>ACARE</td>
<td>Advisory Council for Aeronautics Research in Europe</td>
</tr>
<tr>
<td>CO₂</td>
<td>Carbon Dioxide</td>
</tr>
<tr>
<td>DEA</td>
<td>Dielectric Elastomer Actuator</td>
</tr>
<tr>
<td>DNS</td>
<td>Direct Numerical Simulation</td>
</tr>
<tr>
<td>DSP</td>
<td>Digital Speckle Photogrammetry</td>
</tr>
<tr>
<td>EAP</td>
<td>Electro-Active Polymer</td>
</tr>
<tr>
<td>EB</td>
<td>Equibiaxial tension</td>
</tr>
<tr>
<td>FE</td>
<td>Finite Element</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Modelling</td>
</tr>
<tr>
<td>MAV</td>
<td>Micro-Air Vehicles</td>
</tr>
<tr>
<td>MEMS</td>
<td>Micro-Electro-Mechanical Systems</td>
</tr>
<tr>
<td>PT</td>
<td>Planar Tension</td>
</tr>
<tr>
<td>PZT</td>
<td>Piezoceramic Zirconate Titanate</td>
</tr>
<tr>
<td>SMA</td>
<td>Shape Memory Alloy</td>
</tr>
<tr>
<td>ST</td>
<td>Simple Tension</td>
</tr>
<tr>
<td>UAV</td>
<td>Unmanned Air Vehicle</td>
</tr>
</tbody>
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Chapter 1

Introduction

1.1 Overview

Controlling turbulence is a major aim in aeronautical engineering. A key objective of such control is that of skin-friction drag reduction. Skin-friction drag accounts for about 50% of the total drag (Gal-El-Hak, 1994) in aerial vehicles ranging from Unmanned Air Vehicles (UAVs) to commercial aircraft. Reducing drag on aircraft results directly in a reduction of the fuel consumption thereby enabling larger flight range and endurance. Reducing fuel consumption also provides environmental and economic benefits.

From an environmental point of view, all forms of aviation release carbon dioxide (CO$_2$) into the atmosphere, very likely contributing to the acceleration of global warming. While the principal greenhouse gas emission from powered aircraft in flight is CO$_2$, other emissions may include nitric oxide and nitrogen dioxide and particulates. All these emissions harm the environment and contribute to climate change. An increase in awareness of the problems associated with climate change has arisen in the past decade. Various regulations have therefore been put in place. For example, the European Commission has set targets of reducing CO$_2$ emissions by 80-95% by 2050 (Commission, 2010). Also, the Advisory Council for Aeronautics Research in Europe (ACARE) is suggesting that a reduction of 85 % in global CO$_2$ emissions is necessary in order to limit the global mean temperature increase to a range of 2°C-2.4°C by 2050 (ACARE, 2013). Aircraft manufacturers need to therefore invest in research that would lead to fuel consumption reductions. The
necessary drag reduction required to reach such fuel savings is unlikely to be achieved with existing technologies; hence there is a real need for new ones.

From an economic point of view, fuel savings result in direct cost reductions. In the current economic climate, financial savings are more important than ever as the aviation industry is facing a crisis and oil prices keep on rising. For the first time ever, in 2006, fuel replaced labour as the largest single cost item for the global airline industry (IATA, 2007).

After several decades of research, it has been shown that the large frictional drag in turbulent flows can be attributed primarily to the existence of near-wall vortical structures and their associated events (Kline et al., 1967; Robinson, 1991). The focus of turbulence control is to suppress or counteract these near-wall coherent structures. The challenge for such a control is the fabrication of actuators and sensors functional at the spatial scales of the coherent structures (10 \( \mu \)m to 0.1 mm) and temporal scales (100 kHz) of the turbulence (Kasagi et al., 2005) at the Reynolds numbers appropriate for full-size aircraft. It is only in the past two decades that the development of the MEMS technology has made possible the fabrication of such micro-sensors and micro-actuators. To operate efficiently, actuators need to be controlled using information from sensors. Researchers have documented the most important advances achieved in the past decades on the subject of feedback control. In 1994, Moin and Bewley (1994) published a review of the approaches to feedback control of turbulence. Similarly, Gad-el-Hak published in 1996 a review of the advances in the field of flow control over the six years prior to 1996, see Gad-el Hak (1996). At Imperial College, all the different types of feedback control methods have been reviewed by Koberg (2007). As expected, it is the reactive flow control method which is shown to be the most efficient for turbulence control. A typical system consists of sensors capturing flow information, transmitting it to a controller which then determines the necessary actuation, see Figure 1.1.

In the early 1920s, research on dolphins showed that based on the anticipated muscle power of a dolphin, their swimming speeds were unusually high. This is known as Gray’s paradox (Parry, 1948). An explanation suggested at the time was that their skin actively controls the turbulence to improve their performance. Research has now shown that Gray’s research was based on many flaws (Fish, 2006). Gray estimated the swimming speed of dolphins based on a dolphin swimming near
a ship and during a sprint, two facts that affect considerably the performance of a
dolphin. Fish (2006), when addressing the controversy around this paradox, shares
results of live dolphin studies (Fish, 2006). It is suggested that neither passive nor
active compliance mechanisms match the technique used by dolphins. This is based
on the fact that while gliding, dolphins display a turbulent boundary layer whereas
it is an incomplete turbulent boundary layer that is observed while swimming (Fish,
2006). There is no reason to expect different mechanisms at different times (Fish,
2006). Despite being flawed, drag reduction studies were inspired by Gray’s findings.
Based on Gray’s paradox, the aerodynamicist Max Kramer, claimed that a dolphin
ensured a low level of friction drag by maintaining the laminar flow over most parts of
his body. Kramer suggested that retaining a stable laminar boundary layer over high
speed bodies was possible using a new approach called ‘stabilisation by distributed
damping’. Kramer designed compliant surfaces which he claimed in his article could
reduce drag by up to 60%, see Kramer (1960). As scientists struggled to replicate
such a high drag reduction, Kramer’s work was discredited by aerodynamicists for
a few decades. However, more recently, considerable progress has been achieved in
the theory and practice of compliant walls. Passive methods to reduce surface skin
friction using compliant surfaces have been successfully reported by many, includ-
ing Gad-el Hak (1986) and Choi et al. (1997). These methods suggest the use of
compliant surfaces to delay the laminar-to-turbulence transition. Using compliant
surfaces as passive mechanisms to reduce drag in turbulent boundary layers has yet to be proved, and is likely to require active control. Indeed, Direct Numerical Simulation (DNS) studies (Fukagata et al., 2007) are still struggling to prove the ability of passive compliant surfaces to reduce drag by a significant amount. In 2007 for example, Fukagata et al. (2007) achieved a maximum of 8% reduction using an anisotropic compliant surface.

The example of dolphins in nature together with the advances in MEMS technology have inspired its application to aircraft technology and the conception of a ‘smart’ skin for aerodynamic control based on local time-dependent wall deformation and reactive flow control. Indeed, actuators can now be manufactured at a micro and even nano scale (Khoshnoud, 2012).

A typical illustration of the implementation of such a system on an aeroplane is illustrated in Figure 1.2. Developing such a surface requires expertise from various engineering fields. In the Flow Control research group at Imperial College, London, research has been carried out since 2003 to try and develop such a ‘smart skin’.

![Envisioned skin-friction drag reduction by wall deformation on aircraft](image)

**Figure 1.2:** Envisioned skin-friction drag reduction by wall deformation on aircraft (Koberg, 2007)

Computational work carried out by Koberg (2007) showed that flow perturbations induced by active wall deformation exhibit features of flow control. An efficient
control algorithm based on the opposition control scheme of Choi et al. (1994) has been identified and tested numerically by Koberg (2007). Extensive work carried out by Dearing (2007) led to the choice of the high-performance Electro-Active Polymers (EAP) materials for the fabrication of micro-sensors and micro-actuators for this ‘smart skin’ project. Indeed, EAPs are an ideal material for smart surfaces. They offer high strain rates, high deflections and a bandwidth higher than 1kHz for silicone made actuators and a high electro-mechanical efficiency (Dearing, 2007). Also, EAP actuators are equally suitable for actuators as well as sensors, as discussed in the thesis of Fox (2007), making the idea of a ‘smart skin’ realistic. In terms of manufacturing, the potential ink-jet printing of EAPs is a low-cost fabrication technique that would offer accuracy and reliability as well as the availability to distribute them over a wide surface, a further advantage that led to the choice of their use. A review of the existing technology and applications of ink-jet printing has been carried out by De Gans et al. (2004). Most of the ink-jet printing work reported so far in the literature has been two-dimensional (De Gans et al., 2004). The potential for a layer by layer three-dimensional printing of ceramics has been identified (De Gans et al., 2004). Ongoing work is being conducted by the Flow Control research group to develop and improve EAP performances as well as fabrication techniques.

Dielectric Elastomer Actuators (DEAs), a specific type of EAP actuators, were chosen for the ‘smart skin’ project. Being designed as a thin sheet of elastomer coated with a top and bottom electrode, DEAs extend when a voltage is applied across a device: details of the mechanism of DEAs can be found in section 2.2. This actuating mechanism together with the incompressibility of the elastomer material results in coupled out-of-plane and in-plane strains. Strains as high as 360% have been recorded (Pelrine et al., 1998). Some of the advantages DEAs have over other types of electroactive devices are their high strains, low cost and ease of fabrication (Dearing, 2007). A more comprehensive comparison of DEAs versus other smart actuators will be presented in section 2.1.

Using the appropriate boundary conditions, the design of DEAs makes it possible to design actuators that provide out-of-plane wall deformations similar to the ones that have been identified by Koberg (2007) with the potential of reducing skin-friction drag. When clamped around its edges, a circular actuator buckles when a high enough voltage is applied across its electrodes, thus providing a significant out-
of-plane deflection compared to the size of the actuator. Dearing (2007) achieved deflections of a few hundred microns for dimple shaped actuators of 5 to 10 mm in diameter as will be detailed in section 4.3.

The present project mainly focused on the study of actuators and a novel pressure sensor, first suggested by Lavoie et al. (2007) and modelled by the author. As will be reviewed in the next chapter, sensors functioning at the spatio-temporal scales required for the coherent structures and able to measure the pressure fluctuations of turbulent flow are still lacking (Lavoie et al., 2007).

Most of the work undertaken on EAP actuators and sensors has largely been numerical and theoretical in nature and is experimentally at a very early stage. An accurate modelling technique would provide a low cost and fast method for optimising the design of these devices so improving their performance. The essential task of acquiring an accurate modelling technique for EAP actuators and sensor devices is the aim of this work.

1.2 Objectives and outline

As a first step, a technique to accurately model EAP materials is being developed, taking into account its non-linearities as well as its behaviour at large deformations (hyperelasticity). This technique is presented and validated in Chapter 3.

The main challenge in modelling dielectric elastomer devices is the modelling of their electro-mechanical behaviour. Commercially available electro-mechanical simulation software do not apply to non-linear materials such as EAPs. The ANSYS Finite Element (FE) software will be used in this work to develop a novel modelling technique applicable for DEAs. The modelling procedure, along with simulations and experimental results, is presented in Chapter 4.

Designs for actuators and sensors can be studied and optimised to suit the required applications using the novel dielectric elastomer model presented in this work. Various means of optimising the design of dielectric elastomer devices are suggested in the sixth chapter. Parameters such as material thickness, the amount of pre-stretch (stretching applied to the material during the fabrication process before it is used) and boundary conditions can have considerable impact on the performance of devices as will be discussed.
A reactive flow-control system requires sensors as well as actuators. A novel design of an EAP-based pressure sensor is currently under development in the Flow Control group and is presented in Chapter 7. An FE model is developed to study the abilities and performance of such a device and how best to optimise it.

To complete the model, its time-dependent properties are examined by a modal analysis. This is examined in a eighth chapter including a comparison with experimental data.

The thesis is completed by examining the potential for dielectric elastomer devices in providing a ‘smart’ surface for aerodynamic control.
Chapter 2

Literature review

2.1 Smart actuators

The most commercialised and used smart actuator is the piezoelectric actuator (Trease, 2001). The most common type of piezoceramic material is the lead zirconate titanate (PZT) (Trease, 2001). Piezoceramics have a crystal internal lattice structure, undergoing displacements when an electric field is applied. The advantage of this type of actuator is their large bandwidth, even able to reach the gigahertz frequency (Trease, 2001). The range of material strain typically observed is 0.1-0.2% (Tabib-Azar, 1998). Voltages of 1kV/mm to 2kV/mm can be typically applied to such actuators (Robinson, 2006). To achieve higher strains than a typical actuator without increasing the applied electric field, thin layers of piezoelectric actuators can be glued together to form stacks. Voltage is applied to each layer individually, and the displacement is found to be the sum of all the individual displacements of the layers (Robinson, 2006). The assembling and wiring of the actuators gets more challenging as the number of layers increases. A new type of piezoceramics being developed is the single crystal piezoceramic actuator (Trease, 2001). This type of device offers 5 to 10 times greater strains as well as lower hysteresis and strains, 10-15% (Trease, 2001). Table 2.1 presents some of their key attributes. As reported by Dearing (2007), adding prestress to such materials enables displacements of up to 1000 µm. However, for dimple actuators the actuators would have to act as linear pistons so the stacking design can unfortunately not be used. Furthermore, ceramic PZT actuators are quite brittle (Trease, 2001), making them difficult to use.
for dimple actuators.

Magnetostrictive actuators are ferromagnetic materials that are actuated by magnetisation. On the atomic level, small internal magnetic domains align when a magnetic field is applied. The displacement increases per unit of magnetic field, it is therefore favoured for large scale actuators. The most popular magnetostrictor material is Terfenol-D. Strains between 0.1-0.6% can be achieved (Gauthier et al., 2006). Levels of hysteresis are low, around 2%. Among the disadvantages of these actuators is the need of a magnetic field delivered to an embedded actuator. Also, such actuators are highly stiff but can be made more flexible combining them with materials such as silicone (Trease, 2001).

Shape memory alloys (SMA) provide low-stiffness, high displacement, thermally driven actuators. When plastically deformed SMA actuators, recover their original state when heated. A major disadvantage of this actuation mechanism is the limited rate of heating and cooling. Also, as they are thermally driven, their performance will be degraded in an aerodynamic environment by the forced convection heat loss. Their range of operating frequencies is quite low 0.5-5 Hz (Trease, 2001), they are therefore not suitable for high frequency applications. Further disadvantages reported by Dearing (2007) are their large hysteretic effects, weight and poor fatigue properties.

Interest in organic polymers for an increasing number of mature and cutting-edge technologies has risen in the past few decades due to their versatile properties, low cost and robustness (Plante and Dubowsky, 2007a). Most recently, interest has grown in EAPs, soft and flexible polymers capable of transforming electrical energy into mechanical energy. EAP materials are often qualified as ‘smart’ materials as they provide tunable deflections (Shankar et al., 2007). EAP materials were first proposed as actuators in 1989 (Baughman and Shaklette, 1989). In 1994, one of the first articles on electrically induced strains appeared, a strain of 3% was reported (Kofod, 2001). This was then considered a significant step forward compared to piezoelectric materials that were only offering strains of up to 0.1%. In 1999, Bar-Cohen initiated an annual SPIE conference on EAPs, as part of the Smart Structures and Materials Symposium, to promote international cooperation among the developers, users and potential sponsors. Extensive research on EAP materials has been carried out since and higher maximum strains are constantly being
achieved. Strains of several hundreds of percent are now available. In 2000, Pelrine et al. reached strains of up to 117% with silicone elastomers, and up to 215% with acrylic elastomers (Pelrine et al., 1998). By 2001, Kornbluh et al. reached strains of 300% and by 2004 they reached 380% (Bar-Cohen, 2004).

EAP materials can be configured in a variety of shapes and designs and can be used as actuators or sensors (Bar-Cohen, 2004). Emerging technologies such as Micro-Air Vehicles (MAV) are looking into their use to achieve technological improvements. Indeed, the ongoing work on improving the performance of MAVs has recently been focusing on trying to match the performance of insect flight. This is done by trying to reproduce the insect-like flapping motion that would provide the MAV with increased manoeuvrability. A solution that has been proposed by Conn et al. (2008) is the development of an ‘artificial muscle’ using EAP actuators to provide a lightweight and under-constrained flapping mechanism. Prototypes have shown similar performance to muscles. EAPs mimic biological muscles through their electro-responsive properties and they are often referred to as ‘artificial muscles’ (Bar-Cohen, 2004). Key attributes such as toughness, actuation stress and strain, coupled with high fatigue resistance, scalability and reliability are the properties that have been critical in choosing these materials to reproduce the muscular behaviour of animals, insects and humans. They have been used for research into medical implants, responsive prosthetics and the human-machine interface (Shankar et al., 2007).

There are two major groups of EAPs, ionic EAPs and electric EAPs. Ionic EAPs are driven by the mobility and diffusion of ions and therefore have to be wet. It is electric fields that drive electric EAPs. The electro-mechanical coupling of such materials make it possible to design EAP sensing devices as well as actuators. Both actuators and sensors could be ink-jet printed onto surfaces to provide active control feedback of turbulence (Bar-Cohen, 2002), the ultimate goal of the ‘smart skin’ project (Dearing et al., 2010). This manufacturing method is currently being studied at Imperial College, London as well as outside of Imperial College, see Pabst et al. (2011). Table 2.1 summarises some key properties of the smart actuators mentioned above as well as DEAs.
<table>
<thead>
<tr>
<th>Properties</th>
<th>DEA</th>
<th>SMA</th>
<th>Magnetostrictive</th>
<th>Piezoelectric</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Silicone</td>
<td>Acrylic</td>
<td>PZT</td>
<td>Single Crystal</td>
</tr>
<tr>
<td>In-plane Strains %</td>
<td>120(^1)</td>
<td>380(^1)</td>
<td>5(^2)</td>
<td>0.1 – 0.6</td>
</tr>
<tr>
<td></td>
<td>38(^3)</td>
<td>10(^4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thickness Strains %</td>
<td>33(^4)</td>
<td>≈ 30(^4)</td>
<td>8(^4)</td>
<td>0.1 – 0.2</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>10</td>
<td></td>
<td>0.5 – 1.0</td>
</tr>
<tr>
<td>Bandwidth Hz- 3dB</td>
<td>1400(^2)</td>
<td>10/100(^2)</td>
<td>0.5 – 5(^3)</td>
<td>1(^5)</td>
</tr>
<tr>
<td></td>
<td>&lt; 10(^k)</td>
<td>1</td>
<td></td>
<td>10(^k)</td>
</tr>
<tr>
<td>Work density J/cm(^3)</td>
<td>0.75(^6)</td>
<td>3.4</td>
<td>&gt; 100(^2)</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>100(^3)</td>
<td></td>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td>Electro-Mechanical coupling %</td>
<td>54(^7)</td>
<td>60</td>
<td>5</td>
<td>81</td>
</tr>
<tr>
<td>Operational electric field MV/m</td>
<td>(O(100))</td>
<td>(O(100))</td>
<td>thermally activated(^8)</td>
<td>O(0.1-1.0)</td>
</tr>
<tr>
<td></td>
<td>(≈ 5V)</td>
<td>magnetic field (≈ 600kA/m)</td>
<td></td>
<td>O(0.1-1.0)</td>
</tr>
<tr>
<td>Young’s modulus MPa</td>
<td>0.1 to 3.0(^9)</td>
<td>1.0 to 3.0(^9)</td>
<td>20,000(^2) to 83,000(^3)</td>
<td>10,000</td>
</tr>
<tr>
<td></td>
<td>25,000</td>
<td>35,000</td>
<td>4,000 to 8,000</td>
<td></td>
</tr>
<tr>
<td>Operating temperature °C</td>
<td>–100 to 150(^6)</td>
<td>–10</td>
<td>–100 to 200(^6)</td>
<td>–40 to 200</td>
</tr>
<tr>
<td></td>
<td>–20 to 180</td>
<td>–20</td>
<td>–20 to 180(^6)</td>
<td>–40 to 200</td>
</tr>
</tbody>
</table>

Table 2.1: Comparison of different smart actuators, based on Table 3.1 (Dearing, 2007) Superscripts refer to: 1- with prestretch, 2- typical, 3-maximum, 4- without prestretch
2.2 Dielectric Elastomer Actuators (DEAs)

A dielectric elastomer can be described as a compliant capacitor, as illustrated in Figure 2.1. The dielectric medium consists of a thin sheet of incompressible, highly deformable elastomer material, sandwiched between two compliant electrodes. As for any capacitor, when an electric field is applied, positive charges appear on one electrode and negative charges appear on the other, generating Coulomb forces between the electrodes. For a voltage, $V$, applied between the electrodes, the Coulomb forces generates a pressure, $\sigma_e$, between the electrodes known as the Maxwell stresses, according to Equation 2.1.

$$\sigma_e = \epsilon_e \epsilon_0 \left(\frac{V}{t}\right)^2$$  \hspace{1cm} (2.1)

where $\epsilon_e$ is the elastomer relative permittivity, $\epsilon_0$ is the free space permittivity and $t$ is the elastomer thickness.

The Maxwell stresses draw the electrodes nearer to each other, resulting in a thinning of the elastomer sheet. Being incompressible, an expansion of the material in a direction perpendicular to the one of the Coulomb forces occurs, as illustrated in Figure 2.1.

2.3 Modelling DEAs

In their 10 years of development, DEAs have not been widely used in commercial applications. On the one hand, this is due to a lack of understanding of the mechanisms governing their performance, which has increased the interest in the modelling
of DEA actuators. For example, Kofod (2001), tried to validate the use of different hyperelastic models using uniaxial tensile test results to model the VHB 4910 elastomer. At the same time, Kim et al. (2001) used FEM to look into how parameters such as Young’s modulus, pre-strain, thickness, frequency and temperature affect the performance of electrostrictive polymers. In 2004, Carpi and Rossi presented an analytical model for dielectric elastomer cylindrical actuators based on linear elastic theories, which was validated by experimental work (Carpi and Rossi, 2004). Further analytical models such as the one developed by Goulbourne et al. (2004) for the modelling of cardiac pumps or by Sommer-Larsen and Kofod (2002) for strip actuators were not validated experimentally.

On the other hand, in experimental setups, DEAs have not provided performances as high as expected due to various technical limitations. In the research carried out at Imperial College by the author it was found for example that an internal prestretch is introduced in EAPs during the manufacturing process, see section 4.4. The amount of prestretch added during manufacturing is hard to predict and makes the voltages required for actuation very high, as will be discussed further in this thesis in Chapter 5. A further delay in using DEAs is the struggle to achieve repeatable results when their performance is tested, see Dearing (2007). When used under real conditions, the performances of DEAs were lower than the ones predicted by the various theoretical studies as discussed by Plante and Dubowsky (2007a). An accurate theoretical model of the actuating mechanism of DEAs would be an efficient tool for optimising their use and design.

The challenges in modelling the actuation of dielectric elastomers are two-fold. Firstly, the characterisation of the constitutive behaviour of the elastomer material is needed. Secondly, the coupling between electrostatic forces developed in the elastomer and its dimensions, expressed by Equation 2.1, also has to be modelled (Wissler and Mazza, 2005a).

Studies first concentrated on the modelling of actuators with simple geometries such as planar strip DEAs. An analytical model based on a visco-hyperelastic film model has been developed by Lochmatter et al. (2007). A different model based on a Gent hyperelastic model was studied by Yang et al. (2005). Simulations from both studies agreed well with the experimental data available, their application is however restricted to the strip actuator geometry. Further work to improve these models is
still required but the accuracy of the modelling of planar strip actuators has moti-
ved the modelling of more complicated geometries. In 2007, Plante and Dubowsky
(2007a) developed an analytical model of a diamond shape actuator made of mate-
rial VB4905/4910 enabling the study of viscoelasticity and current-leakage effects.
The polymer and energy-based models verified experimental results explaining ex-
perimental observations over a wide range of working conditions: stretches of the
material of up to 5, velocities varying by more than three orders of magnitude and
under load reversal as well (Plante and Dubowsky, 2007a). The limitation of this
model is to the diamond shape geometry pictured in Figure 2.2. A diamond shaped
structure is preloaded by two elastic bands as seen in Figure 2.2, an elastomer film
covers the inside of the actuator. The film is pressured when voltage is applied due
to the Maxwell stresses developing. As a result of this pressure, the planar area of
the film increases thus generating a uniform stress/strain during actuation (Plante
and Dubowsky, 2007a).

![Diamond shape actuator](image)

**Figure 2.2: Diamond shape actuator (Plante and Dubowsky, 2007b)**

Models of DEAs providing out-of-plane actuation have been reported in the
literature. Goulbourne et al. (2005) aiming to model cardiac pumps, developed the
model of actuation for an inflatable membrane. The use of this model is restricted
to the circular membrane configuration (Goulbourne et al., 2005). Carpi and Rossi
modelled the cylindrical actuator (Carpi and Rossi, 2004), as mentioned earlier.
Their modelling also applies specifically to actuators of that precise configuration as
are the majority of published models as they are all based on uniaxial tensile data
only.

Wissler and Mazza (2005a,b, 2007) have carried out significant modelling and
simulation work using FEM. Focusing on the best choice of hyperelastic model for
the study of EAPs, the Yeoh, Ogden and Mooney-Rivlin models are investigated (Wissler and Mazza, 2005b). At first, material parameters are found by fitting uniaxial data only. The chosen actuator used to develop this work is a biaxially pre-strained circular device, fixed to a circular frame. The top and bottom centre part of the actuator are coated with electrodes as seen in Figure 2.3. When voltage is applied across the electrodes, the actuator expands causing a thinning of the material, see Figure 2.3. In their work, Wissler et al. modelled the electro-mechanical coupling of the elastomer material by decoupling the electrostatic and structural problems (Wissler and Mazza, 2005b). The electrostatic problem is solved numerically while the inverse structural problem is then solved using FE iterative simulations while the elastomer is modelled using a time-dependent hyperelastic model. The contraction of the circular device in the thickness direction is imposed as a boundary condition, the compressive forces required are calculated based on the FE results of mechanical pressure in the thickness direction. Voltage is then calculated using the Maxwell equation 2.1.

![Figure 2.3: Circular actuator configuration (Wissler and Mazza, 2005b)](image)

The three models predict uniaxial tension accurately, however show very different results when simulating circular actuation. The Ogden and Mooney-Rivlin models predicted respectively 30% and 200% higher deflections. The Yeoh model predicted accurate results, highlighting the very important outcome of this research, that the choice of hyperelastic model plays a key role in the quality of the modelling (Wissler and Mazza, 2005b).

The Yeoh model is then used to model a circular actuator together with a Prony series to model the time-dependent material properties of the material in further
work, see Wissler and Mazza (2005a). FE and experimental results are collected using the ABAQUS 2003 FE software. Results are collected over 175 seconds when a prestrained actuator is activated by a constant voltage of 3.5 kV. Calculated output voltages are plotted together with the voltage input, see Figure 2.4. An error of 20% was initially collected but reduced gradually and disappeared after 80 sec.

![Figure 2.4: Simulation vs. experimental results (Wissler and Mazza, 2005a)](image)

A couple of years later, Wissler at al. compared their improved analytical and simulation models to experimental data (Wissler and Mazza, 2007), thus providing a way to determine constitutive model parameters as well as a validation for the models suggested previously by the authors (Wissler and Mazza, 2005a,b, 2007).

Dubois et al. (2006) present a preliminary geometrically non-linear multi-physics displacement FE model in ANSYS of a diaphragm actuator providing a qualitative appreciation of the shape of the out-of-plane deflection, see Figure 2.5. Although it is based on multiple assumptions, including the linearity of the elastomers and use of previous experimental data, the study showed strong dependence on boundary conditions, prestretch (initial material imperfection leading to internal material strains), electric permittivity and Young’s modulus.
Figure 2.5: FE simulation of a 30 µm thick diaphragm actuated with 400 V, measured in µm (Dubois et al., 2006)
2.4 FE software

The FE method is a well established technique for analysing the behaviour of structures subjected to a variety of loads. These may be static or dynamic, and the structural responses may be linear or non-linear, with varying degrees of complexity. The software most used in aeronautical engineering is ABAQUS 6.11 (ABAQUS, 2012). However, the multiphysics capabilities of ABAQUS only model the following physical couplings: fluid-mechanical, piezoelectric-mechanical, structural-acoustic, electrical-thermal, thermal-mechanical, thermal-fluid-mechanical and structural-pore pressure (ABAQUS, 2012). An electrostatic-structural solver is not available and therefore it is not the appropriate tool for electro-mechanical modelling. External electromagnetic software or analytical equations can be coupled with structural simulations of ABAQUS 6.4.1 to model the functioning of DEAs. This has been done by Wissler and Mazza (2005a) as mentioned earlier. In this work, a direct modelling procedure is required, the ABAQUS software is therefore not suited.

ANSYS is commercially available FE software providing electrostatic-structural as well as flow-structural coupling options (ANSYS, 2009a). Flow-electro-mechanical simulations can therefore be carried out as well providing a means for the optimisation of DEA devices for flow control. ANSYS Multiphysics delivers two proven solution techniques for solving coupled-physics problems, the directly coupled-field elements and the ANSYS multi-field solver, as will be discussed in Chapter 4. Together, the two solution techniques in ANSYS Multiphysics provide the appropriate solution technology to solve an extremely broad range of industry applications such as DEAs. Therefore, ANSYS is the software chosen in this work.

2.5 Hyperelastic modelling

There is very limited work available in the literature dealing with elastomer modelling. A standard modelling procedure has yet to be developed. Recent work has confirmed that hyperelastic modelling of elastomers provides the most accurate results. Wadham-Gagnon et al. (2006) showed that with some preconditioning introduced, high-order hyperelastic models such as a third-order Yeoh or Ogden model, provide good agreement with the experiments. Wadham-Gagnon et al. (2006) introduced a preconditioning iteration to take into account the strain-induced stress-
softening of the material, a material property that will be mentioned later as the Mullin effect in section 8.1.2. In the work of Wissler and Mazza (2005a), the Yeoh model has been used to model the behaviour of a pre-strained circular actuator. Results agreed with the experimental work to a great extent as discussed in section 2.3, see Figure 2.4.

Hyperelastic materials are non-linear materials that can experience large deformations. They are characterised through a strain-energy potential, $W$, representing the strain energy of the material as a function of deformation. The material response is assumed to be isotropic. This assumption allows the strain-energy potentials to be expressed in terms of strain invariants or principal-stretch ratios. The stretch ratio, $\lambda_i$, is the ratio of the final length to initial length in the direction of the $i$-strain axis. $\lambda_i$ equals 1 when there is no deformation. The three strain invariants, $I_i$, of the stretch-ratio tensor are expressed as:

$$
I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \\
I_2 = (\lambda_1 \lambda_2)^2 + (\lambda_2 \lambda_3)^2 + (\lambda_3 \lambda_1)^2 \\
I_3 = (\lambda_1 \lambda_2 \lambda_3)^2. 
$$

(2.2)

The constant volume assumption is made, therefore:

$$
I_3 = (\lambda_1 \lambda_2 \lambda_3)^2 = 1. 
$$

(2.3)

Different hyperelastic models are available to help find the best fit to the material data. The Mooney-Rivlin model (Mooney, 1940) is the hyperelastic model most in use today and is the model chosen in this work. It presents many advantages in terms of being able to handle behaviour of different kinds of rubbers. It also offers an ability to increase the number of modes permitting the handling of large strains. In the present work, the two-constant, and five-constant Mooney-Rivlin models are used. The strain-energy function for the two-constant Mooney-Rivlin model is expressed as:

$$
W = C_{10}(I_1 - 3) + C_{01}(I_2 - 3) 
$$

(2.4)
where \( C_{10} \) and \( C_{01} \) are empirically determined material parameters. The material parameters differ for every material, even for the same type of material manufactured in different ways. The strain energy function for the five-constant Mooney-Rivlin model is expressed as:

\[
W = C_{10}(I_1 - 3) + C_{01}(I_2 - 3) + C_{20}(I_1 - 3)^2 + C_{11}(I_1 - 3)(I_2 - 3) + C_{02}(I_2 - 3)^2 \quad (2.5)
\]

where \( C_{10}, C_{01}, C_{20}, C_{02} \) and \( C_{11} \) are also empirically determined material parameters.

In this work, the Mooney-Rivlin constants will be determined through the curve fitting procedure of ANSYS v.10.0. They are calculated by performing a least squares fit analysis between the experimental stress-strain data and the Cauchy predicted stress values presented in the next section for the three elementary deformation modes, see section 2.5.2.

2.5.1 Experimental testing

Successful modelling and design of elastomers depends on the right selection of strain-energy function as well as on an accurate determination of the material constants. As mentioned by Miller (2002), there are no national or international standards on the type of experiments required for the hyperelastic modelling of an elastomer. To achieve stability, it is recommended by the software used in this study, ANSYS 10.0, that the material constants be fitted using test data in at least as many deformation states as will be experienced in the analysis. Six deformation modes are identified and illustrated in Figure 2.6. As described in Figure 2.7, we find that upon the addition of hydrostatic stresses, the following modes are identical:

- uniaxial tension and equibiaxial compression
- uniaxial compression and equibiaxial tension
- planar tension and planar compression.

2.5.1.1 Uniaxial tensile test

Uniaxial tensile tests are performed using a tensile machine. A typical tensile machine can be seen in Figure 2.8. To ensure pure tension, the length of the specimens
Figure 2.6: Elementary deformation modes (ANSYS, 2009a)
Figure 2.7: Equivalent deformation modes (ANSYS, 2009a)
must be at least ten times bigger than their width and thickness. The specimen is held by two grips attached to crossheads as illustrated in Figure 2.8. During the experiment, the strain is applied to the material by the displacement of the lower crosshead while the upper crosshead stays fixed. A load-cell located on the top of the upper crosshead is used to measure the force developing inside the specimen while it is being strained. The crosshead speed and the frequency of the measurements are fixed by the user during the experiment. The data collected consists of the specimen elongation and force measurements applied on the load cell due to the stretching for every time step. Once extracted the data are plotted as a strain-stress curve.

![Uniaxial tensile machine](image)

Figure 2.8: Uniaxial tensile machine (Miller, 2002)

Based on the orientations of the axis in Figure 2.6, the hyperelastic stress relationship of the uniaxial deformation can be expressed by Equation 2.6 and the corresponding engineering stress by Equation 2.7. Details concerning the derivation of these equations can be found in ANSYS (2009a).

\[
\sigma_{11} = 2(\lambda_1^2 - \lambda_1^{-1}) \left[ \frac{\partial W}{\partial I_1} + \lambda_1^{-1} \frac{\partial W}{\partial I_2} \right] \\
T_1 = \sigma_{11} \lambda_1^{-1}
\]

Equations 2.6 and 2.7 are used to evaluate the material constants of the strain energy potential.
2.5.1.2 Biaxial tensile test

A specimen subjected to biaxial tension is simultaneously stretched in the horizontal and vertical directions. Biaxial tension experiments have not been standardised yet as mechanical testing machines with two independent axes are rare in test laboratories due to their expense. A few techniques are available: these include radial extension, biaxial test fixtures in a tensile machine, double-axes tensile test or a bulge test (Dearing, 2007). Bulge tests are inexpensive to carry out and give accurate results for strains up to 40%. The theory behind why a bulge test is equivalent to a biaxial tensile test can be found in Appendix A.

Bulge test

In a bulge test, pressure is applied under a circularly clamped membrane. The deflections of the membrane are collected as a measurement for the strains and stresses that develop in the membrane. An illustration of the experimental setup used for bulge test experiments by Dearing (2007) is given in Figure 2.9. A pressure chamber linked to a piston provides the pressure that is applied on the bottom of the circular clamped membrane. Digital Speckle Photogrammetry (DSP) equipment is used to take photos of the inflated membrane and measure the out-of-plane deflections. Details of the setup can be found in Dearing (2007).

Based on the direction of the axis in Figure 2.6, the hyperelastic stress relationship of the biaxial deformation mode, can be expressed as follows:

\[ \sigma_{11} = 2(\lambda_1^2 - \lambda_1^{-4}) \left[ \frac{\partial W}{\partial I_1} + \lambda_1^2 \frac{\partial W}{\partial I_2} \right] \]  \hspace{1cm} (2.8)

\[ T_1 = \sigma_{11} \lambda_1^{-1} \]  \hspace{1cm} (2.9)

The principal true stress is expressed by Equation 2.8 and the corresponding engineering stress is expressed by Equation 2.9. As for the uniaxial test, these expressions are necessary for evaluating the material constants of the strain-energy functions used to model the material.
Figure 2.9: Bulge test experimental set-up (Dearing, 2007)

Figure 2.10: Planar tension test (Miller, 2002)
2.5.1.3 Planar tension test

A typical setup for a planar tension test is presented in Figure 2.10. Specimens are required to be at least ten times wider than the thickness or the length. The high aspect ratio imposes plane strain conditions on the test specimen by preventing the edges of the specimen from contracting. As in the tensile test, the data collected from the planar tension test is post-processed to obtain a strain-stress curve.

The principal pure shear stress is expressed by Equation 2.10, the corresponding engineering stress is expressed by Equation 2.11.

\[ \sigma_{11} = 2(\lambda_1^2 - \lambda_1^{-2}) \left[ \frac{\partial W}{\partial I_1} + \frac{\partial W}{\partial I_2} \right] \]  
\[ (2.10) \]

\[ T_1 = \sigma_{11} \lambda_1^{-1} \]  
\[ (2.11) \]

2.5.2 Material constants

As discussed earlier, the most accurate constants are achieved when the three strain-stress sets of data are used, from the uniaxial, biaxial and planar tensile test. A typical set of data is presented in Figure 2.11. To extract the material constants from the experimental data, these are fitted to the chosen constitutive model using the ANSYS least squares fit analysis. Comparing the experimental data to simulated data, using the constants acquired, is a check of the consistency of the curve-fitting process with the simulation and hyperelastic models. Details can be found in Appendix B.

2.6 Dimple actuators

Dimpled surfaces have long been used successfully as flow control devices. Golf balls are an example of the use of such surfaces (Choi et al., 2006), where the presence of dimples on their surface delays the flow separation thus improving their performance. These dimpled surfaces act as passive devices, modifying the flow without any external energy output. Active dimples are devices able to adjust their height over time for flow control purposes. The typical dimples studied in this work are made of a clamped electroactive circular membrane coated with top and bottom
graphite electrodes. When voltage is applied across the electrodes, the material thickness reduces, due to the Maxwell stresses expressed by Equation 2.1. Owing to the clamping boundary conditions and the material being incompressible, the material cannot expand, see Figure 2.12. Compressive stresses build up until they reach the buckling limit. The detailed analytical buckling analysis can be found in Chapter 5. In buckling cases, the direction of deflection is controlled by external directional bias. In laboratory conditions the directional bias is part of the design of the dimple. Indeed, the weight of the material and electrodes provide the structure the mechanical bias it needs to deflect downwards. For the application of dimples onto aircrafts, a directional bias other than weight will have to be applied to make sure the buckling occurs in the desired direction. Indeed, if this is not the case, the air flow might be generating an unwanted mechanical bias.

Over the past few years, experimental and computational work has been undertaken by the Flow Control group at Imperial College, London to study the effect of active dimples on flow. Experimental work has been carried out on single dimples (Lambert et al., 2005; Dearing et al., 2007) in a water channel and in a wind tunnel (Dearing, 2007).

Flow fields generated by the actuation of dimples in a water channel are pictured for various frequencies, as a function of the non-dimensional frequencies, the Strouhal number in Figure 2.13, \( St = fD/U_\infty \), where \( f \) is the forcing frequency of the dimple,
$D$ the diameter of the dimple and $U_\infty$ the free stream velocity. Downstream of the dimples, the dark regions of dye swept towards the centre-line show a common flow away from the surface, see Figure 2.13. At the highest frequencies, a jet-like structure is formed. It was shown by Lambert and Morrison (2006) that a flux of vorticity can be created by local time-dependent surface deformation. A piston driven dimple was used during the experimental work carried out by Lambert and Morrison (2006). The local acceleration resulted in a large increase of the streamwise and spanwise components of vorticity thus generating a counter rotating pair of vortices. Further designs will be developed aiming to produce a single vortex. The dimple geometry is believed to be able to control the penetration length making the dimple a promising direction for developing a way of artificially producing a vortex aiming to reduce drag (Lambert and Morrison, 2006). Dearing et al. (2007) used phase-averaged velocity contours to show that the vorticity and velocity move in phase. The magnitude of the vorticity is found to be controllable through appropriate combinations of Strouhal and Reynolds numbers. Further research by McKeon et al. (2004) showed that dimple design and frequencies can be varied for effective flow control.

Experimental work in a wind tunnel carried out by Dearing has shown that prototypes are not functioning at the size and frequency response necessary to achieve turbulence control yet. More work on the fabrication of dimple actuators especially to achieve more robustness and frequency response is required for better perfor-
Figure 2.13: Flow visualisation of smooth dimple (a) $St=13$; (b) $St=24$; (c) $St=46$; (d) $St=68$; (e) $St=90$; (f) $St=112$ (Lambert et al., 2005)
2.7 Pressure sensors

Pressure sensors are commercially available, but devices operating at spatial scales of the order of magnitude of turbulent coherent structures at high Reynolds numbers are still lacking. Recent MEMS developments have enabled the development of devices operating at smaller scales. In this work, a novel capacitive pressure sensor is presented and studied, see Chapter 7.

Capacitive pressure sensors use local changes in pressure to deflect or deform a thin membrane which forms one plate of a capacitor; deflection of the membrane may then be detected by sensing the small change in the capacitance. A typical structure of a capacitive pressure sensor is shown in Figure 2.14 (Wang and Ko, 1999a). Capacitive pressure sensors have the advantage of being low power-consumption devices that are robust and highly sensitive (Wang and Ko, 1999b). They have been developed for multiple industrial applications such as aerodynamic measurements (Zagnoni et al., 2005), bio-medics (Caronti et al., 2006) and measurements in harsh environments.

![Figure 2.14: Basic structure of a capacitive pressure sensor (Wang and Ko, 1999a)](image)

Various studies have been carried out to optimise the design of capacitive pressure sensors. Peters et al. (1999) developed an analytical model to show that the sensitivity of a device can be increased by changing parameters such as the length of the edges of the membrane, the thickness of the membrane as well as the height of the pressure chamber. A common disadvantage of capacitive pressure sensors is their non-linear output to pressures. Also, their sensitivity is not high enough to ignore the stray capacitance effects (Wang and Ko, 1999a). FE simulations validated by experimental work were carried out by Wang and Ko (1999a) and Rosengren
et al. (1992). Both showed the benefits of touch-mode capacitive sensing. Such sensors operate in much the same way as the classical capacitive sensor described above, except that the membrane is allowed to be in contact with the other plate of the capacitor and become mechanically constrained by it. This is described as a ‘touch-mode’ design. The two plates remain electrically isolated by a thin film of insulation. A typical capacitive sensor operating in touch-mode is illustrated in Figure 2.15. The diaphragm membrane is always collapsed, touching the insulating layer. Capacitive pressure sensors operating in touch mode have shown to not only offer a nearly linear response to pressure (Rosengren et al., 1992; Wang and Ko, 1999a), see Figure 2.16, but also provide higher sensitivity (Wang and Ko, 1999a,b) and robustness over classical deforming or deflecting membranes. Industrial applications have also been researched (Wang and Ko, 1997), and these devices have been deployed commercially for the measurement of high-amplitude pressure fluctuations. However, it appears that pressure sensors have not yet been developed for applications at operational Reynolds numbers.

A pressure sensor using the properties of electroactive materials and the increased sensitivity of the touch-mode has been suggested by Lavoie et al. (2007). Such a novel device can be used as a pressure sensor on a ‘smart skin’. To optimise its design and performance, modelling of this device has been carried out by the author and is presented in Chapter 7.

![Figure 2.15: Structure of a touch mode capacitive pressure sensor (Wang and Ko, 1999a)](image)
Figure 2.16: Capacitance vs. pressure for a capacitive pressure sensor (Wang and Ko, 1999a)
Chapter 3

Modelling elastomers

Modelling accurately elastomers is essential to design elastomer made devices such as dimple actuators or pressure sensors. As discussed in section 2.5, successful elastomer modelling can be provided by hyperelastic models, a FE curve-fitting process and experimental data from the three elementary deformation modes: uniaxial, biaxial and planar tension test. Experimental tests and simulations are carried out in this chapter to check the ability of the ANSYS software to model accurately the behaviour of elastomers. In this chapter, elastomers are modelled using time-independent models. Viscoelastic and other time-dependent effects will be discussed in Chapter 8.

3.1 Material testing

Five different silicone elastomers are studied in this work: MED4905 of thicknesses 250 µm, 100 µm and 50 µm and MED4930 of thicknesses 100 µm and 50 µm. The MED4905, 250 µm material was purchased from Nusil while the four other materials were purchased from SSFAB. The experimental data used in this work has been collected by an external laboratory, Axel Labs inc. According to the Axel testing policy, each test was carried out three times for repeatability. However, a single set of data was sent to the author for each material and testing mode according to the purchased testing package. A typical set of data collected for a MED4905 specimen of 250 µm thickness tested in uniaxial, biaxial and planar tension is presented in Figure 3.1. To validate the accuracy of the results collected by Axel, a number of
experiments were carried out by the author at Imperial College, London.

A quasi-static bulge test was carried out by the author on a 250 \( \mu m \) thick MED4905 specimen. The experimental setup used was designed by Dearing (2007), see section 2.5.1.2. Due to equipment availability, the experiment was only carried out once by the author. The experimental data is compared to a set of the Axel data collected for the same material. Both sets of data agree well as seen in Figure 3.2, thus giving us confidence to rely on the results provided by Axel.

Uniaxial tests have also been carried out at Imperial College, by the author, to verify the accuracy of the Axel tensile test results. An initial experiment was performed on the existing tensile testing setup usually used for testing rubber materials, an Instron machine equipped with metallic clamps and a 1 kN load cell. The collected data was very noisy, see Figure 3.3. Enquiries about the causes of such noise led to the consideration that the clamps and load cell were not suited to elastomer testing as the measured stresses are very low, below 1 N for strains up to
Figure 3.2: Bulge test data, True Stress vs. True Strain, both are the raw and single set of data collected respectively by the author and Axel

700 %. For a better understanding of the importance of the load cell choice, Axel conducted tensile test experiments using three different load cells: 50 N, 250 N and 400 N. The results are presented in Figure 3.4. As this testing had to stay within our initial budget with Axel, this is unfortunately the only set of data collected with different load cells. Based on the results available, Axel and the author agreed that the use of a 50 N load cell would be appropriate for accurate testing. All further data collected by Axel was carried out using the 50 N load cell.

In this chapter, the author is working with time-independent models. Materials need to be tested at a speed which is low enough to capture their quasi-static properties. To establish an appropriate testing speed, cyclic uniaxial tests were carried out by Axel at both 0.83mm/sec and 0.083mm/sec. Results are plotted in Figure 3.5. In the cyclic loading of the material seen in Figure 3.4 and Figure 3.5, material properties show much greater stiffness for the first cycle than the subsequent ones. This can be explained by the fact that high forces are required to stretch the polymer chains for the first time. Once the polymer chains have been stretched, it is much easier to stretch them again. This mechanism, known as hysteresis, is similar to the one observed while filling up air balloons and will
Figure 3.3: Tensile test data using a 1 kN load cell, True Stress vs. True Strain, single set of data collected by the author at Imperial College, London

be discussed further in Chapter 8. After a few cycles, it seems that results are similar for both testing speeds, suggesting that at both speeds it is the quasi-static properties that are captured. A further specimen was tested for one cycle only at a speed of 0.083mm/sec, see red curve in Figure 3.5. Results suggest that testing conditions and material properties of the MED4905 material cannot be drawn based on its first stretching cycle. As will be discussed further, the behaviour of this elastomer especially the first testing cycle is highly dependent on the specific batch and handling during testing. Judging by the second testing cycle and above, the speed of 0.83mm/sec was chosen for the Axel testing.

A load cell of 10 N was chosen for the experiments carried out by the author. This 10 N load cell from Instron was available for testing and was the only one available to the author for testing that was below 400 N. To accommodate this very light load cell, clamps made of Delrin, a light weight but hard black plastic material, weighing less than 500 g were designed, see Figure 3.7. Sketches of the clamps in the PTC - Creo Elements/Pro 5.0 software are attached in Appendix C.

A uniaxial tensile test is carried out on a 3 mm-wide and 30 mm-long specimen. The stress-strain response is collected for the first, second and fourth cycle of loading.
Figure 3.4: Load cell sensitivity analysis: uniaxial test, data collected by Axel Labs inc., single set of data collected. Raw data plotted by Axel, purchased by Imperial College, London.
Figure 3.5: Tensile tests at different strain rates, raw data plotted by Axel, purchased by Imperial College, London
The data collected by Axel is compared to the one collected by the author in Figure 3.6. For the first loading cycle, results collected by the author are about 14% higher, while the loading curves have a very similar shape. For further cycles, a similar behaviour is observed until the material reaches strains of over 300% where the curves start showing different speeds of stress increase.

Figure 3.6: Comparison between cyclic testing - Axel vs. Author, raw data

A few reasons have been identified as the cause of these discrepancies. Firstly, the author used a rectangular strip of material for testing rather than the standard specimen shape, ASTM D412, used by Axel. Further work would require testing using the standard shape, however at this stage the author wanted a quick way to check the accuracy of the results provided by Axel. Also, a laser extensometer was used by Axel thus capturing strains undergone by the central area of the specimen. The author however used the elongation of the whole specimen to calculate strains, as provided by the Instron testing machine. It is thought that the central area of the specimen will provide more accurate results, as it is the specimen area least affected by the clamping conditions. After the testing was completed, it was noticed that all experiments carried out by the author involved stretching the material by 600%, an arbitrary choice. Axel only stretched the materials by 400% before unloading the specimens. As Figure 8.4 shows, the history of loading levels highly affects material
properties. Also, as the loading and unloading speeds were not identical for both experiments, results might have been affected by the material potentially exhibiting creep properties that will be discussed further in Chapter 8. As will be discussed further in this work, slight material imperfections can highly affect the performance of the material and therefore explain the discrepancies between the results collected by Axel and the author. Extensive work carried out by Gouder (2011) on the MED4905 material, showed that material properties are extremely dependent on the process of fabrication as well as the handling of the specimen prior to testing. A further factor affecting results is the way in which samples are clamped in the testing machine. Due to the material being so thin and sticky once removed from its backing, the author found it very hard to test the material without adding any prestretch. While testing these samples, it was realised that the precision in the way the backing of the material was removed, the placement of the specimen of the sample in the machine as well as various other parameters could not be monitored well enough by the author to ensure precision. It was therefore decided that in this work, results from Axel will be used as the experimental data provided was obtained using more advanced techniques than the ones available at Imperial College, London. The data collected by Axel, for the five different types of material studied in this work, are plotted in Appendix C.

Figure 3.7: Clamps for tensile test
3.2 Hyperelastic modelling

In this section, experimental and simulation results are compared in order to assess the accuracy of the ANSYS curve-fitting modelling procedure presented in section 2.5. A specific set of experimental data collected by Axel for the MED4930 material of 50 μm thickness is chosen. The ANSYS curve-fitting process, see section 9.2 of the ANSYS manual (ANSYS, 2009a), is used to build five-constant Mooney-Rivlin models of this material. Six different sets of material constants are calculated using different combinations of the uniaxial, biaxial and planar tension test data. The six different sets of constants are collected in Table 3.1. In the next paragraphs, all the stress and strain FE measurements are taken as local measurements taken at the most central point of the specimen, (x=0, y=0, z=0).

<table>
<thead>
<tr>
<th>Material constants (Pa)</th>
<th>Uniaxial</th>
<th>Biaxial</th>
<th>Uniaxial, biaxial</th>
<th>Uniaxial, biaxial planar</th>
<th>Uniaxial, planar</th>
<th>Biaxial, planar</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{10}$</td>
<td>−73424</td>
<td>+705341</td>
<td>+20959</td>
<td>+5421</td>
<td>+22300</td>
<td>−43032</td>
</tr>
<tr>
<td>$C_{01}$</td>
<td>+159761</td>
<td>−479630</td>
<td>+80426</td>
<td>+85119</td>
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<td>+7177</td>
<td>+93022</td>
</tr>
<tr>
<td>$C_{11}$</td>
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<td>−114987</td>
<td>−17528</td>
<td>+515</td>
<td>−21305</td>
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</tr>
<tr>
<td>$C_{02}$</td>
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<td>+22969</td>
<td>+3386</td>
<td>−3002</td>
<td>+15557</td>
<td>+28704</td>
</tr>
</tbody>
</table>

Table 3.1: Material parameters for a specific MED4930 50 μm specimen

3.2.1 Uniaxial tensile test

A tensile test of the 50 μm thick MED4930 material is simulated in ANSYS. Three five-constant Mooney-Rivlin models are used. The three different sets of material constants are found using respectively the uniaxial only, uniaxial and biaxial, and uniaxial, biaxial and planar tensile data collected by Axel for this material, see Table 3.1. A 5 mm-wide and 50 mm-long specimen is modelled using the ‘Solid186’ element, the boundary and loading conditions are illustrated by Figure 3.8. Solid186 is a 3D solid element with 20 nodes, each having three degrees of freedom: translations in the nodal $x$, $y$, and $z$ directions. An illustration of the element can be
found in Appendix E. The element supports plasticity, hyperelasticity, creep, stress stiffening, large deflections and large strain capabilities.

The loading is applied in steps as a displacement constraint to the bottom area of the specimen. This loading is similar to the one carried out in the tensile machine and pictured in Figure 3.7. Mesh sensitivity studies can be found in Appendix F, and the ANSYS simulation itself can be found in Appendix G. This study was carried out using 80 elements, 20 along the length of the specimen, two along its width and two along the depth. Simulation results are compared to experimental data in Figure 3.9, no wrinkling was seen at any point during the ANSYS simulations. As mentioned earlier, FE measurements are taken as local measurements taken at the centre of the specimen, \((x=0, y=0, z=0)\). Running the curve-fitting process with uniaxial data alone is found to provide a material model sufficient for an accurate tensile test simulation. Using the biaxial data together with the uniaxial data for the curve-fitting process does not make a significant difference to the accuracy. Using the three sets of data: uniaxial, biaxial and planar tension for the curve-fitting process does reduce the accuracy slightly for strains above 300%. The accuracy remains high for strains below that value.

Figure 3.8: Boundary and loading conditions for tensile test simulation
Figure 3.9: True stress vs. True strain plot for uniaxial tension experimental data for a 50 \( \mu \)m thick MED4930 specimen vs. simulated data using Mooney-Rivlin models of the same material.

### 3.2.2 Biaxial tensile test

A biaxial test simulation is run in ANSYS for three different material models obtained using respectively, one, two or three sets of data. The FE model is built using the ‘Solsh190 element’, a 3D 8-node layered solid shell element pictured in Appendix D. The loading and boundary conditions can be seen in Figure 3.10. The loading was applied in steps. The simulation does not replicate the bulge test experiment. However, as discussed in section 2.5.1.2, a bulge test is equivalent to a planar biaxial tensile test. Simulation and experimental results are plotted in Figure 3.11. As mentioned earlier and similarly to the uniaxial tensile test simulations, FE measurements are taken as local measurements taken at the centre of the specimen, \( (x=0, y=0, z=0) \). A mesh sensitivity analysis can be found in Appendix F and the ANSYS simulation itself in Appendix G. In the same way as for the uniaxial test, the mesh sensitivity analysis showed that the study shows the same level of accuracy whether only one element is used or a greater number. Simulations were carried out with 100 elements by the author, ten on each side of the area biaxially stretched, and one element along the thickness.

The most accurate simulation is achieved when biaxial results are the only data.
Figure 3.10: Boundary and loading conditions for FE biaxial tensile test simulation

Figure 3.11: True stress vs. True strain plot for biaxial tension experimental data for a 50 µm thick MED4930 specimen vs. simulated data using Mooney-Rivlin models of the same material, data from one specimen only is used
set used in the curve-fitting process discussed in section 2.5.2. As in the uniaxial simulation, this confirms the consistency of the ANSYS curve-fitting process. When using two or three sets of data, the simulation is slightly less accurate. Indeed, when the uniaxial and/or planar tensile experimental data are used together with the biaxial tensile data, material properties other than the biaxial tension properties are taken into account.
3.2.3 Planar tensile test

Planar tests are simulated in ANSYS in the same way as the uniaxial tensile tests. Three material models are used, the sets of material constants are found using the planar tension data together with the uniaxial, biaxial, or uniaxial and biaxial sets of data. Simulation results of a tensile test are then compared to experimental data in Figure 3.12. As mentioned earlier and similarly to the uniaxial and biaxial tensile test simulations, FE measurements are taken as local measurements taken at the centre of the specimen, (x=0, y=0, z=0).

![Figure 3.12: True stress vs. True strain plot for planar tension experimental data for a 50 µm thick MED4930 specimen vs. simulated data using Mooney-Rivlin models of the same material, data from one specimen only is used](image)

Experimental results compare well with all the simulations based on planar tension test data combined with either uniaxial tensile data or biaxial tensile data or with both.

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3.2.4 Analysis

The uniaxial, biaxial and planar tension simulations reported in this section confirm the consistency of the ANSYS hyperelastic modelling procedure and curve-fitting process. It also confirms that, to acquire the most accurate model, data from the three elementary deformation modes is needed.

3.3 Elastomers as linear materials

The elastomer hyperelastic modelling process is complex, requires lengthy experimental testing as well as significant post-processing work. Therefore, the option of modelling elastomers as linear materials for very low strains is investigated. Such an approximation would provide a basic tool for preliminary design of elastomer-made devices. Following a first approximation, a full modelling procedure taking into account the hyperelastic properties of elastomers would be required for further development. A set of tensile test data for four types of materials is plotted in Figure 3.13. It is seen that at low strain levels below 10\%, see Figure 3.14, these materials can be accurately modelled as linear materials. The linearity assumption is used as a first approximation in this work as the buckling of dimples occurs for strains lower than 10\% due to the clamping conditions.
Figure 3.13: Tensile test data, linear approximation of true Stress and Strain

Figure 3.14: Linear approximation of true Stress and Strain, enlargement of Fig 3.13
3.4 Conclusion

Simulations and experimental data of the MED4930 50 \( \mu \text{m} \) material confirmed that a five-constant Mooney-Rivlin model with constants estimated using the ANSYS curve-fitting process provides a model able to simulate accurately the behaviour of the material. The curve-fitting process requires the use of the uniaxial, biaxial as well as planar tensile test data. The comparison was only carried out for one set of specimens, a single specimen per material and testing speed. A range of tests will be needed to validate our conclusions further. Further accuracy checks require the comparison of simulations and experiments for situations involving a combination of deformation modes such as a non-uniform biaxial tensile test. Also, as the data was purchased externally, this work was carried out by the author using one specific set of data rather than an average. Let us remind the reader that Axel Labs inc. according to their policy carry out three sets of experiments to make sure results are consistent but only provide the customer with a single set. Further validation work will require in-house testing of a few specimens.
Chapter 4

Modelling dielectric elastomers

4.1 ANSYS for coupled-field analyses

4.1.1 Overview

ANSYS is a FE software providing structural, thermal, electromagnetic and flow single-field analyses as well as coupled-field analyses. In a coupled-field analysis, the input of one field analysis depends on the results from another field. Simple engineering problems in which the field solutions are coupled can be modelled with a one-way coupling and do not require iterations between the two field solutions. More complicated cases, such as the dielectric elastomer modelling, require iterations between the two physics fields for convergence. Such a coupling can be accomplished by a direct or load-transfer method.

Direct coupling involves one analysis only using a coupled-field element containing all necessary degrees of freedom. Direct coupling is advantageous when the coupled-field interaction involves strongly-coupled physics or is highly non-linear and is best solved in a single solution using a coupled formulation. An electro-mechanical direct coupling is not available for hyperelastic materials using commercially available FE softwares. The load-transfer method provides a more efficient coupled-field solution because the two analyses can be performed independently and for any type of material. Coupling may be recursive; iterations between the different physics are performed until the desired level of convergence is achieved. The load transfer method requires more user-intervention than the direct coupling, but at the same
time offers more flexibility and can be applied to hyperelastic materials.

A load-transfer analysis between a structural and electrostatic field is used to model the electro-mechanical coupling of dielectric elastomer materials in this work. The ANSYS package was chosen over other software because of its ability to provide electromagnetic-structural interactions for any type of material as well as flow-structural interactions. The latter will be required for further work to optimise DEAs for flow control by providing potential flow-electro-mechanical simulations.

### 4.1.2 Structural analysis

In a typical FE structural simulation, the primary unknowns calculated are displacements. Other quantities such as strains, stresses and reaction forces are then derived from the nodal displacements. The overall equilibrium equation for a linear structural analysis solved by ANSYS is expressed by Equation 4.1, see ANSYS (2009b).

\[
[K]\{u\} = \{F\} 
\]

or

\[
[K]\{u\} = \{F^a\} + \{F^r\} 
\]

where \([K]\) is the total stiffness matrix \([K] = \sum_{m=1}^{N}[K_e]\),
\(
\{u\}\) is the nodal displacement vector,
\(N\) is the number of elements,
\([K_e]\) is the element stiffness matrix,
\(
\{F^a\}\) is the total applied vector
and \(\{F^r\}\) is the reaction load vector.

Most ANSYS element types are structural, ranging from simple spars and beams to more complex layered shells and large strain solids. In this work, the element used in structural simulations is the ‘Plane82’ element. This element is defined by eight nodes each having two degrees of freedom with translations in the nodal \(x\) and \(y\) directions, see Appendix D. The element may be used as a plane or axi-symmetric element and can simulate the effects of plasticity, creep, swelling, stress stiffening,
large deflection and large strain.

### 4.1.3 Electrostatic analysis

In ANSYS, an FE electrostatic analysis determines the electric field and electric scalar potential or voltage distribution caused by voltage and charge densities. Loads can be applied either on the solid model (keypoints, lines and areas) or on the finite-element model (nodes and elements). In this work, the 2D element used is ‘Plane121’. It is a quadrilateral 8-node element with voltage as the degree of freedom at each node as illustrated in Appendix D. The reduced Maxwell equation, expressed by Equation 4.3, is solved by ANSYS for an electrostatic solution.

\[
-\nabla \cdot ([\epsilon] \nabla V) = \rho \tag{4.3}
\]

where \(V\) is the scalar potential,
\(\rho\) is the electric-charge density
and \([\epsilon]\) is the permittivity matrix, see Equation 4.4.

The permittivity matrix used in this study is presented in Equation 4.4.

\[
[\epsilon] = \begin{pmatrix}
\epsilon_{xx} & 0 & 0 \\
0 & \epsilon_{yy} & 0 \\
0 & 0 & \epsilon_{zz}
\end{pmatrix} \tag{4.4}
\]

The matrix equation of Equation 4.3 is expressed by Equation 4.5.

\[
[K^{VS}]V_e = L_e \tag{4.5}
\]

where \([K^{VS}]\) is the dielectric permittivity matrix = \(\int_{vol} (\nabla N^T)[\epsilon](\nabla N^T) dvol\)

\(L_e = L_n^e + L_c^e + L_{sc}^e\),
\(L_c^e = \int_{vol} \rho N^T dvol\),
\(L_{sc}^e = \int_{vol} \rho_s N^T dvol\),
\(L_n^e\) is the nodal charge vector,
\(N\) is the element shape function,
\(V_e\) is the nodal electric scalar potential,
\[ V = N^T V_e \] is the electric potential,
\[ \rho \] is the charge density vector
and \[ \rho_s \] is the surface charge density vector.

In this work the permittivity is considered to be time and strain independent. Such an assumption might not be fully accurate. Further experimental work is required to fully evaluate the material permittivity dependence on time and strain, such effects will be discussed in greater detail in Chapter 8. These time and strain dependent effects could be implemented in the sequential modelling technique proposed in section 4.1.5. Being sequential, the modelling method allows changes to be made to the material parameters as iterations and therefore deflections increase. Also, the permittivity in this study is considered temperature independent which is not always the case for elastomers, see Qiang et al. (2012). Similarly to the time and strain effects on permittivity, temperature effects if and when present can be taken into account during the sequential modelling technique of section 4.1.5. As deflections increase between consecutive iterations, the speed of actuation can theoretically be checked and if necessary the permittivity could be amended accordingly. No permittivity changes between consecutive iterations have been carried out by the author at this stage but will be necessary in future work to validate the ability of the model to accommodate such changes.

### 4.1.4 Direct coupled-field electro-mechanical analysis

Direct electro-mechanical simulations are performed in this work using the electroelastic ‘Plane 223’ element. This 2D element has eight nodes with up to four degrees of freedom per node: translation in the \( x, y \) and \( z \) direction, temperature and voltage. Its structural capabilities are elastic only and include large deflections and stress stiffening. The element may be used as a plane element or as an axi-symmetric element. An illustration of the element can be found in Appendix D. In an ANSYS electroelastic simulation, the electrostatic body force that causes the deformation is derived from the Maxwell stress tensor \( [\sigma^M] \) presented in Equation 4.6.
\[
[\sigma^M] = \begin{pmatrix}
\sigma_x^M & \sigma_{xy}^M & \sigma_{xz}^M \\
\text{sym} & \sigma_y^M & \sigma_{yz}^M \\
\text{sym} & \text{sym} & \sigma_z^M
\end{pmatrix} = 1/2\{E\}\{D\}^T + \{D\}\{E\}^T - \{D\}^T\{E\}\{I\} \quad (4.6)
\]

where \(\{E\}\) is the electric field intensity vector,
\(\{D\}\) is the electric flux density vector,
and \(\{I\}\) is the identity matrix.

Applying the variational principle to the stress equation of motion with the electrostatic body force loading the charge equation of electrostatics, produces the finite element equation for electroelasticity presented in Equation 4.7 solved by ANSYS when an electroelastic simulation is run.

\[
\begin{pmatrix}
[M] & {[0]} \\
[0] & [0]
\end{pmatrix}
\begin{pmatrix}
\{\ddot{u}\} \\
\{\ddot{v}\}
\end{pmatrix} +
\begin{pmatrix}
[C] & {[0]} \\
[0] & [0]
\end{pmatrix}
\begin{pmatrix}
\{\dot{u}\} \\
\{\dot{v}\}
\end{pmatrix} +
\begin{pmatrix}
[K] & {[0]} \\
[0] & [K_d]
\end{pmatrix}
\begin{pmatrix}
\{u\} \\
\{v\}
\end{pmatrix} =
\begin{pmatrix}
\{F + F_e\} \\
\{L\}
\end{pmatrix} \quad (4.7)
\]

where \([M]\) is the element mass matrix,
\([C]\) is the element structural damping matrix,
\(\{u\}\) is the nodal displacement in the \(y\) direction,
\(\{F\}\) is the vector of nodal and surface forces,
\(\{F_e\}\) is the vector of nodal electrostatic forces \(= \int_{\text{vol}} [B]^T \sigma^M dvol\),
\([B]\) is the strain displacement matrix,
\([K_d]\) is the element dielectric permittivity coefficient matrix,
and \([L]\) is the vector of nodal, surface and body charges.

The direct electro-mechanical simulations can only be run for linear materials. The modelling of DEAs, which are made of hyperelastic materials, requires the development of an electro-mechanical modelling method based on the load-transfer coupling method as both the structural and electrostatic analyses support hyperelastic materials.
4.1.5 Load-transfer coupled-field electro-mechanical analysis

A sequential modelling procedure based on the load-transfer coupling method of ANSYS is proposed in this work for DEA modelling and was published in 2008, see Appendix I. The ANSYS code of this simulation process can be found in Appendix F. Further in this work, this modelling method will be referred to as the ‘sequential method’. The different steps of this modelling technique are described in Figure 4.1. The first step is to create a finely meshed FE model of the actuator. Voltage is then applied across the actuator model and an electrostatic simulation is run. The electrostatic forces created by this voltage are provided by postprocessing these results. In the second step, using the load-transfer method, the electrostatic forces are applied as external forces to a structural simulation of the actuator. The displacements obtained from the postprocessing of the structural simulation are then integrated in an updated structure on which the two steps described above can be reiterated. The iterations are controlled by a user-specified convergence criterion. The criterion used in this work is that the maximum displacement collected be smaller than 1% of the material thickness. When this happens, a simulation is considered converged.

Figure 4.1: Electro-mechanical simulation process, N - number of elements in FE model, t - Material thickness

The iteration process described in Figure 4.1 is required due to the hyperelasticity of the material. Indeed, as mentioned earlier, the direct coupling method is not
available for such materials in commercially available FE softwares such as ANSYS (2009a) or ABAQUS (2012).

4.2 Validating simulations on dimple actuators

The accuracy of the sequential coupling modelling method is checked by comparing FE simulation results for linear materials collected from simulations run using both the sequential coupling, presented in section 4.1.5, and the direct-coupling method, presented in section 4.1.4. The validating simulations are run for clamped dimple actuators, modelled using an axi-symmetric model. The boundary conditions applied to the model are expressed by Equation 4.8, see Figure 4.2. A mesh sensitivity analysis can be found in Appendix F, results converge when the number of elements reaches 100. The model is finely meshed with 250 elements as seen in Figure 4.3.

\[ u_x(x = R) = u_y(x = R) = 0 \]  

Figure 4.2: Boundary conditions of axi-symmetric dimple

The material parameters used in the simulations of this section are presented in Table 4.1. All the parameters apart for the Young’s modulus were provided by the manufacturer, while the value for the Young’s modulus was chosen based on preliminary experimental work carried out by Dearing (2007) on material provided by Nusil. 0.2 MPa was the value that was expected to be measured at the time when this part of the work was carried out. As the work on dimple actuators progressed, experimental work led to a change in choice of material and manufacturer. Material was bought from SSFAB and further studies carried out by Gouder (2011) showed
that the Young’s modulus was not a value of 0.2 MPa as expected. The value of
the Young’s modulus will be discussed further, in section 5.5. Also, it has to be
noted that in further work, the material chosen was of a smaller thickness, 100 µm.
The 100 µm thick material provided higher Maxwell stresses for the same applied
voltage, see Equation 2.1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>5,10,15 mm</td>
</tr>
<tr>
<td>$t$</td>
<td>250 µm</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1130 kg/m³</td>
</tr>
<tr>
<td>$\varepsilon_r$</td>
<td>2.8</td>
</tr>
<tr>
<td>$E$</td>
<td>0.2 MPa</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Table 4.1: Parameters for dimple simulations using both the direct and se-
quential modelling methods

A first set of direct and sequential electro-mechanical simulations is run for volt-
ages in the range of 0 V to 1 kV applied across the electrodes of a 5 mm radius
dimple. The memory space required for direct simulations at voltages higher than 1
kV was beyond the capacity of the available computational equipment. The dimple
is initially mechanically biased due to the gravity effect, a downward deflection is
therefore observed as seen in Figure 4.3. When voltages are applied, the downward
deflection increases. A typical dimple deflection, as captured in ANSYS, is pictured
in Figure 4.4.

Figure 4.3: Meshed dimple in a deformed and undeformed shape
Maximum deflections at the centre of the dimple from both direct and sequential simulations are collected and presented in Figure 4.5.

Sequential coupling results are consistent with results from the direct coupling. Further dimple simulations are run for the radii $R=10$ mm and $R=15$ mm using the parameters presented in Table 4.1. Results are plotted in Figures 4.6 and 4.7. Analysis of Figures 4.6 and 4.7 confirms further the accuracy of the sequential modelling method developed in this work. It can be noted that the larger the dimple, the
higher the accuracy. The error between the direct and sequential modelling is the largest when the radius is $R=5$ mm. Indeed, for high voltages the error is of 3\% whereas it is only of 1.5\% when the radius is 10 mm. This might be due to a limitation of the sequential load transfer as the accuracy of the transferred loads is dependent on the available storage space of the computational equipment. Further work will be required to verify this.
4.3 Validating experiments on dimple actuators

Experiments on dimple actuators have been carried out by Dearing (2007). The dimples were fabricated using the 100 $\mu$m MED4930 material, and designed to match as well as possible the clamped boundary conditions of Equation 4.8. The material was glued using a silicone based glue, to a perspex substrate with a circular hole of the dimple actuator radius. The gluing of the material was carried out following the removal of its bottom backing. The top backing remained on the elastomer until the glue cured in order to avoid material stretching. Graphite powder electrodes were brushed on the actuator and two foil leads respectively attached to the edge of the top and bottom electrodes as pictured in Figure 4.8. Three sets of the maximum out-of-plane deflections, collected by Dearing, for dimples of radii $R=5$ mm, $R=10$ mm and $R=15$ mm are presented in Figures 4.9, 4.10 and 4.11, respectively.

Dimple simulations are run using the sequential load-transfer method presented in section 4.1.5. The model and material parameters remain identical to the ones used in section 4.2 apart from the material thickness, now taken to be 100 $\mu$m. At this stage of the research, the initial experimental work on the 250 $\mu$m thickness
Figure 4.8: Hand-made dimple (Dearing, 2007)

Figure 4.9: Dimple experimental deflections $R=5$ mm (Dearing, 2007)
Figure 4.10: Dimple experimental deflections $R=10$ mm (Dearing, 2007)

Figure 4.11: Dimple experimental deflections $R=15$ mm (Dearing, 2007)
material was stopped as a new material of 100 \( \mu \text{m} \), manufactured by SSFAB, was found and expected to provide higher out-of-plane deflections. Simulations were run again for the appropriate thickness, see Table 4.2. Experimental results of the dimple maximum deflections for various voltages are shown in Table 4.2 for a radius of \( R=5 \) mm and \( R=15 \) mm. Experimental deflections are much smaller than simulation deflections. The effect of material prestretch needs to be taken into account to explain these differences, as will be discussed in the next section. The hysteresis effects exhibited for the three dimples will be dealt with in Chapter 8.

<table>
<thead>
<tr>
<th>Volts (V)</th>
<th>Simulation deflections(( \mu \text{m} )) ( R=5 ) mm</th>
<th>Simulation deflections(( \mu \text{m} )) ( R=15 ) mm</th>
<th>Experimental deflections(( \mu \text{m} )) ( R=5 ) mm</th>
<th>Experimental deflections(( \mu \text{m} )) ( R=15 ) mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>174</td>
<td>840</td>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td>250</td>
<td>206</td>
<td>908</td>
<td>25</td>
<td>6</td>
</tr>
<tr>
<td>500</td>
<td>320</td>
<td>1113</td>
<td>30</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 4.2: Comparison of deflections for dimples made of 100 \( \mu \text{m} \) thick MED4930, simulation and experiment, \( R=5 \) mm and \( R=15 \) mm

### 4.3.1 Buckling threshold and prestretch

When no voltage is applied across the dimple actuators, experimental results show minor deflections, see Figures 4.9, 4.10 and 4.11. According to the simulations, however, the effect of gravity is on its own responsible for an initial deflection of a few hundred microns when no voltage is applied, see Table 4.2. The dimples deflect further when voltages are applied across the electrodes. A closer analysis of the experimental results of Figure 4.9, 4.10 and 4.11 shows that experimentally, the dimples do not actuate significantly until a certain voltage threshold is reached, at which point deflections rise rapidly as the voltage applied increases. In contrast to simulation results, a buckling effect seems to be observed in the experimental results collected by Dearing. A further analysis of the experimental results is required for a better understanding of this dimple deflection mechanism.

Multiple qualitative experiments were carried out by the author verifying the existence of the buckling threshold, however no quantitative data was collected. The
only experimental data available to the author was collected by Dearing (2007) and is presented in Figures 4.9, 4.10 and 4.11. The presence of initial material prestretch is expected to be the cause of the buckling thresholds observed. Following an initial stage where the voltage-induced stresses contribute to removing the prestretch in the dimples, the structure starts actuating. The thresholds observed in Figures 4.9, 4.10 and 4.11 can be explained as representing the points where electro-mechanical forces have balanced the internal material stresses. This effect was not expected to be present when the material was chosen for the study of dimple actuation, however repeated experiments showed that the presence of the buckling threshold was common to all the samples tested.

According to simulation results, dimples of radius and thicknesses seen in this chapter should show an initial deflection even before actuation, due to their own weight. This was not observed by Dearing (2007) nor the author. This might have been caused by an initial material prestretch present in the purchased material or might have been added during the setting up of the different experiments. The dimples were designed to be flat clamped circular membranes covered in electrodes. During the experimental setup, efforts were made to make sure the membrane was flat as at the time of the experiments there was a lack of awareness to effect of the weight of the membrane. It is important to note that the reason there was a lack of awareness to the weight is that when the backing material was glued to the dimple edges and the backing material was then removed, no deflections were detected. This visual observation in itself confirms the presence of initial material prestretch in the materials used for the dimple experiments described in this chapter.

4.4 Conclusion

In this chapter, a sequential FE-based electro-mechanical coupling model, applicable to any type of material, is suggested for DEA modelling. The first validation step was carried out by comparing direct and sequential simulations of dimple actuators where a high accuracy was demonstrated. As a second validation step, simulation results were compared to experimental results. This has highlighted the presence of a buckling threshold when voltage is applied across dimple actuators. This is linked to the presence of initial prestretch in the material used by Dearing to manufacture
the dimple actuators, a hypothesis that will be discussed and studied in the next chapter.
Chapter 5

Buckling of dimple actuators and initial material prestretch

In the previous chapter, experimental results for the deflection of dimple actuators suggested the presence of a buckling threshold when voltage is applied across the electrodes of the dimple. Under that critical load value, the dimples did not show any sign of deflection unlike the results initially suggested by simulations. At first glance, this effect was attributed to the presence of material prestretch. A further study of the buckling of dimples together with the study of the effect of material prestretch is presented in this chapter.

5.1 Analytical buckling analysis of dimples

For a further understanding of the effect of initial material prestretch on the buckling of dimples and to verify that it is indeed a buckling effect that is observed in the experimental results presented in section 4.3, an analytical buckling analysis is conducted. In this purely analytical section, no experimental or simulation work is used. The background for this section is to understand what buckling values are theoretically expected for dimple actuators, thus enabling us to verify that it is indeed buckling that is observed in the simulation work carried out in ANSYS for dimple actuators. In this section, the material used for dimple actuators is assumed to be linear. The buckling of a membrane involves low amounts of strain, thus justifying this assumption.
5.1.1 Buckling stress for clamped circular plate

A simple circular plate with clamped edges is studied, as seen in Timoshenko and Gere (1961) and presented in Figure 5.1. In this subsection, a slightly different notation is used in order to match the available literature (Timoshenko and Gere, 1961): polar coordinates are used. The thickness of the plate is noted \( t \) and the Poisson ratio of the plate material is \( \nu \).

![Figure 5.1: Model of circular clamped plate (Timoshenko and Gere, 1961)](image)

To determine the critical stress for buckling, \( \sigma_{cr} \), the assumption that a slight buckling has occurred is made. The differential equation of the deflection surface of the plate is used. Assuming that the deflection surface is a surface of revolution and denoting by \( \phi \) the angle between the axis of revolution and any normal to the plate, the required equation is found in Timoshenko and Gere (1961) to be:

\[
r^2 \frac{d^2 \phi}{dr^2} + r \frac{d \phi}{dr} - \phi = - \frac{Qr^2}{D},
\]

where \( D \) is the bending stiffness of the plate expressed by Equation 5.2,

\[
D = \frac{E t^2}{12(1 - \nu^2)},
\]

\( r \) is the distance of any point measured from the centre of the plate, \( Q \) is the shearing force per unit of length, the positive direction of which is shown in Figure 5.1. Since there are no lateral loads acting on the plate:
\[ Q = \sigma_{cr}\phi. \]  

(5.3)

Using the notation,

\[ \frac{\sigma_{cr}}{D} = \alpha^2, \]  

(5.4)

the differential equation can be written:

\[ r^2 \frac{d^2\phi}{dr^2} + r \frac{d\phi}{dr} + (\alpha^2 r^2 - 1)\phi = 0. \]  

(5.5)

A new variable is introduced: \( u = \alpha r \). The fact that \( \alpha \) is a constant, means we can write:

\[ dr = d\left(\frac{u}{\alpha}\right) = \frac{1}{\alpha} du; dr^2 = \frac{1}{\alpha^2} d^2 u. \]  

(5.6)

We can therefore write:

\[ \frac{d\phi}{dr} = \frac{1}{\alpha} \frac{d\phi}{du}; \frac{d^2\phi}{dr^2} = \frac{1}{\alpha^2} \frac{d^2\phi}{du^2}. \]  

(5.7)

Equation 5.5 can therefore be written,

\[ u^2 \frac{d^2\phi}{du^2} + u \frac{d\phi}{du} + (u^2 - 1)\phi = 0. \]  

(5.8)

The general solution of this equation is:

\[ \phi = A_1 J_1(u) + A_2 Y_1(u), \]  

(5.9)

where \( J_1(u) \) and \( Y_1(u) \) are Bessel functions of first order of the first and second kind respectively. At the centre of the plate, where \( r = u = 0 \), the angle \( \phi \) must be zero to satisfy the condition of symmetry. Since the function \( Y_1(u) \) becomes infinite as \( u \) approaches zero, the above conditions require \( A_2 = 0 \). To satisfy the condition at the clamped edge of the plate:

\[ (\phi)_{r=a} = 0. \]  

(5.10)

Therefore,

\[ J_1(\alpha a) = 0. \]  

(5.11)
The roots of equation 5.11 are calculated using the Matlab 7.0.4 software and Halley’s root-finding algorithm. The first four roots are:

\[
\text{roots} = 3.832; 7.016; 10.173; 13.324.
\]  

(5.12)

The smallest root of equation 5.11 is used to evaluate the first critical stress, see Equation 5.13.

\[
\sigma_{cr} = \frac{14.68D}{a^2} = 1.22 \frac{E}{1 - \nu^2} \left( \frac{t}{r} \right)^2
\]  

(5.13)

The first critical stress will be used to study the dimple deformations collected by Dearing (2007) in her work presented in Figures 4.9, 4.10, 4.11. Dearing (2007) carried out her experiments at the low frequency of 1 Hz. The existence of higher roots explains why when actuated at higher frequencies, dimples show different vibration modes as will be seen in Chapter 8, see section 8.2.2.

5.1.2 Buckling voltage

As discussed earlier in section 2.2, a dielectric elastomer consists of a thin sheet of elastomer compressed between two electrodes. When voltage is applied across the electrodes, an actuation pressure develops in the out-of-plane direction according to Equation 5.14.

\[
\sigma_y = \sigma_e = \epsilon_e \epsilon_0 \left( \frac{V}{l} \right)^2
\]  

(5.14)

The membrane, clamped around its edges, cannot expand laterally. The electrically induced vertical stress \( \sigma_e \) is therefore integrally transmitted to the lateral axes, according to Equation 5.15, see Rosset et al. (2008).

\[
\sigma_x = \sigma_e
\]  

(5.15)

When a residual stress \( \sigma_0 \), also called initial material prestretch, is present in the material due to the fabrication process or intentionally applied by prestretch, the electrostatic pressure adds to the residual stress in the \( x \) direction according to Equation 5.16.

\[
\sigma_x = \sigma_e - \sigma_0 = \epsilon_e \epsilon_0 \left( \frac{V}{l} \right)^2 - \sigma_0
\]  

(5.16)
When the applied voltage is high enough, the lateral stress reaches the compressive
buckling limit of the membrane which then buckles as seen in section 5.1.1. By
combining Equation 5.16 and 5.13, the buckling voltage $V_{cr}$ of a clamped circular
membrane can be expressed by equation 5.17, see Rosset et al. (2008).

$$V_{cr} = \frac{t}{\sqrt{\varepsilon_0 \cdot \varepsilon_r}} \sqrt{\frac{1.22 E}{1 - \nu^2} \left( \frac{t}{r} \right)^{2} + \sigma_0}$$

(5.17)

When the initial material prestretch is significantly higher than the critical stress,
most of the electrical energy will be used to cancel it, see Equation 5.17. In such a
case, the critical voltage is only slightly influenced by the radius of the membrane, as
was observed in the experimental results collected by Dearing (2007) seen in Figures
4.10 and 4.11. This will be discussed further in the next section.

It should be noted that for the equations mentioned in this section, the effect of
gravity is neglected. The validity of this assumption will be discussed further in this
chapter.

5.2 Buckling simulations compared to analytical results

The boundary conditions of the clamped dimple presented in Figure 4.2 are used
throughout this section to create the FE model of dimples in ANSYS. The Poisson
ratio is chosen to be 0.4999 as a representation of the incompressibility of the mate-
rial as the value 0.5 cannot be used for computational reasons. It can be noted that
in other places of this thesis, values such as 0.48 or 0.49 are used. In such places
these values were a suitable choice as simulations were not compared to experimental
results but to other simulations. In this section the material is assumed to be linear,
therefore the FE simulations are based on the direct electro-mechanical modelling
method, presented in section 4.1.4.

A value of 1 MPa was chosen for the Young’s modulus of the elastomer. By
comparing simulation and experimental work, it became clear at this stage of the
study, that the value of 0.2 MPa was inaccurate. Many reasons could be behind
this change. It seems that the specimens provided to Dearing (2007) were much
stiffer than the manufacturer had expected. As will be explained in section 5.5,
the author and Gouder (2011) realised that the manufacturing process of elastomer sheets as well as the fabrication of elastomer actuators could stiffen the material and prevent sheets of elastomer to reach the same flexibility as the advertised material properties. The material properties provided by the manufacturers were obtained through testing of specimens much thicker than the ones the present work is working on, thicknesses of 50 to 250 $\mu$m. A Young’s modulus value that seemed to provide simulation results close to the experimental results was 1 MPa. It is only later, that work carried out jointly between the author and Gouder showed that a choice of 1 MPa was realistic. Doubts over the Young’s modulus of this material were due to the lack of repeatability between the testing results for different batches of the same material, see section 5.5.

Buckling simulations are run using dimensions and parameters chosen to match as well as possible the dimple experiments carried out by Dearing (2007), they are presented in Table 5.1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>15 mm</td>
</tr>
<tr>
<td>$t$</td>
<td>100 $\mu$m</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1130 kg/m$^3$</td>
</tr>
<tr>
<td>$\epsilon_r$</td>
<td>2.8</td>
</tr>
<tr>
<td>$E$</td>
<td>1 MPa</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.4999</td>
</tr>
</tbody>
</table>

Table 5.1: Parameters for dimple simulation of buckling

5.2.1 Eigenvalue buckling

In ANSYS, the critical buckling stress of a structure can be obtained by running an eigenvalue or a non-linear buckling analysis. From a computational perspective, the eigenvalue analysis provides results that are theoretically most accurate as it does not require the intervention of the user. It is rare however, that cases in real life reproduce exactly these buckling values. Slight imperfections in the geometry, material or boundary conditions can highly affect the results. Due to software limitations,
an eigenvalue buckling analysis cannot be carried out to directly evaluate the critical voltage for prestretched materials. The presence of prestretch is simulated by a temperature load, therefore a critical loading analysis is made impossible by the presence of multiple loads on the structure. The eigenvalue buckling analysis can only be used to evaluate the critical stress of a structure without prestretch. This will explain why the results found in the following sections provide much lower values for the buckling pressure and buckling voltage, compared to experimental results. Despite not including prestretch, simulations and analytical results are still studied to better understand the mechanism of actuation of the dimple actuators as seen in the ANSYS simulations. Equation 5.18 is solved by ANSYS for the eigenvalue buckling analysis.

\[ [K]\phi_i = \lambda_i[S]\phi_i \]  

(5.18)

where \([K]\) is the structure stiffness matrix, 
\(\phi_i\) is the eigenvector, 
\(\lambda_i\) is the eigenvalue and \([S]\) is the stress stiffness matrix. The stress stiffness matrix accounts for stiffening effects, it is automatically calculated by ANSYS based on the previous equilibrium iteration.

An axi-symmetrical model of the dimple is first created using the parameters presented in Table 5.1 and the model presented in Figure 4.2. The edges are required to be fully clamped and a unit of pressure is applied to the edge of the dimple to simulate the effect of the in-plane stresses described in Equation 5.16. Due to the impossibility of loading and constraining the dimple at the same location, the nodes at the edge of the dimple are only coupled in the \(x\) direction as a unit of pressure is applied in the \(x\) direction. Nodes are fixed in the \(y\) direction. The detailed ANSYS code for this simulation can be found in Appendix G.4.

The first four eigenvalues output by the FE simulation are collected in Table 5.2 alongside the expected analytical results obtained using Equations 5.4 and 5.12 and the parameters of Table 5.1. As expected, ANSYS simulations and the buckling theory presented in section 5.1.1, provide buckling loads that match well thus providing us with an understanding of the buckling equations solved by the ANSYS software when an eigenvalue buckling analysis is conducted. However, it is important to note
that buckling values are low due to the fact that the current study is undertaken for materials with no initial prestretch as such a study is not supported by the ANSYS software. The model run for the eigenvalue buckling analysis is two-dimensional. The first two axi-symmetrical modes are expanded and pictured in Figure 5.2

<table>
<thead>
<tr>
<th>Mode</th>
<th>Analytical critical stress (Pa)</th>
<th>2D FE critical stress(Pa)</th>
<th>Difference from analytical model(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>71.35</td>
<td>71.70</td>
<td>1.9</td>
</tr>
<tr>
<td>Second</td>
<td>240.30</td>
<td>243.72</td>
<td>1.4</td>
</tr>
<tr>
<td>Third</td>
<td>502.85</td>
<td>512.15</td>
<td>1.8</td>
</tr>
<tr>
<td>Fourth</td>
<td>862.61</td>
<td>877.62</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Table 5.2: Critical stress for clamped dimple using an eigenvalue buckling analysis compared to analytical results for the first four deformation modes

5.2.2 Non-linear buckling analysis

A non-linear buckling analysis is a static analysis where the large deflection effects are taken into account. The approach is to constantly increment the applied loads until the solution begins to show an extremely high deflection change for a very small load increment. The load increment needs to be fine enough as the expected critical load is approached. If the load increment is too coarse, the buckling load predicted may not be accurate. In a non-linear analysis, an initial imperfection has to be added to the model for the buckling to occur.

Three non-linear buckling analyses of a dimple, described by Figure 4.2, are carried out in ANSYS to find successively its buckling voltage, buckling pressure and buckling temperature. An initial imperfection of the shape of the first mode of deformation and an amplitude equal to 1% of material thickness is introduced. Simulations are run using the same parameters as the section above. Buckling values were estimated to be the intersection of the two straight lines drawn respectively along the curve before and after buckling. It is important to note that in this section, it is assumed that there is no initial material prestretch which explains why the buckling values are low compared to the experimental results mentioned in the previous chapter.
Figure 5.2: First two axi-symmetrical modes of a clamped dimple, colours representing increasing out-of-plane displacements.
5.2.2.1 Voltage

Voltages are applied across the top and bottom electrodes of the dimple, displacement measurements collected at the centre of the specimen are collected and presented in Figure 5.3. The critical voltage obtained by the simulation, according to the technique described earlier in this section, is presented in Table 5.3 and compared to the theoretical expected value obtained using Equation 5.17 in which the imperfection is assumed negligible. Simulations and theoretical results match well thus confirming the buckling equations solved by ANSYS when evaluating critical voltages, Equation 5.17.

![Figure 5.3: Non-linear buckling analysis - Displacement vs. Voltage. Red lines showing the lines drawn along the curve before and after buckling used to estimate the buckling value](image)

5.2.2.2 Pressure

To validate the ability of ANSYS to simulate buckling accurately, pressure induced simulations are run. As explained earlier in this section, the applied voltage across a dimple translates into in-plane stresses according to Equation 5.16 due the boundary
Table 5.3: Critical Voltage for clamped dimple conditions of the clamped dimple. Using the same parameters as in section 5.2.2.1, simulations are run when in-plane pressures are gradually applied to the FE model presented in Figure 5.4. Displacement measurements collected at the centre of the dimples are presented in Figure 5.5

<table>
<thead>
<tr>
<th>Solution</th>
<th>Critical Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical</td>
<td>169</td>
</tr>
<tr>
<td>FE</td>
<td>170</td>
</tr>
</tbody>
</table>

The critical pressure obtained by the simulation is presented in Table 5.3 and compared to the theoretical expected value obtained using Equation 5.16 in which the imperfection is assumed negligible. Simulations and theoretical results match well for pressure induced buckling.

Table 5.4: Critical pressure for clamped dimple

<table>
<thead>
<tr>
<th>Solution</th>
<th>Critical Pressure (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical</td>
<td>71.35</td>
</tr>
<tr>
<td>FE</td>
<td>70</td>
</tr>
</tbody>
</table>

Figure 5.4: Boundary conditions for pressure induced buckling

The critical pressure obtained by the simulation is presented in Table 5.3 and compared to the theoretical expected value obtained using Equation 5.16 in which the imperfection is assumed negligible. Simulations and theoretical results match well for pressure induced buckling.

Table 5.4: Critical pressure for clamped dimple
Figure 5.5: Non-linear buckling analysis - Displacement vs. Pressure, Red lines showing the lines drawn along the curve before and after buckling used to estimate the buckling value
5.2.2.3 Orthotropic thermal buckling

For a better understanding of how voltage induced simulations are solved by ANSYS, an orthotropic thermal expansion simulation is run. It has been seen earlier that when voltage is applied across the electrodes of a dimple, out-of-plane Maxwell stresses expressed by Equation 5.14 develop.

The constitutive equation translating these stresses into strains is Hooke’s law:

\[
\begin{pmatrix}
\varepsilon_x \\
\varepsilon_y \\
2\varepsilon_{xy}
\end{pmatrix} = \frac{1}{E} \begin{pmatrix}
1 & -\nu & 0 \\
-\nu & 1 & 0 \\
0 & 0 & 2(1 + \nu)
\end{pmatrix} \begin{pmatrix}
\sigma_x \\
\sigma_y \\
\sigma_{xy}
\end{pmatrix}
\] (5.19)

Combining Equations 5.14 and 5.19, the voltage applied across the dimple is found to have the same effect as applying the in-plane and out-of-plane strains presented in Equation 5.20.

\[
\begin{align*}
\varepsilon_x &= -\frac{1}{E} \nu \sigma_y \\
\varepsilon_y &= \frac{1}{E} \sigma_y
\end{align*}
\] (5.20)

Such strains can be reproduced through orthotropic thermal simulations using an FE model with the boundary conditions presented in Figure 5.6.

In an effort to reproduce the relationship of Equation 5.20, the dimple material is given thermal expansion coefficients in the in-plane and out-of-plane directions that obey the relationship of Equation 5.21.
\[ \alpha_y = -\frac{1}{\nu} \alpha_x \]  \hspace{1cm} (5.21)

Applying a uniform temperature change gradually to an FE model of a dimple with the boundary conditions presented in Figure 5.6 translates into initial strains applied to the model in both in-plane and out-of-plane directions according to Equation 5.22.

\[
\begin{cases}
\epsilon_x = \alpha_x \Delta T \\
\epsilon_y = -\frac{1}{\nu} \alpha_x \Delta T
\end{cases}
\]  \hspace{1cm} (5.22)

As seen in section 5.1.2, it is believed that the Maxwell stresses transform into in-plane stresses due to the clamped boundary conditions of the dimple. These stresses induce buckling of the dimple when they reach a critical value \( \sigma_{cr} \) presented in Table 5.4. If our understanding of the voltage induced buckling of dimples is correct, buckling is therefore expected to occur, based on Equation 5.20 and Equation 5.22 when:

\[
\frac{E}{\nu} \alpha_x \Delta T = \sigma_{cr}
\]  \hspace{1cm} (5.23)

For simplification purposes, simulations are run when the thermal coefficient is chosen so that it obeys Equation 5.24.

\[
\frac{E}{\nu} \alpha_x = 1
\]  \hspace{1cm} (5.24)

Buckling is therefore expected to occur when \( \Delta T = \sigma_{cr} = 71.35 \).

FE simulations are run in ANSYS when the temperature difference between the structure and its surroundings is gradually increased. Results are collected at the centre of the dimple and plotted in Figure 5.7.

Using the intersection between the lines following the curve respectively before and after buckling, we get an approximate value of 72 Pa for the critical temperature difference. If we run the eigenvalue buckling analysis of the orthotropic expansion we also get a value of 71.30 Pa. The eigenvalue buckling analysis result confirms the results from the non-linear buckling analysis presented in Figure 5.7. Resulted are collected in Table 5.5. It can be therefore concluded that the orthotropic thermal expansion simulations produce the same effect as applying voltage across the electrodes of a clamped dimple.
Figure 5.7: Non-linear buckling analysis - Displacement vs. Temperature, Red lines showing the lines drawn along the curve before and after buckling used to estimate the buckling value.

<table>
<thead>
<tr>
<th>Eigenvalue buckling analysis ANSYS</th>
<th>Non-linear buckling analysis ANSYS</th>
<th>Expected theoretical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>71.30</td>
<td>72.00</td>
<td>71.35</td>
</tr>
</tbody>
</table>

Table 5.5: Critical temperature for clamped dimple
5.2.3 Conclusion

Numerical calculations together with ANSYS simulations have provided an extensive understanding of the behaviour of clamped dimples when voltage is applied across their electrodes. The critical pressure obtained through thermally induced simulations matches the critical voltage and critical pressure obtained when voltage and pressure induced simulations are respectively run for clamped dimples, see Figures 5.3, 5.5 and 5.7. All these critical measurements also match the buckling pressure obtained analytically and through the ANSYS eigenvalue buckling analysis carried out earlier in this chapter, see Table 5.2.

It can therefore be concluded that when voltage is applied across the electrodes of a fully clamped dimple, an out-of-plane stress develops according to the Maxwell Equation 5.14. Due to clamping conditions this entirely transfers to in-plane stresses, see Equation 5.15, that cause buckling of the structure when a critical stress value is reached. These stresses can also simulated by applying a set of in-plane and out-of-plane strains using an orthotropic thermal simulation.

5.3 The effects of prestretch on buckling

5.3.1 Internal stresses

The prestretching of a membrane can be simulated in FE by applying temperature differences between the membrane and its surroundings while the membrane is given orthotropic thermal properties. Simulating material prestretch in this work is carried out by giving an expansion ratio of \( \alpha \) to the material in its in-plane direction, while an expansion coefficient of zero is given to the out-of-plane direction, see Equation 5.25.

\[
\begin{align*}
\alpha_x &= \alpha \\
\alpha_y &= 0
\end{align*}
\]  

(5.25)

For a flat isotropic membrane without any constraint at the membrane edge and radius \( R \), the radial displacement caused by an increase of temperature \( \Delta T \) to such an orthotropic membrane, is \( \alpha R \Delta T \). If the flat membrane is clamped, a radial
thermal stress, $\sigma_f$, develops at the edges of the membrane according to Equation 5.26. The amount of prestretch can be noted $Pst = \alpha \Delta T$.

$$\sigma_f = -\frac{Pst \cdot E}{1 - \nu}$$  \hspace{1cm} (5.26)

Using this equation, internal stresses have been calculated for a few different values of material prestretch and Young’s modulus. The material being incompressible, a Poisson ratio of 0.5 is used. Results are summarised in Table 5.6.

<table>
<thead>
<tr>
<th>Pst / E</th>
<th>1 MPa</th>
<th>2 MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 %</td>
<td>20,000 Pa</td>
<td>40,000 Pa</td>
</tr>
<tr>
<td>2 %</td>
<td>40,000 Pa</td>
<td>80,000 Pa</td>
</tr>
</tbody>
</table>

Table 5.6: Internal stresses due to prestretch

Simulations of prestretched dimples are carried out to validate the accuracy between ANSYS calculated internal stress values against values obtained using Equation 5.26, see Table 5.6. Dimples of the geometry described in Figure 4.2 are used. Apart from the Young’s modulus, the parameters presented in Table 5.1 are used. FE results are found to validate the theoretical values collected in Table 5.6 with an accuracy of 99 %.

According to simulations results discussed in Chapter 4, gravity is expected to generate an initial deflection for dimple actuators when the material parameters are taken from Table 5.1. Such a deflection did not show during experimental work. Chapter 4 was concluded by suggesting that it is the presence of initial material prestretch that stops the material from deflecting under its own weight. Simulations are therefore run when the material is given a 1% material prestretch, gravity effects are turned on. The FE model of Figure 4.2 and the material parameters of Table 5.1 are used to run simulations. Internal stresses are collected at the centre of the specimen. The measured internal stresses presented above in Table 5.6 are only reduced by up to 1 kPa. Therefore we can conclude that despite neglecting the gravity effects, Equation 5.17 is a good tool for a first approximation of the buckling loads for dimple structures that are made of prestretched materials when the prestretch is of 1% or more. Due to fabrication process, such imperfections seem to always be present in elastomer sheets as dimples have never been seen to deflect.
under their own weight.

### 5.3.2 Buckling voltage

The theoretical buckling voltage for a prestretched membrane, found by combining Equation 5.17 and 5.26, is expressed by Equation 5.27.

\[
V_{cr} = \frac{t}{\sqrt{\epsilon_0 \cdot \epsilon_f}} \sqrt{\frac{E \cdot P_{st}}{1 - \nu}} + 1.22 \frac{E}{1 - \nu^2} \left(\frac{t}{r}\right)^2
\]  

(5.27)

When slight material initial stresses are present in the material, for example of 1% and 2%, the material induced stresses are much more significant than the critical stress of the structure. Indeed, in these cases the critical stress represents less than 1% of the total internal stress in the circular membrane and the radius does not play an important role in the buckling voltage as was observed in the experimental results presented in Figures 4.10 and 4.11.

### 5.4 Experimental results analysis

Predicting accurately buckling loads using buckling test data is challenging. Southwell proposed in 1932 a graphical solution for elastic columns (Southwell, 1932). In a Southwell plot, the compliance (deflection/load) is plotted against the deflection, and the buckling load is determined from the inverse slope of the plot around the buckling point while the imperfection is represented by the intersection of the asymptotic line with the \( y \) vertical axis. The imperfection in this work is most likely to be caused by the initial material prestretch. This theory is based on the fact that Southwell observed that the following approximation could be made when writing the deflection of a beam, \( \delta \), when the load approaches the buckling load.

\[
\delta \approx \frac{K}{\frac{P_{cr}}{r} - 1}
\]  

(5.28)

where \( P \) is the load applied to the column, \( P_{cr} \) the critical load and \( K \) a constant. Further studies have showed that the Southwell method can be applied for the buckling of a plate. In his thesis, Majumdar (1968) developed the equations necessary to show that the Southwell approximation is valid for circular plates.
In the current study, the load created by the applied voltage is:

\[ P_{cr} = \varepsilon \varepsilon_0 V^2 \]  

(5.29) according to Equation 2.1. The relative and free space permittivities are assumed to be constant during actuation, the Southwell plots are therefore drawn using the displacement for the horizontal axis, and the displacement/$V^2$ for the vertical axis. The Southwell plot is expected to be linear around the buckling point, the slope is expected to be equal to the inverse of the critical voltage squared, $V_{cr}^2$.

In the coming sections, the experimental results presented in section 4.3 and the simulations results presented in section 4.2 are used jointly to evaluate the amount of initial prestretch present in the materials used for the dimple actuators.

The Southwell plot obtained using the experimental data collected by Dearing (2007) for a 15 mm radius dimple according to the method presented in section 5.4, is shown in Figure 5.8. The curve is, as expected, asymptotic to a line around the buckling point.

In an effort to get the most accurate value for the critical voltage, the asymptotic line is found using a selective set of data. The equation of the asymptotic line can be seen in Figure 5.8. The slope measured translates into a critical voltage of:

\[ V_{cr} = 4.08 \text{ kV}. \]  

(5.30)

Comparison of that result and the buckling experimental data itself, see Figure 4.11, shows that the buckling voltage estimated through the Southwell method is over-estimated by around 20 %. Also, Figure 5.8 suggests an initial imperfection of around 50 \( \mu \text{m} \), which is not the case in Figure 4.11. Further work will be needed to identify the material properties, clamping or testing conditions that make the dimple deflection collected by Dearing (2007) not conform to the predictions of Southwell (1932), as was expected by the author.

A graphical approach is used to evaluate the buckling voltages of the 10 and 15 mm radii dimples, as described in section 5.2.2. Based on the results presented in Figures 4.10 and 4.11, the critical voltages are found to be:

\[ V_{cr}(R = 15\text{mm}) = 2.8 \text{ kV} ; \quad V_{cr}(R = 10\text{mm}) = 3.3 \text{ kV} \]  

(5.31)
5.5 Comparing experimental, FE and theoretical results

Experimental work carried out at Imperial College has showed that while carrying out a clamped dimple experiment, the exact value of two parameters remains unknown: the Young’s modulus and the initial material prestretch. A high dependence on the specific specimen studied as well as the handling of it was identified, as discussed in section 3.1. Based on Equation 5.27, an experimentally collected buckling voltage can potentially be matched to an infinity of combinations for the Young’s modulus and internal prestretch of the material.

In the experiments carried out by Dearing (2007), the MED4930 material was used. Due to the imperfections introduced to the MED4930 material along the manufacturing process, it is only possible to have an estimate of both the Young’s modulus and initial prestretch. Material testing carried out by Gouder (2011) has showed that the Young’s modulus of the MED4930 material varies from one sheet of material to the other. The Young’s modulus varies between 0.5 MPa and 1.5 MPa,
thus generating an umbrella of possibilities for the prestretch and Young’s modulus values. A further factor of uncertainty in the material properties is the Mullin’s effect present in the material, which will be discussed in section 8.1.2.

FE simulations have been run for dimples using values for the Young’s modulus and prestretch that are realistic for the experimental results collected by Dearing and presented in graphs 4.10 and 4.11. The FE model and material parameters are kept identical to the previous section. The most relevant prestretch and Young’s modulus values are presented in Table 5.7. Critical voltages are estimated based on the FE simulations as well as Equation 5.27, giving us the theoretical buckling values. They indicate that the material used to carry out the experiment, had most probably a Young’s modulus of between 0.8 MPa and 1 MPa and a prestretch of between 1 % and 2 %. It also confirmed the fact that the radius does not play a significant role in the determination of the critical voltage for such amounts of prestretch.

<table>
<thead>
<tr>
<th>Pst ; E</th>
<th>$(V_{cr})_{R=15}$ FE</th>
<th>$(V_{cr})_{R=10}$ FE</th>
<th>$(V_{cr})_{R=15}$ Theory</th>
<th>$(V_{cr})_{R=10}$ Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>E=1 MPa Pst=1%</td>
<td>2.8 kV</td>
<td>2.8 kV</td>
<td>2.8 kV</td>
<td>2.8 kV</td>
</tr>
<tr>
<td>E=1.5 MPa Pst=1%</td>
<td>3.5 kV</td>
<td>3.3 kV</td>
<td>3.4 kV</td>
<td>3.4 kV</td>
</tr>
<tr>
<td>E=1 MPa Pst=1.5%</td>
<td>3.4 kV</td>
<td>3.4 kV</td>
<td>3.4 kV</td>
<td>3.4 kV</td>
</tr>
<tr>
<td>E=0.8 MPa Pst=2%</td>
<td>3.6 kV</td>
<td>3.6 kV</td>
<td>3.6 kV</td>
<td>3.6 kV</td>
</tr>
</tbody>
</table>

Table 5.7: Critical voltage, FE and theoretical results calculated using Equation 5.27

Unfortunately, due to the limited amount of data available, there is no knowledge of the exact prestretch or Young’s modulus of the material used for the experimental data collected by Dearing. Such data would confirm the actuation mechanism of the dimple actuator. Further experimental data, including the multiple repeat of the same experiment are required to confirm the MED4930 material properties.
5.6 Conclusion

In this chapter, simulation results were studied together with experimental results to evaluate the Young’s modulus of the material as well as the initial material pre-stretch. Working with EAP elastomers is very delicate and not fully predictable as material parameters can change easily due to manufacturing or handling processes. FE simulations are a useful tool to attempt to fully understand the mechanism behind a specific type of actuator and can help optimise the design of actuators. Experiments will always be needed to confirm experimental results.
Chapter 6

Designing DEAs

Optimising the design of DEAs enhances their performance such as deflection shape, height and direction. Studying the influence of parameters such as material thickness, boundary conditions and amount of prestretch on the performance of DEAs can be carried out efficiently using the ANSYS software.

6.1 Material thickness

Elastomers are hyperelastic materials. A full material model is obtained using an FE curve-fitting process together with material experimental data for three elementary deformation modes: uniaxial (ST), biaxial (EB) and planar tension (PT) as outlined in section 2.5. A typical set of experimental data for material MED4930 of two different thicknesses, 50 µm and 100 µm, is presented in Figure 6.1, to study the effect of material thickness on material performance. The data was collected by Axel labs inc. The data was collected using specimens cut out from elastomer sheets coming from SSFAB. The sheets were part of the same batch and therefore should have as similar properties as possible. The effect of material thickness is found to be different for each of the deformation modes, see Figure 6.1. While in planar tension, the material thickness has a greater impact on the stress-strain curve than when in simple tension. The effect of the change in thickness on simple tension is in turn greater than when the material is in biaxial tension. A general actuator movement is a combination of the elementary deformation modes. The impact of the material thickness can therefore not be predicted based on experimental data alone. Hyper-
elastic simulations using the material properties in the three deformation modes are necessary.

Figure 6.1: Effect of thickness on engineering stress-strain curve; ST: Simple Tension - EB: Equibiaxial tension - PT: Planar tension

To acquire a better understanding of the effect of hyperelastic material thickness on the design of actuators, an initial study, based on ANSYS simulation results only, is carried out. Hyperelastic simulations are run for dimple actuators made of material MED4930 of 50 µm and 100 µm thickness. The sequential FE modelling method developed in section 4.1.5 is used. The dimple model is fully clamped as seen in Figure 4.2. The effect of gravity is turned on, the structure is therefore mechanically biased and deflects downwards as soon as voltage is applied across the electrodes. At this stage, no material imperfection or prestretch is taken into consideration and the materials are modelled using the Mooney-Rivlin five-constants model described in section 2.5. The material parameters used are presented in Table 6.1, based on the material information provided by the manufacturer of the materials, Nusil.
### Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>5 mm</td>
</tr>
<tr>
<td>$t$</td>
<td>50/100 $\mu$m</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1130 kg/m$^3$</td>
</tr>
<tr>
<td>$\epsilon_r$</td>
<td>2.8</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Table 6.1: Parameters for dimple simulation of buckling

Maximum deflections, measured at the centre of the actuator, are presented in Table 6.2 for different voltages applied across its electrodes. It is found that the thinner the material, the larger the deflections. This general trend was expected based on the Maxwell equation governing the functioning of DEAs, see Equation 2.1. Indeed, the lower the thickness of the material, the higher the electric field developing across the material and therefore the higher the Maxwell stresses. However, there is no proportionality between the two sets of results and the respective material thickness used for the simulation. This was expected based on the hyperelastic material properties of the material, see Figure 6.1.

<table>
<thead>
<tr>
<th>$V$  (Volts)</th>
<th>Deflections ($\mu$m)</th>
<th>Deflections ($\mu$m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$t=50$ $\mu$m</td>
<td>$t=100$ $\mu$m</td>
</tr>
<tr>
<td>0</td>
<td>174</td>
<td>76</td>
</tr>
<tr>
<td>250</td>
<td>206</td>
<td>78</td>
</tr>
<tr>
<td>500</td>
<td>320</td>
<td>84</td>
</tr>
</tbody>
</table>

Table 6.2: Effect of thickness, $t$, on dimple deflection, FE simulation

In this section, the material has been modelled with the same material properties irrespective of its thickness and has not been given any initial prestretch. Both these assumptions are not fully accurate. This section has however demonstrated that a good start for choosing the thickness of a hyperelastic material when designing DEA actuators is to use FE simulations as there is no proportionality between deflections and the thickness of an elastomer. Including the effect of initial material prestretch and material thickness in simulations will be required in future work for full DEA
6.2 Prestretching

6.2.1 Introduction

Prestretching elastomers improves their performance. Indeed, it increases the speed of response as well as the breakdown voltage (Kofod and Sommer-Larsen, 2005). The prestretching of a material also reduces its thickness, thus reducing the required actuation voltages, see Equation 2.1. This effect benefits applications limited by the amount of voltage than can be supplied. These advantages are however accompanied by disadvantages and handling challenges (Biddiss and Chau, 2008). Elastomers are very fragile and soft materials. Prestretching increases the risk of stress concentrations and related failures. Further, prestretching causes a stiffening of the material and therefore a deterioration of its strain response. Experimentally, the process is delicate and challenging to carry out with precision. No commercial apparatus is available. A jig similar to the one developed by Gouder (2011) at Imperial College is pictured in Figure 6.2.

Figure 6.2: Biaxial stretching apparatus, (Deben, 2012)
6.2.2 FE modelling

In this section, simulations are run using ANSYS to capture the effect that prestretching has on the stress-strain relationship of the MED4930 hyperelastic material. Such simulations can help optimise the performance of elastomer devices, a process quicker and cheaper than experimental testing. Modelling prestretched hyperelastic materials using ANSYS can be carried out in two different ways. The first approach consists of modelling the experimental steps of the prestretching described in Figure 6.3. As a first stage, the material is stretched in the $x$ direction by an amount $\delta x_1$ at each one of the two extremities while it is allowed to shrink in the $y$ direction by $\delta y_1$ at both the top and bottom extremities. In the second step, the specimen is stretched by $\delta y_2$ on both sides while the width of the specimen is kept constant.

The second means of modelling prestretch is through the use of ‘virtual temperatures’. Such an approach is limited to isotropic prestretching of hyperelastic materials or non-isotropic prestretching of linear materials. This technique has been considered in the previous chapter in relation to the modelling of prestretch in dimple actuators which was found to delay the buckling of dimple actuators, see section 5.3.1.

The prestretching of six different MED4930, 50 $\mu$m specimens is simulated in ANSYS. The material is modelled using a Mooney-Rivlin five-constants function. The material constants can be found in Table 3.1. The specimens are chosen to be square and of identical initial dimensions, 30 mm square. The amounts of prestretch in the $x$ and $y$ direction are recorded in Table 6.3. The amounts of prestretch are chosen to ensure an identical final thickness of 30 $\mu$m for all six specimens using the assumption of material incompressibility.

Simulations of horizontal ‘strip actuators’ made of the six different prestretched specimens are run to assess the effect of prestretch on actuation. A typical ‘strip actuator’ is illustrated in Figure 6.4, for which the elastomer is modelled with a horizontal top and bottom electrode. The edges of the actuator are clamped, thus restricting movement inside the actuator between the active and passive areas. When voltage is applied across the actuator, the active area expands in the $y$ direction by an amount $\delta y$, see Figure 6.4. In this study, the width of the electrode is arbitrarily
Figure 6.3: Prestretching: the experimental procedure

<table>
<thead>
<tr>
<th>Case</th>
<th>Prestretch in the $x$ direction (%)</th>
<th>Prestretch in the $y$ direction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>50.00</td>
<td>36.00</td>
</tr>
<tr>
<td>B</td>
<td>58.33</td>
<td>24.50</td>
</tr>
<tr>
<td>C</td>
<td>66.67</td>
<td>29.00</td>
</tr>
<tr>
<td>D</td>
<td>83.33</td>
<td>23.00</td>
</tr>
<tr>
<td>E</td>
<td>100.00</td>
<td>17.80</td>
</tr>
<tr>
<td>F</td>
<td>150.00</td>
<td>5.40</td>
</tr>
</tbody>
</table>

Table 6.3: Cases A-F: Prestretch amounts for FE simulations
chosen to be one third of the specimen width. In future work, simulations will be run to evaluate the optimum electrode size for maximum deflection.

The FE analysis of the six different cases is carried out using purely structural simulations. To simulate actuation, equivalent displacements are applied to the active area of the specimen rather than a voltage. Running purely structural simulations enables the use of direct hyperelastic simulations thus requiring minimal computational resources. Internal material stresses required for actuation are collected for the six specimens and plotted in Figure 6.5. These internal stresses are proportional to the voltage squared required to reach such structural deformations, see the Maxwell stress equation 2.1.

![Figure 6.4: Strip actuator model](image)

The material becomes stiffer in the $y$ direction as the amount of prestretch in the $x$ direction increases, see Figure 6.5. At the same time, material prestretching offers valuable performance improvements by reducing the material thickness as discussed earlier. This trade-off between reduced voltage and stiffness increase makes DEA optimisation challenging. FE simulations offer the opportunity to predict actuation shapes and amplitudes and, therefore, to optimise the use of prestretching in DEAs. Experimental work on material prestretching is under way, see Gouder (2011); Gouder et al. (2013); Potter et al. (2011). Although at an early stage, the simulation results seem to validate the trends observed experimentally. Actuator prototypes stiffen when the amount of prestretching is increased.
Figure 6.5: Prestretching simulation results presented as True Stress Vs. True Strain, Cases A-F
6.3 Imperfections and boundary conditions

The effect of boundary conditions and material imperfections on the performance of DEAs is studied on dimple actuators, a design used extensively in this work. A 2D model of a fully clamped dimple with no prestretch is simulated, as seen in Figure 4.2. In this section, the elastomer material is modelled as linear. As the buckling only involves a limited amount of material strain, this assumption provides accurate results for a first performance approximation. The direct electro-mechanical modelling method described in section 4.1.4 is used in this section. The dimple parameters used for the simulations are presented in Table 6.4.

6.3.1 Material imperfections

At first, simulations are run without any imperfection being introduced. Increasing voltages are applied to the 15 mm dimples. The deflections collected at the centre of the dimple are very small until a critical voltage of more than 1.5 kV is reached, see Figure 6.6. For voltages higher than this critical voltage, deflections increase rapidly. Indeed, no deflections are measured for voltages up to 1.5 kV. By 2 kV, deflections have reached over 140 microns. It is very rare in real life cases that no imperfections are present. Imperfections are therefore introduced in the following FE simulations. The imperfections chosen in this section are in the form of a uniform linear acceleration applied to the structure, noted $g$. The $g=0.001$ value provides the dimple with an initial downwards deflection, which at the centre of the dimple is of 1% of the material thickness. To check the impact this imperfection has on the buckling of dimples, as well as to validate this choice of value, a few simulations have been run for different acceleration values. The deflection at the centre of the dimple is collected and presented in Figure 6.7 for three different degrees of imperfection, $g=0.01$, $g=0.001$ and $g=0.0001$, as the voltage applied across the dimple is gradually increased. For comparison purposes, the deflection curve for the imperfection $g=0.01$ is also compared to the deflection curve obtained when no imperfection is taken into account, see Figure 6.6.

For dimples with $g=0.001$ and $g=0.0001$ imperfections, there is none or almost no movement before a critical voltage is reached, as seen in Figure 6.7. The different degrees of imperfection only affect the shape of the deflection curve, the critical
<table>
<thead>
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<th>Parameters</th>
<th>Value</th>
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<tr>
<td>$R$</td>
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<tr>
<td>$E$</td>
<td>1 MPa</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.49</td>
</tr>
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</table>

Table 6.4: Parameters for dimple simulations with different degrees of imperfection

Figure 6.6: Buckling of dimple with/without imperfection
Figure 6.7: Deflection vs. voltage for dimples with different degrees of imperfection

Voltage is almost identical, see Table 6.5. For an imperfection of $g=0.01$, there is a downwards deformation of a few microns even before any voltage is applied. We can therefore not consider that the $g=0.01$ case is one involving small imperfections. The critical voltages for the $g=0.001$ and $g=0.0001$ deflection curves are found geometrically, as the intersection of two straight lines following the curve before and after the critical voltage. Values are compared in Table 6.5 to the theoretical critical voltage values. Using Equation 5.17, the theoretical critical voltage for the buckling of a dimple with no imperfection is expressed by Equation 6.1.

$$V_{cr} = \frac{t}{\sqrt{\epsilon_r} \cdot \epsilon_0} \sqrt{\frac{1.22}{1 - \nu^2} \left(\frac{t}{r}\right)^2}$$  \hspace{1cm} (6.1)

<table>
<thead>
<tr>
<th>Imperfection</th>
<th>Geometrical FE (V)</th>
<th>Theory</th>
<th>Difference (%)</th>
</tr>
</thead>
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<td>$g=0.001$</td>
<td>155</td>
<td>149</td>
<td>4</td>
</tr>
<tr>
<td>$g=0.0001$</td>
<td>152</td>
<td>149</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 6.5: Comparison of critical voltage for clamped dimple FE vs. Theory

Buckling voltages from FE simulation using small imperfections, simulated using
a linear acceleration of $g=0.001$ or $g=0.0001$, reproduce well the expected theoretical result. For higher imperfections, the buckling voltage is more affected and matches less well the theoretical result. Simulation results have confirmed that using an imperfection of the order of $g=0.001$ does not affect the buckling values significantly. However, as discussed earlier, adding these imperfection to dimple simulations are essential to observe deflections for voltages of a few hundred volts, see Figure 6.6. Further in this work, imperfections will therefore be simulated using the linear acceleration of $g=0.001$.

6.3.2 Boundary conditions

6.3.2.1 Simply supported dimple

The effect of boundary conditions on DEA actuators is studied on dimples by comparing simulation results from simply supported and clamped dimples. A 2D model of a simply supported dimple is created in ANSYS as seen in Figure 6.8. The material parameters used in section 6.3.1 are also used in this section, see Table 6.4. The material is assumed to be linear, the direct electro-mechanical modelling method described in section 4.1.4 is therefore used. Simulations are run when increasing voltages are applied across the dimple electrodes, while the maximum deflection is collected at the centre of the dimple for each voltage.

![Figure 6.8: Boundary conditions of simply supported dimple](image)

Due to the asymmetrical clamping conditions, the dimple is not subject to any type of buckling and deflects as soon as voltage is applied across its top and bottom electrodes as seen in Figure 6.9 for dimples with different degrees of material
imperfection. The small change in boundary conditions from clamped to simply supported is seen to have a major impact on dimple deflections. FE simulations are an effective tool in identifying such effects, and are therefore very useful for initial studies on different actuator designs while searching for the appropriate boundary conditions.

6.3.2.2 Material imperfections

In this section, simulations of simply supported dimple actuators are run using identical parameters as in the section above, including material imperfections simulated using linear accelerations of $g=0$, $g=0.01$, $g=0.001$ and $g=0.0001$. Results are collected and plotted in Figure 6.9. It is seen that the amount of deflection affects the amount of initial deflection as well as the amount of deflection collected when gradually increasing voltages are applied across the actuators. Based on FE results, it is seen that material imperfections can highly affect the performance of simply supported dimple actuators. FE simulations are a useful tool to identify such effects and when used in conjunction with experimental results, can be used to provide information about the amount of material imperfection present in the material used for a specific actuator. Further experimental work will be required to validate FE results.
6.4 Design of double-layer dimple

To provide a bi-directional response to a dimple actuator (a downward as well as upward movement in a controllable manner) a double-layer dimple design is suggested by the author. A 2D model of such a design is seen in Figure 6.10, electrodes are located in such a way that each of the two layers can be actuated independently. A finely meshed FE model of the double-layer dimple can be seen in Figure 6.11. The material parameters chosen for the simulations are presented in Table 6.6 and are identical for both layers. Due to software unavailability, simulations are run using orthotropic thermal expansion in the in-plane and out-of-plane directions rather than when applying voltages across the layers of the dimple. Indeed, the license of the ANSYS version supporting electromechanical elements had expired at this stage. A less powerful version of ANSYS was available to the author, although it did not support electromechanical simulations. The author concluded in section 5.2.2.3 that applying temperatures created the same effect as the Maxwell stresses induced by voltage across the electrodes. Similarly to section 5.2.2.3, in order to reproduce accurately the Maxwell stresses, the thermal expansion coefficients of the actuated layer are chosen for the actuated layer to be:
\[
\begin{cases}
\epsilon_x \\
\epsilon_y = -\frac{1}{\rho} \alpha_x
\end{cases}
\] (6.2)

The thermal expansion coefficients of the non-actuated layer are chosen to be zero. The boundary conditions of the FE model can be seen in Figure 6.12.

The thermal strains are successively applied to the top and bottom layer of the structure by applying a temperature difference between the structure and its surroundings. The double-layer dimple deflects respectively upwards and downwards once a critical temperature difference has been reached. A typical deflection of a double-layer dimple is represented in Figure 6.13. Deflections are collected at the centre of the specimen when the temperature differences are gradually increased, see Figure 6.14. Due to boundary conditions, no significant deflections are observed when lower
Table 6.6: Parameters for dimple simulation of voltage induced deflections

<table>
<thead>
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<th>Parameters</th>
<th>Value</th>
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<tbody>
<tr>
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<td>100 $\mu$m</td>
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<td>1 MPa</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.49</td>
</tr>
</tbody>
</table>

Figure 6.12: FE model of double-layer actuator
temperature differences are applied. When the top layer is actuated, it contracts in the out-of-plane direction and expands in the in-plane direction, as seen for the single layer actuator in section 5.2.2.3. An eccentricity is generated resulting in an upwards deflection of the structure. The mechanism is identically reversed when it is the bottom layer that is actuated.

Figure 6.13: Double-layer dimple FE model: out-of-plane deflections (m)

Figure 6.14: Deflection vs. Temperature across top and bottom layer of dimple actuator when respectively actuated

The critical temperature difference is evaluated using the Southwell method presented in section 5.4. The critical temperature difference is found graphically as
the slope of the Southwell plot, see Figure 6.15. That critical temperature difference is then compared to the critical temperature difference found using the ANSYS eigenvalue buckling analysis. Results are presented in Table 6.7 and shows a 20% difference which is difficult to explain. The Southwell plot being linear around the critical temperature difference confirms that it is a buckling phenomenon that we are witnessing. The computer used to run these simulations has a limited memory storage which might have affected approximations. Further work is therefore required to confirm the buckling value of a double-layer dimple. Also, voltage induced simulations will be required in future work for further conclusions on the double-layer mechanism. The main conclusion the author has reached here is that it is possible to design a double-layer dimple that provides upwards or downwards deflections in a controllable manner.

Figure 6.15: Deflection/Temperature difference vs. Deflection across top and bottom layer of dimple actuator when respectively actuated
Table 6.7: Critical temperature difference for double-layer dimple

<table>
<thead>
<tr>
<th>Southwell method</th>
<th>Southwell method</th>
<th>ANSYS Eigenvalue analysis</th>
<th>ANSYS Eigenvalue analysis</th>
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</thead>
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<td>Top layer</td>
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<td>Top layer</td>
<td>Bottom layer</td>
</tr>
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<td>actuated</td>
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</tr>
<tr>
<td>769</td>
<td>769</td>
<td>605</td>
<td>605</td>
</tr>
</tbody>
</table>

6.5 Conclusion and future work

The choice of material, actuator geometry, boundary conditions and prestretch are key parameters affecting the performance of DEAs. FE simulations are useful for finding the optimum combination of these parameters for the specific application. They provide fast, affordable and accurate information. Such simulations are not a substitute for experimental work but they do provide an efficient and effective means in guiding the direction of future experimental work.

FE design optimisation of DEAs is only complete when a full flow-electro-mechanical simulation is run. DEAs used as flow control devices can only be fully optimised through an evaluation of their effect of the surrounding flow. Future work will be carried out in ANSYS, using the developed electro-mechanical coupling, and coupling it to the relevant flow simulations.
Chapter 7

Pressure sensor

A novel pressure sensor using the properties of electroactive materials and the increased sensitivity of touch-mode devices has recently been developed by the Flow Control group. In this work, an FE model has been created for a better understanding of the performance of this novel sensor.

7.1 Design and operation principle

7.1.1 Design

![Design of the novel pressure transducer](image)

Figure 7.1: Design of the novel pressure transducer (Lavoie et al., 2007)

A novel pressure sensor design has been developed and is pictured in Figure 7.1. An elastomer sheet of thickness $t$ is placed over a circular gap of height $h$, providing space for the membrane to deflect onto a substrate. A compliant electrode is printed on the top surface of the membrane as well as on the substrate, as illustrated in
Figure 7.1. This device is used in touch-mode, as illustrated in Figure 7.2, thus providing measurements of greater sensitivity and robustness as discussed in Chapter 2.

Figure 7.2: Pressure sensor used in touch-down mode (Lavoie et al., 2007)

7.1.2 Operation principle

The elastomer membrane of a pressure device is built flush on top of the substrate as illustrated in Figure 7.1. When a voltage, $V$, is applied between the electrodes of the device, electrostatic forces develop across the elastomer and air-gap leading to the deflection of the elastomer membrane into the gap. The electrostatic pressure at a position $r$ along the radius of the membrane is expressed by Equation 7.1 before the touch-down of the membrane onto the substrate and by Equation 7.2 after touch-down. Before touch-down, electrostatic pressure and membrane deflection are shown to be coupled parameters, see Equation 7.1. Iterations will therefore be needed to find the equilibrium position of the membrane for any given voltage.

$$p_e(r) = \frac{\epsilon_0 \epsilon_a \epsilon_e V^2}{2[\epsilon_a t + \epsilon_e [g - w(r)]]^2}$$  \hspace{1cm} (7.1)

$$p_e(r) = \frac{\epsilon_0 \epsilon_e V^2}{2t^2}$$  \hspace{1cm} (7.2)

The air gap and elastomer membrane can be seen as two capacitors in series. The total capacitance $C_{eq}$ of the sensor is obtained by integrating a series of annular
capacitances of radius $r$ and width $dr$, see Equation 7.3. A detailed analysis of this equation is found in the work of Lavoie et al. (2007).

$$C_{eq} = \int_0^a \frac{2\pi\epsilon_0\varepsilon_a\varepsilon_e}{\varepsilon_a\ell + \varepsilon_e[g - w(r)]} r dr$$

(7.3)

For the sensor to operate in touch-down mode, an initial bias voltage is applied. Its value will depend on the geometry of the device and the area of touch-down required, according to Equation 7.1. In operation, the device is placed in a fluid with pressure fluctuations. As the amplitude of the pressure increases, the loading on the diaphragm rises, the membrane deflects further thus increasing the size of the touch-down area. Figure 7.3 is a 2D axi-symmetrical plot of a typical set of membrane positions. ‘Position 1’ represents a touch-down position of the device before it is used for measurements and ‘Positions 2 and 3’, the membrane positions while increasing pressures are applied. The membrane being part of the capacitor in this design, see Figure 7.1, the deflection of the membrane is path dependent. Let us consider the initial position of the membrane to be described by the function $w_1(r)$ when a voltage $V_1$ is present across the electrodes. When an additional external pressure, $P$, is applied to the device, the membrane deflects further. Iterations are needed to find the new equilibrium membrane position, $w_2(r)$, according to Equation 7.1. This new position is not only dependent on the external pressure but also on the membrane position before the pressure was applied. Moreover, when the external pressure, $P$, is removed, the membrane will not go back to its position before the pressure was applied but to a new equilibrium position $w_3(r)$ found using Equation 7.1. This device shows path dependence properties.

A simplified analytical model of this novel pressure sensor has been developed by Lavoie et al. (2007). The model is based on the assumption that the electrostatic forces acting on the top elastomer membrane are uniform and acting in the vertical direction only. The diaphragm is modelled before touch-down as a clamped circular plate. After touch-down, the collapsed area is also modelled as clamped. The shape of the remaining part of the membrane is approximated as an annulus with a guided inner edge and a fixed outer edge. Full details of the analytical model can be found in (Lavoie et al., 2007).
7.2 FE simulations

Preliminary simulations of the sensor are run in ANSYS. The simplifying assumption of neglecting the material initial prestretch is made. To date, the simulations have not yet been validated by experimental data. There are three types of simulations. The first is a purely structural set of simulations. The second is based on the direct electro-mechanical coupling modelling method described in section 4.1.4, a procedure limited to linear materials. The third type of simulation is based on the sequential coupling method described in section 4.1.5. The 2D model of the pressure sensor used for FE simulations in this section is illustrated by Figure 7.4.

7.2.1 Structural simulations

Structural 2D simulations use the 'Plane182' element, which is a 2D element that can be used as either a plane or an axi-symmetric element. It is defined by four nodes having two degrees of freedom at each node, translations in the nodal $x$ and $y$ directions, see illustration in Appendix D. The element has plasticity, hyperelasticity, stress stiffening, large deflections and large strain capabilities.
The axi-symmetric model of the pressure sensor is finely meshed, see Figure 7.5. The top and bottom rectangles are used to model the elastomer membrane and the substrate respectively. In this section, the air elements are not modelled. Since the resistance of the air gap to the movement of the membrane is minimal, such an approximation is accurate. Simulations are run for large deflections and include stress-stiffening effects. The parameters chosen for the simulations are presented in Table 7.1. Pressures are applied to the top membrane while maximum deflections are collected at the centre of the diaphragm. A typical out-of-plane deflection is shown in Figure 7.6. Simulation results have been compared to analytical results collected by Lavoie et al. (2007) in Figure 7.7. A mesh sensitivity analysis of the simulation results can be found in Appendix E and the simulation itself can be found in Appendix F.

While the membrane deflections are smaller than the size of the air-gap, simulations are consistent with analytical results. The discrepancy between the two sets of results increases once the membrane reaches touch-down. Simulations show a reduced area of touch-down compared to the analytical case. This disagreement is likely due to the assumptions made in the analytical modelling in which the effects of in-plane deformation such as stress-stiffening have been neglected. These are minor for small deflections, but increase for higher deflections. ANSYS simulations
<table>
<thead>
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<th>Value</th>
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<tr>
<td>$\epsilon_r$</td>
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</tr>
<tr>
<td>$\nu$</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Table 7.1: Parameters for structural pressure sensor simulation

Figure 7.5: Pressure sensor FE structural model

Figure 7.6: Pressure sensor out-of-plane deflections (m)
confirmed this assumption. When the effects of large deformations are turned off, analytical and simulation results agree with more than 99% accuracy for all the range of pressures and positions along the radius.

7.2.2 Electro-elastic simulations

Direct electro-elastic simulations are run to simulate the response of the sensor to voltage. The 2D Plane223 element, described in chapter 4, is used to create an axi-symmetrical three-layer model of the sensor as shown in Figure 7.4.

In this section, the air elements need to be modelled. Indeed, when voltage is applied to the pressure sensor, electrostatic forces develop across the elastomer as well as across the air-gap. The air is therefore modelled as an electro-elastic material with $\varepsilon_r = 1$ while the Young’s modulus of air is chosen to be vanishingly small. The parameters used for the simulations are presented in Table 7.2. The air gap to thickness ratio, $h/t$, is chosen to be 1 and 2 for two consecutive sets of simulations. Voltages are applied across the electrodes. The deflections collected at the centre of the device are plotted in Figures 7.10 and 7.11, together with the analytical results reported by Lavoie et al. (2007).

The “pull-in voltage” of the device can be defined as the voltage at which any
### Table 7.2: Parameters for electro-elastic pressure sensor simulations

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>5 mm</td>
</tr>
<tr>
<td>$t_{elastomer}$</td>
<td>100 $\mu$m</td>
</tr>
<tr>
<td>$\epsilon_{elastomer}$</td>
<td>2.8</td>
</tr>
<tr>
<td>$h_{air-gap}$</td>
<td>100/200 $\mu$m</td>
</tr>
<tr>
<td>$\epsilon_{air}$</td>
<td>1</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1130 kg/m$^3$</td>
</tr>
<tr>
<td>$E_{elastomer}$</td>
<td>0.2 MPa</td>
</tr>
<tr>
<td>$E_{air}$</td>
<td>$10^{-6}$ MPa</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Figure 7.8: Structural model of the pressure sensor

Figure 7.9: FE model of deflections of the pressure sensor approaching touchdown (m)
Figure 7.10: Electro-elastic simulation and analytical estimates of displacement vs. voltage for $h/t=1$

Figure 7.11: Electro-elastic simulation and analytical estimates of displacement vs. voltage for $h/t=2$
slight increase will cause immediate collapse of the membrane onto the substrate. At this voltage, the internal elastic forces within the membrane are no longer sufficient to overcome the increasing electrostatic forces. This lack of equilibrium causes the membrane to touch-down abruptly. In the present design it occurs when the membrane deflection reaches approximately one third of the air-gap according to analytical results, Lavoie et al. (2007).

Figures 7.10 and 7.11 show very good agreement between simulation and analytical results for voltages less than about half of the pull-in voltage. For voltages near the pull-in voltage, a difference of up to 20% is observed between the deflections collected using the FE simulations and the ones collected using the analytical model. The analytical model shows smaller deflections than the FE simulations, and seems to model the touch-down process as a gradual one. This can be explained by the fact that, in the analytical model, electrostatic pressures have been assumed to be uniform over the whole membrane (Lavoie et al., 2007). In reality, electrostatic pressures vary along the radius of the membrane according to Equation 7.1. Electrostatic pressures are the highest at the centre of the membrane, as seen in Figure 7.12, leading to the abrupt pull-in effect.

![Figure 7.12: Radial profile of the electrostatic pressure](image)

The analytical model can be improved so that it may be used for voltages around,
and higher than, the pull-in voltage. An analytical model, using Equation 7.1 to model the electrostatic pressures for every radius is run in Matlab. This updated analytical model is compared to the ANSYS results for a gap of $h/t=1$ and 2 in Figures 7.13 and 7.14, and is called the “updated analytical model”. The Matlab code used for this analytical simulation can be found in Appendix H.

![Graph: Electro-elastic simulation and updated analytical estimates of displacement vs. voltage for $h/t=1$](image)

Figure 7.13: Electro-elastic simulation and updated analytical estimates of displacement vs. voltage for $h/t=1$

As in the previous section, the FE simulations results show smaller deflections than those calculated from the analytical model thus leading to a later touch-down. This is due to the fact that in-plane deformation effects such as stress-stiffening are not included in either of the analytical models. Such effects can be neglected for small deformations. For higher voltages, these effects need to be taken into account. Simulations were run in ANSYS for the same cases when the large deformation effects was turned off. Unfortunately, these results did not provide a significantly better agreement with the analytical model. This can be due to a few factors. The analytical model was written for small deflections only which is not the case of the simulations carried out in this section. Also, the presence of elements to model the air-gap might introduce a delay in the collapse of the structure. Further analytical
work is required to develop a model that is not limited to small deflections.

For voltages higher than the pull-in voltage, no FE results are available due to limitations of the electro-elastic simulations of ANSYS: Figure 7.15 shows the high deformations undergone by air-elements while approaching the touch-down mode. To model the collapse of the membrane onto the substrate correctly, air elements have to be reduced to a vanishingly small thickness, such that elements have an infinite aspect ratio.

As indicated in Chapter 4, direct electro-mechanical simulations are limited to linear materials. A fully accurate FE model of the sensor requires the modelling of the elastomer membrane as a hyperelastic material.

### 7.2.3 Load-transfer modelling

#### 7.2.3.1 Theory

A modelling procedure based on the sequential method described in section 4.1.5 is developed to provide a hyperelastic pressure sensor model able to run when the
voltages applied are larger than the pull-in value. In this method, electrostatic and structural simulations are solved sequentially as separate physics fields. The electrostatic forces output from the electrostatic simulations are used as external loads to the structural simulations. Following each set of electrostatic-structural simulations, the geometry is updated to its deformed shape. The electrostatic-structural sequential calculation is repeated until the solution converges.

The electrostatic model is built using the ‘Plane121’ element. Plane121 is a 2D, 8-node, charge-based electric element with one degree of freedom, and a voltage at each node. The structural model is created using Plane183 elements. It is a 2D, six or eight-node structural element. The element has two degrees of freedom at each node, translation in the $x$ and $y$ direction. See Appendix D for illustration. The size and shape of the sensor modelled are identical to those of the previous section.

The challenge in this modelling procedure lies in the fact that, as seen in the two previous sections, the pressure sensor structural simulation is carried out without modelling the air elements to enable the FE model of the membrane to reach touch-down, see Figure 7.5. However, the electro-elastic simulations cannot be run without modelling the air-gap, as it is a conductor for the electric forces, see Figure 7.8. User intervention is thus necessary to mesh the deformed air-gap between each set of iterations to enable electrostatic simulations to be carried out on the newly updated geometries. Two options are suggested.

The first option is to create a new mesh manually for the deformed air-gap for each set of iterations. The air-gap elements are deleted before each structural
simulation and the mesh is regenerated. This enables the electrostatic simulation to be run on the newly updated structure which now includes air elements. This process is iterated until convergence. This technique requires manual tracking of the air-gap nodes as well as reconstruction of the deformed air-gap for each iteration. Such a procedure is computationally intensive but is the only known direct solution. In the present study, this approach is not taken, as the necessary computational resources are not available.

The second option is based on analytical modelling. A first electrostatic-structural iteration is run and then the air elements are removed. Further iterations are purely structural and run until convergence. At the end of each structural simulation the mesh is updated. The increased electrostatic forces are evaluated analytically and then applied as the external load to the structural simulation in the next iteration. Equation 7.1 shows that the electrostatic pressures at iteration \( i \) can be expressed as a function of the electrostatic pressure at the first iteration, see Equation 7.4,

\[
p_{ei} = \frac{(\epsilon_a t + \epsilon_e [g - w_i(r)])^2}{(\epsilon_a t + \epsilon_e [g - w_1(r)])^2},
\]

where: \( p_{e1} \) is the electrostatic pressure collected after the first iteration, \( p_{ei} \) the electrostatic pressure at iteration \( i \). Though this is only a linear approximation, the error incurred will be minimal providing that the gradients are reasonably small.

### 7.2.3.2 Results and analysis

Analytical and ANSYS simulations are compared in Figure 7.16 for a sensor of radius \( R=5 \) mm and 100 \( \mu \)m thickness. The analytical calculations used in this section are obtained using the updated analytical model described in the previous section. A first set of simulations is run for \( h/t=1 \).

Both curves are in good agreement at low voltages, see Figure 7.16. The discrepancy increases gradually after the pull-in voltage. For a further understanding of this discrepancy, simulations are also run for \( h/t=2 \), see Figure 7.17.

For both air-gap sizes, simulation and analytical results agree well for low voltages. This validates the accuracy of both models for small deflections. As the applied voltage approaches the pull-in voltage, solutions for the analytical and FE
Figure 7.16: Updated analytical and ANSYS sequential displacement vs. voltage for $h/t=1$

Figure 7.17: Updated analytical and ANSYS sequential deflection vs. Voltage for $h/t=2$
simulations diverge. This inaccuracy is due to a combination of factors. Firstly, the mechanical instability occurring around the pull-in voltage of the system introduces error into all the models. Also, from an analytical point of view, the model has been based on the flat plate theory, see Young and Budynas (2002), which is only valid for deflections of less than half the thickness of the material. Analytical results can therefore not be expected to be accurate for large deflections. Also, the electrostatic forces have been assumed to be vertical only, thus neglecting in-plane deformation effects so leading to a touch down at lower voltages as discussed in the previous section.

The simplifying assumptions used in the analytical model do not provide accurate data for voltages above the pull-in voltages. More sophisticated models will be developed in future work and compared to the ANSYS results.

7.3 Conclusion and future work

The ability of ANSYS to model the pressure sensor, although preliminary, is a tool for a better understanding of its design parameters and performance. A combined numerical and FE approach has been shown to be the best approach for an accurate model. ANSYS simulations and the analytical model show good agreement at low voltages. For higher voltages, an improved analytical model is required. Experimental work will also be needed in future work to validate the accuracy of both the analytical and simulation models.

The functioning process of this novel pressure sensor highlights that deflection measurements and therefore capacitance measurements cannot be used for external pressure measurements. As a result, this device cannot be used as a standard capacitive device, see Equation 7.3. A feedback control system needs to be created to make the device a null-reading device, capable of measuring external pressures. The control system will ensure that the area of touch-down remains unchanged thus maintaining the total capacitance identical during the measurements. This is achieved by varying the voltage across the electrodes. A calibration curve specific to each pressure sensor design will then be used to translate voltage differences into pressure measurements. A null-reading device is also likely to be inherently more accurate and not have the frequency limitations set by a process in which the finite
mass of the diaphragm is being moved.
Chapter 8

Viscoelasticity, modal and vibration analysis of DEAs

In this work, elastomers have been modelled as time-independent materials. However, when actuated, elastomer materials exhibit various time-dependent effects such as viscoelasticity. Such effects need to be taken into account for accurate material modelling. A modal analysis, evaluating the natural frequencies and modal shapes of a structure ensures the desired actuation by selecting the appropriate operating frequency. Carrying out a vibration analysis enables the identification, suppression, or modification of unwanted vibrations in a system thereby improving the performance of the system. Material viscoelasticity is studied in the first part of this chapter. Modal and vibration analysis are dealt within the second and third part of the chapter.

8.1 Frequency response

The response speed of an actuator is essential for applications that will require deflections that can be performed quickly. DEAs act as capacitors, their response speed will therefore be dependent on the electrical response time of the actuator as well as the geometry of the actuator and the viscoelastic properties of the material the actuator is made from.
8.1.1 Electrical response time of a capacitor

When capacitors are charged or discharged, we define the time it will take them to reach 37% of their initial charge or voltage as the ‘RC time constant’, it is marked $\tau$. That time constant is a measure of the electrical response time of the capacitor. It is known to be equivalent to $RC$ where $R$ is the resistance of the circuit across which the capacitor is discharged and $C$ the capacitance of the actuator. When a DEA is charged or discharged, the resistance of the circuit used to evaluate the time constant is calculated based on the resistance of the capacitor itself as well as the resistance of the electrodes, wires and all the electrical circuitry involved. The total resistance of all these factors can be combined into a single resistance, labelled the equivalent resistance $R_{eq}$. The equations of charge and discharge of capacitors are respectively Equation 8.1 for the charge

\[
\begin{align*}
Q(t) &= CV_0(1 - e^{-t/RC}) \\
V(t) &= V_0(1 - e^{-t/RC})
\end{align*}
\tag{8.1}
\]

when the capacitor is charged across a source of potential $V_0$ and Equation 8.2 for the discharge.

\[
\begin{align*}
Q(t) &= CV_0e^{-t/RC} \\
V(t) &= V_0e^{-t/RC}
\end{align*}
\tag{8.2}
\]

Rosset et al. (2012) carried out some research to evaluate the influence of the type of electrode on the performance of three different silicone actuators. Results showed that the type of electrode affects the strain response through stiffening and viscous losses, as well as the response speed, see Rosset et al. (2012). Ion implantation electrodes showed a good speed response, while carbon grease leads showed a high viscoelastic drift.

A study of the response speed of the dimple actuators presented in this work will be required to develop them further. The effect of different types of electrodes and the circuitry need to be measured for an efficient actuator design as well as the time-dependence of the elastomer material itself. Work by Molberg et al. (2009) shows that for three different types of elastomers, lower strains are obtained when they are used at high frequencies. This was shown to be mainly caused by the viscoelastic response of the material.
8.1.2 Viscoelasticity

Elastic materials strain instantaneously when loaded and return to their initial state when unloaded. The stress-strain behaviour in such materials obeys Hooke’s law, see Equation 8.3 and Figure 8.1(a).

\[ \sigma = E\epsilon. \] (8.3)

However, the stress in a viscous fluid under shear stress with a material viscosity of \( \eta \) obeys Equation 8.4.

\[ \sigma = \eta \frac{d\epsilon}{dt}. \] (8.4)

Viscoelastic materials exhibit both elastic and viscous properties when undergoing deformation, their stress-strain response is time-dependent. Some phenomena exhibited by viscoelastic materials are hysteresis, Mullin’s effect, the creep and relaxation effects as well as a material strain-rate dependence.

8.1.3 Hysteresis

A system with hysteresis is path-dependent. From a stress-strain aspect, unlike purely linear material, see Figure 8.1(a), the stress-strain response differs between loading and unloading, see Figure 8.1(b). The red area represents the hysteresis loop between a loading and unloading cycle.

![Stress-strain curves](image)

Figure 8.1: Stress-strain curves for (a) a purely linear material and (b) a viscoelastic material

Cyclic uniaxial tensile tests have been carried out on the MED4905 material of 250 \( \mu \)m thickness at low speeds. Such experimental data will provide information on
the quasi-static properties of the material. Results are plotted in Figure 8.2. In real conditions, the DEAs will be actuated at high frequencies, similar to the frequency of the coherent structures present in the turbulent boundary layer of aircrafts for example. This will require operating at frequencies of a few hundred Herzs. As a result, in future work the materials properties will have to be checked when tested at high frequencies. In Figure 8.2, the amount of hysteresis observed is significant between the first and second loadings but reduces between further consecutive loadings. The hysteresis of this material is time-dependent. A material is considered to be ‘preconditioned’ once the time-dependence effects between successive loadings and unloading cycles have both reduced and reached a minimal value. A loading-unloading cycle of the material in a preconditioned position are plotted in Figure 8.3.

Figure 8.2: Cyclic uniaxial loading - MED4905 250 µm - Engineering Stress vs. Engineering Strain, single set and raw data collected by Axel Labs inc.

The energy loss associated with the hysteretic loop was calculated by determining the difference in area between the loading and unloading curves of Figure 8.3: an energy loss of 16.6 kJ/m³ is derived. This corresponds to a temperature increase of less than one degree Kelvin per cycle based on a conservative adiabatic assumption and Equation 8.5 where \( Q \) is the heat energy, \( c_p \) the specific heat capacity of the
Further experiments are needed, but this preliminary study suggests that in order to model the 250 $\mu$m thick MED4905 material in a preconditioned state, including hysteresis effects is not essential when the material is stretched at low speeds. Further experimental data, involving stretching the material at different speed rates, is required for further conclusions on the behaviour of the material. Also, any cumulative temperature increase may degrade the polymer properties, thus a detailed measurement of the temperature during cyclic testing should be investigated.

### 8.1.4 Mullin’s effect

Recent experiments carried out on the 100 $\mu$m thick MED4905 material showed a clear Mullin’s effect. Figure 8.4 is a plot of the stress-strain curve of a cyclic tensile test carried out on a specimen stretched cyclically and successively to 100%, 200%...
and 400%. For each of these strain levels, the experiment is carried out until the material is preconditioned. The positions reached after 100%, 200% and 400% are respectively noted ‘Preconditioned 1’, ‘Preconditioned 2’ and ‘Preconditioned 3’ in Figure 8.4.

Figure 8.4: Mullin’s effect on MED4905 250 μm - Engineering Stress vs. Engineering Strain, single set and raw data collected by Axel Labs inc.

For material modelling purposes, similar cyclic biaxial and planar tensile tests are collected. Such data has been collected by Axel, for the materials used in this work, and are presented in Appendix C. For future modelling work, a new hyperelastic model can be evaluated for each of the preconditioned cases using the procedure described in section 2.5 together with the appropriate preconditioned set of uniaxial, biaxial and planar tensile data.

8.1.5 Creep and relaxation

It is essential for dielectric elastomer actuators to remain structurally stable. A material is said to exhibit creep properties if it suffers an increase in strain at a constant stress, as illustrated in Figure 8.5 (a).

\[ H(t) \] is the heaviside function defined as zero for \( t \) less than zero, one for \( t \) greater
than zero and 1/2 for \( t=0 \). For a one-dimensional system, when the stress is a step function dependent on time, it is expressed by:

\[
\sigma = \sigma_0 H(t) \quad \text{(8.6)}
\]

and the strain increases in time according to the creep compliance ratio of Equation 8.7,

\[
J(t) = \frac{\epsilon(t)}{\sigma_0}. \quad \text{(8.7)}
\]

![Figure 8.5: Creep and relaxation effects](image)

A material exhibits relaxation effects if it suffers a decrease in stress over time when a constant strain is applied, as seen in Figure 8.5 (b). When the stress is a step function beginning at time \( t=0 \) expressed by:

\[
\epsilon = \epsilon_0 H(t), \quad \text{(8.8)}
\]

the stress will decrease according to the relaxation modulus ratio of Equation 8.9,

\[
E(t) = \frac{\sigma(t)}{\epsilon_0}. \quad \text{(8.9)}
\]

In this work, experimental creep and relaxation studies have not been carried out, but will be in future work.
8.1.6 Strain-rate dependency

The performance of rubber materials is dependent on their strain-rate as seen in Figure 8.6, a typical graph of strain-rate dependency for uniaxial tests (Dalrymple et al., 2007). The study proposes an FE model in ABAQUS able to support both the viscoelastic and hyperelastic properties of an elastomer. Strain-rate effects were accurately predicted for loading by Dalrymple et al. (2007) when stress relaxation data was used for the calibration of a Prony series model.

![Rubber, Constant Strain-Rate Testing](image)

Figure 8.6: Typical family of constant strain-rate curves, loading only (Dalrymple et al., 2007)

Strain-rate effects have not been included in the current work but will be important in future work to achieve a fully accurate elastomer model.

8.1.7 Conclusion

The viscoelastic properties of elastomers need to be taken into account for accurate DEA modelling. In further work, more experimental data on hysteresis effects, Mullin’s effect, creep and relaxation effects is required to achieve a fully accurate viscoelastic elastomer model.
8.2 Modal analysis

The ANSYS software provides a tool for running 2D and 3D modal analysis for linear materials with or without prestretch. In this work, the accuracy of the ANSYS modal analysis is studied on dimple actuators by comparing simulation results to experimental data collected by Dearing (2007). The modal analysis in ANSYS, for a typical undamped system, solves the classical Equation 8.10.

\[ [K] \phi_i = \omega_i^2 [M] \phi_i \]  

(8.10)

where \( \omega_i \) is the oscillation frequency and \( \phi_i \) the phase displacement of the oscillation with respect to the exciting force.

8.2.1 Simulations

An axi-symmetric and a full 3D model of a dimple actuator are both created in ANSYS, based on the model presented earlier in Figure 4.2. For the axi-symmetric model, the Plane223 element. The 3D model is built using the ‘Solid226’ element. Solid226 is a 3D, 20-node coupled field solid element with up to four degrees of freedom per node: translation in the \( x \), \( y \) and \( z \) direction, temperature and voltage. An illustration of this element is available in Appendix D. For computational purposes, the axi-symmetric model is more effective to run than the full 3D model by reducing significantly the number of equations solved and therefore allowing more sophisticated problems to be solved. However, axi-symmetric models are obviously limited to the modelling of axi-symmetrical modes.

Before choosing axi-symmetric models for the continuation of this work, an accuracy check is carried out by comparison to the 3D modal analysis for the two first axi-symmetrical modes of a dimple in actuation. Results are collected for a dimple simulation using the parameters presented in Table 8.1. The Young’s modulus is chosen to be 1 MPa as it was concluded in section 5.5.2 that this was the most likely value for the Young’s modulus of the material tested by Dearing (2007). The first two results for axi-symmetric modes are collected in Table 8.2 and indicate that the axi-symmetric and 3D model compare favourably.
Table 8.1: Parameters for dimple simulation of frequency modes

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>5 mm</td>
</tr>
<tr>
<td>$t$</td>
<td>100 µm</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1130 kg/m³</td>
</tr>
<tr>
<td>$\varepsilon_r$</td>
<td>2.8</td>
</tr>
<tr>
<td>$E$</td>
<td>1 MPa</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.48</td>
</tr>
<tr>
<td>Presretch</td>
<td>0%</td>
</tr>
</tbody>
</table>

Table 8.2: Dimple first two axi-symmetric modes of $R=5$ mm dimple - FE results

<table>
<thead>
<tr>
<th>Mode</th>
<th>2D frequency (Hz)</th>
<th>3D frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 01</td>
<td>57</td>
<td>61</td>
</tr>
<tr>
<td>Mode 02</td>
<td>224</td>
<td>239</td>
</tr>
</tbody>
</table>
The four first modal shapes obtained from a typical 3D dimple model are pictured in Figure 8.7 and will be compared to experimental data in the following section. Mode 01 and Mode 02 match well the modes pictured earlier in Figure 5.2.

![Figure 8.7: First four dimple mode shapes - FE simulation](image)

8.2.2 Experiments

Experimental data are available for dimple actuators made of 100 $\mu$m thick MED4930 elastomer with a voltage of 2 kV applied across, see (Dearing, 2007). The first four modes of three dimples of radii $R=15$ mm, $R=10$ mm and $R=5$ mm are pictured using a Laser Doppler Vibrometer (LDV) and presented in Figures 8.8, 8.9 and 8.10 respectively.

The effect of various boundary conditions are observed. The first modal shapes are least affected, but at higher frequencies, the edges of the dimple lifted significantly from their mounting. As discussed in chapter 4, slight changes in boundary conditions affect actuation results significantly.
Figure 8.8: Vibrational modes - $R=15$ mm (Dearing, 2007)

Figure 8.9: Vibrational modes - $R=10$ mm (Dearing, 2007)

Figure 8.10: Vibrational modes - $R=5$ mm (Dearing, 2007)
8.2.3 Analysis

Simulations are run for dimples of radii $R=5$ mm, $R=10$ mm and $R=15$ mm to provide us with simulation results to compare to the experimental results presented in the previous section. The dimple parameters used in these simulations are presented in Table 8.3.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>5/10/15 mm</td>
</tr>
<tr>
<td>$t$</td>
<td>100 $\mu$m</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1130 kg/m$^3$</td>
</tr>
<tr>
<td>$\varepsilon_r$</td>
<td>2.8</td>
</tr>
<tr>
<td>$E$</td>
<td>1 MPa</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.48</td>
</tr>
<tr>
<td>Prestretch</td>
<td>2%</td>
</tr>
</tbody>
</table>

Table 8.3: Parameters for dimple simulations of frequency modes with initial prestretch

As a voltage of 2 kV is applied to the dimples, experimental and simulation results of the first mode are presented in Table 8.4. They agree within an error of less than 5%.

<table>
<thead>
<tr>
<th>$R$ (mm)</th>
<th>Frequency (Hz) Experiment</th>
<th>Frequency (Hz) Simulation</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>405</td>
<td>392</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>201</td>
<td>191</td>
<td>5</td>
</tr>
<tr>
<td>15</td>
<td>132</td>
<td>127</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 8.4: First dimple axi-symmetric mode - Simulation and experimental results

The natural frequency of the second axi-symmetric mode obtained experimentally is compared to simulation results in Table 8.5. The inaccuracy between simulation and experimental data is still low for a radius of $R=15$ mm but increases to
about 30% for smaller dimples. This is in agreement with the experimental observation that the boundary condition inaccuracies due to imperfections decrease with the increase of the radius, see Figures 8.8, 8.9 and 8.10.

<table>
<thead>
<tr>
<th>$R$ (mm)</th>
<th>Frequency (Hz) Experiment</th>
<th>Frequency (Hz) Simulation</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1261</td>
<td>891</td>
<td>29</td>
</tr>
<tr>
<td>10</td>
<td>662</td>
<td>430</td>
<td>35</td>
</tr>
<tr>
<td>15</td>
<td>327</td>
<td>284</td>
<td>13</td>
</tr>
</tbody>
</table>

Table 8.5: Second dimple axi-symmetric mode - Simulation and experimental results

ANSYS modal analysis provides simulation results matching accurately experimental work. Carrying out experiments is resource intensive and challenging. For the first steps of a DEA design, FE is therefore the most convenient tool.

### 8.3 Vibration analysis

Two types of vibrations can be distinguished, free vibrations and forced vibrations. Free vibrations are the natural response of a structure to an impact or displacement. Following the impact, the system vibrates at one or more of the natural frequencies of the system. When external forces are applied to the system, the system is under forced vibrations. Sinusoidal excitation causes the system to vibrate at the same frequency as the excitation itself. If the frequency of the external excitation coincides with one of the natural frequencies of the system, resonance will occur. While developing systems, appropriate analysis needs to be carried out to avoid resonance. Study of the impact of free vibrations as well as forced vibrations on the system are very revealing about the system properties and performance. Vibration analyses are carried out in ANSYS using the transient dynamic solver. This solver determines the dynamic response of a structure under the action of any general time-dependent load. Transient dynamic analysis differs from static analysis by taking into account the inertia and damping of the system. The basic equation solved by a transient dynamic analysis is the following differential equation, 8.11:
\[ [M]\ddot{u} + [C]\dot{u} + [K]u = F(t) \] (8.11)

where \([M]\) is the mass matrix, \([C]\) the damping matrix and \([K]\) the stiffness matrix. ANSYS uses the Newmark time integration method to solve these equations at discrete time points.

### 8.3.1 Step input - Impulse excitation test

#### 8.3.1.1 Theory

The response of a single degree-of-freedom system to free vibrations is fully characterised by its mass, stiffness and damping. The movement of the system is defined by Equation 8.11 where \(F(t)=0\). As an impulse is applied to the system, it oscillates around the equilibrium position. After every oscillation, the amplitude of the oscillations weakens, until the system reaches a stable position. The oscillation frequency, called the natural frequency, is defined by Equation 8.12,

\[ w_n = \sqrt{\frac{K}{M}}. \] (8.12)

The decay of the oscillations is due to the damping of the system. The damping can be described in terms of damping ratio, \(\xi\), a dimensionless quantity expressed by Equation 8.13,

\[ \xi = \frac{C}{2Mw_n}. \] (8.13)

The behaviour of the system depends on the natural frequency and the damping ratio. The damping ratio can take a wide range of values. When the damping ratio is small, the system is considered to be ‘under-damped’. In such a state, when the system is excited, it will take a long time to reach the equilibrium state, see Figure 8.11. When the damping is higher than 1, the system is defined as being ‘over-damped’. It will not oscillate and might take some time to reach equilibrium as seen in Figure 8.11. When the damping ratio is 1, the system is ‘critically-damped’ and will return to the equilibrium position in the shortest time possible, see Figure 8.11. Theoretical results of impulse excitation simulations are used to verify the ability of the FE software to model such vibrations.
8.3.1.2 Simulations and experiments

Different types of damping are available in ANSYS. In this work, material-dependent constant damping is chosen. By definition, this damping constant is such that when the damping is chosen as $\beta_i$ for material $i$, the damping matrix is expressed as:

$$\sum \beta_i \cdot [M].$$  \hfill (8.14)

FE dimple simulations are run when a step input is applied. A 2D model of the dimple is used, as seen in previous chapters, see Figure 4.2. The parameters of the simulations are presented in Table 8.6. Three different damping coefficients are chosen. As for the static dimple simulations, a mesh sensitivity and stability analysis is carried out for each of the simulations.

In order to observe the system in an under-damped state, the first damping coefficient chosen is very low, $\beta = 10^{-4}$. Results are plotted in Figure 8.12, the dimple oscillates around its stable position before converging to it. Such a result is consistent with the theory. The next damping coefficient is chosen a hundred times bigger, $\beta = 10^{-2}$, the dimple response is plotted in Figure 8.13. As expected, the response is now over-damped and converges in a direct and more asymptotic way to the equilibrium position. A damping constant which is ten times smaller than the
Table 8.6: Parameters for transient dynamic dimple simulation of step response

Dimple simulations using different time steps or damping coefficients respond differently to impulse excitations, as seen in Figures 8.11, 8.12, 8.13 and 8.14 but converge to the same deflection as expected from the theory. The dimple deflection is checked for a static case to compare it to this asymptotic deflection value. The deflection simulated in ANSYS for the static case is pictured in Figure 8.15 and is found to be about 200 µm.

Simulated responses for different damping coefficients agree well with the expected theoretical results. Experimental data collected by Dearing (2007) is compared to some FE results for further validation purposes. The response of a dimple of radius $R=5\text{ mm}$ to a $V=4.1\text{ kV}$ step impulse has been obtained experimentally by Dearing (2007), see Figure 8.16. A simulation reproducing this experimental setup as much as possible is run in ANSYS, the parameters used are presented in Figure

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>5 mm</td>
</tr>
<tr>
<td>$t$</td>
<td>100 µm</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1130 kg/m$^3$</td>
</tr>
<tr>
<td>$\epsilon_r$</td>
<td>2.8</td>
</tr>
<tr>
<td>$E$</td>
<td>1 MPa</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.48</td>
</tr>
<tr>
<td>Load type</td>
<td>Step</td>
</tr>
<tr>
<td>Load</td>
<td>250 V</td>
</tr>
</tbody>
</table>
Figure 8.12: Dimple response for a step input, $\beta = 10^{-4}$

Figure 8.13: Dimple response for a step input, $\beta = 10^{-2}$, $dt = 10^{-3}$ sec and $dt = 10^{-4}$ sec
Figure 8.14: Dimple response for a step input, $\beta = 10^{-3}$

Figure 8.15: FE static dimple deflection (m)
8.7. Due to limitation in computational resources, simulations are only run for a voltage of 1 kV. FE results are plotted in Figure 8.17, the trend of this impulse response is compared to the experimental data.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
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</tr>
<tr>
<td>$t$</td>
<td>100 $\mu$m</td>
</tr>
<tr>
<td>$\rho$</td>
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<tr>
<td>$\varepsilon_r$</td>
<td>2.8</td>
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<tr>
<td>$E$</td>
<td>1 MPa</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.48</td>
</tr>
<tr>
<td>Load type</td>
<td>Step</td>
</tr>
<tr>
<td>Load</td>
<td>1 kV</td>
</tr>
<tr>
<td>Prestretch</td>
<td>2%</td>
</tr>
<tr>
<td>Damping $\beta$</td>
<td>0.01</td>
</tr>
<tr>
<td>$dt$</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Table 8.7: Parameters for step input dimple simulation

Both responses, experimental and simulated, follow the step input with a small delay in a similar way. As expected, due to the lower voltage applied, FE results show smaller dimple deflections than those observed experimentally. The accuracy of the trend between the results illustrates the potential of FE simulations to predict the behaviour of actuators to impulse excitations.

### 8.3.2 Harmonic excitation

#### 8.3.2.1 Theory

Pure harmonic excitation is less likely to occur than periodic or other types of excitation. However, understanding the behaviour of a system under the effect of harmonic excitation reveals how the system will respond to more general types of excitations.
Figure 8.16: Dimple response to step input, Experimental data (Dearing, 2007)
A system subjected to a harmonic vibration is forced to vibrate at the same frequency as the vibration, Equation 8.11 can be updated by $F(t) = F_0 \cos(\omega t)$. The steady-state solution of this problem can be written:

$$x = X \cdot \cos(\omega t - \phi).$$  \hspace{1cm} (8.15)

The system oscillates at the same frequency as the applied force but with a phase shift $\phi$. The amplitude of the vibration $X$ is defined as:

$$X = \frac{F_0}{K} \frac{1}{\sqrt{(1 - r^2)^2 + (2\xi r)^2}}$$  \hspace{1cm} (8.16)

where $r$ is defined as the ratio of the harmonic force frequency over the undamped natural frequency of the system: $r = \frac{f}{f_n}$.

The phase shift is defined as:

$$\phi = \arctan\left(\frac{2\xi r}{1 - r^2}\right).$$  \hspace{1cm} (8.17)

The plot of these functions called “frequency response of the system”, see Figure 8.17: Dimple response to step input, Simulation data.
8.18, presents the important features in forced vibration. The behaviour of amplitude and phase shift of a typical forced harmonic vibration response are plotted against frequencies for various damping ratios. Comparing experimental results from a harmonically forced system to theoretical results is revealing of the system properties. In the following section ANSYS simulations are compared to experimental data obtained by Dearing (2007) as well as to the theoretical behaviour.

![Figure 8.18: Theoretical frequency response of the system: Amplitude and phase shift vs. frequency (Wikipedia, 2012)](image)

### 8.3.2.2 Simulations and experiments

Experimental work has been carried out on dimples of radius $R = 15$ mm, made of MED4930 100 $\mu$m thickness, forced between 1-100 Hz with voltages between 2.8-4.2 kV (Dearing, 2007). The frequency response was measured using a laser scanning unit pointing at the centre of the actuator. For 3.8 kV sinusoidal forcing at 1 Hz, experimental results are presented in Figure 8.19. A simulation as similar as possible to this experiment, is run in ANSYS using the parameters presented in Table 8.8. Unfortunately it was not possible to reach a 3.8 kV input due to computational limitations. Indeed, the deflections involved and therefore the amount of substeps required exceeded the capacity offered by the resources available for this work. However, a very close input voltage of 3.7 kV was successfully simulated, results are plotted together with the experimental results on Figure 8.19. Both responses are sinusoidal of amplitude 500-700 $\mu$m and slightly phase-shifted from the input signal. The simulations reproduce accurately the forced harmonic response of
this dimple actuator.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
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</tr>
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<tr>
<td>$\epsilon_r$</td>
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<td>1 MPa</td>
</tr>
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</tr>
<tr>
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<tr>
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<td>1 Hz</td>
</tr>
<tr>
<td>Load</td>
<td>1 kV</td>
</tr>
<tr>
<td>Damping $\beta$</td>
<td>0.01</td>
</tr>
<tr>
<td>$dt$</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Table 8.8: Dimple under forced vibration, FE parameters

The experimental phase shift between the input and response signals of the dimple has been obtained for a range of frequencies when a voltage of 3 kV is applied across a dimple, see Figure 8.20. Experimental data for frequencies higher than 80 Hz was not available thus limiting our study to these frequencies. The FE model used earlier in this section is used to run simulations of forced vibrations. Results are plotted in Figure 8.20. For both the experimental and simulation cases, the phase shifts increase with increasing frequencies as expected from the theory, see Figure 8.18. The phase angle will reach values higher than 90 degrees when the damping term is dominant in the frequency response which is not the case for the frequencies considered in this study. For frequencies below 80 Hz it seems that the stiffness of the material dominates the response.

In a time-dependent analysis, the frequency of the input signal affects the performance of actuators by affecting the amplitude of the response as seen in Figure 8.18. In the case of dimples, the parameter used to measure such an effect is the
Figure 8.19: Forced harmonic vibration on $R=15$ mm dimple, experimental and simulation data

Figure 8.20: Phase angle vs. frequency for dimple $R=15$ mm, Experimental and simulation data
non-dimensional ratio of the maximum deflection of the dimple to its diameter. This depth-to-diameter ratio is collected experimentally for a 15 mm radius dimple by Dearing (2007), for frequencies between $f=1-100$ Hz, see Figure 8.21 and voltages of 3kV and 3.5 kV. The higher the frequency, the lower the collected deflections and therefore the lower the performance. Unfortunately, the experimental data available limits our study to frequencies up to the first resonant frequency. The post-processed results of ANSYS simulations of the above experiment provide results plotted in the same figure. The trend observed in the simulation and experimental data are similar, the higher the frequency the lower the depth-to-diameter ratio. The performance of the dimple is strongly affected by its actuation frequency as expected from the theory, see Figure 8.18.

![Figure 8.21: Depth to diameter vs. frequency, $R=15$ mm](image)

For both the phase and depth-to-diameter ratio results, it can be noted that there is a difference between the experimental and simulation deflections collected for high frequencies, simulations predict higher deflections than the experimental results collected as well as a lower phase shift. This can be explained by the fact that material properties have been modelled as time-independent in the current simulations. In future work, material properties will need to be time-dependent and include viscoelastic effects such as creep, relaxation, and hysteresis. Also, it should be noted that at high frequencies the deflections that are measured are of the order of
tens of microns, thus the accuracy of the measurements can highly affect the results. As discussed earlier in chapter 4, it is extremely difficult to define accurately the amount of prestretch in the material as well as the Young’s modulus of the material. Simulations were run for what has been defined as the best suited combination for the studied material. However, there are still inaccuracies in the modelling of the experiments that are themselves very sensitive to any fabrication imperfection.

The graph also shows that for lower frequencies the discrepancy between simulated and experimental results is lower than for higher frequencies. As discussed earlier, it is very challenging to evaluate with precision the Young’s modulus of a material such as the elastomer used in the work of Dearing. It is therefore possible that the material was stiffer than the 1 MPa expected and used in simulations. It is then possible that while being actuated at higher frequencies, the material becomes softer and the approximation of 1 MPa is more accurate. This will require more experimental work to validate as this is a suggestion based on the qualitative experience the author has of the material, which is not backed up by any quantitative data. Future work will address this issue.

The response of a structure to harmonically forced vibrations has showed the influence of the actuation frequency of the performance of structures on the phase shift as well as the depth-to-diameter ratio. Harmonically forced vibrations provide essential information for actuator design. This can be found experimentally or is accurately reproduced using FE simulations.

8.4 Conclusion

A full understanding of the time-dependent properties of elastomers is essential for effective DEA design as they have been seen to affect the speed, shape and amplitude of the response. Experimental work is a way of evaluating the performance of an actuator when subjected to different time-dependent conditions, though inconvenient as the manufacturing of DEA devices is challenging, time-consuming and expensive. ANSYS simulations reproduce accurately this experimental behaviour and it is therefore suggested as a first step in device designing. Future work will require prototypes to be tested in actuation for a full device performance assessment.
Chapter 9

Conclusion and outlook

The goal of this thesis has been to formulate an ANSYS based FE modelling and optimisation procedure for EAP devices. Such a procedure enables a better understanding of their functionality and allows the creation of an effective design optimisation tool. The following novel achievements were attained both when constructing the model itself and subsequently when seeking to optimise it.

• Development of modelling techniques for hyperelastic materials
  
  DEAs are made of a thin sheet of elastomer sandwiched between two electrodes. Their modelling therefore requires an accurate modelling of the elastomer material itself which is best modelled using hyperelastic models. However, there is no standard hyperelastic modelling procedure available. A procedure combining the use of experimental data of uniaxial, biaxial and planar tension test together with a FE curve-fitting process has been developed in this work. Experimental data collected for different types of elastomers have been used to validate the consistency of this modelling technique.

• Development of a novel algorithm for electro-mechanical coupling for hyperelastic materials
  
  A DEA actuates when voltage is applied across its electrodes. Accurate modelling of this electro-mechanical behaviour requires software supporting the modelling of electro-mechanical coupling for hyperelastic materials. The lack of commercially available software for this purpose led to the develop-
ment of a novel, ANSYS based, sequential model based on the load transfer method between structural and electrostatic simulations. The efficiency of this novel modelling process has been validated against the ANSYS in-built direct electro-elastic simulations as well as against experimental results collected in previous work for dimple actuators. The new model has also shown good agreement when used for time-dependent simulations. Analyses of frequency modes and shapes as well as response to step input and sinusoidal signals have been validated against experimental data collected for dimple actuators.

• Identification of prestretch as an important design parameter in the design of flexible surfaces for both sensing and actuation

The validation of simulation results against experimental data for EAP dimple actuators revealed the existence of internal prestretch inside the materials used in the experimental work. Prestretches as low as 1-2% delayed the buckling of dimple actuators by hundreds of volts and caused a reduction of the amount of deflection collected after buckling by hundreds of microns. A simple adjustment to the simulations with the new model has been successfully implemented to take account of such effects. This technique uses virtual negative temperatures and the appropriate thermal expansion factor.

• Identification of the uncertainty in evaluating the material properties of elastomers

The main difficulty in modelling and optimising the use of elastomers is the challenge in evaluating their Young’s modulus and the amount of initial material prestretch present. Elastomer materials are sensitive to their manufacturing process, different batches demonstrate different material properties. When handling them or even by simply removing the material from its backing, on which they are placed due to fragility, an unknown amount of prestretch will be introduced. This affects significantly the material properties in an unpredictable manner. Such an uncertainty in material properties makes experimental validation as well as the design of actuators challenging.
• Evaluation of the importance of design parameters in actuator performance using the electro-mechanical model.

The effect of design parameters on the performance of devices was studied. The performance of actuators was shown to be highly sensitive to slight changes in boundary conditions, size, type of material as well as material prestretch. Modelling is shown to be a powerful tool allowing a low-cost and efficient optimisation process. It can be used to design an actuator to suit the requirements of a specific application prior to experimentation. An optimisation of dimple actuators for turbulence flow control has been carried out and a new double-layer device has been proposed.

• Innovative design of double-layer actuator that provides a bi-directional response - a fundamental requirement in the design of smart surfaces for flow control

Actuators providing upward as well as downward movement combined with a feedback control scheme on a smart surface might well prove to provide effective flow control. A novel dimple actuator design made of two separate layers of elastomer providing a bi-directional response has been developed in this work. The actuation principle lies in using the non-actuated layer as a mechanical bias for the actuated layer. By actuating the appropriate layer, upward or downward movement can be achieved. Experimental work will be required in future work to validate the efficiency of such devices.

• Validation of a novel pressure sensor, of which two prototypes are currently under development

A novel pressure sensor based on elastomer-based capacitive sensing has been suggested. Such sensors offer increased sensitivity and robustness in turbulence control as well as permitting miniaturisation. Their modelling made it possible to conduct an analysis of the benefits and drawbacks of such a design leading to the suggestion of an improved model combining the advantages of the touch-mode capacitive sensing with the high accuracy of a null-reading device. Further experimental work is required to assess the performance of
such a sensor.

The future of this work is to integrate EAP devices for accurate sensing and controllable actuation together with a feedback control scheme in a ‘smart skin’ for effective flow control. Sensors placed on the surface of aircraft are of particular interest for flow control. The modelling technique developed in this work provides a low cost and reliable method to optimise the design of such a surface for optimal flow control. Such work will involve developing the DEA electro-mechanical model from this work a step further. Adding a flow-structural interface to these simulations would enable the running of flow-electro-mechanical simulations. The effect of the various actuator and sensor design parameters could be measured directly against their effect on flow turbulence. The ANSYS software supports such couplings, adding such an interface to the existing simulations would therefore constitute the natural way forward to progress the optimisation of the model developed in this work.
Bibliography


ANSYS. Release 10.0 structural guide. 2009b.


IATA. Iata economic briefing. 2007.


Appendix A

Bulge test theory

The thin membrane theory neglects bending stresses, it is quite accurate for axisymmetric systems. Therefore, membrane equations for a bulge test are found using the thin membrane theory. During a bulge test, pressure is applied under a thin flat sheet as illustrated in Figure A.1. As a result, stresses develop both in the hoop direction, $\sigma_\theta$, and in the meridional direction, $\sigma_l$.

The flat sheet is forced into the shape of an axisymmetric shell of revolution. It is reasonable to assume the section as the cap of a sphere of radius $R$, as illustrated in Figure A.1. By making this assumption, meridional and hoop forces are found to be equal. The membrane is therefore under biaxial tension. The radius of the section can be found:

$$ R = \frac{a^2 + \delta^2}{2\delta} $$  \hspace{1cm} (A.1)

where:

$\delta$ is the central deflection

$a$ is the radius of the circular sheet.

The strains that develop in the sheet as a result of the inflation can be expressed as:

$$ \epsilon = \epsilon_\theta = \epsilon_l = \ln(1 + \delta^2/a^2) $$  \hspace{1cm} (A.2)

where:
Figure A.1: Bulge test
\( \varepsilon_{\theta} \) is the hoop strain and \\
\( \varepsilon_{l} \) is the meridional strains \\
The stress at the top of the sheet is:

\[
\sigma = \sigma_{\theta} = \sigma_{l} = \frac{PR}{2h_0} \left(1 + \frac{\delta^2}{a^2}\right) \tag{A.3}
\]

where \( h_0 \) is the original thickness of the membrane and \( \sigma_{\theta} \) is the hoop stress and \( \sigma_{l} \) is the meridional stress.
Appendix B

Hyperelastic curve fitting- Least Squares Fit analysis (ANSYS, 2009a)

The least squares fit minimises the sum of squared error between experimental and Cauchy predicted stress values by Equations .

\[
E = \sum_{i=1}^{n} (T_i^E - T_i(c_j))^2
\]  

(B.1)

where:

- E = least squares residual error
- \( T_i^E \) = experimental stress values
- \( T_i(c_j) \) = engineering stress values
- \( n \) = number of experimental data points
Equation B.1 is minimised by setting the variation of the squared error to zero: 
\[ \partial^2 E = 0. \] The set of simultaneous equations that needs to be solved are:

\[
\begin{align*}
\frac{\partial E^2}{\partial C_1} &= 0 \\
\frac{\partial E^2}{\partial C_2} &= 0 \\
\frac{\partial E^2}{\partial C_3} &= 0 \\
\end{align*}
\] (B.2)

\[
\begin{align*}
\vdots
\end{align*}
\] (B.5)

\[
\begin{align*}
\vdots
\end{align*}
\] (B.6)

\[
\begin{align*}
\vdots
\end{align*}
\] (B.7)

etc. (B.8)
Appendix C

Tensile machine clamps
Figure C.1: Clamps pro-E assembly
Figure C.2: Part 1
Figure C.4: Part 3
Appendix D

Material testing data

This appendix presents the full cyclic data from the testing in the three deformation modes of the MED 4905 material, with a 250 µm thickness. The results represent the raw single set of data provided by Axel Labs inc an purchased by Imperial College, London.

D.1 Simple tension cyclic test

![Simple tension cyclic test graph](image)

Figure D.1: Simple tension cyclic test
D.2 Equibiaxial tension cyclic test

Figure D.2: Equibiaxial tension cyclic test
D.3 Planar tension cyclic test

Figure D.3: Planar tension cyclic test
Appendix E

Element models

This appendix shows the models of the different elements used in the simulations carried out in this work. All the figures are taken from the ANSYS manual (ANSYS, 2009a).

E.1 Solid186

Solid186 is a higher order 3D 20-node solid element that exhibits quadratic displacement behavior. The element is defined by 20 nodes having three degrees of freedom at each node: translation in the x, y and z directions. The element supports plasticity, hyperelasticity, creep, stress stiffening, large deflection and large strain capabilities. It has the ability to model fully incompressible hyperelastic materials. The element may have any spatial orientation.

Figure E.1: Solid186 element model
E.2 Solsh190

Solsh190 is a 3D 8-node solid shell with three degrees of freedom at each node, translations in the x, y and z direction. It is used to simulate shell structures with a wide range of thicknesses. The element has plasticity, hyperelasticity, stress stiffening, creep, large deflection and large strain capabilities. The element formulation is based on logarithmic strain and true strain measures.

E.3 Plane82

Plane82 is a 2D 8-node solid element. It has two degrees of freedom at each node: translation in the x and y directions. The element may be used as a plane or axysymmetric element. It has plasticity, creep, swelling, stress stiffening, large deflection and large strain capabilities.
E.4 Plane121

Plane121 is a 2D 8-node electrostatic element. The element has one degree of freedom at each node, voltage. This element is suited to 2D electrostatic and time-harmonic quasistatic electric field analyses.

![Figure E.4: Plane121 element model - 2D - 8 node Electrostatic Solid](image)

E.5 Plane223

Plane223 is a 2D 8-node coupled-field element. The element has eight nodes with up to four degrees of freedom per node. Structural capabilities are elastic only and include large deflection and stress stiffening.

![Figure E.5: Plane223 element model - 2D - 8 node Couple-Field Solid](image)

E.6 Plane182

Plane182 is a 2D 4-node structural element. The element can be used either as a plane element or an axysymmetric element. It is defined by four nodes having
two degrees of freedom at each node: translations in the nodal x and y directions. The element has plasticity, hyperelasticity, stress stiffening, large deflection and large strain capabilities. It also has a mixed formulation capability for simulating deformations of nearly compressible elastoplastic materials and fully incompressible hyperelastic materials.

![Figure E.6: Plane182 element model - D - 4 node Structural Solid](image)

### E.7 Plane183

Plane183 is a 2D 8-node structural element. It is defined by eight nodes having two degrees of freedom at each node: translations in the nodal x and y directions. The element may be used as a plane element or as an axisymmetric element. This element has plasticity, hyperelasticity, creep, stress stiffening, large deflection and large strain capabilities. It has a mixed formulation capability for simulating deformations of nearly incompressible elastoplastic materials and fully incompressible hyperelastic materials. Initial stress import is supported.

![Figure E.7: Plane183 element model - 2D - 8 node Structural Solid](image)
E.8 Solid226

Solid 226 is a 3D 20-node coupled field structural element. The element has structural-thermal, piezoresistive, piezoelectric, thermal-electric, structural-thermoelectric and thermal-piezoelectric capabilities. The element has twenty nodes with up to five degrees of freedom per node. Structural capabilities are elastic only and include large deflection and stress stiffening.

Figure E.8: Solid226 element model
Appendix F

Mesh sensitivity

F.1 Tensile test simulation, section 3.2.1

For the tensile test simulation, the mesh sensitivity is studied when the specimen has been stretched by 100% of its length. The quantity chosen to be compared is the internal stress developing in the specimen in the direction of the length of the specimen.

![Graph showing mesh sensitivity analysis for uniaxial tension test](image)

Figure F.1: Mesh sensitivity analysis for uniaxial tension test
F.2 Biaxial tension test simulation, section 3.2.2

For the tensile test simulation, the mesh sensitivity is studied when the specimen has been stretched by 15% biaxially. The quantity chosen to be compared is the internal stress developing in the specimen.

Figure F.2: Mesh sensitivity analysis for biaxial tension test
F.3 Dimple in actuation, section 4.3

The mesh sensitivity is checked for a 2D model of a dimple. A voltage of 1 kV is applied to the dimple, and the maximum deflection is collected and compared for various mesh sizes.

![Graph showing mesh sensitivity for dimple in actuation](image)

Figure F.3: Mesh sensitivity for dimple in actuation
F.4 Pressure sensor, section 6.4

The mesh sensitivity is checked when a pressure of $P=0.26$ Pa is applied. Maximum deflections are collected and compared for different mesh sizes.

Figure F.4: Mesh sensitivity analysis for pressure sensor
Appendix G

ANSYS simulations

This chapter documents some of the key simulations that were run by the author in ANSYS to provide the results used in this work. The codes presented in this chapter need to be inserted directly in the command line of the software.

G.1 Uniaxial tension simulation, section 3.2.1

This code needs to be input in ANSYS to see results of a simulated uniaxial tensile test.

finish
/clear
/title,Electrostatic simulation
/prep7

Creating the electrostatic model
et,1,solid186

tb,hyper,,5,mooney
tbdata,1,-73424
tbdata,2,159761
tbdata,3,13927
tbdata,4,-61830
tbdata,5,10664
Structural model

H=50e-3
L=5e-3
t=50e-6

Creating thin sheet of material
BLC5,0,0,L,H,t

lesize,1,,20
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lesize,6,,20
lesize,8,,20

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lesize,5,,2
lesize,2,,2
lesize,7,,2

lesize,9,,2
lesize,10,,2
lesize,11,,2
lesize,12,,2

aatt,1,1,1
vmesh,all

nsel,s,loc,y,H/2
d,all,uy,0
allsel,all

nsel,s,loc,x,0
d,all,ux,0
allsel, all

nsel, s, loc, z, 0
d, all, uz, 0
allsel, all

/solu
dim, loadtab, table, 100, 1, 1, time
tread, loadtab, disp{x}, csv ! read table from file
toper, deltax2, loadtab, add, loadtab, -5, 0, 0

nsel, s, loc, y, -H/2
d, all, uy, %deltax2%
allsel

*do, i, 1, 10
time, i
nlgeom, on
cnvtol, f, 1, 1e-8
solcontrol, on
solve
save
enddo

/post1
plns, u, y
G.2 Biaxial tension test simulation, section 3.2.2

This code needs to be input in ANSYS to see results of a simulated biaxial tensile test.

```
finish
/clear
/title, Electrostatic simulation
/prep7

Creating the electrostatic model
et,1,190
keyopt,1,6,1

tb, hyper,,5, mooney
tbdata,1,5421
tbdata,2,85119
tbdata,3,2431
tbdata,4,515
tbdata,5,-3002

Structural model
H=30e-3
L=30e-3
t=250e-6

Creating thin sheet of material
BLC5,0,0,L,H,t

lesize,1,,10
lesize,3,,10
lesize,6,,10
lesize,8,,10
lesize,4,,10
```
lesize, 5
lesize, 2
lesize, 7

lesize, 9
lesize, 10
lesize, 11
lesize, 12

aatt, 1, 1, 1
vmesh, all

nsel, s, loc, x, 0
d, all, ux, 0
allsel, all

nsel, s, loc, y, 0
d, all, uy, 0
allsel, all

nsel, s, loc, z, 0
d, all, uz, 0
allsel, all

/solu
*dim, loadtab, table, 100, 1, 1, time
tread, loadtab, disp, csv
toper, deltax, loadtab, add, loadtab, 0.25, 0, 0

*dim, loadtab, table, 100, 1, 1, time
tread, loadtab, disp, csv
toper, deltax2, loadtab, add, loadtab, -0.25, 0, 0
nse1,s,loc,y,H/2
d,all,uy,%deltax%
allsel

nse1,s,loc,y,-H/2
d,all,uy,%deltax2%
allsel
nse1,s,loc,x,H/2
d,all,ux,%deltax%
allsel

nse1,s,loc,x,-H/2
d,all,ux,%deltax2%
allsel

*do,i,1,10
time,i
nlgeom,on
cnvotol,f,1,1e-8
solcontrol,on
solve
save
enddo

/post1
plns,u,y
G.3 Dimple in actuation, section 4.3

This code run in ANSYS will provide results of a dimple being actuated when voltages are applied across its top and bottom electrodes.

```plaintext
finish
/clear
/prep7

emunit,mks
et,1,plane223,1001.1
mp,perx,1,2.8
MP,nuxy,1,0.48
MP,ex,1,0.2e6
mp,dens,1,1130

H=250e-6
L=5e-3

Blc4,0,0,L,H

type,1
asel,s,area,,1
aatt,1,,1
LESIZE,2,,10
LESIZE,4,,10
LESIZE,1,,20
LESIZE,3,,20
amesh,1
allsel

nsel,s,loc,x,L
nsel,r,loc,y,0,H
d,all,ux,0
d,all,uy,0
```
allsel

nsel,s,loc,y,1*H
d,all,volt,1000
allsel

nsel,s,loc,y,0
d,all,volt,0
allsel

acel,,9.8

/sol
nlgeom,on
solve
finish

/post1
plns,u,y
### G.4 Eigenvalue buckling, 2D model

This code will provide an eigenvalue buckling analysis for a 2D, 15 mm radius dimple.

Finish
/Clear
/dele,all
/PREP7
emunit,mks

Defining material properties
et,1,PLANE223,1001,1
mp,perx,1,2.8
MP,NUXY,1,0.4999
MP,EX,1,1e6
mp,dens,1,1130
mp,alpy,1,0
mp,alpx,1,1e-2

Geometry of the dimple
H=100e-6
L=15e-3
Blc4,0,0,L,H

Defining meshing details
type,1
asel,s,area,,1
aatt,1,,1
LESIZE,2,,3
LESIZE,4,,3
LESIZE,1,,100
LESIZE,3,,100
amesh,1
allsel
Applying boundary conditions
lsel,s,line,2
nsll,s
cpintf,ux
d,all,uy,0
allsel
finish

/sol
antype,static
pstres,on
sfl,2,pres,1
solcontrol,on
autots,on
nsubst,1000,1000,10
outpr,,all
solve
finish

Starting the buckling analysis
/Solu
Antype,buckle
bucopt,subsp,20
mxpand,20
solve

Plotting the first mode shape
/Post1
set,first
pldisp,1
finish
G.5 Pressure sensor, section 6.4

This code will provide a simulation of a pressure sensor, radius 5mm, when increasing pressures are applied, see section 6.4.

finish
/clear
/prep7

emunit,mks
eps0=8.854e-12

et,1,plane182,,1
mp,ex,1,0.2e6
mp,nuxy,1,0.48

et,3,plane182,,1
mp,ex,3,7e10
mp,nuxy,3,0.48

Structural model
H=100e-6
L=5e-3

Creating thin sheet of material
type,1
Blc4,0,H,L,H
aatt,1,,1
lesize,1,,200
lesize,3,,200
lesize,2,,20
lesize,4,,20
mshkey,1
amesh,1
Copy area and mesh make concrete
agen,2,all,,−2*H
asel,s,area,,2
esla
emodif,all,mat,3
emodif,all,type,3
alls

Use contact elements
et,4,169 !Target
et,5,171 !Contact

lset,s,loc,y,H
nsll,s,1
type,4
real,4
esurf

lset,s,loc,y,0*H
nsll,s,1
type,5
esurf

nset,s,loc,x,L
d,all,ux,0
d,all,uy,0
allsel

da,2,all,0

*dim,loadtab,table,20,1,1,time
tread,loadtab,pres,csv ! read table from file
toper,pel,loadtab,add,loadtab,1,0,0
lsel,s,loc,y,2*H
sfl,all,pres,%pel%
dl,1„all,0

finish

/solu
nlgeom,on

*do,i,1,10
time,i
solve
enddo

finish
/post1
set,10
/window
/dscale„off
plns,u,y
G.6 Sequential electro-mechanical coupling for a dimple, section 4.2

This code shows the sequential electro-mechanical coupling of a dimple of 5 mm radius.

```
finish
clear
dele,all
title,Electrostatic simulation
/prep7
emunit,mks

Creating the structural model
et,1,plane121,,1

Structural model
H=100e-6
L=5e-3

Creating thin sheet of material
Blc4,0,0,L,H
lesize,1,,100
lesize,3,,100
lesize,2,,10
lesize,4,,10

aatt,1,1,1
amesh,all
get,N,node,,count
mp,ex,1,0.8e6
mp,prxy,1,0.48
mp,perx,1,2.8
mp,dens,1,1130
```
nsel, s, loc, y, H

d, all, volt, 200

allsel

nsel, s, loc, y, 0
d, all, volt, 0

allsel

nlgeom, on

acel,, 9.8

physics, write, elect

physics, clear

/title, struc

Creating the structural model

et, 1, plane42,, 1

mp, ex, 1, 0.8e6

mp, prxy, 1, 0.48

mp, perx, 1, 2.8

mp, dens, 1, 1130

nsel, s, loc, x, L

nsel, r, loc, y, 0, H
d, all, ux, 0
d, all, uy, 0

allsel

nlgeom, on

acel,, 9.8

physics, write, struct

physics, clear
Solving the first iteration of electrostatic followed by structural simulation
/solu physics,read,elect
solve
finish

/post1
vget,fx1,node,fmag,x
vget,fy1,node,fmag,y

/solu
physics,read,struct
ldread,forc,rth
nlgeom,on
acel,9.8
solve
finish

/post1
plns,u,y

End of first iteration of solving
Adding loops for convergence criteria
last=1
imax=10
dim,delta,array,imax,1
do,i,1,imax,1
nsort,u,y,1
get,uymax,sort,0,max

cdelta(i)=abs(uymax)

/prep7
upgeom,1,,file,rst

/solu
physics,read,elect
time,i+1
solve

/post1
vget,fxnew,node,,fmag,x
voper,temp1x,fx1,mult,-1
voper,temp2x,fxnew,mult,1
voper,fx2,temp1x,add,temp2x

*vget,fynew,node,,fmag,y
voper,temp1y,fy1,mult,-1
voper,temp2y,fynew,mult,1
voper,fy2,temp1y,add,temp2y
finish

/solu
physics,read,struct
do,inc,1,N,1
f,inc,fx,fx2(inc)
f,inc,fy,fy2(inc)
enddo

cnvtol,f,1,1e-8
solcontrol,on
time,i+1
solve

/post1
plns,u,y
new = abs(uymax)/H
if, abs(new), GT, 1e-2, then
last = abs(uymax)
doinc2, 1, N, 1
fx1(inc2) = fx2(inc2) + fx1(inc2)
fy1(inc2) = fy2(inc2) + fy1(inc2)
fxnew(inc2) = 0
temp1x(inc2) = 0
temp1y(inc2) = 0
temp2x(inc2) = 0
temp2y(inc2) = 0
fx2(inc2) = 0
fy2(inc2) = 0
enddo
cycle
else
exit
dendif
enddo

/post1
plns, u, y
G.7 Simulation of dimple with prestretch

This code simulates the actuation of a 15 mm radius dimple, when different amounts of prestretch are applied.

finish
/clear
/dele,all

/prep7
emunit,mks
et,1,plane223,1001,1
mp,perx,1,2.8
mp,nuxy,1,0.48
mp,ex,1,1e6
mp,dens,1,1130
mp,alpx,1,1e-2

H=100e-6
L=15e-3

Blec4,0,0,L,H
type,1
asel,s,area,1
aatt,1,1
lesize,2,5
lesize,4,5
lesize,1,50
lesize,3,50
amesh,1
allsel

nsel,s,loc,x,L
nsel,r,loc,y,0,H
d,all,ux,0
d,all,uy,0
allsel

acel,9.8
finish

/sol antype,static
pstres,on

*dim,loadtab,table,17,1,1,time
tread,loadtab,voltageprestretchT,csv
toper,VT,loadtab,add,loadtab,1,0,0

nsel,s,loc,y,1*H
d,all,volt,%VT%
allsel
nsel,s,loc,y,0
d,all,volt,0
allsel

nsel,s,loc,x,0
d,all,ux,0
allsel

bfunif,temp,-3

*do,i,1,17
time,i
nlgeom,on
sstif,on
solcontrol,on
autots,on
nsubst,1000,1000,10 solve
enddo
finish

/post1
plns,u,y
G.8 Pressure sensor, section 6.4

This code presents the simulation of a pressure sensor of 5 mm radius. Deflections are collected when different amounts of pressure are applied.

```
finish
/clear
/prep7

emunit,mks
eps0=8.854e-12

et,1,plane182,,1
mp,ex,1,0.2e6
mp,nuxy,1,0.48

et,3,plane182,,1
mp,ex,3,7e10
mp,nuxy,3,0.48

Structural model
H=100e-6
L=5e-3

Creating thin sheet of material
type,1
Blc4,0,H,L,H
aatt,1,,1
lesize,1,,50
lesize,3,,50
lesize,2,,5
lesize,4,,5
mshkey,1
amesh,1
```
Copy area and mesh make concrete agen, 2, all, 2*H
asel, s, area, 2
esla
emodif, all, mat, 3
emodif, all, type, 3
alls

Use contact elements
et, 4, 169 Target
et, 5, 171 Contact

lsel, s, loc, y, H
nsll, s, 1
type, 4
real, 4
esurf

lsel, s, loc, y, 0*H
nsll, s, 1
type, 5
esurf

nsel, s, loc, x, L
d, all, ux, 0
d, all, uy, 0
allsel

da, 2, all, 0

*dim, loadtab, table, 20, 1, 1, time
tread, loadtab, pres, csv ! read table from file
toper, pel, loadtab, add, loadtab, 1, 0, 0
lsel,s,loc,y,2*H
sfl,all,pres,%pel%
dl,1,,all,0

finish

/solu
nlgeom,on

*do,i,1,10
time,i
solve
endo
do

finish
/post1
set,10
/window
/dscale,,off
plns,u,y
Appendix H

Matlab 6.0.4 updated analytical model - Code

clear all
close all

a=5e-3;
thick=100e-6;
E=0.2*1e6;
ni=0.5;
D=E * thick^3/12/(1 - ni^2);
eps0=8.854e-12;
epsa=1;
epsc=2.8;
N=100;
M=15;
w(1:N+1,1:M)=0;
w2f(1:N+1,1:M)=0;
i0(1:M)=0;
deltax=5e-6;

l=1;
h=1;
\[ g(h) = \text{thick} \times h \times 0.5; \]
\[ V(l) = 10 \times l; \]

Starting first iteration
\[ u = 1; \]
Starting i loop
for \( i = 1: \text{N} + 1 \)
\[ P(i, u) = \frac{(\varepsilon_0 \times \varepsilon_{sa} \times \varepsilon_{se} \times \varepsilon_{se} \times V(l) \times V(l))}{2 \times ((\varepsilon_{sa} \times \text{thick} + \varepsilon_{se} \times (g(h)))^2)}; \]

\[ A = \frac{0.488}{\text{thick}/\text{thick}}; \]
\[ B = 0; \]
\[ C = 1; \]
\[ D_d = -P(i, u) \times a^4/64/D; \]
\[ t = [A \ B \ C \ D_d]; \]
\[ w0(:, i) = \text{roots}(t); \]

for \( j = 1:3 \)
if angle(w0(j, i)) < 1e-10
\[ w02(i) = \text{real}(w0(j, i)); \]
end
end
\[ r(i) = a/N*(i-1); \]
\[ w(i, u) = w02(i) \times ((1 - (r(i)/a)^2))^2; \]
end
Ending i loop

if max(abs(w(:, u))) < g(h)
for \( i = 1: \text{N} + 1 \)
\[ w2f(i, u) = w(i, u); \]
end
elseif max(abs(w(:, u))) >= g(h)
for \( i = 1: \text{N} + 1 \)
if abs(w(i, u)) >= g(h)
\[ w_{2f}(i,u) = g(h); \]
else
\[ w_{2f}(i,u) = w(i,u); \]
end
end

Finishing first iteration
Starting iterations from \( u = 2 \) to \( M \)
for \( u = 2 : M \)
for \( i = 1 : N + 1 \)
if \( \text{abs}(w_{2f}(i,u-1)) < g(h) \)
P(i,u) = \\
\left( \varepsilon_0 \varepsilon_a \varepsilon_e \varepsilon e V(l) V(l) / 2 / ((\varepsilon_a \varepsilon e + \varepsilon e (g(h) - w_{2f}(i, u - 1)))^2) \right); \\
elseif \text{abs}(w_{2f}(i,u-1)) \geq g(h) \\
P(i,u) = \varepsilon_0 \varepsilon e V(l) V(l) / 2 / \text{thick}\text{thick}; \\
end
end

If no touchdown- Option 1
for \( i = 1 : N + 1 \)
A = 0.488 / \text{thick}\text{thick}; \\
B = 0; \\
C = 1; \\
Dd = -P(i, u) a^4 / 64 / D; \\
t = [A B C Dd]; \\
w0(:,i) = \text{roots}(t); \\
for \( j = 1 : 3 \)
if \( \text{angle}(w0(j,i)) < 1\mathrm{e}-10 \)
w02(i) = \text{real}(w0(j,i)); \\
end
end
\[ w(i,u) = w02(i) \times ((1 - (r(i)/a)^2))^2; \]

end

if max(abs(w(:,u))) < g(h)
for i=1:N+1
w2f(i,u) = w(i,u);
end

If touchdown - option 2
elseif max(abs(w(:,u))) >= g(h)

for i=1:N+1
if abs(w(i,u)) > g(h) + deltax
w2f(i,u) = g(h);
elseif abs(w(i,u)) <= g(h) + deltax & abs(w(i,u)) > g(h) - deltax
r0(u) = r(i);
i0(u) = i;
w2f(i,u) = g(h);
end
end

for i=1:N+1
if abs(w(i,u)) < g(h) - deltax
C2 =
0.25 \times (1 - (r0(u)/a)^2) \times (1 + 2 \times \log(a/r0(u)));
C5 =
0.5 \times (1 - (r0(u)/a)^2);
L11 = (1/64) \times (1 + 4 \times (r0(u)/a)^2 - 5 \times (r0(u)/a)^4 - 4 \times (r0(u)/a)^2 \times \log(a/r0(u)));
L14 =
(1/16) \times (1 - (r0(u)/a)^4 - 4 \times (r0(u)/a)^2 \times \log(a/r0(u)));
F2(i) =
(1/4) \times (1 - (r0(u)/r(i))^2) \times (1 + 2 \times \log(r(i)/r0(u)));
end
\[ G_{11}(i) = \frac{1}{64} \times \left( 1 + 4 \times \frac{r_0(u)}{r(i)} \right)^2 - 5 \times \left( \frac{r_0(u)}{r(i)} \right)^4 - 4 \times \left( \frac{r_0(u)}{r(i)} \right)^2 \times \left( 2 + \left( \frac{r_0(u)}{r(i)} \right)^2 \right) \times \log \frac{r(i)}{r_0(u)}; \]

\[ f_1 = C_2 \times L_{14}/C_5 - L_{11}; \]

\[ f_2(i) = L_{14}/C_5 \times F_2(i); \]

\[ f_3(i) = G_{11}(i); \]

\[ w_{2}(i,u) = 4 \times \left( P(i,u) \times a^4/D \times f_1 - P(i,u) \times a^2 \times r(i)^2/D \times f_2(i) + P(i,u) \times r(i)^4/D \times f_3(i) \right)^4; \]

end

end

damp(u) = g(h)/w_{2}(i0(u)+1,u);
for i=i0(u)+1:N+1
  w_{2f}(i,u) = w_{2}(i,u) \times \text{damp}(u);
end
for i=1:i0(u)
  w_{2f}(i,u) = g(h);
end
end
end of option 2 end

Checking for convergence
if max(abs((abs(w_{2f}(.;u)))-(abs(w_{2f}(.;u-1))))) < \text{deltax} \times 3
  break;
elseif max(abs((abs(w_{2f}(.;u)))-(abs(w_{2f}(.;u-1))))) > \text{deltax} \times 3
  continue;
end
end for u
end

figure(2)
plot(r,-w_{2f}(.;u));

Adding external pressure
for k=1:5
Pext=10*k;;
for v=1:M
    for i=1:N+1
        if v==1
            P2(i,v)=P(i,u)+Pext;
        elseif v>1
            if abs(w2f2(i,v-1))<g(h)
                P2(i,v)=(eps0 * epsa * epse * epse * V(l) * V(l))/2/
                         ((epsa * thick + epse * (g(h) - w2f2(i, v - 1)))^2);
            elseif abs(w2f2(i,v-1))>=g(h)
                P2(i,v)=eps0*epse*V(l)*V(l)/2/thick/thick;
            end
        end
    end
end

for i=1:N+1
    A=0.488/thick/thick;
    B=0;
    C=1;
    Dd=-P2(i,v)*a^4/64/D;

    t=[A B C Dd];
    w0(:,i)=roots(t);
    for j=1:3
        if angle(w0(j,i))<1e-10
            w02(i)=real(w0(j,i));
        end
    end
    w(i,v)=w02(i) * ((1 - (r(i)/a))^2);
end

if max(abs(w(:,v)))<g(h)
    for i=1:N+1

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w2f2(i,v)=w(i,v);
end
elseif max(abs(w(:,v)))>=g(h)
    for i=1:N+1
        if abs(w(i,v))>g(h)+deltax
            w2f2(i,v)=g(h);
        elseif abs(w(i,v))<=g(h)+deltax & abs(w(i,v))>g(h)-deltax
            r0(v)=r(i);
i0(v)=i;
w2f2(i,v)=g(h);
        end
    end
end

for i=1:N+1
    if abs(w(i,v))<g(h)-deltax
        C2=0.25 * (1 - (r0(v)/a)^2 * (1 + 2 * log(a/r0(v))));
        C5=0.5 * (1 - (r0(v)/a)^2);
        L11=(1/64) * (1 + 4 * (r0(v)/a)^2 - 5 * (r0(v)/a)^4 - 4 * (r0(v)/a)^2 * (2 + (r0(v)/a)^2) * log(a/r0(v)));
        L14=(1/16) * (1 - (r0(v)/a)^4 - 4 * (r0(v)/a)^2 * log(a/r0(v)));
        F2(i)=(1/4) * (1 - (r0(v)/r(i))^2 * (1 + 2 * log(r(i)/r0(v))));
        G11(i)=(1/64) * (1 + 4 * (r0(v)/r(i))^2 - 5 * (r0(v)/r(i))^4 - 4 * (r0(v)/r(i))^2 * (2 + (r0(v)/r(i))^2) * log(r(i)/r0(v)));
        f1=C2*L14/C5-L11;
        f2(i)=L14/C5*F2(i);
        f3(i)=G11(i);
        w2(i,v)=(P2(i,v) * a^4/D * f1 - P2(i,v) * a^2 * r(i)^2/D * f2(i) + P2(i,v) * r(i)^4/D * f3(i));
    end
end

damp(v)=g(h)/w2(i0(v)+1,v);
for i=i0(v)+1:N+1
    w2f2(i,v)=w2(i,v)*damp(v);
end
for i=1:i0(v)
    w2f2(i,v)=g(h);
end
end

Checking for convergence
if v>1
    if max(abs((abs(w2f2(:,v)))-(abs(w2f2(:,v-1)))))<deltax*3
        break;
    elseif max(abs((abs(w2f2(:,v)))-(abs(w2f2(:,v-1)))))>deltax*3
        continue;
    end
end

figure(3)
if k==1
    plot(r,-w2f2(:,v));
    hold on;
end

End for v
end

figure(4)
if k==1
    plot(r,-w2f2(:,v),'r');
    hold on;
elseif k==2
    plot(r,-w2f2(:,v),'b');
hold on;
elseif k==3
plot(r,-w2f2(:,v),'y');
hold on;
elseif k==4
plot(r,-w2f2(:,v),'g');
hold on;
end
hold on;
plot(r,-w2f(:,u),'r.');

for i=1:N+1
C(i,k)=(2*pi*eps0*epsa*epse/(epsa*thick+epse*(g(h)-w2f2(i,v))))*r(i);
Cmax=2*eps0*epse*pi*a*a/thick;
C(i,k)=C(i,k)/(Cmax);
end

S(k)=sum(C(:,k));
End for k
end
Appendix I

Author’s article- SPIE 2008
Modelling Electroactive Polymer (EAP) Actuators: Electro-Mechanical Coupling using Finite Element Software

F. Rosenblatt, J.F. Morrison & L. Iannucci
Department of Aeronautics, Imperial College, London, SW7 2AZ, UK

ABSTRACT

Controlling turbulence is a major aim for many engineering disciplines. Decades of research, have shown that the large frictional drag in turbulent flows is attributed to the existence of near-wall coherent structures\(^1\). Turbulence control is therefore likely to be achieved by manipulating these coherent structures. The challenge this presents is to find actuators that are functional at the spatial scales of those coherent structures (10 \(\mu\)m to 0.1 mm) and their temporal scale (100 kHz). Recent advances in MEMS technology have made possible the construction of such actuators. Electroactive polymers (EAP) provide excellent performance, are lightweight, flexible, and inexpensive. Therefore EAPs, and in particular dielectric elastomers (DEAs), provide many potential applications as micro-actuators. The modelling and simulating of EAP actuators are a cost-effective way of providing a better understanding of the material itself in order to optimise designs. A technique to accurately model DEA materials, taking into account its non-linearities as well as its large deformations, is being developed in this study.

Keywords: Actuators, dielectric elastomer, hyperelastic, finite element, model

1. INTRODUCTION

Modelling DEA actuators is an essential prerequisite for their development. They have been the subject of extensive studies\(^2\)-\(^10\) and many devices have been developed. However, after a decade of research, a full understanding, simulation and complete model of DEA actuators is still lacking. The existing models are mainly analytical ones\(^8\)-\(^12\), thus limiting their use for specific actuator configurations. Finite Element (FE) simulations are applicable to any actuator geometry; the use of FE is therefore used in this work for modelling. Achieving an accurate DEA model presents a twofold challenge. The first lies in finding an accurate model of the elastomer itself, which will be discussed in a first section. The second is posed by the electromechanical behaviour of DEA materials, the implementation of a model is described in the second section. A third section addresses the necessity of developing more sophisticated models.

2. MODELLING ELASTOMERS

A DEA consists of a thin sheet of elastomer compressed between two compliant electrodes. The dielectric elastomer deforms when a voltage is applied across its electrodes. The elastomers used in this study are MED-4905 and MED-4930. Their thickness can be as low as 50 \(\mu\)m and they exhibit strains of as much as a few hundred percent of their thickness. By operating at frequencies as high as a few hundred Hz in a repeatable way, these materials appear suitable for use as turbulence control actuators, up to frequencies of one or two kHz.

An accurate model of the elastomer itself is an essential first step. Previous studies have shown that hyperelastic models are the most appropriate way to model their behaviour accurately\(^13\)-\(^14\). Hyperelastic materials are characterised through a strain-energy potential, \(W\), which represents the strain energy of the material as a function of deformation. If the material response is assumed to be isotropic, the strain-energy potential can be expressed as a function of the strain invariants, \(I_i\) of the stretch-ratio tensor. Various models are available such as Yeoh\(^15\), Mooney-Rivlin\(^16\) and others. In this study, the models used are the first and second-order Mooney-Rivlin models. They are respectively expressed by:

\[
W = C_{10}(I_1 - 3) + C_{01}(I_2 - 3) \tag{1}
\]
where $C_{10}, C_{01}$ are material parameters and

$$W = C_{10}(I_1 - 3) + C_{01}(I_2 - 3) + C_{20}(I_1 - 3)^2 + C_{11}(I_1 - 3)(I_2 - 3) + C_{02}(I_1 - 3)^2$$

(2)

where $C_{10}, C_{01}, C_{20}, C_{02}, C_{11}$ are material constants.

Strain-energy functions are a function of material parameters that can be evaluated using experimental data. They are specific to each material which exhibits hyperelastic behaviour. Many different approaches have been taken to evaluate these constants in the literature\(^\text{12}\). However, there is still no standard set of experiments to define them.

The accuracy of a model is determined by its ability to predict all types of material deformations using the same material constants. They must therefore be estimated using test data from as many deformation modes as would be experienced in a complex three-dimensional problem.

Data from uniaxial tensile tests, biaxial tensile tests and shear tests, have been shown to provide sufficient information about a material to predict any material deformation\(^\text{19}\). Indeed, any deformation is a superposition of one, two or three of these three elementary deformation modes. Therefore, in this work, the data sets used to determine material properties are those from the uniaxial, biaxial and shear tests. Data of the MED-4905 material, collected by Axel Products Inc., is plotted in Figure 1. Conventionally, the engineering stresses are plotted versus the engineering strains.

![Stress-Strain for MED-4905, 250 µm thickness](image)

Figure 1. Experimental results of Axel MED-4905, 250 µm thickness

Following characterisation of the material, it is necessary to select a specific hyperelastic model. The choice of model is guided by the experimental data, the level of accuracy required as well as the optimum curve-fitting. Equipped with a set of experimental data and a choice of hyperelastic model, the material constants can now be determined. Given a set of experimental data, closed-source curve-fitting processes evaluate the best fitted material constants for a chosen model. It is important to note that FE curve-fitting processes provide time and temperature independent hyperelastic material constants. Also, the experimental data collected are obtained from quasi-static experiments. Therefore the use of the models developed in this work is limited to quasi-static simulations. Dynamic simulations will be developed in future work.

The ANSYS software is selected in this study. The ANSYS curve-fitting process uses a least-square fit between the expected theoretical value $T_i^{th}$ and the experimental data $T_i^{exp}$ to optimise the hyperelastic material constants.

Using the set of data of Figure 1, ANSYS is used to find the material parameters for the MED-4905 elastomer. The first–order and second-order Mooney-Rivlin models are used. Results are presented in Table 1.
Experimental validation is essential for accuracy checks. Simulations of a uniaxial and biaxial test, using the material constants of Table 1, have been compared to the experimental results of these same tests, see Figure 2. The tensile and biaxial tests were respectively carried out on rectangular and circular specimens. Accordingly, the FE simulations were run on densely meshed rectangular and circular actuator models.

The proximity of the experimental and simulation results observed in Figure 2, indicates the reliability of the model. It should be pointed out that the uniaxial and biaxial tests are elementary deformation modes. Therefore, future accuracy checks will concentrate on more sophisticated experiments, involving a larger combination of deformation modes.

### 2. ELECTROMECHANICAL COUPLING

When a voltage $U$ is applied across a DEA actuator, an electric field is generated across the electrodes generating an electrostatic pressure $P_{el}$ between the electrodes. This electrostatic pressure is defined:

$$P_{el} = \varepsilon_0 \varepsilon_r \left( \frac{U}{d} \right)^2$$  \hspace{1cm} (3)

where $\varepsilon_0$ is the permittivity of free space, $\varepsilon_r$ the relative permittivity of the elastomer and $d$ the distance between the electrodes.
Due to the electrostatic pressure, electrodes are drawn nearer each other according to Equation 3. As the distance $d$ between electrodes reduces, the electrostatic pressure $P_{el}$ increases. This increase of electrostatic pressure results in a further decrease of the distance $d$. The distance $d$ between the electrodes and the electrostatic pressure are coupled; a variation of one induces a variation of the other. This coupling is known as the electromechanical coupling. Iterations are needed to reach equilibrium between displacement and electric field variations.

To our knowledge, commercial software modelling of this electromechanical coupling only exists for elastic materials, not hyperelastic materials. The steps required for this DEA model are summarised in the diagram below.

![Electro-mechanical simulation process diagram](image_url)

Figure 3. Electro-mechanical simulation process

As a first step, an FE model of the actuator is created. An electrostatic simulation of the FE actuator model is performed and the electrostatic forces created by the applied voltage are calculated. In a second step, these electrostatic forces are applied to the structural model as external forces. The displacements obtained from the structural simulation are then integrated in an updated structure and this process is repeated. Indeed, for each subsequent iteration, the new electrostatic field is calculated when the structure has encountered deformations. This iteration loop, controlled by convergence criterion 2, replicates precisely the electrostatic-structural interactions that occur in DEAs.

The difficulty in modelling DEA is that an additional iterative process within the electromechanical iteration process is required due to the hyperelastic properties of elastomers. Indeed, in addition to its non-linear material properties, hyperelastic materials also present geometric non-linearities when the material undergoes high deformations. A constant update of the element stiffness matrix is therefore required and the structural simulation requires an iterative process for global stiffness updates. This process takes place between the second and fourth step of the algorithm in Figure 3. For each structural simulation, the stiffness matrix is updated as the material deflects until convergence. Convergence criterion 1, illustrated in Figure 3, determines when convergence is reached.

### 2.1 Validating simulations

The accuracy of the sequential coupling described above is first verified by running and comparing two sets of simulations. The first is run using the in-built electroelastic option of ANSYS (“Direct” coupling). The second uses the
sequential coupling developed in this work. Accuracy is expected to be confirmed by obtaining similar results from these two sets of simulations.

A two-dimensional rectangular actuator, of length 125 µm and width 250 µm, is first simulated. The material is treated as linear with a 0.2 MPa Young modulus. Voltages applied are between 500 V and 2500 V. Convergence criterion 2 was fixed to limit the deflection to 1% of the material thickness. Results are summarised in Table 2.

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Table 2. Simulation results – Displacements vs. Voltages

Results obtained from the two simulations do not differ by more than 1%, suggesting the sequential coupling process to be accurate for electromechanical simulations of linear elastic materials. The importance of the accuracy of the electromechanical sequential coupling presented here, lies in the fact that it is applicable to hyperelastic materials, unlike commercially available FE software. It has not yet been possible to validate the electromechanical coupling with hyperelastic materials. However, an analytical model is currently under development.

To confirm the accuracy of this sequential coupling method, a three-dimensional circular DEA dimple actuator is also simulated. Dimple actuators, are circular actuators clamped along their boundary as illustrated in Figure 4. When a voltage is applied between the top and bottom electrodes, the material between them expands. The boundary conditions prevent any planar extension, so that the actuator buckles causing out-of-plane displacements, see Figure 4.

Figure 4. Dimple actuator

For the simulations, the 250 µm thick material is modelled using a 0.2 MPa Youngs modulus. The material is assumed to be incompressible, a Poisson ratio of nearly 0.5 is used for the simulations. Deflections, collected at the centre of a 15 mm radius dimple, are of the order of a few hundred microns. Simulations from both the direct electroelastic coupling and the sequential coupling, are plotted in Figure 5.
In this case as well, results agree to an error of 1%. The two-dimensional and three-dimensional simulations both confirm the accuracy of the electromechanical coupling for linear elastic materials. As discussed above, a numerical validation is not yet available for hyperelastic materials. Hence, experimental validation is required to confirm the applicability of this model to the hyperelastic case.

2.2. Validation experiments
DEA actuator experiments are carried out while simulations of identical experiments are run using the hyperelastic electromechanical coupling developed in this work. Both sets of results are then compared in order to confirm the accuracy of the modelling technique. DEA actuators are commonly used in two actuation modes, in-plane and out-of-plane. Experiments of in-plane and out-of-plane actuations are therefore being studied.

2.3.1 In-plane actuation
A planar square actuator, as illustrated in Figure 6, is tested. The actuator edges are clamped on all four corners. Square electrodes are painted on the top and bottom of the elastomer. Various actuator and electrode sizes will be tested.

As the voltage is applied, a planar extension of the electrode area will be seen. Measurements will be collected with a Digital Speckle Photogrammetry (DSP) imaging system. A full three-dimensional strain field will be collected and compared, in future work, to simulation results. The corner regions provide a discontinuity which would not exist with a circular in-plane actuator. This will validate the coupling for corner regions in three dimensional models.

2.3.2 Out-of-plane actuation
Circular dimple actuators, as illustrated in Figure 4, provide out-of-plane displacements such as the ones shown in Figure 7. Such actuators made of different materials and different sizes, have been tested; experimental results have been collected and summarised by Dearing\textsuperscript{20}. Experimental results are currently being compared to simulations.
3. MODELLING OF VISCOELASTIC AND TIME-DEPENDENT EFFECTS

The models presented in this work do not include viscoelastic effects nor permittivity changes due to material stretching. Various studies, over the last decade, have stressed the importance of such effects. The modelling process, suggested in this work, has the benefit of using a sequential solver. Additional effects can therefore be easily implemented in an effort to reach a more complete model. Time-independent effects can be directly incorporated into the quasi-static simulations. Their implementation requires running dynamic simulations. Materials used in this study are checked for the necessity of including such effects in their constitutive models.

3.1 Time-independent effects

3.1.1 Hysteresis

Hysteresis, a time-independent effect, appears here as the difference in the stress-strain relationship between loading and unloading cycles. In the present work, hysteresis can also be described as a strain-induced stress softening phenomenon. Results of cyclic tensile tests, carried out by Axel Products Inc. on the MED-4905 material used in this work, are plotted in Figure 8.

When a material is cyclically loaded and unloaded, the stress softening effect does not necessarily stabilise immediately but rather after a few cycles. When the loading cycle has stabilised, the material is considered to be preconditioned. A loading and an unloading cycle of this material, are plotted in Figure 9, once it is preconditioned.
The energy loss associated with the hysteretic loop, between the loading and unloading cycles, was calculated by determining the difference in area between the loading and unloading curves of Figure 9. An energy loss of 16.6 kJ/m$^3$ is derived, corresponding to a temperature increase of less than one degree Kelvin per cycle based on a conservative adiabatic assumption. In reality temperature dissipation modes will reduce this predicted temperature increase. Also, no significant residual strain is measured. Further experiments are needed, but this preliminary study suggests that, in order to model the 250 µm thick MED-4905 accurately, inclusion of hysteresis effects is not essential. However, any cumulative temperature increase may degrade the polymer properties, thus a detailed measurement of the temperature during cyclical testing should be investigated.

### 3.1.2 Mullin’s effect

A material presents a Mullin’s effect if its stress-strain response depends on the maximum loading it previously encountered. In such a case, the material is said to present a ‘history’ effect. Recent experiments carried out on the 100 µm thick MED-4905 material, showed a clear Mullin’s effect, see Figure 10. The graph below plots the stress-strain curve of a cyclic tensile test, successively carried out to 100%, 200% and 400%.

By collecting similar cyclic biaxial and shear tests, similar preconditioned data are obtained. Based on the appropriate preconditioned set of data, the hyperelastic material constants can be calculated for each of these preconditioned cases. In future work, cyclic tests will be carried out in a way to provide data on enough preconditioned cases, for an interpolation algorithm to be written. Such an algorithm would provide material constants for any strain case and strain history.
Modelling accurately the MED-4905 material, requires the implementation of Mullin’s effect. A procedure for updating material constants based on present and past strains, can be used at the same time as the structure is updated in the initial step of the modelling process described in Figure 3.

3.1.3 Permittivity changes
It has been suggested recently⁹, that the permittivity of an elastomer changes as it is being stretched. The sequential solver developed in this work, enables an immediate implementation of such an effect. Simulations and experiments will be carried out to validate the implementation of the time-independent properties in the model developed.

3.2 Time-dependent and rate dependent effects
Time-dependent effects have to be implemented in dynamic simulations. The quasi-static simulations run in this work cannot include those effects. Work is currently undergoing to run dynamic simulations. Creep tests are planned in future work in an effort to define the time-dependent material properties.

4. CONCLUSION
A model, currently quasi-static, has been developed and investigated in this work. Simulations have successfully validated the DEA modelling process, while a series of experimental validation is planned for further accuracy checks. Square planar actuators as well as dimple actuators of different sizes, creating respectively, in-plane and out-of-plane deflections, will be tested for various voltages. Experimental results are expected to validate the simulation results and subsequently the material modelling process.

A preliminary assessment of viscoelastic and time-independent material properties, would indicate the necessity of their implementation in the material model. Being sequential, a straightforward implementation is possible in the material modelling process. Implementing such effects together with time-dependent effects is planned in future work, in a goal to reach a fully accurate time-dependent DEA modelling process.

REFERENCES


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Figures 2.6, 2.7 and Appendix E
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