Cooperative Non-Orthogonal Multiple Access in Cognitive Radio

Lu Lv, Jian Chen, *Member, IEEE,* and Qiang Ni, *Senior Member, IEEE*

***Abstract*—This letter studies the application of non-orthogonal multiple access to a downlink cognitive radio (termed CR-NOMA) system. A new cooperative transmission scheme is proposed, aiming at exploiting the inherent spatial diversity offered by the CR-NOMA system. Closed-form analytical results are developed to show that the cooperative transmission scheme gives better performance when more secondary users participate in relaying, which helps achieve the maximum diversity order at secondary user and a diversity order of two at primary user. Simulations are performed to validate the performance of the proposed scheme and the accuracy of the analytical results.**

***Index Terms*—Non-orthogonal multiple access (NOMA), cooperative NOMA, cognitive radio, multicast.**

I. Introduction

NON-ORTHOGONAL multiple access (NOMA) is emerging as a spectrally efficient multiple access technique for the fifth generation (5G) mobile networks, where multiple users are served at the same time or frequency, but with different power levels [1]. Most recently, NOMA has been used in various systems, i.e. uplink transmissions [2], coordinated systems [3], and wireless power transfer networks [4].

On the other hand, cognitive radio (CR) has been recognized as a promising approach to improve spectral efficiency [5]. By introducing NOMA to CR systems (termed CR-NOMA), the opportunity of secondary users (SUs) to access the licensed spectrum and the system throughput can be largely increased. For example, if conventional multiple access is employed, the orthogonal spectrum allocated to primary user (PU) cannot be access by SUs, even if PU may be in poor channel condition, thus leading to a long delay for SUs to be served and low system throughput. The use of NOMA ensures that both PU and SUs can be served simultaneously, without causing too much performance degradation at PU, which effectively improves the spectrum utilization. In [6], NOMA has been applied to underlay CR networks, and its performance is evaluated by using stochastic geometry.

For multicast services in CR-NOMA systems, it still remains an open issue to use cooperative transmission to further enhance the system performance. The key idea of cooperative CR-NOMA lies in the fact that, multicast SUs can provide cooperation for the compensation of obtaining the opportunity to access PU's spectrum. In a little more detail, with the help of successive interference cancellation (SIC) at SUs, the message intended to PU can be decoded by SUs. As a result, these SUs can serve as potential relays to improve the performance of the CR-NOMA systems via diversity techniques. To the best of our knowledge, the above cooperative transmission scheme has not been investigated in the current literature on CR-NOMA, such as [6].

This work was supported in part by the EU FP7 CROWN project under Grant No. PIRSES-GA-2013-610524, by the National Natural Science Foundation of China under Grant No. 61540046, and by the "111" project of China under Grant No. B38038.

L. Lv and J. Chen are with the School of Telecommunications Engineering, Xidian University, Xi'an 710071, China (email: lulv@stu.xidian.edu.cn, jianchen@mail.xidian.edu.cn).

Q. Ni is with the School of Computing and Communications, Lancaster University, Lancaster LA1 4WA, U.K. (email: q.ni@lancaster.ac.uk).

In this letter, we consider unicast/multicast transmissions in a downlink CR-NOMA system, where two different messages are sent by a base station (BS) to a unicast PU and a group of multicast SUs. A cooperative transmission scheme is proposed for the system, where the best SU acts as a relay to forward the PU's and SUs' messages. Recall that the performance of the multicast group degrades with an increased number of SUs [7], as thus, a question naturally arises: *is it beneficial for the CR-NOMA system to increase the number of SUs?* This letter is to answer this question by evaluating the performance improvement of the CR-NOMA system when increasing the number of SUs. Closed-form expressions for the exact and asymptotic outage probabilities are developed to facilitate the performance analysis of the proposed cooperative transmission scheme. Our study demonstrates that: 1) Different from non-cooperative scheme for the CR-NOMA system, the proposed cooperative transmission scheme ensures that a diversity order of the number of SUs is achieved at the SUs, and a diversity order of two is obtained at the PU; 2) When increasing the number of multicast SUs, the CR-NOMA system can achieve superior outage performance by the proposed scheme.

II. System Model

Consider a downlink CR-NOMA system, a BS provides unicast/multicast services to a PU and a group of multicast SUs, denoted by $\mathcal{N} = \{1, 2, \cdots, N\}$. By employing the NOMA signaling, both messages of high priority for the PU and low priority for the SUs are transmitted simultaneously from the BS. We assume all nodes are equipped with a single antenna and operate in a half-duplex mode. All channels experience independent but not necessarily identically distributed (i.n.i.d.) Rayleigh block fading. The total transmit power at each node is limited by P, and the additive white Gaussian noise (AWGN) is represented by a zero-mean, complex Gaussian variable with variance N_0. The proposed cooperative transmission scheme is described as follows.

During the first time slot, the BS transmits the superimposed signals $(a_{p,1}x_p + a_{s,1}x_s)$ to the PU and the SUs with unit power, where x_p and x_s are the messages for the PU and the SUs, and $a_{p,1}$ and $a_{s,1}$ denote the power allocation coefficients at the BS. It is assumed that $E[|x_p|^2] = E[|x_s|^2] = 1$. The use of NOMA implies that $a_{p,1} > a_{s,1}$ with $a_{p,1}^2 + a_{s,1}^2 = 1$, in order to guarantee high priority of the PU's message. The observation at SU n is given by $y_n = \sqrt{P} h_{b,n}(a_{p,1}x_p + a_{s,1}x_s) + \omega_n$, where $h_{b,n}$ denotes the channel gain between the BS and SU n, and ω_n denotes the AWGN.

The conditions for SU n to successfully decode the messages x_p and x_s follow: $\frac{1}{2}\log\left(1 + \frac{a_{p,1}^2|h_{b,n}|^2}{a_{s,1}^2|h_{b,n}|^2 + \frac{1}{\rho}}\right) > R_p$, and $\frac{1}{2}\log\left(1 + \rho a_{s,1}^2 |h_{b,n}|^2\right) > R_s$, where $\rho = P/N_0$ denotes the transmit signal-to-noise ratio (SNR). Note that the SUs including in a set $\mathcal{D}$, namely *decoding set*, are able to successfully decode the messages x_p and x_s, whose target rates are R_p and R_s, respectively.

The observation at the PU during the first time slot follows: $y_{p,1} = \sqrt{P}h_{b,p}(a_{p,1}x_p + a_{s,1}x_s) + \omega_{p,1}$, and the signal-to-interference-plus-noise ratio (SINR) can be expressed as

$$SINR_{p,1} = \frac{a_{p,1}^2|h_{b,p}|^2}{a_{s,1}^2|h_{b,p}|^2 + \frac{1}{\rho}} \tag{1}$$

where $h_{b,p}$ is the channel gain between the BS and the PU, and $\omega_{p,1}$ is the AWGN.

During the second time slot, the best SU n^* is selected from $\mathcal{D}$ (if $\mathcal{D} \neq \emptyset$) to re-encode and forward $(a_{p,2}x_p + a_{s,2}x_s)$ with unit power, where $a_{p,2}$ and $a_{s,2}$ are the power allocation coefficients at SU n^*, satisfying $a_{p,2} > a_{s,2}$ with $a_{p,2}^2 + a_{s,2}^2 = 1$. In particular, the proposed optimal user selection criterion can be obtained as: $n^* = \arg\max_{n\in\mathcal{D}} \left\{ \min_{m\in\bar{\mathcal{D}}}\{|h_{n,m}|^2\}\right\}$, where $h_{n,m}$ denotes the channel gain between SU n and SU m, and $\bar{\mathcal{D}} = \mathcal{N} - \mathcal{D}$ denotes the set of all unsuccessful SUs (who fails in decoding the messages x_p and x_s). A distributed user selection is implemented, each SU maintains a timer and sets an initial value of the timer in inverse proportional to $\min_{m\in\bar{\mathcal{D}}}\{|h_{n,m}|^2\}$, resulting in the optimal SU with the smallest initial value for its timer. Finally, the optimal SU exhausts its timer earliest compared with the other SUs, and then broadcasts a control packet to notify the BS and other SUs. The unsuccessful SU m tries to decode the messages transmitted from SU n^* by using SIC, and the received SNR for its own message is expressed as $SNR_{m,c} = \rho a_{s,2}^2|h_{n^*,m}|^2$.

The PU receives the signals: $y_{p,2} = \sqrt{P}h_{n^*,p}(a_{p,2}x_p + a_{s,2}x_s) + \omega_{p,2}$, where $h_{n^*,p}$ is the channel gain between SU n^* and the PU, and $\omega_{p,2}$ is the AWGN. The received SINR during this time slot is given by

$$SINR_{p,2} = \frac{a_{p,2}^2|h_{n^*,p}|^2}{a_{s,2}^2|h_{n^*,p}|^2 + \frac{1}{\rho}}. \tag{2}$$

To reduce the complexity of the receiver structure, it is supposed that selection combining is performed. Therefore, the end-to-end SINR at the PU can be written as

$$SINR_{\text{end},2} = \max\Big\{SINR_{p,1}, SINR_{p,2} \mid n^* \in \mathcal{D}\Big\}. \tag{3}$$

On the other hand, if $\mathcal{D} = \emptyset$, the PU only receives the message from the BS, and the end-to-end SINR is given by

$$SINR_{\text{end},1} = SINR_{p,1}. \tag{4}$$

III. Performance Analysis

The performance of the considered CR-NOMA system will be characterized as following.

1) Outage Probability of the Group of Multicast SUs: The capacity of multicast transmission is dominated by the SU with the weakest channel in order to minimize outage and retransmission. Moreover, the use of NOMA poses the following two conditions [4]: $a_{p,1}^2 > a_{s,1}^2\tau_p$ and $a_{p,2}^2 > a_{s,2}^2\tau_p$, where $\tau_p = 2^{2R_p} - 1$. The outage probability of the group of multicast SUs can be formulated as

$$\begin{aligned} P_{out,s} &= P_{out,s}\big(\mathcal{D}=\emptyset\big)\Pr\big(\mathcal{D}=\emptyset\big) \\ &+ \sum_{k=1}^{N-1}\sum_{\substack{\mathcal{D}_k\subseteq\mathcal{N}\\|\mathcal{D}_k|=k}} P_{out,s}\big(\mathcal{D}=\mathcal{D}_k\big)\Pr\big(\mathcal{D}=\mathcal{D}_k\big) \end{aligned} \tag{5}$$

where $P_{out,s}(\mathcal{D}=\mathcal{D}_k)$ denotes the outage probability for the decoding set $\mathcal{D}_k$, $\Pr(\mathcal{D}=\mathcal{D}_k)$ represents the probability that the SUs belong to the decoding set, and $\mathcal{D}_k$ is a subset of $\mathcal{N}$ with cardinality $|\mathcal{D}_k| = k$.

Lemma 1: For i.n.i.d. channels, the exact outage probability of the group of multicast SUs can be expressed as

$$\begin{aligned} P_{out,s} =& \prod_{n\in\mathcal{N}}\left(1-\exp\left(-\frac{\theta}{\rho\lambda_{b,n}}\right)\right) \\ &+\sum_{k=1}^{N-1}\sum_{\substack{\mathcal{D}_k\subseteq\mathcal{N}\\|\mathcal{D}_k|=k}}\prod_{n\in\mathcal{D}_k}\left(1-\exp\left(-\sum_{m\in\bar{\mathcal{D}}_k}\frac{\phi}{\rho\lambda_{n,m}}\right)\right) \\ &\times\prod_{m\in\bar{\mathcal{D}}_k}\left(1-\exp\left(-\frac{\theta}{\rho\lambda_{b,m}}\right)\right)\exp\left(-\sum_{n\in\mathcal{D}_k}\frac{\theta}{\rho\lambda_{b,n}}\right) \end{aligned} \tag{6}$$

where $\theta = \max\{\frac{\tau_p}{a_{p,1}^2 - a_{s,1}^2\tau_p}, \frac{\tau_s}{a_{s,1}^2}\}$, $\phi = \max\{\frac{\tau_p}{a_{p,2}^2 - a_{s,2}^2\tau_p}, \frac{\tau_s}{a_{s,2}^2}\}$, $\tau_s = 2^{2R_s} - 1$, $\lambda_{b,n} = E[|h_{b,n}|^2]$, and $\lambda_{n,m} = E[|h_{n,m}|^2]$.

Proof: Please refer to Appendix A. ■

2) Outage Probability of the PU: Similar to (5), the outage probability of the PU can be obtained by

$$\begin{aligned} P_{out,p} &= P_{out,p}\big(\mathcal{D}=\emptyset\big)\Pr\big(\mathcal{D}=\emptyset\big) \\ &+ \sum_{k=1}^{N}\sum_{\substack{\mathcal{D}_k\subseteq\mathcal{N}\\|\mathcal{D}_k|=k}} P_{out,p}\big(\mathcal{D}=\mathcal{D}_k\big)\Pr\big(\mathcal{D}=\mathcal{D}_k\big). \end{aligned} \tag{7}$$

Lemma 2: For i.n.i.d. channels, the exact outage probability of the PU can be expressed as

$$\begin{aligned} P_{out,p} =& \prod_{n\in\mathcal{N}}\left(1-\exp\left(-\frac{\theta}{\rho\lambda_{b,n}}\right)\right)\left(1-\exp\left(-\frac{\varpi}{\rho\lambda_{b,p}}\right)\right) \\ &+\sum_{k=1}^{N}\sum_{\substack{\mathcal{D}_k\subseteq\mathcal{N}\\|\mathcal{D}_k|=k}}\sum_{n\in\mathcal{D}_k}\Pr(n^*=n) \\ &\times\left(1-\exp\left(-\frac{\zeta}{\rho\lambda_{n,p}}\right)\right)\left(1-\exp\left(-\frac{\varpi}{\rho\lambda_{b,p}}\right)\right) \\ &\times\prod_{m\in\bar{\mathcal{D}}_k}\left(1-\exp\left(-\frac{\theta}{\rho\lambda_{b,m}}\right)\right)\exp\left(-\sum_{n\in\mathcal{D}_k}\frac{\theta}{\rho\lambda_{b,n}}\right) \end{aligned} \tag{8}$$

where $\varpi = \frac{\tau_p}{a_{p,1}^2 - a_{s,1}^2\tau_p}$, $\zeta = \frac{\tau_p}{a_{p,2}^2 - a_{s,2}^2\tau_p}$, $\lambda_{b,p} = E[|h_{b,p}|^2]$, and $\lambda_{n,p} = E[|h_{n,p}|^2]$. In addition, $\Pr(n^* = n)$ is given by

$$\begin{aligned} \Pr(n^*=n) =& 1 + \sum_{r=1}^{k-1}\sum_{\substack{\mathcal{A}_r\subseteq\mathcal{D}_k-n\\|\mathcal{A}_r|=k-1}}(-1)^r \\ &\times\frac{\sum_{m\in\bar{\mathcal{D}}_k}\frac{1}{\lambda_{n,m}}}{\sum_{m\in\bar{\mathcal{D}}_k}\frac{1}{\lambda_{n,m}} + \sum_{i\in\mathcal{A}_i}\sum_{m'\in\bar{\mathcal{D}}_k}\frac{1}{\lambda_{r,m'}}}. \end{aligned} \tag{9}$$

Proof: Please refer to Appendix B. ■

To obtain further insights, we provide an asymptotic outage analysis as follows.

3) Asymptotic outage probability of the group of multicast SUs: As suggested by the fact that $1 - e^{-x} \simeq x$ for $x \to 0$, the outage probability in (6) can be expressed as

$$P_{out,s}^{\infty} \simeq \frac{1}{\rho^N}\sum_{k=0}^{N-1}\sum_{\substack{\mathcal{D}_k\subseteq\mathcal{N}\\|\mathcal{D}_k|=k}}\prod_{n\in\mathcal{D}_k}\Bigg(\sum_{m\in\bar{\mathcal{D}}_k}\frac{\phi}{\lambda_{n,m}}\Bigg)\prod_{m\in\bar{\mathcal{D}}_k}\frac{\theta}{\lambda_{b,m}} \propto \frac{1}{\rho^N} \tag{10}$$

for a sufficiently large SNR, i.e. $\rho \to \infty$. Known from (10), the proposed cooperative transmission scheme can achieve a full diversity order of N at the group of multicast SUs.

4) Asymptotic outage probability of the PU: By applying the same approximation rationale used in (10), the asymptotic expression of $P_{out,p}$ is given by

$$\begin{aligned} P_{out,p}^{\infty} &\simeq \frac{1}{\rho^{N+1}} \prod_{n\in\mathcal{N}} \frac{\varpi\theta}{\lambda_{b,p}\lambda_{b,n}} + \frac{1}{\rho^2} \sum_{n=1}^{N} \Pr(n^* = n) \frac{\zeta\varpi}{\lambda_{b,p}\lambda_{n,p}} \\ &\propto \frac{1}{\rho^2} + o\Big(\frac{1}{\rho^2}\Big) \end{aligned} \tag{11}$$

where $\lim_{x\to 0} o(x)/x = 0$. At high SNRs, all the factors with $\frac{1}{\rho^k}$ for $k \geq 3$ can be ignored, and the dominant factor of (11) will be the term of the order of $\frac{1}{\rho^2}$. Therefore, a diversity order of two can be achieved at the PU.

IV. Numerical Examples

Without loss of generality, the BS and the PU are located at $(0, 0)$ and $(1, 1)$, respectively, and N SUs are uniformly distributed in the first quadrant of the 1×1 rectangular region. The average channel gain is set as $\lambda_{i,j} = d_{i,j}^{-\eta}$, with $d_{i,j}$ being the normalized distance between node i and node j, and η the path loss exponent. It is assumed that $\eta = 3$ (which corresponds to an urban cellular network environment [8]). The power allocation coefficients are $a_{p,1}^2 = a_{p,2}^2 = 0.8$ and $a_{s,1}^2 = a_{s,2}^2 = 0.2$.

Fig. 1 shows the outage probability of the proposed cooperative transmission scheme for the i.n.i.d scenario, where $R_p = 1$ bps/Hz and $R_s = 1.5$ bps/Hz. It can be observed that the analytical curves perfectly agree with the simulated ones, and the asymptotes give tight bounds in the high-SNR region, which verifies the accuracy of our analytical results. For the considered CR-NOMA system, the proposed cooperative scheme ensures that the PU obtains a diversity order of two and the SUs achieve a full diversity order of N. Whereas, only a diversity order of one is obtained at both PU and SUs with the non-cooperative scheme, thus validating the availability of the proposed scheme. Furthermore, the outage probability of the PU sightly decreases when increasing the number of SU in the medium SNR region, since the probability of a non-empty decoding set becomes large with an increased N, as indicated by (14). Therefore, it is helpful to implement cooperative CR-NOMA due to its superior performance gain.

Fig. 2 plots the outage probability of the cooperative/non-cooperative transmission schemes versus the number of SUs. Note that the proposed cooperative scheme for the CR-NOMA system outperforms the non-cooperative scheme, due to the increased diversity orders for both the SUs and the PU. It can be observed that, when N increases, the outage probability of the SUs achieved by the proposed scheme significantly gets better, whereas that of the non-cooperative scheme becomes worse. Due to the fact that a diversity order of N is achieved at the SUs by the proposed scheme while the outage performance of the non-cooperative scheme is dominated by the worst link. In addition, it can be also observed that there exists the outage floor for the PU by the proposed scheme when $N \geq 3$. This is because the probability of a non-empty decoding set is approaching to one, and thus the outage probability is determined by the transmit SNR and the target data rate.

V. Conclusion

In this letter, a cooperative transmission scheme has been proposed for a downlink CR-NOMA system, aiming to exploit the maximal spatial diversity. Closed-form expressions of the exact and asymptotic outage probabilities have been derived to demonstrate the achievable performance gain of the proposed cooperative scheme over non-cooperative scheme under CR-NOMA system. Simulation results have proved the accuracy of the analytical results, and have also shown that the proposed scheme ensures a full diversity order at the SUs, and a diversity order of two at the PU, respectively.

Appendix A

When there is no SUs in the decoding set (i.e. $\mathcal{D} = \emptyset$), it is derived that $P_{out,s}(\mathcal{D} = \emptyset) = 1$. Conditioned on $a_{p,1}^2 > a_{s,1}^2\tau_p$, the probability of $\Pr(\mathcal{D} = \emptyset)$ can be calculated as

$$\begin{aligned} &\Pr\left(\mathcal{D} = \emptyset\right) \\ &= \prod_{n\in\mathcal{N}} \left(\Pr\left(\frac{a_{p,1}^2|h_{b,n}|^2}{a_{s,1}^2|h_{b,n}|^2 + \frac{1}{\rho}} < \tau_p \right) \right. \\ &\qquad \left. + \Pr\left(\frac{a_{p,1}^2|h_{b,n}|^2}{a_{s,1}^2|h_{b,n}|^2 + \frac{1}{\rho}} > \tau_p, \rho a_{s,1}^2|h_{b,n}|^2 < \tau_s \right) \right) \\ &= \prod_{n\in\mathcal{N}} \Pr\left(|h_{b,n}|^2 < \frac{\theta}{\rho} \right) = \prod_{n\in\mathcal{N}} \left(1 - \exp\left(-\frac{\theta}{\rho\lambda_{b,n}} \right) \right) \end{aligned} \tag{12}$$

where θ and λ_n are defined in Lemma 1. Furthermore, when $\mathcal{D} = \mathcal{D}_k \neq \emptyset$, $P_{out,s}(\mathcal{D} = \mathcal{D}_k)$ can be given by

$$\begin{aligned} &P_{out,s}(\mathcal{D} = \mathcal{D}_k) \\ &= \Pr\left(\min_{m\in\bar{\mathcal{D}}_k} \left\{ \frac{a_{p,2}^2|h_{n^*,m}|^2}{a_{s,2}^2|h_{n^*,m}|^2 + \frac{1}{\rho}} \right\} < \tau_p \mid n^* \in \mathcal{D}_k \right) \\ &\quad + \Pr\left(\min_{m\in\bar{\mathcal{D}}_k} \left\{ \frac{a_{p,2}^2|h_{n^*,m}|^2}{a_{s,2}^2|h_{n^*,m}|^2 + \frac{1}{\rho}} \right\} > \tau_p, \right. \\ &\qquad \left. \min_{m\in\bar{\mathcal{D}}_k} \{\rho a_{s,2}^2|h_{n^*,m}|^2\} < \tau_s \mid n^* \in \mathcal{D}_k \right) \\ &= \prod_{n\in\mathcal{D}_k} \left(1 - \Pr\left(\min_{m\in\bar{\mathcal{D}}_k} \left\{ |h_{n,m}|^2 \right\} > \frac{\phi}{\rho} \right) \right) \\ &= \prod_{n\in\mathcal{D}_k} \left(1 - \exp\left(-\sum_{m\in\bar{\mathcal{D}}_k} \frac{\phi}{\rho\lambda_{n,m}} \right) \right) \end{aligned} \tag{13}$$

conditioned on $a_{p,2}^2 > a_{s,2}^2\tau_p$. In addition, the probability of $\Pr(\mathcal{D} = \mathcal{D}_k)$ can be derived as

$$\begin{aligned} &\Pr\left(\mathcal{D} = \mathcal{D}_k\right) \\ &= \prod_{n\in\mathcal{D}_k} \Pr\left(\frac{a_{p,1}^2|h_{b,n}|^2}{a_{s,1}^2|h_{b,n}|^2 + \frac{1}{\rho}} > \tau_p, \rho a_{s,1}^2|h_{b,n}|^2 > \tau_s \right) \\ &\quad \times \prod_{m\in\bar{\mathcal{D}}_k} \left(1 - \Pr\left(\frac{a_{p,1}^2|h_{b,n}|^2}{a_{s,1}^2|h_{b,n}|^2 + \frac{1}{\rho}} > \tau_p, \rho a_{s,1}^2|h_{b,n}|^2 > \tau_s \right) \right) \\ &= \exp\left(-\sum_{n\in\mathcal{D}_k} \frac{\theta}{\rho\lambda_{b,n}} \right) \prod_{m\in\bar{\mathcal{D}}_k} \left(1 - \exp\left(-\frac{\theta}{\rho\lambda_{b,m}} \right) \right). \end{aligned} \tag{14}$$

Now, substituting (12)–(14) into (5), Lemma 1 can be proved straightforwardly.

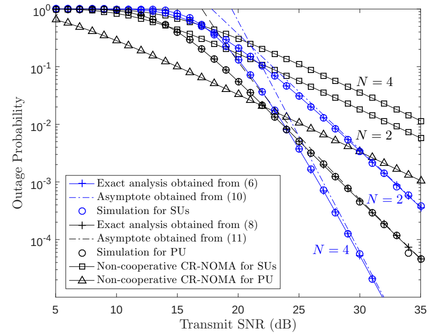

Fig. 1. Outage probability for the primary and secondary systems versus transmit SNR.

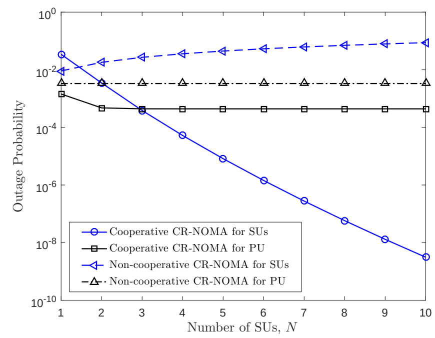

Fig. 2. Outage probability for the secondary system versus number of SUs.

Appendix B

If $\mathcal{D} = \emptyset$, the PU receives x_p only from the BS, in this case, the outage probability of $P_{out,p}(\mathcal{D} = \emptyset)$ is solved as

$$\begin{aligned} P_{out,p}(\mathcal{D} = \emptyset) &= \Pr\left(\frac{a_{p,1}^2 |h_{b,p}|^2}{a_{s,1}^2 |h_{b,p}|^2 + \frac{1}{\rho}} < \tau_p\right) \\ &= \Pr\left(|h_{b,p}|^2 < \frac{\varpi}{\rho}\right) = 1 - \exp\left(-\frac{\varpi}{\rho \lambda_{b,p}}\right). \end{aligned} \quad (15)$$

On the other hand, the outage probability of $P_{out,p}(\mathcal{D} = \mathcal{D}_k)$ with a non-empty decoding set can be rewritten as

$$\begin{aligned} &P_{out,p}(\mathcal{D} = \mathcal{D}_k) \\ &= \Pr\left(\max\left\{\frac{a_{p,1}^2 |h_{b,p}|^2}{a_{s,1}^2 |h_{b,p}|^2 + \frac{1}{\rho}}, \frac{a_{p,2}^2 |h_{n^*,p}|^2}{a_{s,2}^2 |h_{n^*,p}|^2 + \frac{1}{\rho}}\right\} < \tau_p\right) \\ &\overset{(i)}{=} \Pr\left(|h_{b,p}|^2 < \frac{\varpi}{\rho}\right) \sum_{n \in \mathcal{D}_k} \Pr(n^* = n) \Pr\left(|h_{n,p}|^2 < \frac{\zeta}{\rho}\right) \end{aligned} \quad (16)$$

where step (i) is due to the independence of the channels and the total probability theorem. Specifically, $\Pr(n^* = n)$ can be expressed as

$$\Pr(n^* = n) = \Pr\left(\bigcap_{l \neq n}^{\mathcal{D}_k}\left(\min_{m \in \bar{\mathcal{D}}_k}\left\{|h_{n,m}|^2\right\} > \min_{m' \in \bar{\mathcal{D}}_k}\left\{|h_{l,m'}|^2\right\}\right)\right). \quad (17)$$

Denote $X = \min_{m \in \bar{\mathcal{D}}_k}\left\{|h_{n,m}|^2\right\}$, whose cumulative density function and probability density function can be easily obtained as: $F(x) = 1 - \exp(-\sum_{m \in \bar{\mathcal{D}}_k} x/\lambda_{n,m})$, and $f(x) = \sum_{m \in \bar{\mathcal{D}}_k} 1/\lambda_{n,m} \exp(-\sum_{m \in \bar{\mathcal{D}}_k} x/\lambda_{n,m})$. Using the law of conditional probability, (17) can be rewritten as

$$\Pr(n^* = n) = \int_0^\infty \underbrace{\prod_{l \in \mathcal{D}_k - n} \Pr\left(x > \min_{m' \in \bar{\mathcal{D}}_k}\left\{|h_{l,m'}|^2\right\}\right)}_{\chi} f(x) dx. \quad (18)$$

In (18), χ can be calculated as

$$\chi = 1 + \sum_{r=1}^{k-1} \sum_{\substack{\mathcal{A}_r \subseteq \mathcal{D}_k - n \\ |\mathcal{A}_r| = k-1}} (-1)^r \exp\left(-\sum_{r \in \mathcal{A}_r} \sum_{m' \in \bar{\mathcal{D}}_k} \frac{x}{\lambda_{r,m'}}\right) \quad (19)$$

where the multinomial expansion identity is used. Substituting (19) into (18) and performing the required integral, a closed-form expression for $\Pr(n^* = n)$ is obtained in (9). Therefore, combining (18) with (16), we have $P_{out,p}(\mathcal{D} = \mathcal{D}_k)$.

Moreover, the probabilities of $\Pr(\mathcal{D} = \emptyset)$ and $\Pr(\mathcal{D} = \mathcal{D}_k)$ can be obtained from (12) and (14). Hence, by substituting the foregoing results into (7), one can readily prove Lemma 2.

References

[1] S. Timotheou and I. Krikidis, "Fairness for non-orthogonal multiple access in 5G systems," *IEEE Signal Process. Lett.*, vol. 22, no. 10, pp. 1647–1651, Oct. 2015.

[2] M. Al-Imari, P. Xiao, M. Imran, and R. Tafazolli, "Uplink non-orthogonal multiple access for 5G wireless networks," in *Proc. 11th Int. Symp. Wireless Commun. Syst. (ISWCS)*, Aug. 2014, pp. 781–785.

[3] J. Choi, "Non-orthogonal multiple access in downlink coordinated two-point systems," *IEEE Commun. Lett.*, vol. 18, no. 2, pp. 313–316, Feb. 2014.

[4] Y. Liu, Z. Ding, M. Elkashlan, and H. V. Poor, "Cooperative non-orthogonal multiple access with simultaneous wireless information and power transfer," *IEEE J. Sel. Areas Commun.*, vol. 34, no. 4, pp. 938–953, Apr. 2016.

[5] A. Goldsmith, S. A. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *Proc. of the IEEE*, vol. 97, no. 5, pp. 894–914, May 2009.

[6] Y. Liu, Z. Ding, M. Elkashlan, and J. Yuan, "Non-orthogonal multiple access in large-scale underlay cognitive radio networks," *IEEE Trans. Veh. Technol.*, vol. PP, no. 99, pp. 1–1, 2016.

[7] L. Yang, J. Chen, Y. Kuo, and H. Zhang, "Outage performance of DF-based cooperative multicast in spectrum-sharing cognitive relay networks," *IEEE Commun. Lett.*, vol. 18, no. 7, pp. 1250–1253, July 2014.

[8] A. Goldsmith, *Wireless Communications*, U.K.: Cambridge Univ. Press, 2005.