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Nonlinear hydro turbine model having a surge tank

Yun Zeng[a,b] , Yakun Guo[b*], Lixiang Zhang[a], Tianmao Xu[c], Hongkui Dong[d]

[a] Department of Engineering Mechanics, Kunming University of Science and Technology, Yunnan Kunming 650051, China

[b] School of Engineering, University of Aberdeen, Aberdeen AB24 3UE, UK

[c] Computing Centre, Kunming University of Science and Technology, Yunnan Kunming 650051, China

[d] Department of Hydraulic Energy Yunnan Electric Power Science Test and Research Institute, Kunming, Yunnan Kunming 650217, China

Abstract: This paper models a hydro turbine based on the dynamic description of the hydraulic system having a surge tank and elastic water hammer. The dynamic of the hydraulic system is transformed from transfer function form into the differential equation model in relative value. This model is then combined with the motion equation of the main servomotor to form the nonlinear model of the hydro turbine, in which the power of the hydro turbine is calculated using algebraic equation. A new control model is thus proposed in which the dynamic of the surge tank is taken as an additional input of control items. As such, the complex hydraulic system is decomposed into a classical one penstock and one machine model with an additional input control. Therefore, the order of the system is descended. As a result, the feasibility of the system is largely improved. The simulated results show that the additional input of the surge tank is effective and the proposed method is realizable.

Keywords: Additional input; elastic water hammer; nonlinear hydro turbine; surge tank

1. Introduction

The motion equation of the hydro turbine generator unit is built under the condition that the elastic deformation of the shafting is ignored and the hydro turbine is considered as a rigid component. Therefore, the dynamic features of the hydro turbine are determined by the dynamic of the hydraulic systems. The influence of the hydraulic system on the hydro turbine is the variation of water head and flow at the inlet of the hydro turbine. The dynamic of the hydraulic system of various types of hydro turbines is converted to the variation of water head and flow at the inlet, thus achieving the connection with the hydro turbine [1-3]. Therefore, the model of hydro turbine includes both the hydraulic dynamic and the description of torque.

For the hydraulic system having a surge tank, the dynamic equations of the surge tank are usually considered as a part of the dynamic of the system. The dynamic of the system and the surge tank then compose the model for the whole system [4-6]. This approach is also applied to simulate the case which has two surge tanks upstream and downstream of the hydro turbine [7].

[*] Corresponding author: Dr Yakun Guo, School of Engineering, University of Aberdeen, Aberdeen, AB24 3UE, UK. Email: y.guo@abdn.ac.uk

The dynamic of the penstock with elastic water hammer is calculated using the classical equations derived by Oldenburger [8]. However, such method increases the number of the differential equations, thus becomes more complex. Furthermore, new models are required for describing the different hydraulic system, which limits its application.

There are two nonlinear models for the description of the turbine torque. One is the model proposed by IEEE Working Group as well as its similar form. Such model expresses the power output using algebraic equations [9, 10]. Another approach expresses the torque as a set of differential equations derived from the IEEE Working Group model with non-elastic water hammer [11-15]. Since the hydro turbine is a rigid component, the algebraic model is equivalent to the differential equations model when the torque is in transient state. This means that the dynamic of the turbine torque depends only on the dynamic of the hydraulic system [16]. Most studies usually take the hydraulic system as rigid water hammer. One of the reasons is that the hydraulic dynamic with elastic hammer is described by transfer function [17, 18]. Consequently, it is difficult and inconvenient to apply such models in the nonlinear analysis and design method.

The objective of this study is to include the dynamic of the complex hydraulic system in the hydro turbine model, thus providing the detailed differential equations model for the design of the controller of the unit. To this end, the nonlinear differential equations model of the hydraulic system with elastic water hammer will be firstly derived. A descended order method is then proposed. Finally, the surge tank is considered as an additional input. As such, the complex hydraulic system is decomposed into a classical one penstock and one machine system with an additional input of the surge tank. The simulated results show that the proposed method is feasible and effective.

2. Hydraulic system

The complex hydraulic system shown as in Fig.1 is used for establishing the model. Fig. 1 shows that the hydraulic system has *n* conduits downstream of the surge tank. The outlet (end) of each conduit connects with *m* bifurcation penstocks. Therefore, the total number of sets is $n\times m$.

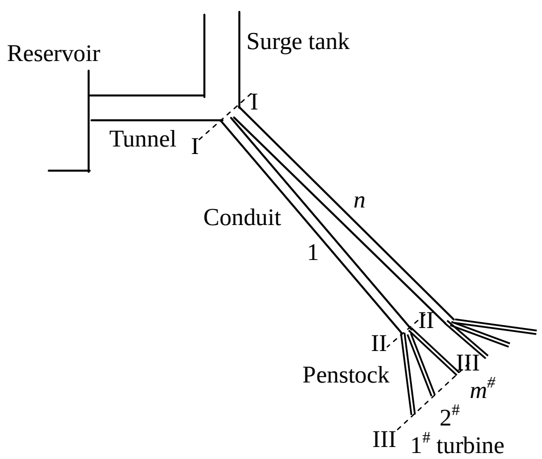

Fig.1 Sketch of the hydraulic system

The hydraulic system includes the tunnel, the surge tank, the conduit and the bifurcated penstock. Each component has its descriptive equation. The description of the dynamic of the conduit is similar to that of the bifurcated penstock except that the flow is different. For the convenience of analysis, the conduit and the bifurcated penstock are considered as an equivalent penstock that connects the surge tank and the inlet of the hydro turbine.

Define the surge impedance of the penstock in per unit as following:

$$Z_{\mathrm{n}} = \frac{\alpha}{Aa_{\mathrm{g}}}\left(\frac{Q_{\mathrm{base}}}{H_{\mathrm{base}}}\right) \tag{1}$$

where a_{g} is the acceleration due to gravity (m/s^2), α is the wave velocity (m/s), A is the area of penstock (m^2), H_{base} is the head base (m), Q_{base} is the flow base (m^3/s). Usually, the hydro turbine generator units in a hydropower plant are the same type and have the same parameters. In order to connect to generator, the rated head H_{r} is chosen as the head base, i.e. $H_{\mathrm{base}}=H_{\mathrm{r}}$, and the rated flow Q_{r} is chosen as the flow base, i.e. $Q_{\mathrm{base}}=Q_{\mathrm{r}}$.

Define the elastic time as:

$$T_{\mathrm{e}} = \frac{L}{\alpha} \tag{2}$$

where L is the length of the penstock (m).

The water start time constant is:

$$T_{\mathrm{w}} = Z_{\mathrm{n}} T_{\mathrm{e}} \tag{3}$$

According to [8], the head changes between section I and section II, section II and section III in Fig.1 can be expressed as following, respectively:

$$h_{\mathrm{II}} = h_{\mathrm{I}} \sec h(T_{\mathrm{eC}}s) - Z_{\mathrm{nC}} q_{\mathrm{II}} \tanh(T_{\mathrm{eC}}s) - f_{\mathrm{pC}} q_{\mathrm{II}}^2 \tag{4}$$

$$h_{\mathrm{III}} = h_{\mathrm{II}} \sec h(T_{ei}s) - Z_{ni} q_{\mathrm{III}} \tanh(T_{ei}s) - f_{pi} q_{\mathrm{III}}^2 \tag{5}$$

where s is the Laplace operator, $T_{\mathrm{e}i}$ and T_{eC} are the elastic time (s) of the i^{th} bifurcated penstock and the conduit respectively, $Z_{\mathrm{n}i}$ and Z_{nC} are the surge impedance of the i^{th} bifurcated penstock and the conduit in per unit, $f_{\mathrm{p}i}$ and f_{pC} are the head loss coefficients of the i^{th} bifurcated penstock and the conduit, h_{I} and h_{II} are the head of section I and II in per unit, h_{III} is the head of section III (inlet of the hydro turbine) in per unit. Thus, this head is the turbine head, i.e. $h_{\mathrm{t}i}=h_{\mathrm{III}}$. q_{II} is the flow at section II in per unit, which is the flow of the conduit; q_{III} is the flow of the i^{th} set, $q_i=q_{\mathrm{III}}$. Therefore, q_{II} and q_{III} ($=q_i$) satisfy the following relation:

$$q_{\mathrm{II}} = q_i + \sum_{k \neq i} q_k \tag{6}$$

Substituting (6) and (5) into (4) yields:

$$\begin{aligned} h_{ti} &= h_{\mathrm{I}} \sec h(T_{eC}s)\sec h(T_{ei}s) - Z_{\mathrm{nC}}(\sum_{k\neq i} q_k)\tanh(T_{eC}s)\sec h(T_{ei}s) \\ &- Z_{\mathrm{nC}}q_i \tanh(T_{eC}s)\sec h(T_{ei}s) - Z_{\mathrm{n}i}q_i \tanh(T_{ei}s) - f_{pi}q_i^2 - f_{pC}(q_i + \sum_{k\neq i} q_k)^2 \sec h(T_{ei}s) \end{aligned} \tag{7}$$

Equation (7) contains much information, but it is too complex to use. For convenience of application (7), some simplifications on equation (7) are conducted as following.

① The T_{ei} is the elastic time of the bifurcated penstock, which is small as the actual length of the penstock is relatively short. As such, we have $\sec h(T_{ei}s) = 2/(e^{T_{ei}s} + e^{-T_{ei}s}) \approx 1$. The first item on the right hand side of (7) can be expressed using the head h_s of the surge tank::

$$h_s \approx h_{\mathrm{I}} \sec h(T_{eC}s) \tag{8}$$

From the point of view of head composition, the head of section II in Fig.1 is equal to the difference of the head of surge tank and the dynamical head and friction loss head of conduit. This relation corresponds to the head relation described by equation (4) in which the head of the surge tank is $h_{\mathrm{I}} \sec h(T_{eC}s)$. Therefore, equation (8) can be obtained using approximation $\sec h(T_{ei}s) \approx 1$.

② Due to $\sec h(T_{ei}s) \approx 1$, the second item on the right hand side of (7) can be written as $Z_{\mathrm{nC}}(\sum_{k\neq i} q_k)\tanh(T_{eC}s)$, which is the head variation in the conduit induced by the flow variation of other turbine units and is called as the dynamic head or the hydraulic coupling. Assume that other turbine units are in steady operation and flows are constant, this item can then be ignored as it is zero in steady condition.

③ Both the third and fourth items on the right hand side of (7) are the head variations induced by the variation of flow q_i. Their expanded forms are complex. As $\sec h(T_{ei}s) \approx 1$ the third and fourth items on the right hand side of (7) can be simplified as $-\{Z_{\mathrm{nC}}q_i \tanh(T_{eC}s) + Z_{\mathrm{n}i}q_i \tanh(T_{ei}s)\}$. Because the general form of dynamic water head is: $h_q = Z_n q \tanh(T_e s)$, the sum of the third and fourth items on the right hand side of (7) is the sum of water heads of the conduit and the bifurcated penstock. For simplification, we use a penstock with equivalent parameters to represent both the conduit and the bifurcated penstock. According to the basic principle that whole elastic time is equal to the sum of elastic time of all subsections, and whole water starting time is equal to the sum of water starting time of all subsections, the equivalent parameters are therefore defined as following:

$$\begin{cases} T_{(i)} = T_{ei} + T_{e\mathrm{C}} \\ Z_{(i)} = \dfrac{Z_{ni}L_{0i} + Z_{nC}L_{0C}}{L_{0i} + L_{0C}} \end{cases} \tag{9}$$

where $T_{(i)}$ is the elastic time of the equivalent penstock in seconds, $Z_{(i)}$ is the surge impedance in per unit of the equivalent penstock, L_{0i} and L_{0C} are the length of the i^{th} bifurcated penstock and the conduit in m.

As such, the third and fourth items can be combined into one item as following:

$$h_{q(i)} = Z_{(i)} q_i \tanh(T_{(i)} s) \tag{10}$$

Note that equation (10) is an approximate expression and is not directly derived from (7) and (9).

④ Since $\sec h(T_{ei} s) \approx 1$, the last two items on the right hand side of (7) which represent water head loss could be expressed as following:

$$\begin{aligned} - f_{pi} q_i^2 - f_{pC} (q_i + \sum_{k \neq i} q_k)^2 &\approx -(f_{pi} + f_{pC}) q_i^2 \\ &= - f_{p(i)} q_i^2 \end{aligned} \tag{11}$$

where $f_{p(i)}=f_{pi}+f_{pC}$ is the loss coefficient of the equivalent penstock. Note that equation (11) is a simplification in which $f_{pC} \sum_{k \neq i} q_k$ (the friction loss generated in conduit by q_k($k \neq i$)) is ignored. The simplification is based on the fact: the flow rate of other penstocks remains almost the same during the adjustment of the unit. Therefore, the corresponding friction loss remains almost the same. This means that such the friction loss has little influence on the transient state of the unit. For simplification of description, we assume $f_{pC} \sum_{k \neq i} q_k =0$.

After adopting above approximation and simplification, equation (7) can be rewritten as:

$$h_{ti} = h_s - Z_{(i)} q_i \tanh(T_{(i)} s) - f_{p(i)} q_i^2 \tag{12}$$

Assume that the water level of upstream reservoir is constant, the hammer in the tunnel is rigid, and by referring to the IEEE Working Group model [4], the hydraulic dynamic of the i^{th} turbine unit at the inlet is then shown in Fig.2.

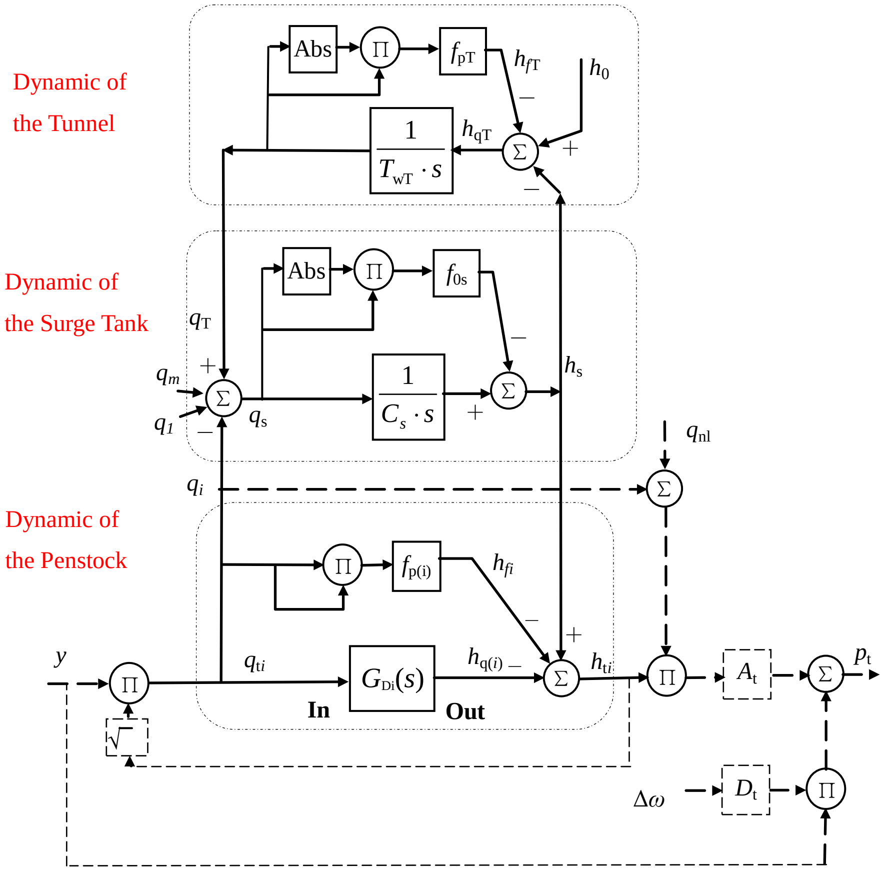

Fig.2 Hydraulic dynamic of turbine with surge tank

In Fig.2, f_{0s} is the coefficient of the head loss at the surge tank, q_T and q_s are the flow of the tunnel and the surge tank in per unit respectively, q_{ti} is the flow of the hydro turbine in per unit, $h_{q(i)}$ and h_{qT} are the dynamic heads of the equivalent penstock and the tunnel in per unit respectively, h_0 is the static head of the hydropower plant in per unit, which is defined as the difference of water level between upstream and downstream, C_s is the storage constant of the surge tank in seconds, defined as $C_s = A_s H_r / Q_r$, A_s is the section area of the surge tank (m^2), T_{wT} is the water starting time constant of the tunnel (s), and $G_{Di}(s) = Z_{(i)} \tanh(T_{(i)} s)$ is the transfer function between flow and head, y is the main servomotor displacement in per unit, p_t is the power output of the hydro turbine in per unit, A_t is the gain coefficient of the hydro turbine, q_{nl} is the no-loading flow in per unit, D_t is the mechanical damping coefficient of the hydro turbine, and $\Delta\omega$ is the difference of angular velocity and the parts circled by dotted lines are used for calculating the power of hydro turbine.

From Fig.2, the water head distribution can be constructed and shown as in Fig.3.

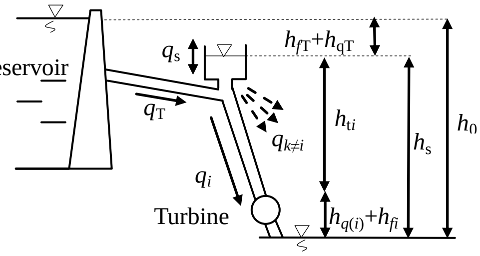

Fig.3 Schematic diagram of the water head and flow rate distributions

Fig.3 shows that the influence of complex hydraulic system has been represented using the equivalent water head at the inlet of the hydro turbine.

The following two issues need consideration when one applies above equations.

Remark 1: The flow base is the rated flow Q_r.

Equations (4) and (5) are applied to calculate the surge impedance. According to [8], the coefficient of the second item on the right hand side of (4) is rewritten as following:

$$\frac{\alpha}{Aa_g}Q_{II} = \frac{\alpha}{Aa_g}\frac{Q_{\text{base}}}{H_{\text{base}}}\frac{Q_{II}}{Q_{base}} \tag{13}$$

When Q_r is chosen as the flow base, each item on the right hand side of (6) is the relative value of flow of each unit. Similarly, Q_{base} is taken as Q_r for the calculation of Z_{ni}. Meanwhile, each q_i is the relative value with respect to the base Q_r. This is consistent with the definition of flow in traditional model.

Remark 2: The water head loss per meter in circular section pipe can be expressed as follows:

$$I = \frac{N^2}{R^{\frac{4}{3}}}V^2 = \frac{N^2}{(\frac{1}{4}D)^{\frac{4}{3}}}(\frac{4Q}{\pi D^2}\frac{Q_r}{Q_r})^2 \tag{14}$$

where I is the water head loss per meter (m/m), N is the Manning roughness coefficient, N=0.012-0.014 for steel pipe and 0.03-0.04 for tunnel, V is the flow velocity (m/s), R is the hydraulic radius (m), D is the diameter (m).

Selecting Q_r as base value, the loss coefficient used in (4) may be evaluated as (N=0.014):

$$f_p = L\frac{0.014^2}{(\frac{1}{4}D)^{\frac{4}{3}}}(\frac{4Q_r}{\pi D^2})^2\frac{1}{H_r} \tag{15}$$

where L is the length of the pipe (m). The head loss coefficient f_p is the total hydraulic losses for the whole length of pipe in relative value.

3. Dynamic equations

3.1 Dynamic equations of the penstock

The hydraulic dynamic of the i^{th} bifurcated penstock of the first conduit is expressed by equation (10). Expanding the term tanh($T_{(i)}$s) in equation (10) and ignoring higher order items yields:

$$h_{\mathrm{q}(i)}(s) = Z_{(i)} \frac{\pi^2 T_{(i)} s + T_{(i)}^3 s^3}{\pi^2 + 4T_{(i)}^2 s^2} q_i(s) \tag{16}$$

Let:

$$R(s) = \frac{1}{\pi^2 + 4T_{(i)}^2 s^2} q_i(s) \tag{17}$$

Equation (16) can then be written as:

$$h_{\mathrm{q}(i)}(s) = Z_{(i)} \pi^2 T_{(i)} R(s) s + Z_{(i)} T_{(i)}^3 R(s) s^3 \tag{18}$$

where $r(t)$ is a time-dependent variable while $R(s)$ is its Laplace transform. Both $r(t)$ and $R(s)$ are temporary variables in the Laplace transform. The initial value theorem of the Laplace transform shows $\lim_{t\to 0^+} r(t) = \lim_{s\to\infty} sR(s)$, indicating that the initial value of $r(t)$ is zero. Taking the Laplace inverse transform for equations (17) and (18) yields:

$$q_i = \pi^2 r(t) + 4T_{(i)}^2 \frac{\mathrm{d}^2 r(t)}{\mathrm{d}t^2} + q_{i0} \tag{19}$$

$$h_{\mathrm{q}(i)} = Z_{(i)} \pi^2 T_{(i)} \frac{\mathrm{d}r(t)}{\mathrm{d}t} + Z_{(i)} T_{(i)}^3 \frac{\mathrm{d}^3 r(t)}{\mathrm{d}t^3} \tag{20}$$

Denoting $x_1 = r(t)$, $x_2 = \mathrm{d}r(t)/\mathrm{d}t$, $x_3 = \mathrm{d}^2 r(t)/\mathrm{d}t^2$, (19) and (20) can then be written as:

$$\begin{cases} \dfrac{\mathrm{d}x_1}{\mathrm{d}t} = x_2 \\ \dfrac{\mathrm{d}x_2}{\mathrm{d}t} = x_3 \\ \dfrac{\mathrm{d}x_3}{\mathrm{d}t} = -\dfrac{\pi^2}{T_{(i)}^2} x_2 + \dfrac{1}{Z_{(i)} T_{(i)}^3} h_{\mathrm{q}(i)} \end{cases} \tag{21}$$

$$q_i = \pi^2 x_1 + 4T_{(i)}^2 x_3 + q_{i0} \tag{22}$$

Each variable x_i is the deviation in relative value in (21). According to the equilibrium point condition, $\mathrm{d}\bar{x}/\mathrm{d}t = 0$, $\bar{x}_{10} = 0$、$\bar{x}_{20} = 0$、$\bar{x}_{30} = 0$, namely the initial values of each variable are zero. Equation (21) remains the same form while variables are expressed using relative value.

Differentiating (22) and using (21), one obtains the dynamic equation of the penstock

$$\frac{\mathrm{d}q_i}{\mathrm{d}t} = -3\pi^2 x_2 + \frac{4}{Z_{(i)}T_{(i)}} h_{\mathrm{q}(i)} \tag{23}$$

3.2 Dynamic equation of the tunnel

All variables shown in Fig.2 are in relative value. q_{T} is the flow of the tunnel in relative value. Therefore, the dynamic equation of the tunnel can be written as:

$$\frac{\mathrm{d}q_T}{\mathrm{d}t} = \frac{1}{T_{\mathrm{wT}}}(h_0 - h_{\mathrm{s}} - f_{\mathrm{p}T} q_{\mathrm{c}}^2) \tag{24}$$

The balance equation of flow in the surge tank is:

$$q_{\mathrm{s}} = q_{\mathrm{T}} - q_{\mathrm{i}} - \sum_{k \neq i}^{nm} q_k \tag{25}$$

The third item on the right hand side of (25) is a sum of flow except that of the unit under investigation. In this study, we assume that the other units are in steady operation and their flows are constant. Therefore, the third item in (25) is also a constant. The equivalent flow in the tunnel of the i^{th} unit is:

$$q_{\mathrm{T}i} = q_{\mathrm{T}} - \sum_{k \neq i}^{nm} q_k \tag{26}$$

Rewriting (24) yields the dynamic equation of the tunnel:

$$\frac{\mathrm{d}q_{\mathrm{T}i}}{\mathrm{d}t} = \frac{1}{T_{\mathrm{wT}}}[h_0 - h_{\mathrm{s}} - f_{\mathrm{pT}}(q_{\mathrm{T}i} + \sum_{k \neq i}^{nm} q_k)^2] \tag{27}$$

3.3 Dynamic equation of the surge tank

The equation of the surge tank is:

$$h_{\mathrm{s}} = \frac{1}{C_{\mathrm{s}}}\int q_{\mathrm{s}}\mathrm{d}t - f_{0s} q_{\mathrm{s}}|q_{\mathrm{s}}| \tag{28}$$

Differentiating (28) yields:

$$\begin{aligned} \frac{\mathrm{d}h_{\mathrm{s}}}{\mathrm{d}t} &= q_{\mathrm{s}}\frac{1}{C_{\mathrm{s}}} - 2 f_{0s}|q_{\mathrm{s}}|\frac{\mathrm{d}q_{\mathrm{s}}}{\mathrm{d}t} \\ &= (q_{\mathrm{Ti}} - q_{\mathrm{i}})\frac{1}{C_{\mathrm{s}}} - 2 f_{0s}|q_{\mathrm{Ti}} - q_{\mathrm{i}}|[\frac{\mathrm{d}q_{\mathrm{Ti}}}{\mathrm{d}t} - \frac{\mathrm{d}q_{\mathrm{i}}}{\mathrm{d}t}] \end{aligned} \tag{29}$$

The second item on the right hand side of (29) can be expanded using the differential equations (23) and (24). However, this will make (29) too complex and difficult to use. From the point of view of physics, this item is the damping loss induced by the flow variation at the surge tank which is in transient state and is usually very small (see example in Section 5). For simplification, this item can be ignored and equation (29) is reduced to:

$$\frac{\mathrm{d}h_{\mathrm{s}}}{\mathrm{d}t} = (q_{\mathrm{Ti}} - q_{\mathrm{i}})\frac{1}{C_{\mathrm{s}}} \tag{30}$$

3.4 Motion equation of the main servomotor displacement

In the study of the dynamic of the hydraulic system, the controllable variable is the flow of the hydro turbine. The variation of the flow is controlled by the displacement of the main servomotor. Therefore, the main servomotor displacement should be introduced into the dynamic equations.

Taking H_{r}, Q_{r} and Y_{max} as base values, the flow of the hydro turbine can be written as following:

$$q_i = \frac{1}{y_{\mathrm{r}}} y\sqrt{h_{\mathrm{t}i}} \tag{31}$$

where y is the main servomotor displacement in per unit, y_{r} is the value of y at rated load, Y_{max} is the maximum of the main servomotor displacement.

From Fig.2, the head loss of the equivalent penstock can be written as following:

$$h_{fi} = f_{\mathrm{p(i)}} q_i^2 \tag{32}$$

From equations (31), (32), (10) and (12), one obtains the dynamic head as following:

$$h_{\mathrm{q}(i)} = h_s - (f_{\mathrm{p}(i)} + \frac{{y_r}^2}{y^2}) q_{\mathrm{i}}^2 \tag{33}$$

Substituting (33) into (21) and (23) yields the differential equation model of the hydro turbine.

The differential equation of the electro-hydraulic servo system can be written as:

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{1}{T_{\mathrm{y}}}(u - y + y_0) \tag{34}$$

Because the electro-hydraulic converter is a linear element, the input control u can be approximated as the output signal of the governor. T_{y} is the time constant of the main servomotor (in s), y_0 is an initial value of the main servomotor displacement.

3.5 Uniform form

Let x_4=q_{i}, x_5=y, x_6=q_{Ti}, x_7=h_{s}, and $\boldsymbol{x}=[x_1,x_2,x_3,x_4,x_5,x_6,x_7]^{\mathrm{T}}$, equations (21), (23), (27), (30) and (34) can then be rewritten as a uniform form:

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}u \tag{35}$$

where:

$$\boldsymbol{f}(\boldsymbol{x})=\begin{bmatrix} x_2 \\ x_3 \\ -\dfrac{\pi^2}{T_{(i)}^2}x_2+\dfrac{1}{Z_{(i)}T_{(i)}^3}[x_7-(f_{\mathrm{p}(i)}+\dfrac{y_r^{\ 2}}{x_5^2})x_4^2] \\ -3\pi^2x_2+\dfrac{4}{Z_{(i)}T_{(i)}}[x_7-(f_{\mathrm{p}(i)}+\dfrac{y_r^{\ 2}}{x_5^2})x_4^2] \\ -\dfrac{1}{T_{\mathrm{y}}}(x_5-y_0) \\ \dfrac{1}{T_{\mathrm{wT}}}[h_0-x_7-f_{\mathrm{pT}}(x_6+\sum_{k\neq i}^{nm}q_k)^2] \\ (x_6-x_4)\dfrac{1}{C_s} \end{bmatrix},\quad \boldsymbol{g}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \dfrac{1}{T_y} \\ 0 \\ 0 \end{bmatrix}$$

The above differential equation can be used to calculate the variation of the head and flow at the inlet of the hydro turbine. Based on these results, the output of the power of the hydro turbine can be calculated using the algebraic equation of IEEE Working Group model:

$$p_{\mathrm{t}}=A_{\mathrm{t}}h_{\mathrm{t}}q_i(q_i-q_{\mathrm{nl}})-D_{\mathrm{t}}\Delta\omega y \tag{36}$$

The nonlinear hydro turbine model is composed of (35) and (36), which are a differential algebraic system. This system is the nonlinear differential equations with seven orders and can be used to obtain the characteristics and details of the transient state of the system. However, they are too complex to use for the design of controller. Therefore, they need to be descended for the practical convenience of application.

Remark 3: Comparing with the IEEE Working Group model, the expression of power p_{t} in this model has the following advantages and improvements: (i) the dynamic of the hydraulic system can be directly calculated by means of the differential equations (35), and (ii) the proposed model is easier to use.

4. Descending order

Figure 2 shows that the dynamic of both the tunnel and surge tank take the flow q_{t} as input and the head h_{s} as output, therefore, from the point of view of mathematics, the dynamic of the tunnel and surge tank can be combined into a dynamic element. This means that the dynamic of the surge tank could be converted into an additional input. As such, the whole complex system is decomposed into one penstock and one machine system with additional input of the surge tank. Consequently, many studies on one penstock and one machine system can be applied to this system, thereby improving the feasibility of the system.

Based on above analysis, the main system includes the dynamic of the equivalent penstock and the motion equation of the main servomotor. In order to distinguish (35), the state variables are chosen as v_1=x_1, v_2=x_2, v_3=x_3, v_4=x_4, v_5=x_5, $\boldsymbol{v}$=$[v_1,v_2,v_3,v_4,v_5]^{\mathrm{T}}$. Selecting differential equations of x_1,x_2, x_3, x_4, x_5 from (35) and recomposing yields the following differential equations:

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \boldsymbol{F}(\boldsymbol{v}) + \boldsymbol{g}_u u + \boldsymbol{g}_s h_s \tag{37}$$

where

$$\boldsymbol{F}(\boldsymbol{v}) = \begin{bmatrix} v_2 \\ v_3 \\ -\dfrac{\pi^2}{T_{(i)}^2} v_2 - \dfrac{1}{Z_{(i)} T_{(i)}^3}(f_{\mathrm{p}(i)} + \dfrac{y_r^{\ 2}}{v_5^2}) v_4^2 \\ -3\pi^2 v_2 - \dfrac{4}{Z_{(i)} T_{(i)}}(f_{\mathrm{p}(i)} + \dfrac{y_r^{\ 2}}{v_5^2}) v_4^2 \\ -\dfrac{1}{T_{\mathrm{y}}}(v_5 - y_0) \end{bmatrix}; \boldsymbol{g}_u = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \dfrac{1}{T_y} \end{bmatrix}; \boldsymbol{g}_s = \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{Z_{(i)} T_{(i)}^3} \\ \dfrac{4}{Z_{(i)} T_{(i)}} \\ 0 \end{bmatrix}.$$

The water head h_s of the surge tank in equation (37) is an additional input control. In the hydropower plant the water level at the surge tank is usually measured. These measured data can be used to calculate the water head h_s. Therefore, taking the water head h_s as an additional input is achievable for practical application. As the dynamic of the surge tank is a servo system with fixed parameters, such treatment is feasible and simplifies the system design.

Remark 4: When the water level of the surge tank exceeds the top of the surge tank, overflow will take place. This is referred as the nonlinearity of level variation of the surge tank. Although this nonlinearity is not directly reflected in the differential model (37), the water level of the surge tank is taken as an additional input control. As such, this nonlinearity has been indirectly considered in the proposed differential model.

Remark 5: In the design stage of the governor, h_s can be calculated from the surge tank equation and is then taken as an additional input signal. In practical power plant, h_s can be measured and used as the addition input signal of the governor. When the nonlinear theory is applied to design the governor, the complexities of the governor control significantly increases with the increase of its differential equations orders. Therefore, descending the model's order (equation (37)) can greatly decrease the complexity of the system. Furthermore, it provides a reference for decoupling the complex hydraulic system, namely, various hydraulic couplings can be decoupled into additional inputs in the system. As such, the generality of the model is enhanced.
This approach is generic and can be applied to describe the other forms of hydraulic disturbance and coupling.

5. Examples

5.1 background and parameters

The LBG hydroelectric power plant with surge tank and Francis turbines in the southwest of China is chosen as the example for validating the model. The layout of the hydraulic system is shown as in Fig.1. The parameters of the plant are: the tunnel connecting the reservoir to the surge tank has a diameter of 8m and length of 9382m. The surge tank of 13m inner diameter is 65.8m high. There are two conduits with diameter 4.6m downstream of the surge tank with the length being 517m and 490m, respectively. Two penstocks of diameter of 3.2m connect each conduit at its outlet. Penstocks from 3.2m to 2.2m connect with four hydro turbines.

Choose the water wave velocity α=1100(m/s). The penstock parameters and characteristics are listed in Table 1.

Table1 Penstock parameters and characteristic values

	L_i (m)	Z_{0i}	T_{ei} (s)	f_{pi}
Tunnel	9382	0.3825	8.5291	0.0166
Conduit	517	1.1570	0.4700	0.0028
Penstock 1	50	5.0581	0.0455	0.0138
Surge Tank	65.8	0.1449	0.0598	0.00001

Other parameters are: H_r=312 m, Q_r=53.5 m^3/s, N_r=150 MW, n_r=333.3(r/min), C_s=774.06(s).

5.2 Accuracy of the model

The classic/traditional method applies the transfer function description for hydraulic system dynamic. Reference [2] simulated the response of the transfer function based model to various disturbances and compared with the measurements. The results show that the simulation of transfer function model agrees well with the measurements, indicating that the transfer function model is accurate. Therefore, this study uses the well verified transfer function model to verify the proposed method due to the lack of field data.

In the description of transfer function，the hyperbolic function can be expanded. The transfer function in Fig.2 can then be written as:

$$G_{Di}(s) = Z_{(i)}\tanh(T_{(i)}s) = Z_{(i)}\frac{sT_{(i)}\prod_{n=1}^{n=\infty}\left[1+\left(\frac{sT_{(i)}}{n\pi}\right)^2\right]}{\prod_{n=1}^{n=\infty}\left[1+\left(\frac{2sT_{(i)}}{(2n-1)\pi}\right)^2\right]} \tag{38}$$

For n=1：

$$G_{Di}(s) = Z_{(i)}\frac{sT_{(i)}(\pi^2+s^2T_{(i)}^2)}{\pi^2+4s^2T_{(i)}^2} \tag{39}$$

For n=2：

$$G_{Di}(s) = Z_{(i)}\frac{sT_{(i)}(\pi^2+s^2T_{(i)}^2)(\pi^2+\frac{1}{4}s^2T_{(i)}^2)}{(\pi^2+4s^2T_{(i)}^2)(\pi^2+\frac{4}{9}s^2T_{(i)}^2)} \tag{40}$$

In simulation, the friction loss of the surge tank is neglected. The nonlinear overflow of the surge tank is not taken into account. Four units initially operate at 100% of the rated load. The input control of No.1 unit suddenly decreases with the step input u=−0.4, while the other three units keep steady operation at the rated load.

The input control is u=−0.4 in model (37). The head h_{t1} and flow q_{t1} can be calculated directly using the differential model (37). The output power of turbine can be calculated using (36) while the damping item $D_t\Delta\omega y$ is neglected.

In traditional method, the step input control u should be transformed into the displacement of the main servomotor by means of the transfer function $y(s)=\frac{1}{T_y s+1}u(s)$, the hydraulic parameters can then be calculated using the transfer function or Simullink method. Fig. 4 shows the comparison of the responses of turbine head h_{t1} with time using the proposed differential model and the traditional transfer function method (n=1).

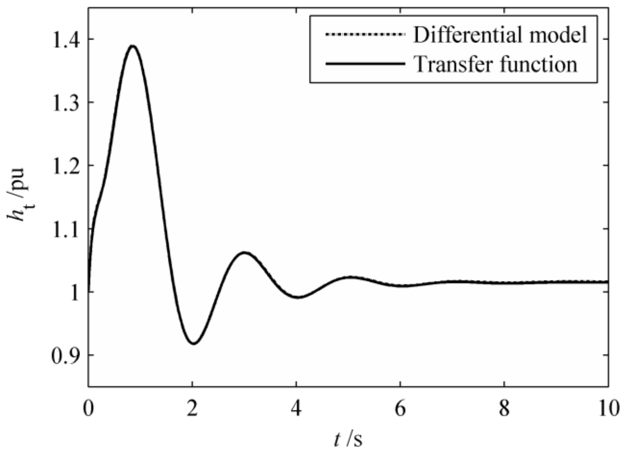

Fig. 4 Comparison of responses of the head h_{t1} using the proposed model and traditional one (n=1)

It is seen from Fig.4 that the responses from two models are almost identical. However, the calculation using the proposed model is much simpler than that using the traditional one.

Now taking n=2 in the expansion of hyperbolic function (e.g. equation (40), the comparison of the responses of head h_t and output power calculated using this model and traditional one are shown in Fig. 5 and Fig.6, respectively.

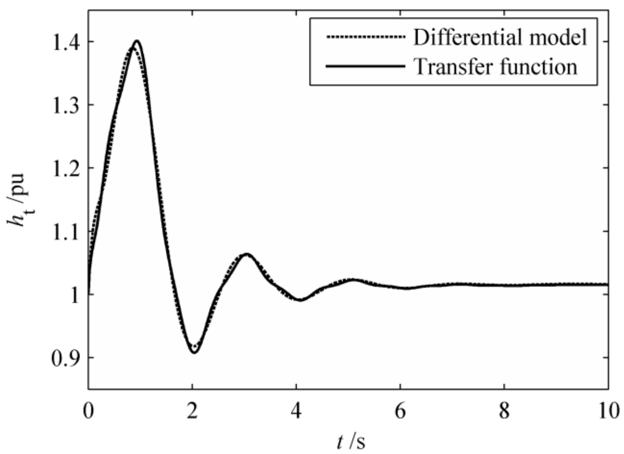

Fig. 5 Comparison of responses of the head h_{t1} using the proposed model with traditional one (n=2)

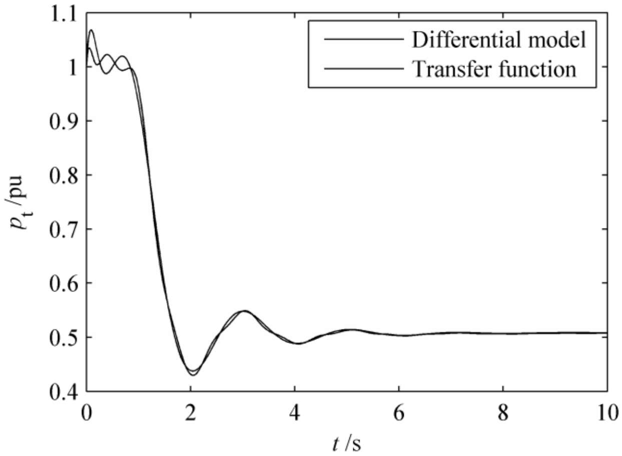

Fig. 6 Comparison of the responses of the output power p_{t1} using the proposed model with traditional one (n=2)

It is seen from Fig.5 and Fig.6 that the calculation error of the head of hydro turbine is small using higher order transfer function (complex) and the proposed differential model in this study. Though slight deviation of the calculated power of hydro turbine takes place at the beginning (the first second), the deviation becomes smaller and smaller with time. Therefore, selected hydraulic transient in simulation is appropriate and the proposed model has general application.

5.3 Feasibility of decoupling method

The principle of decoupling model is to take the water level of surge tank as an additional input control. However, the governing system of the hydro turbine is a real-time control system, if the water level of surge tank changes rapidly, then taking the water level of surge tank as an additional input may not meet the requirement of the governing speed of the turbine. In this situation, the decoupling input is impracticable. To overcome this, the variation of the water level at the surge tank is simulated.

In order to simulate the operation characteristics of the hydro turbine units under the control of the controller, we choose the simulation system which is shown in Fig.7. The dynamic of the hydraulic system might affect the power and speed, which are related with the governor. Therefore, a classical parallel PID controller of governor is used in Fig.7. The parameters of the controller are: K_P=5.0, K_D=2.5, K_I=1.5, and b_p=0.04. Meanwhile, the excitation system regulates according to its operation mode during the adjustment of the active power. In Fig.7, the operation mode of the excitation is the constant power factor mode with the power factor being 0.9. The excitation is PI controller of reactive power with K_{P1}=1, K_{I1}=1.5. Four units have the same characteristics and parameters.

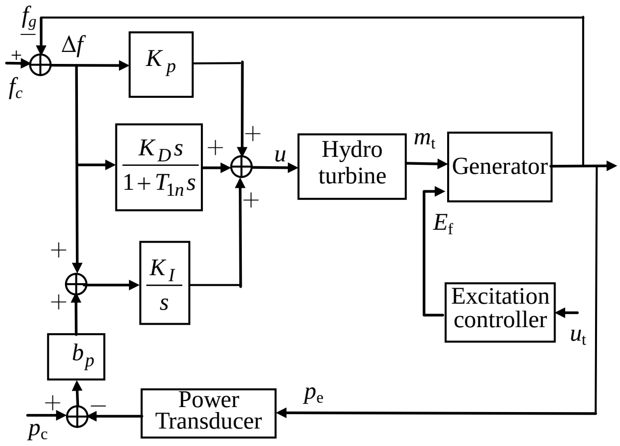

Fig. 7 Sketch of the simulated system

Simulation operation: Four units initially operate at 100% of the rated load. No.1 unit suddenly decreases load to 50% of the rated load due to some reasons while the other three units keep steady operation at the rated load.

The head responses of hydro turbine and the water level at the surge tank are shown in Fig.8.

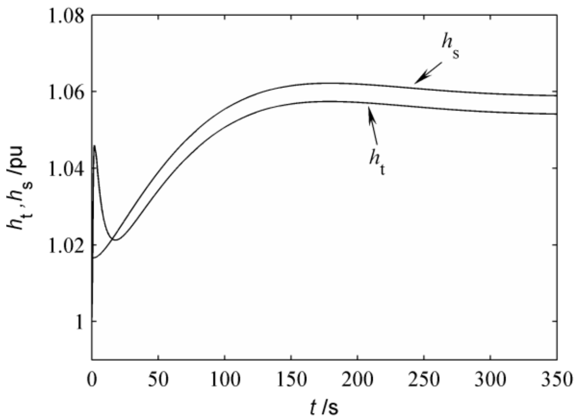

Fig.8 Responses of the head of hydro turbine and the water level at the surge tank

It is seen from Figs 8 that the flow decreases in the long tunnel and penstock, resulting in the reduction of the head loss. Therefore, the heads of both the surge tank and hydro turbine slightly increase after reaching the steady state. This is consistent with the practical situation.

Fig.8 also shows a phenomenon that the head change of the surge tank is slow in transient. This is because it has a long adjustment time required by the long diversion tunnel. This feature indicates that it is possible to use the water level of the surge tank which can be easily monitored using the sensor as an additional input. In practical hydropower plant with the surge tank, the water level measurement of the surge tank is an essential requirement. Therefore, the descent order method proposed in this paper which takes the head of surge tank as an additional disturbance input is realizable in practice.

6. Conclusions

The hydraulic dynamic, a part of hydro turbine generating units object system, need to combine with other parts of system for the study of stability and control design. Each part of the system is described as differential equation model while the nonlinear theory is applied to investigate the stability and control of hydro turbine. Therefore, hydraulic dynamic described by differential equation proposed in this study can directly combine the models of generator and controller. This overcomes the difficult of transfer function form applied to the nonlinear theory. The calculated results using the simpler differential equation model agree well with the results obtained using the complex transfer function model, indicating that the proposed method can satisfy requirements of stability analysis and control design of hydro turbine generating units.

The method and ideas proposed in this paper is effective and useful for modeling the hydro turbine with a complex hydraulic system. It is expected that the proposed approach could be extended to deal with general nonlinear hydro turbine model with various hydraulic conditions. This will require further investigation.

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