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## **Passive Investment Strategies and Financial Bubbles**

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# Passive Investment Strategies and Financial Bubbles

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## Abstract

In this paper, a model of bounded rational investors investing their portfolio in a passive investment vehicle (e.g., an Exchange Traded Fund replicating a broad index) or an actively managed fund is presented. The model proposes that the quick reswitching of these short-term oriented investors induces momentum behavior in prices. Investors prefer passive funds in time of low risk-free rates and when active funds charge high management costs. Actively managed funds have a lower volatility but are only able to outperform the passive funds in downturns. Simulations confirm the emergence of two regimes: a regime where prices are close to fundamentals and another regime with a positive bubble. The size and the length of this bubble increases for low market liquidity and high switching speed of investors. The market volatility increases for strong reswitching activities and short-term thinking of bounded rational investors. Negative bubbles (market prices lower than fundamentals) tend to occur if active portfolio managers exhibit high risk aversion, but are less frequent than positive bubbles.

*JEL classification: G11 - C15 - C62 - D58*

*Keywords: stock market - passive trading - financial stability - arbitrage trading - financial bubbles - Heterogeneous Agent Model*

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# 1 Introduction

Recent years have seen a sharp increase in passive investment vehicles. This increase was motivated by the classic portfolio theory results in the sense of Markowitz (1952) showing that by holding the market portfolio idiosyncratic risk can be hedged away and overall risk can be reduced to the market risk. Furthermore, empirical studies such as Lakonishok et al. (1992) or Malkiel (1995) show that active portfolio management underperformed a broad index such as the S&P 500. These results are also closely related to the Efficient Market Hypothesis (EMH): in its strong form, it states that the return of an actively managed fund should equal the return of a passively managed fund *not* accounting for fund management cost. This is because the hypothesis states that markets are efficient in the sense that there are no excess returns to be achieved by stock picking. The semi-strong form (which accounts for the fact of inside information) states that the return of an actively managed fund should equal the return of a passively managed fund *after* accounting for fund management costs. This at least accepts that active portfolio management does not destroy value, though it does not yield excess returns. This argument was put forward in the theoretic model of Grossman and Stiglitz (1980) showing the higher returns of informed investors are perfectly offset by the costs of acquiring the information.

Backed by economic literature, the so-called Exchange Traded Funds (ETFs) became one of the best-selling financial innovations in recent years. Initially created in Northern America and then spilling over into Europe, these products allow non-institutional investors to participate in various broad markets such as equity, fixed investment, and commodities (Kosev and Williams, 2011). Beside the diversification aspect and the low management costs, these products provide tax advantages. Compared to standard passive investment products (such as mutual funds), ETFs also exhibit high trading liquidity which goes along with low tracking error risk. ETF trading also allows for naked short-selling, stop-loss orders, and buying on margin (Deville, 2008). In fact, ETFs are more liquid than their underlying stocks (Deville, 2008).

A very important aspect of ETFs is their creation process. Classic plain vanilla ETFs similar to standard passive investment vehicles require the buying of the underlying shares for the creation of new stocks of ETFs (Ramaswamy, 2011). The current trend (especially in Europe) leads to a replacement of these classic physical ETFs by new synthetic forms. These forms do not require holding the asset replicated by the product but allow for holding an optimized basket of securities. This may generate additional income to the issuer of the ETF and requires the use of more complex financial products

such as swaps and other derivatives. Synthetic ETFs pose the advantage that they help to invest in illiquid markets (e.g., emerging markets). This higher liquidity also contributes to lower tracking error risk<sup>1</sup>. On the other hand, these benefits go along with a counterparty risk. Private households trying to invest in a broad stock index in order to hedge away idiosyncratic risk of stock picking expose their portfolio to opaque risks of complex derivatives by investing in ETFs. These subjects are extensively discussed in current literature (e.g., Ramaswamy (2011)). This should be kept in mind even though it is not the focus of the model presented in this paper.

We model the interaction of active and passive trading in a framework similar to a Heterogeneous Agent Model (HAM) now widely used in finance. These models mostly consider the interaction of trend-following chartist traders and stabilizing fundamentalist traders to explain the behavior of asset prices<sup>2</sup>. For our model, we borrow the two basic assumptions, namely that agents are heterogeneous and exhibit bounded rationality. We assume that funds manage other people's money and thereby introduce a delegation problem into the model. Non-professional investors subject to bounded rationality and short-term thinking decide whether to invest their portfolio in an active vehicle or to hold it in a passive fund. The model is able to replicate the empirical behavior found in a recent paper by Raddatz and Schmukler (2011), showing that injections into and redemptions out of passive funds are procyclical and thereby impose possibly destabilizing behavior into markets<sup>3</sup>. This is especially the case for emerging countries suffering from sudden stops of capital inflow in times of crises (Raddatz and Schmukler, 2011). The major result of the model is that the interaction of these traders in the presence of bounded rational investors can generate a stable cycle in asset prices with positive bubbles. The degree and the length of the bubble is mainly driven by the illiquidity of the market and the reswitching speed of the investors. Negative bubbles (prices below fundamentals) are less frequent and only appear for strong risk aversion of active traders. The volatility increases for short-term thinking and strong reswitching of the bounded rational investors. A financial transaction tax in the sense of Tobin (1978) on the secondary market might reduce the portfolio rebalancing activities, while the higher illiquidity

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<sup>1</sup>The mispricing of mutual funds relative to their underlying index, also known as *mutual fund puzzle*, is partly explained by the illiquidity of the asset (Shleifer, 2000).

<sup>2</sup>These models are intensely discussed in recent surveys such as Hommes and Wagener (2009), Lux (2009) and Chiarella et al. (2009).

<sup>3</sup>A similar rationale has been presented in the paper of Vayanos and Woolley (2008) explaining the behavior of over and underreaction in a framework where investors reshift their portfolio between active and passive funds.

on the primary market due to the tax eventually increases the size of the bubble.

The remainder of this paper is organized as follows: the next section presents the basic model formulation, whilst section 3 presents basic simulation results. In section 4 and 5, we present an analytical and simulation-based discussion of the attractiveness of different strategies driving the price dynamics and the bubble state and dynamics. The last section concludes and gives indication for future research.

## 2 Formulation of the two-trader model with bounded rational investors

In addition to the classic representation of a HAM, we furthermore introduce a delegation problem. Funds do not manage their own holdings but other people's money. These *other people* may be thought of as small investors that trust in the expertise of their fund managers. They have to choose between either investing in a passive fund (e.g., an ETF) or an actively managed fund. These small investors are subject to bounded rationality in the sense of Simon (1955): due to their limited resources of time and money they choose a suboptimal strategy. The portfolio weights  $W_t^i$  at time  $t$  of the strategy  $i \in \{p, a\}$  with  $p$  indicating a passive and  $a$  indicating an active strategy are calculated according to the Multinomial Logit Model as presented in Manski and McFadden (1981):

$$W_t^i = \frac{e^{\gamma A_t^i}}{\sum_{i=1}^n e^{\gamma A_t^i}} \quad (1)$$

The application of the Multinomial Logit Model as a strategy-switching model was introduced in Brock and Hommes (1997), whilst its application in the financial market context dates back to Brock and Hommes (1998). This weight depends on the attractiveness  $A_t^i$  of a strategy and the rationality  $\gamma > 0$  of the agents. In the case where  $\gamma$  equals zero, the weights of the groups are constant and amount to  $1/2$ . The other extreme case with  $\gamma$  converging to infinity represents the case in which all individuals choose the optimal forecast. De Grauwe and Grimaldi (2006) therefore interpret this parameter as a model of the behavioral effect of Status Quos Bias as presented in Kahneman et al. (1991). This effect implies that individuals find it difficult to change a decision rule they used in the past.

The attractiveness of a strategy is measured by its returns. Accordingly, the attractiveness of the passive strategy is a function of the market return, which in a first order approximation equals the difference of log-prices  $p^4$ :

$$\begin{aligned} A_{t+1}^p &= r_t + \lambda A_{t-1}^p \approx (p_t - p_{t-1}) + \lambda A_t^p = (\ln(P_t) - \ln(P_{t-1})) + \lambda A_t^p \\ &= \ln\left(\frac{P_t}{P_{t-1}}\right) + \lambda A_t^p = \ln(1+r) + \lambda A_t^p \end{aligned} \quad (2)$$

In this case, capital letters ( $P_t = \exp(p_t)$ ) represent real prices. The parameter  $0 < \lambda < 1$  can be seen as the memory of agents. High values of  $\lambda$  indicate long-term memory. More formally, the expression  $(1 - \lambda)$  can be interpreted as the time discounting  $i$  of individuals<sup>5</sup>. This implies that the value of  $\lambda$  should be close to one.

The attractiveness of the active strategy is more complicated, since fund managers actively shift between risky stocks and risk-free asset with a return of  $r_{f,t}$ . The weight of assets holdings  $w_t^q$  is given by the following equation<sup>6</sup>:

$$w_t^q = \frac{q_t^A P_t}{q_t^A \cdot P_t + C_t} \quad (3)$$

It represents the ratio of the number of assets held by active traders  $q_t^A$  in real market prices  $P_t$  and cash  $C_t$  (in the unit of money). By also considering the management costs of active portfolio management  $\kappa$  (in base points), this yields the following attractiveness:

$$A_{t+1}^A = w_{t-1}^q r_t + (1 - w_{t-1}^q) r_{f,t} - \kappa + \lambda A_t^A \quad (4)$$

The trading costs  $\kappa$  can initially be attributed to transaction costs active portfolio managers face due to active shifting between risk-free and risky asset. As we will see in the following, active portfolio managers have features of fundamental traders of classic HAMs who try to buy undervalued and sell overvalued securities. In order to do so, they have to know the fundamental value of securities. To gain this *inside* information (inside in the sense of not

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<sup>4</sup>In a first-order Taylor approximation the following relation holds:  $\ln(1+r_t) = r_t + O(r_t^2)$ .

<sup>5</sup>If we interpret the rate of return as a cash-flow  $CF$  and assume the transversality condition, the following result can be derived:  $A_t = \lambda A_{t-1} + CF_t = \lambda(\lambda A_{t-2} + CF_{t-1}) + CF_t = \sum_{n=0}^t CF_{t-n} \lambda^n$ . If this equals a standard discounted cash flow with time preference rate  $i$ , the following relation becomes apparent:  $\sum_{n=0}^t CF_{t-n} \lambda^n = \sum_{n=0}^t \frac{CF_{t-n}}{(1+i)^n} \Rightarrow \lambda = \frac{1}{1+i} \approx 1 - i$ .

<sup>6</sup>Note that capital letters  $W_t^i$  measure the portfolio weight of investors in active and passive strategy, whilst lower case letters  $w_t^i$  describe the portfolio composition of active traders consisting of risk-free cash and risky asset.

being publicly available), they hire professionals to conduct market research. These research costs to unravel the log-fundamental value  $f_t$  are also reflected in the costs  $\kappa$ . Note that, more precisely, the costs  $\kappa$  are the excess costs of active trading relative to passive trading. Passive trading also yields costs, but usually at a small neglectable order of parts per thousands annually.

The demand of the active trader is very similar to the demand of fundamental traders in classic HAMs. In fact, it models the strategy of so-called alpha-seeking of hedge funds, implying that they buy undervalued, whilst selling overvalued securities since they expect that prices converge to their log-fundamental value  $f_t$ . The difference of log-fundamental value and topic log-price  $p_t$  in a first-order approximation can therefore be thought of as an expected return. If this expected return is higher than the risk-free rate  $r_{f,t}$ , they shift their holdings to risky stocks, implying a positive demand ( $d_t^A > 0$ ):

$$d_t^A = \alpha(f_t - p_t - r_{f,t}) + a_t \quad (5)$$

The parameter  $\alpha > 0$  can be interpreted as a the inverse risk aversion of active traders. In case they are risk-averse (low values of  $\alpha$ ) they refrain from taking strong positions. Furthermore, the demand process is superimposed by a noisy process  $a_t \sim N(0, \sigma_a)$ . This captures the feature of noise trading as advocated in the seminal work by Black (1986)<sup>7</sup>. Noise plays a crucial part in financial markets. Even smart traders (in our case the active traders) have a noisy component which on mean should cancel itself out (Shleifer, 2000)<sup>8</sup>. The noise term therefore relaxes the strong assumption that active traders know the fundamental value exactly, in the form that they only know it correctly on mean and are subject to noise.

Since the demand for risky asset represents the change in stock of risky asset, equation 5 describes the flow of risky asset for active traders. The stock of risky asset held by active trades  $q_t^A$  is presented in the following equation:

$$q_t^A = q_{t-1}^A + W_t^A d_t^A + w_{t-1}^q DW_t^A \quad (6)$$

The second term in this equation shows that the demand of active trading is weighted with their market share  $W_t^A$ . This assumption commonly used in HAMs ensures that a group that does not have a market share cannot execute trading activities. Conversely, a group that has a strong market share can take strong positions in the market. The new aspect of this model is the third term: investors can shift their holding into or out of actively managed funds

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<sup>7</sup>More technically, the introduction of noise leads to endogenous price movements. Since we assume that prices are initially equal to their fundamental value, there would not be any price movements in the simulation without noise.

<sup>8</sup>This is taken into account by assuming  $E(a_t) = 0$ .

$DW_t^A$ . If there is a cash-inflow into the actively managed funds ( $DW_t^A > 0$ ), managers build up positions in risky asset without changing their portfolio composition of risky and risk-free assets.

On the other side, active funds hold cash  $C_t$ . If they buy an amount  $q_t^A$  of risky asset at current market price  $P_t$ , this reduces cash and vice versa. Moreover, the cash-inflow  $DW_t^A$  is partly held in cash in order to keep the ratio of cash and risky asset constant. These ideas can be summarized in the following equation<sup>9</sup>:

$$C_t = C_{t-1} - W_t^A d_t^A P_t + w_{t-1}^C DW_t^A P_t = C_{t-1} - W_t^A d_t^A P_t + (1 - w_{t-1}^q) DW_t^A P_t \quad (7)$$

These two equations imply that the total flow (in number of assets) in the active fund equals the flow of assets due to portfolio shifting of the investors between passive and active funds<sup>10</sup>:

$$\frac{\dot{C}_t}{P_t} + \dot{q}_t^A = DW_t^A \quad (8)$$

This also implies that only the demand of active trading  $d_t^A$  changes the composition of the portfolio of active traders<sup>11</sup>. These basic ideas of the complete model are also illustrated in figure 1.

Using the flow equations of cash and risky asset we can also implement short-sale and cash-constraints in our model. However, as we will see in the following simulation results they are not of importance.

We now want to investigate the passive traders. Since passive traders follow a *buy and hold strategy*, they do not engage in active trading. They only increase their demand for risky asset (they do not hold risk-free cash) if they have inflow into their funds. Since we only assume two investment opportunities, an inflow in passive investment equals an outflow of active funds and vice versa ( $DW_t^P = -DW_t^A$ ):

$$q_t^P = q_{t-1}^P + DW_t^P = q_{t-1}^P - DW_t^A \quad (9)$$

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<sup>9</sup>Note that this formulation neglects the effect of the risk-free return. If we take it into account, the equation should be as follows:  $C_t = (1 + r_{f,t})C_{t-1} - W_t^A d_t^A P_t + (1 - w_{t-1}^q) DW_t^A P_t$ . The risk-free rate is neglected in this case since (at least on a daily basis) it is close to zero.

<sup>10</sup>To transform the difference equation into a differential equation we take the following assumption for flow of cash (and similar for flow of assets):  $\dot{C}_t = C_t - C_{t-1}$ .

<sup>11</sup>This can be understood if we insert the flow of assets equation 6 and the flow of cash equation 7 into equation 3 representing the portfolio weights of the active traders:  $w_t^q = \frac{P_t(q_{t-1}^A + W_t^A d_t^A + w_{t-1}^q DW_t^A)}{C_{t-1} + P_t(q_{t-1}^A + DW_t^A)}$ . In absence of active demand  $d_t^A$ , the composition of the portfolio remains constant ( $w_t^q = w_{t-1}^q$ ).



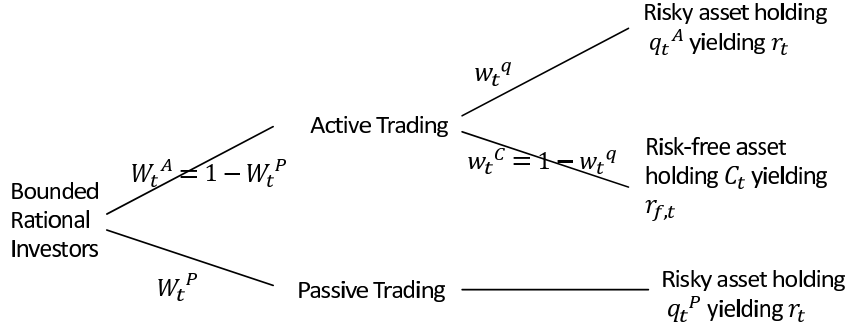


Figure 1: Structure of the model

Total demand equals the sum of flow in risky asset by active and passive traders. Excess demand feeds back into returns and price levels. We follow the conventional modeling approach in the HAM literature (cp. e.g., Chiarella et al. (2006), Westerhoff (2008)) and model market clearing with a stylized version of a market maker. The key idea is that there is an institution named market maker that takes an offsetting long or short position to assure that excess demand in period  $t$  equals zero. In the next period, the market maker announces a new log-price  $p_{t+1}$  to reduce excess demand. The parameter  $\mu$  can thereby be interpreted as market illiquidity. In illiquid times,  $\mu$  is very high, yielding strong price changes for a given excess demand:

$$\begin{aligned}
 p_{t+1} &= p_t + \mu(\dot{q}_t^A + \dot{q}_t^P) = p_t + \mu(W_t^A d_t^A + w_{t-1}^q DW_t^A - DW_t^A) \\
 &= p_t + \mu(W_t^A d_t^A - (1 - w_{t-1}^q) DW_t^A) = p_t + \mu(W_t^A d_t^A + (1 - w_{t-1}^q) DW_t^P)
 \end{aligned} \tag{10}$$

The presented equation implies that prices not only increase if active traders try to take long positions (in case of undervalued securities), but also if there is a flow of funds into passive trading. The latter effect is promoted if active funds hold large proportions in cash (which happens if they believe that

risky asset is overvalued and will fall soon). Note that the whole model is stock-flow consistent, meaning that the total holding of investors in risky and risk-free asset is constant<sup>12</sup>:

$$\begin{aligned} P_t(\dot{q}_t^A + \dot{q}_t^P) + \dot{C}_t &= P_t(W_t^A d_t^A + (1 - w_{t-1}^q) DW_t^P - W_t^A d_t^A + (1 - w_{t-1}^q) DW_t^A) \\ &= (1 - w_{t-1}^q)(DW_t^A + DW_t^P)P_t = 0 \end{aligned} \quad (11)$$

The key component of the model is the change in investors' portfolio composition  $DW_t^P$ :

$$DW_t^P = \frac{1}{\tau}(W_t^P - W_{t-1}^P) \quad (12)$$

Basically, this measures the change in amount of investment held when pursuing the passive strategy. Its weight increases if bounded rational investors attribute a high attractiveness to the passive strategy. We further introduce the parameter  $\tau > 1$  which captures the frequency with which investors reevaluate their portfolio holdings. The change of portfolio composition is therefore diluted over  $\tau$  periods of time. The lower this parameter, the stronger the investors react to changes in perceived attractiveness.

This section presented the basic model. In the following, we want to investigate the results of the model both on a simulation basis and analytically.

### 3 Basic simulation results

The model presented consists of many parameters and initial conditions - both technical and behavioral. The calibration of the latter is complicated since they cannot be observed directly in empirical data. One strong simplifying assumption of the simulation is that the log-fundamental value is constant and equals zero ( $f = 0$ ). We calibrate the model so that one discrete step size represents one trading day. In the first instance, the daily risk-free rate is assumed to be constant, implying that the risk-free asset is in infinite supply ( $r_{f,t} = r_f = 0.02\%$ ). Daily costs of active trading are slightly below the risk-free rate ( $\kappa = 0.01\%$ )<sup>13</sup>. The behavioral parameters are set according to several HAM (cp. e.g., De Grauwe and Grimaldi (2006), Westerhoff (2008)). We set the memory to  $\lambda = 0.98$ , rationality to  $\gamma = 500$ , and inverse risk aversion of active traders to  $\alpha = 0.1$ . The latter parameter can also be interpreted in the way that active traders expect the prices to converge to fundamental value within  $\frac{1}{\alpha} = 10$  trading days. The investor

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<sup>12</sup>This results from equations 7, 6, and 9.

<sup>13</sup>On an annual basis, these values represent  $r_f \approx 5\%$  and  $\kappa \approx 2.5\%$ .

valuation frequency is set to  $\tau = 5$ , implying that investors rebalance their portfolio weekly (e.g., on the weekend) and traders can thereby dilute the consequential demand over one week. Noise variance  $\sigma_a^2$  is set to 0.005 and market liquidity  $\mu$  to 1. We assume that the initial attractiveness of both strategies equals zero, resulting in the fact that both strategies have the same initial market share. Therefore, we impose that they have the same amount of asset holding, where active funds hold 60 % percent in cash and the rest in risky asset ( $w_0^q = 0.4 = 1 - w_0^C$ ). We further assume that simulation initially starts in the fundamental value ( $p = f = 0$ ).

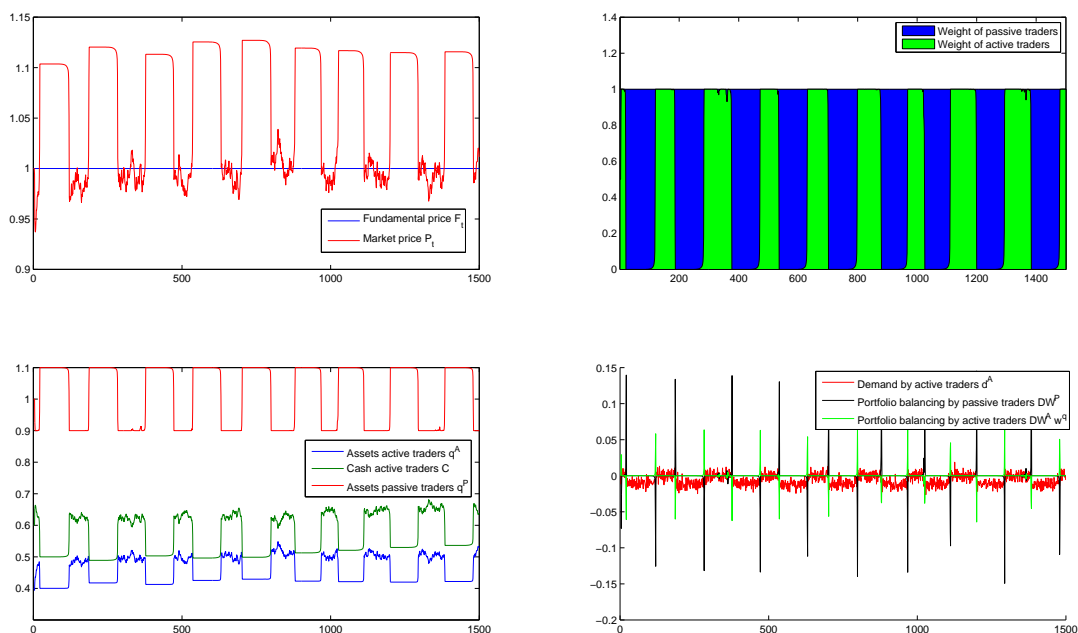


Figure 2: Panel 1: Prices  $P_t$  and fundamental value  $F_t$ ; Panel 2: Portfolio weight of active  $W_t^A$  and passive  $W_t^P$  trade strategies; Panel 3: Assets and cash held by active and passive traders (stock); Panel 4: Demand by active traders and portfolio balancing by both strategies (flow)

One exemplary simulation result is presented in figure 2. Two different regimes emerge: in one case the market consists of active traders only and prices are close to fundamentals. In the opposite case, there are only passive traders and risky asset is overvalued<sup>14</sup>. The risky asset portfolio balancing

<sup>14</sup>Note that the simulation shows the fast growing of a bubble with a slower decrease back to fundamental value, implying a positive skewness of the return distribution. This result

of active and passive traders have the opposite sign, while the balancing of passive traders has a higher amplitude. This can be explained by the fact that the latter only hold risky asset and in contrast to active traders are not invested in cash. It is also worth noticing that the amount of assets and cash also swings around two states. Both cash constraint and short-sale constraint do not become binding. A cash constraint might come into effect for the active traders when they try to take long positions. This is the case when they believe that the asset is undervalued. But as seen in the simulation in periods of low prices, there is a flow of funds into active funds. Moreover, as prices are low the existing cash can be utilized more effectively<sup>15</sup>. On the other side active traders try to go short in times of bubbles. Yet, as seen in the numerical results, in these times they have no market weight and therefore cannot engage in trading activity making the short-sale constraint not binding<sup>16</sup>.

Now we want to compare the attractiveness of both strategies (see figure 3). The passive investment strategy shows higher volatility than the active strategy. This implies that active portfolio balancing in economic downturns is more profitable than a simple *buy and hold* strategy. This can be explained by the fact that active portfolio managers also hold risk-free asset which most of the time yields lower returns than the risky asset but pay out in downturns. On the other side, the market is fairly efficient and active portfolio managers are only *sucking nickels* by trying to profit from arbitrage opportunities. Consistent with the EMH, in the long run the market cannot be outperformed. In business practice, actively managed funds *lever up* their investment return by replacing equity with debt. This important effect is not discussed here but should be considered in future research.

Until now, we considered risk-neutral investors since they only decided on portfolio composition according to returns. In the following, we want to introduce a risk-adjusted measurement to compare active and passive trading. This is also closer to the definition of the EMH in the sense of Jensen (1978), who clarifies that in an efficient market it is not possible to make *risk-adjusted* profits by arbitrage trading.

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is disconfirmed by empirical studies (and standard HAMs e.g., Lux (2009)) measuring negative skewness of return distribution and thereby represents a major shortcoming of this model. This effect can be attributed to the properties of the weighting mechanism and will be further discussed in the following sections.

<sup>15</sup>This can also be seen if we look at the cash constraint equation resulting from the flow of cash equation 7:  $d_t^A \leq \frac{1}{P_t W_t^A} (C_{t-1} + (1 - w_{t-1}^q) DW_t^A)$ .

<sup>16</sup>This also becomes clear from the inequality describing the short-sale constraint and resulting from equation 6:  $d_t^A \geq -\frac{1}{W_t^A} (q_{t-1}^A + w_{t-1}^q DW_t^A)$

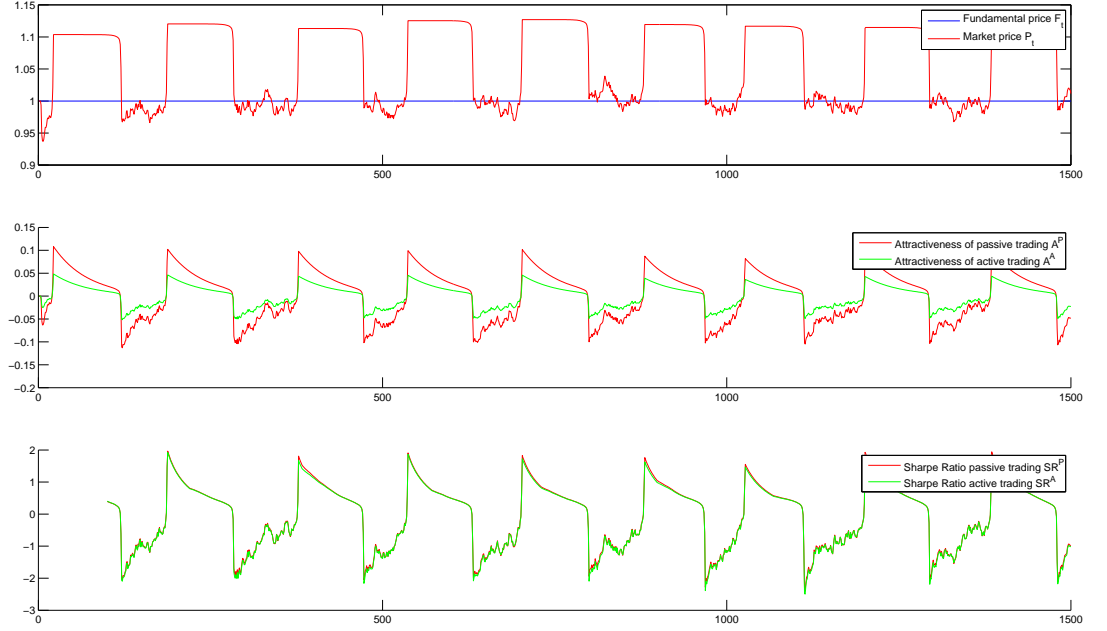


Figure 3: Panel 1: Prices  $P_t$  and fundamental value  $F_t$ ; Panel 2: Attractiveness of passive  $A_t^P$  and active trading strategy  $A_t^A$ ; Panel 3: Sharpe Ratio of passive  $SR_t^P$  and active strategy  $SR_t^A$

A combined risk-return measure frequently used in business practice is the so-called *Sharpe Ratio* ( $SR_t^i$ ), considering a ratio of excess return and volatility as measured in standard deviation of returns<sup>17</sup>:

$$SR_t^i = \frac{A_t^i - r_{f,t}}{\sigma_t(A^i)} \quad (13)$$

We plot the Sharpe Ratio for the different strategies in the third panel of figure 3. The intriguing result is that the Sharpe Ratios are nearly identical for both strategies as alluded by the EMH definition of Jensen (1978).

We can also feedback the Sharpe Ratio in the model. In order to do so, we replace the attractiveness  $A_t^i$  of a strategy with its Sharpe Ratio  $SR_t^i$  in the

<sup>17</sup>For estimating the volatility, we use the maximum likelihood estimator for constant variance (Hull, 2010)  $\sigma_t^2(A^i) = \frac{1}{n} \sum_{j=1}^n (A_{t-j}^i)^2$ . This is actually the moving average of the squared returns. Long window lengths  $n$  result in low perceived volatilities. We set  $n = 100$ . This leads to the effect that we cannot give values for the Sharpe-Ratio in the first 100 simulation periods. The measure itself varies over time.

weighting equation 1. The case with weights according to the attractiveness can therefore also be interpreted as the case with risk-neutral investors. The simulation results are depicted in figure 4<sup>18</sup>. Once again, the Sharpe Ratios of both strategies are close together. On average, the active trading is more attractive leading to the fact that prices are close to fundamentals. Nevertheless, once again periods of overpricing associated with high attractiveness and therefore also high weight of passive traders emerge.

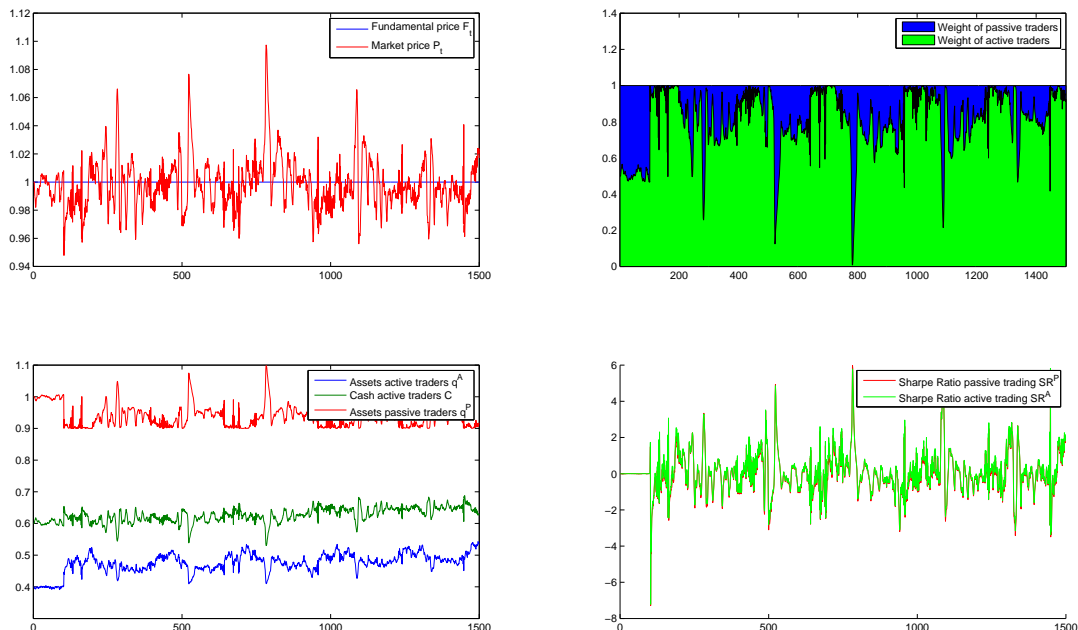


Figure 4: Panel 1: Prices  $P_t$  and fundamental value  $F_t$ ; Panel 2: Portfolio weight of active  $W_t^A$  and passive  $W_t^P$  trade strategies; Panel 3: Assets and cash held by active and passive traders (stock); Panel 4: Sharpe Ratio  $SR_t^i$  of active and passive strategy

In the next section, we want to take a closer look at what drives the attractiveness of the different strategies and thereby also governs the price dynamics.

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<sup>18</sup>This simulation assumes the same parameters as in the first case, except for  $\gamma = 25$ . The latter is made to scale down the Sharpe Ratio  $SR_t^i$  to a size comparable to the attractiveness  $A_t^i$ . Furthermore, we also hold the random seed of the noisy process constant to control for this effect.

## 4 Determinants of long-run trading success

To analyze the success of a trading strategy we want to go one step back and account for the simple attractiveness measure instead of the Sharpe Ratio. In the first instance, we want to assume extreme short-term thinking or no memory ( $\lambda = 0$ ). Starting from equation 2, this yields the following attractiveness for passive traders:

$$A_t^P = r_t = \dot{p}_t \quad (14)$$

This means that attractiveness is high in cases of high returns leading to a higher weight of passive strategy in investors' portfolios. Another intriguing result is found for perfect memory or no discounting ( $\lambda = 1$ )<sup>19</sup>:

$$\dot{A}_t^P = r_t = \dot{p}_t \Rightarrow A_t^P = p_t \quad (15)$$

This implies that the passive strategy is always more attractive if prices are at a high level. Since, in contrast to active traders, investors do not know the fundamental value, they are not aware of potential mispricings. This approach is more appropriate than the first one since the value of  $\lambda$  is close to one, reflecting that the daily discount rate  $i \approx 1 - \lambda$  is close to zero.

In the model itself rather than the absolute value  $A_t^i$  the relative value of attractiveness  $U_t = A_t^P - A_t^A$  is important. This also translates to the fact that instead of the absolute weight  $W_t^i$  the difference in weights  $m_t = W_t^P - W_t^A$  matters. We can follow the well-established approach of Brock and Hommes (1998) and replace the Logit Model with a tanh-function:

$$m_t = \frac{e^{\gamma A_t^P} - e^{\gamma A_t^A}}{e^{\gamma A_t^P} + e^{\gamma A_t^A}} = \frac{e^{\gamma(A_t^P - A_t^A)} - 1}{e^{\gamma(A_t^P - A_t^A)} + 1} = \frac{e^{\gamma U_t} - 1}{e^{\gamma U_t} + 1} = \tanh\left(\frac{\gamma}{2} U_t\right) \quad (16)$$

In this equation, positive difference in attractiveness ( $U_t = A_t^P - A_t^A > 0$ ) leads to a higher weight of passive traders ( $W_t^P > W_t^A$ ) and vice versa. This difference in attractiveness is given as follows:

$$U_t = A_t^P - A_t^A = p_t - p_{t-1} + \lambda A_{t-1}^P - [w_{t-1}^q(p_t - p_{t-1}) + (1 - w_{t-1}^q)r_f - \kappa + \lambda A_{t-1}^A] = (1 - w_{t-1}^q)(p_t - p_{t-1} - r_f) + \kappa + \lambda(A_{t-1}^P - A_{t-1}^A) \quad (17)$$

In this case, we can once again consider the extreme cases for the memory  $\lambda$ . We start by analyzing the situation with no memory ( $\lambda = 0$ ):

$$U_t = (1 - w_{t-1}^q)(r_t - r_f) + \kappa = w_{t-1}^C(r_t - r_f) + \kappa \quad (18)$$

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<sup>19</sup>This result can be derived if we consider the initial conditions of the simulation  $A_0^P = p_0 = 0$ .

In this scenario, active trading can only be more successful ( $U_t < 0$ ) if the risk-free return is higher than the return of the risky asset ( $r_f > r_t$ ). This is only the case in downturns. This effect is pronounced if active traders hold large proportions in cash  $w_{t-1}^C$  and only charge low management costs  $\kappa$ . The exact condition requires excess return of cash to risky asset weighted with its portfolio weight to be higher than the management costs:

$$(r_f - r_t)w_{t-1}^C \geq \kappa \quad (19)$$

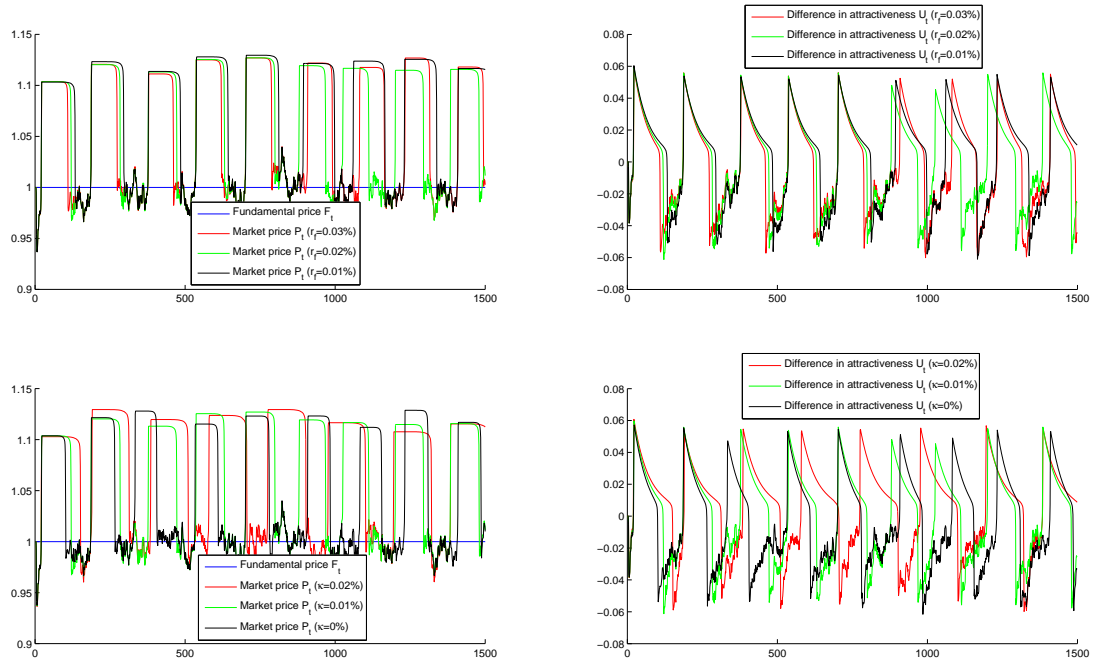


Figure 5: Panel 1: Prices  $P_t$  and fundamental value  $F_t$  with varying risk-free rate  $r_f$ ; Panel 2: Relative attractiveness  $U_t$  with varying risk-free rate  $r_f$ ; Panel 3: Prices  $P_t$  and fundamental value  $F_t$  with varying management costs  $\kappa$ ; Panel 4: Relative attractiveness  $U_t$  with varying management costs  $\kappa$

In the other extreme case, agents have perfect memory  $\lambda = 1$ . We can derive the following result<sup>20</sup>:

$$\dot{U} = w^C(\dot{p} - r_f) + \kappa \Rightarrow U = w^C(p - r_f \cdot t) + \kappa \cdot t = w^C \cdot p + (\kappa - w^C \cdot r_f)t \quad (20)$$

<sup>20</sup>Note that we skip the time indices. This is the case since we consider a differential equation instead of a difference equation. Furthermore, we assume that (as seen in the simulation) in the long-run the weight of cash is constant ( $w_{t-1}^C = w^C$ ).



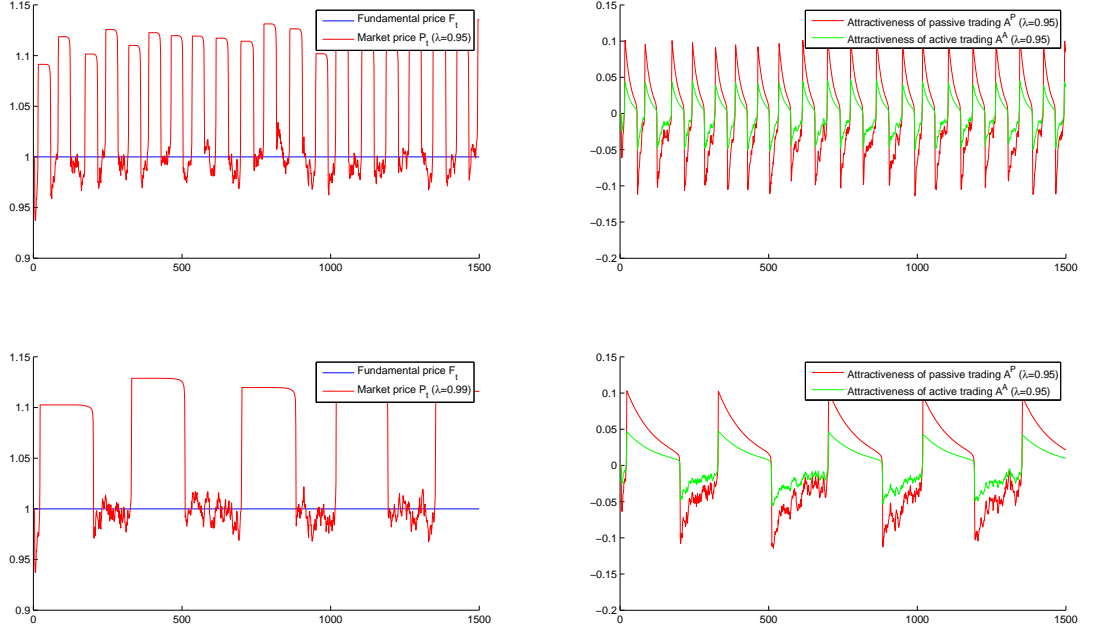


Figure 6: Panel 1: Prices  $P_t$  and fundamental value  $F_t$  with memory  $\lambda = 0.95$ ; Panel 2: Attractiveness with memory  $\lambda = 0.95$ ; Panel 3: Prices  $P_t$  and fundamental value  $F_t$  with memory  $\lambda = 0.99$ ; Panel 4: Attractiveness with memory  $\lambda = 0.99$ ;

As already explained, this second approach is closer to our simulation results. The long run attractiveness of the active trading strategy basically depends on the management costs, as well as the weight of cash and the risk-free rate. It requires the risk-free return weighted with its portfolio share to be higher than the managing costs<sup>21</sup>:

$$w^C \cdot r_f > \kappa \quad (21)$$

Moreover, given this condition, starting from equation 20 we can calculate the time it takes for active traders to take over the market:

$$t \geq \frac{w^C \cdot p}{r_f \cdot w^C - \kappa} = \frac{p}{r_f - \frac{\kappa}{w^C}} \quad (22)$$

This results confirms that in times of high management costs  $\kappa$ , low cash weights  $w^C$ , and low risk-free interest rates  $r_f$  the bubble last longer. Fur-

<sup>21</sup>Note that this condition was satisfied in our first simulation results.

thermore, a stronger bubble (as characterized by higher values of  $p$ ) also lasts longer.

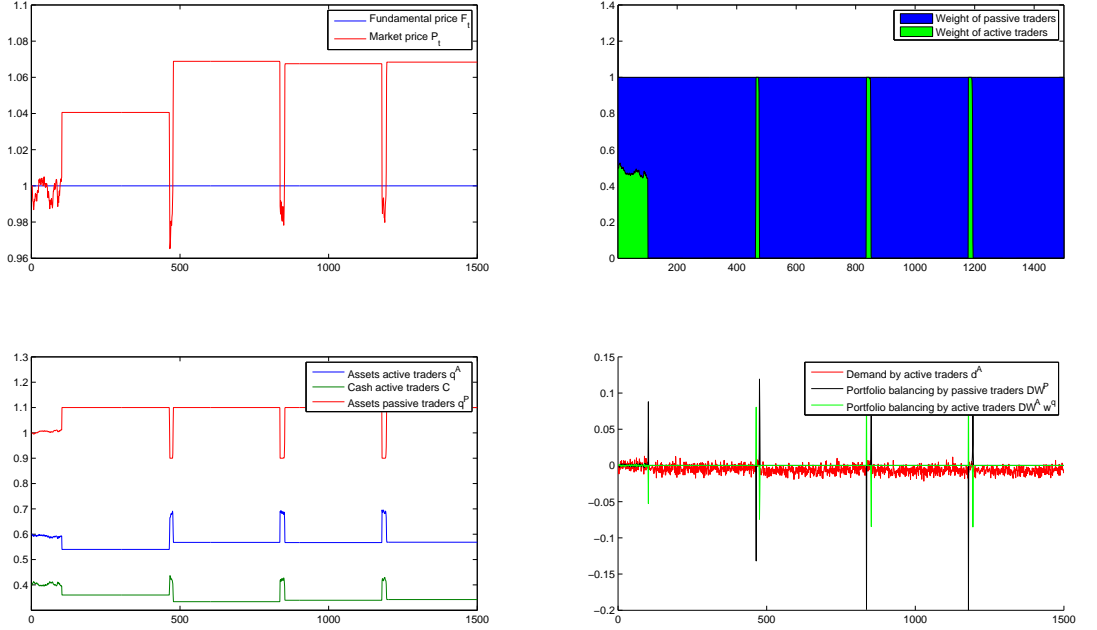


Figure 7: Panel 1: Prices  $P_t$  and fundamental value  $F_t$ ; Panel 2: Portfolio weight of active  $W_t^A$  and passive  $W_t^P$  trade strategies; Panel 3: Assets and cash held by active and passive traders (stock); Panel 4: Sharpe Ratio  $SR_t^i$  of active and passive strategy

We now want to investigate these *back of the envelope calculations* based upon numerical simulations. First, we vary the parameters management costs  $\kappa$  and risk-free rate  $r_f$  with fixed random seeds (see figure 5). As predicted by the calculations, high management costs as well as low risk-free rates *ceteris paribus* lengthen the boom periods resulting from the difference in attractiveness. Similar results emerge if we vary the memory  $\lambda$  of the agent as shown in figure 6. Long-term thinking as accounted for by high values of  $\lambda$  lengthens the boom and bust periods and thereby decreases market volatility. Technically, this can be explained the longer time it takes for the attractiveness of active and passive trading to intersect.

By assuming  $r_f = 0.01\%$  and  $\kappa = 0.02\%$  and keeping everything else equal, we violated condition 21. This means that in this configuration passive trading is more attractive than active trading. In figure 7, we take both

assumptions and apply them to the model with the Sharpe Ratio. Compared to our first simulation with Sharpe Ratio depicted in figure 4 this dramatically changes the results: the market is dominated by passive traders and most of the time at a bubbly equilibrium.

In the next section we want to determine the behavior of the other behavioral parameters and also more intensively discuss the bubble and bust dynamics.

## 5 States and dynamics of bubbles and busts

Numerical simulations confirmed the existence of two regimes. In the first regime there are only active traders ( $W^A = 1$ ). In this steady state there are no changes in the portfolio composition of investors ( $DW^P = 0$ ). The steady state can be calculated starting from the following equation:

$$\dot{p} = \mu(1 \cdot d^A + w^C \cdot 0) = \mu(\alpha(f - p - r_f) + a) \quad (23)$$

By setting noise to its expected value of zero ( $E(a) = 0$ ), the steady state ( $\dot{p} = 0$ ) can be derived<sup>22</sup>:

$$\dot{p} = 0 = \mu(\alpha(f - p - r_f)) \Rightarrow p = f - r_f \Rightarrow P \approx \frac{F}{1 + r_f} \quad (24)$$

This means prices equal fundamentals discounted with the risk-free rate. The bubble scenario is the more interesting case. Here active traders have zero market weight ( $W_A = 0$ )<sup>23</sup>. Meanwhile there is flight to the passive trading strategy  $DW_t^P$  resulting in the following price equation:

$$\dot{p} = \mu(0 \cdot d^A + w^C DW^P) = \mu \cdot w^C \frac{\dot{W}^P}{\tau} \quad (25)$$

Since in the ascent of a bubble the weight of passive traders changes from zero to one, we can insert  $\dot{W}^P = 1$ . Therefore, the bubbly equilibrium can be characterized as follows<sup>24</sup>:

$$p = \frac{\mu w^C}{\tau} \quad (26)$$

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<sup>22</sup>To transform the log-values back to real values, we assume the first-order approximation for the interest rate  $\log(1 + r_f) = r_f$ .

<sup>23</sup>The simulation results still plots their demand positions  $d_t^A$  in the market. Their fundamental trading strategy suggest them to incur short positions in the positive bubble. However, since they have no market weight at the current situation they do not engage in trading.

<sup>24</sup>This results is derived by assuming that the initial price equals the fundamental value  $p_0 = 0$ , which is the case for active traders only ( $W^A = 1$ ) and a neglectable daily risk-free rate ( $r_f \approx 0$ ).

Note that by assuming that log-fundamental value is zero ( $f = 0$ ), the log-price can be interpreted as a percentage deviation from fundamentals. This implies that this bubbly equilibrium is higher for illiquid markets (high values for  $\mu$ ) in which investors reallocate their portfolios with a high frequency (low values for  $\tau$ ). Combining the results of equation 26 and 22 we can also make a statement about the length of the bubble:

$$t \geq \frac{\mu w^C}{\tau(r_f - \frac{\kappa}{w^C})} \quad (27)$$

This condition declares that illiquid markets (high values of  $\mu$ ) with strong switching speed of bounded rational investors (low values for  $\tau$ ) are subject to longer lasting bubbles. In practice, this is especially the case for emerging markets. The result for the weight of cash is ambiguous. Strong cash holdings lead to stronger excess demand of the passive fund for the risky asset leading to a stronger bubble which is more persistent. On the other side, strong cash weights (given the condition  $w^C r_f > \kappa$ ) make the active strategy more attractive leading to a stronger reswitching. The cash weight for a minimum length bubble period is  $w_*^C = \frac{2\kappa}{r_f}$ <sup>25</sup>.

Figure 8 further investigates the variation of different behavioral parameters. In the first panel, we show prices with different degrees of rationality  $\gamma$ . As pointed out in equation 16, this parameter links the attractiveness with the market weight and thereby also with the flow between trading strategies. High rationality amplifies the flow between strategies and thereby shortens the length of the boom/fundamental periods also leading to increased financial volatility. This result contradicts the findings of Fischer (2011) investigating the effect of rationality in a fundamentalist/chartist-framework as a reaction to news fundamentals and showing that high rationality  $\gamma$  increases financial stability. In the context of Fischer (2011), higher rationality increases the speed at which prices converge to the new steady state with a different fundamental value. In our case, however, we do not have a single steady state but two. High rationality thereby increases the switching between these states. The positive skewness in return distribution can also be alluded to the switching mechanism presented as a tanh-function. The prices are mainly driven by the flow between funds. In fact, the returns (being the derivative of log-prices) are proportional to the derivative of relative weights as presented in equation 16<sup>26</sup>. The concavity of the tanh-function leads to the

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<sup>25</sup>This result can be derived with standard derivation for a local minimum  $\frac{\partial t}{\partial w^C} \stackrel{!}{=} 0$  and  $\frac{\partial^2 t}{\partial w^{C2}}|_{w_*^C} > 0$ .

<sup>26</sup>The price equation using the difference in attractiveness approach can be presented in the form of the following differential equation:  $\dot{p} = \frac{\mu}{2} [(1 - m)d^A + mw^C]$ .

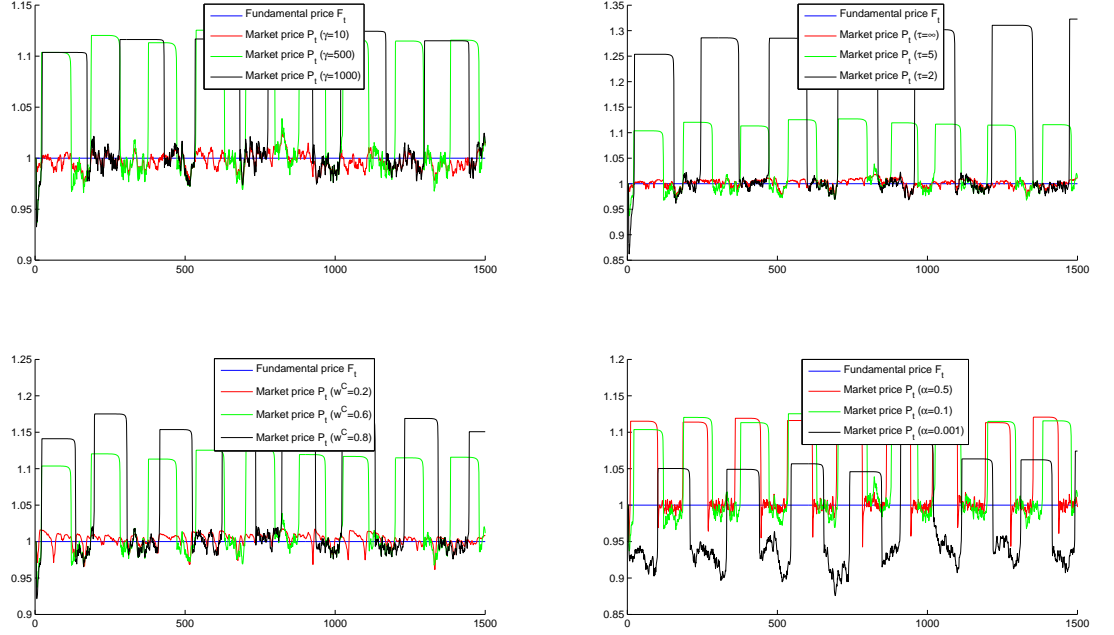


Figure 8: Model with attractiveness Panel 1: Prices  $P_t$  with variation of rationality  $\gamma$ ; Panel 2: Prices  $P_t$  with variation of reevaluation time of portfolio  $\tau$ ; Panel 3: Prices  $P_t$  with variation of memory  $\lambda$ ; Panel 4: Prices  $P_t$  with variation of active traders aggressiveness  $\alpha$

fact that investors strongly switch into the passive strategy going along with strong positive returns. The flow out of the passive strategy on the other side is rather slow resulting in low absolute values for the negative returns.

Furthermore, we vary the rebalancing time  $\tau$ . Note that the case with  $\tau = \infty$  represents the case with no rebalancing and therefore with active trading-only. In this case, price movements follow a random walk around fundamentals as induced by the noise trading of active traders. As demonstrated in the calculations, the low rebalancing time increases the size of the bubble as well as its length.

The weight of cash  $w^C$  is an important source of nonlinearities in the model. In a scenario where active traders are strongly invested in the risky asset, the two regimes do not emerge. This can be explained by the fact that in this framework the destabilizing flow into passive funds leading to the bubble can be offset by the stabilizing trading of active fundamental traders. When, however, a certain threshold is crossed, the bipolar scenario

emerges. As already presented, strong cash holdings also increase the size and the length of the bubble.

Lastly, we vary the risk aversion of active traders. High risk aversion of active traders (as implied by low values for  $\alpha$ ) can eventually also produce negative bubbles (prices lower than warranted by fundamentals). This is because active traders do not stem strongly against undervaluation of the risky asset. Moreover, high risk aversion of active traders also decreases positive bubbles. Since active traders are not very aggressive, their action does not contribute to high returns which in a second round effect does not contribute to a strong switching to the passive strategy driving the positive bubble.

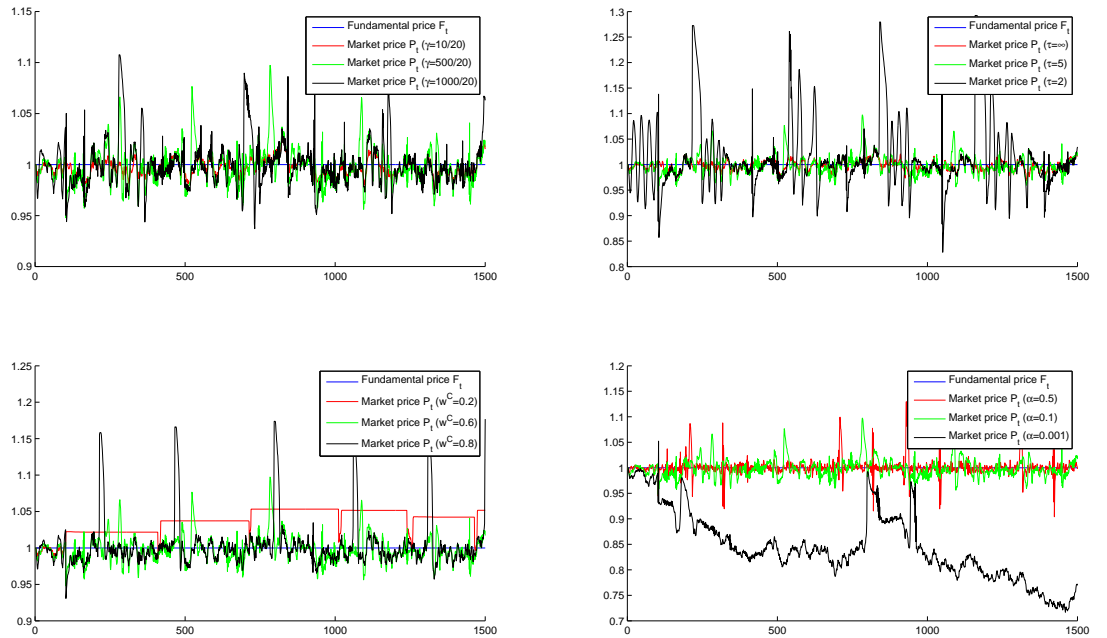


Figure 9: Model with Sharpe Ratio Panel 1: Prices  $P_t$  with variation of rationality  $\gamma$ ; Panel 2: Prices  $P_t$  with variation of reevaluation time of portfolio  $\tau$ ; Panel 3: Prices  $P_t$  with variation of memory  $\lambda$ ; Panel 4: Prices  $P_t$  with variation of active traders aggressiveness  $\alpha$

We now repeat the same simulations now with non-risk-neutral investors valuing a strategy based upon its Sharpe Ratio (see figure 9). The results of the risk-neutral case are mostly reproduced. High rationality  $\gamma$  enforces

reswitching and therefore also market volatility<sup>27</sup>. The same holds true for short rebalancing times  $\tau$ . The most interesting result is visible for variation for weight of cash in the active traders portfolio. Very low cash holdings of active traders makes their strategy unattractive (as presented in equation 21) leading to strong positions of passive traders going along with a bubble. On the other side, high cash weight leads to strong bubbles as presented in equation 26. High aggressiveness of active traders  $\alpha$  (low risk aversion) increases financial volatility, while low aggressiveness leads to negative bubbles. Therefore both, too little and too much risk aversion of arbitrage traders, are not beneficial for financial markets.

The model is still highly stylized. Therefore, any policy conclusions have to be taken with a pinch of salt. The model suggests that, even without the presence of chartist traders, market prices can be above fundamentals. This is especially the case if bounded rational investors can quickly switch their portfolios between active and passive holdings. Plain vanilla ETFs that actually hold their underlying assets and can be exchanged quickly (low  $\tau$  in the model sense) introduce a positive feedback behavior in the markets<sup>28</sup>. A financial transaction tax in the sense of Tobin (1978) on private speculation might reduce the problem of quick switching of assets (low values of  $\tau$ ) and therefore lower the dimension of the bubble. On the other hand, a Tobin tax in the primary market will lower its liquidity (high values for  $\mu$ ) and thereby increase the bubbly outcome.

## 6 Summary and outlook

The main result of the model presented is that market prices can be above fundamental value even though the market only consists of stabilizing fundamental traders and passive *buy and hold* traders. The rationale is that bounded rational investors switch to a passive investment strategy because of recent price increases. This effect is emphasized if we assume that agents frequently revalue the portfolio and reshift strong proportions of their asset holdings. Investment companies have to create shares of passively invested funds (e.g., ETFs) by buying the underlying shares. In contrast to the passive funds that only holds risky asset, actively managed funds have an optimized portfolio of risk-free and risky assets. Therefore, the flow of investor money from actively managed to passive funds creates excess demand for the risky

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<sup>27</sup>Once again, to keep the results of the model with attractiveness and Sharpe Ratio comparable, we scaled down the rationality parameter with the factor 20.

<sup>28</sup>Note that, as already argued in the first section, synthetic ETFs might, on the other hand, impose severe risks on the financial system by exposing holders to counterparty risk.

asset leading to higher stock prices. We showed that active funds can only be successful if they charge low management costs. Moreover, they have a stronger market weight in downturns since they are also invested in a risk-free asset. The size of the bubble mainly depends on the illiquidity of the market and the rebalancing time of the bounded rational investors. A Tobin tax might be beneficial by reducing the trading activity on the secondary market, but also leads to higher illiquidity on the primary which in turn increases the size of the positive bubble. Negative bubbles only seem to occur in the case that active traders are very risk-averse.

This model is still highly stylized. A consequential extension would be the introduction of a destabilizing trend-following strategy. If, on the other hand, we explicitly model the market for risk-free asset a stabilizing effect is expected. The rationale for this is that if active traders believe that the risky asset is overvalued, they sell them and start holding risk-free asset. The increased demand for risk-free asset inflates its price and thereby reduces the risk-free returns. This again leads to a reshift into the risky asset by active traders and thereby has a stabilizing effect. A strong price reaction by the risk-free market to excess demand thereby acts similarly to a high risk aversion of active traders by reducing the reshifting between risky and risk-free asset. These further extensions will also provide a more rigorous framework for evaluating policy decisions.



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