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Modeling and Control of Formations of Nonholonomic Mobile Robots

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Abstract

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Modeling and Control of Formations of Nonholonomic Mobile Robots

Jaydev P. Desai, James P. Ostrowski, and Vijay Kumar

Abstract—This paper addresses the control of a team of nonholonomic mobile robots navigating in a terrain with obstacles while maintaining a desired formation and changing formations when required, using graph theory. We model the team as a triple, $(g, \ r, \ \mathcal{H})$, consisting of a group element g that describes the gross position of the lead robot, a set of shape variables r that describe the relative positions of robots, and a control graph $\mathcal{H}$ that describes the behaviors of the robots in the formation. Our framework enables the representation and enumeration of possible control graphs and the coordination of transitions between any two formations.

Index Terms—Formation control of mobile robots, graph theory, nonlinear control.

Fig. 1. Formation of robots changing shapes.

Fig. 2. Notation for $l - \psi$ and $l - l$ control.

I. INTRODUCTION

In this paper, we discuss the fundamental issues underlying the control and coordination of multiple autonomous robots. We formulate the problem of modeling a formation of nonholonomic mobile robots and develop a framework for transitioning from one formation to another. We focus on tasks in which the robots are required to follow a trajectory while maintaining a desired formation and avoiding obstacles. In a situation such as the one shown in Fig. 1, for example, it may be necessary to change the formation in order to negotiate the obstacle, and then reform the original formation.

While there are many approaches to solving such a problem [1], we are interested in a method that scales with the number of robots and obstacles in the environment. For this reason, we pursue in the current work easily computable, decentralized feedback laws that can be used in conjunction with a higher level (but lower complexity) motion planner.

We model a team of robots in formation as a triple $(g, \ r, \ \mathcal{H})$, where $g \in SE(N)$ represents the *gross* position and orientation of the lead robot in N dimensions (N equals two or three), r is a set of *shape* variables that describe the relative positions of the robots in the team, and $\mathcal{H}$ is a *control graph* which describes the control strategy (or behavior) used by each robot and the dependence of its trajectory on that of one or more of its neighbors. When viewed in this framework, the problem of locomotion can be broken down into three subproblems: 1) trajectory planning (g); 2) robot control; and 3) formation control $(r, \ \mathcal{H})$.

Most previous work in motion planning has focused on obtaining the path and, in some cases, designing feedback architecture and model-independent coordination strategy. For example, see [2]–[4]. When the actuator inputs for each robot are concurrently planned [5], the computational complexity of the planning t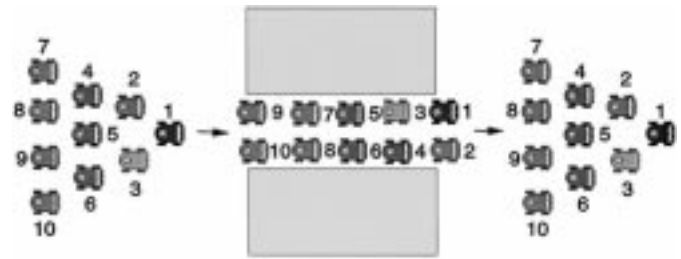ask inc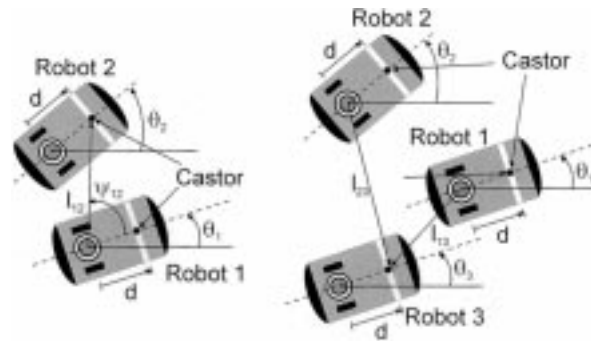reases exponentially with the number of robots and obstacles and quickly becomes intractable. In contrast, we address the problem of coordinating a number of robots, each with their own sensors and feedback controllers. One possible approach to designing independent controllers is to use simple control laws based on the potential field theory [6]. In Arkin's behavior-based control paradigm [7], this approach is used to coordinate formations of robots. It is possible to synthesize an impressive array of group behaviors [7] and coordinate robots for such tasks. However, the interaction between the controllers and planners for independently controlled robots is complex and the performance analysis of such systems is very difficult.

Another related concept for a formation of multiple mobile robots is the concept of string stability, which has been addressed by several researchers in the context of automated highway systems (AHS) [8], [9]. We realize the importance of dynamic analysis, such as is done in understanding string stability. However, our focus is on kinematic issues for nonholonomic mobile robots. Though string stability is important, the primary goal of this paper is to develop a new framework for modeling a formation of mobile robots using graph theory and relating the changes in formation to changes in the graph structure.

II. CONTROL LAWS FOR SHAPE VARIABLES

We first develop a set of decentralized control laws that allows each robot to maintain a desired position within a formation and to enable changes in the shape of a team. We develop two types of feedback controllers for maintaining a formation of a team of mobile robots. These control laws are useful for maintaining either: 1) the desired separation and relative angle between the leader and the follower robot or 2) the desired separation of the follower robot from its two leaders. These two types of controllers are shown in Fig. 2.

In the case of $l - \psi$ control, the state of the follower robot can be written in coordinates relative to the lead robot as $(l_{12}, \ \psi_{12}, \ \theta_2)^T$. Similarly, in the case of $l - l$ control, the state of the follower robot can be written in terms of the two leader robots as: $(l_{13}, \ l_{23}, \ \theta_3)^T$. In the $l - \psi$ control mode for two mobile robots, the aim is to maintain

a desired length l_{12}^d and a desired relative angle ψ_{12}^d between the two robots. The kinematic equations for the follower robot having $l - \psi$ control is given by

$$\dot{l}_{12} = v_2 \cos \gamma_1 - v_1 \cos \psi_{12} + d\omega_2 \sin \gamma_1$$
$$\dot{\psi}_{12} = \frac{1}{l_{12}} (v_1 \sin \psi_{12} - \gamma v_2 \sin \gamma_1 + d\omega_2 \cos \gamma_1 - l_{12}\omega_1)$$
$$\dot{\theta}_2 = \omega_2$$

where $\gamma_1 = \theta_1 + \psi_{12} - \theta_2$ and v_i, ω_i $(i = 1, 2)$ are the linear and angular velocities at the center of the axle of each robot.

In $l - l$ control, this requires regulating the desired lengths, l_{13}^d and l_{23}^d, of the third robot from its two leaders. The kinematic equations for the follower robot is expressed as

$$\dot{l}_{13} = v_3 \cos \gamma_1 - v_1 \cos \psi_{13} + d\omega_3 \sin \gamma_1$$
$$\dot{l}_{23} = v_3 \cos \gamma_2 - v_2 \cos \psi_{23} + d\omega_3 \sin \gamma_2$$
$$\dot{\theta}_3 = \omega_3$$

for the third robot, where $\gamma_i = \theta_i + \psi_{i3} - \theta_3$ $(i = 1, 2)$. Using input/output linearization, we can guarantee exponential convergence for the controlled variables and derive the control law for the inputs ω and v for the follower robot in $l - \psi$ and $l - l$ control [10].

III. ENUMERATION AND TRANSITIONS IN FORMATIONS

In our formulation, a team of n robots has one designated *lead robot* labeled R_1 that directly or indirectly controls all other (follower) robots in the formation. Within the formation, the follower robots depend on other robots for their motion. Thus there are many *leaders* that "lead" other follower robots, but there is a unique lead robot, R_1. For example, in Fig. 3, R_1 is the lead robot and R_2, R_3, R_4, and R_5 are all leaders each with one or more followers.

In our work, we define *control graphs* to be labeled digraphs with each vertex having a uniquely assigned integer number and subject to the following three constraints.

Constraint A: Every vertex (robot) except one has at least one incoming edge. There is one vertex (robot) with no incoming edges, and at least one outgoing edge, which is labeled R_1. This is the lead robot in the formation.

Constraint B: Every directed edge in the digraph goes from a lower vertex label to a higher vertex label.

Constraint C: The number of incoming edges for any vertex R_i $(i > 1)$ is less than or equal to an integer $p \leq \dim(SE(N))$, that describes the number of output variables for a given robot. $p = 2$ for Hilare type robots and we will work with this p for the rest of this paper.

A convenient method for representing control graphs is through an $n \times n$ adjacency matrix [11]. It can be shown that if there are n vertices in a control graph, there are exactly $M(n) = n!(n-1)!/2^{n-1}$ distinct control graphs (based on the constraints stated above) [11], [12].

It is possible to classify control graphs based on the number of $l - \psi$ and $l - l$ controllers. We first introduce some notation. Since we will need to count the number of possible graphs, we will need to investigate permutations of the robot's indices. Given n_ψ robots with $l - \psi$ controllers, let $\{\alpha\}$ represent the set of all possible orderings of n_ψ of the integers (robot indices) $\{1, \ldots, n\}$. Thus, there are γ possible sets, α^1, α^2, $\ldots$, α^γ, where α^i and γ are given by

$$\alpha^i = \{N_1^i, N_2^i, \ldots, N_{n_\psi}^i\} = \{2, N_2^i, \ldots, N_{n_\psi}^i\}$$
$$\gamma = \binom{n - 2}{n_\psi - 1} = \frac{(n-2)!}{(n_\psi - 1)!(n - n_\psi - 1)!}$$
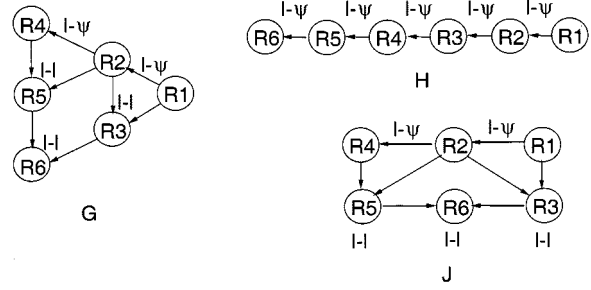

Fig. 3.   Examples of formations.

and $N_j^i \in \{1, \cdots, n\}$ denotes the index of the robot in the formation with $l - \psi$ control, i.e., $R_{N_j^i}$. The 2 appears in each α^i because R_2 always has $l - \psi$ control.

The set $\{\alpha\}$ uniquely determines a corresponding set, $\{\beta\}$, defined as the collections of the γ sets with the remaining robot indices. We denote these by β^1, β^2, $\ldots$, β^γ where

$$\beta^i = \{L_1^i, L_2^i, \ldots, L_{n_l}^i\}$$

and L_j^i denotes the index of the robot in the formation with $l - l$ control, i.e., $R_{L_j^i}$.

Theorem 1: Given n robots in the formation with $n_\psi \geq 1$ robots having $l - \psi$ control and the remaining $n_l = (n - n_\psi - 1)$ robots having $l - l$ control, with $n_l > 0$, there are exactly $\Gamma(n_\psi)$ control graphs where $\Gamma(n_\psi)$ is given by

$$\Gamma(n_\psi) = \left(\frac{1}{2}\right)^{n_l} (n-1)! \sum_{i=1}^{\gamma} \prod_{j=1}^{n_l} (L_j^i - 2). \qquad (1)$$

Proof: a) Given that R_1 is the lead robot in any formation, and R_2 by constraint B always has $l - \psi$ control, the sets $\{\alpha\}$ and $\{\beta\}$ give all possible combinations of formations with n_ψ robots having $l - \psi$ and n_l robots having $l - l$ controllers. Let us consider one such possible combination, α^i for robots with $l - \psi$ control and β^i the corresponding set for robots with $l - l$ control. Based on constraint B, each number L_j^i has $\binom{L_j^i - 1}{2}$ possible choices for the index of the leader robots. Thus, the total number of $l - l$ formations possible for these n_l robots is given by

$$\Omega^i = \prod_{j=1}^{n_l} \binom{L_j^i - 1}{2} = \prod_{j=1}^{n_l} \frac{(L_j^i - 1)(L_j^i - 2)}{2}.$$

Similarly for each index N_j^i, in the set α^i denoting $l - \psi$ control for the robot, there are $(N_j^i - 1)$ choices of a leader. Thus, the total number of $l - \psi$ formations possible for these n_ψ robots is

$$\Psi^i = \prod_{j=1}^{n_\psi} (N_j^i - 1).$$

Finally, combining the above two results, we obtain the total number Θ^i of formations possible for a given set α^i and the corresponding set β^i

$$\Theta^i = \Psi^i \cdot \Omega^i.$$

Since this computation is true for all the sets α^1, α^2, $\ldots$, $\alpha^\gamma \in \{\alpha\}$ (and the corresponding sets β^1, β^2, $\ldots$, $\beta^\gamma \in \{\beta\}$), the total number of control graphs for n robots with n_ψ robots having $l - \psi$ control and all other robots having $l - l$ control is given by

$$\Gamma(n_\psi) = \sum_{i=1}^{\gamma} \Psi^i \cdot \Omega^i = \left(\frac{1}{2}\right)_l^n (n-1)! \sum_{i=1}^{\gamma} \prod_{j=1}^{n_l} (L_j^i - 2).$$

∎
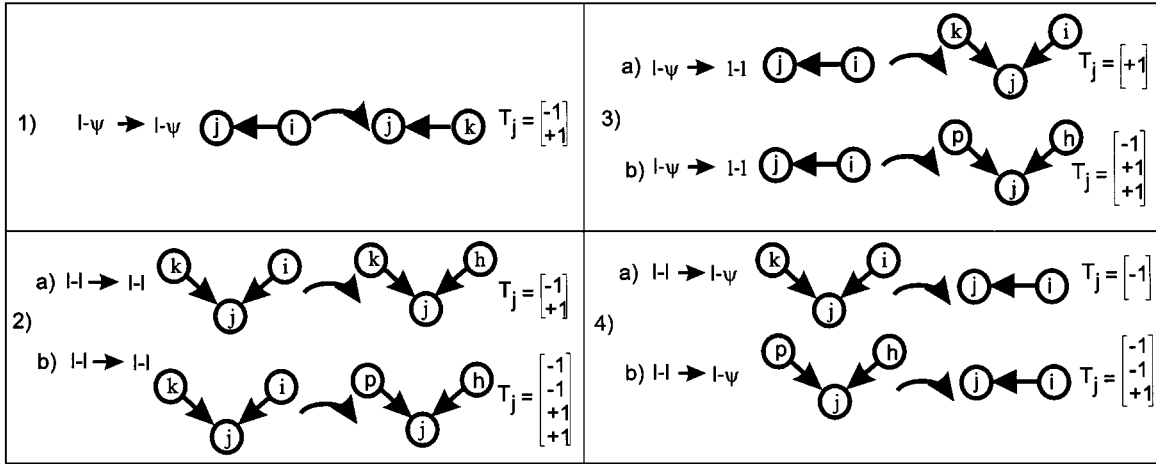
Fig. 4.   Enumeration of transitions in control laws for robot R_j.

Given the total number of robots in the formation and n_ψ, we can easily compute all possible sets α^i and the corresponding set β^i. For $n = 3$, there are only two control graph with two $l - \psi$ controllers and one control graph for one $l - \psi$ (i.e., R_2) and R_3 having $l - l$ control. So there are totally three control graphs.

While Theorem 1 allows the classification of control graphs, it does not give us any information about the equivalence classes of control graphs. On the other hand, since control graphs are derived from digraphs, it is productive to enumerate the possible digraphs for an n robot formation and establish equivalence classes of digraphs. An upper bound on the digraphs for n robots is given by the polynomial [11], [12]

$$Q_n(x) = \sum_{k=(n-1)}^{(2n-3)} a_k x^k. \qquad (2)$$

Based on (2), there are $(n - 1)$ equivalence classes of digraphs, where the kth equivalence class has a_k members, with a_k given by Polya's theorem [13]. A more detailed description of the mathematical tools for enumeration of control graphs can be found in [11] and [12].

In summary, the procedure for enumerating control graphs involves the following two steps.

1) Use Polya's Theorem to enumerate all digraphs of order n, i.e.,

$$d_n(x) = 1 + a_1 x + a_2 x^2 + \cdots + a_{n(n-1)} x^{n(n-1)}.$$

Equation (2) provides a tighter bound on allowable digraphs, i.e., $Q_n(x)$:

$$Q_n(x) = a_{(n-1)} x^{(n-1)} + a_n x^n + \cdots + a_{(2n-3)} x^{(2n-3)}.$$

2) Choose any control graph G, enumerated by $Q_n(x)$, and label the vertices arbitrarily. Based on this labeling, construct its adjacency matrix. By trial and error, obtain the control graphs for which the adjacency matrix is upper triangular.

As the exponent of x in the expression for $Q_n(x)$ varies from $(n-1)$ to $(2n - 3)$, there is a spectrum of control graphs ranging from pure $l - \psi$ control to pure $l - l$ control with a medley of combined $l - \psi$ and $l - l$ control falling in between the two exponents (except for $n = 3$ which only has pure $l - \psi$ and pure $l - l$ control).

The transition from one control graph to another is modeled by a *transition matrix*. We define the transition matrix, T, as the difference between the final (F) and the initial (I) adjacency matrices. The appearance of (-1) in the transition matrix denotes that the edge connecting the vertices representing the robots in the formation needs to be broken to achieve the transition. Similarly, the appearance of a $(+1)$ denotes the addition of an edge (i.e., communication setup between the robots).
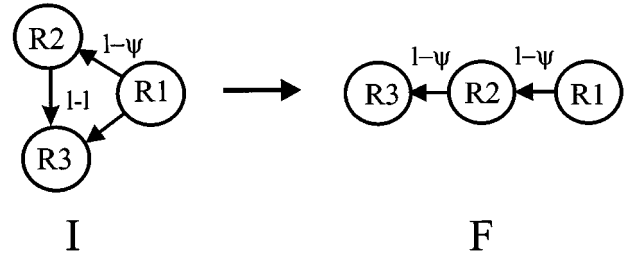

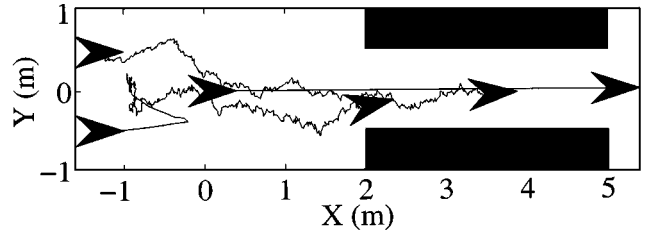Fig. 5.   Initial and final control graph for Example 1.

Fig. 6.   Formation change for three robots in the presence of sensory noise.

Fig. 4 is an exhaustive list of all possible transitions in the control graph for the jth robot, R_j $(j \neq 1)$.

IV. SIMULATION RESULTS

In this section, we will demonstrate the use of the proposed control laws in the presence of obstacles and show how we can transition from one formation to another based on the transitions enumerated in Fig. 4.

Example 1: In this example, we examine the effect of making changes in the control graph. We take the example of three robots moving originally in a triangular formation and transitioning to a straight line formation (see Fig. 5). Fig. 6 shows the path of the robots, R_2 and R_3, in the presence of sensory noise, where the noise is modeled as a uniformly distributed random noise. It is seen that, even in the presence of noise, the formation converges to the desired shape while maintaining the constraints.

Example 2: In this example, the task is to transition from a triangular formation to a rectangular formation. Fig. 7 illustrates the initial and final control graphs. The change in controller for each robot is illustrated through the various transition laws in Fig. 4. Fig. 8 illustrates the path of the various robots during the transition from the triangular formation to a rectangular formation while avoiding the obstacles.
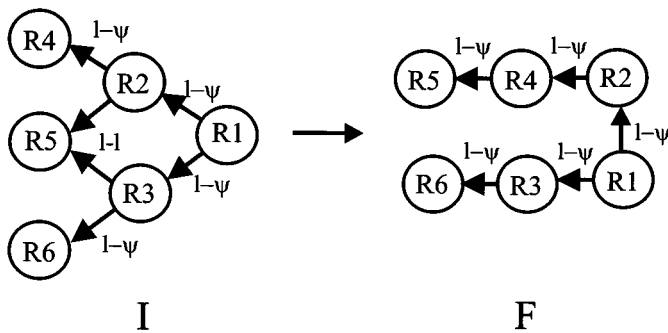
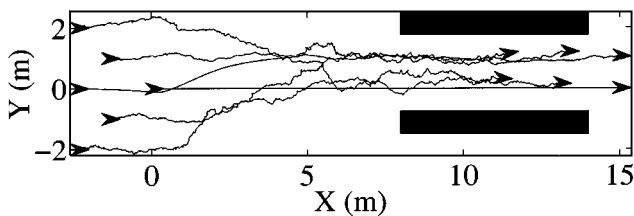Fig. 7.   Initial and final control graph for Example 2.

Fig. 8.   Formation change for six robots in the presence of sensory noise.

V. CONCLUSION

In this paper, we have studied strategies for controlling formations of mobile robots using methods from nonlinear control theory and graph theory. We have focused on decomposing the problem of controlling a formation of nonholonomic mobile robots into: 1) controlling a single lead robot and 2) controlling other follower robots in the team. We used the terms $l - \psi$ and $l - l$ control to reflect whether the control laws are based on tracking the position and orientation of the robot relative to a leader, or the position relative to two leaders, respectively. We also defined the concept of a transition matrix, which governs the addition and deletion of edges in the control graph and hence the change in the communication protocol. Based on this, we presented an exhaustive list of all possible transitions that can occur within the robots in the formation and the corresponding transition matrix column.

There are several important issues that need to be addressed in future research in this area, including: 1) how to choose a control graph and the desired shape based on the constraints in the environment; 2) how to plan changes in $(g, r, \mathcal{H})$ depending on sensor constraints; 3) how to allow formations to be split into sub-formations, leading to multiple lead robots; and 4d) though the transition matrix gives us the information needed to change formations, it is not clear if there is an optimal way for carrying out these changes, rather than the sequential algorithm presented here. Some of these topics are the focus of our present research.

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Multisensor Fusion for Simultaneous Localization and Map Building

J. A. Castellanos, J. Neira, and J. D. Tardós

Abstract—This paper describes how multisensor fusion increases both reliability and precision of the environmental observations used for the simultaneous localization and map-building problem for mobile robots. Multisensor fusion is performed at the level of landmarks, which represent sets of related and possibly correlated sensor observations. The work emphasizes the idea of partial redundancy due to the different nature of the information provided by different sensors. Experimentation with a mobile robot equipped with a multisensor system composed of a 2-D laser rangefinder and a charge coupled device camera is reported.

Index Terms—Correlation, landmark, mobile robot, multisensor fusion, simultaneous localization and map building.

I. INTRODUCTION

Reliable and accurate sensing of the environment of a mobile robot is an important task both in localizing the robot and in building a detailed map of such an environment. One of the fundamental ideas to achieve this reliability is the use of redundancy, that is, to combine environmental information obtained by several sensors [1]–[3]. Dealing with redundancy requires both the availability of a systematic description of uncertain geometric information and a consistent multisensor fusion mechanism [4].

Different approaches to the simultaneous localization and map-building (SLAM) problem for mobile robots have been reported in the literature after the seminal paper of Smith *et al.* [5] and the early