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**Abstract**

Two logically distinct and permissive extensions of iterative weak dominance are introduced for games with possibly vector-valued payoffs. The first, *iterative partial dominance*, builds on an easy-to-check condition but may lead to solutions that do not include any (generalized) Nash equilibria. However, the second and intuitively more demanding extension, *iterative essential dominance*, is shown to be an equilibrium refinement. The latter result includes Moulin's (1979) classic theorem as a special case when all players' payoffs are real-valued. Therefore, essential dominance solvability can be a useful solution concept for making sharper predictions in multicriteria games that feature a plethora of equilibria.

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## 1 Introduction

This paper is concerned with dominance solvability in games where the players' payoffs are generally multidimensional, or vector-valued. Allowing for such multidimensionality is intuitive and in principle desirable, but generally comes at the cost of diminished analytical tractability relative to the standard case where payoffs are captured by real numbers. Despite this added degree of complexity, however, recent examples suggest that the analysis of such games can be fruitful in economically relevant settings (Bade, 2005; Lejano and Ingram, 2012).

The modelling approach we adopt in this paper builds on the assumption that, whenever vector-valued, all payoffs are ordered by the usual (Pareto) partial ordering. The assumption is obviously suitable when all players' preferences do actually coincide with this particular partial ordering and this fact is commonly known, but its relevance is not limited to this special case. When some players' preferences involve a scalarization of their vector payoffs and this fact is privately known by these players, the game could obviously be modelled as one of incomplete information by introducing a type for each possible scalarization that each player might employ and common beliefs over the distribution of these types. However, in the present setting, the minimal rationality requirement that "weakly more in every dimension and strictly more in some dimension is strictly better" provides a salient benchmark for the players' beliefs about each other's preferences. Consequently, one can imagine the players responding to type uncertainty by adhering to this benchmark or focal point, and playing the game as if *all* players' preferences coincided with this partial ordering, irrespective of what their own true preferences actually are.

When games with vector payoffs are analysed by assuming that payoffs are partially ordered in the above sense, a difficulty that one expects to generally arise more frequently compared to games with scalar payoffs is multiplicity of (a suitably generalized notion of) equilibria. Our main motivation for studying dominance solvability in this class of games is to provide an equilibrium refinement that is analogous to the notion of *sophisticated equilibrium* introduced in Moulin (1979, 1986) for standard scalar-payoff games, and thereby help alleviate this problem by eliminating generalized equilibria in which some player's strategy is (iteratively) dominated.

In the present setting, a player's strategy $A$ is said to *weakly dominate* $B$ if $A$ yields a weakly better payoff (i.e. with weakly more in every dimension) for every opponent profile and a strictly better payoff (i.e. with weakly more in every dimension and strictly more in some) for some opponent profile. This is the straightforward but very restrictive extension of conventional weak dominance in higher dimensions. In response to this restrictiveness, we introduce two intuitive generalizations of this concept that are considerably more permissive. First, strategy $A$ is said to *partially dominate* $B$ if there is no opponent profile where $B$ results in a better payoff than $A$ and there is some opponent profile where $A$ yields a strictly better payoff than $B$. Moreover, $A$ is said to *essentially dominate* $B$ if there is no linear completion of the player's payoffs according to which

$B$ weakly dominates $A$ in the conventional sense, and there is some such completion under which $A$ weakly dominates $B$.

Partial and essential dominance are logically distinct and motivate different notions of dominance solvability, both of which generalize the one proposed in Moulin (1979). With regard to our main questions concerning whether the solution set of a game that is dominance solvable according to either notion is a subset of the game's generalized equilibria or not, it is shown that the answer is *yes* for iterative essential dominance but, perhaps surprisingly, *no* for iterative partial dominance. It is also shown, however, that when an easily testable restriction is added to the requirements of partial dominance, this suffices for the latter to imply essential dominance and hence for this discrepancy to disappear.

## 2 Multicriteria Games and Generalized Equilibrium

A ***multicriteria game*** is a collection $(S_i, v_i)_{i=1}^I$, where $S_i$ is player $i$'s pure strategy set, $S = \prod_{i=1}^I S_i$, and $v_i : S \to \mathbb{R}^{n_i}$, $n_i \geq 1$, is player $i$'s payoff function. For $s, s' \in S$, we write

$$v_i(s) \geq v_i(s'), \quad \text{if } v_i^j(s) \geq v_i^j(s') \text{ for all } j \leq n_i$$

$$v_i(s) > v_i(s'), \quad \text{if } v_i(s) \geq v_i(s') \text{ and } v_i(s) \neq v_i(s')$$

For the two reasons laid out in the introduction, it will be assumed throughout that all players payoffs are ordered in this way, and that this fact is common knowledge.

A strategy profile $s \in S$ is a pure strategy ***generalized Nash equilibrium*** if, for all $i \leq I$,

$$v_i(s_i', s_{-i}) \not> v_i(s_i, s_{-i}) \tag{1}$$

for all $s_i' \in S_i$. If $s$ is a generalized equilibrium, then a unilateral deviation by some player will result in a loss for that player in at least one payoff dimension. When $n_i = 1$ for all $i$ this definition reduces to that of ordinary pure-strategy Nash equilibrium. The generalization goes back to Shapley (1959).[1]

## 3 Partial Dominance and Non-Equilibrium Solutions

Given a multicriteria game $(S_i, v_i)_{i=1}^I$, a strategy $s_i'$ ***weakly dominates*** another strategy $s_i$ if

$$\begin{array}{rcll} v_i(s_i', s_{-i}) & \geq & v_i(s_i, s_{-i}) & \text{for all } s_{-i} \in S_{-i} \\ & \& & & \\ v_i(s_i', s_{-i}) & > & v_i(s_i, s_{-i}) & \text{for some } s_{-i} \in S_{-i} \end{array} \tag{2}$$

[1] Voorneveld, Vermeulen, and Borm (1999) and Voorneveld, Grahn, and Dufwenberg (2000) refer to this concept as *Pareto equilibrium* instead.

If $n_i = 1$, then the above obviously coincides with the standard definition of weak dominance. If $n_i > 1$, then (2) provides the straightforward extension of the concept in higher payoff dimensions. This extension, however, is obviously very restrictive and not very likely to be applicable in games of economic relevance. We will therefore consider more permissive notions of dominance, starting with the following:

**Definition 1.** *A strategy $s_i'$* ***partially dominates*** *another strategy $s_i$ if*

$$\begin{array}{rcll} v_i(s_i, s_{-i}) & \not> & v_i(s_i', s_{-i}) & \text{for all } s_{-i} \in S_{-i} \\ & \& & & \\ v_i(s_i', s_{-i}) & > & v_i(s_i, s_{-i}) & \text{for some } s_{-i} \in S_{-i} \end{array} \tag{3}$$

In words, $s_i'$ partially dominates $s_i$ if for no opponents' choices does $s_i$ result in a higher payoff than $s_i'$, while for some opponents' choices $s_i'$ results in a higher payoff than $s_i$. Partial dominance clearly generalizes weak dominance when $n_i > 1$, while it reduces to it when $n_i = 1$.

For $S' \subseteq S$, let $P(S') \subseteq S'$ denote the set of all strategy profiles that obtain after every partially dominated strategy has been removed from $S'$ by every player. Also, write $P_i(S')$ for the set of player $i$'s strategies in $S_i'$ that are not partially dominated.

**Definition 2.** *A finite multicriteria game $(S_i, v_i)_{i=1}^I$ is* ***partially dominance solvable*** *if there exist $S^1, S^2, \ldots, S^k$ such that $S = S^1$, $S^{j+1} = P(S^j)$ for all $j \leq k-1$, $P(S^k) = S^k$, and for every player $i$, $v_i(s) \not\gtrless v_i(s')$ for all $s, s' \in S^k$.*

Partial dominance solvability generalizes standard dominance solvability à la Moulin (1979), both in terms of the dominance criterion employed and also in terms of what constitutes a solution set. Like Moulin (1979), the dominated strategies of *all* players are eliminated in each round. As is well known, if this condition is not imposed when weakly dominated strategies in standard games with scalar payoffs are iteratively deleted, the solution ultimately obtained will depend on the order of elimination. Unlike Moulin (1979), the players' payoff functions at the solution set are not required to be constant. Intuitively, when payoffs are scalar-valued, the players' indifference between two strategies conditional on an opponent strategy profile is captured by equality of their payoffs. When the latter are multidimensional, the partial dominance solution concept essentially expands the notion of indifference to vector *equality or incomparability*. Thus, the requirement that each player must be indifferent between any two profiles at the solution set translates into the requirement that their payoffs be either equal incomparable according to the vector dominance relation.

Although straightforward to formulate and not particularly demanding on the players from a computational point of view, the following normative drawback is associated with iterative partial dominance.

**Observation 1.** *The solution set of a partially dominance solvable finite multicriteria game may not include any generalized Nash equilibria.*

To illustrate, consider the following game:

| | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|
| $U$ | $(2,3),\begin{pmatrix}1\\1\end{pmatrix}$ | $(2,5),\begin{pmatrix}3\\3\end{pmatrix}$ | $(2,5),\begin{pmatrix}3\\1\end{pmatrix}$ | $(3,5),\begin{pmatrix}3\\2\end{pmatrix}$ |
| $M$ | $(2,4),\begin{pmatrix}3\\2\end{pmatrix}$ | $(5,3),\begin{pmatrix}3\\1\end{pmatrix}$ | $(4,4),\begin{pmatrix}2\\3\end{pmatrix}$ | $(3,4),\begin{pmatrix}3\\2\end{pmatrix}$ |
| $D$ | $(2,2),\begin{pmatrix}3\\3\end{pmatrix}$ | $(3,5),\begin{pmatrix}2\\3\end{pmatrix}$ | $(3,4),\begin{pmatrix}3\\2\end{pmatrix}$ | $(4,3),\begin{pmatrix}2\\3\end{pmatrix}$ |

The pure strategy equilibria in this game are $(M,a)$ and $(M,c)$. In the first round of elimination, $M$ dominates $D$ and $d$ dominates $c$. In the second round, $d$ dominates $a$. In the third round, $U$ dominates $M$, while in the final round, $b$ dominates $d$. Thus, the unique partial dominance solution is $(U,b)$. At this profile, the row player profitably deviates to $D$.

A logically distinct extension of weak dominance is introduced in the next section that motivates a solution concept which on the one hand is strong enough to restore the relationship between generalized dominance solutions and equilibria and on the other hand remains permissive enough to relax weak dominance.

## 4 Essential Dominance and Sophisticated Equilibria

The following definition is key in what follows:

**Definition 3.** *Given a vector-valued function $v_i : S \to \mathbb{R}^{n_i}$, a function $u_i : S \to \mathbb{R}$ is a* ***completion*** *of $v_i$ if there exists a vector $\alpha \in \mathbb{R}^{n_i}_{++}$ such that $\sum_{i=1}^{n_i} \alpha_i = 1$ and, for all $s \in S$,*

$$u_i(s) = \sum_{j=1}^{n_i} \alpha_j v_i^j(s). \tag{4}$$

The motivation for requiring the vector $\alpha$ of scalarization weights in the above definition to be an element of $\mathbb{R}^{n_i}_{++}$ and not merely of $\mathbb{R}^{n_i}_{+}$ comes from noting that if a player was willing to scalarize his payoffs in such a way that $\alpha_j = 0$ was true for some $j \leq n_i$, then the $j$-th payoff dimension would be considered redundant for that player, and therefore his payoff dimensionality would have to be decreased in proportion to the number of such irrelevant dimensions. However, all players' true payoff dimensionalities are assumed to be common knowledge in the game. Hence, for this assumption to remain conceptually compatible with the one that allows players to scalarize their payoffs, the weights must be assumed strictly positive.

**Definition 4.** *A strategy $s_i'$* ***essentially dominates*** *another strategy $s_i$ if $s_i'$ is not weakly dominated by $s_i$ under any completion of player i's payoffs and $s_i'$ weakly dominates $s_i$ under some such completion.*

As with the concept of partial dominance, the player here is portrayed as employing two criteria before considering one strategy to dominate another. The first criterion (undomination under all completions) must be satisfied universally, while the second (dominance under some completion) only partially. Unlike that concept, however, in which the criteria were defined in terms of the vector dominance relation, here the player is assumed to engage in a considerably more demanding computational task that involves comparing his strategies by weak dominance relative to all possible scalarized payoff values these may be associated with.

Although partial and essential dominance are logically distinct, it is shown next that the former implies the latter once a structural condition on the player's payoffs is satisfied.

**Observation 2.** *A strategy $s_i'$ essentially dominates another strategy $s_i$ if $s_i'$ partially dominates $s_i$ and, for each $j \leq n_i$,*

$$\operatorname{sgn}\left(v_i^j(s_i', s_{-i}) - v_i^j(s_i, s_{-i})\right) = \operatorname{sgn}\left(v_i^j(s_i', s_{-i}') - v_i^j(s_i, s_{-i}')\right) \tag{5}$$

*for all $s_{-i}, s_{-i}'$.*

Indeed, suppose $s_i'$ partially dominates $s_i$ and (5) also holds. Since, by assumption, $v_i(s_i', s_{-i}) > v_i(s_i, s_{-i})$ for some $s_{-i} \in S_{-i}$, there is no completion $\widehat{u}_i$ of $v_i$ under which $s_i$ weakly dominates $s_i'$. Now let

$$\widehat{v}_i^j := \min_{s_{-i} \in S_{-i}} \left(v_i^j(s_i', s_{-i}) - v_i^j(s_i, s_{-i})\right).$$

Without loss of generality, suppose

$$\widehat{v}_i^1 \leq \ldots \leq \widehat{v}_i^{n_i}.$$

If $\widehat{v}_i^1 \geq 0$, then $\widehat{v}_i^j \geq 0$ for all $j \leq n_i$, which implies that $s_i'$ dominates $s_i$ in the stronger sense of (2). Now suppose $\widehat{v}_i^j < 0$ for some $j < n_i$. Since (5) holds and $s_i'$ partially dominates $s_i$, it follows that $\widehat{v}_i^k > 0$ for some $k$, where $j < k \leq n_i$. Let $\overline{u}_i := \alpha_1 v_i^1 + \ldots + \alpha_{n_i} v_i^{n_i}$ be a completion of $v_i$. Since $v_i(s_i', s_{-i}) > v_i(s_i, s_{-i})$ for some $s_{-i} \in S_{-i}$, for $s_i'$ to weakly dominate $s_i$ under $\overline{u}_i$ it suffices to choose the $\alpha_j$s so that $\overline{u}_i(s_i', s_{-i}) \geq \overline{u}_i(s_i, s_{-i}) \geq 0$ for all $s_{-i} \in S_{-i}$. Since $\widehat{v}_i^{n_i} > 0$ and $\widehat{v}_i^{n_i} \geq \widehat{v}_i^j$ for all $j \leq n_i$, this is achieved by choosing $\alpha_{n_i}$ to be sufficiently close to 1.

The restriction imposed by (5) in relation to strategies $s_i'$ and $s_i$ is that as the opponents' strategies change, there can be no reversal in the dominance direction for any fixed payoff dimension of player $i$ associated with playing $s_i$ or $s_i'$. For example, if $s_i'$ is better than $s_i$ in the first payoff dimension and worse in the second when the opponents play $s_{-i}$, then the condition states that neither of these relations can be strictly reversed when the opponents play any other $s_{-i}'$.

The converse implication in Proposition 2 is not true in general. Indeed, consider the following example:

| | $a$ | $b$ | $c$ |
|---|---|---|---|
| $U$ | $(3,1),\ \cdot$ | $(5,3),\ \cdot$ | $(3,5),\ \cdot$ |
| $D$ | $(1,2),\ \cdot$ | $(3,5),\ \cdot$ | $(5,3),\ \cdot$ |

Here, $U$ does not partially dominate nor is it partially dominated by $D$, and (5) also fails. Clearly, however, under the row player's completion with weight $\alpha_1 = 0.5$, $U$ weakly dominates $D$, whereas for no $\alpha_1 \in (0,1)$ does $D$ weakly dominate $U$.

Now, for $S' \subseteq S$, let $E(S') \subseteq S'$ denote the set of all strategy profiles that come about after every essentially dominated strategy has been removed from $S'$ by every player, and define $E_i(S')$ as the set of player $i$'s strategies that are essentially undominated in $S'$.

**Definition 5.** *A finite multicriteria game $(S_i, v_i)_{i=1}^I$ is* ***essentially dominance solvable*** *if there exist $S^1, S^2, \ldots, S^k$ such that $S = S^1$, $S^{j+1} = E(S^j)$ for all $j \in \{1, \ldots, k-1\}$, $E(S^k) = S^k$, and for every player $i$, $v_i(s) \nleq v_i(s')$ for all $s, s' \in S^k$.*

Essential dominance solvability allows for different completions to be employed by the same player across different rounds of elimination. It is worth stressing, however, that what is implicitly assumed to be common knowledge in the game is the general *decision rule* by which players choose strategies.[2] As such, the solution concept implicitly requires all players to carry out all computations necessary for checking if any of their own as well as their opponents' strategies are iteratively essentially dominated. Thus, all possible completions by all players are assumed to be checked in this process, and this is done consistently across all rounds of elimination.

The solution set associated with an essentially dominance solvable game with strategy profile set $S$ will be denoted $D^e(S)$, while its set of pure strategy generalized Nash equilibria will be denoted $N(S)$.

**Proposition 1.** *If a finite multicriteria game $(S_i, v_i)_{i=1}^I$ is essentially dominance solvable, then $D^e(S) \subseteq N(S)$.*

This result includes Moulin's (1979) Proposition 1 as a special case when all players' payoffs are scalar-valued. Also, in view of this result, Observation 2 with its easy-to-check condition (5) is informative as to how far partial dominance solvability is from being an equilibrium refinement as well. In the spirit of Moulin (1986), in those essentially dominance solvable games where the solution set is a proper subset of the game's set of equilibria, one may refer to those selected by the proposed solution concept as *sophisticated generalized equilibria*.

Putting iterated essential dominance at work in the example of the previous section, one notes that in the first round $M$ essentially dominates $U$ and $D$ and that $c$ is dominated by $d$. In the second round, $b$ is dominated by $a$ and $d$. The surviving profiles are

[2] We abstract from an analysis of the epistemic conditions for either partial or essential dominance solvability. For the state of the art concerning such conditions for iterated admissibility in the standard case of real-valued payoffs the reader is referred to Dekel, Friedenberg, and Siniscalchi (2014) and references therein.

therefore $(M, a)$ and $(M, d)$. However, the game is not essentially dominance solvable because $v_1(M, d) > v_1(M, a)$.

Now consider the following slightly modified version of this game (the new payoffs for the row player at $(U, a)$ and $(U, d)$ are $(3, 2)$ and $(3, 3)$, respectively). As before, the game's equilibria are $(M, a)$ and $(M, c)$.

| | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|
| $U$ | $(2,3), \binom{3}{2}$ | $(2,5), \binom{3}{3}$ | $(2,5), \binom{3}{1}$ | $(3,5), \binom{3}{2}$ |
| $M$ | $(2,4), \binom{3}{2}$ | $(5,3), \binom{3}{1}$ | $(4,4), \binom{2}{3}$ | $(3,4), \binom{3}{2}$ |
| $D$ | $(2,2), \binom{3}{3}$ | $(3,5), \binom{2}{3}$ | $(3,4), \binom{3}{2}$ | $(4,3), \binom{2}{3}$ |

$U$ and $D$ are essentially dominated by $M$ in the first round, but now $c$ and $d$ are dominated by $a$ as well. In the second round, $b$ is dominated by $a$, which leads to $(M, a)$ as the unique sophisticated equilibrium.

## 5 Concluding Remarks

The analysis of multicriteria games can be complicated by the presence of a possibly large number of equilibria. To mitigate this multiplicity, strong assumptions about the players' knowledge (e.g. on how they scalarize their vector payoffs) may need to be employed. In this paper we showed that even when the analysis is restricted to the original vector-payoff formulation of the game, sharper predictions than those based on equilibrium alone can in principle still be made in such games when the proposed notion of *iterative essential dominance* is employed as the relevant solution concept.

## Appendix

***Proof of Proposition 1***

Following Moulin (1979), it will first be shown that for any $\widehat{S} \subseteq S$,

$$N(E(\widehat{S})) \subseteq N(\widehat{S}).$$

Assume to the contrary that there is $s \in \widehat{S}$ such that $s \in N(E(\widehat{S}))$ and $s \notin N(\widehat{S})$. Then, for some player $i$ and strategy $s_i' \in \widehat{S}_i \setminus E(\widehat{S}_i)$ it holds that

$$v_i(s_i', s_{-i}) > v_i(s_i, s_{-i}). \tag{6}$$

This implies that, for every completion $u_i$ of $v_i$,

$$u_i(s_i', s_{-i}) > u_i(s_i, s_{-i}). \tag{7}$$

Indeed, let $u_i$ be an arbitrary completion of $v_i$. Then, for some $\alpha \in \mathbb{R}^{n_i}_{++}$ such that $\sum_{j=1}^{n_i} \alpha_j = 1$, $u_i(s) = \sum_{j=1}^{n_i} \alpha_j v_i^j(s)$ for all $s \in \widehat{S}$. From (6), $v_i^j(s'_i, s_{-i}) \geq v_i^j(s_i, s_{-i})$ for all $j \leq n_i$ and $v_i^j(s'_i, s_{-i}) > v_i^j(s_i, s_{-i})$ for some $j$. The latter and the fact that $\alpha_j > 0$ for all $j$ establishes the claim.

In view of $s'_i \notin E(\widehat{S}_i)$, there exists $s''_i \in E(\widehat{S}_i)$ that essentially dominates $s'_i$. This implies that, for some completion $\overline{u}_i$ of $v_i$, $\overline{u}_i(s''_i, \overline{s}_{-i}) \geq \overline{u}_i(s'_i, \overline{s}_{-i})$ for all $\overline{s}_{-i} \in \widehat{S}_{-i}$ and, in particular,

$$\overline{u}_i(s''_i, s_{-i}) \geq \overline{u}_i(s'_i, s_{-i}). \tag{8}$$

From (7) and (8), we get $\overline{u}_i(s''_i, s_{-i}) > \overline{u}_i(s_i, s_{-i})$. In view of $s''_i \in E(\widehat{S})$, this contradicts the postulate $s \in N(E(\widehat{S}))$.

Since the game is essentially dominance solvable, there exist $S^1, \ldots, S^k$ such that $S = S^1$, $S^{j+1} = E(S^j)$ for all $j \in \{1, \ldots, k-1\}$, and $E(S^k) = S^k$. Since $N(E(S')) \subseteq N(S')$ for all $S' \subseteq S$ and $S^{j+1} = E(S^j)$, it follows that

$$N(S) = N(S^1) \supseteq N(S^2) \supseteq \ldots \supseteq N(S^k). \tag{9}$$

Thus, $D^e(S) = S^k = E(S^k) = N(E(S^k)) \subseteq N(S)$. ■

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