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ABSTRACT

The Uzawa (1961) theorem applied to finance and growth suggests that a long-run positive correlation between financial efficiency and depth is only present when variations in the extent of access to financial services are considered. Improvements in financial efficiency can lead to new capital augmenting technologies along the balanced path, but only improvements in financial efficiency directed towards labor can change the rate of growth in the long-run. These findings suggest ways to understand some of the more nuanced relationships between finance and growth observed in the data and point in a number of directions for future research.

JEL Classification: O11, O16, O40.

Keywords: Finance and Growth; Endogenous Growth; Uzawa Theorem.

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1 Introduction

The literature on the relationship between financial development and economic growth has become one of the most important in applied economics. The early empirical work of King and Levine (1993a,b) suggested that financial depth is an important part of what explains variations in growth across countries. The robustness of that connection has been supported by subsequent studies such as Demirgüç-Kunt and Maksimovic (1998), Levine et al. (2000) and Rousseau and Sylla (2005).¹ A number of other results have suggested that the empirical evidence is not consistently positive, however. Important variations have been shown to exist in the significance and direction of finance-growth connection across countries (Demetriades and Hussein, 1996), across stages of development (Rioja and Valev, 2004), and across time (Rousseau and Wachtel, 2010).

The empirical link between financial development and growth has been supported by the emergence of a theoretical literature on financial intermediation and endogenous growth. Works such as King and Levine (1993b), de la Fuente and Marín (1996), Blackburn and Hung (1998) and Aghion et al. (2005) offer a large number of ways in which financial efficiency can impact on the equilibrium rate of technological progress and so on long-run growth.² More recently, Michalopoulos et al. (2009) have looked to incorporate endogenous financial innovation in a model of endogenous growth, finding that permanent improvements in financial efficiency are necessary for positive economic growth to exist in equilibrium.³ While the early empirical evidence on finance and growth largely used measures of financial depth as a proxy for financial development, the theoretical literature has focused on the impact that financial efficiency has on, for example, the ease with which firms can find good entrepreneurs to fund.

¹See Levine (2005) and Beck (2008) for surveys.

²See Capasso (2004) for a survey of the theoretical literature.

³Though, on aggregate, financial efficiency is constant in steady state.

That focus on the efficiency of intermediation has been driven by the nature of the endogenous growth framework which places emphasis on that part of the economy which produces new ideas and new technologies. The complementarity between the empirical and theoretical findings thus rests upon the implicit assumption that efficiency and depth are both good proxies for the financial development that matters for economic growth in the long run.

The objective of this paper is to study that connection between financial efficiency and financial depth when the economy is growing in the long-run. That is, are efficiency and depth positively correlated along a balanced growth path, and how do they interact when the long-run growth rate changes? This focus on balanced path outcomes leads us also to consider the implications of the Uzawa (1961) theorem for the way in which financial efficiency can interact with growth over the long-run; if all technological change must be purely labor-augmenting in the long-run, what is the implication for how financial efficiency and economic growth should interact?

In order to address these questions, Section 2 develops a simple neoclassical model with a role for financial efficiency in determining the level of technological progress. Using a version of the Uzawa theorem, it becomes clear that balanced growth restricts both how financial efficiency can relate with financial depth and how financial efficiency can relate with economic growth. First, sustained growth cannot come about from sustained progress in financial efficiency if financial efficiency is related with depth. Second, if financial efficiency is not related with depth, it can contribute to long-run technological change but only if that progress in financial efficiency is purely labor augmenting.

Section 3 develops the implications for the efficiency-depth connection in the context of an endogenous growth model. While depth must be constant along the balanced growth path, we find that when the growth rate changes depth and growth must be negatively related. That result is qualified, however, when ‘access’

to financial intermediation is considered as a part of financial development. The model suggests that when access is low and growing, a positive correlation between depth and growth and between efficiency and growth can exist. When access is high or not growing, the positive relationship between efficiency and growth remains, but depth and growth can be inversely related. These results relate to some aspects of the theoretical literature such as Greenwood and Jovanovic (1990) and Rousseau (1998), but they also give greater force to the importance of a very recent empirical literature (Beck and Demirgüç-Kunt, 2008; Kendall et al., 2010) that looks to measure differences in access to financial services across countries.

Section 4 develops the balanced path implications for finance and growth in the directed technical change framework with capital and labor. Again, the permanent technological change which occurs on a balanced growth path must be purely labor augmenting; only those changes in financial efficiency which impact on the discovery of labor-augmenting technologies can influence the long-run rate of growth. Changes in financial efficiency toward capital can encourage innovations in capital augmenting technologies in the short-run, but have no effect on the balanced path rate of growth.

Section 5 offers some concluding remarks. While this paper principally contributes to our understanding of how finance and growth can interact in theory, the results point to a number of ways in which the empirical estimation of the importance of finance might develop.

2 Balanced Growth and Finance

Consider the following neoclassical production function,

$$Y_t = F(K_t, L_t, A_t(\zeta_t)) \tag{1}$$

where capital, K_t , and labor, L_t , combine with a measure of technological progress, A_t , to produce output and where $F(\cdot)$ has the usual characteristics, including linear homogeneity. The parameter ζ_t denotes financial efficiency, which enters as an input to the level of technological progress. In theoretical analyses of finance and growth, something like ζ_t is generally the object of interest. Exactly how it represents financial matters and how it enters into A_t can vary; for the purposes of this model, we simply assert that greater financial efficiency increases the level of technology, $\partial A_t / \partial \zeta_t > 0$.

Typically not modelled are the costs associated with that financial efficiency. Financial intermediary services provide, for example, specialized screening of entrepreneurs of unknown type at a cost that reflects the efficiency of finance. When firms pay those financial intermediaries they allocate resources *away* from production. Financial depth is then the sum of such costs as a proportion of aggregate output.⁴ As such, when those costs change, the sum of resources going to financial services can also change; that is, financial efficiency, financial depth and the level of technological progress are all connected. While efficiency is exogenous here, we can see what happens if we permit the connection between the efficiency of financial services and the sum of resources allocated away from production to be non-zero.

Financial depth so defined enters into the capital accumulation equation as follows,

$$\dot{K}_t = Y_t - C_t - D(\zeta_t)Y_t - \delta K_t, \quad (2)$$

where $D(\zeta_t)$ is financial depth based on the costs of financial services and where there are no a priori restrictions on D' .

The economy is on a balanced growth path (BGP) after some time $\tau > 0$ if and only if output, capital, consumption and depth are all positive and all grow at

⁴This is a more focused definition of depth than tends to be captured in the data, indeed depth here has to be less than one where it is often not empirically. Clearly, there is a lot in the empirical measure that is absent here but this conception of financial depth focuses on that part of finance which is relevant to growth.

constant exponential rates g_y , g_k , g_c and g_d respectively, for all $t \geq \tau$,

$$Y_t = Y_\tau e^{g_y(t-\tau)}, \quad (3)$$

$$K_t = K_\tau e^{g_k(t-\tau)}, \quad (4)$$

$$C_t = C_\tau e^{g_c(t-\tau)}, \quad (5)$$

$$D_t = D_\tau e^{g_d(t-\tau)}. \quad (6)$$

where D_t is shorthand for $D(\zeta_t)$. Assume that there is no growth in the labor supply, $L_t = L$ for all $t \geq \tau$. With the BGP so defined, the first part of the following proposition makes clear that if financial depth is related with financial efficiency, then improvements in financial efficiency cannot be a part of what causes improvements in technology.

Proposition 1 *On the BGP, i) financial efficiency cannot cause sustained technological progress unless it does not interact with financial depth; and, ii), if financial efficiency contributes to long-run technological change without interacting with depth, it must be labor-augmenting.*

Proof. The proof uses a variant of that of the Uzawa (1961) theorem found in Schlicht (2006).⁵ We can re-write (2) for $t \geq \tau$ using the definition of a BGP,

$$(\delta + g_k)K_\tau = e^{(g_y - g_k)(t-\tau)}Y_\tau + e^{(g_y + g_d - g_k)(t-\tau)}Y_\tau D_\tau - e^{(g_c - g_k)(t-\tau)}C_\tau. \quad (7)$$

Taking the derivative of (7) with respect to time, we can see that on a BGP,

$$0 = (g_y - g_k)e^{(g_y - g_k)(t-\tau)}Y_\tau + (g_y + g_d - g_k)e^{(g_y + g_d - g_k)(t-\tau)}Y_\tau D_\tau + \\ - (g_c - g_k)e^{(g_c - g_k)(t-\tau)}C_\tau. \quad (8)$$

If $g_d + g_y = g_k$ then it must be that $g_y = g_c$ and $(g_y - g_k)(Y_\tau - C_\tau) = 0$ which

⁵See also Jones and Scrimgeour (2008).

cannot be true on a BGP since it would imply $K_\tau < 0$. As such, we must have that $D_\tau > 0$, $g_d = 0$ and that $g_c = g_y = g_k \geq 0$.

Consider that technology A_t augments one or both of the factors of production with coefficients of technological progress $A_{K,t}$ and $A_{L,t}$. Then let $G(A_{K,t}K_t, A_{L,t}L_t) := F(\cdot)$. Using homogeneity of the production function, we can write,

$$Y_t = G(e^{(g_y - g_k)(t - \tau)} K_t, e^{g_y(t - \tau)} L), \text{ for any } t \geq \tau. \quad (9)$$

Since $g_y = g_k$, equation (9) shows that all technological progress, $\dot{A}_t/A_t = g_y$, is purely labor-augmenting.

If $D' \neq 0$, part *i)* of the proposition follows from the BGP requirement that $g_d = 0$. If $D' = 0$, part *ii)* follows from requirement that technological progress, including that out of financial efficiency, be Harrod-neutral on BGP. ■

The proposition has two main implications, one for the relationship between financial efficiency and depth in an economy on a balanced growth path, another for the relationship between financial efficiency and technological progress on balanced growth. Sustained progress in financial efficiency can only be part of technological change if, first, financial efficiency does not affect financial depth and, second, it is labor-augmenting. Both of these requirements might be considered somewhat restrictive. Either way, the implication for financial depth is clear: Away from balanced growth path depth can vary but in the long-run it must be constant.

Sections 3 and 4 respectively elaborate on each of these two implications in the context of models of growth where the pace of technological change is endogenous to both underlying incentives to innovate and the efficiency of financial intermediation. This permits a more complete specification of the underlying financial problem and reveals a number of additional insights.

3 Finance and Endogenous Growth

In models of endogenous growth where financial efficiency matters, intermediaries exist to screen potential entrepreneurs, to monitor research effort, or to evaluate the quality of discoveries; that is, they sit between those that wish to acquire new technologies or blueprints for intermediate goods and those who have the ideas for those new technologies or blueprints.⁶ The expanding inputs model recently expounded in Acemoglu (2009) serves as a useful framework within which to explore the implications of the Uzawa theorem for finance and growth. That model is adapted to include a screening problem with which the intermediary can be engaged. Other features of the model are standard, so we leave some detail to Acemoglu (2009).

Preferences and technology

The representative household has CRRA preferences, choosing consumption levels to maximise the present value of discounted future utility,

$$\int_0^\infty e^{-\rho t} \left(\frac{C_t^{1-\theta} - 1}{1-\theta} \right) dt. \quad (10)$$

Agents consume a perfectly competitive final good, Y_t , the production of which takes a Dixit-Stiglitz form and uses labor, L , and a variety of machines, x_t , as inputs,

$$Y_t = \frac{1}{1-\beta} \left(\int_0^{N_t} x_t(v)^{1-\beta} dv \right) L^\beta, \quad (11)$$

where v indexes input variety and N_t is the total number of machine input varieties in existence at time t . The number N_t is thus our measure of the extent of technological progress. The machines used as inputs to final good production are supplied by the holders of blueprints for those machines. The discovery of new blueprints

⁶Consider the screening of entrepreneurial type in King and Levine (1993b) and Morales (2003) and the monitoring of effort in de la Fuente and Marín (1996) and Blackburn and Hung (1998).

requires investment, Z_t , in lab equipment. Where $X_t = \int_0^{N_t} x_t(v)dv$ is the total spend on machines in period t , the economy-wide resource constraint is,

$$Y_t \geq C_t + X_t + Z_t. \quad (12)$$

Technological progress and finance

Lab equipment can itself be of different levels of quality: Capable (incapable) scientists have created good (bad) lab equipment. A proportion $\phi \in (0, 1)$ of scientists are capable, the remaining $(1 - \phi)$ are not. Only by investing in the good equipment, that created by capable scientists, can new blueprints be discovered.

Machine producers can see who created what equipment but it is infinitely costly for them to see the type of scientist. Financial intermediaries exist to reveal type by charging a fee for screening potential researchers.⁷ The screening mechanism is perfect (the truth is always known, post-screen), but the efficiency of the screening, ζ_t , determines its cost. For a given ζ_t , the evolution of new blueprints then follows,

$$\dot{N}_t = \eta_N Z_t [1 - f(\zeta_t)], \quad (13)$$

where $\eta_N > 0$ and $f(\zeta_t)$ is the fee charged for intermediation. The efficiency level ζ_t is exogenous but not necessarily constant over time. Financial depth is then,

$$\mathcal{D}_t = \frac{f(\zeta_t)\eta_N Z_t}{Y_t}. \quad (14)$$

The fee charged is based on the screening problem faced by the intermediary. Intermediaries face a cost $z(\zeta_t) > 0$ of screening scientists from the pool of applicants, where $z' < 0$. An intermediary chooses the fee that maximises expected

⁷We are also restricting ourselves throughout to equilibria where $[1 - f(\zeta_t)] > \phi$, i.e., where screening is always better than not screening.

profits from providing screening services,

$$E[profit] = \phi[fZ_t - z(\zeta_t)] + (1 - \phi)[-z(\zeta_t)]. \quad (15)$$

Assuming competitive intermediation, the fee charged is simply,

$$f^*(\zeta_t) = \frac{z(\zeta_t)}{\phi Z_t}. \quad (16)$$

Evidently, an increase in financial efficiency will, other things equal, reduce the fee charged for intermediation, increase the rate of discovery of new blueprints and *decrease* the level of financial depth. Despite running counter to evidence, it is intuitive in a set-up where financial intermediaries do nothing other than funnel research funding to the right research activities. As such, this implication is consistent with other finance and endogenous growth models, and Section 3.1 shows that reconciling the efficiency and depth correlation with data requires us to modify the model only slightly.

Equilibrium and the BGP

Profit maximization by the final goods producer yields the following demands for machines,

$$x_t(v) = p_t^x(v)^{-\beta} L, \quad (17)$$

where p_t^x is the machine price set by the blueprint holder.

Let $\pi_t(v) \equiv p_t^x(v)x_t(v) - \psi x_t(v)$ be the instantaneous profits which accrue from selling machine variety v at time t . Given that the blueprint owner is a monopolist in producing the machine variety, the price $p_t^x(v)$ is chosen to maximise $\pi_t(v)$. Because the demand for machines is iso-elastic, the price $p_t^x(v)$ is invariant across time and machine variety so we normalize the marginal cost of machine production to $\psi = (1 - \beta)$. As such, the profit-maximising price is $p_t^x(v) = 1$ and monopoly

profits are $\pi_t(v) = \beta L$. The final good production function can then be written as,

$$Y_t = \frac{1}{1 - \beta} N_t L. \quad (18)$$

Equation (18) implies that any balanced growth equilibrium where Y_t grows at a constant rate will also have that N_t grows at a constant rate.

The net present discounted value of a perpetual license to a blueprint can be written as,

$$V_t(v) = \frac{\pi_t(v) + \dot{V}_t(v)}{r_t}, \quad (19)$$

where r_t is the interest rate and $\dot{V}_t(v)$ is the change in the net present value of a blueprint over time (see Acemoglu, 2009).

When there is positive technological progress, free entry to research requires that one unit of spending on equipment leads to an equal net present discounted return to the blueprint holder,

$$[1 - f^*(\zeta_t)] \eta_N V_t(v) = 1. \quad (20)$$

An economy on a BGP is one where the growth rate of output (and consumption) is constant. Consumer optimization yields,

$$g_c = \frac{\dot{C}_t}{C_t} = \frac{1}{\theta} (r_t - \rho). \quad (21)$$

This Euler equation implies that a balanced growth path is characterized by a constant interest rate. Since profits to blueprint ownership are also constant over time, it must be that $\dot{V}_t(v) = 0$. Combining this with equations (19) and (20), the equilibrium interest rate along a BGP is,

$$r^* = [1 - f^*(\zeta_t)] \eta_N \beta L, \quad (22)$$

which makes equilibrium balanced growth rate equal to,

$$g^* = \theta^{-1} ([1 - f^*(\zeta_t)]\eta_N\beta L - \rho). \quad (23)$$

Equation (28) implies that the higher is the efficiency with which financial intermediaries screen scientists, the higher is the equilibrium growth rate.

Financial efficiency and financial depth along the BGP

A direct result of equation (18) is that along a balanced growth path, N_t must also grow at a constant rate $\dot{N}_t/N_t = g^*$. We then have the following Proposition, which is analogous to the first part of Proposition 1,

Proposition 2 *Financial depth is constant on a balanced growth equilibrium of the endogenous growth model.*

Proof. The growth rate of new blueprints is,

$$\frac{\dot{N}_t}{N_t} = \frac{\eta_N [Z_t - z(\zeta_t)/\phi]}{N}, \quad (24)$$

which must be a constant on a BGP. Since total spend on machines is $X_t = (1 - \beta)N_tL$, the BGP combined with the economy-wide resource constraint implies that the growth rate of Z_t must be constant also which is not possible unless $z(\zeta_t)$ is proportional to Z_t . With $z(\zeta_t)$ proportional to Z_t and using equation (16), then financial depth, equation (14), is a constant on BGP. ■

Proposition 2 shows that a BGP requires that financial costs as a proportion of research spending, $z(\zeta_t)/Z_t$, is constant; that is, *relative* financial efficiency is constant. This can happen if ζ_t falls over time in such a way to make $z(\zeta_t)$ increase along with Z_t , but it is more typical for something like ζ_t itself to reflect that relative

financial efficiency. As such, let z take the following form,

$$z(\zeta_t) = \frac{\eta_Z Z_t}{\zeta_t}, \quad (25)$$

with $\eta_Z > 0$. Then the optimal fee charged by intermediaries is constant on BGP, $f^*(\zeta_t) = \eta_Z / (\phi \zeta_t)$.

This analysis provides a justification for focusing on *relative* financial efficiency, but it also gives an idea about what must be happening to financial depth along a balanced growth path. There is, in addition, an implied relationship between depth and efficiency when the balanced growth path itself changes. Consider, for example, that relative financial efficiency increases. Since there are no transition dynamics, the rate of growth of output and consumption along the balanced growth path immediately increases, but financial depth *decreases* – though consumption growth is faster, the resources allocated to finance as a proportion of output are smaller.

There are a number of problems in contrasting this theoretical depth-efficiency relationship with the empirical evidence. First, economies are likely not *on* a balanced growth path a lot of the time and, second, our conception of financial depth clearly misses out some aspects of that measured in the data. Nonetheless, there is empirical evidence that shows that the nexus between finance and growth is not a simple one. The model as laid-out so far suggests that it is not possible to have anything other than a negative relationship between efficiency and depth on the balanced growth path. Section 3.1 look at what relaxations of the model are required in order for different results to exist on a balanced growth path.

3.1 Access to Finance

A number of models of finance and growth consider that a part of financial development can be an increase in the number of agents using financial services in addition

to the efficiency with which they do so. A simple exposition is Townsend (1983), where development can be characterized as a proportion q of all non-financed agents being ‘thrown’ into the financial sector at the start of each period. Additionally, Greenwood and Jovanovic (1990) model the relationship between inequality, growth and the *extent* of financial development while Acemoglu and Zilibotti (1997) model barriers to risk sharing that are progressively overcome through time.

Suppose that only some fraction $\lambda_t \in [0, 1]$ of scientists are able to be screened, regardless of their type. The proportion λ_t affects both the cost of screening and the rate of new discoveries. First, the larger the pool of scientists (the higher is λ_t) among whom the intermediary screens, the more costly it is to screen out incapable scientists.⁸ The following formulation simply generalizes (25),

$$z(\zeta_t, \lambda_t) = \frac{\lambda_t \eta_Z Z_t}{\zeta_t}. \quad (26)$$

Second, since the rate of increase in new blueprints is limited to that portion which can be screened by intermediaries, the evolution of blueprints follows,

$$\dot{N}_t = \lambda_t \eta_N \left(1 - \frac{\lambda_t \eta_Z}{\phi \zeta_t} \right) Z_t, \quad (27)$$

and the balanced path rate of growth is,

$$g^* = \theta^{-1} \left[\lambda_t \eta_N \left(1 - \frac{\lambda_t \eta_Z}{\phi \zeta_t} \right) \beta L - \rho \right]. \quad (28)$$

Compared with the model where $\lambda_t = 1$ for all t , the nature of financial development along the balanced growth path can be very different. As already established, along a BGP the ratio Z_t/N_t is constant. The effect of an increase in financial efficiency is to speed up the rate of discovery of new blueprints; the effect of increasing access to financial services is ambiguous, however. Suppose that access and effi-

⁸For a given amount of research funding to allocate, the higher is λ_t the more screening the intermediary is required to do.

ciency can both change exogenously over time at rates $\dot{\lambda}_t$ and $\dot{\zeta}_t$, respectively. The BGP can then be characterized by Proposition 3.

Proposition 3 *On the balanced growth path, financial depth increases at a rate proportional to that at which access increases.*

Proof. The BGP requires that $\frac{\dot{N}_t}{N_t}$ is constant, which implies,

$$\left(1 - \frac{2\lambda_t\eta_Z}{\phi\zeta_t}\right)\dot{\lambda}_t = -\frac{\lambda_t^2\eta_Z}{\phi\zeta_t^2}\dot{\zeta}_t. \quad (29)$$

Financial depth is $\mathcal{D}_t = \lambda_t f(\zeta_t, \lambda_t) \eta_N \psi$ where $\varphi = Z_t/Y_t$ is the constant share of output allocated to research. Taking the derivative of depth with respect to time and using (29), the change of depth on BGP is,

$$\dot{\mathcal{D}}_t = \varphi \eta_N \dot{\lambda}_t. \quad (30)$$

Financial depth will grow while the economy is on the BGP if $\dot{\lambda}_t > 0$. ■

One solution to (29) is for the BGP to be characterized by no development in access or efficiency, $\dot{\zeta}_t = \dot{\lambda}_t = 0$. However, it is possible to have some forms of financial development on BGP. If $2\lambda_t\eta_Z > \phi\zeta_t$, then it is possible to have $\dot{\zeta}_t > 0$ and $\dot{\lambda}_t > 0$ on the BGP; i.e., depth and efficiency can both be growing on a balanced growth path. Clearly, there is a limit to this when access becomes high (i.e., close to $\lambda = 1$). Moreover, countries with low (high) access and high (low) efficiency are less (more) likely to observe $2\lambda_t\eta_Z > \phi\zeta_t$.

Perhaps more interestingly, consider again what happens when changes in financial efficiency and access cause the balanced growth path to change. Suppose that financial efficiency and access both increase over time, but that the following holds,

$$\left(1 - \frac{2\lambda_t\eta_Z}{\phi\zeta_t}\right)\dot{\lambda}_t + \frac{\lambda_t^2\eta_Z}{\phi\zeta_t^2}\dot{\zeta}_t = d > 0, \quad (31)$$

Where $2\lambda_t\eta_Z > \phi\zeta_t$, equation (31) implies that the effect of increasing financial efficiency pushes up the rate of discovery of new blueprints faster than the effect of increasing access pushes it down. Using (31), the change in depth is simply,

$$\dot{\mathcal{D}}_t = \varphi\eta_N (\dot{\lambda}_t - d). \quad (32)$$

In other words, when access can change over time as the growth rate of the economy increases, the correlation between efficiency and depth can be of either sign depending on the difference between the growth-effects of efficiency and access. Where $\lambda_t = 1$ it must be that $\dot{\mathcal{D}}_t \leq 0$ and so a positive correlation is not possible.

Incorporating access to finance appears to be an important part of understanding the empirical finding that efficiency, depth and growth are all correlated positively. The model suggests that when access is low and growing, there is a positive correlation between depth and growth, and between efficiency and growth. When access is high or not growing, the positive correlation between efficiency and growth persists, but a negative correlation between depth and growth can appear. By appreciating the role that access plays, it might be possible to classify particular empirical results in the light of the data on access to financial services.

4 Directed Technical Change and Financial Development

The implication of the Uzawa theorem, that all technological progress must be labor-augmenting in the long run, can be captured in an endogenous growth model with directed technical change. This section explores the implications of two types of financial efficiency, one supplementing the discovery of blueprints for labor-augmenting machines, the other blueprints for capital-augmenting machines. Given the findings of Section 3, and to simplify the exposition, this section does not con-

sider long-run change in efficiency in the presence of incomplete access.

While retaining the CRRA preferences specification from Section 3, there are now two types of intermediate good that go into producing the final good,⁹

$$Y_t = \left[\gamma_L (Y_{L,t})^{\frac{\varepsilon-1}{\varepsilon}} + \gamma_K (Y_{K,t})^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad (33)$$

where Y_L is the intermediate good produced with labor and Y_K is that produced with capital. The parameter ε is the elasticity of substitution between the intermediate goods, so assume throughout that $\varepsilon < 1$. The difference between labor and capital is that capital can accumulate permanently, whereas the labor supply is fixed at, L . The structure of each type of intermediate good production follows the machine-blueprint technology of Section 3,

$$Y_{L,t} = \frac{1}{1-\beta} \left(\int_0^{N_{L,t}} x_{L,t}(v)^{1-\beta} dv \right) L^\beta \quad (34)$$

$$Y_{K,t} = \frac{1}{1-\beta} \left(\int_0^{N_{K,t}} x_{K,t}(v)^{1-\beta} dv \right) K_t^\beta \quad (35)$$

where now $x_{L,t}(v)$ is a labor-augmenting machine, and $x_{K,t}(v)$ a capital-augmenting one. The numbers $N_{L,t}$ and $N_{K,t}$ thus correspond to the extent of progress in labor- and capital-augmenting technologies.

Financial intermediation again enters into the blueprint discovery sector. Consider that there is full access to financial services ($\lambda_t = 1$) as in the first model of Section 3, but that there are sector-specific financial efficiencies, $\zeta_{K,t}$ and $\zeta_{L,t}$, that affect the fees charged for intermediary services in capital- and labor-augmenting technologies, respectively. In the light of Section 3, assume for simplicity that the level of financial efficiency is constant in each sector, $\zeta_{K,t} = \zeta_K$ and $\zeta_{L,t} = \zeta_L$. There exists a fixed pool of scientists, separate from the labor that enters into in-

⁹Much of this is exposition of a now-familiar framework for understanding directed technical change, so the model here follows Acemoglu (2009); again, we leave the interested reader to find details there.

intermediate good production, which can be engaged to discover either capital- or labor-augmenting blueprints, $S_L + S_K = S$,

$$\dot{N}_{K,t} = \eta_K N_{K,t} [1 - f(\zeta_K)] S_K \quad (36)$$

$$\dot{N}_{L,t} = \eta_L N_{L,t} [1 - f(\zeta_L)] S_L \quad (37)$$

The resource constraint in the economy is,

$$Y_t \geq C_t + X_t + I_t, \quad (38)$$

where I_t is capital investment, and X_t is now the sum of spending on all types of machines. Following Acemoglu (2003), capital accumulates without depreciating,

$$\dot{K}_t = I_t. \quad (39)$$

Solving for intermediate demands and the value of holding a blueprint in each sector follows as previously, but now equilibrium in the R&D sector depends upon relative factor supplies. First, the free-entry requirement for blueprints implies,

$$\omega_{S,t} = \eta_L [1 - f(\zeta_L)] N_{L,t} V_{L,t} = \eta_K [1 - f(\zeta_K)] N_{K,t} V_{K,t} \quad (40)$$

where $\omega_{S,t}$ is the wage rate of scientists and, using same argument as above, $V_L = \pi_{L,t}/r_t$ and $V_K = \pi_{K,t}/r_t$ where r_t is the interest rate. Again normalizing the marginal cost of machine production, the prices of machines are unity and profits are a function of the prices of the intermediate goods, $\pi_{L,t} = \beta p_{L,t}^{1/\beta} L$ and $\pi_{K,t} = \beta p_{K,t}^{1/\beta} K$. The relative intermediate prices are,

$$\frac{p_K}{p_L} = \left(\frac{\gamma_K}{\gamma_L} \right)^{\frac{\varepsilon\beta}{\sigma}} \left(\frac{N_{K,t} K_t}{N_{L,t} L} \right)^{-\frac{\beta}{\sigma}} \quad (41)$$

where $\sigma \equiv 1 + (\varepsilon - 1)\beta$ is the elasticity of substitution between capital and labor (and, since $\varepsilon < 1$, it is the case that $\sigma < 1$). With the free-entry condition, we arrive at the equilibrium relative technology levels along a BGP,

$$\left(\frac{N_K}{N_L}\right)^* = \left[\frac{\eta_K [1 - f(\zeta_K)]}{\eta_L [1 - f(\zeta_L)]}\right]^{-\frac{\sigma}{\sigma-1}} \left(\frac{\gamma_K}{\gamma_L}\right)^{-\frac{\varepsilon}{\sigma-1}} \left(\frac{K_t}{L}\right)^{-1} \quad (42)$$

Capital deepening means that technical change is relatively more labor-augmenting along the balanced growth path.

It can be shown that the BGP interest rate reduces to,

$$r_t = \beta \gamma_K N_{K,t} \left[\gamma_L \left(\frac{\eta_K [1 - f(\zeta_K)]}{\eta_L [1 - f(\zeta_L)]} \right) \left(\frac{\gamma_K}{\gamma_L} \right)^{\frac{\varepsilon}{\sigma}} + \gamma_K \right]^{\frac{1}{\sigma-1}}. \quad (43)$$

By the Euler equation along a BGP it must be that r is constant. As such, where ζ_L and ζ_K are constant, it is the case that all technological progress must be labor-augmenting in the long-run; $\dot{N}_{K,t}/N_{K,t} = 0$ and $\dot{N}_{L,t}/N_{L,t} = \dot{K}_t/K_t$.

Given that the only BGP equilibrium is one with purely Harrod-neutral technological change, it must be that $S = S_L$. The rate of growth along the balanced path is then simply,

$$g^* = \eta_L [1 - f(\zeta_L)] S. \quad (44)$$

So, from the Euler equation, in equilibrium the interest rate is $r^* = \rho + \theta g^*$. Since the growth rate, (44), is invariant to ζ_K , it must be that the equilibrium interest rate is also invariant to ζ_K .

Proposition 4 *Labor augmenting financial efficiency cannot change along a BGP; capital augmenting financial efficiency can.*

Proof. Clearly ζ_L cannot change by itself along a BGP. Equation (43) shows that ζ_K and ζ_L can vary in opposite directions such that r_t is constant, but (44) shows that g^* has to change nevertheless since only ζ_L enters into the BGP rate of growth.

When g^* changes, r^* must change also. If ζ_K changes holding ζ_L constant, g^* and r^* are unaffected but (42) implies that $(N_K/N_L)^*$ increases at the point in time when ζ_K changes. The model is stable where $\sigma < 1$ in the sense that starting with some $(N_{K,t}/N_{L,t}) \neq (N_K/N_L)^*$ the economy will converge to the BGP equilibrium (see Acemoglu, 2009). ■

The permanent technological progress that occurs along a BGP must be purely labor-augmenting. That means that labor-augmenting financial efficiency cannot change along a BGP, but it also means that only labor-augmenting financial efficiency can influence the long-run rate of growth.

Despite that, the model contains interesting implications for capital-augmenting financial efficiency. Because $\sigma < 1$, increasing ζ_K means, by equation (43), that the number of capital augmenting blueprints must also increase for r^* to remain constant in the long run, i.e., N_K must increase. So while technological progress is still purely labor-augmenting in the long-run, there can be capital-augmenting technical progress in response to changes in financial efficiency toward capital. In transition to the new the BGP, there will be some $S_K > 0$ allocated to discovering new capital-augmenting blueprints. In other words, the appearance of technological progress in response to an increase in financial efficiency in the capital sector has, in the long-run, no effect on balanced growth. *Because* growth is purely labor-augmenting, the financial efficiency which enters into the discovery of capital-augmenting blueprints can vary along a constant BGP, but cannot change what that BGP is.

5 Concluding Remarks

A variant of the Uzawa theorem permits us to qualify the generally assumed complementarity between empirical and theoretical results on finance and growth. Along a constant growth path, a long-run positive correlation between financial efficiency and depth is present only when we consider that access to financial services can

vary. Moreover, the ways in which efficiency and depth can be related when the long-run growth rate changes are also limited: A positive relationship between growth, efficiency and depth is only observed when access to financial services is increasing from a low level.

The results point in a number of potentially fruitful research directions. The importance of access in explaining a positive correlation between efficiency and depth suggests one route to a more nuanced understanding of the ways in which finance can matter for growth. Data such as Kendall et al. (2010) which measures access to finance across countries is well placed to enable such research. The importance of a distinction between financial efficiency which separately benefit the discovery of capital- and labor-augmenting technologies also suggests directions for future theoretical and empirical research. Models in which the direction of financial innovation matters could follow from work such as Michalopoulos et al. (2009), and might lead to a richer understanding of the nature of financial development in economies which go through a period of takeoff in long-run growth.

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