Reconfigurable Aperiodic Array Synthesis by Compressive Sensing

Carlo Bencivenni[1], Marianna V. Ivashina[1], and Rob Maaskant[1]

[1]Department of Signals and Systems, Chalmers University of Technology, Gothenburg, Sweden
Email: carlo.bencivenni@chalmers.se, marianna.ivashina@chalmers.se, rob.maaskant@chalmers.se

***Abstract*—Aperiodic arrays represent an attractive technology for applications requiring multiple pencil beams or contour beams, such as in radars, satellite communication and mw-sensor systems. These antennas are typically designed to either produce high-directivity beams over a given scan range or a single beam with a specified complex shape. In this manuscript we present a CS approach for the synthesis of a single aperiodic array layout capable of radiating multiple beams with different shapes The approach aims at designing reconfigurable arrays with least number of elements as well as the optimal excitation set for each of the desired beams. Preliminary results for an array providing both pencil and a flattop coverage are presented.**

***Index Terms*—sparse array, aperiodic array, Compressive Sensing, multi-beam, shaped beam, reconfigurable array.**

I. Introduction

Antenna arrays offer many attractive capabilities including electronic beam scanning and shaping, redundancy and power pooling. However arrays are typically more expensive than single antennas due to the higher complexity. Especially in active arrays, where each element has dedicated electronics, the total cost is strongly dependent on the number of antenna elements. Aperiodic arrays can significantly reduce the number of elements and thus the costs, however designing an optimal array is challenging. Generally two approaches have been successfully adopted: (i) general-purpose global optimization methods (such as Genetic Algorithm [1], Invasive Weed Optimization [2] and Particle Swarms [3]), and; (ii) deterministic techniques (such as Density Tapered method [4], Matrix Pencil Method [5] and Iterative Fourier Technique [6]). The first are flexible and accurate but often too computational expensive to design large arrays, the second are effective and intuitive but often too simplistic to include additional effects such as mutual coupling.

Compressive Sensing (CS) has been recently proposed as an alternative approach to the synthesis of aperiodic arrays [7]. This formulation admits a fast and deterministic solution while preserving the flexibility of the method, thus challenging the limitations of the other two approaches. In previous publications the authors have: (i) hybridized the method with full-wave EM analysis to include mutual coupling effects in the design of the array [8]; (ii) extended the formulation for accurate beam scanning optimization [9]; (iii) considered the properties of symmetrical and modular designs [9]; and (iv) studied multi element-type arrays [10].

In this manuscript we focus on the synthesis of reconfigurable aperiodic arrays, i.e. arrays designed for an arbitrary number of desired beams. Regardless of the approach adopted, the array layout (and the reduction of number of elements) is dependent on the desired radiation pattern. Since most aperiodic arrays layouts are designed for one beam shape (or beam direction) only, they are not necessarily optimal to provide multiple types of beams, so that their reconfigurability capabilities are reduced. The problem is thus that of designing a unique array layout with the least number of elements, capable of switching between the desired beams by appropriately changing the excitation coefficients. Research in this direction have been using global optimization methods [11] as well as on deterministic methods such as the Extended Matrix Pencil Method [12] and phase-only reconfigurable Density tapered arrays [13], or limited to zoomable beams [14], [15]. In this manuscript we propose an alternative method to the problem of multi-beam synthesis based on Compressive Sensing. The method can be coupled with the other extensions already introduced in the previous manuscripts to obtain a powerful framework for the design of complex aperiodic arrays. Preliminary results for an array providing both pencil and flat top converge are presented.

II. Method

The problem of designing a Maximally Sparse Array (MSA), i.e. the array with the least number of elements for a desired radiation mask, can be interpreted according to Compressive Sensing as finding the minimum set of *samples* required to obtain a desired radiation pattern [7].

Let $\mathbf{w} \in \mathbb{C}^N$ be the excitation vector of the (very dense) space samples and $\mathbf{f}(\hat{\boldsymbol{r}})$ the vector of the corresponding scalar (copolar) far-field patterns. The problem of designing a MSA can be stated as solving argmin $\|\mathbf{w}\|_{\ell_0}$, subject to some constraint on the radiation pattern $f(\hat{\boldsymbol{r}}) = \mathbf{w}^T\mathbf{f}(\hat{\boldsymbol{r}})$. Unfortunately such problem cannot be solved directly, thus an iterative approximation is adopted as argmin $\|\mathbf{Z}^i\mathbf{w}^i\|_{\ell_1}$, where $\mathbf{Z}^i$ is the diagonal matrix for iteration i, whose elements $\mathbf{z}^i = 1/(|\mathbf{w}^{(i-1)}| + \epsilon)$ are chosen to maximally enhance the sparsity of the solution by suppressing redundant element but allowing an evolution of the solution during the iteration [8]. The latter formulation is convex, thus general-purpose fast and deterministic methods can be used, additionally only few

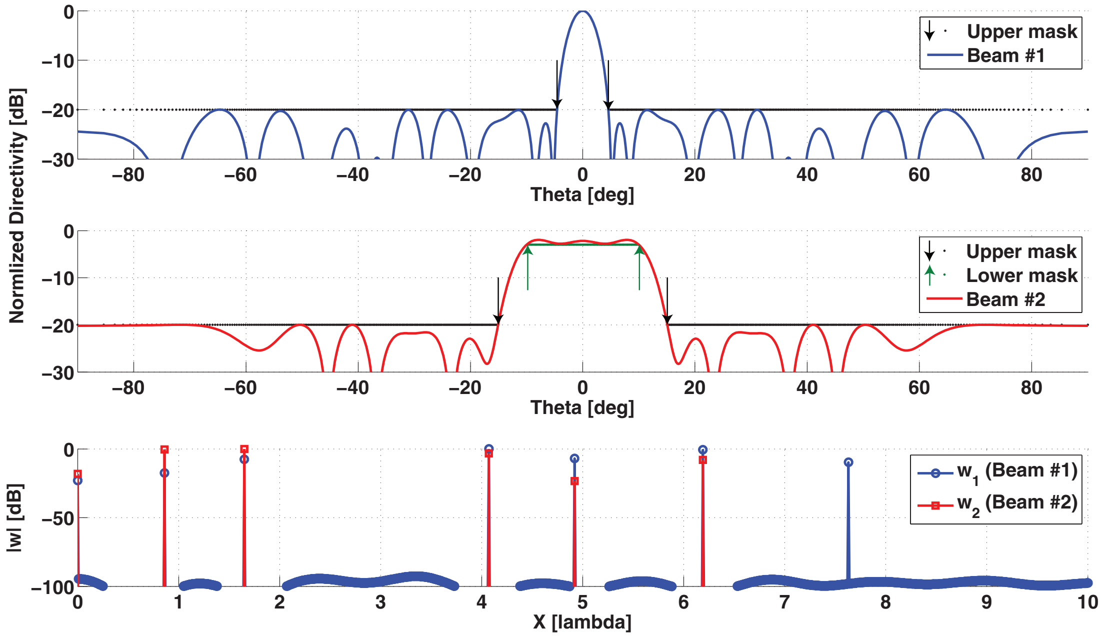

Fig. 1. Far-field pattern for beam (i) and (ii) and their respective excitation amplitudes.

iterations are required to reach convergence.

To extend the analysis to multibeam arrays, multiple excitation coefficients and their respective radiation pattern are introduced in the synthesis. Additionally, the formulation is modified to enforce a common element position for all beams while preserving the convexity of the problem, so that fast deterministic methods can be used.

Let us consider $b = 1, \ldots, B$ shaped beams and their respective excitation coefficients $\mathbf{w}_b$, then the synthesis problem can be written as:

$$\underset{\mathbf{w}^i \in \mathbb{C}^N}{\operatorname{argmin}} \max \left\{ \|\mathbf{Z}_1^i \mathbf{w}_1^i\|_{\ell_1}, \|\mathbf{Z}_2^i \mathbf{w}_2^i\|_{\ell_1}, \ldots, \|\mathbf{Z}_B^i \mathbf{w}_B^i\|_{\ell_1} \right\} \quad \text{(1a)}$$

$$\text{subject to} \begin{cases} f^b(\hat{\boldsymbol{r}}_0) = 1, & \text{normalize} \quad \text{(1b)} \\ |f^b(\hat{\boldsymbol{r}})|^2 \leq M^b(\hat{\boldsymbol{r}}), & \text{upper mask} \quad \text{(1c)} \\ 2\Re(f^b(\hat{\boldsymbol{r}})) \geq L^b(\hat{\boldsymbol{r}}), & \text{lower mask} \quad \text{(1d)} \\ |f^b(\hat{\boldsymbol{r}}) - F^b(\hat{\boldsymbol{r}})|^2 \leq \delta, & \text{ref. pattern} \quad \text{(1e)} \\ \quad b = 1, \ldots, B \end{cases}$$

where constraint (1b) is the normalized broadside directivity, M^b in (1c) is the maximum radiation mask, L^b in (1d) is the minimum radiation mask (valid for symmetric conjugate arrays only) and F^b in (1e) is a reference pattern.

Note that this formulation allows for independent amplitude and phases for the different sets of excitation coefficients, thus allowing in theory an arbitrary number B of shaped beams to be realized. Additionally, it is worth noticing that the goal of the optimization is to minimize the number of total antenna elements required to realize all the beams, thus layouts where most elements are used by each beams are in general preferred, although neither elements are precluded from being dedicated to one or a part of the beams only.

III. Results

Preliminary results of the proposed approach are shown for a reconfigurable symmetrical linear array of isotropic radiating sources. An aperture of diameter $d = 20\lambda$ is densely sampled with a step size $\Delta x = \lambda/100$. The array is designed to provide two beams: (i) a pencil beam with 20 dB SLL from $4.5°$; and (ii) a flat top beam with a maximum variation of 1 dB over $\pm 10°$ and a 20dB SLL from $15°$. Accordingly, constraints (1b) and (1c) have been used for the first beam and (1c) and (1d) for the second beam. In Figure 1 the target masks and far-field patterns for beam (i) and (ii) are shown, as well as the respective element's excitation amplitudes for the positive element positions. Both patterns are compliant with the desired radiation mask and the resulting element positions are 13, with 11 elements being shared by both beams while the two most outer ones are dedicated to the narrow pencil beam only.

IV. Conclusion

An extension of the earlier developed Compressive Sensing approach to the synthesis of reconfigurable arrays has been presented. The method aims at designing an array with the minimum number of elements capable of switching between an arbitrary set of shaped beams with the appropriate excitation coefficients. The method has been demonstrated in the synthesis of a linear array reconfigurable between pencil beam and flat top coverage. The algorithm successfully returned the common element positions and the relative excitation coefficients for the two beams.

In the final paper the authors will asses the effectiveness of the proposed approach based upon the most important aspects of the final array, such as the element number, pattern shape and number of beams. Results will be compared both with the earlier single-beam Compressive Sensing design approach as well as with other published work on reconfigurable aperiodic arrays [12], [13]. Preliminary results suggest that the method should be very effective in finding the minimum number of elements as well as creating multiple beams with respect to other approaches. However, the choice of variable amplitude excitations for each beam poses additional complexity on the array design with respect to phase only reconfigurable solutions, thus the authors will provide an analysis of the advantages and limitations of the approach.

REFERENCES

[1] R. L. Haupt, "Thinned arrays using genetic algorithms," *IEEE Trans. Antennas Propag.*, vol. 42, no. 7, pp. 993–999, 1994.

[2] G. G. Roy, S. Das, P. Chakraborty, and P. N. Suganthan, "Design of non-uniform circular antenna arrays using a modified invasive weed optimization algorithm," *IEEE Trans. Antennas Propag.*, vol. 59, no. 1, pp. 110–118, 2011.

[3] M. M. Khodier and C. G. Christodoulou, "Linear array geometry synthesis with minimum sidelobe level and null control using particle swarm optimization," *IEEE Trans. Antennas Propag.*, vol. 53, no. 8, pp. 2674–2679, 2005.

[4] O. Bucci, S. Perna, and D. Pinchera, "Advances in the deterministic synthesis of uniform amplitude pencil beam concentric ring arrays," *IEEE Trans. Antennas Propag.*, vol. 60, no. 7, pp. 3504–3509, 2012.

[5] Y. Liu, Z. Nie, and Q.-H. Liu, "Reducing the number of elements in a linear antenna array by the matrix pencil method," *Antennas and Propagation, IEEE Transactions on*, vol. 56, no. 9, pp. 2955–2962, 2008.

[6] W. P. M. N. Keizer, "Large planar array thinning using iterative fft techniques," *IEEE Trans. Antennas Propag.*, vol. 57, no. 10, pp. 3359–3362, Oct 2009.

[7] G. Prisco and M. D'Urso, "Maximally sparse arrays via sequential convex optimizations," *IEEE Antennas Wireless Propag. Lett.*, vol. 11, pp. 192–195, 2012.

[8] C. Bencivenni, M. Ivashina, R. Maaskant, and J. Wettergren, "Design of maximally sparse antenna arrays in the presence of mutual coupling," *IEEE Antennas Wireless Propag. Lett.*, vol. 14, pp. 159–162, 2015.

[9] ——, "Fast synthesis of wide-scan-angle maximally sparse array antennas using compressive-sensing and full-wave em-analysis," *IEEE Trans. Antennas Propag.*, Submitted 2015.

[10] C. Bencivenni, M. Ivashina, and R. Maaskant, "Multi-element aperiodic array synthesis by compressive sensing," in *ICEAA*, Sep. 2015.

[11] Y. Liu, Y.-C. Jiao, Y.-M. Zhang, and Y.-Y. Tan, "Synthesis of phase-only reconfigurable linear arrays using multiobjective invasive weed optimization based on decomposition," *International Journal of Antennas and Propagation*, vol. 2014, no. 630529, p. 11, 2014.

[12] Y. Liu, Q. H. Liu, and Z. Nie, "Reducing the number of elements in multiple-pattern linear arrays by the extended matrix pencil methods," *IEEE Trans. Antennas Propag.*, vol. 62, no. 2, pp. 652–660, Feb. 2014.

[13] A. Morabito, A. Massa, P. Rocca, and T. Isernia, "An effective approach to the synthesis of phase-only reconfigurable linear arrays," *IEEE Trans. Antennas Propag.*, vol. 60, no. 8, pp. 3622–3631, Aug. 2012.

[14] O. Bucci, S. Perna, and D. Pinchera, "Simultaneous radiation of narrow and wide beams exploiting two concentric isophoric sparse arrays," in *Antennas and Propagation (EuCAP), 2015 9th European Conference on*, apr 2015, pp. 1–4.

[15] ——, "Synthesis of isophoric sparse arrays allowing zoomable beams and arbitrary coverage in satellite communications," *IEEE Trans. Antennas Propag.*, vol. 63, no. 4, pp. 1445–1457, Apr. 2015.