Synthesis of Circular Isophoric Sparse Arrays by using Compressive-Sensing

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***Abstract*—A design approach for large-scale sparse arrays based on Compressive Sensing has been recently introduced in the literature and extended to include complex EM effects and scan performance. However, that approach cannot directly control the number of excitation amplitudes. Here, we apply a two-step procedure that first synthesizes continuous rings with unconstrained amplitudes using an iterative ℓ_1-norm minimization approach, and then replaces them with a circular isophoric ring array with a number of elements proportional to the original amplitude of each ring. The procedure is demonstrated for an isotropic array of a 10λ radius, for which a reference solution based on the analytical density-taper approach is available in the literature. Results show the capability of the proposed method to achieve a significant reduction of the array aperture (20%) with 25% less elements or 4dB lower peak side lobe level.**

***Index Terms*—sparse array antennas, isophoric array, compressive sensing.**

I. Introduction

Active arrays offer many attractive capabilities over classical antenna systems, including beam agility, power pooling and redundancy. However these are expensive and complex systems where the main cost driver are the number of active elements. Aperiodic sparse array can drastically reduce the number of elements and thus are very attractive. Furthermore, it is important to operate at saturation in transmitting active arrays in order to achieve maximum amplifier efficiency. Due to the complexity of deploying one amplifier type for each single excitation level it is often desirable to limit the number of amplitude levels in the array design. In this regard, isophoric arrays, i.e. those employing one single amplitude level, are the most attractive from a design complexity point of view.

Aperiodic array synthesis methods include global optimization techniques, e.g. Genetic Algorithms [1], and analytical methods, e.g. the Density Taper [2]. The former ones are most effective in minimizing the number of elements and are also flexible, but highly computationally expensive, and thereby suitable only for small-to-medium sized arrays only; the latter, on the other hand, are very effective in handling large problems, but less performing in reducing the number of elements and limited to simple design specifications. A recently proposed Compressive Sensing (CS) [3] has been successfully used and proven to optimally combine the advantages of both types of methods [4]. However, the method cannot directly control the number of excitation amplitudes.

In this manuscript we demonstrate a two-step procedure for the synthesis of sparse circular isophoric arrays which combines the ℓ_1-norm minimization with the density taper approach, following the lines of [5]. First we employ CS in the synthesis of an array of continuous rings to find the optimal ring positions and excitations. Then we approximate the continuous rings with arrays of regular isophoric elements distributed proportionally to the original amplitude.

II. Method

A. Step I: Continuous ring array synthesis

The first step of the approach involves the synthesis of a continuous ring array. The scalar far-field pattern of an array of R continuous rings in the direction θ can be written as

$$f(\theta) = \sum_{r=1}^{R} w_r J_0(ka_r \sin\theta), \tag{1}$$

where w_r and a_r are the excitation coefficient and radius of ring r, respectively, and J_0 is the zero-order Bessel function.

According to [3], the problem of finding the sparsest array can be cast in an iterative convex form, where the i^{th} iteration reads

$$\underset{\mathbf{w}^i \in \mathbb{C}^N}{\text{argmin}} \ \|\mathbf{Z}^i \mathbf{w}^i\|_{\ell_1}, \text{ subject to } \begin{cases} f(\theta_\text{s}) = 1, \\ |f(\theta)|^2 \le M(\theta), \theta \in \text{mask} \end{cases}$$

where $\mathbf{w} = [w_1, w_2, \ldots, w_R]^T$ is the excitation vector and $\mathbf{Z}^i$ is a diagonal matrix with entries $\mathbf{z}^i = 1/(|\mathbf{w}^{(i-1)}| + \epsilon)$, chosen as to maximally enhance the sparsity of the solution and is selected based on the recommendation of [3]. Note that this returns both the position as well as the excitation of each ring.

B. Step II: Isophoric ring array synthesis

The continuous rings are then transformed into discrete isophoric ring arrays. The pattern of an array of rings of uniformly excited equi-spaced isotropic elements can be written as [6]

$$F(\theta,\phi) = \sum_{r=1}^{R} W_r N_r \sum_{t=-\infty}^{\infty} J_{tN_r}(ka_r \sin\theta) e^{jtN_r(\pi/2-\phi)} \tag{2}$$

where N_r and W_r are the number of elements and the prescribed excitation level of ring r respectively. It is worth noticing that expressions (1) and (2) are equivalent if:

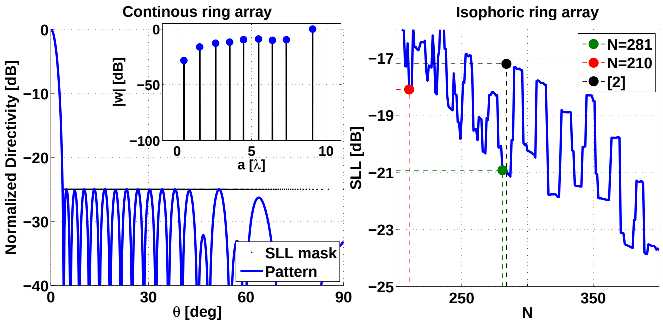

Fig. 1. (left) Continuous ring array far-field pattern and excitation amplitudes. (right) Isophoric ring array sidelobe level against the number of elements N.

1) *Only the zero-order term of t in* (2) *is significant.* Bessel functions rapidly tend to zero when the argument is smaller theirs the order. The first non-zero term is negligible if $\sin\theta \ll N_r/ka_r$.
2) *The excitation coefficients are chosen as $W_r N_r \propto w_r$*

Then, for the isophoric case, i.e. $\{W_r = 1\}_{r=1}^{R}$, the number of elements per ring should be chosen proportional to the respective ring amplitude. Thus, given an arbitrary total number of elements N, the number of elements per ring are [5]

$$N_r = \left\lfloor \left(N - \sum_{l=1}^{l<r} N_l \right) \frac{|w_r|}{\sum_{m=1}^{K} |w_m|} \right\rfloor, \quad r = 1...R \quad (3)$$

where $\lfloor \bullet \rfloor$ indicates the floor function. It is then possible to sweep the total number of elements and select the array layout with the appropriate number of elements.

III. Results

The proposed method has been demonstrated for an isophoric circular array of isotropic sources with a constant side lobe level (SLL) of -25dB for $\theta > 3.45°$. While in [2] and [5] a reference Taylor distribution of radius $R = 10\lambda$ is used, we can reduce the aperture radius to $R = 9.1\lambda$ with uniform mask constraint and meet the target radiation mask.

The initial continuous ring array far-field pattern (ϕ invariant) and excitation amplitudes are shown in Fig. 1 (left). Fig. 1 (right) illustrates the resulting peak side lobe (over every ϕ) versus the total number of elements, distributed according to (3). As expected, the SLL converges to the target value for increasing number of elements, although the constructive/destructive summation of the higher order terms of (2) is the cause for the oscillatory component.

From Fig. 1 (right), two designs (green and red) are selected and compared to [2] (black). The green-line design, shown in Fig. 2, has approximately the same number of elements as that in [2] (281 and 286), but exhibits better performance in the peak side lobe (-21db against -17dB) as well as total radiated radiated power above the target level. The red-line design, shown in Fig. 3, has a similar peak side lobe level (-18dB and -17dB) with a significantly reduced number of elements (210 against 286).

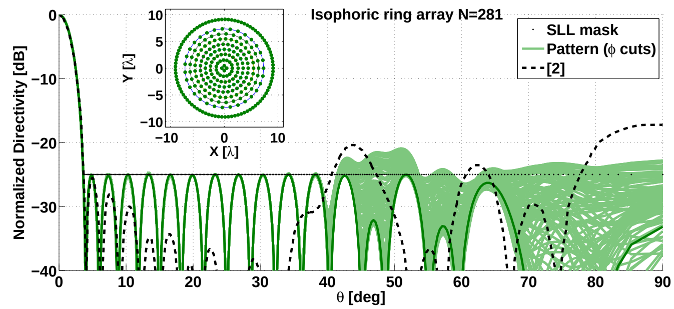

Fig. 2. Far-field pattern and array layout for $N = 281$ elements.

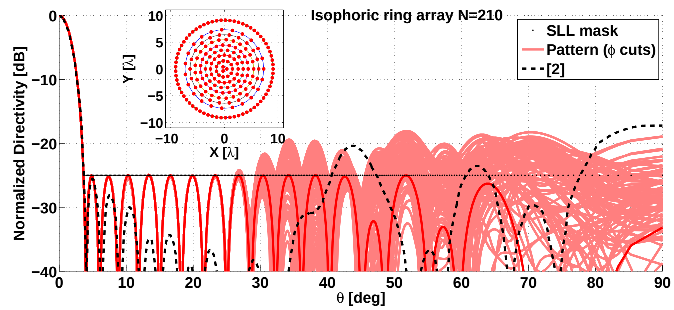

Fig. 3. Far-field pattern and array layout for $N = 210$ elements.

IV. Conclusions

We have demonstrated the synthesis of isophoric ring array by using the ℓ_1-norm minimization method combined with a density taper approach. Results show a 20% relative reduction of the aperture area w.r.t. the reference case obtained with the analytical density taper approach [2] along with a SLL improvement from -17dB to -21dB for the same number of elements. Another way of interpreting these results is that for a given SLL, one can achieve the aforementioned aperture reduction as well as 25% reduction in the number of antenna elements. Results show how this approach can outperform well-established analytical techniques in terms of element reduction while offering high computational efficiency w.r.t. global optimization methods. Further work includes extending the method to an arbitrary number of amplitudes to further decrease the number of elements and improve the SLL.

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