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The Provision and Pricing of Excludable Public Goods: Ramsey-Boiteux Pricing versus Bundling*

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Abstract

This paper studies the relation between Bayesian mechanism design and the Ramsey-Boiteux approach to the provision and pricing of excludable public goods. For a large economy with private information about individual preferences, the two approaches are shown to be equivalent if and only if, in addition to incentive compatibility and participation constraints, the final allocation of private-good consumption and admission tickets to public goods satisfies a condition of *renegotiation proofness.* Without this condition, a mechanism involving *mixed bundling*, i.e. combination tickets at a discount, is superior.

Key Words: Mechanism Design, Excludable Public Goods, Ramsey-Boiteux Pricing, Renegotiation Proofness, Bundling

JEL Classification: D61, H21, H41,H42

1 Introduction

This paper studies the relation between the Bayesian mechanism design approach to the provision and financing of public goods and the Ramsey-Boiteux approach to public-sector pricing under a government budget constraint. In the tradition of public economics, these two approaches have

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developed separately and focus on different issues. However, there is an area of overlap. For *excludable* public goods such as parks and highways, one can charge admission fees and exclude people who do not pay these fees. For such goods, therefore, the question arises what fees are appropriate. This paper investigates the conditions under which the two approaches give the same answer to this question. It also investigates why they might sometimes give different answers.

The mechanism design approach focuses on the revelation of preferences as a basis determining the level of public-good provision, as well as each participant's financial contribution. The Ramsey-Boiteux approach focuses on the tradeoff between revenue contributions and efficiency losses associated with different financing instruments. In the mechanism design approach, the form of a payment scheme is determined endogenously, as part of the solution to the given incentive problem. In the Ramsey-Boiteux approach, the form of the payment scheme is taken as given.

However, under some conditions, the subject of enquiry of the two approaches is exactly the same. To see this, consider a large economy in which people have private information about their public-goods preferences (independent private values), but through a large-numbers effect, the cross-section distribution of preferences is fixed and commonly known. In this case, the assessment of alternative levels of public-good provision is unencumbered by information problems. However, any attempt to relate financial contributions to the benefits that people draw from the public goods is hampered by the fact that information about their preferences is private. If everybody is allowed free access to all public goods, the only incentive-compatible financing scheme stipulates equal lump-sum payments from all individuals, whether they benefit from the public goods or not. Under such a financing scheme, people that have no desire for the public goods at all are negatively affected by their provision, and their participation must be based on coercion rather than voluntary agreement.[1] If coercion is to be avoided, one must take recourse to admission fees supported by the possibility of individual exclusion.

Thus, Schmitz (1997) and Norman (2004) have argued that, to avoid coercion, an excludable public good should be financed by admission fees if this is possible. When there is nonrivalry in consumption, admission fees induce an inefficiency, but, in the absence of other sources of funds, this inefficiency is unavoidable if the public good is to be provided at all.

[1] Thus, Mailath and Postlewaite (1990) show that, in a large economy, there is no way at all to provide a non-excludable public good on a voluntary basis.

Without coercion, the public good has to be financed from payments that people are willing to make in order to enhance their prospects of benefitting from it. In the large economy, the only such motivation comes from the desire to avoid being excluded, i.e., the willingness to pay for admission. By using admission fees to finance the public good, one avoids the inefficiency arising from not having the public good at all. Having the public good and excluding the people who are not willing to pay the fee is still better than not having it at all.

The argument is the same as the one that underlies the Ramsey-Boiteux theory of optimal deviations from marginal-cost pricing when fixed costs of production must be recovered.[2] In fact, the distinction between the two specifications is merely a matter of semantics. If we think about "access to the public good" as a private good, then the production of this private good involves a fixed cost, namely the cost of installing the public good, and zero variable costs.

This connection deepens our understanding of both approaches. Regarding the analysis of Bayesian mechanisms, one learns that the imposition of interim participation constraints (in addition to feasibility) is equivalent to the imposition of a "government budget constraint" with a ban on lump-sum taxation. Regarding the Ramsey-Boiteux analysis, one learns that, when there is a single excludable public good, Ramsey-Boiteux pricing can be identified with a second-best incentive mechanism. Whereas the Ramsey-Boiteux analysis takes the form of the payment scheme - and thereby the form of the allocation mechanism - as given, the analysis of Schmitz (1997) and Norman (2004) shows that the Ramsey-Boiteux solution cannot be improved upon, even if one allows for more general payment schemes and allocation mechanisms. In particular, nothing is to be gained by allowing for the possibility of providing people with lotteries over admissions and having them pay in accordance with the admission probabilities that are generated by the lotteries.

The case of multiple public goods is more interesting. For a single excludable public good, the Ramsey-Boiteux analysis implies only that, if the public good is provided at all, then the admission fee should be set equal to the lowest value at which provision costs can be covered.[3] For multiple excludable public goods, there is an additional degree of freedom because the government budget constraint requires only that total revenues cover

[2] The link between excludable public goods and the Ramsey-Boiteux pricing problem has been pointed out by Samuelson (1958, 1969) and Laffont (1982/1988); see also Drèze (1980).

[3] This prescription of course much older. It goes back at least to Dupuit (1844).

total costs. This constraint allows for the possibility of cross-subsidization between the different public goods, a possibility that has traditionally not been considered in the analysis of public-good provision.

This paper shows that, when there are *multiple* excludable public goods, the Bayesian mechanism design approach and the Ramsey-Boiteux approach are equivalent *if and only if*, in addition to incentive compatibility and participation constraints, the final allocation of private goods and of admission tickets for the excludable public goods is required to satisfy a condition of *renegotiation proofness.* An allocation satisfies this condition if it does not leave any room for Pareto improvements through incentive-compatible side-trading among the participants.

The new equivalence result further deepens our understanding of both approaches. For the mechanism design approach to public-goods provision, one learns that cross-subsidization between public goods may be useful. Traditional models of a single public good do not make room for this possibility. Such models therefore miss an important aspect of the problem, important, that is, in a second-best world where public-goods finance must not rely on coercion.

The equivalence result of this paper also shows that, for multiple public goods, the Ramsey-Boiteux approach is in an important way more restrictive than the general second-best mechanism design approach. In assuming that payments are given by a vector of admission fees, the Ramsey-Boiteux approach excludes many mechanisms *a priori*, mechanisms that involve lotteries, as well as mechanisms that involve bundling, e.g., a mechanism that provides combination tickets to an opera performance and a football match at a discount relative to the prices of the separate tickets. The mechanisms that are thus excluded are important because, by contrast to the case of a single public good, they tend to dominate the Ramsey-Boiteux solution. Such mechanisms are, however, vulnerable to side-trading among the participants.

If the mechanism designer is unable to prevent people from frictionless side-trading, the allocation of private goods and of admission tickets for the excludable public goods that he stipulates will be the final one if and only if it is renegotiation-proof, i.e., it does not leave any room for incentive-compatible, Pareto-improving trades among the participants. In the large economy, this renegotiation proofness condition is satisfied if and only if the final allocation of private goods and of admission tickets for public goods is Walrasian. The associated price vector is precisely the vector of consumer prices that the Ramsey-Boiteux theory is concerned with. A Bayesian mechanism that satisfies renegotiation proofness, as well as interim incentive compatibility and individual rationality, is thus identified with a vector of admission prices. The mechanism design problem then is equivalent to the corresponding Ramsey-Boiteux problem.

In this problem, a vector of optimal admission fees must satisfy a version of the well-known inverse-elasticities rule, i.e., across the different public goods, markups over marginal costs (zero) must be inversely proportional to the elasticities of demand. Given this rule, there usually is cross-subsidization between the public goods, i.e., there is no presumption that, for any one of them, admission fee revenues just cover costs.

Admission fee revenues on excludable public goods can also be used to finance nonexcludable public goods. This cross-subsidization eliminates the problem identified by Mailath and Postlewaite (1990) that, in a large economy with private information, the provision of nonexcludable public goods cannot be financed at all without coercion. The inverse-elasticities rule for excludable public goods is unaffected.

If renegotiation proofness is *not* imposed, a second-best mechanism can require pure or mixed bundling and even a randomization of admissions. Fang and Norman (2003/2006) have shown that under certain assumptions about the underlying data, the Ramsey-Boiteux solution sometimes is dominated by a mechanism involving *pure bundling* in the sense that consumers are offered admission to all public goods at once or to none. This paper shows that, if the valuations for the different public goods are stochastically independent, then a mechanism involving *mixed bundling,* i.e. a scheme where bundles of public goods come at a discount relative to their individual components, *always* dominates the optimal renegotiation-proof mechanism, i.e. the optimal Ramsey-Boiteux solution. The reason is that the demand for a bundle is more elastic than the demands for the individual components. A discount on the bundle is therefore mandated by the very logic that underlies the inverse-elasticities rule of the Ramsey-Boiteux approach itself. The result parallels similar results of McAfee *et al.* (1989) and Manelli and Vincent (2006 a) for a multiproduct monopoly. It implies that the requirement of renegotiation proofness is *necessary*, as well as sufficient, for the applicability of the Ramsey-Boiteux approach.

Altogether, the analysis combines three ideas. First, the combination of interim participation and feasibility constraints in the Bayesian mechanism design problem with private information is equivalent to the imposition of a "government budget constraint" à la Ramsey-Boiteux. Second, with frictionless side-trading, the final allocation in the large economy must be Walrasian, i.e. supported by a price system which does not leave any room for arbitrage. Third, if side-trading is infeasible, mixed bundling can be used to raise profits. Each of these ideas has been around before: The equivalence of interim participation constraints and the "government budget constraint" is pointed out in Hellwig (2003). The constraints that frictionless side-trading imposes on mechanism design have been studied by Hammond (1979, 1987) and Guesnerie (1995). In the multiproduct monopoly literature, the advantages of mixed bundling have been pointed out by McAfee *et al*. (1989) and Manelli and Vincent (2006 a). This paper's contribution is to pull these ideas together for a precise characterization of the relation between the Ramsey-Boiteux approach and the Bayesian mechanism design approach to the provision and financing of multiple excludable public goods.

The Ramsey-Boiteux approach itself has been criticized by Atkinson and Stiglitz (1976) because it ignores the possibility of raising funds through from lump sum taxes or income taxes and because, therefore, the government budget constraint that it imposes induces unnecessary inefficiencies. In principle, this critique also applies in the context of public-good provision.[4] However, in the context of public-goods provision, it is natural to assume that different people have different preferences and that their preferences are their private information. After all, this is the essence of the standard mechanism design problem for public-goods provision. When this assumption is imposed, the Atkinson-Stiglitz critique necessitates the government's using its powers of coercion so that even people who don't benefit from the public goods at all are made to pay the taxes.[5]

The use of coercion to levy contributions from people who do not benefit from the public goods at all raises concerns about equity as well as the possibility of power abuse.[6] The use of coercion is also incompatible with the contractarian approach to government that stood behind Lindahl's (1919) development of the theory of public goods.[7] Given that governments usually do have a power of coercion, Lindahl's approach may seem unrealistic. Even so, there is some interest in understanding its implications. One may also wish to follow Boiteux (1956) and look at a self-financing requirement for providers of public goods as a way to avoid the (unmodelled) adverse

[4] See Christiansen (1981), Boadway and Keen (1993).

[5] This assessment is robust to th e introduction of an income tax. Hellwig (2004) expands the analysis developed here to allow for endogenous production with different people having different productivities, as in Mirrlees (1971). The availability of funds from income taxation eases the government budget constraint, but it does not, in general, eliminate the inefficiencies that it induces. Incentive and participation constraints can still preclude the attainment of a first best allocation, in which case it is desirable to have admission fees, as well as income taxes for public-goods finance.

[6] On equity, see Hellwig (2005); on power abuse, see Bierbrauer (2002).

[7] For an extensive account, see Musgrave (1959), Ch. 4.

incentive effects that would come from an unlimited access to the public purse.

In the following, Section 2 lays out the basic model of a large economy with private information about individual preferences. Section 3 establishes the equivalence of the Bayesian mechanism design problem with interim participation constraints and the Ramsey-Boiteux problem under the renegotiation proofness condition. Section 4 shows that the equivalence of the two approaches breaks down and that some form of bundling dominates Ramsey-Boiteux pricing if renegotiation proofness fails. Section 5 shows that the inverse-elasticities rule of the original Ramsey-Boiteux analysis is replaced by a *weighted* inverse-elasticities rule if the mechanism designer is *inequality averse,* the weights taking account of differences in marginal social valuations attached to the consumers of the different public goods. If inequality aversion is sufficiently large, then, as in Hellwig (2005), the desire for redistribution may replace the interim participation constraints and the "government budget constraint" that they induce as a rationale for admission fees. Proofs are given in Appendix A.

2 Bayesian Mechanism Design in a Model with Multiple Public Goods

2.1 The Model

I study public-good provision in a large economy with one private good and m public goods. The public goods are excludable. An allocation must determine provision levels $Q_1, ..., Q_m$ for the public goods and, for each individual h in the economy, an amount c^h of private-good consumption and a set J^h of public goods to which the individual is admitted. Given c^h, J^h, and $Q_1, ..., Q_m$, the consumer obtains the payoff

$$c^h + \sum_{i \in J^h} \theta_i^h Q_i. \tag{2.1}$$

The vector $\boldsymbol{\theta}^h = (\theta_1^h, ..., \theta_m^h)$ of parameters determining the consumer's preferences for the different public goods is the realization of a random variable $\tilde{\boldsymbol{\theta}}^h$, taking values in $[0,1]^m$, which is defined on some underlying probability space $(X, \mathcal{F}, P)$. Private-good consumption and public-goods admissions will typically be made to depend on $\boldsymbol{\theta}^h$. In addition, they will also be allowed to depend on the realization ω^h of a further random variable $\tilde{\omega}^h$, taking values in $[0,1]$. This random variable is introduced to allow for the possibility of individual randomization in public-good admissions and private-good consumption.

The random variables $\tilde{\boldsymbol{\theta}}^h$ and $\tilde{\omega}^h$ are assumed to be independent. Their distributions F and ν are assumed to be the same for all agents. Moreover, the distribution F of the vector $\tilde{\boldsymbol{\theta}}^h$ of preference parameters has a strictly positive, continuously differentiable density $f(\cdot)$. For $i = 1, ...m$, the marginal distribution of $\tilde{\theta}_i$, the i-th component of the random vector $\tilde{\boldsymbol{\theta}}$, is denoted as F_i, its density as f_i.

The set of participants is modelled as an atomless measure space $(H, \mathcal{H}, \eta)$. I assume a large-numbers effect whereby the cross-section distribution of the pair $(\tilde{\boldsymbol{\theta}}^h(x), \tilde{\omega}^h(x))$ in the population is P-almost surely equal to the probability distribution $F \times \nu$. Thus, for almost every $x \in X$, I postulate that

$$\frac{1}{\eta(H)} \int_H \varphi(\tilde{\boldsymbol{\theta}}^h(x), \tilde{\omega}^h(x)) d\eta(h) = \int_{[0,1]^{m+1}} \varphi(\boldsymbol{\theta}, \omega) dF(\boldsymbol{\theta}) d\nu(\omega) \tag{2.2}$$

for every $F \times \nu$-integrable function φ from $[0, 1]^{m+1}$ into $\Re$.[8]

I restrict the analysis to allocations that satisfy an *ex-ante neutrality* or *anonymity* condition. The level c^h of an individual's private-good consumption and the set J^h of public goods to which the individual is admitted are assumed to depend on h and on the state of the world x *only* through the realizations $\tilde{\boldsymbol{\theta}}^h(x) = \boldsymbol{\theta}^h$ and $\tilde{\omega}^h(x) = \omega^h$ of the random variables $\tilde{\boldsymbol{\theta}}^h$ and $\tilde{\omega}^h$. In principle, c^h and J^h should also depend on the cross-section distribution of the other agents' parameter realizations $\tilde{\boldsymbol{\theta}}^{h'}(x) = \boldsymbol{\theta}^{h'}$ and $\tilde{\omega}^{h'}(x) = \omega^{h'}$ in the population, but because this cross-section distribution is constant and independent of x, there is no need to make this dependence explicit. This is a major advantage of working with the large-economy specification with the law of large numbers.

An *allocation* is thus defined as an array

$$(\mathbf{Q}, c(\cdot, \cdot), \chi_1(\cdot, \cdot), ..., \chi_m(\cdot, \cdot)), \tag{2.3}$$

such that $\mathbf{Q} = (Q_1, ..., Q_m)$ is a vector of public-good provision levels, and $c(\cdot, \cdot), \chi_1(\cdot, \cdot), ..., \chi_m(\cdot, \cdot)$ are functions which stipulate for each $(\boldsymbol{\theta}, \omega) \in [0, 1]^{m+1}$, a level $c(\boldsymbol{\theta}, \omega)$ of private-good consumption and indicators $\chi_i(\boldsymbol{\theta}, \omega)$ for admission to public goods $i = 1, ..., m$, to be applied to participant h in

[8] As discussed by Judd (1985), the law-of-large-numbers property (2.2) is consistent with, though not implied by, the stochastic independence of the random pairs $(\tilde{\boldsymbol{\theta}}^h, \tilde{\omega}^h)$, $h \in H$. For a large-economy specification with independence in which the law of large numbers holds as a theorem, see Al-Najjar (2004).

the state x if $(\tilde{\boldsymbol{\theta}}^h(x), \tilde{\omega}^h(x)) = (\boldsymbol{\theta}, \omega)$. The indicator $\chi_i^A(\boldsymbol{\theta}, \omega)$ takes the value one if the consumer is given access and the value zero, if he is not given access to public good i.

The economy has an exogenous production capacity permitting the aggregate consumption Y of the private good if no public goods are provided. If a vector $\mathbf{Q}$ of public-good provision levels is to be provided, an aggregate amount $K(\mathbf{Q})$ of private-good consumption must be foregone. An allocation is *feasible* if

$$\frac{1}{\eta(H)} \int_H c(\tilde{\boldsymbol{\theta}}^h(x), \tilde{\omega}^h(x)) d\eta(h) + K(\mathbf{Q}) \leq Y \tag{2.4}$$

for almost every $x \in X$, so the sum of aggregate consumption and public-good provision costs does not exceed Y. By the large-numbers condition (2.2), this requirement is equivalent to the inequality

$$\int_{[0,1]^{m+1}} c(\boldsymbol{\theta}, \omega) f(\boldsymbol{\theta}) d\boldsymbol{\theta} d\nu(\omega) + K(\mathbf{Q}) \leq Y. \tag{2.5}$$

The cost function $K(\cdot)$ is assumed to be strictly increasing, strictly convex, and twice continuously differentiable, with $K(\mathbf{0}) = 0$, and with partial derivatives $K_i(\cdot)$ such that $\lim_{k \to \infty} K_i(\mathbf{Q}^k) = 0$ for any sequence $\{\mathbf{Q}^k\}$ with $\lim_{k \to \infty} Q_i^k = 0$ and $\lim_{k \to \infty} K_i(\mathbf{Q}^k) = \infty$ for any sequence $\{\mathbf{Q}^k\}$ with $\lim_{k \to \infty} Q_i^k = \infty$.

The allocation $(\mathbf{Q}, c(\cdot,\cdot), \chi_1(\cdot,\cdot), ..., \chi_m(\cdot,\cdot))$ provides consumer h with the *ex ante* expected payoff

$$\int_{[0,1]^{m+1}} [c(\boldsymbol{\theta}, \omega) + \sum_{i=1}^{m} \chi_i(\boldsymbol{\theta}, \omega) \theta_i Q_i] f(\boldsymbol{\theta}) d\boldsymbol{\theta} d\nu(\omega). \tag{2.6}$$

Because of the *ex ante* neutrality property of allocations, (2.6) is independent of h. All participants are therefore in agreement about the *ex ante* ranking of allocations. Taking this ranking as a normative standard, I refer to an allocation as being *first-best* if it maximizes (2.6) over the set of feasible allocations. By (2.2), the *ex ante* expected payoff for any one participant is equal to the aggregate *per capita* payoff

$$\frac{1}{\eta(H)} \int_H [c(\tilde{\boldsymbol{\theta}}^h(x), \tilde{\omega}^h(x)) + \sum_{i=1}^{m} \chi_i(\tilde{\boldsymbol{\theta}}^h(x), \tilde{\omega}^h(x)) \tilde{\theta}_i^h(x) Q_i] d\eta(h) \tag{2.7}$$

for almost every $x \in X$. A first-best allocation therefore is also best if the mechanism designer is concerned with this cross-section aggregate of payoffs in the population. In taking (2.7) or (2.6) to be a suitable welfare indicator, I implicitly assume that there is no risk aversion on the side of participants and no inequality aversion on the side of the mechanism designer.

2.2 First-Best Allocations, Incentive Compatibility, and Individual Rationality

In a slightly more compact notation, the first-best welfare problem is to choose an allocation so as to maximize

$$\int_{[0,1]^m} \left[C(\boldsymbol{\theta}) + \sum_{i=1}^{m} \pi_i(\boldsymbol{\theta})\theta_i Q_i \right] f(\boldsymbol{\theta})d\boldsymbol{\theta} \tag{2.8}$$

under the constraint that

$$\int_{[0,1]^m} C(\boldsymbol{\theta})f(\boldsymbol{\theta})d\boldsymbol{\theta} + K(\mathbf{Q}) \leq Y, \tag{2.9}$$

where

$$C(\boldsymbol{\theta}) := \int_{[0,1]} c(\boldsymbol{\theta},\omega)d\nu(\omega), \tag{2.10}$$

$$\pi_i(\boldsymbol{\theta}) := \int_{[0,1]} \chi_i(\boldsymbol{\theta},\omega)d\nu(\omega) \tag{2.11}$$

are the conditional expectations of a consumer's private-good consumption and admission probability for public good i, given the information that $\tilde{\boldsymbol{\theta}}^h = \boldsymbol{\theta}$.

By standard arguments, one obtains:

Lemma 2.1 *An allocation is first-best if and only if it satisfies the feasibility condition (2.5) with equality and, for $i = 1, ..., m$, one has*

$$K_i(\mathbf{Q}) = \int_0^1 \theta_i \, dF_i(\theta_i), \tag{2.12}$$

and $\pi_i(\boldsymbol{\theta}) = 1$ for almost all $\boldsymbol{\theta} \in [0,1]^m$.[9]

[9] If there was risk aversion on the side of consumers or inequality aversion on the side of the mechanism designer, the conditions of Lemma 2.1 would have to be augmented by a condition equating the social marginal utility of private-good consumption across agents. For details, see Hellwig (2005, 2004).

In a first-best allocation, the ability to exclude people from the enjoyment of a public good is never used. Moreover, the levels of public-good provision are chosen so that, for each i, the marginal cost $K_i(\mathbf{Q})$ of increasing the level at which public good i is provided is equal to the aggregate marginal benefits that consumers in the economy draw from the increase. Given the assumption that $\lim_{k\to\infty} K_i(\mathbf{Q}^k) = 0$ for any i and any sequence $\{\mathbf{Q}^k\}$ with $\lim_{k\to\infty} Q_i^k = 0$, it follows that, in a first-best allocation, provision levels of all public goods are positive, and so is $K(\mathbf{Q})$.

Turning to the specification of information, I assume that each consumer knows the realization $\boldsymbol{\theta}$ of his own preference parameter vector, but, about the random variable $\tilde{\omega}$, he knows nothing beyond the measure ν. The information about $\boldsymbol{\theta}$ is private. Apart from the distribution $F \times \nu$, nobody knows anything about the pair $(\tilde{\boldsymbol{\theta}}, \tilde{\omega})$ pertaining to somebody else. Given this information specification, an allocation is said to be *incentive-compatible* if and only if, for all $\boldsymbol{\theta}$ and $\boldsymbol{\theta}' \in [0,1]^m$,

$$v(\boldsymbol{\theta}) \geq C(\boldsymbol{\theta}') + \sum_{i=1}^{m} \pi_i(\boldsymbol{\theta}')\theta_i Q_i, \tag{2.13}$$

where

$$v(\boldsymbol{\theta}) := C(\boldsymbol{\theta}) + \sum_{i=1}^{m} \pi_i(\boldsymbol{\theta})\theta_i Q_i. \tag{2.14}$$

From Rochet (1987), one has

Lemma 2.2 *An allocation is incentive-compatible if and only if the expected-payoff function $v(\cdot)$ that is given by (2.14) is convex and has partial derivatives $v_i(\cdot)$ satisfying*

$$v_i(\boldsymbol{\theta}) = \pi_i(\boldsymbol{\theta}) Q_i \tag{2.15}$$

for all i and almost all $\boldsymbol{\theta} \in [0,1]^m$.

For a first-best allocation, with $\pi_i(\boldsymbol{\theta}) = 1$ for almost all $\boldsymbol{\theta} \in [0,1]^m$, (2.15) is equivalent to the requirement that $C(\boldsymbol{\theta})$ be independent of $\boldsymbol{\theta}$. From Lemmas 2.1 and 2.2, one therefore obtains:

Proposition 2.3 *A first-best allocation is incentive-compatible if and only if*

$$C(\boldsymbol{\theta}) = Y - K(\mathbf{Q}) \tag{2.16}$$

for almost all $\boldsymbol{\theta} \in [0,1]^m$.

Proposition 2.3 indicates that, in the large economy studied here, a first-best allocation can be implemented if and only if public-good provision is entirely financed by a lump-sum payment, which people make regardless of their preferences.[10] This lump-sum payment amounts to $K(\mathbf{Q}) > 0$ per person. Public-goods provision then hurts people who do not care for the public goods and benefits people who care a lot for them. People who do not care for the public goods are strictly worse off than they would be if they could just have the private-good consumption Y.

To articulate this concern formally, I introduce a concept of individual rationality. I assume that each participant h has the capacity to produce Y units of the private good (at no further cost to himself) and that, without any agreement on the provision of public goods, each participant simply consumes these Y units of the private good out of his own production. Given this assumption, an allocation is said to be *individually rational* if the expected payoff (2.14) satisfies $v(\boldsymbol{\theta}) \geq Y$ for all $\boldsymbol{\theta} \in [0,1]^m$.[11] With this definition, Proposition 2.3 implies that *a first-best, incentive-compatible allocation cannot be individually rational.* Indeed, any incentive-compatible allocation with $K(Q) > 0$ and $\pi_i(\theta) = 1$ for almost all $\theta \in [0,1]^m$ fails to be individually rational.[12] This result does not depend on either the exogeneity of production or the homogeneity of individuals. Whenever the government refrains from using taxes on people's production capacities or people's outputs to finance the public goods, the impossibility result goes through without change.

The assumption that public-goods provision is *not* financed from taxes on production capacities or production activities stands in the tradition of Lindahl (1919). Lindahl's (1919) creation of the theory of public goods was designed as an interpretation of government activities in terms of *voluntary* exchanges, culminating the development of the benefit approach to public finance as part of a *theory of the state built on contracts.* In Lindahl's

[10] This result is moot if participants are risk averse or the mechanism designer is inequality averse. In this case, (2.16) is incompatible with the requirement that the social marginal utility of private-good consumption be equalized across agents. See Hellwig (2005, 2004).

[11] This specification encompasses the requirement that nobody should have an incentive to reject the proposed allocation in order to consume his own output Y *and* at the same time to enjoy the public goods provided by others? If one combines the incentive compatibility condition (2.13) for any $\boldsymbol{\theta}$ and $\boldsymbol{\theta}' = \mathbf{0}$ with the participation constraint $v(\mathbf{0}) \geq Y$, one finds that $v(\boldsymbol{\theta}) \geq Y + \sum_{i=1}^{m} \pi_i(\mathbf{0})\theta_i Q_i$, which is precisely the requirement in question.

[12] These observations provide a large-economy, multiple-public-goods extension of the finite-economy, single-public-good impossibility theorems of Güth and Hellwig (1986), Rob (1989), Mailath and Postlewaite (1990), Norman (2004).

analysis, the government was considered to be first providing for equity through redistribution and then providing for efficient public-goods provision through *voluntary* contracting on a *do-ut-des* basis. The voluntariness of exchange at the second stage of government activity corresponds precisely to the individual-rationality condition introduced here.[13]

2.3 The Problem of the Second-Best

A *second-best allocation* is defined as an allocation that maximizes the aggregate surplus (2.8) over the set of all feasible, incentive-compatible and individually rational allocations. As discussed by Rochet and Choné (1998), the problem of finding such an allocation can be formulated in terms of the provision levels $Q_1, ..., Q_m$ and the expected-payoff function $v^A(\cdot)$. Lemma 2.2 implies that, for any $\mathbf{Q} = (Q_1, ..., Q_m)$ and any convex function $v(\cdot)$ with partial derivatives satisfying $v_i(\boldsymbol{\theta}) \in [0, Q_i]$ for all i and almost all $\boldsymbol{\theta} \in [0,1]^m$, an incentive-compatible allocation is obtained by setting

$$\pi_i(\boldsymbol{\theta}) = \frac{1}{Q_i} \lim_{\theta_i' \downarrow \theta_i} v_i(\theta_i', \boldsymbol{\theta}_{-i}) \text{ if } Q_i > 0, \tag{2.17}$$

$$\pi_i(\boldsymbol{\theta}) = 0 \text{ if } Q_i = 0, \tag{2.18}$$

and

$$C(\boldsymbol{\theta}) = v(\boldsymbol{\theta}) - \sum_{i=1}^{m} \theta_i v_i(\boldsymbol{\theta}), \tag{2.19}$$

and specifying $c^A(\cdot,\cdot)$ and $\chi_1^A(\cdot,\cdot), ..., \chi_m^A(\cdot,\cdot)$ accordingly. By (2.19), (2.5) is equivalent to the inequality

$$\int_{[0,1]^m} \left[v(\boldsymbol{\theta}) - \sum_{i=1}^{m} \theta_i v_i(\boldsymbol{\theta}) \right] f(\boldsymbol{\theta}) d\boldsymbol{\theta} \leq Y - K(\mathbf{Q}). \tag{2.20}$$

The problem of finding a second-best allocation is therefore equivalent to the problem of choosing $\mathbf{Q}$ and a convex function v with partial derivatives satisfying $v_i(\boldsymbol{\theta}) \in [0, Q_i]$ for all i and almost all $\boldsymbol{\theta}$ so as to maximize (2.8) subject to (2.20) and the participation constraint $v(\boldsymbol{\theta}) \geq Y$ for all $\boldsymbol{\theta}$.

For a single excludable public good, i.e., when $m = 1$, Schmitz (1997) and Norman (2004) have shown that this problem has a simple solution: For any second-best allocation $(Q, c(\cdot,\cdot), \chi_1(\cdot,\cdot))$, there is some $\hat{\theta} \in (0,1)$ such

[13] This individual-rationality condition is central to the entire literature on the relevant version of the Myerson-Satterthwaite theorem for public-goods provision; see, e.g., Güth and Hellwig (1986), Rob (1989), Mailath and Postlewaite (1990), Norman (2004).

that $C(\theta) = Y$, $\pi(\theta) = 0$ for $\theta < \hat{\theta}$ and $C(\theta) = Y - \hat{\theta}Q$, $\pi(\theta) = 1$ for $\theta > \hat{\theta}$. There is a user fee $p = \hat{\theta}Q$, so that people for whom the benefit θQ from the enjoyment of the public good exceeds p pay the fee and are given access, and people for whom the benefit θQ is less than p do not pay the fee and are not given access. The critical $\hat{\theta}$ and the fee $p = \hat{\theta}Q$ are chosen so that the aggregate revenue $p(1 - F(\hat{\theta}))$ just covers the cost $K(Q)$ of providing the public good.[14]

For $m > 1$, a general characterization of second-best allocations does not seem to be available. As usual in problems of multi-dimensional mechanism design, the second-order conditions for incentive compatibility (convexity of the expected-payoff function $v(\cdot)$) and integrability conditions (equality of the cross derivatives $v_{ij}(\boldsymbol{\theta})$ and $v_{ji}(\boldsymbol{\theta})$, i.e. of the derivatives $\frac{\partial \pi_i(\boldsymbol{\theta})}{\partial \theta_j} Q_i$ and $\frac{\partial \pi_j(\boldsymbol{\theta})}{\partial \theta_i} Q_j$) are difficult to handle analytically. For the case of two excludable public goods, i.e., $m = 2$, with independent preference parameters having identical two-point distributions, a complete characterization of second-best allocations in finite economies is provided by Fang and Norman (2003/2006). The large-economy limits of these allocations involve nonseparability and genuine randomization in admission rules. The complexity of the particular admission rules that they obtain may be due to their working with a two-point distribution of preference parameters.[15] However, the work of Thanassoulis (2004), as well as Manelli and Vincent (2006 a, b) on the closely related problem of profit maximization by a multi-product monopolist suggests that, for $m > 1$, nonseparability and genuine randomization in admissions are the rule, rather than the exception.[16]

[14] If the function $\theta \to g(\theta) := \theta - \frac{1-F(\theta)}{f(\theta)}$ is increasing, the solution is actually unique. If the function $\theta \to g(\theta)$ is not increasing, there may be more than one solution, including solutions that involve randomized admissions. However, even in this case, at least one solution has the simple interpretation in terms of an entry fee that is given in the text; see Manelli and Vincent (2006 b).

[15] Under this distribution, some randomization of admissions is desirable even in the case $m = 1$.

[16] Manelli and Vincent (2006 b) show that the set of optimal mechanisms must contain an extreme point in the set of admissible mechanisms. They provide an algebraic procedure to determine whether a given mechanism satisfies this requirement and show that the set of extreme points contains a rich set of "novel" mechanisms, including mechanisms that involve randomized admissions for all goods. Under an additional undominatedness criterion, such a mechanism is profit maximizing for some distribution of the hidden characteristics.

3 Renegotiation Proofness and the Optimality of Ramsey-Boiteux Pricing

The price characterization of second-best mechanisms that Schmitz (1997) and Norman (2004) have provided for the case of a single public good relate the mechanism design problem to the Ramsey-Boiteux tradition of public-sector pricing. The admission fee that characterizes a second-best mechanism is in fact the same as the second-best price in the Ramsey-Boiteux analysis, the lowest price at which the costs of public-goods provision are covered.

With multiple public goods, it is usually not possible to characterize second-best allocations by admission fees. In this case, the question of what is the relation between the second-best mechanism design problem and Ramsey-Boiteux pricing seems moot. Instead of pursuing this question, I therefore consider *third-best allocations*, defined as allocations that maximize aggregate surplus subject to feasibility, incentive compatibility, individual rationality, and an *additional condition of renegotiation proofness*.

The latter condition reflects the idea that the agency which implements the chosen mechanism is unable to verify the identities of people who present tickets for access to the enjoyment of a public good. In particular, the agency is unable to check whether the people who present tickets for access to a public good are in fact the same people to whom the tickets have been issued. It is also unable to prevent people from trading these tickets, as well as the private good, among each other. If the initial allocation of tickets leaves room for a Pareto improvement through such trading, then, as discussed by Hammond (1979, 1987) and Guesnerie (1995), in the absence of transactions costs, such trading will occur, and the initial allocation will not actually be the final allocation.

Underlying the imposition of renegotiation proofness is the notion that, regardless of the allocation that is initially chosen by the mechanism designer, in the absence of transactions costs, any allocation that is finally implemented will itself be renegotiation proof. If the mechanism designer is aware of the possibility of renegotiation, and if he cares about the allocation that is finally implemented rather than the one that is initially chosen, his choice may be directly expressed in terms of the final renegotiation-proof allocation. Indeed if he chooses a renegotiation proof allocation from the beginning, this initial allocation will also be the final allocation. Given these considerations, I refer to an allocation as *third-best* if and only if it maximizes the aggregate surplus (2.8) over the set of all feasible, incentive-compatible, individually rational, and renegotiation proof allocations.[17]

To define renegotiation proofness formally, I say that a *net-trade allocation* for private-good consumption and public-good admission tickets is an array $(z_c(\cdot,\cdot), z_1(\cdot,\cdot), ..., z_m(\cdot,\cdot))$ such that for each $(\boldsymbol{\theta}, \omega)$, $z_c(\boldsymbol{\theta}, \omega)$ and $z_1(\boldsymbol{\theta}, \omega), .., z_m(\boldsymbol{\theta}, \omega)$ are the net additions to private-good consumption and holdings of public-good admission tickets of a consumer with preference parameter vector $\boldsymbol{\theta}$ and indicator value ω. Given the initial allocation (2.3), a net-trade allocation $(z_c(\cdot,\cdot), z_1(\cdot,\cdot), ..., z_m(\cdot,\cdot))$ is said to be *feasible* if $\chi_i(\boldsymbol{\theta}, \omega) + z_i(\boldsymbol{\theta}, \omega) \in \{0, 1\}$ for all $(\boldsymbol{\theta}, \omega) \in [0, 1]^{m+1}$ and, moreover,

$$\int_H z_i(\tilde{\boldsymbol{\theta}}^h(x), \tilde{\omega}^h(x)) d\eta(h) = 0 \tag{3.1}$$

for $i = c, 1, ..., m$ and almost all $x \in X$, which by (2.2) is equivalent to the requirement that

$$\int_{[0,1]^{m+1}} z_i(\boldsymbol{\theta},\omega) f(\boldsymbol{\theta}) d\boldsymbol{\theta} d\nu(\omega) = 0 \tag{3.2}$$

for $i = c, 1, ..., m$.[18] Given (2.3), the net-trade allocation $(z_c(.,.), z_1(.,.), .., z_m(.,.))$ is said to be *incentive-compatible*, if

$$z_c(\boldsymbol{\theta}, \omega) + \sum_{i=1}^{m} z_i(\boldsymbol{\theta}, \omega)\theta_i Q_i \geq z_c(\boldsymbol{\theta}', \omega') + \sum_{i=1}^{m} z_i(\boldsymbol{\theta}', \omega')\theta_i Q_i \tag{3.3}$$

for all $\boldsymbol{\theta}$ and $\boldsymbol{\theta}'$ in $[0, 1]^m$ and all ω and ω' in Ω for which $\chi_i(\boldsymbol{\theta}, \omega) + z_i(\boldsymbol{\theta}', \omega') \in \{0, 1\}$ for all i. The idea is that the holdings $(c(\boldsymbol{\theta}, \omega), \chi_1(\boldsymbol{\theta}, \omega), ..., \chi_m(\boldsymbol{\theta}, \omega))$ of private-good consumption and public-goods admission tickets of a given agent, as well as the realization $(\boldsymbol{\theta}, \omega)$ of his preference parameter vector $\tilde{\boldsymbol{\theta}}$ and randomization device $\tilde{\omega}$ are not known by anybody else.[19] Therefore, if

[17] By contrast, Hammond (1979, 1987) and Guesnerie (1995) treat the mechanism design problem in terms of a two-stage game with a revelation game in the first stage determining an allocation which provides the starting point for side-trading in the second stage, leading to a Walrasian outcome. The approach taken here collapses the two stages into one by imposing a renegotiation proofness constraint on the mechanism designer.

[18] To keep matters simple, I assume that Y is large enough that nonnegativity of private-good consumption is not an issue.

[19] One might argue that the mechanism designer knows the consumer's actual holdings as well as ω, and therefore the incentive constraints on net-trade allocations might be alleviated. Such loosening of incentive constraints would tend to enhance the scope for renegotiations and make the condition of renegotiation proofness even more restrictive. In the large economy considered here, it does not actually make a difference because the characterization of renegotiation proofness in Lemma 3.1 remains valid. In a finite economy, there would be a difference.

the agent claims that his preference parameter vector is $\boldsymbol{\theta}'$, he obtains the net trade $(z_c(\boldsymbol{\theta}', \omega'), z_1(\boldsymbol{\theta}', \omega'), ..., z_m(\boldsymbol{\theta}', \omega'))$ that is available to an agent with parameter vector $\boldsymbol{\theta}'$ when his randomization variable takes the value ω'. Incentive compatibility of the net-trade allocation requires that such a claim not provide the agent with an improvement over the stipulated net trade $(z_c(\boldsymbol{\theta}, \omega), z_1(\boldsymbol{\theta}, \omega), ..., z_m(\boldsymbol{\theta}, \omega))$.

An allocation is said to be *renegotiation proof* if, starting from it, there is no feasible and incentive-compatible net-trade allocation that provides a Pareto improvement in the sense that for all $(\boldsymbol{\theta}, \omega) \in [0,1]^{m+1}$, the utility gain from the net trade $(z_c(\boldsymbol{\theta}, \omega), z_1(\boldsymbol{\theta}, \omega), ..., z_m(\boldsymbol{\theta}, \omega))$ is nonnegative, i.e.

$$z_c(\boldsymbol{\theta}, \omega) + \sum_{i=1}^{m} z_i(\boldsymbol{\theta}, \omega)\theta_i Q_i \geq 0, \tag{3.4}$$

and the aggregate utility gain is strictly positive, i.e.,

$$\int_H [z_c(\tilde{\boldsymbol{\theta}}^h, \tilde{\omega}^h) + \sum_{i=1}^{m} z_i(\tilde{\boldsymbol{\theta}}^h, \tilde{\omega}^h)\tilde{\theta}_i^h Q_i] d\eta(h) > 0 \tag{3.5}$$

with positive probability; by (2.2), the latter inequality is equivalent to to the inequality

$$\int_{[0,1]^{m+1}} [z_c(\boldsymbol{\theta}, \omega) + \sum_{i=1}^{m} z_i(\boldsymbol{\theta}, \omega)\theta_i Q_i] \; dF(\boldsymbol{\theta}) d\nu(\omega) > 0, \tag{3.6}$$

which actually implies that (3.5) holds with probability one.

As introduced here, the concept of renegotiation proofness presumes that tickets to all public goods can be traded separately. A weaker concept of renegotiation proofness would be obtained if the mechanism designer were unable to prevent sidetrading, but able to prepare tickets to bundles of public goods in such a way that unbundling is impossible. This weaker concept is briefly discussed at the end of Section 4 and in Appendix B.

For the strong concept introduced here, the following lemma shows that renegotiation proofness holds if and only if there exists a price system which supports the allocation as a competitive equilibrium of the exchange economy in which people trade the private good, as well as admission tickets for the different public goods, taking the vector $\mathbf{Q}$ of public-good provision levels as given.

Lemma 3.1 *An allocation (2.3) is renegotiation proof if and only if there exist numbers* $p_1, ...p_m$ *such that for* $i = 1, ..., m$, *and almost all* $(\boldsymbol{\theta}, \omega) \in [0,1]^m \times \Omega$, *one has*

$$\chi_i(\boldsymbol{\theta}, \omega) = 0 \quad \textit{if} \quad \theta_i Q_i < p_i \tag{3.7}$$

and

$$\chi_i(\boldsymbol{\theta}, \omega) = 1 \quad \textit{if} \quad \theta_i Q_i > p_i. \tag{3.8}$$

Renegotiation proofness implies, for each public good i, a simple dichotomy between a set of participants with high θ_i, who get admission to public good i with probability one, and a set of participants with low θ_i, who do not get admission to public good i. This leaves no room for randomized admissions.

The simplicity of the admission rule in Lemma 3.1 provides for a drastic simplification of incentive compatibility conditions. The numbers $p_1, ..., p_m$ can be interpreted as prices, i.e., as *admission fees.* In an incentive-compatible allocation, a consumer is granted access to public good i if and only if he pays the fee p_i. Consumers with $\theta_i Q_i > p_i$ pay the fee and enjoy the public good for a net benefit equal to $\theta_i Q_i - p_i$, consumers with $\theta_i Q_i < p_i$ do not pay the fee and are excluded from the public good. Formally, one obtains:

Lemma 3.2 *An allocation (2.3) is renegotiation proof and incentive-compatible if and only if there exist prices* $p_1, ...p_m$ *such that for all* $\boldsymbol{\theta} \in [0,1]^m$, *the admission probabilities* $\pi_i(\boldsymbol{\theta})$, $i = 1..., m$, *and the conditional expectation* $C(\boldsymbol{\theta})$ *of private-good consumption satisfy*

$$\pi_i(\boldsymbol{\theta}) = 0 \quad \textit{if} \quad \theta_i Q_i < p_i, \tag{3.9}$$

$$\pi_i(\boldsymbol{\theta}) = 1 \quad \textit{if} \quad \theta_i Q_i > p_i, \tag{3.10}$$

and

$$C(\boldsymbol{\theta}) = C(\mathbf{0}) - \sum_{i=1}^{m} \pi_i(\boldsymbol{\theta})\, p_i. \tag{3.11}$$

The associated expected payoff is

$$v(\boldsymbol{\theta}) = C(\mathbf{0}) + \sum_{i=1}^{m} \max(\theta_i Q_i - p_i, 0) \tag{3.12}$$

for $\boldsymbol{\theta} \in [0,1]^m$.

For any incentive-compatible and renegotiation proof allocation, the aggregate surplus (2.8) and the feasibility constraint (2.5) take the form

$$C(\mathbf{0}) + \sum_{i=1}^{m} \int_{\hat{\theta}_i(p_i^A, Q_i^A)}^{1} (\theta_i Q_i - p_i)\, dF_i(\theta_i) \tag{3.13}$$

and

$$C(\mathbf{0}) - \sum_{i=1}^{m} p_i(1 - F_i(\hat{\theta}_i(p_i, Q_i))) + K(\mathbf{Q}) \leq Y, \tag{3.14}$$

where $p_1, ..., p_m$ are the competitive prices associated with the allocation and, for any i,

$$\hat{\theta}_i(p_i, Q_i) := \frac{p_i}{Q_i} \text{ if } Q_i > 0 \text{ and } \hat{\theta}_i(p_i, Q_i) := 1 \text{ if } Q_i = 0. \tag{3.15}$$

The problem of finding a third-best allocation is therefore equivalent to the problem of choosing an expected base consumption $C(\mathbf{0})$, as well as public-good provision levels $Q_1, ..., Q_m$, and prices $p_1, ..., p_m$, with associated critical preference parameter values $\hat{\theta}_1, ..., \hat{\theta}_m$ satisfying $\hat{\theta}_i Q_i = p_i$, so as to maximize (3.13) subject to the feasibility constraint (3.14) and the participation constraint $C(\mathbf{0}) \geq Y$.

In this maximization, the participation constraint is binding. Otherwise the problem would be solved by the incentive-compatible first-best allocation of Lemma 2.1 and Proposition 2.3, which is obviously renegotiation proof, but violates the participation constraint.

Given that the participation constraint is binding, the base consumption $C(\mathbf{0})$ in (3.13) and (3.14) can be replaced by the constant Y, and one obtains:

Proposition 3.3 *The third-best allocation problem is equivalent to the problem of choosing public-good provision levels $Q_1, ..., Q_m$ and prices $p_1, ..., p_m$ so as to maximize*

$$\sum_{i=1}^{m} \int_{\hat{\theta}_i(p_i, Q_i)}^{1} (\theta_i Q_i - p_i)\, dF_i(\theta_i) \tag{3.16}$$

under the constraint that

$$\sum_{i=1}^{m} p_i(1 - F_i(\hat{\theta}_i(p_i, Q_i))) \geq K(\mathbf{Q}). \tag{3.17}$$

The problem of finding a third-best allocation has thus been reformulated solely in terms of the public-good provision levels and prices. Given the fees $p_1, ...p_m$, for any i, there are $(1 - F_i(\hat{\theta}_i(p_i, Q_i)))$ participants with $\theta_i Q_i$ who are willing to pay the fee p_i for access to public good i. The aggregate admission fee revenue from public good i is therefore $p_i(1 - F_i(\hat{\theta}_i(p_i, Q_i)))$, and the aggregate admission fee revenue from all public goods is $\sum_{i=1}^{m} p_i(1 - F_i(\hat{\theta}_i(p_i, Q_i)))$. The constraint (3.17) requires that this revenue cover the cost $K(\mathbf{Q})$.

Proposition 3.3 provides an analogue to Hammond (1979, 1987) and Guesnerie (1995), where the possibility of unrestricted side-trading reduces the general problem of mechanism design for optimal taxation to a Diamond-Mirrlees (1971) problem of finding optimal consumer prices. Here, the third-best allocation problem is equivalent to the Ramsey-Boiteux problem of choosing public-good provision levels and prices so as to maximize aggregate surplus under the constraint that admission fee revenues be sufficient to cover the costs of public-good provision. Because individual-rationality constraints preclude the imposition of lump-sum taxes, the costs of public-good provision must be fully financed by payments that people make in order to gain the benefits of the public goods. Renegotiation proofness implies that these payments are characterized by admission fees $p_1, ..., p_m$, as they are in the Ramsey-Boiteux approach to public-sector service provision and pricing.

The equivalence stated in Proposition 3.3 indicates that the difference between the allocation problem for an excludable public good and the allocation problem for a good whose production involves significant fixed costs is purely one of semantics: The enjoyment of the public good by any one individual can be treated as a private good, the production of which involves only a fixed cost and no variable costs.

The following characterization of third-best allocations in terms of first-order conditions is now straightforward.

Proposition 3.4 *If $Q_1, ..., Q_m$, and $p_1, ..., p_m$ are the public-good provision levels and admission fees in a third-best allocation, then $Q_i > p_i > 0$ for $i = 1, ..., m$; moreover, there exists $\lambda > 1$, such that*

$$\frac{1}{\lambda} \int_{\hat{\theta}_i^A}^{1} \theta_i dF_i(\theta_i) + \frac{(\lambda - 1)}{\lambda} \hat{\theta}_i^A (1 - F_i(\hat{\theta}_i)) = K_i(\mathbf{Q}) \qquad (3.18)$$

and

$$p_i \; f_i(\hat{\theta}_i) \; \frac{1}{Q_i} = \frac{\lambda - 1}{\lambda} \; (1 - F_i(\hat{\theta}_i)) \qquad (3.19)$$

for $i = 1, ..., m$, where $\hat{\theta}_i = \hat{\theta}_i(p_i, Q_i)$ is given by (3.15).

Condition (3.19) is the usual Ramsey-Boiteux condition for "second-best" consumer prices. The term $(1-F_i(\frac{p_i}{Q_i}))$ on the right-hand side indicates the level of aggregate demand for admissions to public good i when the price is p_i and the "quality", i.e., the provision level, is Q_i. The term $f_i(\frac{p_i}{Q_i})\ \frac{1}{Q_i}$ on the left-hand side indicates the absolute value of the derivative of demand with respect to p_i. Condition (3.19) requires admission fees to be chosen in such a way that the elasticities

$$\eta_i := \frac{p_i}{(1 - F_i(\frac{p_i}{Q_i}))}\ f_i(\frac{p_i}{Q_i})\ \frac{1}{Q_i}$$

of demands for admissions to the different public goods are locally all the same, i.e. that

$$1 = \frac{\lambda - 1}{\lambda}\ \frac{1}{\eta_i}$$

for all i. This is the degenerate form taken by the Ramsey-Boiteux inverse-elasticities formula when variable costs are identically equal to zero.[20]

Condition (3.18) is the version of the Lindahl-Samuelson condition for public-good provision that is appropriate for the third-best allocation problem.[21] Third-best public-good provision levels are determined so that for each i, the marginal cost of providing public good i is equated to a weighted average of the aggregate marginal benefits that are obtained by users and the aggregate marginal revenues that are obtained by the mechanism designer if the admission fee p_i is raised in proportion to Q_i so that the critical $\hat{\theta}_i(p_i, Q_i)$ is unchanged. If provision costs are additively separable, i.e., if the marginal cost $K_i(\mathbf{Q})$ depends only on Q_i, then each of of the third-best provision levels $Q_1, ..., Q_m$ will be lower than the corresponding first-best level given by (2.12). The reason is, first, that there are fewer users of the public good than in the first-best allocation, and second, that the mechanism designer is unable to fully appropriate the benefits from additional

[20] See, e.g., equation (15-23), p. 467, in Atkinson and Stiglitz (1980). If variable costs are positive, e.g., if costs take the form $K(\mathbf{Q},U_1, ..., U_m)$, where, for $i = 1, ..., m$, $U_i := \int \pi_i dF$ is the aggregate use of public good i, equation (3.19) takes the form

$$(p_i - \frac{\partial K}{\partial U_i})\ f_i(\hat{\theta}_i)\ \frac{1}{Q_i} = \frac{\lambda - 1}{\lambda}\ (1 - F_i(\hat{\theta}_i)),$$

which yields the usual nondegenerate form

$$\frac{p_i - \frac{\partial K}{\partial U_i}}{p_i} = \frac{\lambda - 1}{\lambda} \frac{1}{\eta_i}$$

of the inverse-elasticities formula.

[21] For the case $m = 1$, this condition is also obtained by Norman (2004).

public-good provision so that the aggregate marginal revenues accruing to him are less than the aggregate marginal benefits accruing to users.

The analysis is easily extended to a situation with nonexcludable as well as excludable public goods. Suppose, for example, that $n < m$ public goods $1, ..., n$ are nonexcludable and public goods $n+1, ..., m$ are excludable. Nonexcludability of a public good i is equivalent to the requirement that the admission probability $\pi_i(\boldsymbol{\theta})$ be equal to one for all $\boldsymbol{\theta}$. In terms of the Ramsey-Boiteux analysis, this in turn is equivalent to the requirement that the admission fee for this public good be equal to zero. The third-best allocation problem then is to maximize (3.16) subject to (3.17) *and* the constraint that $p_i = 0$ for $i = 1, ..., n$. Except for the fact that admission fees for nonexcludable public goods are zero, the conditions for a third-best allocation are the same as before, i.e., admission fees for excludable public goods satisfy an inverse-elasticities formula, and provision levels for all public goods satisfy an appropriate version of the Lindahl-Samuelson condition.

For the nonexcludable public goods, the Lindahl-Samuelson condition takes the form

$$\frac{1}{\lambda}\int_0^1 \theta_i dF_i(\theta_i) = K_i(\mathbf{Q}), \tag{3.20}$$

so, by the same reasoning as before, a third-best allocation also involves strictly positive provision levels of nonexcludable public goods. In the large economy, provision of the nonexcludable public goods does not generate any revenue, but they are provided nevertheless. Admission fees from the excludable public goods provide finance for the nonexcludable public goods as well. This cross-subsidization is desirable because, with $\int_0^1 \theta_i dF_i(\theta_i) > 0$ and $K_i(\mathbf{Q}) = 0$ when $Q_i = 0$, the benefits of the first (infinitesimal) unit that is provided always exceed the costs. This finding is unaffected by the fact that the cross-subsidization of nonexcludable public goods requires higher admission fees and creates additional distortions for excludable public goods. Concern about these additional distortions will reduce but not eliminate the provision of nonexcludable public goods.[22]

This discussion of nonexcludable public goods stands in contrast to the assessment of Mailath and Postlewaite (1990) that, in a large economy with asymmetric information and interim participation constraints, a nonexcludable public good will not be provided at all. The Mailath-Postlewaite result presumes a single nonexcludable public good, the costs of which have to be

[22] Given that $\lambda > 1$, a comparison of (3.20) and (2.12) shows that if the cost function K is additively separable, the third-best provision level for public goods i is strictly lower than the corresponding first-best level given by (2.12). On this point, see also Guesnerie (1995).

covered by revenues coming from this very public good itself. Here there is no such requirement. Interim participation constraints for individual consumers and the induced aggregate budget (feasibility) constraint allow for cross-subsidization between public goods. For nonexcludable public goods, this cross-subsidization eliminates the Mailath-Postlewaite problem.

The cross-subsidization of nonexcludable public goods can be seen as an instance of the well known general principle that, in the Ramsey-Boiteux framework, there is no presumption that any one good should be self-financing. Even if the cost function $K(\cdot)$ is additively separable and there is no ambiguity in assessing the cost of providing public good i at a level Q_i, there is still no presumption that this cost $K^i(Q_i)$ should be altogether covered by the revenue $p_i(1-F_i(\hat{\theta}_i(p_i,Q_i)))$ that is attributable to this particular public good.[23]

4 Mixed Bundling Dominates Ramsey-Boiteux Pricing

In the preceding analysis, the requirement of renegotiation proofness has served to reduce a complex problem of multidimensional mechanism design to a simple m-dimensional pricing problem. The key to this simplification lies in the observation that renegotiation proofness restricts admission rules so that the expected-payoff function $v(\cdot)$ takes the form (3.12), which is additively separable and convex in $\theta_1, ..., \theta_m$. The integrability condition $v_{ij}^A = v_{ji}^A$ and the second-order condition for incentive compatibility (convexity) are then automatically satisfied.

However, renegotiation proofness is restrictive. If the mechanism designer is able to control the identities of people presenting admission tickets to the different public goods, or if there are some impediments to renegotiation, an optimal allocation will typically *not* have the simple structure that is implied by renegotiation proofness. Second-best allocations, which maximize aggregate surplus subject to feasibility, incentive compatibility and individual rationality, *without* renegotiation proofness, tend to involve *bundling* and, possibly, randomized admissions. These devices reduce the efficiency losses that are associated with the use of admission fees to reduce the participants' information rents.

[23] If there are multiple private, as well as public, goods, the Ramsey-Boiteux formalism also mandates some additional cross-subsidization from private goods. For analysis putting this cross-subsidization into the context of second-best mechanism design, see Fang and Norman (2005).

Thus, Fang and Norman (2003/2006) have noted that, if the random variables $\tilde{\theta}_1, ..., \tilde{\theta}_m$ are independent and identically distributed, the weighted sum $\sum_{i=1}^m \tilde{\theta}_i Q_i$ has a lower coefficient of variation than any one of its summands, and therefore, under certain additional assumptions about the distribution of $\tilde{\theta}_i$, an allocation involving *pure bundling* can dominate a third-best allocation because it involves a lower incidence of participants being excluded.[24] Pure bundling refers to a situation where participants are admitted either to *all* public goods at once or to none.

In the following, I show that, if $\tilde{\theta}_1, ..., \tilde{\theta}_m$ are mutually independent, a third-best allocation is *always* dominated by an allocation involving *mixed bundling,* i.e., an allocation where participants can obtain admission to each public good separately, as well as admission to different public goods at the same time, through a combination ticket which comes at a discount relative to the individual tickets. Even if tastes for the different public goods are completely unrelated, the allocation that is induced by the best pricing scheme à la Ramsey-Boiteux is dominated by some other allocation with nonrandom admission rules. Renegotiation proofness is *necessary*, as well as sufficient, for Ramsey-Boiteux pricing to be equivalent to optimal mechanism design under interim incentive compatibility and individual-rationality constraints. This finding complements the results of Fang and Norman. The argument involves a straightforward adaptation of corresponding arguments in the multiproduct monopoly models of McAfee et al. (1989) or Manelli and Vincent (2006 a).

To fix notation and terminology, let $M = \{1, ..., m\}$ be the set of public goods, and let $\mathcal{P}(M)$ be the set of all subsets of M. As in Manelli and Vincent (2006 a), a function $P : \mathcal{P}(M) \to \Re$ is called a *price schedule,* with the interpretation that for any set $J \subset M$, $P(J)$ is the amount of private-good consumption that a participant has to give up in order to get a combination ticket for admission to the public goods in J. Given $\mathbf{Q}$, the allocation $(\mathbf{Q}, c_P(\cdot,\cdot), \chi_{1P}(\cdot,\cdot), ..., \chi_{mP}(\cdot,\cdot))$ is said to be *induced by the price schedule* P if, for any $\boldsymbol{\theta} \in [0,1]^m$, there exists a vector[25] $\mathbf{q}_P(\boldsymbol{\theta}) =$

[24] However, their characterization of second-best mechanisms for specifications with two public goods and two-point distrib butions of preferences for each public good also shows that under the conditions that favour pure bundling over separate provision, both pure bundling and mixed bundling are dominated by a randomized admission rule that is also nonseparable across goods, but has no simple price characterization.

[25] In the notation used here and in (4.4) below, the dependence of consumers' choices and payoffs on $\mathbf{Q}$ is suppressed because it does not play a role in the analysis.

$(q_P(\emptyset;\boldsymbol{\theta}),...,q_P(M;\boldsymbol{\theta}))$ of probabilities on $\mathcal{P}(M)$ such that

$$C_P(\boldsymbol{\theta}) := \int_\Omega c_P(\boldsymbol{\theta},\omega)d\nu(\omega) = Y - \sum_{J\in\mathcal{P}(M)} q_P(J;\boldsymbol{\theta})P(J),$$

$$\pi_{jP}(\boldsymbol{\theta}) := \int_\Omega \chi_{jP}(\boldsymbol{\theta},\omega)d\nu(\omega) = \sum_{J\in\mathcal{P}(M)} \delta_{jJ}\, q_P(J;\boldsymbol{\theta}),$$

for $j = 1, ..., m$, where $\delta_{jJ} = 1$ if $j \in J$, $\delta_{jJ} = 0$ if $j \notin J$, and, finally,

$$Y+\sum_{J\in\mathcal{P}(M)} q_P(J;\boldsymbol{\theta})\left[\sum_{j\in J}\theta_j Q_j - P(J)\right] \geq Y+\sum_{J\in\mathcal{P}(M)} q(J)\left[\sum_{j\in J}\theta_j Q_j - P(J)\right] \tag{4.1}$$

for all probability vectors $\mathbf{q}$ on $\mathcal{P}(M)$. The idea is that, for the given $\mathbf{Q}$ and P, each consumer is free to choose a set J of public goods that he wants to enjoy at a price $P(J)$. He may also randomize this choice. For generic price schedules though, there is a single set $J_P(\boldsymbol{\theta})$ which he strictly prefers to all others; in this case, the incentive compatibility condition (4.1) becomes $q_P(J_P(\boldsymbol{\theta});\boldsymbol{\theta}) = 1$, i.e., he simply chooses the set $J_P(\boldsymbol{\theta})$.

A price schedule P is said to be *arbitrage free* if it satisfies the equation

$$P(J) = \sum_{j\in J} P(\{j\}) \tag{4.2}$$

for all J, so each set $J \subset M$ is priced as the sum of its components. One easily verifies that, if an allocation is induced by an arbitrage-free price schedule P, then it is also renegotiation proof and incentive-compatible, with associated prices $p_j = P(\{j\})$, $j = 1, ..., m$. Conversely, an allocation that renegotiation proof and incentive-compatible, with associated prices $p_1, ..., p_m$, is induced by the arbitrage-free price schedule P satisfying

$$P(J) = \sum_{j\in J} p_j \tag{4.3}$$

for any $J \subset M$.

The set of renegotiation proof and incentive-compatible allocations is thus a proper subset of the set allocations that are induced by any price schedules. However, this set does *not* include allocations that are induced by price schedules with discounts for *bundles* of public goods, i.e. with $P(J) < \sum_{j\in J} p_j$ for some $J \subset M$. Such allocations leave room for Pareto-improving renegotiations. For instance, a person who found it barely worthwhile to buy the combination ticket for J could gain by reselling its different components to different people, each of whom cares for only one public good j, but has not bought the separate ticket to this public good because the price p_j is just a little bit too high.

Given that the set of renegotiation-proof and incentive-compatible allocations is a strict subset of the set of allocations induced by price schedules, it is of interest to compare optimal price schedules with the arbitrage-free price schedules that correspond to third-best allocations and optimal Ramsey-Boiteux prices. For an allocation that is induced by a price schedule, the resulting payoffs are given as

$$v(\boldsymbol{\theta}|P) = Y + \sum_{J \in \mathcal{P}(M)} q_P(J; \boldsymbol{\theta}) \left[\sum_{j \in J} \theta_j Q_j - P(J) \right]. \tag{4.4}$$

Given a vector $\mathbf{Q} >> \mathbf{0}$ of public-good provision levels, the price schedule P is said to be *optimal* if it maximizes the aggregate surplus

$$\int_{[0,1]^m} v(\boldsymbol{\theta}|P) f(\boldsymbol{\theta}) d\boldsymbol{\theta} \tag{4.5}$$

over the set of all price schedules.

The following result shows that, if the preference parameters $\tilde{\theta}_1, ..., \tilde{\theta}_m$ are independent, an optimal price schedule is *never* arbitrage free, so a third-best allocation is *always* dominated by another allocation. In particular, a third-best allocation is always dominated by an allocation that is induced by a price schedule with mixed bundling, e.g., an offer involving combination tickets for an opera performance and a football game at a discount relative to the sum of the prices for separate tickets.

Proposition 4.1 *Let $m > 1$, and assume that the density f takes the form*

$$f(\boldsymbol{\theta}) = \prod_{j=1}^{m} f_j(\theta_j). \tag{4.6}$$

For any $\mathbf{Q} >> \mathbf{0}$, an optimal price schedule is not *arbitrage free. In particular, if $p_1, ..., p_m$ are the admission fees associated with a third-best allocation, there exists a price schedule $\hat{P}$ satisfying $\hat{P}(M) < \sum_{j=1}^{m} p_j$ and $\hat{P}(\{j\}) > p_j$ for $j = 1, ..., m$ such that, given $\mathbf{Q}$, the allocation that is induced by $\hat{P}$ is feasible and individually rational and generates a higher aggregate surplus than the third-best allocation.*

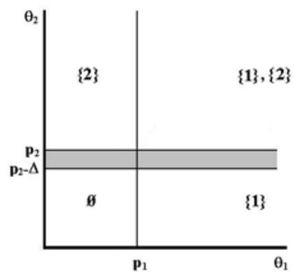

Figure 1: Effects of a decrease in p_2 under arbitrage-free pricing

Proposition 4.1 is based on the same reasoning as the corresponding results of McAfee et al. (1989) or Manelli and Vincent (2002)[26] for a multi-product monopolist. In principle, the choice of a price schedule $P(\cdot)$ involves the same elasticities considerations as the choice of a price vector $(p_1, ..., p_m)$ in the Ramsey-Boiteux analysis of the preceding section. However, these considerations are now applied to the prices of bundles, as well as the prices of separate admission tickets. The relevant elasticities are therefore different. Moreover, one must take account of cross-elasticities between combination tickets and separate admission tickets.

Two effects are particularly important. First, the demand for a bundle is likely to be more elastic than the demands for the components of the bundle under Ramsey-Boiteux pricing. The effect is illustrated in Figures 1 and 2. Both figures concern the case $m = 2$, $Q_1 = Q_2 = 1$, under the assumption that the price schedule is initially arbitrage free. At this price schedule, the admission ticket to public good 1 is purchased by anybody with $\theta_1 > p_1$, the admission ticket to public good 2 by anybody with $\theta_2 > p_2$. People with $\theta_1 > p_1$ and $\theta_2 > p_2$ buy admission tickets to both public goods.

The shaded area in Figure 1 shows the effects of a decrease Δ in the admission price p_2 for public good 2 under arbitrage-free pricing; the shaded area in Figure 2 the effects of a decrease in the bundle price $P(\{1, 2\})$ from $p_1 + p_2$ to $p_1 + p_2 - \Delta$, while $P(\{1\})$ and $P(\{2\})$ remain fixed at p_1 and

[26]Proposition 4.1 is slightly more general than the statement of Theorem 2 in Manelli and Vincent (2002), which involves a monotonicity assumption on the functions $\theta_i \to \theta_i f_i(\theta_i)$. If the range of the random variable $\tilde{\theta}_i$ extends all the way down to zero, this monotonicity assumption is not needed.

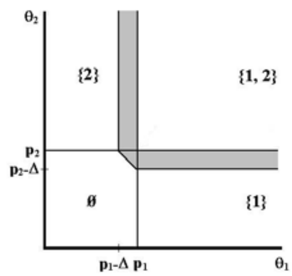

Figure 2: Effects of a price decrease for the bundle $\{1, 2\}$

p_2. The price decrease considered in Figure 1 gains customers for good 2 on one margin, people with $\theta_2 \in (p_2 - \Delta, p_2)$, who previously did not buy an admission ticket for good 2. By contrast, the price decrease considered in Figure 2 gains customers for the bundle on *two margins,* people with $\theta_2 \in (p_2 - \Delta, p_2)$, who previously only bought tickets for public good 1 and people with $\theta_1 \in (p_1 - \Delta, p_1)$, who previously only bought tickets for public good 2.[27] To be sure, these gains of customers for the bundle are accompanied by losses of customers for the separate tickets. However, the net effects are still positive. The people who previously only bought tickets for public good 1 and now buy the combination ticket pay an additional amount $p_2 - \Delta \approx p_2$; the people who previously only bought tickets for public good 2 and now buy the combination ticket pay an additional amount $p_1 - \Delta \approx p_1$.

As usual, the assessment of these price decreases depends on how the revenue gains from having additional customers compare to the revenue losses from charging existing customers less. In each case, this comparison depends on the relation between the size of the group of customers one gains and the size of the group of customers one already has. If the density f takes the form (4.6), then in Figure 1, the size of the group of customers gained is approximately $\Delta\ f_2(p_2)$, and the size of the group of customers one already has is approximately $(1 - F_2(p_2))$. The corresponding revenue effects are approximately $\Delta\ p_2\ f_2(p_2)$ and $\Delta\ (1 - F_2(p_2))$. By contrast, in Figure 2,

[27] A third margin involves people who previously purchased no ticket at all. However, because the initial price schedule was arbitrage-free, this group is small (in relation to Δ) and can be neglected.

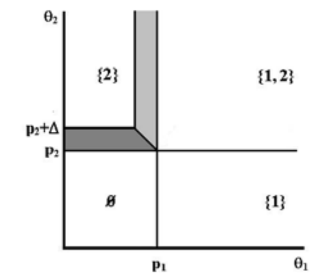

Figure 3: Effects of an increase in $P(\{2\})$

the sizes of the groups that switch are approximately $\Delta\ f_2(p_2)(1 - F_1(p_1))$ and $\Delta\ f_1(p_1)(1 - F_2(p_2))$; the total revenue gain from attracting additional customers to the combination ticket is approximately equal to

$$\Delta\ p_2\ f_2(p_2)(1 - F_1(p_1)) + \Delta\ p_1\ f_1(p_1)(1 - F_2(p_2)). \tag{4.7}$$

The existing customer base to which the price reduction applies is of size $(1 - F_1(p_1))(1 - F_2(p_2))$, and the revenue loss from charging these people a lower price is

$$\Delta\ (1 - F_1(p_1))(1 - F_2(p_2)). \tag{4.8}$$

From (4.7) and (4.8), one immediately sees that the ratio of revenue gains to revenue losses from a price decrease is *more* favourable for the bundle than the corresponding ratio $p_2 f_2(p_2)/(1 - F_2(p_2))$ for arbitrage-free pricing. The independence assumption (4.6) guarantees that elasticities considerations concerning people who react to the price decrease by adding public good 2 to the things they want are the same in both settings; the additional consideration - that there are also people who react to the price decrease by adding public good 2 to the things they want - comes on top of that, making a price decrease for the bundle appear attractive even when a decrease in p_2 under arbitrage-free pricing is not.

Second, because of cross-elasticities considerations, the effects of raising the price $P(\{j\})$ for separate admission to a single public good j are likely to be less harmful than the effects of raising the Ramsey-Boiteux price p_j. The argument is illustrated in Figure 3. For the same initial arbitrage-free price constellation as in Figures 1 and 2, this figure shows the effects of an increase in the price $P(\{2\})$ while $P(\{1\})$ and $P(\{1, 2\})$ remain unchanged.

The increase in $P(\{2\})$ induces a loss of customers for the separate admission ticket for public good 2 on two margins. One margin, represented by the darkly shaded area in the figure, involves people who stop buying any admission ticket at all. The other margin, represented by the lightly shaded area in the figure, involves people who switch from buying the separate admission ticket for public good 2 to buying the combination ticket. This latter group ends up paying $P(\{1,2\})$ rather than $P(\{2\})$, so their switching causes total revenue from admission tickets to go up rather than down. This cross-effect of the price $P(\{2\})$ on the demand for the bundle $\{1,2\}$ makes an increase in $P(\{2\})$ appear more attractive. Without this cross-effect, under the independence assumption (4.6), elasticities considerations for setting $P(\{2\})$ would be the same as for setting p_2 under arbitrage-free pricing.[28] The cross-effect makes an increase in the separate admission fee $P(\{2\})$ appear attractive, even when an increase in p_2 under arbitrage-free pricing is not.

These considerations suggest that, at least under the independence assumption, optimal price schedules should always involve some bundling. The following proposition, which is again inspired by Manelli and Vincent (2006 a), confirms this notion for the case $m = 2$. For $m > 2$, unfortunately, the combinatorics of different bundles are so complicated that a similarly clear-cut result does not seem to be available.

Proposition 4.2 *Let $m = 2$, and assume that the density f takes the form $f(\theta_1, \theta_2) = f_1(\theta_1)f_2(\theta_2)$. Assume further that, for $i = 1, 2$, the function $\theta_i \to \frac{\theta_i f_i(\theta_i)}{1-F_i(\theta_i)}$ is nondecreasing. Then, for any $\mathbf{Q} >> \mathbf{0}$, an optimal price schedule P^* satisfies $P^*(\{1,2\}) < P^*(\{1\}) + P^*(\{2\})$.*

In Propositions 4.1 and 4.2, the assumption that $\tilde{\theta}_1, ..., \tilde{\theta}_m$ are mutually independent should be interpreted as providing a focal point. The arguments of McAfee et al. (1989) indicate that the superiority of mixed bundling holds *a fortiori* if valuations are negatively correlated. Thus, one easily verifies that the above comparison of elasticities considerations for setting the prices p_2 for admission to public good 2 and for setting the price $P(\{1,2\})$ for the bundle $\{1,2\}$ remains valid if the conditional hazard rate $\frac{f_2(p_2|\theta_1)}{1-F_2(p_2|\theta_1)}$ is a decreasing function of θ_1. Whereas the independence assumption (4.6) had guaranteed that elasticities considerations concerning people who react to the price decrease by adding public good 2 to the things they want are the

[28] The ratio of the mass $\Delta f_2(p_2)F_1(p_1)$ of the darkly shaded area in Figure 3 to the mass $(1-F_2(p_2))F_1(p_1)$ of the rectangle above this shaded area is the same as the corresponding ratio $\Delta f_2(p_2)/(1 - F_2(p_2))$ for Figure 1.

same in both settings, the monotonicity assumption on $\frac{f_2(p_2|\theta_1)}{1-F_2(p_2|\theta_1)}$ enhances the difference between the price elasticities of demand for the bundle $\{1,2\}$ and demand for admission to public good 2.

If it is actually possible to enforce combination tickets being *more,* rather than less, expensive than the sum of individual tickets, one gets an even stronger result. In this case, the arguments of McAfee et al. (1989) imply that, for generic taste distributions, the Ramsey-Boiteux solution is dominated by a price schedule that is not arbitrage free. If all price schedules are admitted, it could only be by a fluke that first-order conditions for the pricing of bundles are all satisfied at the arbitrage-free price schedule that corresponds to the vector of Ramsey-Boiteux prices.

Whereas the preceding analysis has contrasted third-best allocations with allocations induced by price schedules, the reader may wonder about the relation between the latter and second-best allocations. For the multiproduct monopoly problem, the examples of Thanassoulis (2004) as well as Manelli and Vincent (2006 a, b) show that price schedules themselves are likely be dominated by more complicated schemes involving nontrivial randomization over admissions to the different public goods. Given that the formal structure of the second-best welfare problem is very similar to the monopoly problem, the lesson from these examples should apply in the current setting as well.[29]

A focus on allocations that are induced by price schedules can be justified by the weakening of the renegotiation proofness condition that was mentioned in the preceding section. If the mechanism designer is unable to prevent side-trading, but "bundles" of admission tickets to different public goods can be prepared in such a way that "unbundling" is impossible, then an argument parallel to the one given before can be used to show that an allocation is renegotiation proof in this weaker sense and incentive-compatible if and only if it is induced by a price schedule.[30] An allocation that is induced by an *optimal price schedule* may thus be said to be 2.5^{th} best, i.e. *optimal in the set of all feasible, incentive-compatible and weakly renegotiation proof allocations.*

[29] On this point, see also Fang and Norman (2003/2006).

[30] The proof of this proposition proceeds by redefining goods so that each bundle is treated as a separate good and then applying the same arguments as in the preceding sections. For details, the reader is referred to Appendix B.

A Appendix: Proofs

Proof of Lemma 3.1. The "if" part of the lemma is an instance of the first welfare theorem. To prove the "only if" part, let $(\mathbf{Q}, c(\cdot,\cdot), \chi_1(\cdot,\cdot), ..., \chi_m(\cdot,\cdot))$ be a renegotiation proof allocation. For $i = 1, ..., m$, let $\hat{\theta}_i$ be the unique solution to the equation

$$1 - F_i(\hat{\theta}_i) = \int_{[0,1]^m \times \Omega} \chi_i(\boldsymbol{\theta}, \omega)\, dF(\boldsymbol{\theta}) d\nu(\omega). \tag{A.1}$$

Consider the net-trade allocation $(z_c(\cdot,\cdot), z_1(\cdot,\cdot), ..., z_m(\cdot,\cdot))$ satisfying

$$z_i(\boldsymbol{\theta}, \omega) = -\chi_i(\boldsymbol{\theta}, \omega) \ \text{ if } \ \theta_i < \hat{\theta}_i, \tag{A.2}$$

$$z_i(\boldsymbol{\theta}, \omega) = 1 - \chi_i(\boldsymbol{\theta}, \omega) \ \text{ if } \ \theta_i \geq \hat{\theta}_i, \tag{A.3}$$

for $i = 1, ..., m$, and

$$z_c(\boldsymbol{\theta}, \omega) = -\sum_{i=1}^{m} z_i(\boldsymbol{\theta}, \omega)\hat{\theta}_i Q_i. \tag{A.4}$$

One easily verifies that, for the given vector $\mathbf{Q}$ of public-good provision levels, the price system $(1, p_1, ..., p_m)$ and the allocation $(c(\cdot,\cdot), \chi_1(\cdot,\cdot), ..., \chi_m(\cdot,\cdot)) + (z_c(\cdot,\cdot), z_1(\cdot,\cdot), ..., z_m(\cdot,\cdot))$ correspond to a competitive equilibrium of the exchange economy involving trade in the private good and in admission tickets for the public goods, with initial endowments given by $(c(\cdot,\cdot), \chi_1(\cdot,\cdot), ..., \chi_m(\cdot,\cdot))$. Feasibility and net-trade incentive compatibility of the net-trade allocation $(z_c(\cdot,\cdot), z_1(\cdot,\cdot), ..., z_m(\cdot,\cdot))$ follow immediately, as does the dominance condition (3.4). Reallocation proofness of the allocation therefore implies that (3.6) does not hold. Given that (3.4) does hold, it follows that

$$z_c(\boldsymbol{\theta}, \omega) + \sum_{i=1}^{m} z_i(\boldsymbol{\theta}, \omega)\theta_i Q_i = 0 \tag{A.5}$$

for almost all $(\boldsymbol{\theta}, \omega) \in [0,1]^m \times \Omega$. From (A.4), one has

$$z_c(\boldsymbol{\theta}, \omega) + \sum_{i=1}^{m} z_i(\boldsymbol{\theta}, \omega)\theta_i Q_i = \sum_{i=1}^{m} z_i(\boldsymbol{\theta}, \omega)(\theta_i - \hat{\theta}_i)Q_i. \tag{A.6}$$

By (A.2) and (A.3), each of the summands on the right-hand side of (A.6) is nonnegative, so (A.5) implies that, for each i and almost all $(\boldsymbol{\theta}, \omega) \in [0,1]^m \times \Omega$, one has

$$z_i(\boldsymbol{\theta}, \omega)(\theta_i - \hat{\theta}_i)Q_i = 0, \tag{A.7}$$

hence, by (A.2) and (A.3),

$$[\max(\theta_i - \hat{\theta}_i, 0) - \chi_i(\boldsymbol{\theta}, \omega)(\theta_i - \hat{\theta}_i)]Q_i = 0. \tag{A.8}$$

Now (A.8) implies $\chi_i(\boldsymbol{\theta}, \omega) = 0$ if $\theta_i Q_i < \hat{\theta}_i Q_i$ and $\chi_i(\boldsymbol{\theta}, \omega) = 1$ if $\theta_i Q_i > \hat{\theta}_i Q_i$. Upon setting, $p_i = \hat{\theta}_i Q_i$, one finds that the claim of the lemma is established. ■

Proof of Lemma 3.2. The "if" part of the lemma is trivial. To prove the "only if" part, let $(\mathbf{Q}, c(\cdot,\cdot), \chi_1(\cdot,\cdot), ..., \chi_m(\cdot,\cdot))$ be a renegotiation proof and incentive-compatible allocation, and let $v(.)$ be the associated expected-payoff function as given by (2.14). By (2.14), one has $v(\mathbf{0}) = C(\mathbf{0})$. Moreover, Lemmas 2.2 and 3.1, and (2.11) imply that for almost any $\boldsymbol{\theta} \in [0,1]^m$, the function $v(.)$ has first partial derivatives satisfying

$$v_i(\boldsymbol{\theta}) = 0 \;\text{ if }\; \theta_i Q_i < p_i \tag{A.9}$$

and

$$v_i(\boldsymbol{\theta}) = Q_i \;\text{ if }\; \theta_i Q_i > p_i, \tag{A.10}$$

where $p_1, ..., p_m$ are the prices given by Lemma 3.1. By integration, one then obtains (3.12), so (2.15) implies that (3.9) and (3.10) hold for all $\boldsymbol{\theta} \in [0,1]^m$. From (3.12) and (2.14), one also obtains

$$C(\boldsymbol{\theta}) = C(\mathbf{0}) + \sum_{i=1}^{m} \max(\theta_i Q_i - p_i, 0) - \sum_{i=1}^{m} \pi_i(\boldsymbol{\theta})\theta_i Q_i, \tag{A.11}$$

so (3.11) follows from (3.9) and (3.10). ■

The proof of Proposition 3.3 is trivial and is left to the reader.

Proof of Proposition 3.4. By Proposition 3.3, $Q_1, ..., Q_m$ and $p_1, ..., p_m$ maximize (3.16) subject to (3.17). For some $\lambda \geq 0$ therefore, $Q_1, ..., Q_m$ and $p_1, ..., p_m$ maximize the Lagrangian expression

$$\sum_{i=1}^{m} \int_{\hat{\theta}_i(p_i,Q_i)}^{1} (\theta_i Q_i - p_i)\, dF_i(\theta_i) + \lambda \left(\sum_{i=1}^{m} p_i(1 - F_i(\hat{\theta}_i(p_i, Q_i))) - K(\mathbf{Q}) \right). \tag{A.12}$$

Given that $p_i = \hat{\theta}_i(p_i, Q_i)Q_i$ for all i, the problem of maximizing (A.12) with respect to $Q_1, ..., Q_m$ and $p_1, ..., p_m$ is equivalent to the problem of maximizing

$$\sum_{i=1}^{m} \int_{\hat{\theta}_i}^{1} (\theta_i - \hat{\theta}_i)Q_i\, dF_i(\theta_i) + \lambda \left(\sum_{i=1}^{m} \hat{\theta}_i Q_i(1 - F_i(\hat{\theta}_i)) - K(\mathbf{Q}) \right) \tag{A.13}$$

with respect to $Q_1, ..., Q_m$ and $\hat{\theta}_1, ..., \hat{\theta}_m$.

To prove that $Q_i > 0$, it suffices to observe that, for $Q_i = 0$, (A.13) is independent of $\hat{\theta}_i$, and, for $\hat{\theta}_i < 1$, at $Q_i = 0$, (A.13) is strictly increasing in Q_i. Any pair $(Q_i, \hat{\theta}_i)$ with $Q_i = 0$ is therefore dominated by the pair $(\varepsilon, \frac{1}{2})$, provided that $\varepsilon > 0$ is sufficiently small.

Given that public-good provision levels must be positive, the first-order conditions for maximizing (A.13) are given as

$$\int_{\hat{\theta}_i}^{1} (\theta_i - \hat{\theta}_i)\, dF_i(\theta_i) + \lambda \hat{\theta}_i (1 - F_i(\hat{\theta}_i)) - \lambda K_i(\mathbf{Q}) = 0 \tag{A.14}$$

for Q_i and

$$-\int_{\hat{\theta}_i}^{1} Q_i\, dF_i(\theta_i) + \lambda Q_i (1 - F_i(\hat{\theta}_i)) - \lambda \hat{\theta}_i Q_i f_i(\hat{\theta}_i) \leq 0 \tag{A.15}$$

for $\hat{\theta}_i$, with a strict inequality only if $\hat{\theta}_i = 0$. With $Q_i > 0$, (A.15) simplifies to:

$$(\lambda - 1)(1 - F_i(\hat{\theta}_i)) - \lambda \hat{\theta}_i f_i(\hat{\theta}_i) \leq 0, \tag{A.16}$$

with a strict inequality only if $\hat{\theta}_i = 0$.

The Lagrange multiplier must exceed one. For $\lambda \leq 1$, (A.16) would imply $\hat{\theta}_i = 0$, hence $p_i = 0$ for all i, and, by the constraint (3.17), $K(\mathbf{Q}) = 0$, which is impossible if $Q_i > 0$ for all i. Therefore one cannot have $\lambda \leq 1$. For $\lambda > 1$, (A.16) implies $1 > \hat{\theta}_i > 0$, hence $Q_i > p_i > 0$ for all i. Now (3.19) follows from (A.16) by substituting for $\hat{\theta}_i = p_i/Q_i > 0$. By a rearrangement of terms, (3.18) follows from (A.15). ∎

Proof of Proposition 4.1. By contradiction, suppose that the first statement of Proposition 4.1 is false. Then there exist $\mathbf{Q} >> \mathbf{0}$ and P^* such that P^* is arbitrage-free and, given $\mathbf{Q}$, P^* maximizes (4.5) over the set of price schedules inducing individually rational and feasible allocations.

For any price schedule P, an allocation induced by P is individually rational if $P(\emptyset) = 0$. By (2.20), the allocation is also feasible if

$$\int_{[0,1]^m} [v(\boldsymbol{\theta}|P) - \sum_{i=1}^{m} \theta_i v_i(\boldsymbol{\theta}|P)] f(\boldsymbol{\theta}) d\boldsymbol{\theta} \leq Y - K(Q). \tag{A.17}$$

Through integration by parts, as in McAfee et al. (1989), (A.17) is seen to be equivalent to the inequality

$$\int_{[0,1]^m} v(\boldsymbol{\theta}|P)[(m+1)f(\boldsymbol{\theta}) + \boldsymbol{\theta} \cdot \nabla f(\boldsymbol{\theta})] d\boldsymbol{\theta} - \sum_{i=1}^{m} \int_{[0,1]^{m-1}} v(1, \boldsymbol{\theta}_{-i}|P) f(1, \boldsymbol{\theta}_{-i}) d\boldsymbol{\theta}_{-i}$$
$$\leq Y - K(\mathbf{Q}). \tag{A.18}$$

Thus P^* maximizes (4.5) over the set of price schedules P that satisfy $P(\emptyset) = 0$ and (A.18).

For some $\lambda \geq 0$ and some μ, therefore, P^* maximizes the Lagrangian expression

$$\int_{[0,1]^m} v(\boldsymbol{\theta}|P)f(\boldsymbol{\theta})d\boldsymbol{\theta} - \lambda \int_{[0,1]^m} v(\boldsymbol{\theta}|P)[(m+1)f(\boldsymbol{\theta}) + \boldsymbol{\theta} \cdot \nabla f(\boldsymbol{\theta})]d\boldsymbol{\theta}$$
$$+\lambda \sum_{i=1}^{m} \int_{[0,1]^{m-1}} v(1, \boldsymbol{\theta}_{-i}|P)f(1, \boldsymbol{\theta}_{-i})d\boldsymbol{\theta}_{-i} + \lambda(Y - K(\mathbf{Q})) + \mu P(\emptyset). \quad \text{(A.19)}$$

Given that any price schedule P is characterized by the finite list of numbers $P(\emptyset), P(\{1\}), ..., P(\{m\}), ..., P(M)$, it follows that, for any nonempty set $J \subset M$, the first-order condition

$$\frac{\partial}{\partial P(J)} \int_{[0,1]^m} v(\boldsymbol{\theta}|P)f(\boldsymbol{\theta})d\boldsymbol{\theta} - \lambda \frac{\partial}{\partial P(J)} \int_{[0,1]^m} v(\boldsymbol{\theta}|P)[(m+1)f(\boldsymbol{\theta}) + \boldsymbol{\theta} \cdot \nabla f(\boldsymbol{\theta})]d\boldsymbol{\theta}$$
$$+\lambda \frac{\partial}{\partial P(J)} \sum_{i=1}^{m} \int_{[0,1]^{m-1}} v(1, \boldsymbol{\theta}_{-i}|P)f(1, \boldsymbol{\theta}_{-i})d\boldsymbol{\theta}_{-i} \leq 0, \quad \text{(A.20)}$$
$$\text{with equality unless } P^*(J) = 0,$$

must be satisfied at $P = P^*$. By an argument of Manelli and Vincent (2002),[31] (A.20) can be rewritten as

$$-\int_{A_J(P^*)} f(\boldsymbol{\theta})d\boldsymbol{\theta} + \lambda \int_{A_J(P^*)} [(m+1)f(\boldsymbol{\theta}) + \boldsymbol{\theta} \cdot \nabla f(\boldsymbol{\theta})]d\boldsymbol{\theta}$$
$$-\lambda \sum_{i \in J} \int_{B_J^i(P^*)} f(1, \boldsymbol{\theta}_{-i})d\boldsymbol{\theta}_{-i} \leq 0, \quad \text{(A.21)}$$
$$\text{with equality unless } P^*(J) = 0,$$

where, for any price schedule P, $A_J(P) = cl\ \{\boldsymbol{\theta} \in [0,1]^m |\ q_P(J;\boldsymbol{\theta}) > 0\}$, and, for any $i \in J$, $B_J^i(P) = cl\ \{\boldsymbol{\theta}_{-i} \in [0,1]^{m-1} |\ q_P(J; 1, \boldsymbol{\theta}_{-i}) > 0\}$. Given

[31] The idea behind the argument is that the derivatives of the integrals in (A.20) with respect to $P(J)$ can be equated with the integrals of the derivatives of the integrands on the interiors of the sets $A_J(P)$ and $B_J^i(P)$. The boundaries of these sets do not matter because they have Lebesgue measure zero.

that f takes the form $f(\boldsymbol{\theta}) = \prod_{j=1}^m f_j(\theta_j)$, (A.21) can be rewritten as:

$$(\lambda - 1)\int_{A_J(P^*)} \prod_{j=1}^m f_j(\theta_j) d\boldsymbol{\theta} + \lambda \sum_{i=1}^m \int_{A_J(P^*)} (f_i(\theta_i) + \theta_i f_i'(\theta_i)) \prod_{j\neq i} f_j(\theta_j) d\boldsymbol{\theta}$$

$$-\lambda \sum_{i\in J} \int_{B_J^i(P^*)} f_i(1) \prod_{j\neq i} f_j(\theta_j) d\boldsymbol{\theta}_{-i} \leq 0, \qquad \text{(A.22)}$$

with equality unless $P^*(J) = 0$.

I first show that, because P^* is arbitrage free, (A.22) implies the Ramsey-Boiteux first-order condition (3.19). For any j, I define $\mathcal{J}(j) := \{J \subset M | \; j \in J\}$ as the set of all sets J that contain public good j. Summation of (A.22) over $J \in \mathcal{J}(j)$ yields

$$(\lambda - 1)\int_{\cup_{J\in\mathcal{J}(j)} A_J(P^*)} \prod_{j=1}^m f_j(\theta_j) d\boldsymbol{\theta}$$

$$+\lambda \sum_{i=1}^m \int_{\cup_{J\in\mathcal{J}(j)} A_J(P^*)} (f_i(\theta_i) + \theta_i f_i'(\theta_i)) \prod_{j\neq i} f_j(\theta_j) d\boldsymbol{\theta}$$

$$-\lambda \sum_{i\in M} \int_{\cup_{J\in\mathcal{J}(j)\cap\mathcal{J}(i)} B_J^i(P^*)} f_i(1) \prod_{j\neq i} f_j(\theta_j) d\boldsymbol{\theta}_{-i} \leq 0, \qquad \text{(A.23)}$$

with equality unless $P^*(J) = 0$ for some $J \in \mathcal{J}(j)$.

Because P^* is arbitrage free, one has $\cup_{J\in\mathcal{J}(j)} A_J(P^*) = \{\boldsymbol{\theta} \in [0,1]^m | \; \theta_j \geq \hat{\theta}_j\}$, $\cup_{J\in\mathcal{J}(j)} B_J^j(P^*) = [0,1]^{m-1}$, and, for $i \neq j$, $\cup_{J\in\mathcal{J}(j)\cap\mathcal{J}(i)} B_J^i(P^*) = \{\boldsymbol{\theta}_{-i} \in [0,1]^{m-1} | \; \theta_j \geq \hat{\theta}_j\}$, where $\hat{\theta}_j = \frac{P(\{j\})}{Q_j}$. Thus (A.23) can be rewritten as

$$(\lambda - 1)(1 - F_j(\hat{\theta}_j)) + \lambda \int_{\hat{\theta}_j}^1 [f_j(\theta_j) + \theta_j f_j'(\theta_j)] d\theta_j$$

$$+\sum_{\substack{k=1\\k\neq j}}^m (1 - F_j(\hat{\theta}_j)) \int_0^1 [f_k(\theta_k) + \theta_k f_k'(\theta_k)] d\theta_k - \lambda f_j(1) - \sum_{\substack{i=1\\i\neq j}}^m (1 - F_j(\hat{\theta}_j)) f_i(1) \leq 0,$$

with equality unless $\hat{\theta}_j = 0$.

Upon computing the integrals and cancelling terms involving $f_j(1)$ or $f_i(1)$, one can rewrite this condition as

$$(\lambda - 1)(1 - F_j(\hat{\theta}_j)) - \lambda \hat{\theta}_j f_j(\hat{\theta}_j) \leq 0, \qquad \text{(A.24)}$$

with equality unless $\hat{\theta}_j = 0$.

This is identical with condition (A.16) in the proof of Proposition 3.4. The same argument as was given there shows that one must have $\lambda > 1$ and $1 > \hat{\theta}_i > 0$, and hence

$$(\lambda - 1)(1 - F_j(\hat{\theta}_j)) - \lambda\hat{\theta}_j f_j(\hat{\theta}_j) = 0 \tag{A.25}$$

for all j.

Because P^* is arbitrage free, for $j = 1, ..., m$, one also has $A_{\{j\}}(P^*) = [\hat{\theta}_j, 1] \times \prod_{i \neq j}[0, \hat{\theta}_i]$ and $B^j_{\{j\}}(P^*) = \prod_{i \neq j}[0, \hat{\theta}_i]$, where again $\hat{\theta}_j = \frac{P^*(\{j\})}{Q_j}$ and $\hat{\theta}_i = \frac{P^*(\{i\})}{Q_i}$. Since $\hat{\theta}_j > 0$ implies $P^*(\{j\}) > 0$, for $J = \{j\}$, (A.22) becomes

$$\begin{aligned} -(1-\lambda)(1 - F_j(\hat{\theta}_j)) \prod_{\substack{i=1 \\ i \neq j}}^m F_i(\hat{\theta}_i) + \lambda \int_{\hat{\theta}_j}^1 [f_j(\theta_j) + \theta_j f_j'(\theta_j)] d\theta_i \prod_{\substack{i=1 \\ i \neq j}}^m F_i(\hat{\theta}_i) \\ + \lambda(1 - F_j(\hat{\theta}_j)) \sum_{\substack{k=1 \\ k \neq j}}^m \int_{[0,\hat{\theta}_k]} [f_k(\theta_k) + \theta_k f_k'(\theta_k)] d\theta_k \prod_{\substack{\ell=1 \\ \ell \neq j,k}}^m F_\ell(\hat{\theta}_\ell) \\ - \lambda f_j(1) \prod_{\substack{i=1 \\ i \neq j}}^m F_i(\hat{\theta}_i) = 0. \end{aligned} \tag{A.26}$$

Upon computing the integrals in (A.26), cancelling terms involving $f_j(1)$ and dividing by $\prod_{i \neq j} F_i(\hat{\theta}_i)$, one further obtains

$$(\lambda - 1)(1 - F_j(\hat{\theta}_j)) - \lambda\hat{\theta}_j f_j(\hat{\theta}_j) + \lambda(1 - F_j(\hat{\theta}_j)) \sum_{\substack{k=1 \\ k \neq j}}^m \frac{\hat{\theta}_k f_k(\hat{\theta}_k)}{F_k(\hat{\theta}_k)} = 0. \tag{A.27}$$

By (A.24), it follows that

$$\lambda(1 - F_j(\hat{\theta}_j)) \sum_{\substack{k=1 \\ k \neq j}}^m \frac{\hat{\theta}_k f_k(\hat{\theta}_k)}{F_k(\hat{\theta}_k)} = 0,$$

which is impossible because $\hat{\theta}_j < 1$ and $\hat{\theta}_k > 0$ for all k. The assumption that the first statement of the proposition is false has thus led to a contradiction.

Turning to the second statement of the proposition, the argument just given implies that, at the arbitrage-free price schedule P^* that induces a third-best allocation, the derivative of the Lagrangian (A.19) with respect to the singleton price $P(\{j\})$ is strictly positive. As for the bundle M, because

P^* is arbitrage free, one has $A_M(P^*) = \prod_{j=1}^m [\hat{\theta}_j, 1]$, and, for $i = 1, ..., m$, $B^i_M(P^*) = \prod_{j \neq i}[\hat{\theta}_j, 1]$, so, at $P = P^*$, the derivative of the Lagrangian (A.19) with respect to the price $P(M)$ takes the form

$$-(1-\lambda)\prod_{j=1}^m (1 - F_j(\hat{\theta}_j)) + \lambda \sum_{i=1}^m \prod_{\substack{j=1 \\ j \neq i}}^m (1 - F_j(\hat{\theta}_j)) \int_{\hat{\theta}_i}^1 [f_i(\theta_i) + \theta_i f_i'(\theta_i)] d\theta_i$$
$$-\lambda \sum_{i=1}^m f_i(1) \prod_{\substack{j=1 \\ j \neq i}}^m (1 - F_j(\hat{\theta}_j)),$$

which simplifies to

$$(\lambda - 1)\prod_{j=1}^m (1 - F_j(\hat{\theta}_j)) - \lambda \sum_{i=1}^m \prod_{\substack{j=1 \\ j \neq i}}^m (1 - F_j(\hat{\theta}_j))\hat{\theta}_i f_i(\hat{\theta}_i). \tag{A.28}$$

Upon using (A.24) to substitute for $\lambda \hat{\theta}_i f_i(\hat{\theta}_i)$, $i = 1, ..., m$, one finds that (A.28) is equal to

$$(\lambda - 1)(1 - m)\prod_{j=1}^m (1 - F_j(\hat{\theta}_j)),$$

which is strictly negative because $m > 1$. The dominating price schedule $\hat{P}$ in the vicinity of P^* may thus be chosen with $\hat{P}(M) < P^*(M)$, as well as $\hat{P}(\{j\}) > P^*(\{j\})$ for $j = 1, ..., m$. ■

Proof of Proposition 4.2. Suppose that the proposition is false. For $m = 2$, let $\mathbf{Q} >> \mathbf{0}$ and P^* be such that P^* is an optimal price schedule, given $\mathbf{Q}$ and $P^*(\{1,2\}) \geq P^*(\{1\}) + P^*(\{2\})$. Optimality of P^* implies that, for some $\lambda \geq 0$ and some μ, P^* satisfies the first-order condition (A.22). Feasibility implies that $P^*(\{1,2\}) > 0$.

If $P^*(\{1,2\}) \geq P^*(\{1\}) + P^*(\{2\})$, one has $A_{\{1,2\}}(P^*) = [\bar{\theta}_1, 1] \times [\bar{\theta}_2, 1]$, where, for $i = 1, 2$, $\bar{\theta}_i := P^*(\{1,2\}) - P^*(\{i\})$. For $J = \{1,2\}$, with $P^*(\{1,2\}) > 0$, (A.22) then yields:

$$(\lambda - 1)(1 - F_1(\bar{\theta}_1))(1 - F_2(\bar{\theta}_2)) - \lambda \bar{\theta}_1 f_1(\bar{\theta}_1)(1 - F_2(\bar{\theta}_2))$$
$$-\lambda \bar{\theta}_2 f_2(\bar{\theta}_2)(1 - F_1(\bar{\theta}_1)) = 0,$$

or

$$(\lambda - 1) = \lambda \sum_{i=1,2} \frac{\bar{\theta}_i f_i(\bar{\theta}_i)}{1 - F_i(\bar{\theta}_i)}. \tag{A.29}$$

If $P^*(\{1,2\}) \geq P^*(\{1\}) + P^*(\{2\})$, one also has

$$A_{\{1\}}(P^*) = \{(\theta_1, \theta_2)|\ \theta_2 \leq \bar{\theta}_2 \text{ and } \theta_1 \geq \hat{\theta}_1(\theta_2)\},$$

where $\hat{\theta}_1(\theta_2) := P^*(\{1\}) + \max(\theta_2 - P^*(\{2\}), 0)$. For $J = \{1\}$, therefore, (A.22) implies

$$(\lambda - 1) \int_0^{\bar{\theta}_2} (1 - F_1(\hat{\theta}_1(\theta_2)) f_2(\theta_2) d\theta_2 - \lambda \int_0^{\bar{\theta}_2} \hat{\theta}_1(\theta_2) f_1(\hat{\theta}_1(\theta_2)) f_2(\theta_2) d\theta_2 \leq 0$$

or

$$\int_0^{\bar{\theta}_2} \left[(\lambda - 1) - \lambda \frac{\hat{\theta}_1(\theta_2) f_1(\hat{\theta}_1(\theta_2))}{(1 - F_1(\hat{\theta}_1(\theta_2))} \right] (1 - F_1(\hat{\theta}_1(\theta_2)) f_2(\theta_2) d\theta_2 \leq 0. \quad \text{(A.30)}$$

For $\theta_2 \leq \bar{\theta}_2$, one has

$$\begin{aligned} \hat{\theta}_1(\theta_2) &\leq P^*(\{1\}) + \max(\bar{\theta}_2 - P^*(\{2\}), 0) \\ &= P^*(\{1\}) + \max(P^*(\{1,2\}) - P^*(\{1\}) - P^*(\{2\})), 0) \\ &= P^*(\{1,2\}) - P^*(\{2\}) = \bar{\theta}_1. \end{aligned}$$

Given the assumed monotonicity of the functions $\theta_i \to \frac{\theta_i f_i(\theta_i)}{1 - F_i(\theta_i)}$, it follows that (A.30) implies

$$(\lambda - 1) \leq \lambda \frac{\bar{\theta}_1 f_1(\bar{\theta}_1)}{1 - F_1(\bar{\theta}_1)}. \quad \text{(A.31)}$$

By a precisely symmetric argument for the set $\{2\}$, one also has

$$(\lambda - 1) \leq \lambda \frac{\bar{\theta}_2 f_2(\bar{\theta}_2)}{1 - F_2(\bar{\theta}_2)}. \quad \text{(A.32)}$$

Upon combining (A.31) and (A.32) with (120), one obtains

$$(\lambda - 1) \geq 2(\lambda - 1),$$

which implies $\lambda \leq 1$. By (A.31), it follows that $\bar{\theta}_i = 0$ for $i = 1, 2$, hence $P^*(\{1,2\}) = 0$, which is impossible if the cost $K(\mathbf{Q}) > 0$ is to be covered. The assumption that $P^*(\{1,2\}) \geq P^*(\{1\}) + P^*(\{2\})$ thus leads to a contradiction and must be false. ■

B Appendix: Weak Renegotiation Proofness

In this appendix, I consider the possibility that the mechanism designer prepares bundles of admission tickets to the different public goods in such a way that participants are unable to unbundle them. In this case, an *allocation* will be an array

$$(\mathbf{Q}, c(\cdot,\cdot), \{\chi_J(\cdot,\cdot)\}_{J\subset M}), \tag{B.1}$$

such that $\mathbf{Q} = (Q_1, ..., Q_m)$ is the vector of public-good provision levels, $c(\cdot,\cdot)$ is a function which stipulates for each $(\boldsymbol{\theta}, \omega) \in [0,1]^{m+1}$, a level $c(\boldsymbol{\theta}, \omega)$ of private-good consumption for consumer h in the state x if $(\tilde{\boldsymbol{\theta}}^h(x), \tilde{\omega}^h(x)) = (\boldsymbol{\theta}, \omega)$, and, for each subset J of the set M of public goods, $\chi_J(\cdot,\cdot)$ is a function which stipulates for each $(\boldsymbol{\theta}, \omega) \in [0,1]^{m+1}$ whether a consumer h gets a ticket to the bundle J if $(\tilde{\boldsymbol{\theta}}^h(x), \tilde{\omega}^h(x)) = (\boldsymbol{\theta}, \omega)$ or whether he does not get such a ticket. In the first case, $\chi_J(\boldsymbol{\theta}, \omega)$ takes the value one, in the second, the value zero. Assuming that the mechanism designer specifies exactly[32] one bundle per consumer, one also has $\sum_{J\subset M} \chi_J(\boldsymbol{\theta}, \omega) = 1$.

For any $(\boldsymbol{\theta}, \omega) \in [0,1]^{m+1}$, the allocation $(\mathbf{Q}, c(.,.), \{\chi_J(.,.)\}_{J\subset M})$ provides consumer h with the payoff

$$c(\boldsymbol{\theta}, \omega) + \sum_{J\subset M} \chi_J(\boldsymbol{\theta}, \omega) \sum_{j\in J} \theta_j Q_j \tag{B.2}$$

if $(\tilde{\boldsymbol{\theta}}^h(x), \tilde{\omega}^h(x)) = (\boldsymbol{\theta}, \omega)$.

A *net-trade allocation* is now defined as an array

$$(z_c(.,.), \{z_J(.,.)\}_{J\subset M}) \tag{B.3}$$

such that, for any $(\boldsymbol{\theta}, \omega) \in [0,1]^{m+1}$, $z_c(\boldsymbol{\theta}, \omega)$ and $z_J(\boldsymbol{\theta}, \omega), J \subset M$, are the net additions to private-good consumption and admission tickets to bundle J which are stipulated for consumer h if $(\tilde{\boldsymbol{\theta}}^h(x), \tilde{\omega}^h(x)) = (\boldsymbol{\theta}, \omega)$. Given the initial allocation (B.1), a net-trade allocation is said to be *feasible* if $\chi_J(\boldsymbol{\theta}, \omega) + z_J(\boldsymbol{\theta}, \omega) \in \{0,1\}$ for all J, $\sum_{J\subset M}[\chi_J(\boldsymbol{\theta}, \omega) + z_J(\boldsymbol{\theta}, \omega)] = 1$, and, moreover,

$$\int_{[0,1]^{m+1}} z_i(\boldsymbol{\theta}, \omega) f(\boldsymbol{\theta}) d\boldsymbol{\theta} d\nu(\omega) = 0 \tag{B.4}$$

for $i = c, \{1\}, \{2\}, ..., M$. Given the initial allocation (B.1), the net-trade allocation is *incentive-compatible* if

$$z_c(\boldsymbol{\theta}, \omega) + \sum_{J\subset M} z_J(\boldsymbol{\theta}, \omega) \sum_{j\in J} \theta_j Q_j \geq z_c(\boldsymbol{\theta}', \omega') + \sum_{J\subset M} z_J(\boldsymbol{\theta}', \omega') \sum_{j\in J} \theta_j Q_j \tag{B.5}$$

[32] In this formalism, the empty set is one of the bundles that can be assigned.

for all $(\boldsymbol{\theta}, \omega)$ and $(\boldsymbol{\theta}', \omega')$ in $[0,1]^{m+1}$ for which $\chi_J(\boldsymbol{\theta}, \omega) + z_J(\boldsymbol{\theta}', \omega') \in \{0,1\}$ and $\sum_{J \subset M}[\chi_J(\boldsymbol{\theta}, \omega) + z_J(\boldsymbol{\theta}', \omega')] = 1$.

An allocation is said to be *weakly renegotiation proof* if there is no feasible and incentive-compatible net-trade allocation that provides a Pareto improvement over this allocation, i.e., there is no $(z_c(\cdot,\cdot), \{z_J(\cdot,\cdot)\}_{J \subset M})$ satisfying

$$z_c(\boldsymbol{\theta}, \omega) + \sum_{J \subset M} z_J(\boldsymbol{\theta}, \omega) \sum_{j \in J} \theta_j Q_j \geq 0 \tag{B.6}$$

for all $(\boldsymbol{\theta}, \omega) \in [0,1]^{m+1}$ and

$$\int_{[0,1]^{m+1}} [z_c(\boldsymbol{\theta}, \omega) + \sum_{J \subset M} z_J(\boldsymbol{\theta}, \omega) \sum_{j \in J} \theta_j Q_j] f(\boldsymbol{\theta}) d\boldsymbol{\theta} d\nu(\omega) > 0, \tag{B.7}$$

as well as (B.4) and (B.5).

The following lemmas provide analogues of Lemmas 3.1 and 3.2 for this weaker concept of renegotiation proofness.

Lemma B.1 *The allocation (B.1) is weakly renegotiation proof if and only if there exists a price schedule $P(\cdot)$ such that, for almost all $(\boldsymbol{\theta}, \omega) \in [0,1]^{m+1}$, the vector $\{\chi_J(\boldsymbol{\theta}, \omega)\}_{J \subset M}$ is a solution to the problem*

$$\max_{\{\chi_J\}_{J \subset M}} \sum_{J \subset M} \chi_J \left[\sum_{j \in J} \theta_j Q_j - P(J) \right] \tag{B.8}$$

under the constraints that $\chi_J \in \{0,1\}$ for all $J \subset M$ and $\sum_{J \subset M} \chi_J = 1$.

Proof Sketch. The argument is the same as for Lemma 3.1. The "if" part of the lemma is again an instance of the first welfare theorem. As for the "only if" part, one easily verifies that, for any allocation and any price schedule P, the set of solutions to problem (B.8) is nonempty. Moreover, the solution correspondence is upper hemi-continuous in P. Given that the measure $F \times \nu$ is atomless, it follows that the maximizer correspondence for problem (B.8) and the endowment specification in (B.1) generate an upper hemi-continuous and convex-valued aggregate excess demand correspondence. Moreover, one easily verifies that, if P is required to take values in $[0, m]$, the aggregate excess demand correspondence satisfies a suitable boundary condition. For any initial allocation, a standard fixed-point argument therefore yields the existence of a competitive-equilibrium price schedule P. If $\{\chi_J(\boldsymbol{\theta}, \omega)\}_{J \subset M}$ fails to be a solution to problem (B.8), the associated competitive-equilibrium net-trade allocation is feasible and incentive-compatible and provides Pareto improvement over the allocation (B.1).

Hence if this allocation is weakly renegotiation proof, then $\{\chi_J(\boldsymbol{\theta},\omega)\}_{J\subset M}$ must be a solution to problem (B.8). ■

Lemma B.2 *The allocation (B.1) is weakly renegotiation proof and incentive-compatible if and only if there exists a price schedule $P(\cdot)$. such that, for all $\boldsymbol{\theta}\in[0,1]^m$, the purchase probabilities*

$$q(J;\boldsymbol{\theta}) := \int_{[0,1]} \chi_J(\boldsymbol{\theta},\omega)d\nu(\omega) \tag{B.9}$$

maximize

$$\sum_{J\subset M} q_J \left[\sum_{j\in J}\theta_j Q_j - P(J)\right] \tag{B.10}$$

under the constraints that $q_J \geq 0$ for all $J\subset M$ and $\sum_{J\subset M} q_J = 1$, and moreover, there exists $\bar{C}$, such that

$$C(\boldsymbol{\theta}) := \int_{[0,1]} c(\boldsymbol{\theta},\omega)d\nu(\omega) = \bar{C} - \sum_{J\subset M)} q(J;\boldsymbol{\theta})P(J) \tag{B.11}$$

and

$$v(\boldsymbol{\theta}) = \bar{C} + \max_{J\subset M}\left[\sum_{j\in J}\theta_j Q_j - P(J)\right] \tag{B.12}$$

for all $\boldsymbol{\theta}\in[0,1]^m$.

Proof. As in the proof of Lemma 3.2, the "if" part of the lemma is trivial. As for the "only if" part, I first note that, if the allocation (B.1) is weakly renegotiation proof and incentive-compatible, then Lemma B.1 and (B.9) imply that there exists a price schedule $P(\cdot)$, such that the vector $\{q(J;\boldsymbol{\theta})\}_{J\subset M}$ of purchase probabilities maximizes (B.10) under the constraints that $q_J\geq 0$ for all $J\subset M$ and $\sum_{J\subset M} q_J = 1$.

Conditional on $\tilde{\boldsymbol{\theta}} = \boldsymbol{\theta}$, the allocation generates the expected payoff

$$\begin{aligned} v(\boldsymbol{\theta}) &= \int_{[0,1]}[c(\boldsymbol{\theta},\omega) + \sum_{J\subset M}\chi_J(\boldsymbol{\theta},\omega)\sum_{j\in J}\theta_j Q_j]d\nu(\omega) && \text{(B.13)} \\ &= C(\boldsymbol{\theta}) + \sum_{J\subset M} q(J;\boldsymbol{\theta})\sum_{j\in J}\theta_j Q_j. && \text{(B.14)} \end{aligned}$$

Trivially, (B.14) implies $v(\mathbf{0}) = C(\mathbf{0})$. Moreover, Lemma 2.2 implies, that for almost any $\boldsymbol{\theta} \in [0,1]^m$, the function $v(\cdot)$ has first partial derivatives satisfying

$$v_i(\boldsymbol{\theta}) = \sum_{J \subset M} \delta_{iJ}\, q(J;\boldsymbol{\theta})\, Q_i, \tag{B.15}$$

where again $\delta_{iJ} = 1$ if $i \in J$ and $\delta_{iJ} = 0$ if $i \notin J$.

Define $\lambda_0 = 0$ and, for $k = 1, ...$ let λ_k, J_k be such that, for $\lambda \in [\lambda_{k-1}, \lambda_k]$,

$$J_k \in \arg\max_J \left[\sum_{j \in J} \lambda \theta_j Q_j - P(J) \right]. \tag{B.16}$$

Further, let $\bar{k}$ be such that $\lambda_{\bar{k}} < 1$ and $\lambda_{\bar{k}+1} \geq 1$. From (B.14) - (B.16), one obtains

$$v(\boldsymbol{\theta}) - v(\lambda_{\bar{k}}\, \boldsymbol{\theta}) = \sum_{J \subset M} q(J;\boldsymbol{\theta}) \sum_{j \in J} [\theta_j Q_j - \lambda_{\bar{k}} \theta_j Q_j]$$

$$= \sum_{J \subset M} q(J;\boldsymbol{\theta}) \left(\sum_{j \in J} \theta_j Q_j - P(J) \right) - \sum_{J \subset M} q(J;\boldsymbol{\theta}) \left(\sum_{j \in J} \lambda_{\bar{k}} \theta_j Q_j - P(J) \right)$$

$$= \sum_{j \in J_{\bar{k}+1}} \theta_j Q_j - P(J_{\bar{k}+1}) - \left(\sum_{j \in J_{\bar{k}+1}} \lambda_{\bar{k}} \theta_j Q_j - P(J_{\bar{k}+1}) \right)$$

$$= \sum_{j \in J_{\bar{k}+1}} \theta_j Q_j - P(J_{\bar{k}+1}) - \left(\sum_{j \in J_{\bar{k}}} \lambda_{\bar{k}} \theta_j Q_j - P^A(J_{\bar{k}}) \right),$$

the last equation following from the maximization property of the vectors $\{q^A(J;\lambda\boldsymbol{\theta})\}_{J \subset M}$ and the sets $J_{\bar{k}+1}$ and $J_{\bar{k}}$. By a precisely analogous argument, one also obtains

$$v(\lambda_k\, \boldsymbol{\theta}) - v(\lambda_{k-1}\, \boldsymbol{\theta}) = \sum_{j \in J_k} \lambda_k \theta_j Q_j - P(J_k) - \left(\sum_{j \in J_{k-1}} \lambda_{k-1} \theta_j Q_j - P(J_{k-1}) \right)$$

for $k = \bar{k}, \bar{k}-1, ..., 1$. Upon adding these equations, one concludes that

$$\begin{aligned} v(\boldsymbol{\theta}) - v(\mathbf{0}) &= \sum_{j \in J_{\bar{k}+1}} \theta_j Q_j - P(J_{\bar{k}+1}) - (-P(J_0)). \\ &= \sum_{j \in J_{\bar{k}+1}} \theta_j Q_j - P(J_{\bar{k}+1}) + \sum_{J \subset M} q(J;\mathbf{0}) P(J), \end{aligned}$$

or, equivalently,

$$v(\boldsymbol{\theta}) = C(\mathbf{0}) + \sum_{J \subset M} q(J;\mathbf{0})P(J) + \max_{J \subset M} \left[\sum_{j \in J} \theta_j Q_j - P(J) \right].$$

Upon setting $\bar{C} := C(\mathbf{0}) + \sum_{J \subset M} q(J;\mathbf{0})P(J)$, one obtains (B.12). From (B.14), one then also obtains (B.11). ■

In view of Lemma B.2, the problem of finding an allocation that maximizes welfare over the set of allocations that are feasible, incentive-compatible, weakly renegotiation proof and individually rational is equivalent to the problem of finding an optimal price schedule.

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