Pieter Omtzigt

Money Demand in the Netherlands

2002/26

UNIVERSITÀ DELL'INSUBRIA
FACOLTÀ DI ECONOMIA

http://eco.uninsubria.it

In questi quaderni vengono pubblicati i lavori dei docenti della Facoltà di Economia dell’Università dell’Insubria. La pubblicazione di contributi di altri studiosi, che abbiano un rapporto didattico o scientifico stabile con la Facoltà, può essere proposta da un professore della Facoltà, dopo che il contributo sia stato discusso pubblicamente. Il nome del proponente è riportato in nota all'articolo. I punti di vista espressi nei quaderni della Facoltà di Economia riflettono unicamente le opinioni degli autori, e non rispecchiano necessariamente quelli della Facoltà di Economia dell'Università dell'Insubria.

These Working papers collect the work of the Faculty of Economics of the University of Insubria. The publication of work by other Authors can be proposed by a member of the Faculty, provided that the paper has been presented in public. The name of the proposer is reported in a footnote. The views expressed in the Working papers reflect the opinions of the Authors only, and not necessarily the ones of the Economics Faculty of the University of Insubria.

Printed in Italy in July 2003
Università degli Studi dell'Insubria
Via Ravasi 2, 21100 Varese, Italy

Money Demand in the Netherlands

Pieter Omtzigt *

July 17, 2003

Abstract

In this paper we discuss money demand in the Netherlands over the period 1979-1999. The model it with a VAR integrated of order two and use full maximum likelihood for inference of testing. We find a stable money demand function over the period considered. It depends on the short term interest rate available to private investors, not the rate in the money market.

1 Introduction

The introduction[1] of the Euro in 12 of the 15 members of the European Union, has led to a large debate, scholarly, political and popular, on the costs and benefits of a monetary union. Discussion still takes place on the costs and benefits of joining for the three old EU members, that have so far stayed out, and the ten new members, who will join the EU in 2004. The most frequently cited cost is probably the inability to react differently to asymmetric shocks. Different transmission mechanisms in the different countries are a second source of asymmetry: due to institutional differences, like a prevalence of fixed rate mortgages (the Netherlands) or floating rate mortgages (the UK) and differences in structural parameters of the economy can cause a monetary intervention to have different effects even to economies hit by exactly the same shock.

The Netherlands has in the last few years often been praised for its economic successes of the last two decades: a rapidly declining unemployment rate, increasing participation rate and a massive public deficit, which had turned into a small surplus before the latest economic slowdown. This success took place, while the Netherlands had a de facto monetary union with Germany, by far its most important trade partner. The Dutch central bank had rendered control of its monetary polity to the German Bundesbank. Within the Euro-area, countries relinquish virtually all their monetary autonomy to the European Central Bank in exchange for a very small say in the actual running of monetary policy.

*University of Amsterdam, Faculty of Economics and Econometrics, Roetersstraat 11, 1018WB Amsterdam, The Netherlands, Email: P.H.Omtzigt@uva.nl

[1] I am grateful to Katarina Juselius for her help and patience in the econometric modelling of the data. She is not responsible for any errors or policy statements.

Gijs de Bruin (postbank), Annelies van Rhenen (spaarbeleg), IJda Geerts (NVM), Dhr. Elferich (De Nederlandsche Bank) kindly provided data for this paper.

Studying Dutch money demand and monetary transmission mechanism may therefore provide a valuable insight in this success story and provide understanding as to how a monetary union can work or fail, a question which is still relevant.

In the next section, the Dutch macro-economic and political situation are described together with the monetary policy pursued in the period under study, 1979 1st quarter until 1998, 4th quarter, when the Euro was introduced. From 1999 there are no publicly available statistics on the money supply in the individual countries inside the Euro area, such that the study cannot be extended.

Then follows section 3 with a short overview of the literature and a description of the data. Section 4 contains an overview of the relevant economic literature. In section 5 we discuss the I(2) and I(1) methodology, the nominal to real transformation and automated model selection as well as small sample properties of the estimators used. The analysis of the Dutch data follows in section 6 before final conclusions are drawn.

A technical result concerning bootstrapping linear within equation restrictions in the cointegration space is given in section A.

Throughout this paper we shall argue that the bootstrap (of tests on the cointegration parameters) should be based on the unrestricted estimate. We then need to find alternative null hypotheses for the bootstrap samples. In the appendix to this paper an alternative null hypothesis for linear restrictions within each cointegration vector is given.

2 The Netherlands

In this long section we given an overview of the economic and political situation in the Netherlands in the period we study. We then proceed with stating the monetary objectives, the instruments that were used to achieve them and the exchange rate policy followed. Then follows a short account of the liberalization process.

2.1 Political and Economic situation

During the second world war, the Netherlands had undergone extensive economic damage. After the war, a policy of national reconstruction was started, strongly led by the central government, which channelled credit to specific sectors, set explicit targets for the number of houses to be built each year, and in agreement with the social partners, implemented a policy of moderate wage increases until 1963. After that year a wage-price spiral erupted, which was fuelled by the revenue of newly discovered natural gas reserves in the North of the country. The 1960s and 1970s saw a rapid expansion of the Welfare State and even the two oil shocks did not immediately hit the Netherlands as hard as it did other countries: natural gas income provided a rapidly increasing source of income. In 1979 a housing market asset bubble burst and by 1981 the labour share of national income had risen to over 95%. Unemployment started to rise rapidly as the Netherlands entered its severest post-war depression in 1982, during which national output fell to 1978 levels, and unemployment tripled.

From 1977 to 1982, 3 increasingly unstable coalitions governed the Netherlands. Although the social democrats were the big winners of the 1982 general election, a coalition between the Christian Democrats (center) and Conservative Liberals was formed in November 1982, under prime minister Ruud Lubbers, which was to govern until 1989.

Under pressure from the government and the unfavourable economic situation, the leaders of the Dutch employers, C. van der Lede and employees, Wim Kok, agreed on the so-called “Agreement of Wassenaar”, which contained wage restraint, a shorter working week and redivision of existing jobs. The government itself pledged to reduce the taxation on labour, once the deficit, which was heading for double digit figure, would allow so. It also cut nominal wages in the public sector, benefits and minimum wages and embraced the market: privatization processes were started, markets were liberalized and the public sector was gradually sized down.

At the same time, the Netherlands entered the hard ERM in 1983, after an unexpected devaluation in 1983. All in all, the macroeconomic performance of the Netherlands improved remarkably from the second half of the 1980s onwards, when inflation was well below German levels, the unemployment rate declined as did the deficit of the government.

In 1989 a new coalition of the Christian Democrats and Social Democrats, who had decidedly moved to the center, took over with a promise of a more social policy. The fall of the German wall and German unification provided a positive demand shock, but when the business cycle swung down, the government, which was still running a deficit, felt that it was necessary to implement a hugely unpopular austerity package of roughly 3% of GDP in the first few months of 1991. This was accompanied by a large increase in the current account balance. The package was also necessary in the light of the Maastricht treaty, which was being drafted by the Dutch government, and contained debt and deficit criteria, to which the Netherlands would not be able to stick without the package. Furthermore the high nominal and real interest rates on the capital market were aggravating the problem for a country with a debt/GDP ratio of 80%. The treaty was approved by parliament, without much discussion and without a popular vote, meaning that the Netherlands would be in the first wave entrants to Euroland.

During the ERM crisis of 1992-93, the Guilder-DM mark parity was never seriously tested by the market and when the system collapsed the Dutch and German authorities entered into a bilateral agreement to maintain the old parities. As Belgium and France, two other important trade partners, also recovered to their old parities quite quickly, the crisis probably had less of an impact on the Dutch economy than on other European economies.

A new government of Social Democrats, Conservative Liberals and Liberals superseded the old government in 1994 and continued with a socially tinted, neo-liberal economic policy. The Dutch economy flourished more than ever, as its official unemployment was the lowest in the Euro-area, and in 1999 the government was able to record its first surplus in 25 years.

2.2 Monetary Policy

The Dutch Bank Law[2] of 1948 states that:

> “It shall be the task of the Dutch Central Bank to regulate the value of the Dutch currency in such a way as is most conductive for the prosperity of the

[2] Bankwet 1948, published in het Staatsblad I-166 on May 14th 1948. Another unofficial translation in English can be found in the Annual Report of De Nederlandsche Bank over 1948.

nation and in doing so, to stabilize that value as much as possible."[3]

Apparently there are three objectives: a welfare objective, internal price stability and external, exchange rate stability. In fact the last part of the phrase was added by parliament to interpret the first part, see De Jong (1960). The law already curbed the independence of the Central Bank with respect to the pre-war law in two important ways: all its shares were bought by the Dutch state (probably at a price slightly below market value) and an important new article was added, which stated that the Minister of Finance could give binding "directions" to the Bank. They have never been given and the article has been removed from the law in 1998 in preparation for the Monetary Union, but in the frequent informal meetings between the President of the Bank and the Minister of Finance the sheer possibility of them gave considerable leverage to the Minister. So the Dutch central bank was somewhat less independent than the German Bundesbank.

In practice Dutch monetary policy, often referred to as "Moderate Monetarism", aimed at exchange rate stability, deemed very important for a trading nation, first in the Bretton Woods system, later in the snake, ERM and EMU. A broad liquidity ratio was used as the key indicator for monetary policy. Whenever money growth was perceived to be too high, the Dutch monetary authorities hit the break with an over time evolving array of policy instruments, which will be described below, as will the exchange rate arrangements. A very complete overview is given in De Greef et al. (1998).

The banks own view is that path of moderate monetarism was a fairly constant one. Still in the 1970s the Dutch government took recourse to monetary financing of its deficit (1975-1983), inflation was high and the Dutch currency was devalued several times, so in fact an expansionary Keynesian policy was followed between 1972 and 1983. Before and after that period, moderate monetarism is an adequate description of monetary policy.

In 1998 a new bank law superseded the old one, in preparation for monetary union. Under the new law, price stability is the main objective and as long as that is not endangered, contributing to reaching the goals of article 2 of the treaty of Rome. The right of the Minister of Finance to overrule the Bank was abolished.

2.3 Liquidity ratio and monetary instruments

From the 1970s onwards, the liquidity ratio was the key variable watched by the Central Bank in the belief that excess liquidity would ultimately lead to inflation. Whenever the Bank felt that M2-growth was excessive, it used its instruments to bring it down. Over time we see two broad developments in the instruments used: the transition from direct to indirect instruments and a gradual orientation to more market based instruments. Both processes were intrinsically linked to the capital and credit market liberalization, which is described below.

From July 1973 until November 1979, the Central Bank imposed a liquidity reserve requirement system on banks. As the liquidity ratio increased nonetheless and the aforementioned measure had undesirable side effects, the Bank decided to apply a net credit restriction between May 1977 and June 1981. The percentage growth rate allowed varied

[3]Article 9(1) of De Bankwet 1948: "De Bank heeft tot taak de waarde van de Nederlandse geldeenheid te reguleren op zodanige wijze als voor 's lands welvaart het meest dienstig is, en daarbij die waarde zoveel mogelijk te stabiliseren."

over time and also differed somewhat between banks. Consumptive credit was restricted between April 1979 and March 1980.

In 1986 and 1987 there was a gentlemen's agreement with commercial banks to limit the growth of credit. Especially the limit in 1987, 2%, was tight. Note that at various other times, there were discussions between the Central Bank and commercial banks. Due to the small number of banks (four and after a merger three large banks had a market share of well over 80%), a formal agreement was not always necessary to limit credit expansion.

Between 1987 and 1993, the Bank held a small portfolio of government bonds for open market operations: it was used only once in March 1989. Between July 1989 and April 1990, a monetary cash reserve (non-interest bearing) applied.

During the 1980s the liquidity ratio was gradually abandoned and during the 1990s the bank only targeted exchange rates, not so much as a policy choice, but a necessity after the capital market liberalizations of the1980s. It was simply impossible to continue targeting both a fixed exchange rate with Germany and the liquidity ratio, but the bank for a long time paid lip service to targeting liquidity (several issues of the annual report of DNB). The liquidity ratio is reported in figure 1(j) and illustrates this point.

2.4 Exchange rate

The Netherlands have a long history of aiming for exchange rate stability: the gold standard was only abandoned after all other countries had left the gold block in 1936, even though unemployment was over 20% at the time. The first bank president after the war, Holtrop held an almost dogmatic aversion to realignments in the Bretton woods system, in which the Guilder was effectively anchored to the dollar: revaluations of the German mark were only partially followed.

When the system broke down, the Dutch government took the initiative and entered into an agreement with Belgium and Luxembourg, fixing exchange rates. A few month later these countries entered the snake and later the Netherlands became one of the founding members of the European Monetary System. In all these arrangements, the Dutch Guilder was restricted to the smallest possible band. In the EMS, the Dutch Guilder was devalued with respect to the German mark on 18 October 1976, 16 October 1978, 24 September 1979 and 21 March 1983. At each time it was devalued by 2% vis-a-vis the German mark. Especially the last devaluation came as a surprise: despite the massive government deficit at the time and consistently higher inflation than in Germany, the interest rate differential with Germany had been closed. The surprise devaluation of the new center-right government re-opened the interest rate gap with Germany for about five years, in which the government ran expensive deficits.

Cumulative inflation since 1983 has been lower in the Netherlands than in Germany and the 1983-peg was maintained afterwards without problems. After the signature of the Maastricht treaty in 1991, drawn up by the Dutch government, which foresaw the creation of a singly European currency on January 1st 1999 and in which the participation of both Netherlands and Germany was never in doubt, turmoil broke out in the EMS: After two waves of speculative attacks in 1992 and 1993 it effectively collapsed: all bands were widened to 15%. Still the Dutch and German authorities immediately entered into a bilateral agreement in August 1993 to maintain the old bands, which were never challenged by the markets. Just before fixing the Euro-exchange rates, the Guilder was markedly stronger than the Mark on speculation that it might be revalued. Wellink, the president of the central bank since 1998, admitted afterwards that they had considered the idea of a revaluation.[4]

2.5 Liberalization process

Already in 1961, current account transactions had been fully liberalized and foreign direct investment was allowed virtually without restrictions. There never was a period of really strict capital controls. Other capital account restrictions remained strict, but were considerably simplified in 1977. In 1983 restrictions on capital inflows were abolished.

On January 1 1986, October 1 1986 and January 1 1988, the domestic capital market was almost completely deregulated: among other things bullet loans, commercial paper, floating rate notes and bank issues of certificate were allowed, as were deep discount and zero-coupon bonds. Foreigners were also allowed to tap the market. In 1991 the prohibition of indexed loans was finally abrogated.[5]

The abolition of a strict separation between banks and insurance companies on January 1 1990 quickly led to a number of mergers and take-overs, which profoundly changed the market structure: by the mid-1990s, three big financial groups, ABN-Amro, ING and the Rabobank controlled almost the entire banking market in the Netherlands. The first two are now rapidly expanding abroad. On the other hand a few insurance companies entered the banking market, offering postal savings account at very competitive rates. Due to the increased competition, banks and these insurance companies started offering savings accounts with interest rates well above the money market rates. At the time of writing this article, April 2003, large banks are offering 4% on instant saving accounts and some smaller ones even 4.8%. Yet the yield curve on the money market (to one year) is well below 2,75%, whereas the yield on 10 year government bonds is 4,2%. Interest rates on saving accounts are thus effectively used to attract and maintain clients and to entice them to buy other profitable services from the bank.

3 Money demand, data and the Netherlands

In this section we shall discuss the literature on money demand in the Netherlands and the data used in this paper.

3.1 Money demand in the Netherlands

The central question in the Dutch money demand literature of the last decade has been: what is the cause of the rise in the liquidity ratio over the last twenty years.

From a monetarist point of view, excess money should ultimately lead to inflation, which given the fixed exchange rate over the period (and even more now in Euroland) should be a real worry to policy makers.

[4]At the presentation of the annual report of the Dutch Central Bank over 1998, on May 25th 1999, he admitted that the Dutch central bank had entertained the idea of a revaluation. They decided not to revalue for fear of turbulence on the exchange rate market (see NRC Handelsblad, May 26 1999)

[5]Unfortunately no indexed bonds have been issued by the Dutch state so far, which means that real interest rates will have to be approximated in the rest of this paper.

A number of explanations have been put forward in the empirical literature: profit hoarding by firms, who are uncertain about the future (Kuipers and Boertje, 1988), an increased importance of the financial transactions motive (Sterken, 1992) and increased wealth in the Dutch economy (Fase and Winder, 1990). All of these studies leave large misspecification in their estimated equations in the form of extremely low Durbin-Watson statistics, a sign that the non-stationarity of the data has not been completely taken care of.

The Dutch Central Bank decided to do a survey of firms in the late eighties to find out why they held more money. Only from the mid 1980s did the central bank record where the liquidity increase took place. Large part of it was in the business sector and from the survey they conclude that this is due to the preference for internal financing over external financing in firms, which given the increased profitability of the business sector, became easier in the 1980s.

Jacobs and Van der Horst (1996) are from a methodological point of view closest to this paper: they consider a small VAR model with the log of real GDP, annual inflation, short and long term interest rates (but not money) and find that the real long term interest rate in the Netherlands in stationary over the period 1977(1)-1992(4).

3.2 Data vector

The data vector of this study is very similar to that used in a number of other studies, namely Juselius (1998, 2001); Juselius and Toro (1999); Beyer (1998). It consists of quarterly data on national income, money, a short term interest rate, a long term interest rate and the quarterly inflation rate. The last four variables are daily averages (monthly average in the case of money supply) over the whole quarter (and thus not end of quarter data).

The log of real gross domestic product(according to ESA 1995 definitions and with base year 1995) y_t is supplied by the Dutch central statistical agency (CBS, available on-line at statline: www.cbs.nl).

The log of nominal M3, $m3_t$ as supplied by the Dutch central bank was chosen, because it was not targeted over the period (a special Dutch definition of M2 was in the beginning of the period under study) and it is the only broad measure for which data are readily available for the whole period, as measurement of M2 stopped a few years earlier. This measure was not targeted, but it appears subject to a large number of data revisions.

The log of the CPI with base year 1975 p_t is supplied by the Dutch central statistical agency and has been taken from datastream. Unfortunately no chain weighted price index is available and the GDP deflator for the statistical agency suffers from large data problems[6].

Lti_t and sti_t are measures of long and short term interest rates respectively, each divided by 400 to make them comparable to the quarterly inflation rate $\Delta p_t = p_t - p_{t-1}$.

Lti_t is the interest rate on liquid[7] Dutch government bonds with a remaining time to

[6]The GDP deflator supplied by the Central Statistical Office(CBS) fluctuates more than 5% per quarter in the early 1980s. In other parts of the series no similar behaviour exists. The CBS explained they were aware of the problem, caused by linking series, but had no idea of the causes or indeed how to fix it.

[7]Trade has to take place in a certain bond and there is a minimal amount outstanding (source: CBS statline)

maturity between 5 and 8 years and has been taken from statline.

Sti_t is a measure of interest rates available on saving accounts and deposits of less than two years: it has been constructed on purpose for this study by taking the daily maximum over all such savings products of the postbank/rijkspost-girospaarbank/ING and Spaarbeleg. The first used to be the state-owned giro service at the post office, was privatized in the 1980s and became part of the ING group in the early 1990s. Throughout the period it had a fairly constant and consistent market share. The latter is part of an insurance company and aggressively entered the market with a postal savings account in the 1990s. This interest is markedly different from short term interest rates on the money markets. It is a good proxy measure for the savings rate available to households (and some small companies) over the period. A gradual increase in competition lead to banks offering interest rates to private consumers which were well above the money market rate towards the end of the period. They did so to attract other more profitable business. Clearly not all companies did and do not have access to these saving rates.

The 5 data series, $y_t, m3_t, p_t, lti_t$ and sti_t together with inflation Δp_t are plotted in figures 1(a)-1(f). In figures 1(g)-1(j) we report 4 derived data series, namely the interest rate differential $id_t = lti_t - sti_t$, the real short run interest rate $rsti_t = sti_t - \Delta p_t$, the real supply of M3, $m3r_t = m3_t - p_t$ and the log liquidity ratio $lr_t = m3r_t - p_t$. All these four last variables play an important role in the theories, that will be tested in this paper.

Graph 1(g) shows that there is a decline in the interest rate spread over the period and that there is one episode in 1993 of an inverted yield curve. The real short term interest rate also declined in the last years of the sample. The growth in real M3 has been phenomenal an has far outstripped the growth in real income, such that the liquidity ratio increased considerable over the period under study.

4 Economic Theory

A host of economic theories predict constant relationships between the above mentioned variables. The following discussion also purports to show which ones have found empirical support in the empirical literature, which uses cointegrated VAR-models.

Money demand, m^d in its most general form is given by:

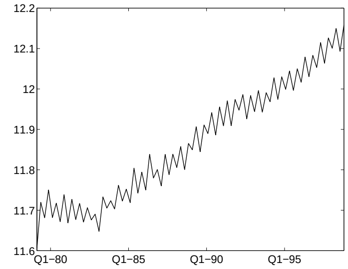
$$m_t^d = b_1 y_t + b_2 p_t + b_3 sti_t - b_4 lti_t - b_5 \Delta p_t + b_6 t + u_t \tag{1}$$

where all parameters, with the exception of b_6, are assumed to be non-negative and u_t is stationary. Often unit price b_2 and income b_1 elasticities are imposed. These are of course testable restrictions and will be treated as such. Furthermore $b_3 = b_4$ is often believed to be necessary, as the differential should give a measure of the opportunity cost of holding money. b_6 is a fairly crude way of including a long liberalization process or alternatively technological innovation.

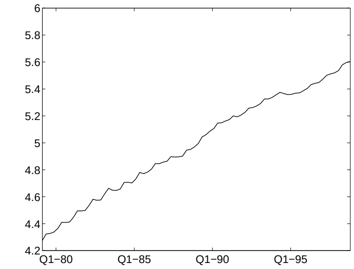
Aggregate income. The standard IS relationship predicts that trend-adjusted real aggregate income is negatively related to the real long term interest rate.

Alternatively trend-adjusted real income may cointegrate with inflation to yield a short-run Phillips curve as in Hendry and Mizon (1993) or Juselius (1996). Both alternatives are captured in the following relationship

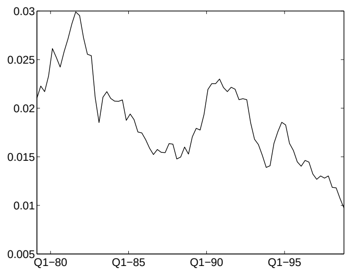
$$y_t = b_1 t - b_2 lti_t + b_3 \Delta p_t + u_t \tag{2}$$



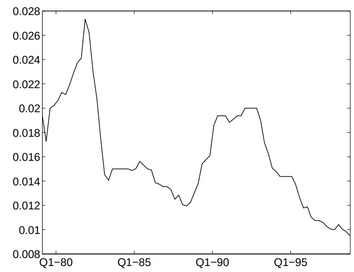
(a) y_t real gdp



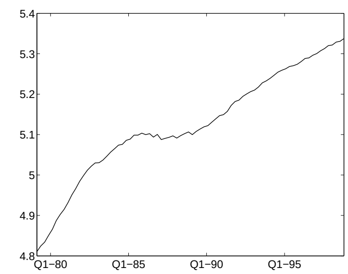
(b) $m3_t$ nominal M3



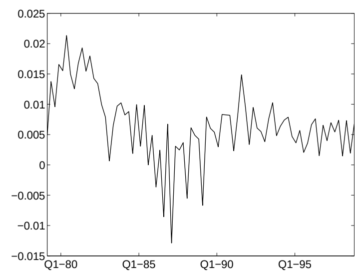
(c) lri_t, long run interest rate



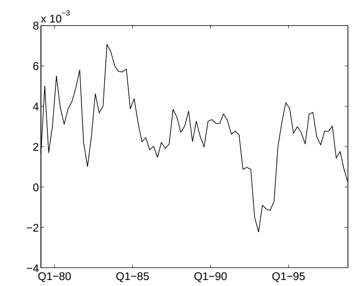
(d) sri_t, short run interest rate



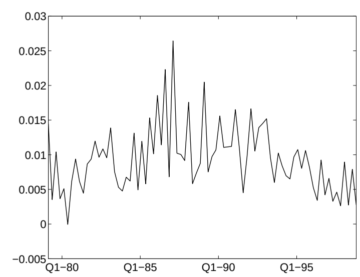
(e) p_t CPI, base year 1975



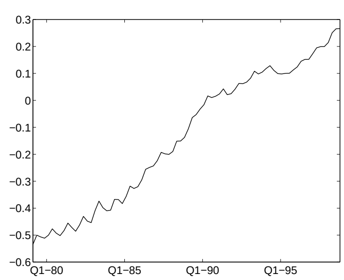
(f) Δp_t inflation



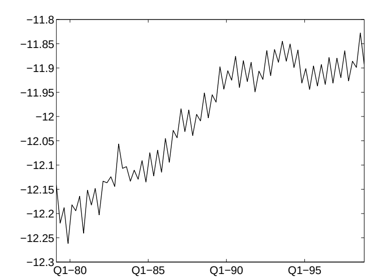
(g) id_t interest rate differential



(h) $rsti_t$ real short term interest rate



(i) $m3r_t$ real M3 ($m3_t - p_t$)



(j) Log liquidity ratio, $lr_t = m3_t - y_t$

Figure 1: The data series

where $b_1 \geq 0,$, $b_2 = b_3$ and $u_t \sim I(0)$ is consistent with an IS curve and $b_2 = 0$ and $b_3 > 0$ specifies a short-run Phillips curve.

Interest rate relations. According to the Fisher parity, the expected real interest rate is a stationary process:

$$sti_t = \mathcal{E}_t(\Delta_8 p_{t+8})/8 + u_{1t} \tag{3}$$

(Here the yield curve is supposed to be increasing over the first two years, such that M3 yields the highest interest rate over 8 quarters). Unfortunately we cannot measure expectations with the present data set, so we have to rely on the outcome. If we make the auxiliary assumptions that:

$$u_{2t} = \mathcal{E}_t(\Delta_8 p_{t+8})/8 - \Delta_8 p_{t+8}/8 \tag{4}$$

and :

$$u_{3t} = \mathcal{E}_t(\Delta_8 p_{t+8})/8 - \Delta p_t \tag{5}$$

where u_{2t} and u_{3t} are stationary, just as u_{1t} is, then we get:

$$\begin{aligned} rsti_t &= sti_t - \Delta p_t & (6) \\ &= u_{1t} + u_{2t} + u_{3t} = u_t & (7) \end{aligned}$$

which is easily testable, but of course heavily dependent on the two auxiliary assumptions. Therefore a rejection does not imply a refutation of the Fisher parity.

The expectations hypothesis, augmented with the two auxiliary assumptions states that the interest differential between long and short term interest rates is stationary:

$$id_t = lti_t - sti_t = u_t \tag{8}$$

where u_t is once more a stationary process.

Central bank policy rules. As the Central Bank targeted the exchange rate in a small open economy with already very liberal capital restrictions in the beginning of the period, it was able to influence neither the (short term) interest rate (which was set by Germany), nor the money supply. We thus do not expect to find a central bank policy rule.

Policy rules are found to yield stationary relationships by Beyer (1998) and Juselius (2001), but in these two cases, the respective countries executed their independent monetary policy.

Monetarist theories Textbook treatment of monetarist theories and also the Dutch moderate monetarism framework assume that the liquidity ratio (or alternatively money velocity, which is its inverse) is stationary:

$$lr_t = m3_t - p_t - y_t = u_t \tag{9}$$

Furthermore excess money will lead to inflation in the medium run and the central bank is assumed to be able to control inflation by increasing the short term interest rates. The last two statements are testable in the moving average representation of the $I(1)$ model.

5 The statistical model

In this section we discuss the statistical models used together with some extra remarks on particular outstanding issues.

5.1 The I(2) model

One representation of the p-dimensional I(2) model (Johansen, 1992) with 2 lags is given by:

$$\Delta^2 X_t = \Pi X_{t-1} - \Gamma \Delta X_{t-1} + \mu_0 + \mu_1 t + \varepsilon_t \tag{10}$$

where ε_t is distributed normally with mean zero and variance-covariance matrix Ω

Define the characteristic polynomial of this process as:

$$F(\lambda) = \lambda^2 I - (\Pi + 2I - \Gamma)\lambda - (\Gamma - I) \tag{11}$$

and let $\lambda_1, \ldots, \lambda_{2p}$ be the roots of $|F(\lambda)| = 0$

The following assumptions apply:

I(2) a $\Pi = \alpha\beta'$ where α and β are $p \times r$ matrices of full column rank. $r < p$

I(2) b $2p-2r-s$ roots λ of the characteristic polynomial (11) equal one $\lambda = 1$. The other $2r+s$ roots are smaller than one in absolute value $|\lambda| < 1$. Let $\{\lambda_i^*\}, i = 1, \ldots, 2r+s$ indicate the roots of the second group. It then follows that $\alpha_\perp' \Gamma \beta_\perp = \xi\eta'$ where ξ and η are full rank matrices of dimension $(p-r) \times s$. Another equivalent way of stating this result is $\bar{\alpha}_\perp \alpha_\perp' \Gamma \bar{\beta}_\perp \beta_\perp' = \alpha_1 \beta_1'$ where α_1 and β_1 are full rank matrices of dimension $p \times s$, $s < p - r$.

I(2) c $\alpha_2' \Theta \beta_2$ where $\Theta = \Gamma \bar{\beta} \bar{\alpha}' \Gamma + I$, $\alpha_2 = (\alpha : \alpha_1)_\perp$ and $\beta_2 = (\beta : \beta_1)_\perp$ is a matrix of full rank $(p - r - s)$.

On the deterministics, we put restrictions to make sure that all variables have a trend in the levels, but no quadratic or cubic trend. This implies that all the variables can be decomposed in a stochastic part Y_t and deterministic part as:

$$X_t = Y_t + m_1 + m_2 t \tag{12}$$

The following particular specification for the deterministic part of the I(2) model was originally proposed by (Rahbek et al., 1999) .

I(2) t1 $\mu_1 = \alpha\beta_0'$ where β_0 is a vector of length r.

I(2) t2 $\alpha_\perp' \mu_0 = \xi\eta_0' + \alpha_\perp' \Gamma \bar{\beta} \beta_0'$, where η_0 is a vector of length $p - r - s$.

No restrictions are placed on $\alpha' \mu_0$.

A few comments on these conditions are warranted: If $s = p - r$ we are back at the well known I(1) model, which is given below. Furthermore all unit roots are at 1 and nowhere else: seasonal unit roots are not considered in this chapter.

The number of I(1) trends equals s, whereas the number of I(2) trends is $p - r - s$.

The advantage of this representation is that all the restrictions are explicitly introduced in the standard VAR-model, one of the main workhorses of modern econometrics. A major disadvantage is that the restrictions are very complicated and non-linear. Furthermore the stationary relations are not obvious in this representation, so this representation does not offer any direct economic interpretation.

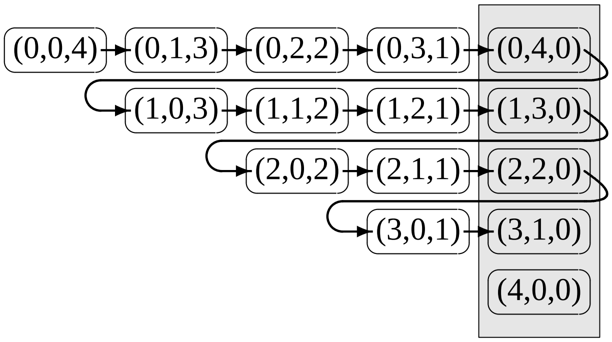


Figure 2: Selection of ranks in the I(2) model

Another problem for the statistician is that no direct methods to explicitly maximize the likelihood function have been derived for this representation: it is difficult to maximize a likelihood under these complicated linear restrictions. So following Paruolo and Rahbek (1999) we reparametrize the model, such that conditions I(2)a, I(2)t1 and I(2)t2 become embedded in the parametrization:

$$\Delta^2 X_t = \alpha \left(\rho' \left(\tau' X_{t-1} + \tau_2 t\right) + \delta \tau_\perp' \Delta X_{t-1}' + \tau_1\right) + \zeta' \left(\tau' \Delta X_{t-1} + \tau_2\right) + \varepsilon_t \qquad (13)$$

5.1.1 Determination of the rank in the I(2) model

In the I(2) model, we need to determine two ranks, namely r and s. We can maximize the likelihood under the restrictions $H_0 : r = r_0, s = s_0$ to obtain $L_{\max}\left(H\left(r, s, p-r-s\right)\right)$ and form the likelihood ratio

$$Q\left(H\left(r,s,p-r-s\right)|H\left(p,0,0\right)\right) = \frac{L_{\max}\left(H\left(r,s,p-r-s\right)\right)}{L_{\max}\left(H\left(p,0,0\right)\right)}$$

Under the null $r = r_0, s = s_0$ the test statistic $-2\ln Q\left(H\left(r,s,p-r-s\right)|H\left(p,0,0\right)\right)$ asymptotically converges to a functional of Brownian motions, which for the deterministic specification in this paper has been tabulated by Rahbek et al. (1999). We start by testing $H\left(0,0,p\right)$. If rejected we test $H\left(0,1,p-1\right)$ to $H\left(0,p,0\right)$ and then test $H\left(1,0,p-1\right)$ as in figure 2. The grey column coincides with the standard I(1) rank tests. The number

of I(2) trends, p-r-s equals zero in that column.

Paruolo (1996) shows that this procedure has an asymptotic rejection probability of 5% if tests performed are 5% tests.

5.1.2 The two step procedure and maximum likelihood estimation

Two approaches have been proposed for the estimation of an I(2) system. Historically the so-called two step procedure (Johansen, 1995c) preceded the full maximum likelihood procedure (Johansen, 1997). Even though most applications so far have used the two step procedure, we shall apply the maximum likelihood procedure, except for the estimation of the rank, where we use both. Me2 (Omtzigt, 2003) is a computer package executing the maximum likelihood procedure.

For the determination of the cointegration ranks, we use the two step estimator as Paruolo (1996) proves that the procedure in figure 2, based on the two step estimator selects the correct integration indices with probability 95% (if 5% tests are used). To our knowledge a similar proof for the maximum likelihood estimator is not available. We shall however calculate the statistics for both procedures.

We shall only need to do inference on the parameters τ and τ_2 in equation (13). Johansen (2002b) derives conditions under which likelihood ratio tests on τ are asymptotically χ^2 distributed[8]. As little is known about the small sample properties of these tests (and nothing is known on the asymptotic distribution of tests on τ_2), we shall also resort to bootstrapping these tests. Small sample properties will be more fully discussed in subsection 5.4.

5.2 The I(1) model

The p-dimensional vector autoregressive model with 2 lags can be represented in its reduced form as:

$$\Delta X_t = \alpha\beta' X_{t-1} + \Gamma_1 \Delta X_{t-1} + \mu_0 + \mu_1 t + \varepsilon_t \tag{14}$$

where ε_t are distributed $N(0, \Omega)$.

Define the characteristic polynomial of this process as:

$$\digamma(\lambda) = \lambda^2 I - (\Pi + I + \Gamma)\,\lambda - \Gamma \tag{15}$$

and let $\lambda_1, \ldots, \lambda_{2p}$ be the roots of $|\digamma(\lambda)| = 0$

The assumptions, which assure that the model is I(1) are:

I(1) a $p - r$ roots λ of the characteristic polynomial (15) equal one: $\lambda = 1$. The other $p+r$ roots are smaller than one in absolute value $|\lambda| < 1$. Let $\{\lambda_i^*\}, i = 1, \ldots, p+r$ indicate the roots of the second group.

I(1) b α and β are full rank $p \times r$ matrices, $r < p$.

The following assumption is placed on the trend variable to assure that no quadratic trend is generated in the data, but that all variables have a trend in their levels is:

I(1) t1 $\alpha'_\perp \mu_1 = 0$ or equivalently $\mu_1 = \alpha\rho'$.

Inference in the I(1) model is thoroughly described in the monograph by Johansen (1995b).

5.3 The nominal to real transformation (NtRT)

The I(2) model is a submodel of the I(1) model: it has the extra rank restriction I(2)c imposed on it. Inference in the I(2) model involves a number of still unknown distributions and maximum likelihood still has not been implemented for general restrictions. Hence a transformation from the I(2) to the I(1) model is highly desirable. Kongsted (2002)

[8]The I(2) model of Johansen (2002b)does not have any deterministics. We shall assume that the results will continue to hold true if these are included.

proposes a testable transformation, in which no information is lost and calls it the nominal to real transformation. The name suggest that the only nominal variables exhibit I(2) behaviour and that it can be removed by subtracting an appropriate price index. This holds true in many cases, but is not an absolute prerequisite, such that the name is slightly misleading.

The transformation starts from the observation that $(\tau' X_t, \tau_\perp' \Delta X) \sim I(1)$ or in fact any transformation $(\tau' X_t, s_\perp' \Delta X) \sim I(1)$ for which $|s_\perp' \tau_\perp| \neq 0$. So the proposed transformation is to analyze $(\tau' X_t, s_\perp' \Delta X)$, which will be shown to have an autoregressive structure and a reduced rank, such that it is an I(1) model.

We shall now derive the autoregressive representation of the new process after the transformation. Let us first define q and x from the following equality, which we shall apply repeatedly:

$$\begin{aligned} I &= s\left(\tau' s\right)^{-1} \tau' + \tau_\perp \left(s_\perp' \tau_\perp\right)^{-1} s_\perp' \\ &= \left(q\tau' + x s_\perp'\right) \end{aligned}$$

If we multiply (13) by τ' and ignore the deterministic part for a moment, we obtain

$$\begin{aligned} \tau' \Delta X_t = {} & \tau' \alpha \rho' \left(\tau' X_{t-1}\right) + \tau' \alpha \delta \tau_\perp' q \left(\tau' \Delta X_{t-1}\right) + \tau' \alpha \delta \tau_\perp' x \left(s_\perp' \Delta X_{t-1}\right) + \tau' \zeta' \left(\tau' \Delta X_{t-1}\right) \\ & + \tau' \Delta X_{t-1} + \tau' \varepsilon_t \end{aligned}$$

and pre-multiplying by $s_\perp'$ we get

$$\begin{aligned} s_\perp' \Delta^2 X_t = {} & s_\perp' \alpha \rho' \left(\tau' X_{t-1}\right) + s_\perp' \alpha \delta \tau_\perp' q \left(\tau' \Delta X_{t-1}\right) + s_\perp' \alpha \delta \tau_\perp' x \left(s_\perp' \Delta X_{t-1}\right) \\ & + s_\perp' \zeta' \left(\tau' \Delta X_{t-1}\right) + s_\perp' \varepsilon_t \end{aligned}$$

Now collecting terms we get the transformed model, which is I(1) with rank r:

$$\begin{aligned} \begin{bmatrix} \tau' \Delta X_t \\ s_\perp' \Delta^2 X_t \end{bmatrix} = {} & \begin{bmatrix} \tau' \alpha \\ s_\perp' \alpha \end{bmatrix} \begin{bmatrix} \rho' & \delta \tau_\perp' x \end{bmatrix} \begin{bmatrix} \tau' X_t \\ s_\perp' \Delta X_t \end{bmatrix} \\ & + \begin{bmatrix} \tau' \alpha \delta \tau_\perp' q + \tau' \zeta' + I & 0 \\ s_\perp' \alpha \delta \tau_\perp' q + s_\perp' \zeta' & 0 \end{bmatrix} \begin{bmatrix} \tau' \Delta X_{t-1} \\ s_\perp' \Delta^2 X_{t-1} \end{bmatrix} \\ & + \begin{bmatrix} \tau' \\ s_\perp' \end{bmatrix} \varepsilon_t \end{aligned}$$

As there is only a trend in the levels of the variables, we see that the new variable $s_\perp' \Delta X_t = s_\perp' \Delta Y_t + s_\perp' m_2$ should only contain a level intercept, but no trend, whereas of course $\tau' X_t = \tau' Y_t + \tau' m_1 + \tau' m_2 t$ should still contain a trend.

So to keep the model exactly the same after the transformation, we should take account of the following two facts:

1. The coefficient to the second lag of $s_\perp' \Delta^2 X_{t-1}$ are zero.

2. $s_\perp' \Delta X_t$ should only contain an intercept, but no trend, whereas $\tau' X_t$ should contain a trend.

Kongsted and Nielsen (2002) study the effect of ignoring the restrictions 1 and 2 in a model which does not contain dummies, seasonal or otherwise. They do so by means of an application on real data and a simulation study, both a 3-dimensional VAR and find that "unrestricted reduced rank regression is shown to yield only a minor loss of efficiency compared to imposing the restrictions in the simulation experiment." They thus argue to transform the model to an I(1) model, ignoring the additional restrictions. They thus transform the I(2) model (10) with restrictions I(2)a-c, I(2)t1-2 to the the I(1) model (10) with restrictions I(1)a-b, I(1)t1.

5.4 Small sample properties of cointegrated VAR models

The small sample properties of most, if not all tests in the cointegrated VAR models (14) and (10) have from the beginning given cause for concern. Most attention has focused on the restrictions on the cointegration parameters β in the I(1) model (14) (see Gonzalo (1994)). Two methods to overcome severe size distortion in small samples have been proposed and applied in the literature, namely Bartlett corrections and bootstrap methods. We shall discuss the implementation of these methods in inference on the cointegration parameters in the I(1) model and then give some comments on small sample properties of the other tests.

Define $\beta^* = (\beta, \rho)'$ and let us consider the general I(1) model (14) and the following hypotheses on β and ρ:

1. $\beta^* = (\beta_0^*, \psi)$ that is $q \leq r$ out of the cointegration relationships are known entirely, including their trend. The other cointegration relations are unknown.

2. $\beta = H\varphi$, that is the same restriction on all cointegration vectors (but no restrictions on the trend parameters).

3. $\beta^* = (H_1\varphi, \ldots, H_r\varphi)$ that is generically identifying, linear restrictions on each of the vectors β^*. If the restrictions are not generically identifying, the algorithm by Omtzigt (2002b) can be used to render them identifying.

Johansen (2000) derives the Bartlett correction for tests 1 and 2 under an assumption on the dummies, which in the current deterministic set-up implies that seasonal dummies can be taken account of, but other dummies not. By means of a Monte Carlo study he shows that the Bartlett corrected test has a size close to 5%.

Gredenhoff and Jacobson (2001) propose to bootstrap all three kinds of tests and do so in a small Monte Carlo study. This method can also be applied if besides seasonal dummies, other dummies are present in the model. They base their bootstrap on the estimate of β^* under the null hypothesis and show by means of a simulation study that the size properties of the bootstrapped test are adequate.

Omtzigt and Fachin (2002) show that both methods can fail in terms of power. They argue that if the null null hypothesis is false, then the estimated model under the (false) null hypothesis does not contain $p - r$ unit roots, but (at least) $p - r + 1$ unit roots, as (at least) one of the cointegration relations becomes non-stationary. If the estimated $(p - r + 1)$th unit root is close to unity, the Bartlett correction factor grows without bound (and becomes undefined when it is unity). Consequently the Bartlett corrected likelihood ratio test statistic becomes very small and the null hypothesis is wrongly accepted. By means of simulations they show that the corrected likelihood ratio can well become a biased test, as can the bootstrapped test statistic.

Let $\hat{\theta}_r = \left(\hat{\beta}_r, \hat{\rho}_r, \hat{\alpha}_r, \hat{\Gamma}_{1r}, \hat{\Omega}_r\right)$ be the estimates under $\mathcal{H}_0$ and $\hat{\theta}_u = \left(\hat{\beta}_u, \hat{\rho}_u, \hat{\alpha}_u, \hat{\Gamma}_{1u}, \hat{\Omega}_u\right)$ the estimates under the alternative. Then Omtzigt and Fachin (2002) propose to base the Bartlett correction on $\hat{\theta}_u$. They also suggest ressampling from the DGP based on $\hat{\theta}_u$ (and not $\hat{\theta}_r$) when applying the bootstrap. $\mathcal{H}_0$ does not neccisarily hold in the bootstrap sample, so propose to take a new null hypothesis $\mathcal{H}_0^\natural$ which holds in the bootstrap sample en equals $\mathcal{H}_0$ if $\mathcal{H}_0$ were to hold true in that sample. They propose $\mathcal{H}_0^\natural$ for the cases 1 and 2, but not for the more frequently applied case 3. In the appendix to this chapter, section A we give one proposal for $\mathcal{H}_0^\natural$ in case 3.

As a general point we shall report $|\lambda^*_{\max}|$, that is the the largest root, that is not restricted to be unity by the model. If $|\lambda^*_{\max}|$ is substantially larger in the restricted model (and close to unity) than in the unrestricted model, then we consider that a sign that $\mathcal{H}_0$ should be rejected. The points raised by Omtzigt and Fachin (2002) are then amply illustrated, as the both the bootstrap and the Bartlett correction cease to function, when $|\lambda^*_{\max}|$ is close to unity. If this is not the case there are no substantial differences between basing the Bartlett correction or bootstrap on the unrestricted estimates.

The asymptotic theory for parameters in the I(2) model has only just been developed and not even for all the tests on τ and τ_2 we perform do we know the asymptotic distribution. So the small sample properties of the estimators are still very much unknown. We therefore bootstrap these tests. The point raised above on the extra unit root(s) is equally valid, but we still bootstrap using the restricted parameter estimates, as we do not have any equivalent null hypotheses for the bootstrap. We do however report the largest root in both the restricted and unrestricted model and note that in the test we perform they are extremely close, such that even with an equivalent hypothesis we are confident that we would obtain very much the same results.

The Bartlett correction of Johansen (2002a) for the trace test in the I(1) model is not valid in the presence of seasonal dummies, which are present in our application. Subsequently we cannot use the correction. An alternative would be to block-bootstrap the residuals and resample under the null as proposed by Van Giersbergen (1996). Yet the length of the block makes a large difference and no clear guidance as to how to select the block length is available.

Two issues are of interest in the rank selection procedure. In the I(2) model the differences between rank selection based on maximum likelihood and rank selection based on the 2 step method has not been commented upon, so we compare the two.

Secondly if we were to apply a nominal-to-real transformation and impose the restrictions 1, 2 in subsection 5.4,if no dummies were present and if we were to use the unrestricted estimate of τ, then the trace statistic for the rank in the I(2) model $\frac{L_{\max}(H(r,0,p-r))}{L_{\max}(H(p,0,0))}$ and the trace test in the I(1) model $\frac{L_{\max}(H(r))}{L_{\max}(H(p))}$ would be the same. We shall find that ignoring these conditions will cause them to be quite different: we use a restricted estimate of τ, seasonal dummies are present in the model and we do not impose the two restrictions in subsection 5.4.

5.5 Automated Model Selection

The identification and restriction of the cointegration space in the I(1) model is a long and fairly arduous process. Following Davidson (1998) who first automates the search for restrictions, Omtzigt (2002a) proposes a procedure for restriction and identification, which mimics the way Juselius (2002) searches for cointegration vectors.

If there is only one cointegration vector, the search procedure is as follows:

1. The program creates $p+1$ unit vectors $h_1, \ldots, h_{p+1}$, corresponding to the p variables and the trend in β^*. The user can specify additional vectors $e_i, i = 1, \ldots, u$. If the first two variables are money and income, then the user may define $e = [1, -1, 0, \ldots, 0]'$ which can be seen as the 'new' variable $m - y$. The program takes any possible combination of maximal p of the vectors $h_1, \ldots, h_{p+1}, e, \ldots, e_u$ and forms the matrices H_1 through to H_K all of full column rank. For each $i \neq j, sp(H_i) \neq sp(H_j)$ and the matrix which only consists of column h_{p+1} (and would thus correspond to testing whether the trend is a stationary relation) is excluded.

2. The programs tests restrictions of the kind $\beta^* = H_v \phi_v$. All those accepted at the 5% level are listed. First the accepted tests with the highest number of over-identifying restrictions (that is with the lowest number of columns in H) are reported. In case of an equal number of over-identifying restrictions, the test with the highest p-value is reported first.

If the rank of the cointegration matrix is 2, then the matrices H in step 1 contain at most $p-1$ columns. In step 2, we first test all the individual restrictions of the form $\beta^* = (H_v \phi_v, \psi)$. Those accepted at the 1% level are then combined in a further step. Let $C_1 = \{1, \ldots c_1\}$ denote the set of accepted restrictions. We then test each combination $i, j \in C_1, i \neq j$ for which the restrictions are generically identifying. We thus test $\beta^* = (H_i \varphi, H_j \varphi)$ Let $C_2 = \{\{i, j\}_l, l = 1, \ldots, c_2\}$ define the set of combined restrictions that are accepted at the 5% level. Then order all the restrictions in C_1, which are accepted at the 5% level and those in C_2 according to the criteria above.

A more detailed account of the procedure can be found in Omtzigt (2002a), who also performs a Monte Carlo study to test the effectiveness of the procedure. He argues that the researcher should chose between the top-5 models selected and shows that under-selection of the lag length leads to a higher probability of recovering the true model.

Even though his simulations show that there is a sizeable size distortion in the procedures, he does not use any corrections (Bartlett or bootstrap) in his Monte Carlo simulations. In this paper we combine automated model search with small sample corrections to gauge whether the resulting procedure is useful for the data set at hand.

6 The empirical analysis

Even though data is available from 1977 quarter 1 (the start of the GDP data series), we effectively use data from 1979 first quarter onwards, as 1977 and 1978 contain a series of outliers, which would require a number of dummies.

Instead of modelling straight away the five data series, we choose to model the following transformation: y_t, p_t, $m3_t, rsti_t$ and id_t. All these variables are plotted in figure

Misspecification tests in the unrestricted VAR						
	Univariate					**Multivariate**
Variable	$y99_t$	p_t	$m3_t$	$rsri_t$	id_t	
Normality	$1.96(0.38)$	$2.64(0.27)$	$0.25(0.88)$	$4.22(0.12)$	$1.51(0.47)$	$11.63(0.31)$
AR1	$0.03(0.87)$	$0.03(0.85)$	$0.01(0.91)$	$0.06(0.81)$	$0.55(0.46)$	$25.08(0.46)$
skewness	0.36	-0.25	-0.02	0.37	-0.08	
kurtosis	3.18	3.50	2.95	3.79	3.30	

Table 1: Misspecification tests of the unrestricted VAR with two lags

1. We needed exactly two lags and centered seasonal dummies, but no other dummies. The residuals of the unrestricted VAR with two lags show no sign of misspecification, see table 1. The normality and AR1 tests have been taken from Doornik and Hansen (1994) and Doornik (1996) respectively. The p-values between brackets indicate that the model is well specified.

6.1 Determination of rank in the I(2) model

The determination of the ranks in the I(2) model is done as described in section 5.1.1. We have calculated the relevant likelihood ratio statistics based on the 2 step estimator and those based on the maximum likelihood estimator. They are reported in tables 2 and 3. The 95% percentile of the asymptotic distribution is given in brackets below the test statistics: we remark once more that we have no formal proof that these percentiles are valid for the maximum likelihood estimator.

From a theoretical point of view we expect there to be just one I(2) trend, which is the nominal trend in $m3_t$ and p_t. Using table 2 the first accepted rank is $(r, s, p-r-s) = (1, 3, 1)$. According to Paruolo (1996) we should now stop and accept this rank.

We do however continue and find that the next hypothesis accepted is $(r, s, p-r-s) = (2, 2, 1)$: from a theoretical perspective we expect at least two cointegration relations to be present in the data set. Furthermore the rank $(r, s, p-r-s) = (1, 3, 1)$ was only just accepted.

We note that 2 step estimation and maximum likelihood give identical results when $r = 0$ and when $p - r - s = 0$: these are the top row and the right hand column in the rank tables. In all other cases the statistic based on maximum likelihood is remarkably lower than the statistic based on the two step estimator.

For two reasons shall we continue to do the rest of the analysis for $(r, s, p-r-s) = (1, 3, 1)$. Firstly it is the first rank accepted in both procedures. Secondly and more importantly, it turns out that using inference from $r = 1$ is beneficial for $r = 2$.

6.2 The nominal to real transformation(s)

The aim of this section is to fully determine τ, for once we have done so, we can apply the nominal to real transformation and continue our analysis in the I(1) model. $\tau' X_t$ contains

2 Step Inference

$p-r$	r						
5	0	593.2 (198.2)	308.0 (167.9)	217.1 (142.2)	151.8 (119.8)	114.8 (101.5)	109.4 (87.2)
4	1		414.9 (137.0)	169.0 (113.0)	103.2 (92.2)	68.8 (75.3)	63.8 (62.8)
3	2			201.7 (87.6)	77.0 (68.2)	42.6 (53.2)	27.1 (42.7)
2	3				118.3 (47.6)	44.8 (34.4)	13.3 (25.4)
1	4					46.0 (19.9)	2.1 (12.5)
$p-r-s$		5	4	3	2	1	0

Table 2: Test statistics of the rank selection procedure, based on two step estimation

Maximum Likelihood Inference

$p-r$	r						
5	0	593.2 (198.2)	308.0 (167.9)	217.1 (142.2)	151.8 (119.8)	114.8 (101.5)	109.4 (87.2)
4	1		198.2 (137.0)	132.0 (113.0)	90.8 (92.2)	68.1 (75.3)	63.8 (62.8)
3	2			85.0 (87.6)	59.0 (68.2)	38.4 (53.2)	27.1 (42.7)
2	3				36.8 (47.6)	23.1 (34.4)	13.3 (25.4)
1	4					12.3 (19.9)	2.1 (12.5)
$p-r-s$		5	4	3	2	1	0

Table 3: Test statistics of the rank selection procedure, based on maximum likelihood estimation

	The nominal to real transformation				
$(r, s, p-r-s)$	I(1) with trend	I(1) without trend	$\lvert\lambda^*_{\max}\rvert$	LR test	BS p-value
$(1,3,1)$			0.55		
	$y_t, m3_t - 0.1p_t, id_t, rsti_t$		0.63	9.29 (0.03)	0.11
	$y_t, m3_t - p_t, id_t, rsti_t$		0.58	16.26 (0.00)	0.03
	$y_t, m3_t - p_t$	$id_t, rsti_t$	0.60	17.07	0.08
$(2,2,1)$			0.76		
	$y_t, m3_t - 0.8p_t, id_t, rsti_t$		0.79	7.67 (0.05)	0.18
	$y_t, m3_t - p_t, id_t, rsti_t$		0.80	7.97 (0.09)	0.29
	$y_t, m3_t - p_t$	$id_t, rsti_t$	0.80	8.88	0.51

Table 4: Testing the nominal to real transformation for r=1 (top) and r=2 (bottom)

all the variables and combinations of variables that are at most I(1). The variables are $X_t = (y_t, m_t, p_t, rsti_t, id_t)$: real GDP, m3, the consumer price index, the real short term interest rate and the interest rate differential between the nominal long term interest rate and the nominal short term interest rate. Let h_1 be the five-dimensional unit vector with 1 as the first element and $e_1 = [0, 1, -1, 0, 0]$.

In the unrestricted I(2) model with ranks $(r, s, p-r-s) = (1, 3, 1)$ the largest root of the characteristic polynomial that is not restricted to 1 equals 0.55 in absolute value, see table 4, where we report the outcome of the tests on τ. The test that $\tau = [h_1, h_4, h_5, \psi]$, where ψ varies freely. This hypothesis implies that that y_t, $rsti_t$ and id_t are (at most) I(1) variables with a linear trend, and that m_t and p_t share the same I(2) trend, but that this trend does not feed proportionally into both variables. It takes a value of 9.82. Since under the null hypothesis this test has a χ^2-distribution with 3 degrees of freedom, its p-value is 0.03 and the test is rejected. We note that the maximum root of the characteristic polynomial is 0.63 and thus close to 0.56 and bootstrap the test statistic to find that the hypothesis is accepted with a p-value of 0.11. The bootstrap procedure has been based on the restricted estimate (and not on the unrestricted estimate as argued before) of the parameters. Yet we shall see later, when testing in the I(1) model, that there may not be a large difference when $|\lambda^*_{\max}|$ does not increase too much.

The next hypothesis $\tau = [h_1, h_4, h_5, e_1]$, which implies that the I(2)-trend feeds proportionally into money and prices, is just rejected with a bootstrapped p-value of 0.03.

The test that $\tau = [h_1, h_4, h_5, e_1], \tau_2 = [*, 0, 0, *]$, which implies that id_t and $rsti_t$ are at most I(1), but do not contain a linear trend is accepted with a p-value of 0.08. Note that Johansen (2002b) does not report the asymptotic distribution for this latest test, such that we can only report the p-value of the bootstrapped test statistic.

For the choice $(r, s, p-r-s) = (2, 2, 1)$ all three hypotheses are accepted with relatively large bootstrapped p-values and $|\lambda^*_{\max}|$ does not increase much in the restricted models.

We thus conclude that the nominal to real transformation, where the I(2) trend feeds proportionally into money and prices is accepted for both choices of ranks.

We accept the transformation for both choices of ranks and proceed with an I(1)-analysis of the transformed data vector $X_t = [y_t, m3r_t, id_t, rsti_t, \Delta p_t]$, where $m3r_t = m3_t - p_t$.

	After transformation				I(2) model	
	Trace	95%	Lmax	95%	Trace	Lmax
$r = 0$	90.3	87.0	39.8	24.8	109.4	45.6
$r = 1$	**50.5**	62.2	21.8	20.0	63.8	36.7
$r = 2$	28.6	42.2	**16.1**	16.7	**27.1**	**13.8**
$r = 3$	12.6	25.5	10.1	13.1	13.3	11.2
$r = 4$	2.5	12.4	2.5	12.4	2.1	2.1

Table 5: Comparison of the rank tests in the I(1) and the I(2) model

6.3 Rank tables after NtR transformation

After the transformation, where following Kongsted and Nielsen (2002) we do not impose the restrictions 1 and 2 on page 14 on the transformed systems, we obtain the trace and Lmax statistics in table 5.(Note that additional differences are caused by the inclusion of seasonal dummies in both the I(2) and I(1) model). For comparison we also report the original statistics from the untransformed I(2) model from table 5. We see that if we use the trace statistic, then we accept $r = 1$, whereas in the I(2) model we would have accepted $r = 2$. Yet is were to use the Lmax statistic, both before and after the transformation, the choice would be $r = 2$. We thus continue with both choices of rank. In general the differences between the test statistics are substantial and thus ignoring the additional restrictions does make a large difference in this data set.

6.4 Hypothesis testing on individual vectors

6.4.1 Hypothesis testing for r=1

Under the assumption that $r = 1$, we test a number of hypotheses on the cointegration space β. Since the all the restrictions are of the type $\beta = H\phi$ and only seasonal dummies are present, we can apply the Bartlett correction to these tests. We report the uncorrected LR-test statistic (with p-value underneath between brackets) and two Bartlett corrected LR-tests: the one based on the unrestricted estimates (only $r = 1$ is imposed) and the restricted estimates. We shall base our decisions on the Bartlett corrected LR-test, which is based on the unrestricted estimate.

$\mathcal{H}_1 - \mathcal{H}_5$ are tests whether individual variables are trend-stationary. All these are soundly rejected. In $\mathcal{H}_6 - \mathcal{H}_9$ we test whether respectively the log-velocity ($y_t - m3r_t$), the real long term interest rate ($rlti_t = lti_t - \Delta p_t$), the nominal short term interest rate or the nominal long term interest rate are stationary. Once more all these hypotheses are rejected. With $\mathcal{H}_{10}$ we reject that any combination between y_t and $m3r_t$ is stationary, whereas $\mathcal{H}_{11}$ rejects any stationary relationship among the interest rates and inflation. We conclude that the stationary relation is thus a combination of y_t and $m3r_t$ on the one hand and interest rates and inflation on the other. This means either a money demand relation or an aggregate income relation. $\mathcal{H}_{12} - \mathcal{H}_{14}$ are different forms of money demand. $\mathcal{H}_{13}$ is the accepted relation with the largest number of restrictions (3) and is a special case of the money demand equation (1) with $b_1 = b_2 = 1, b_3 = b_5$ and $b_4 = 0$. In $\mathcal{H}_{15} - \mathcal{H}_{18}$ we test different forms of aggregate income relations (2) (in $\mathcal{H}_{15}$ the coefficient on $m3r$ is estimated freely, but equals zero) and find that $\mathcal{H}_{17}$, an IS curve (a relation between real

	Restricted estimates of β							.	Unrestricted es		Restricted es		
	y	$m3r$	$rsti$	id	Δp	$1000t$	dof	LR test	BF	LR test	BF	LR test	$\lvert\lambda^*_{\max}$
$\mathcal{H}_1$	1	0	0	0	0	-6.67	4	23.72 (0.00)	1.43	16.54 (0.00)	2.30	10.30 (0.04)	0.84
$\mathcal{H}_2$	0	1	0	0	0	-12.40	4	33.08 (0.00)	1.43	23.06 (0.00)	3.46	9.57 (0.05)	0.96
$\mathcal{H}_3$	0	0	1	0	0	0.04	4	24.56 (0.00)	1.43	17.12 (0.00)	1.96	12.50 (0.01)	0.73
$\mathcal{H}_4$	0	0	0	1	0	0.05	4	29.74 (0.00)	1.43	20.73 (0.00)	2.41	12.36 (0.01)	0.78
$\mathcal{H}_5$	0	0	0	0	1	0.04	4	22.55 (0.00)	1.43	15.72 (0.00)	1.88	11.99 (0.02)	0.77
$\mathcal{H}_6$	1	-1	0	0	0	2.85	4	33.96 (0.00)	1.43	23.67 (0.00)	4.25	7.99 (0.09)	0.95
$\mathcal{H}_7$	0	0	1	1	0	0.08	4	25.04 (0.00)	1.43	17.45 (0.00)	1.99	12.59 (0.01)	0.74
$\mathcal{H}_8$	0	0	1	0	1	0.11	4	25.31 (0.00)	1.43	17.65 (0.00)	1.93	13.09 (0.01)	0.72
$\mathcal{H}_9$	0	0	1	1	1	0.06	4	28.21 (0.00)	1.43	19.67 (0.00)	2.14	13.20 (0.01)	0.86
$\mathcal{H}_{10}$	1	0.05	0	0	0	-7.20	3	23.66 (0.00)	1.46	16.20 (0.00)	2.40	9.84 (0.02)	0.84
$\mathcal{H}_{11}$	0	0	0.12	1	0.83	0.08	2	21.19 (0.00)	1.49	14.26 (0.00)	1.92	11.02 (0.00)	0.71
$\mathcal{H}_{12}$	1	-1	0	-50.84	0	2.07	3	27.57 (0.00)	1.46	18.88 (0.00)	2.25	12.26 (0.01)	0.74
$\mathcal{H}_{13}$	1	-1	15.06	0	0	5.02	3	7.03 (0.07)	1.46	4.81 (0.19)	1.54	4.58 (0.21)	0.50
$\mathcal{H}_{14}$	1	-1	17.00	0	4.59	5.38	2	3.84 (0.15)	1.49	2.58 (0.27)	1.53	2.51 (0.29)	0.49
$\mathcal{H}_{15}$	1	-0.00	0	0.95	-5.71	-6.69	1	8.25 (0.00)	1.51	5.46 (0.02)	1.65	5.00 (0.03)	0.57
$\mathcal{H}_{16}$	1	0	3.33	3.33	-3.56	-6.40	2	5.84 (0.05)	1.49	3.93 (0.14)	1.57	3.72 (0.16)	0.55
$\mathcal{H}_{17}$	1	0	7.12	7.12	0	-5.94	3	9.78 (0.02)	1.46	6.70 (0.08)	1.59	6.14 (0.10)	0.52
$\mathcal{H}_{18}$	1	0	0	0	-5.83	-6.77	3	8.35 (0.04)	1.46	5.72 (0.13)	1.58	5.28 (0.15)	0.57

Table 6: Tests in model with r=1 and 2 lags. All tests are Bartlett corrected in two ways: once the Bartlett factor is based on the unrestricted estimates and the second time it is based on the restricted estimates. For the unrestricted estimate $|\lambda^*_{\max}| = 0.51$.

income and the real long term interest rate) and $\mathcal{H}_{18}$, a short run Phillips curve are both accepted.

We conclude that of the hypotheses considered, we can accept either of $\mathcal{H}_{13}, \mathcal{H}_{17}$ and $\mathcal{H}_{18}$.

All Bartlett corrections are defined, whether they are based on the unrestricted or the restricted estimators, as the maximum unrestricted eigenvalue of the characteristic polynomial (15) never exceeds one in absolute value. Yet we note that the two hypotheses, that are most strongly rejected, $\mathcal{H}_2$ and $\mathcal{H}_6$ are accepted, when the Bartlett correction is based on the restricted estimators. Their $|\lambda^*_{\max}|$ is very close to unity, such that the Bartlett factor is extremely large. This further corroborates the point on power raised in this paper.

6.4.2 Hypothesis testing for r=2

For $r = 2$, we first consider hypotheses of the kind $\beta^* = (H_1\varphi, \psi)$. No Bartlett correction has yet been derived for them, so we bootstrap the test statistics. Once again we have the choice of basing the bootstrap on the unrestricted estimate and the restricted estimate.

To base the bootstrap on the unrestricted estimate, we need to formulate an alternative null hypothesis, which is satisfied by the bootstrap sample. The construction of the alternative hypothesis is given in the appendix of this paper.

We test 37 hypothesis on one of the two vectors (leaving the other unrestricted) and calculate the three corresponding p-values. The results are reported in table 7. The last row of the table contains $|\lambda^*_{\max}|$. The Bartlett correction is not defined, when $|\lambda^*_{\max}| \geq 1$, but from a computational point of view, the bootstrap can be applied. We base our conclusions on the bootstrapped p-value that is based on the unrestricted estimate.

$\mathcal{H}_1 - \mathcal{H}_9$ are tests that exactly one of the variables is stationary, whereas $\mathcal{H}_{10} - \mathcal{H}_{18}$ are the corresponding tests for trend-stationarity. $\mathcal{H}_{19}$ is a test for trend-stationarity of velocity without imposing equal coefficients on real money and prices. $\mathcal{H}_{20} - \mathcal{H}_{27}$ are tests on the stationarity of combinations of interest rates and inflation, $\mathcal{H}_{28} - \mathcal{H}_{31}$ are hypotheses, corresponding to an aggregate demand curve and $\mathcal{H}_{32} - \mathcal{H}_{37}$ hypotheses on a money demand relationship.

The bootstrap based on the restricted estimate accepts all 37 hypotheses, which means the procedure does not have any discriminatory power in this context. The uncorrected likelihood ratio test accepts 10 hypotheses, whereas the bootstrap based on the unrestricted estimate accepts 25 hypotheses, among whom five out of $\mathcal{H}_{10} - \mathcal{H}_{18}$. Under the assumption that $r = 2$, at most two of them can hold true as each of these 9 hypothesis concerned test that one variable or a predefined linear combination of two or three variables is stationary.

Combining two hypotheses at a time out of the 25 accepted, we find that 145 out of 228 possible combinations of restrictions, which are generically identifying, are accepted. The one with the largest number of over-identifying restrictions accepted is $\mathcal{H}_5 + \mathcal{H}_{10}$. Yet the large number of accepted hypotheses indicates that the bootstrapped test has got relatively little discriminatory power.

We have executed a model search as described in section 5.5 with one exception: we have selected only a handful of hypotheses and certainly not every combination of unit vectors $\mathcal{H}_1 - \mathcal{H}_5$ and additional vectors $\mathcal{H}_6 - \mathcal{H}_9$. Yet the large number of accepted hypotheses in table 7 and the computer intensity of the bootstrap would make a full bootstrapped search costly in terms of computing time. Furthermore even in this limited search large numbers of mutually exclusive models were accepted, leading to the conclusion that for the data set at hand, basing the final specification on a bootstrapped automated model search is not the way to proceed.

6.5 Automated Model Selection

Bearing in mind the apparent failure of (semi-)automated model selection, based on the bootstrapped test statistics, we proceed with a full automatic model search, based on $\mathcal{H}_1 - \mathcal{H}_9$ (four user specified variables). That is with the data vector $X_t = [y_t, m3r_t, id_t, rsti_t, \Delta p_t]$, we define the following search directions $e_1 = [1, -1, 0, 0, 0]\,, e_2 = [0, 0, 1, 1, 0]\,, e_3 = [0, 0, 1, 1, 1]$ and $e_4 = [0, 0, 1, 0, 1]$, which correspond to the log-velocity, the long term

	Restricted estimates of β								P-values			
	y	m3r	rsi	id	Δp	1000t	dof	LR test	LR	BS(un)	BS(res)	$\lvert\lambda^*_{\max}\rvert$
$\mathcal{H}_1$	1	0	0	0	0	0	4	19.24	0.00	0.01	0.46	1.009
$\mathcal{H}_2$	0	1	0	0	0	0	4	18.27	0.00	0.02	0.41	1.000
$\mathcal{H}_3$	0	0	1	0	0	0	4	16.65	0.00	0.02	0.26	0.903
$\mathcal{H}_4$	0	0	0	1	0	0	4	14.92	0.00	0.05	0.36	0.824
$\mathcal{H}_5$	0	0	0	0	1	0	4	7.30	0.12	0.48	0.72	0.870
$\mathcal{H}_6$	1	−1	0	0	0	0	4	17.12	0.00	0.02	0.38	0.983
$\mathcal{H}_7$	0	0	1	1	0	0	4	19.62	0.00	0.00	0.20	0.930
$\mathcal{H}_8$	0	0	1	1	1	0	4	11.64	0.02	0.15	0.57	0.941
$\mathcal{H}_9$	0	0	1	0	1	0	4	11.55	0.02	0.26	0.53	0.929
$\mathcal{H}_{10}$	1	0	0	0	0	−6.73	3	8.23	0.04	0.22	0.54	0.898
$\mathcal{H}_{11}$	0	1	0	0	0	−12.34	3	15.23	0.00	0.04	0.23	0.954
$\mathcal{H}_{12}$	0	0	1	0	0	0.11	3	13.21	0.00	0.03	0.29	0.918
$\mathcal{H}_{13}$	0	0	0	1	0	0.05	3	12.46	0.01	0.07	0.29	0.760
$\mathcal{H}_{14}$	0	0	0	0	1	0.02	3	7.13	0.07	0.39	0.59	0.849
$\mathcal{H}_{15}$	1	−1	0	0	0	1.69	3	17.05	0.00	0.02	0.18	0.972
$\mathcal{H}_{16}$	0	0	1	1	0	0.14	3	12.57	0.01	0.03	0.29	0.916
$\mathcal{H}_{17}$	0	0	1	1	1	0.12	3	7.46	0.06	0.24	0.49	0.722
$\mathcal{H}_{18}$	0	0	1	0	1	0.06	3	10.63	0.01	0.21	0.41	0.865
$\mathcal{H}_{19}$	1	0.12	0	0	0	−8.01	2	7.95	0.02	0.11	0.32	0.899
$\mathcal{H}_{20}$	0	0	1	−2.59	0	0	3	13.54	0.00	0.02	0.23	0.826
$\mathcal{H}_{21}$	0	0	1	3.52	0	0.25	2	11.77	0.00	0.01	0.12	0.828
$\mathcal{H}_{22}$	0	0	1	0	23.03	0	3	7.27	0.06	0.38	0.56	0.872
$\mathcal{H}_{23}$	0	0	1	0	4.31	0.15	2	6.91	0.03	0.24	0.33	0.845
$\mathcal{H}_{24}$	0	0	0	1	3.35	0	3	7.20	0.07	0.32	0.56	0.872
$\mathcal{H}_{25}$	0	0	0	1	1.01	0.07	2	6.42	0.04	0.24	0.40	0.798
$\mathcal{H}_{26}$	0	0	1	3.55	12.21	0	2	7.18	0.03	0.24	0.34	0.875
$\mathcal{H}_{27}$	0	0	1	2.30	1.65	0.20	1	4.87	0.03	0.19	0.21	0.704
$\mathcal{H}_{28}$	1	0	1.15	1.15	−5.18	−6.65	1	0.30	0.59	0.70	0.66	0.662
$\mathcal{H}_{29}$	1	0	4.75	4.75	0	−6.20	2	6.51	0.04	0.14	0.29	0.858
$\mathcal{H}_{30}$	1	0	0	0	−6.06	−6.79	2	0.49	0.78	0.89	0.89	0.676
$\mathcal{H}_{31}$	1	0.02	0	0	−6.10	−7.00	1	0.46	0.50	0.68	0.63	0.674
$\mathcal{H}_{32}$	1	−1	0	0	−60.83	2.18	2	7.16	0.03	0.24	0.34	0.839
$\mathcal{H}_{33}$	1	−0.49	8.58	0	0	−0.78	1	0.80	0.37	0.51	0.48	0.651
$\mathcal{H}_{34}$	1	−1	15.24	0	0	5.18	2	5.04	0.08	0.21	0.30	0.818
$\mathcal{H}_{35}$	1	−1	17.44	0	6.13	5.55	1	0.24	0.62	0.75	0.73	0.633
$\mathcal{H}_{36}$	1	0.40	0	16.61	0	−10.39	1	4.98	0.03	0.09	0.23	0.827
$\mathcal{H}_{37}$	1	−1	0	−69.44	0	1.20	2	11.42	0.00	0.02	0.15	0.744

Table 7: Tests on a single cointegration vector in model with $r = 2$ and 2 lags. All tests are Bartlett corrected in two ways: once the Bartlett factor is based on the unrestricted estimates and the second time it is based on the restricted estimates. For the unrestricted estimate $|\lambda^*_{\max}| = 0.642$

Model	Nr Res	LR	P value	y	$m3r$	$rsti$	id	Δp	$trend$
$\mathcal{M}_1$	3	1.14	0.767	1	-0.55	9.26	0	0	0
$\mathcal{M}_2$	3	6.47	0.091	1	-0.52	9.90	9.90	0	0
$\mathcal{M}_3$	3	7.03	0.071	1	-1	15.06	0	0	0.0050
$\mathcal{M}_{3e1}$	3	8.35	0.039	1	0	0	0	-5.83	-0.0067
$\mathcal{M}_{3e2}$	3	9.78	0.021	1	0	7.12	7.12	0	-0.0059
$\mathcal{M}_4$	2	0.62	0.733	1	-0.54	10.03	1.15	1.15	0
$\mathcal{M}_5$	2	0.82	0.662	1	-0.48	8.53	0	0	-0.0008

Table 8: Automated model selection with $r = 1$ and 2 lags

real interest rate, the long term nominal interest rate and the short term nominal interest rate respectively.

We run the algorithm for both $r = 1$ and $r = 2$, and base our decisions on the asymptotic LR-tests, uncorrected for small sample properties.

6.5.1 Rank = 1, 2 Lags

We report the first five models $\mathcal{M}_1 - \mathcal{M}_5$ from the automated model selection in table 8. We also add the two models, which we accepted previously (after Bartlett correction) in subsection 6.4.1: they were the first two models rejected after $\mathcal{M}_3$ as their p-value is below 0.05.

The first two models are difficult to accept, so based on the automated model selection, we choose $\mathcal{M}_3$, a money demand relationship. This corresponds to $\mathcal{H}_{13}$ in table 6. $\mathcal{M}_{3e1}$ and $\mathcal{M}_{3e2}$ are $\mathcal{H}_{18}$ and $\mathcal{H}_{17}$ in the aforementioned table.

6.5.2 Rank = 2 and two lags

With rank=2 and two lags, we obtain the models in table 9. We note that the 71th accepted model is the combination of the restrictions implied by $\mathcal{M}_3$ and $\mathcal{M}_{3e2}$ in table 8. In general we see that models with a large number of over-identifying restrictions are accepted. Each accepted relation has more restrictions that those that are accepted in the model with rank 1.

6.5.3 Rank = 2 and 1 lag

Omtzigt (2002a) noted in a Monte Carlo study that when under-selecting the true lag length, one had a greater chance of recovering the right restrictions: we therefore run the automated model selection procedure with just one lag and note that while $\mathcal{M}_1$ and $\mathcal{M}_2$ are non-interpretable in the light of economic theory outlined earlier in the paper, $\mathcal{M}_{2e2}$ in table 10 corresponds to $\mathcal{M}_{71}$ in table 9. We select this as our final model.

6.6 Selected model

The estimated model is thus a combination of a money demand equation (which is exactly the equation selected, when r=1), depicted in figures 3(c)-3(d) and an IS-curve depicted in figures 3(a)-3(b). The last one The IS curve clearly shows the large disequilibrium

Model	Nr Res	LR	P value	y	$m3r$	$rsti$	id	Δp	$trend$
$\mathcal{M}_1$	6	8.42	0.209	0	0	0	0	1	0
				1	-0.55	9.72	0	9.72	0
$\mathcal{M}_2$	6	10.76	0.096	0	0	0	0	1	0
				1	-1	17.17	0	17.17	0.0054
$\mathcal{M}_3$	6	12.08	0.060	0	0	0	0	1	0
				1	-0.50	11.36	11.36	11.36	0
$\mathcal{M}_4$	6	12.29	0.056	0	0	1	1	1	0
				1	-0.55	9.02	0	0	0
$\mathcal{M}_5$	6	12.52	0.051	0	0	1	0	1	0
				1	-0.55	0	0	-9.18	0
$\mathcal{M}_{65}$	4	6.21	0.184	1	-1	14.92	0	0	0.0051
				1	0	0	0	-6.16	-0.0068
$\mathcal{M}_{71}$	4	6.99	0.136	1	-1	15.27	0	1	0.0053
				1	0	3.66	3.66	0	-0.0063

Table 9: Automated Model Selection with $r = 2$ and 2 lags

Model	Nr Res	LR	P value	y	$m3r$	$rsti$	id	Δp	$trend$
$\mathcal{M}_1$	4	6.24	0.182	1	-0.48	14.87	14.87	0	0
				0	1	-17.65	0	0	-0.0117
$\mathcal{M}_2$	4	8.73	0.068	1	-0.52	17.03	0	0	0
				1	0	9.04	9.04	0	-0.0057
$\mathcal{M}_{2e2}$	4	9.62	0.047	1	-1	32.70	0	0	0.0058
				1	0	11.40	11.40	0	-0.0056
$\mathcal{M}_{2e26}$	4	18.51	0.001	1	-1	16.85	0	0	0.0053
				1	0	0	0	-5.03	-0.0067
$\mathcal{M}_3$	3	1.52	0.678	1	-1	20.73	0	0	-0.0054
				1	0	5.15	8.21	-3.06	-0.0061
$\mathcal{M}_4$	3	2.17	0.537	1	-0.54	12.66	0	0	0
				1	0	4.84	7.03	-2.19	-0.0061
$\mathcal{M}_5$	3	2.30	0.513	1	-0.49	13.52	13.52	0	0
				0	1	-16.95	0	-3.95	-0.0119

Table 10: Automated Model Selection with $r = 2$ and 1 lag

(recession) the Netherlands faced in the beginning of the period and a smaller one in the early 1990s. The money demand relations is far more stable. The only disequilibrium coincides with the only serious policy intervention in 1986-87, when there was a gentlemen's agreement with the Dutch banks to limit credit expansion. It is probably somewhat surprising that it only depends on the own return on money, namely the real short run interest rate. Different groups faced rather different interest rate (the market rate for firms, but a higher rate for small savers), which makes finding this stable relation even more remarkable. The trend is also very important (2.1 percent autonomous growth a year in the real money supply). It is most likely a consequence of the gradual liberalization process.

7 Conclusions

We have studied money demand in the Netherlands in the 1980s and 1990s and modelled it by means of a cointegrated VAR. We found a stable money demand function, which only depends on the own interest rate, not on rates of return on other financial assets considered. With all the shocks and fundamental changes in the economy over the period, this is fairly remarkable.

In modelling the VAR we have applied full maximum likelihood in the I(2) model and found that substantial differences exist between the two step procedure and maximum likelihood for the selection of the rank in the I(2) model. Furthermore ignoring the restrictions of the I(2) model in the I(1) after the nominal to real transformation, does make a large difference.

On the methodology used, we make two points. Firstly the largest stationary root in the characteristic polynomial is very important. If it grows large, then that is a sure sign of misspecification of the model, but contemporaneously makes the small sample corrections (bootstrap and Bartlett corrections) fail miserably, if they are based on the unrestricted estimate. Secondly in selecting the rank of the model, we have selected r=2 in the I(1) model, but for identification, we have used the information from automated model selection with r=1 and 2 lags as well as r=2 and 1 lag. There is no theory yet on the asymptotic distribution of the tests, when the rank and/or lag length are deliberately under-selected and this will be a fruitful alley for further research.

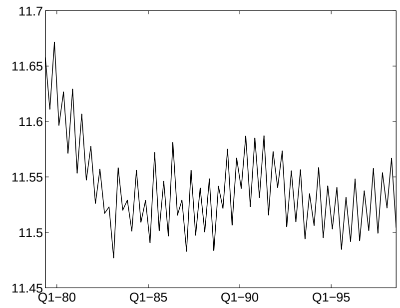
References

Beyer, A. (1998). Modelling money demand in Germany. *Journal of Applied Econometrics 13*, 57–76.

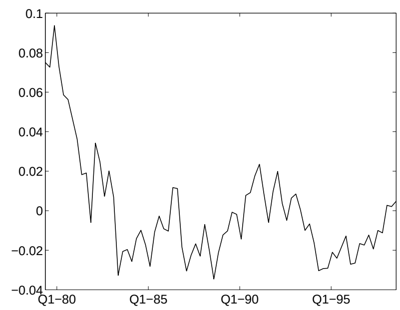
Davidson, J. (1998). Structural relations, cointegration and identification: Some simple results and their applications. *Journal of Econometrics 87*, 87–113.

De Greef, I., P. Hilbers, and L. Hoogduin (1998). Moderate monetarism: A brief survey of Dutch monetary policy in the post-war period. Technical Report 28, Dutch Central Bank.

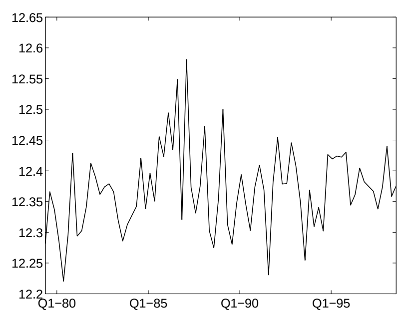
De Jong, A. (1960). *De Wetgeving Nopens de Nederlandsche Bank*. Den Haag: Nijhoff.



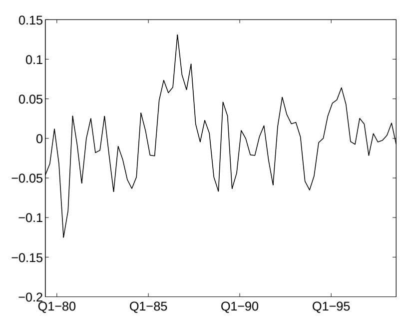
(a) IS curve $\beta_1' X_t$



(b) IS curve $\beta_1' R_t$



(c) Money Demand Relationship $\beta_2' X_t$



(d) Money Demand Relationship $\beta_2' R_t$

Figure 3: The cointegration relationships. R_t is X_t corrected for ΔX_t and D_{t+1}.

De Nederlandsche Bank (1946-2002). Annual report, various issues.

Doornik, J. (1996). Testing vector autocorrelation and heteroscedasticity. Technical report, Nuffield College, Oxford. Available at http://www.nuff.ox.ac.uk/Users/Doornik/.

Doornik, J. and H. Hansen (1994). An omnibus test for univariate and multivariate normality. Technical report, Nuffield College, Oxford.

Fase, M. and C. Winder (1990). The demand for money in the Netherlands revisited. *De Economist 38*, 276–301.

Gonzalo, J. (1994). Five alternative methods of estimating long-run equilibrium relationships. *Journal of Econometrics 64*, 203–233.

Gredenhoff, M. and T. Jacobson (2001). Bootstrap testing linear restrictions on cointegrating vectors. *Journal of Business and Economic Statistics 19*, 63–72.

Hendry, D. and G. Mizon (1993). Evaluating econometric models by encompassing the VAR. In P. Phillips (Ed.), *Models, Methods and Applications of Econometrics*. Blackwell.

Jacobs, J. and A. Van der Horst (1996). VAR-ing the economy of the Netherlands. Technical Report 24, Economics Department of the University of Groningen.

Johansen, S. (1992). A representation of vector autoregressive processes integrated of order 2. *Econometric Theory 8*, 188–202.

Johansen, S. (1995a). Identifying restrictions of linear equations: With applications to simultaneous equation and cointegration. *Journal of Econometrics 69*, 111–132.

Johansen, S. (1995b). *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Oxford: Oxford University Press.

Johansen, S. (1995c). A statistical analysis of cointegration for I(2) variables. *Econometric Theory 11*, 25–59.

Johansen, S. (1997). Likelihood analysis of the I(2) model. *Scandinavian Journal of Statistics 24*, 433–462.

Johansen, S. (2000). A Bartlett correction factor for tests on the cointegrating relations. *Econometric Theory 16*, 740–778.

Johansen, S. (2002a). A small sample correction for the test of cointegrating rank in the vector autoregressive model. *Econometrica 70*, 1929–1961.

Johansen, S. (2002b). Statistical analysis of hypotheses on the cointegrating relations in the I(2) model. Technical Report 13, Department of Theoretical Statistics, University of Copenhagen. Available at http://www.math.ku.dk/˜sjo.

Juselius, K. (1996). An empirical analysis of the changing role of the German Bundesbank after 1983. *Oxford Bulletin of Economics and Statistics 58*, 791–819.

Juselius, K. (1998). A structured VAR for Denmark under changing monetary regimes. *Journal of Business and Economics Statistics 16*, 400–411.

Juselius, K. (2001). European integration and monetary transmission mechanisms: The case of Italy. *Journal of Applied Econometrics 16*, 341–358.

Juselius, K. (2002). *The Cointegrated VAR Model: Econometric Methodology and Macroeconomic Applications*. Unpublished manuscript. Available at http://www.econ.ku.dk/okokj/.

Juselius, K. and J. Toro (1999). The effects of joining the EMS: Monetary transmission mechanisms in Spain. Technical Report 99-22, University of Copenhagen, Institute of Economics. Available at http://www.econ.ku.dk/wpa/.

Kongsted, H. (2002). Testing the nominal-to-real transformation. Technical Report 02/06, Institute of Economics, University of Copenhagen. Available at http://www.econ.ku.dk/wpa/.

Kongsted, H. and H. Nielsen (2002). Analyzing I(2) systems by transformed vector autoregressions. Technical Report 02-20, Institute of Economics, University of Copenhagen. Available at http://www.econ.ku.dk/wpa/.

Kuipers, S. and B. Boertje (1988). On the causes of the rise in the liquidity ratio in the Netherlands in the early eighties. *De Economist 136*, 50–90.

Omtzigt, P. (2002a). Automatic identification and restriction of the cointegration space. Technical Report 2002/16, UvA-Econometrics, University of Amsterdam. Available at http://www.fee.uva.nl/ke/Uva-Econometrics. (Chapter two of this thesis).

Omtzigt, P. (2002b). Automatic identification of simultaneous equations models. Technical Report 2002/14, UvA-Econometrics, University of Amsterdam. Available at http://www.fee.uva.nl/ke/Uva-Econometrics. (Chapter one of this thesis).

Omtzigt, P. (2003). Me2: A computer package for the maximum likelihood estimation of I(2) systems. Technical report, University of Amsterdam.

Omtzigt, P. and S. Fachin (2002). Bootstrapping and Bartlett corrections in the cointegrated VAR model. Technical Report 2002/15, UvA-Econometrics, University of Amsterdam, Amsterdam. Available at http://www.fee.uva.nl/ke/Uva-Econometrics. (Chapter four of this thesis).

Paruolo, P. (1996). On the determination of integration indices in I(2) systems. *Journal of Econometrics 72*, 313–356.

Paruolo, P. and A. Rahbek (1999). Weak exogeneity in I(2) VAR systems. *Journal of Econometrics 93*, 281–308.

Rahbek, A., H. Kongsted, and C. Jørgensen (1999). Trend stationarity in the I(2) cointegration model. *Journal of Econometrics 90*, 265–289.

Sterken, E. (1992). The financial transactions motive in the Dutch money demand function. *Kredit und Kapital 25*, 386–405.

Van Giersbergen, N. (1996). Bootstrapping the trace statistic in VAR models: Monte Carlo results and applications. *Oxford Bulletin of Economics and Statistics 58*, 391–408.

A Equivalent hypotheses

In this appendix we propose a solution for the following problem:

A.1 problem

We have the null hypothesis of linear within-equation restrictions[9]

$$\mathcal{H}_0 : \beta = (H_1\varphi_1, \ldots, H_r\varphi_r) \tag{16}$$

where the matrices $H_1, \ldots, H_r$ are generically identifying in the sense defined by Johansen (1995a). (If they do not, render them generically identifying, see Omtzigt (2002b)). Let matrix H_i possess s_i columns.

We have an unrestricted estimate of $\beta, \hat{\beta}_u$, which does not (necessarily) satisfy the restrictions implied by (16) and need to find an alternative null hypothesis $\mathcal{H}_0^\natural : \beta = \left(\tilde{H}_1^*\varphi_1, \ldots, \tilde{H}_r^*\varphi_r\right)$, which is satisfied by $\hat{\beta}_u$ and equals the restrictions implied by (16) if $\hat{\beta}_u$ satisfies those restrictions.

Subsequently in the bootstrap we can resample from $\hat{\beta}_u$ and impose the new null hypothesis $\mathcal{H}_0^\natural$, see Omtzigt and Fachin (2002).

A.2 solution

We need to rotate the space such that each of the r vectors is as close as possible to its restrictions H_i. We do so as in Johansen (1995b, page 110-111). Solve the following r eigenvalue problems

$$\left|\mu_i\hat{\beta}_u'\hat{\beta}_u - \hat{\beta}_u' H_i \left(H_i'H_i\right)^{-1} H_i'\hat{\beta}_u\right| \qquad i = 1, \ldots, r$$

with r ordered rows of r eigenvalues each $\mu_{i,1} \geq \mu_{i,2} \geq \ldots \geq \mu_{i,r} \geq 0$ and corresponding r sets of eigenvectors $(v_{i,1}, \ldots, v_{i,r})$. Then let

$$\tilde{\beta}_i = \hat{\beta}_u v_{i,1} \qquad i = 1, \ldots, r$$

such that $\tilde{\beta} = \left(\tilde{\beta}_i, \ldots, \tilde{\beta}_r\right)$ is the ordered unrestricted estimate.

Next find the part of H_i which is a close to the null-space of $\tilde{\gamma}_i$, that is $\tilde{\gamma}_{i\perp}$ as possible:

$$\left|\kappa_i H_i'H_i - H_i'\tilde{\beta}_{i\perp}\left(\tilde{\beta}_{i\perp}'\tilde{\beta}_{i\perp}\right)^{-1}\tilde{\beta}_{i\perp}'H_i\right| \qquad i = 1, \ldots, r$$

[9] We take away the star from β^* in the main text to avoid clutter in notation.

with r ordered rows of s_i eigenvalues each $\kappa_{i,1} \geq \kappa_{i,2} \geq \ldots \geq \kappa_{i,s_i} \geq 0$ and corresponding r sets of eigenvectors $(w_{i,1}, \ldots, w_{i,s_i})$.

The new restrictions matrices $\tilde{H}_i$ then read

$$\tilde{H}_i = \left(\tilde{\beta}_i, H_i \left(w_{i,1}, \ldots, w_{i,s_{i-1}}\right)\right) \qquad i = 1, \ldots, r$$

In the last step we have taken the the $s-1$ columns of H_i which are as close to the null space of $\tilde{\beta}_i$ as possible. The reason for which we take these is that if H_i is exactly identifying on a column, then $\tilde{\beta}_i \in sp\left(H_i\right)$ and thus $\tilde{\beta}_i = H_i w_{i,s_i}$. If we were then to include the $H_i w_{i,s_i}$ we would have that $\tilde{H}_i$ is no longer of full column rank.

As a final (optional) step we normalize the new restrictions on the old ones: $\tilde{H}_i^* = \tilde{H}_i \left(H_i' \tilde{H}_i\right)^{-1}$. The new restrictions then read

$$\tilde{H}_i^* \qquad i = 1, \ldots, r$$

and are satisfied by the unrestricted estimate $\tilde{\beta}$.