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Auctions and Efficiency

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1. Introduction

The allocation of resources is an all-pervasive theme in economics. And, I think it is no exaggeration to say that the question of whether there exist mechanisms ensuring efficient allocation--i.e., mechanisms that ensure that resources wind up in the hands of those who value them most--is of central importance in the discipline. Indeed, the very word "economics" connotes a preoccupation with the issue of efficiency.

But economists' interest in efficiency does not end with the question of existence. If efficient mechanisms can be constructed, we want to know what they look like and to what extent they might resemble institutions used in practice.

Understandably, the question of what will constitute an efficient mechanism has been a major concern of economic theorists going back to Adam Smith. But the issue is far from just a theoretical one. It is also of considerable practical importance. This is particularly clear when it comes to privatization, the transfer of assets from the state to the private sector.

In the last fifteen years or so, we have seen a remarkable flurry of privatizations in Eastern Europe, the former Soviet Union, China, and highly industrialized Western nations such as the United States, the United Kingdom, and Germany. An important justification for these transfers has been the expectation that they will improve efficiency. But if efficiency is the rationale, an obvious leading question to ask is: "What sorts of transfer mechanisms will best advance this objective?"

One possible and, of course, familiar answer is "The Market." We know from the First Theorem of Welfare Economics (see Debreu (1959)) that, under certain conditions, the competitive mechanism (the uninhibited exchange and production of goods by buyers and sellers) results in an efficient allocation. A major constraint on the applicability of this result to the circumstances of privatization, however, is the theorem's hypothesis of large numbers. For the competitive mechanism to work properly--to avoid the exercise of monopoly power--there must be sufficiently many buyers and sellers so that no single agent has an appreciable effect on prices. But privatization often entails small numbers. In the recent U.S. "spectrum" auctions--the auctions in which the government sold rights (in the form of licenses) to use certain radio frequency bands for telecommunications--there were often only two or three serious bidders for a given license. The competitive model does not seem readily applicable to such a setting.

An interesting alternative possibility was raised by William Vickrey forty years ago (Vickrey (1961)). Vickrey showed that, if a seller has a single indivisible good for sale, a second-price auction (see Section 2) is an efficient mechanism—i.e., the winner is the buyer whose valuation of the good is highest—in the case where buyers have private values ("private values" mean that no buyer's private information affects any other buyer's valuation). This finding is rendered even more significant by the fact that it can readily be extended to the sale of multiple goods[1], as shown by Theodore Groves (Groves (1973)) and Edward Clarke (Clarke (1971)).

Unfortunately, once the assumption of private values is dropped and thus buyers' valuations do depend on other buyers' information (i.e., we are in the world of common[2] or interdependent values), the second-price auction is no longer efficient, as I will illustrate by means of an example below. Yet, the common-values case is the norm in practice. If, say, a telecommunications firm undertakes a market survey to forecast demand for cell phones in a given region, the results of the survey will surely be of interest to its competitors and thus turn the situation into one of common values.

Recently, a literature has developed on the design of efficient auctions in common-values settings. The time is not yet ripe for a survey; the area is currently evolving too rapidly for that. But I would like to take this opportunity to discuss a few of the ideas from this literature.

[1] Vickrey himself also treated the case of multiple units of the same good.

2. The Basic Model

Because it is particularly simple, I will begin with the case of a single indivisible good. Later I will argue that much (but not all) of what holds in the one-good case extends to multiple goods.

Suppose that there are n potential buyers. It will be simplest to assume that they are risk-neutral (however, we can accommodate any other attitude toward risk if the model is specialized to the case in which there is no residual uncertainty about valuations when all buyers' information is pooled). Assume that each buyer *i's* private information about the good can be summarized by a real-valued signal. That is, buyer *i's* information is reduceable to a one-dimensional parameter.[3] Formally, suppose that each buyer *i's* signal s_i lies in an interval $[\underline{s}_i, \bar{s}_i]$. The joint prior distribution of $(s_1, \ldots, s_n)$ is given by the c.d.f. $F(s_1, \ldots, s_n)$. Buyer i's valuation for the good (i.e., the most he would be willing to pay for it) is given by the function $v_i(s_1, \ldots, s_n)$. I shall suppose (with little loss of generality) that higher values of s_i correspond to higher valuations, i.e.,

$$\frac{\partial v_i}{\partial s_i} > 0. \qquad (1)$$

Let us examine two illustrations of this model.

Example 1: Suppose that

$$v_i(s_1, \cdots, s_n) = s_i .$$

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[2] I am using "common values" in the broad sense to cover any instance where one agent's payoff depends on another's information. The term is sometimes used narrowly to mean that all agents share the same payoff.

[3] Later on I will examine the case of multidimensional signals. As with multiple goods, much will generalize. As we will see, the most problematic case is that in which there are both multiple goods and multidimensional signals.

In this case, we are in the world of private values, not the interesting setting from the perspective of this lecture, but a valid special case.

A more pertinent example is:

Example 2: Suppose that the true value of the good to buyer i is y_i, which, in turn, is the sum of a value component that is common to all buyers and a component that is peculiar to buyer i. That is,

$$y_i = z + z_i,$$

where z is the common component and z_i is buyer i's idiosyncratic component. Suppose, however, that buyer i does not actually observe y_i, but only a noisy signal

$$s_i = y_i + e_i , \qquad (2)$$

where e_i is the noise term, and all the random variables-- z, the z_i's, and the e_i's--are independent. In this case every buyer j's signal s_j provides information to buyer i about his valuation, because s_j is correlated (via (2)) with the common component z. Hence we can express $v_i(s_1, \ldots, s_n)$ as

$$v_i(s_1, \ldots, s_n) = E[y_i \mid s_1, \ldots, s_n], \qquad (3)$$

where the right-hand side of (3) denotes the expectation of y_i conditional on the signals $(s_1, \ldots, s_n)$.

This second example might be kept in mind as representative of the sort of scenario that the analysis is intended to apply to.

3. Auctions

An auction in the model of Section 2 is a mechanism (alternatively termed a "game form" or "outcome function") that, on the basis of the bids submitted, determines (i) who wins (i.e., who--if anyone--is awarded the good), and (ii) how much each buyer pays[4]. Let us call an auction efficient provided that, in equilibrium, buyer i is the winner if and only if

$$v_i(s_1,\ldots,s_n) \geq \max_{j \neq i} v_j(s_1,\ldots,s_n) \qquad (4)$$

(this definition is slightly inaccurate because of the possibility of ties for highest valuation, an issue that I shall ignore). In other words, efficiency demands that, in an equilibrium of the auction, the winner be the buyer with the highest valuation, conditional on all available information (i.e., on all buyers' signals).

This notion of efficiency is sometimes called ex-post efficiency. It assumes implicitly that the social value of the good being sold equals the maximum of the potential buyers' individual valuations. This assumption would be justified if, for example, each buyer used the good (e.g., a spectrum license) to produce an output (e.g., telecommunication service) that is sold in a competitive market without significant externalities (market power or externalities might drive a wedge between individual and social values).

[4] For some purposes—e.g., dealing with risk-averse buyers (see Maskin and Riley (1984)) or liquidity constraints (see Che and Gale (1996) or Maskin (2000) or allocative externalities (see Jehiel and Moldovanu 1998))—one must consider auctions in which buyers other than the winner also make payments. In this lecture, however, I will not have to deal with this possibility.

The reader may wonder why, even if one wants efficiency, it is necessary to insist that the auction itself be efficient. After all, the buyers could always retrade afterwards if the auction resulted in a winner with less than the highest valuation. The problem with relying on post-auction trade, however, is much the same as that plaguing competitive exchange in the first place: these mechanisms do not in general work efficiently when there are only a few traders. To see this, consider the following example[5]:

Example 3: Suppose that there are two buyers. Assume that buyer 1 has won the auction and has a valuation of 1. If the auction is not guaranteed to be efficient, then there is some chance that buyer 2's valuation is higher. Suppose that, from buyer 1's perspective, buyer 2's valuation is distributed uniformly in the interval [0,2]. Now, if there is to be further trade after the auction, someone has to initiate it. Let us assume that buyer 1 does so by proposing a trading price to buyer 2. Presumably, buyer 1 will propose a price p^* that maximizes his expected payoff, i.e., that solves

$$\max_p \frac{1}{2}(2-p)(p-1) \ . \qquad (*)$$

(To understand (*) note that $\frac{1}{2}(2-p)$ is the probability that the proposal is accepted—since it is the probability that buyer 2's valuation is at least p—and that $p-1$ is buyer 1's net gain in the event of acceptance.) But the solution to (*) is $p^* = \frac{3}{2}$. Hence, if buyer 2's valuation lies between 1 and $\frac{3}{2}$, the allocation, even after allowing for ex-post trade, will remain inefficient, since buyer 2 will reject 1's proposal.

I will first look at efficiency in the <u>second-price</u> auction. This auction form (often called the <u>Vickrey</u> auction) has the following rules: (i) each bidder i makes a (sealed) bid b_i, which is a

[5] In this example, buyers have private values, but, as Fieseler, Kittsteiner, and Moldovanu (2000) show, resale

nonnegative number; (ii) the winner is the bidder who has made the highest bid (again ignoring the issue of ties); (iii) the winner pays the second-highest bid, $\max_{j \neq i} b_j$. As I have already noted and will illustrate explicitly below, this auction can readily be extended to multiple goods.

The Vickrey auction is efficient in the case of private values[6]. To see this, note first that it is optimal--in fact, a dominant strategy--for buyer i to set $b_i = v_i$, i.e., to bid his true valuation. In particular, bidding below v_i does not affect buyer *i's* payment if he wins (since his bid does not depend on his own bid); it just reduces his chance of winning—and so is not a good strategy. Bidding above v_i raises buyer *i's* probability of winning, but the additional events in which he wins are precisely those in which someone else has bid higher than v_i. In such events buyer i pays more than v_i, also not a desirable outcome. Thus it is indeed optimal to bid $b_i = v_i$, which implies that the winner is the buyer with the highest valuation, the criterion for efficiency.

Unfortunately, the Vickrey auction does not remain efficient once we depart from private values. To see this, consider the following example.

Example 4: Suppose that there are three buyers with valuation-functions

$$
\begin{aligned}
v_1(s_1, s_2, s_3) &= s_1 + \frac{2}{3} s_2 + \frac{1}{3} s_3 \\
v_2(s_1, s_2, s_3) &= s_2 + \frac{1}{3} s_1 + \frac{2}{3} s_3 \\
v_3(s_1, s_2, s_3) &= s_3.
\end{aligned}
$$

Notice that buyers 1 and 2 have common values, i.e., their valuations do not depend only on their own signals. Assume that it happens that $s_1 = s_2 = 1$ (of course, buyers 1 and 2 would not know that their

can become even more problematic when there are common values.

[6] It is easy to show that the "first-price" auction—the auction in which each buyer makes a bid, the high bidder wins, and the winner pays his bid—is a nonstarter as far as efficiency is concerned. Indeed, even in the case of private values, the first-price auction is never efficient except when buyers' valuations are symmetrically distributed (see Maskin (1992)).

signal values are equal, since signals are private information), and suppose that buyer 3's signal value is either slightly below or slightly above 1. In the former case, it is easy to see that

$$v_1 > v_2 > v_3,$$

and so, for efficiency, buyer 1 ought to win. However, in the latter case

$$v_2 > v_1 > v_3,$$

and so buyer 2 is the efficient winner. Thus the efficient allocation between buyers 1 and 2 turns on whether s_3 is below or above 1. But in a Vickrey auction, the bids made by buyers 1 and 2 cannot incorporate information about s_3 since that signal is private information to buyer 3. Thus the outcome of the auction cannot in general be efficient.

4. An Efficient Auction

How should we respond to the shortcomings of the Vickrey auction as illustrated by Example 3? One possible reaction is to appeal to classical mechanism-design theory. Specifically, we could have each buyer i announce a signal value $\hat{s}_i$, award the good to the buyer i for whom $v_i(\hat{s}_1, \ldots, \hat{s}_n)$ is highest, and choose the winner's payment so as to evoke truth-telling in buyers, i.e., so as to induce each buyer j to set $\hat{s}_j$ equal to his true signal value s_j.

The problem with such a "direct revelation" mechanism is that it is utterly unworkable in practice. In particular, notice that it requires the mechanism designer to know the physical signal spaces $S_1, \ldots, S_n$, the functional forms $v_i(\cdot)$, and the prior distributions of the signals--an extraordinarily demanding constraint. Now, the mechanism designer could attempt to elicit this information from the buyers themselves using the methods of the implementation literature (see Palfrey (1993)). For example, to learn the signal spaces, he could have each buyer announce a vector $(\hat{S}_1, \ldots, \hat{S}_n)$ and assign suitable penalties if the announcements did not match up appropriately. A major difficulty with such a scheme, however, is that in all likelihood the signal spaces S_i are themselves <u>private</u> information. For analytic purposes, we model S_i as simply an interval of numbers.

But, this abstracts from the reality that buyer i's signal corresponds to some physical entity--whatever it is that buyer *i* observes. Indeed, the signal may well be a sufficient statistic for data from a variety of different informational sources. And there is no reason why other buyers should know just what this array of sources is.

To avoid these complications, I shall concentrate on auction rules that do not make use of such details as signal spaces, functional forms, and distributions. Indeed, I will be interested in auctions that work well irrespective of these details, that is, I will adhere to the "Wilson Doctrine" (after Robert Wilson, who has been an eloquent proponent of the view that auction institutions should be "detail-free"). It turns out that a judicious modification of the Vickrey auction will do the trick.

Before turning to the modification, however, I need to introduce a restriction on valuation functions that is critical to the possibility of constructing efficient auctions. Let us assume that for all i and $j \neq i$ and all $(s_1, \ldots, s_n)$,

$$v_i(s_1, \ldots, s_n) = v_j(s_1, \ldots, s_n) \Rightarrow \frac{\partial v_i}{\partial s_i}(s_1, \ldots, s_n) > \frac{\partial v_j}{\partial s_i}(s_1, \ldots, s_n). \tag{5}$$ [7]

In words, condition (5) says that buyer *i's* signal has a greater marginal effect on his own valuation than on that of any other buyer *j* (at least at points where buyer *i's* and buyer *j's* valuations are equal).

Notice that, in view of (1), condition (5)[8] is automatically satisfied by Example 1 (the case of private values): the right-hand side of the inequality then simply vanishes. Condition (5) also holds for Example 2. This is because, in that example, s_i conveys relevant information to buyer $j (\neq i)$ about the common component z but tells buyer i not only about z but also his idiosyncratic component z_i. Thus, v_i will be more sensitive than v_j to variations in s_i.

But whether or not condition (5) is likely to be satisfied, it is, in any event, essential for efficiency. To see what can go wrong without it, consider the following example.

[7] This condition was introduced by Gresik (1991).

Example 5: Suppose that the owner of a tract of land wishes to sell off the rights to drill for oil on her property. There are two potential drillers who are competing for this right. Driller 1's fixed cost of drilling is 1, whereas his marginal cost is 2. In contrast, driller 2 has fixed and marginal cost of 2 and 1, respectively. Assume that driller 1 observes how much oil is

underground. That is, s_1 equals the quantity of oil. Driller 2 obtains no private information. Then if the price of oil is 4 we have

$$v_1(s_1) = (4-2)s_1 - 1 = 2s_1 - 1$$
$$v_2(s_1) = (4-1)s_1 - 2 = 3s_1 - 2.$$

Observe that $v_1(s_1) > v_2(s_1)$ if and only if $s_1 < 1$. Thus, for efficiency, driller 1 should be awarded drilling rights provided that $\frac{1}{2} < s_1 < 1$ (for $s_1 < \frac{1}{2}$, there is not enough oil to justify drilling at all). Driller 2, by contrast, should get the rights when $s_1 > 1$.

In this example, there is no way (either through a modified Vickrey auction or otherwise) of inducing driller 1 to reveal the true value s_1 in order to allocate drilling rights efficiently. To see this, consider, without loss of generality, a direct revelation mechanism and let $t_1(\hat{s}_1)$ be a monetary transfer (possibly negative) to driller 1 if he announces signal value $\hat{s}_1$. Let s_1' and s_1'' be signal values such that

$$\frac{1}{2} < s_1' < 1 < s_1''. \tag{6}$$

Then for driller 1 to have the incentive to announce truthfully when $s_1 = s_1''$, we must have

$$t_1(s_1'') \geq 2s_1'' - 1 + t_1(s_1') \tag{7}$$

[8] Notice that the strictness of the inequality in (5) rules out the case of "pure common values," where all buyers share the same valuation. However, in that case, the issue of who wins does not matter for efficiency.

(the left-hand side is his payoff when he is truthful, whereas the right-hand side is his payoff when he pretends that $s_1 = s_1'$). Similarly, the incentive-constraint corresponding to $s_1 = s_1'$ is

$$2s_1' - 1 + t_1(s_1') \geq t_1(s_1''). \tag{8}$$

Subtracting (8) from (7), we obtain

$$2(s_1' - s_1'') \geq 0,$$

a contradiction of (6). Hence there exists no efficient mechanism.

The feature that interferes with efficiency in this example is the violation of condition (5), i.e., the fact that

$$0 < \frac{\partial v_1}{\partial s_1} < \frac{\partial v_2}{\partial s_1} \ . \tag{9}$$

Inequalities (1) and (9) imply that, as s_1 rises, drilling rights become more valuable to driller 1 but increasingly more likely, from the standpoint of efficiency, to be awarded to driller 2. This conflict makes the task of providing proper incentives for driller 1 impossible.

Assuming henceforth that (5) holds, let us reconfront the task of designing an efficient auction. In Example 4 we saw that the Vickrey auction failed because buyers 1 and 2 could not incorporate pertinent information about buyer 3 in their bids (since s_3 was private information). This suggests that, as in Dasgupta and Maskin (2000), a natural way of amending the Vickrey auction would be to allow buyers to make contingent bids—bids that depend on other buyers' valuations. In Example 4, this would enable buyer 1 to say, in effect, "I don't know what buyer 3's valuation is, but if it turns out to be x, then I want to bid y."

Let us examine how contingent bidding would work in the case of two buyers. Buyer 1 would announce a schedule $\hat{b}_1(\cdot)$, where, for all possible values v_2,

$$\hat{b}_1(v_2) = \text{buyer 1's bid if buyer 2 has valuation } v_2.$$

Similarly, buyer 2 would announce a schedule $\hat{b}_2(\cdot)$, where

$\hat{b}_2(v_1) =$ buyer 2's bid if buyer 1's valuation is v_1.

We would then look for a fixed point

$$(v_1^o, v_2^o) = (\hat{b}_1(v_2^o), \hat{b}_2(v_1^o)), \tag{10}$$

and

$$\text{install buyer 1 as the winner if and only if } v_1^o > v_2^o. \tag{11}$$

To understand the rationale for (10) and (11), imagine that buyers bid truthfully. Since signals are private information and thus buyer 1 will not in general know his own valuation, truthful bidding means that, if his signal value is s_1, he submits a schedule $\hat{b}_1(\cdot) = b_1(\cdot)$ such that

$$b_1(v_2(s_1, s_2')) = v_1(s_1, s_2') \text{ for all } s_2'.\text{[9]} \tag{12}$$

That is, whatever s_2' (and hence v_2) turns out to be, buyer 1 bids his true valuation for that signal value. Similarly, truthful bidding for buyer 2 with signal value s_2 means reporting schedule $\hat{b}_2(\cdot) = b_2(\cdot)$ such that

$$b_2(v_1(s_1', s_2)) = v_2(s_1', s_2) \text{ for all } s_1'. \tag{13}$$

Observe that if buyers bid according to (12) and (13), then the true valuations

$$(v_1(s_1, s_2), v_2(s_1, s_2))$$

constitute a fixed point in the sense of (10).[10]

In view of (10) and (11), this means that if buyers are truthful, the auction will result in an efficient allocation. Thus, the remaining critical issue is how to get buyers to bid truthfully. For this

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[9] I noted in my arguments against direct revelation mechanisms that buyer 1 most likely will not know buyer 2's signal space S_2. But this in no way should prevent him from understanding how his own valuation is related to that of buyer 2, which is what (12) is really expressing (i.e., (12) still makes sense even if buyer 1 does not know what values s_2' can take).

[10] Without further assumptions on valuation functions, there could be additional—non-truthful—fixed points. Dasgupta and Maskin (2000) and Eso and Maskin (2000a) provide conditions to rule such fixed points out. But even if they are not ruled out, the auction rules can be modified so that, in equilibrium, the truthful fixed point results (see Dasgupta and Maskin (2000)).

purpose, it is useful to recall the device that the Vickrey auction exploits to induce truthful bidding, viz., to make the winner's payment equal, not to his own bid, but to the lowest possible bid he could have made and still have won the auction.

This trick cannot be exactly replicated in our setting because buyers are submitting schedules rather than single bids. But let us try to take it as far as it will go. Suppose that when buyers repeat the schedules $\left(\hat{b}_1(\cdot), \hat{b}_2(\cdot)\right)$, the resulting fixed point $\left(v_1^o, v_2^o\right)$ satisfies

$$v_1^o > v_2^o.$$

Then according to our rules, buyer 1 should win. But rather than having him pay v_1^{o}, we will have buyer 1 pay v_1^*, where

$$v_1^* = \hat{b}_2\left(v_1^*\right). \tag{14}$$

This payment rule, I maintain, is the common-values analog of the Vickrey trick in the sense that v_1^* is the lowest <u>constant</u> bid (i.e., the lowest uncontingent bid) that buyer 1 could make and still win (or tie for winning) given buyer 2's bid $\hat{b}_2(\cdot)$. The corresponding payment rule for buyer 2 should he win is v_2^* such that

$$v_2^* = \hat{b}_1\left(v_2^*\right). \tag{15}$$

I claim that, given the payment rules (14) and (15), it is an equilibrium for buyers to bid truthfully. To see this most easily, let us make use of a strengthened version of (5):

$$\frac{\partial v_i}{\partial s_i} > \frac{\partial v_j}{\partial s_i}. \tag{16}$$

Let us suppose that buyer 2 is truthful, i.e., he bids $b_2(\cdot)$ satisfying (13). I must show that it is optimal for buyer 1 to bid $b_1(\cdot)$ satisfying (12).

Notice first that if buyer 1 wins, his payoff is

$$v_1(s_1, s_2) - v_1^*, \text{where } v_1^* = b_2\left(v_1^*\right), \tag{17}$$

regardless of how he bids (since neither his valuation nor his payment depends on his bid). I claim that if buyer 1 bids truthfully, then he wins if and only if (17) is positive. Observe that if this claim is established, then I will in fact have shown that truthful bidding is optimal; because buyer 1's bid does not affect (17), the most he can possibly hope for is to win precisely in those cases where the net payoff from winning is positive.

To see that the claim holds, let us first differentiate (13) with respect to s_1' to obtain

$$\frac{db_2}{dv_1}(v_1(s_1', s_2))\frac{\partial v_1}{\partial s_1}(s_1', s_2) = \frac{\partial v_2}{\partial s_1}(s_1', s_2) \text{ for all } s_1'.$$

This identity, together with (1) and (16), implies that

$$\frac{db_2}{dv_1}(v_1) < 1, \text{for all } v_1. \tag{18}$$

But from (18), (17) is positive if and only if

$$v_1(s_1, s_2) - v_1^* > \frac{db_2}{dv_1}(v_1')\left(v_1(s_1, s_2) - v_1^*\right) \text{for all } v_1'. \tag{19}$$

Now, from the intermediate value theorem, there exists $v_1' \in [v_1^*, v_1(s_1, s_2)]$ such that

$$b_2(v_1(s_1, s_2)) - b_2\left(v_1^*\right) = \frac{db_2}{dv_1}(v_1')\left(v_1(s_1, s_2) - v_1^*\right).$$

Hence (17) is positive if and only if

$$v_1(s_1, s_2) - v_1^* > b_2\left(v_1(s_1, s_2)\right) - b_2\left(v_1^*\right), \tag{20}$$

which, since $v_1^* = b_2\left(v_1^*\right)$, is equivalent to

$$v_1(s_1, s_2) > v_2(s_1, s_2). \tag{21}$$

Now suppose that buyer 1 is truthful. Because $(v_1(s_1, s_2), v_2(s_1, s_2))$ is then a fixed point, 1 wins if and only if (21) holds. So we can conclude that, when buyer 1 is truthful, his net payoff from winning is positive (i.e., (17) is positive) if and only if he wins, which is what I claimed. That is, the modified Vickrey auction is efficient.

An attractive feature of the Vickrey auction in the case of private values is that bidding one's true valuation is optimal <u>regardless</u> of the behavior of other buyers, i.e., it is a <u>dominant</u> <u>strategy</u>. Once we abandon private values, however, there is no hope of finding an efficient mechanism with dominant strategies (this is because, if my payoff depends on your signal, then my optimal strategy necessarily depends on the way that your strategy reflects your signal value, and so is not independent of what you do). Nevertheless, equilibrium in our modified Vickery auction has a strong robustness property. In particular, notice that although, technically, truthful bidding constitutes only a Bayesian (rather than dominant-strategy) equilibrium, equilibrium strategies are <u>independent</u> of the prior distribution of signals F. That is, regardless of buyers' prior beliefs about signals, they will behave the same way in equilibrium. In particular, this means that the modified Vickrey auction will be efficient even in the case in which buyers' signals are believed to be independent of one another.[11]

One might complain that having a buyer make his bid a function of the other buyer's valuation imposes a heavy informational burden on him—what if he doesn't know anything about the connection between the other's valuation and his own? I would argue, however, that the modified Vickrey auction should be viewed as giving buyers an additional <u>opportunity</u> rather than as setting an onerous requirement. After all, the degree to which a buyer makes his bid contingent is entirely up to him. In particular, he always has the option of bidding entirely <u>uncontingently</u>, i.e., of submitting a constant function. Thus, contingency is <u>optional</u> (but, of course, the degree to which the modified Vickrey auction will be more efficient than the ordinary Vickrey will turn on the extent to which buyers are prepared to bid contingently).

[11] Crémer and McLean (1988) exhibit a mechanism that attains efficiency if the joint distribution of signals is common knowledge (including to the auction designer) and exhibits correlation. In very recent work A. Postlewaite has shown how this mechanism can be generalized to the case where the auction designer himself does not know the joint distribution.

I have explicitly illustrated how the modified Vickrey auction works only in the case of two bidders, but the logic extends immediately to larger numbers. For the case of n buyers the rules become:

i) each buyer i submits a contingent bid schedule $\hat{b}_i(\cdot)$, which is a function of v_{-i}, the vector of valuations excluding that of buyer i;

ii) the auctioneer computes a fixed point $\left(v_1^o, \ldots, v_n^o\right)$, where $v_i^o = \hat{b}_i(v_{-i}^o)$ for all i;

iii) the winner is the buyer i for whom $v_i^o \geq v_j^o$ for all $j \neq i$;

iv) the winner pays $\max_{j \neq i} \hat{b}_j(v_{-j}^*)$ where, for all $j \neq i$, v_j^* satisfies $v_j^* = \hat{b}_j(v_{-j}^*)$.

Under conditions (1) and (5), an argument similar to the two-buyer demonstration above establishes that it is an equilibrium in this auction for each buyer to bid truthfully (see Dasgupta and Maskin (2000))[12]. That is, if buyer i's signal value is s_i, he should set $\hat{b}_i(\cdot) = b_i(\cdot)$ such that

$$b_i\left(v_{-i}(s_i, s'_{-i})\right) = v_i(s_i, s'_{-i}) \text{ for all } s'_{-i}.^{13} \tag{22}$$

Furthermore, it is easy to see that, if buyers bid truthfully, the auction results in an efficient allocation.

One drawback of the modified Vickrey auction that I have exhibited is that a buyer must report quite a bit of information (this is an issue distinct from that of the buyer's having to know a great deal, discussed above)--a bid for each possible vector of valuations that others may have. Perry and Reny (1999a) have devised an alternative modification of the Vickrey auction that considerably reduces the complexity of the buyer's report.

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[12] The reader may wonder whether, when (5) is not satisfied and so an efficient auction may not be possible, the efficiency of the final outcome could be enhanced by allowing buyers to retrade after the auction is over. However, any post-auction trading episode could alternatively be viewed as part of a single mechanism that embraces both it and the auction proper. That is, in our search for efficient auctions, we need not consider post-auction trade since such activity could always be folded into the auction itself. Indeed, permitting post-auction trade can, in principle interfere with efficiency in the same way that renegotiation can interfere with the efficiency of a contract (see Dewatripont (1989)).

[13] It is conceivable—although unlikely—that for a given vector v_{-i} there could exist two different signal vectors s'_{-i} and s''_{-i} such that $v_{-i}(s_i, s'_{-i}) = v_{-i}(s_i, s''_{-i}) = v_{-i}$ but $v_i(s_i, s'_{-i}) \neq v_i(s_i, s''_{-i})$, in which case (22) is not well defined. To see how to handle that possibility see Dasgupta and Maskin (2000).

Specifically, the Perry-Reny auction consists of two rounds of bidding. This means that a buyer can make his second-round bid depend on whatever he learned about other buyers' valuations from their first-round bids, and so the auction avoids the need to report bid schedules. In the first round, each buyer *i* submits a bid $b_i \geq 0$. In the second round each buyer *i* submits a bid b^j_i for each buyer $j \neq i$. If some buyer submits a bid of zero in the first round, then the Vickrey rules apply: the winner is the high bidder, and he pays the second-highest bid. If all first-round bids are strictly positive, then the second-round bids determine the outcome. In particular, if there exists a buyer i such that

$$b_i^j \geq b_j^i \text{ for all } j \neq i \tag{23}$$

then buyer *i* wins and pays $\max_{j \neq i} b_j^i$. If there exists no i satisfying (23), then the good is allocated at random.

Perry and Reny show that, under assumption (1) and (5) and provided that the probability a buyer has a zero valuation is zero, there exists an efficient equilibrium of this auction. They also demonstrate that the auction can be readily extended to the case in which multiple identical goods are sold, provided that a buyer's marginal utility from additional units is declining.

5. The English Auction

The reader may wonder why, in my discussion of efficiency, I have not brought up the English auction, the familiar open format in which (i) buyers call out bids publicly (with the proviso that each successive bid exceed the one before), (ii) the winner is the last buyer to make a bid, and (iii) the winner pays his bid. After all, the opportunity to observe other buyers' bids in the English auction would seem to allow a buyer to make a conditional bid in the same way that the modified Vickrey auction does.

However, as shown in Maskin (1992), Eso and Maskin (2000b) and Krishna (2000), the English auction is not efficient in as wide a class of cases as the modified Vickrey auction. To see this, let us consider a variant of the English auction, sometimes called the “Japanese” auction (see Milgrom and Weber (1982)), which is particularly convenient analytically:

(i) all buyers are initially in the auction;

(ii) the auctioneer raises the price continuously starting from zero;

(iii) a buyer can drop out (publicly) at any time;

(iv) the last buyer remaining wins;

(v) the winner pays the price prevailing when the penultimate buyer dropped out.

Now, in this auction, a buyer can indeed condition his drop-out point according to when other buyers have dropped out, allowing bids in effect to be conditional on other buyers’ valuations. However, a buyer can condition only on buyers who have already dropped out. Thus, for efficiency, buyers must drop out in the “right” order in the equilibrium. That this might not happen is illustrated by the following example from Eso and Maskin (2000a):

Example 6: Suppose there are two buyers, where

$$v_1(s_1, s_2) = 2 + s_1 - 2s_2$$

and

$$v_{21}(s_1, s_2) = 2 + s_2 - 2s_1$$

and s_1 and s_2 are distributed uniformly on [0,1]. Notice first that conditions (1) and (5) hold, so that the modified Vickrey auction results in a efficient equilibrium allocation. Indeed, buyers’ equilibrium contingent bids are

$$b_1(v_2) = 6 - 3s_1 - 2v_2$$

and

$$b_2(v_1) = 6 - 3s_2 - 2v_1.$$

Now, consider the English auction. For i=1,2 let $p_i(s_i)$ be the price at which buyer i drops out if his signal value is s_i. If the English auction were efficient, then we would have

$$s_1 > s_2 \text{ if and only if } p_1(s_1) > p_2(s_2). \qquad \textbf{(W)}$$

Hence,

$$\text{if } s_1 = s_2 = s, \text{ then } p_1(s_1) = p_2(s_2) \qquad \textbf{(WW)}$$

But from (**W**) and (**WW**), $p_i(s+)s) > p_i(s)$ and so

$$p_i(\cdot) \text{ is strictly increasing in } s_i. \qquad \textbf{(WWW)}$$

Thus,

$$p_1(s) = v_1(s,s)$$

and

$$p_2(s) = v_2(s,s)$$

(if $v_1(s,s) > p_1(s)$, then buyer 1 drops out before the price reaches his valuation and so would do better to stay in a bit longer; if $v_1(s,s) < p_1(s)$, then buyer 1 stays in for prices above his valuation, and so would do better to drop out earlier). But,

$$v_1(s,s) = 2 + s - 2s = 2 - s,$$

which is <u>decreasing</u> in s, violating our finding that p_1(A) is increasing. In short, efficiency demands that a buyer with a lower signal value drop out first. But if buyer i's signal value is s, he has the incentive to drop out when the price equals $v_1(s,s)$, and this function is decreasing in s. So, in equilibrium buyers will not drop out in the right order. We conclude that the English auction does not have an efficient equilibrium in this example.

In Example 6 each buyer's valuation is decreasing in the other buyer's signal. Indeed, this feature is important: as Maskin (1992) shows, the English auction is efficient in the case n=2 when valuations are nondecreasing functions of signals (and conditions (1) and (5) hold). However, examples due to Perry and Reny (1999b), Krishna (2000), and Eso and Maskin (2000b) demonstrate that this result does not extend to more than two buyers. Nevertheless, Krishna (2000) provides some interesting conditions (considerably stronger than the juxtaposition of (1) and (5)) under which the English auction is efficient with three or more buyers (see also Eso and Maskin (2000b)). Moreover, the Perry and Reny paper shows that the English auction can be modified (in a way analogous to their (1999b) alteration of the Vickrey auction) that renders it efficient under the same conditions as the modified Vickrey auction. In fact, this modified English auction extends to multiple (identical) units, as long as buyers' marginal valuations are decreasing in the number of units consumed (in the multiunit case, the Perry-Reny auction is actually a modification of the Ausubel (1997) generalization of the English auction).

6. Multiple Goods

In the same way that the ordinary Vickrey auction extends to multiple goods via the Groves-Clarke mechanism, so our modified Vickrey auction can be extended to handle more than one good. It is simplest to consider the case of two buyers, 1 and 2, and two goods, *A* and *B*. If there were private values, the pertinent information about buyer *i* would consist of three numbers, $v_{iA}, v_{iB,}$ and v_{iAB}--his valuations, respectively, for good *A*, good *B*, and and both goods together. Efficiency would then mean allocating the goods to maximize the sum of valuations. For example, it would be efficient to allocate both goods to buyer 1 provided that

$$v_{1AB} \geq \max\{v_{1\mathrm{A}} + v_{2B}, v_{1B} + v_{2A}, v_{2AB}\}.$$

The Groves-Clarke mechanism is the natural generalization of the Vickrey auction to a multi-good setting. In this mechanism, buyers submit valuations (in our two-good, private-values model, each buyer *i* submits $\hat{v}_{iA}, \hat{v}_{iB}$, and $\hat{v}_{iAB}$); the goods are allocated in the way that maximizes the sum of the submitted valuations; and each buyer makes a payment equal to his marginal impact on the other buyers (as measured by their submitted valuations). Thus, in the private-values model, if buyer 1 is allocated good A, then he should pay

$$\hat{v}_{2AB} - \hat{v}_{2B}, \tag{24}$$

since $\hat{v}_{2AB}$ would be buyer 2's payoff were buyer 1 absent, $\hat{v}_{2B}$ is his payoff given buyer 1's presence, and so the difference between the two—i.e., (24)-- is buyer 1's marginal effect on buyer 2.

Given private values, bidding one's true valuation is a dominant strategy in the Vickrey auction and the same is true in the Groves-Clarke mechanism. Hence, in view of its allocative rule, the mechanism is efficient in the case of private values. But, as with the Vickrey auction, the Groves-Clarke mechanism is not efficient when there are common values. Hence, I shall examine a modification of Groves-Clarke analogous to that for Vickrey.

As in the one-good case, assume that each buyer i (i=1,2) observes a private real-valued signal s_i. Buyer i's valuations are functions of the two signals:

$$v_{iA}(s_1, s_2), v_{iB}(s_1, s_2), v_{iAB}(s_1, s_2).$$

The appropriate counterpart to condition (1) is the requirement that if H and H' are two bundles of goods for which, given (s_1, s_2), buyer i prefers H, then the intensity of that preference rises with s_i. That is, for all i=1,2 and for any two bundles $H, H' = f, A, B, AB$

$$v_{iH}(s_1, s_2) - v_{iH'}(s_1, s_2) > 0 \Rightarrow \frac{\partial}{\partial s_i}\left(v_{iH}(s_1, s_2) - v_{iH'}(s_1, s_2)\right) > 0 \,. \quad (25)$$

Notice that if, in particular, $H = A$ and $H' = f$, then (25) just reduces to the requirement that if $v_{iA}(s_1, s_2) > 0,$ then $\frac{\partial v_{iA}}{\partial s_i}(s_1, s_2) > 0,$ i.e., to (1).

Similarly, the proper generalization of (5) is the requirement that if, for given signal values, two allocations of goods are equally efficient (i.e., give rise to the same sum of valuations), then an increase in s_i leads the allocation that buyer i prefers to become the more efficient. That is, for all $i = 1,2,$ and any two allocations $(H_1, H_2), (H_1', H_2'),$

$$\begin{aligned}&\text{if} \quad \sum_{j=1}^{2} v_{jH_j}(s_1,s_2) = \sum_{j=1}^{2} v_{jH'_j}(s_1,s_2) \text{ and } v_{iH_i}(s_1,s_2) > v_{iH'_i}(s_1,s_2),\\ &\quad \text{then} \quad \frac{\partial}{\partial s_i}\sum_{j=1}^{2} v_{jH_j}(s_1,s_2) > \frac{\partial}{\partial s_i}\sum_{j=1}^{2} v_{jH'_j}(s_1,s_2).\end{aligned} \tag{26}$$

Notice that, if just one good A were being allocated and the two allocations were $(H_1, H_2) = (A, f)$ and $(H'_1, H'_2) = (f, A)$, then, when i=1, condition (26) would reduce to the requirement

$$\begin{aligned}&\text{if} \quad v_{1A}(s_1,s_2) = v_{2A}(s_1,s_2) \text{ and } v_{1A}(s_1,s_2) > 0,\\ &\quad \text{then } \frac{\partial v_{1A}}{\partial s_1}(s_1,s_2) > \frac{\partial v_{2A}}{\partial s_1}(s_1,s_2),\end{aligned} \tag{27}$$

which is just (5).

An auction is efficient in this setting if, for all (s_1, s_2), the equilibrium allocation $\left(H_1^o, H_2^o\right)$ solves

$$\max_{(H_1,H_2)} \sum_{i=1}^{2} v_{iH_i}(s_1,s_2).$$

Under assumptions (25) and (26), the following rules constitute an efficient auction:

(i) buyer i submits schedules $\hat{b}_{iA}(\cdot), \hat{b}_{iB}(\cdot), \hat{b}_{iAB}(\cdot)$, where for all H=A,B,AB and all v_j

$$\begin{aligned}\hat{b}_{iH}\left(v_j\right) &= \text{buyer } i\text{'s bid for } H \text{ if buyer } j\text{'s } (j \neq i)\\ &\qquad \text{valuations are } v_j = \left(v_{jA}, v_{jB}, v_{jAB}\right);\end{aligned}$$

(ii) the auctioneer computes a fixed point $\left(v_1^o, v_2^o\right)$ such that, for all i and H,

$$v_{iH}^o = \hat{b}_{iH}\left(v_j^o\right) \;;$$

(iii) goods are divided according to allocation $\left(H_1^o, H_2^o\right)$, where

$$\left(H_1^o, H_2^o\right) = \arg \max_{(H_1, H_2)} \sum_{i=1}^{2} v_{iH_i}^o;$$

(iv) suppose that buyer 1 is allocated good A (i.e., $H_1^o = A$); if (a) there exists v_1^* such that

$$v_{1A}^* + \hat{b}_{2B}\left(v_1^*\right) = \hat{b}_{2AB}\left(v_1^*\right), \tag{28}$$

then buyer 1 pays

$$\hat{b}_{2AB}\left(v_1^*\right) - \hat{b}_{2B}\left(v_1^*\right); \tag{29}$$

if instead of (28), (b) there exist $\hat{v}_1^*$ (with $\hat{v}_{1A}^* < v_{1\text{A}}^{\text{o}}$) and v_1^{**} such that

$$\hat{v}_{1A}^* + \hat{b}_{2B}\left(\hat{v}_1^*\right) = \hat{v}_{1B}^* + \hat{b}_{2A}\left(\hat{v}_1^*\right)$$

and

$$v_{1B}^{**} + \hat{b}_{2A}\left(v_1^{**}\right) = \hat{b}_{2AB}\left(v_1^{**}\right),$$

then buyer 1 pays

$$\left(\hat{b}_{2A}\left(\hat{v}_1^*\right) - \hat{b}_{2B}\left(\hat{v}_1^*\right)\right) + \left(\hat{b}_{2AB}\left(v_1^{**}\right) - \hat{b}_{2A}\left(v_1^{**}\right)\right); \tag{30}$$

(v) if buyer 1 is allocated good *B*, then his payment is completely analogous to that of (iv);

(vi) if buyer 1 is allocated goods *A* and *B*, then see the Appendix for his payment;

(vii) buyer 2's payments are completely analogous to those of buyer 1.

Rules (i)-(iii) so closely mirror rules (i)-(iii) of the modified Vickrey auction in section 4 that they do not require further comment. Let us, therefore, focus on rule (iv). If *A* were the only good being allocated, then to compute buyer 1's payment, we would reduce v_{1A} from v_{1A}^o to the point v_{1A}^* where it is no longer uniquely efficient to allocate buyer 1 good *A* (i.e., it becomes equally efficient to allocate *A* to buyer 2) and have him pay his marginal impact at v_1^* on buyer 2: the difference between buyer 2's payoff from getting *A* and that from getting nothing:

$$\hat{b}_{2A}\left(v_{1A}^*\right) - 0 = \hat{b}_{2A}\left(v_{1A}^*\right),$$

which is payment rule (14). Using this same principle in the two-good setting, let us reduce v_{1A} from v_{1A}^{o} to the first point where it is no longer uniquely efficient to allocate A to buyer 1 and B to buyer 2. There are two possible cases. In case (a), at this first switching point it becomes efficient to allocate both goods to buyer 2. Let us denote the switching point in this case by v_{1A}^{*} (choose v_{1B}^{*} and v_{1AB}^{*} to conform with v_{1A}^{*}, i.e., choose them so that $v_1^{*} = \left(v_{1A}^{*}, v_{1B}^{*}, v_{1AB}^{*}\right)$ lies in the domain of $\left(\hat{b}_{2A}(\cdot), \hat{b}_{2B}(\cdot), \hat{b}_{2AB}(\cdot)\right)$). Hence, at v_{1A}^{*}, buyer 1's marginal impact on buyer 2 is the difference between 2's payoff from getting both goods, $\hat{b}_{2AB}\left(v_1^{*}\right)$, and that from getting just B, $\hat{b}_{2B}\left(v_1^{*}\right)$, i.e., (29). In case (b) it becomes efficient at the first switching point $\hat{v}_{1A}^{*}$ (choose $\hat{v}_{1B}^{*}$ and $\hat{v}_{1AB}^{*}$ to conform with $\hat{v}_{1A}^{*}$) to allocate A to buyer 2 but B to buyer 1. Hence, at $\hat{v}_{1A}^{*}$ buyer 1's marginal impact on buyer 2 from being allocated A rather than B is the difference between buyer 2's payoff from A and that from B:

$$\hat{b}_{2A}\left(\hat{v}_1^{*}\right) - \hat{b}_{2B}\left(\hat{v}_1^{*}\right). \tag{31}$$

But (31) does not represent buyer 1's full marginal impact on buyer 2 because it compares buyer 2's payoff from B with that from good A, rather than from both A and B. To obtain the latter comparison, reduce v_{1B} from $\hat{v}_{1B}^{*}$ to the point v_{1B}^{**} where it just becomes efficient to allocate both A and B to buyer 2. The marginal impact on buyer 2 at v_{1B}^{**} (choose v_{1A}^{**} and v_{1AB}^{**} to conform with v_{1B}^{**}) is

$$\hat{b}_{2AB}\left(v_1^{**}\right) - \hat{b}_{2A}\left(v_1^{**}\right). \tag{32}$$

Adding (31) and (32), we obtain buyer 1's full marginal impact on buyer 2, viz., (30). Notice that in the case of private values, where $\hat{b}_{2A}\left(v_1^{**}\right) = \hat{b}_{2A}\left(\hat{v}_1^{*}\right)$, (30) reduces to $\hat{b}_{2AB} - \hat{b}_{2B}$, which is buyer 1's payment for good A in the ordinary Groves-Clarke mechanism.

It can be shown (see Dasgupta and Maskin (2000)) that it is an equilibrium for buyers to bid truthfully in the above auction, i.e., for each i and bundle of goods $H = A, B, AB,$ buyer i should set $\hat{b}_{iH}(\cdot) = b_{iH}(\cdot),$ where

$$b_{iH}\left(v_{jH}(s_i, s'_j)\right) = v_{iH}(s_i, s'_j) \text{ for all } s'_j$$

if buyer i's signal value is s_i. Notice that if, in fact, buyers are truthful, the auction results in an efficient equilibrium.

7. Multidimensional Signals

Up until now, the results I have quoted on efficient auctions with common values have assumed that buyers' signals are one-dimensional. This is for good reason—the results are simply not true otherwise. Indeed, with multidimensional signals, efficiency in the sense I have defined it is generally unattainable with any mechanism (a point found in Maskin (1992) and Jehiel and Moldovanu (1998)). To see this, consider the following example:

Example 7: Suppose that there are two buyers and one good. Assume that buyer 2's signal s_2 is, as usual, one-dimensional but that buyer 1's signal s_1 has two components: $s_1=(s_{11},s_{12})$.

Let

$$v_1(s_{11}, s_{12}, s_2) = s_{11} + s_{12} + a s_2$$

and

$$v_2(s_{11}, s_{12}, s_2) = s_2 + b s_{11} + g s_{12}\ .$$

Because of independence, buyer 1's objective function is the same for any pairs $(s_{11,}\ s_{12})$ that add up to the same constant, and thus, he will behave the same way for any such pairs. In particular, if (s'_{11}, s'_{12}) and (s''_{11}, s''_{12}) are pairs such that $s'_{11} + s'_{12} = s''_{11} + s''_{12}$, then, in any auction, the equilibrium outcome must be identical for the two pairs. But, unless $b=g$, the efficient allocation may turn on which pair obtains—specifically, given s_2, we might have

$$s'_{11} + s'_{12} + a s_2 > s_2 + b s'_{11} + g s'_{12} \tag{33}$$

but

$$s''_1 + s''_2 + a s_2 < s_2 + b s''_{11} + g s''_{12}\ , \tag{34}$$

so that, with $(s_{11}, s_{12}) = (s'_{11}, s'_{12})$, the good should be allocated to buyer 1 and, with $(s_{11}, s_{12}) = (s''_{11}, s''_{12})$, it should be allocated to buyer 2 (if $b=g$, this conflict does not arise; the inequality signs in (33) and (34) must be the same). Hence, an efficient auction is impossible when $b \neq g$.

However, since buyer 1 cares only about the <u>sum</u> $s_{11} + s_{12}$, it is natural to define

$$r_1 = s_{11} + s_{12}$$

and set

$$w_1(r_1, s_2) = r_1 + a s_2$$

and

$$w_2(r_1, s_2) = E_{s_{11}, s_{12}}\left[s_2 + b s_{11} + g s_{12} \mid s_{11} + s_{12} = r_1\right].$$

Notice that we have reduced the two-dimensional signal s_1 to the one-dimensional signal r_1. Furthermore, provided that a, b, and g are all less than 1 (so that condition (5) holds), our modified Vickrey auction <u>is</u> efficient with respect to the "reduced" valuation functions $w_1(\cdot)$ and $w_2(\cdot)$ (because all the analysis of Section 4 applies). Hence, a moment's reflection should convince the reader that, although full efficiency is impossible for the valuation functions $v_1(\cdot)$ and $v_2(\cdot)$, the modified Vickrey auction is <u>constrained</u> efficient, where "constrained" refers to the requirement that buyer 1 must behave the same way for any pair (s_{11}, s_{12}) summing to the same r_1 (in the terminology of Holmstrom and Myerson (1983), the auction is "incentive efficient").

Unfortunately, as Jehiel and Moldovanu (1998) show in their important paper, this trick of reducing a multidimensional signal to one dimension no longer works in general if there are multiple goods. To see the problem suppose that, as in Section 5, there are two goods *A* and *B*, but that now a buyer *i* (i=1,2,3) receives <u>two</u> signals—one for each good. Specifically, let s_{1A} and s_{1B} be buyer *i's* signals for *A* and *B*, respectively, and let his valuation functions be

$$v_{iA}(s_{1A}, s_{2A}, s_{3A}) \text{ and } v_{iB}(s_{1B}, s_{2B}, s_{3B}).$$

Let us first fix the signal values of buyers 2 and 3 at levels such that, as we vary s_{1A} and s_{1B}, either (i) it is efficient to allocate good *A* to buyer 1 and *B* to 2, or (ii) it is efficient to allocate good *A* to 2 and *B* to 3. In case (i), we have

$$v_{1A}(s_{1A}, s_{2A}, s_{3A}) + v_{2B}(s_{1B}, s_{2B}, s_{3B}) > v_{2A}(s_{1A}, s_{2A}, s_{3A}) + v_{3B}(s_{1B}, s_{2B}, s_{3B}),$$

that is,

$$v_{1A}(s_{1A}, s_{2A}, s_{3A}) > v_{2A}(s_{1A}, s_{2A}, s_{3A}) + v_{3B}(s_{1B}, s_{2B}, s_{3B}) - v_{2B}(s_{1B}, s_{2B}, s_{3B}) \quad (35)$$

whereas in case (ii) we have

$$v_{1A}(s_{1A}, s_{2A}, s_{3A}) < v_{2A}(s_{1A}, s_{2A}, s_{3A}) + v_{3B}(s_{1B}, s_{2B}, s_{3B}) - v_{2B}(s_{1B}, s_{2B}, s_{3B}). \quad (36)$$

Notice that buyer 1's objective function does not depend on s_{1B} (s_{1B} affects only buyer 1's valuation for good *B*, but buyer 1 is not allocated *B* in either case (i) or (ii)). Hence, the equilibrium outcome of any auction cannot turn on the value of this parameter. But this means that, if an auction is efficient, which of case (i) or (ii), (i.e., which of (35) or (36)) holds cannot depend on s_{1B}. We conclude, from the right-hand sides of (35) and (36), that

$$v_{3B}(s_{1B}, s_{2B}, s_{3B}) - v_{2B}(s_{1B}, s_{2B}, s_{3B})$$

must be independent of s_{1B}. Expressed differently, we have

$$\frac{\partial}{\partial s_{1B}} v_{3B}(s_{1B}, s_{2B}, s_{3B}) = \frac{\partial}{\partial s_{1B}} v_{2B}(s_{1B}, s_{2B}, s_{3B}).$$

Repeating the argument for all other pairs of buyers and for good B, we have

$$\frac{\partial v_{jH}}{\partial s_{iH}} = \frac{\partial v_{kH}}{\partial s_{iH}}, \text{ for all } j \neq i \neq k \text{ and } H = A, B. \tag{37}$$

Next, let us fix the signal values of buyers 2 and 3 at levels such that, as we vary s_{1A} and s_{1B}, either (iii) it is efficient to allocate A to buyer 1 and B to 2; or (iv) it is efficient to allocate B to buyer 1 and A to 2. In case (iii), we have

$$\begin{aligned} &v_{1A}(s_{1A}, s_{2A}, s_{3A}) + v_{2B}(s_{1B}, s_{2B}, s_{3B}) > \\ &\qquad v_{1B}(s_{1B}, s_{2B}, s_{3B}) + v_{2A}(s_{1A}, s_{2A}, s_{3A}), \end{aligned} \tag{38}$$

and in case (iv),

$$\begin{aligned} &v_{1A}(s_{1A}, s_{2A}, s_{3A}) + v_{2B}(s_{1B}, s_{2B}, s_{3B}) < \\ &\qquad v_{1B}(s_{1B}, s_{2B}, s_{3B}) + v_{2A}(s_{1A}, s_{2A}, s_{3A}). \end{aligned} \tag{39}$$

To simplify matters, let us assume that valuation functions are linear:

$$v_{1A}(s_{1A}, s_{2A}, s_{3A}) = s_{1A} + a_{12}s_{2A} + a_{13}s_{3A} \tag{40}$$

$$v_{1B}(s_{1B}, s_{2B}, s_{3B}) = s_{1B} + b_{12}s_{2B} + b_{13}s_{3B} \tag{41}$$

and similarly for buyers 2 and 3. Then (38) and (39) can be rewritten as

$$s_{1A} - s_{1B} > a_{21}s_{1A} + a_{22}s_{2A} + a_{23}s_{3A} - b_{21}s_{1B} - b_{22}s_{2B} - b_{23}s_{3B} \tag{42}$$

and

$$s_{1A} - s_{1B} < a_{21}s_{1A} + a_{22}s_{2A} + a_{23}s_{3A} - b_{21}s_{1B} - b_{22}s_{2B} - b_{23}s_{3B}. \tag{43}$$

Now (because we have fixed 2's and 3's signal values), buyer 1's objective function depends only on s_{1A}–s_{1B}. That is, for any value of), buyer 1 will behave the same way for signal values (s_{1A}, s_{1B}) as for $(s_{1A} + \Delta\ , s_{1B} + \Delta\ )$. Hence, in any auction, the equilibrium outcome must be the same for any value of Δ. In particular, if the auction is efficient, whether (42) or (43) applies cannot depend on Δ's value. But from the right-hand sides of (42) and (43), this can be the case only if a_{21}=b_{21}, i.e., only if

$$\frac{\partial v_{2A}}{\partial s_{1A}} = \frac{\partial v_{2B}}{\partial s_{1B}}.$$

Repeating the argument for the other buyers, we have

$$\frac{\partial v_{jA}}{\partial s_{iA}} = \frac{\partial v_{jB}}{\partial s_{iB}} \text{ for all } i \text{ and } j \neq i . \tag{44}$$

The necessary conditions (37) and (44), due to Jehiel and Moldovanu (1998), are certainly restrictive. Nevertheless, as shown in Eso and Maskin (2000a), there is a natural class of cases in which they are automatically satisfied. Specifically, suppose that in our two-good model, each buyer wants at most one good (this is not essential). Assume that the true value of good A to buyer i, y_{iA}, is the sum of a component z_A common to all buyers and a component of z_{iA} that is idiosyncratic to him. That is,

$$y_{iA} = z_A + z_{iA} \ .$$

Similarly, assume that buyer i's true valuation of good B, y_{iB}, satisfies

$$y_{iB} = z_B + z_{iB} .$$

Suppose, however, that buyer i does not directly observe his true valuations but only noisy signals of them. That is, he observes s_{iA} and s_{iB}, where

$$s_{iA} = y_{iA} + e_{iA}$$

and

$$s_{iB} = y_{iB} + e_{iB} .$$

It can be shown (see Eso and Maskin (2000a)) that if the random variables z_H, z_{iH}, g_{iH}, $i=1,2,3$, $H=A,B$, are independent, normal random variables and if the variances of g_{iH} and z_{iH} are proportional to that of z_H, i.e., for all i, there exists k_{ig} and k_{iz} such that

$$\text{var}\, e_{iH} = k_{ie} \text{var}\, z_H \text{ and } \text{var}\, z_{iH} = k_{iz} \text{var}\, z_H \ , \ H = A, B ,$$

then (37) and (44) are automatically satisfied and the modified Groves-Clarke mechanism discussed in Section 6 is an efficient auction.

8. Further Work

There is clearly a great deal of work remaining to be done on efficient auctions, including dealing with the multiple good/multidimensional problem in cases where (37) and (44) do not hold. I would like to simply underscore one issue: finding an open auction counterpart to the modified Groves-Clarke mechanism in the case of multiple goods. The task of submitting contingent bids is considerable even for a single good. For multiple goods, it could be formidable. For this reason, as I have already discussed, researchers have sought <u>open</u> auctions –variants of the English auction—as desirable alternatives. Perry and Reny (1999b) have exhibited a lovely modification of the Ausubel (1997) auction (which in turn elegantly extends the English auction to multiple identical goods). However, efficiency in that auction obtains only when all goods are identical and buyers' marginal valuations are declining. It would be an important step, in my judgment, to find a similar result without such restrictions on goods or preferences.

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Appendix: Buyer 1's payment when allocated both goods in a two-good, two-buyer auction.

If

(a) there exists v_1^* such that

$$v_{1AB}^* = \hat{b}_{2AB}\left(v_1^*\right),$$

then buyer 1 pays

$$\hat{b}_{2AB}\left(v_1^*\right);$$

if (a) does not hold and instead

(b) there exists $\hat{v}_1^*$ such that

$$\hat{v}_{1AB}^* = \hat{v}_{1A}^* + \hat{b}_{2B}\left(\hat{v}_1^*\right),$$

then if,

(b1) there exists v_1^{**} such that

$$v_{1A}^{**} + \hat{b}_{2B}\left(v_1^{**}\right) = \hat{b}_{2AB}\left(v_1^{**}\right),$$

buyer 1 pays

$$\hat{b}_{2B}\left(\hat{v}_1^*\right) + \left(\hat{b}_{2AB}\left(v_1^{**}\right) - \hat{b}_{2B}\left(v_1^{**}\right)\right);$$

and if instead

(b2) there exist $\hat{v}_1^{**}$ and $\hat{v}_1^{***}$ such that

$$\hat{v}_{1A}^{**} + \hat{b}_{2B}\left(\hat{v}_1^{**}\right) = \hat{v}_{1B}^{**} + \hat{b}_{2A}\left(\hat{v}_1^{**}\right)$$

and

$$v_{1B}^{***} + \hat{b}_{2A}\left(v_1^{***}\right) = \hat{b}_{2AB}\left(v_1^{***}\right),$$

then buyer 1 pays

$$\hat{b}_{2B}\left(\hat{v}_1^*\right) + \left(\hat{b}_{2A}\left(\hat{v}_1^{**}\right) - \hat{b}_{2B}\left(\hat{v}_1^{**}\right)\right) + \left(\hat{b}_{2AB}\left(v_1^{***}\right) - \hat{b}_{2A}\left(v_1^{***}\right)\right);$$

finally, if

(c) there exists $\hat{\hat{v}}_1^*$ such that

$$\hat{\hat{v}}_{1AB}^* = \hat{\hat{v}}_{1B}^* + \hat{b}_{2A}\left(\hat{\hat{v}}_1^*\right)$$

then if

(c1) there exists v_1^{**} such that

$$v_{1B}^{**} + \hat{b}_{2A}\left(v_1^{**}\right) = \hat{b}_{2AB}\left(v_1^{**}\right),$$

buyer 1 pays

$$\hat{b}_{2A}\left(\hat{\hat{v}}_1^{**}\right) + \left(\hat{b}_{2AB}\left(v_1^{**}\right) - \hat{b}_{2A}\left(v_1^{**}\right)\right);$$

and if instead

(c2) there exist $\hat{\hat{v}}_1^{**}$ and $\hat{v}_1^{***}$ such that

$$\hat{\hat{v}}_{1B}^{**} + \hat{b}_{2A}\left(\hat{\hat{v}}_1^{**}\right) = \hat{\hat{v}}_{1A}^{**} + \hat{b}_{2B}\left(\hat{\hat{v}}_1^{**}\right)$$

and

$$v_{1A}^{***} + \hat{b}_{2B}\left(v_1^{***}\right) = \hat{b}_{2AB}\left(v_1^{***}\right),$$

then buyer 1 pays

$$\hat{b}_{2A}\left(\hat{\hat{v}}_1^{*}\right) + \left(\hat{b}_{2B}\left(\hat{\hat{v}}_1^{**}\right) - \hat{b}_{2A}\left(\hat{\hat{v}}_1^{**}\right)\right) + \left(\hat{b}_{2AB}\left(v_1^{***}\right) - \hat{b}_{2B}\left(v_1^{***}\right)\right).$$