VERTICAL FORECLOSURE AND SPECIFIC INVESTMENTS

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Abstract: Are vertical mergers efficient or restraints to trade? This paper examines this long-standing question in a new setting and reaches new conclusions. We consider a realistic environment where downstream firms can make specific investments in several suppliers at once. In keeping with the "Chicago School" of regulation, we assume inputs are exchanged efficiently regardless of the ownership structure. Nevertheless, we find that vertical merger can be inefficient. A merged firm has an incentive to manipulate its ex ante investments to increase the ex post revenues of its supply unit. It will increase its investment in its internal supplier and decrease its investment in an external supplier relative to the efficient level of investments. The "skewing" is reinforced in equilibrium by other buyers who respond by skewing their own investments. The result is a reduction in the variety of inputs purchased by downstream firms. We relate the theory to studies of vertical mergers in pharmaceuticals and cable television.

I. Introduction

Economists, lawyers, and judges have debated for decades whether vertical mergers are efficient or restraints to trade. In this paper, we examine this question in a new setting and reach new conclusions. We consider vertical merger when downstream firms can invest in assets specific to suppliers. Our environment, with multiple firms and potentially multiple specific investments, captures a reality. Firms invest, often strategically, in vertical supply relations.[1] We find that, even when downstream firms compete for inputs and inputs are allocated efficiently, vertical merger can lead to inefficient investments. These conclusions counter the basic argument against regulation of vertical mergers.

The debate over vertical merger regulation concerns whether merger is inefficient and harms other firms and consumers. One side of the debate, which we will call the "Chicago school," maintains that vertical mergers are neutral. They cannot lead to harmful vertical foreclosure, where a merged firm reduces input supply and raises prices to non-merged downstream firms.[2] In particular, Bork (1978) argues that a vertical merger does not change how a firm decides whether to make or buy inputs nor a firm's incentives to sell inputs to other firms.[3] If there are any efficiency gains from merger, the purchasing unit will optimally favor its internal supply source (p. 207). On the other side of the debate, a series of papers finds that vertical merger is not neutral when downstream firms compete in the output market. Merger does indeed change incentives to sell inputs to rival manufacturers [Salinger (1988), Ordover, Saloner, Salop (1990), Hart and Tirole (1990)]. When, for example, firms are Cournot competitors in the final goods market, a unit of supply translates into an additional unit of final output capacity, and can harm the profits of a merged manufacturer-supplier.[4]

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[1] See, for example, Cawthorne (1995) and Uzzi (1996) for the textile industry, Nichiguchi (1994) for automobiles, and Saxenian (1994) for software. Spencer and Qiu (2001) analyze relationship specific investments within Japanese keiretsu.

[2] Another issue in the debate concerns the impact on the final goods market. Consumers may also suffer from reduced supply and increased prices.

[3] In his discussion of *Ford Motor Co. v. United States*, in which the Supreme Court prevented Ford from purchasing a spark plug supplier, Bork (pg. 236) writes that "The structure of an industry supplying the automotive industry will be whatever is most efficient for the automotive industry. There can be nothing wrong in the automobile manufacturers acquiring all of their suppliers. The decision to make onself or to buy from others is always made on the basis of difference in cost and effectiveness, criteria the law should permit the manufacturer to apply without interference."

[4] A large literature now explains reasons why competition in the output market could change a merged firm's incentives to supply to a rival. See, for example, Ma (1997) and Church & Gandal (2000) and Choi and Yi (2000).

We present here an argument that vertical merger is harmful based not on the final goods market, but on the input market itself. We consider a Chicago school world where the market for inputs is competitive, the allocation of inputs is efficient, and a merged supplier sells to outside buyers whenever their realized values of an input are higher than the value to the internal buyer. In our analysis, we add the feature that firms can make specific investments that affect the value of inputs from different suppliers.

We find that vertical merger can be inefficient because it affects firms' incentives to invest in specific assets. A buyer that merges with a supplier will increase its investment in its internal supplier and decrease its investments in an external supplier, relative to the efficient levels of investments.[5] This "skewing" of investments increases the merged firm's expected value of internal supply relative to external supply. This enhanced value in turn increases the expected revenues the merged firm will earn from outside buyers. Since the supply unit receives higher expected revenues, the skewing of investments increases the merged firm's profits. A merged firm will, ex post, purchase more from its internal supply unit. But the initial investments, which changed the relative value of suppliers' inputs, are not socially optimal.

These inefficiencies arise in equilibrium. In a general model with two upstream suppliers and two downstream buyers, we find that merger between buyer-supplier pairs can be (but need not be) an equilibrium outcome. A merged buyer inefficiently skews its investments away from the external supplier and towards its internal supplier. The other buyer also inefficiently skews its investments, whether or not this buyer is itself merged. Merger is always inefficient, and two mergers lead to more skewed investments than one merger. The inefficiency causes a reduction in the variety of inputs that buyers use to produce final goods. In a model with more than two buyers and sellers, we find that merger is always inefficient although the nature of the distortion on external investments is ambiguous. Hence, in contrast to the Chicago school, mergers can reduce welfare.

This analysis combines themes of three literatures. The literature on vertical foreclosure, cited above, considers how merger affects the supply of inputs and input prices. A second literature, on incomplete contracting, considers how ownership affects incentives to invest in specific assets (e.g.,

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[5]In a model with one buyer and two suppliers, Segal and Whinston (2000) examine how an exclusive dealing contract affects investment incentives. Their problem is different, but they distinguish between external and internal investments in a related way.

Grossman and Hart (1986), Hart and Moore (1990), Bolton and Whinston (1993)). This literature highlights how specific assets affects firms' ex post bargaining positions. A third literature, on networks, considers links or relationships among multiple agents [e.g., Jackson and Wolinsky (1996), Bala and Goyal (2000), Kranton and Minehart (2001)]. Ingredients from each literature appear in our modeling. We consider how vertical merger affects input prices and sales when downstream firms invest in specific assets to potentially multiple suppliers.

In our list of references, Bolton and Whinston's (1993) analysis demands particular attention. They consider vertical merger in a model with two downstream firms and one upstream firm, where a downstream firm can make a specific investment in the upstream firm. They find, as we do, that a merged firm overinvests in the asset specific to the supplier. In their analysis of two downstream and two upstream firms, they restrict a downstream firm to invest in a single supplier. Vertical mergers lead to efficient investments, since merger eliminates a hold-up problem for each pair of firms. In contrast, we consider downstream firms that can invest in multiple suppliers and find that any merger is inefficient.

Our analysis identifies *supply-stealing* and *supply-freeing effects* of multiple specific investments and studies the implication for merger incentives. A downstream firm that increases its investment in a supplier *steals supply* from other buyers. The expected price of this supplier will rise. A downstream firm that decreases an investment in a supplier *frees supply* for other buyers. The expected price of this seller will fall. A merged firm takes advantage of the supply stealing effects: it increases its investment in its internal seller and decreases investment in external sellers. It therefore raises the demand for its supply unit's capacity and raises the expected revenues earned by the supply unit. Other buyers are hurt by the merger and change in investments.

We consider the equilibrium properties of firms' investments. Because of the supply freeing and stealing effects, buyers' investments in the same supplier are strategic substitutes and investments in different suppliers are strategic complements. There are therefore potentially many equilibria in an investment game for a given ownership structure. Nevertheless, we are able to compare equilibria across ownership structures using the theory of monotone comparative statics of Milgrom and Roberts [1990]. We show in our basic model that each merger leads to further skewing of investments away from external suppliers.

Our results yield insights into vertical foreclosure and the effect of ownership on investments.

In our model, ownership affects investments not because of hold-up, but because investments change the value and allocation of inputs and input prices. Indeed, in contrast to the literature on incomplete contracting, merger leads to inefficient investments. A merged firm over-invests in some specific assets and under-invests in others. Relative to the vertical foreclosure literature, we find new incentives for merger. Firms merge in order to benefit from changes in specific investments. The result raises questions about the traditional "supply assurance" incentive for merger. In our model, the merged firm reduces its access to outside supply. As for the regulatory debate, we find that even in a Chicago world, where inputs are allocated efficiently and manufacturers do not compete in the output market, vertical merger can lead to inefficient outcomes.

The analysis may offer a new perspective on recent merger activity. In the early 1990s, for example, there was a wave of mergers between pharmacy benefit management companies (PBM's) and pharmaceutical suppliers.[6] Our results suggest these mergers may have affected incentives for specific investments which change the relative value of drugs from different suppliers. PBM's, such as Merck-Medco, operate pharmaceutical benefit plans for employers and health management organizations. Like the downstream firms in our model, they do not compete for consumers in the short-run. Their client base is fixed; a consumer can obtain her pharmacy benefits only from the PBM that has contracted with her employer. PBM's construct formularies (lists of drugs included in the benefit plan) as well as health management protocols which advise the use of certain combinations of drugs and therapies to battle certain diseases. Throughout the 1990's regulatory authorities were concerned that merged PBM's would favor their parent companies' drugs. The Federal Trade Commision ordered Merck-Medco in 1998, for example, "to take steps to diminish the effects of unwarranted preference that might be given to Merck's drugs over those of Merck's competitors."[7] The FTC's order concerns the ex post distribution and pricing of drugs. It ignores the specific investments that could change a PBM's preference for a supplier's drugs. In particular, specific investments to learn the properties of a parent company's drug can make

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[6] Merck purchased Medco (a PBM) in November 1993 for $6.6 billion. Subsequently, SmithKline purchased Diversified Pharmaceutical Services (DPS) in May 1994 for $2.3 billion and Eli Lilly purchased PCS Health Sytems in November 1994 for $4.1 billion. In 1994 other pharmaceutical companies formed strategic alliances with different PBM's. See Rangan and Bell (1998). To date, SmithKline and Eli Lilly have both sold their PBM units. For detailed discussions of mergers between pharmaceutical suppliers and pharmacy benefit managment companies see, for example, Ragan and Bell (1998), Levy (1999, Chapters II and V), Food and Drug Administration Report (2001, Attachment G).

[7] Federal Trade Commission (1998, p.1). A similar consent agreement preceded the Eli Lilly Merger-PCS merger in 1995 (Federal Trade Commission, 1995).

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that drug more valuable independently or as part of health management protocol.[8] Ex post, the PBM would then *optimally* feature the parent company's drugs. As we show here, however, the investments themselves could be sub-optimal. In the conclusion, we discuss further vertical mergers in pharmaceuticals and other industries.

The paper proceeds as follows. In the next section we present a basic model with two downstream firms and two suppliers that shows our main results: a merged firm over-invests in assets specific to its internal supplier and under-invests in assets specific to external suppliers. Firms have an incentive to merge to benefit from manipulating these specific investments. Each additional merger reduces welfare, and merger occurs in equilibrium. In Section III we extend our results to a general model of two downstream firms and two suppliers; Section IV considers greater numbers of firms and Section V discusses the robustness of our results to different specifications of the model. We conclude in Section VI.

II. An Example for Two Buyers and Two Sellers

There are two downstream units, which we will call buyers, and two upstream units, which we will call sellers. Each seller has the capacity to produce one indivisible unit of an input at zero marginal cost, and each buyer demands one indivisible unit of an input. Inputs are made-to-order, and buyers compete to purchase inputs. We suppose that buyers can have different and random valuations for inputs from different sellers. This uncertainty reflects, for example, final consumers' demand for variety. Buyers may sell several goods in the final good markets, or several varieties of the same good (e.g. different drugs or different models of cars) and have demand for different inputs depending on the ultimate realization of consumer demand.

We assume that specific investments are made before the uncertainty over buyers' valuations is resolved. That is, buyers make long-run investments in anticipation of future, short-run demand for inputs. This assumption captures many industries where investments are of longer duration than the day-to-day or month-to-month fluctuations in consumer demand.[9]

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[8] Medco took on several Merck managers with clinical expertise [Ragan and Bell (1998), pg. 17], and Merck-Medco invested $120 million on information technology to develop health management protocols.

[9] Examples include pharmaceuticals and cable television which we discuss in the conclusion, as well as the garment industry (Uzzi, 1996), electronics and engineering (Nishiguchi, 1994, and Lorenz, 1989), and toys ("The Puppet-master of Toytown," p.88, *Economist*, September 6, 1997).

A. A Merger Game

We consider a three-stage non-cooperative game. In the first stage, firms make merger decisions. In the second stage, buyers invest in assets specific to sellers.[10] The investments determine the distributions of buyers' valuations for sellers' inputs. In the final stage, buyers learn their valuations and production and exchange takes place. This set-up assumes firms can not use long-term contingent contracts to assign investments, future prices, or allocations of goods. The model thus embodies the standard Grossman and Hart (1986) and Hart and Moore (1990) incomplete contracts framework: agents must make investments before uncertainty is resolved, and contingent contracts are not possible.

We consider vertical mergers of single buyers and sellers. In keeping with Grossman and Hart (1986), Hart and Moore (1990), we assume that ownership does not directly affect the production technology. That is, a merged seller can produce an input for its merged buyer or another buyer. Ownership will affect investment incentives, however, and we will contrast the investments of merged buyers and independent buyers. We assume that a merged buyer has claim to both its revenues and its merged seller's revenues. Therefore, when a buyer i is merged with seller j, buyer i chooses investments to maximize the joint profits of buyer i and seller j. When a buyer i is not merged, it maximizes only its own profits.

Stage One: Firms simultaneously decide whether or not to merge vertically. At the end of this stage, the ownership structure is common knowledge. We will consider, without loss of generality, mergers between buyer 1 and seller 1, and mergers between buyer 2 and seller 2. We assume a pair will merge if and only if the merged firm's profits exceeds the sum of their individual un-merged profits, given the merger decision of the other pair.[11] Three possible ownership structures can emerge. In *no merger*, no firms are merged. In *partial merger*, one pair is merged. In *full merger*, both pairs are merged. For partial merger, without loss of generality, we will consider that buyer 1 and seller 1 are merged.

Stage Two: Given the ownership structure, buyers simultaneously choose specific investments to sellers. At the end of this stage, all investments are common knowledge. Let $0 \leq g_{ij} \leq 1$ denote buyer i's investment in seller j. With this notation we capture a buyer's investment in

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[10] Later, we discuss the alternative possibility that sellers invest in assets specific to buyers.

[11] The results, therefore, do not depend on bargaining and how gains from merger are split between the buyer and seller.

any seller, be it a merged seller or an external seller. The cost to buyer i of this investment is $(g_{ij})^2$. The lines in Figure 1 below depict the specific investments between buyers and sellers.
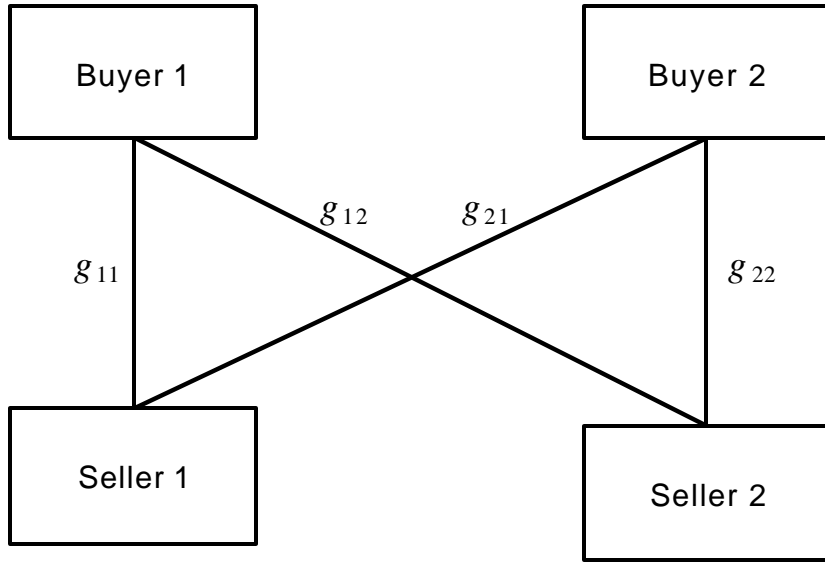

Figure 1. Specific Investments between Downstream and Upstream Firms

Each specific investment determines the probability that a buyer will have positive value for an input from that seller in stage 3. Let v_{ij} be buyer i's value of purchasing an input from seller j. Let g_{ij} be the probability that $v_{ij} = \frac{1}{2}$, and $(1 - g_{ij})$ the probability that $v_{ij} = 0$.

Stage Three: Buyers' valuations of goods are realized, and production and exchange takes place. We assume that allocations of inputs are efficient: buyer 1(2) purchases from seller 1(2) whenever $v_{11} + v_{22} \geq v_{12} + v_{12}$. Otherwise buyer 1(2) purchases from seller 2(1). The price of exchange for seller i is denoted p_i. We do not posit an extensive form game of competition for inputs. Instead, we consider core outcomes, and in particular, the lowest competitive prices that sustain efficient allocations.[12] Seller i receives a price of $p_i = 0$ unless there is excess demand for its capacity. Excess demand occurs only when both buyers have value $\frac{1}{2}$ for an input from seller i and value 0 for an input from seller j. In this case, seller i receives a price of $p_i = \frac{1}{2}$. When a merged buyer purchases the internal input, the price is simply a transfer price. The transfer

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[12]Buyers' valuations need not be common knowledge. When buyers' valuations are private information, the prices we use here would arise from an ascending bid auction for this environment [see Demange, Gale and Sotomayor (1986), Gul and Stacchetti (2000), Kranton and Minehart (2001)]. For the theory of core, i.e., competitive, outcomes in a pairwise setting, see Shapley and Shubik (1972), and Roth and Sotomayor (1990). If prices are competitive, and if buyer 1 obtains an input from seller 2, then there is no higher price that buyer 2 would want to offer seller 2 to induce it to sell it the input instead. Below we formally define competitive prices.

price insures that the input is allocated efficiently. A merged firm will sell the input to the other buyer only if it cannot do better by using the input itself.

We solve for perfect pure strategy Bayesian equilibria of this game. We analyze the game backwards. We consider, first, second-stage equilibrium investments for different ownership structures. We then ask whether firms will merge.

B. Investments for Given Ownership Structures

In the second stage, buyers choose investments to maximize profits given the ownership structure. Profits are third-stage expected revenues minus second-stage investment costs. Let Π_i^b denote buyer i's profits and Π_j^s denote seller j's expected profits. For buyer 1 and seller 1 we have

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\begin{aligned}
\Pi_1^b &= \frac{1}{2}\left[g_{11}g_{12} + g_{11}(1-g_{12})\left(1-g_{21}(1-g_{22})\right) + (1-g_{11})g_{12}\left(1-g_{22}(1-g_{21})\right)\right] - g_{11}^2 - g_{12}^2 \\
\Pi_1^s &= \frac{1}{2}\left[g_{11}\left(1-g_{12}\right)g_{21}\left(1-g_{22}\right)\right]
\end{aligned}
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To derive these payoffs, we consider all possible realizations of buyers' valuations. With probability $g_{11}g_{12}$, buyer 1 values an input either from seller 1 or 2. No seller has excess demand, so the sellers' prices are zero, and buyer 1 receives a payoff of $\frac{1}{2}$. With probability $g_{11}(1-g_{12})$, buyer 1 only values an input from seller 1. With probability $g_{11}(1-g_{12})g_{21}(1-g_{22})$, buyer 2 also only values an input from seller 1, so seller 1 faces excess demand. Seller 1's price is $\frac{1}{2}$, and buyer 1 receives a zero payoff. With probability $g_{11}(1-g_{12})\left(1-g_{21}(1-g_{22})\right)$, seller 1 does not face excess demand, so seller 1's price is 0 and buyer 1 receives a payoff of $\frac{1}{2}$. With probability $(1-g_{11})g_{12}$, buyer 1 only values an input from seller 2. In this case, seller 1 does not face excess demand and so has a price of 0. With probability $(1-g_{11})g_{12}g_{22}(1-g_{21})$, buyer 2 also only values only an input from seller 2, so seller 2 faces excess demand and buyer 1 receives a zero payoff. With probability $(1-g_{11})g_{12}\left(1-g_{22}(1-g_{21})\right)$, seller 2 does not face excess demand and buyer 1 receives a payoff of $\frac{1}{2}$. Buyer 2's and seller 2's profits can be determined similarly.

We consider second-stage equilibrium investments[13] for the three possible ownership structures: no merger, partial merger, and full merger.

No merger. When no firms are merged, buyer i chooses investments to maximize Π_i^b, taking

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[13] That is, we solve for continuation equilibria for each possible outcome of the first stage of the game. For simplicity, we refer to these continuation equilibria as "equilibria."

as given the investments of buyer j. The first order conditions for the investments form a system of four equations. This system yields a unique equilibrium[14] where

$$g_{11} = 0.1811 \qquad g_{12} = 0.1811$$

$$g_{21} = 0.1811 \qquad g_{22} = 0.1811$$

The buyers' equilibrium profits are $\Pi_1^b = \Pi_2^b = .0771$, and the sellers' profits are $\Pi_1^s = \Pi_2^s = .0110$.

Partial merger. When buyer 1 and seller 1 are merged, buyer 1 chooses investments to maximize $\Pi_1^b + \Pi_1^s$. Buyer 2 is not merged and so chooses investments to maximize Π_2^b. As above, there is a unique equilibrium. Here

$$g_{11} = 0.2150 \qquad g_{12} = 0.1658$$

$$g_{21} = 0.1728 \qquad g_{22} = 0.1876$$

The merged firm profits are $\Pi_1^b + \Pi_1^s = 0.0888$, buyer 2's profits are $\Pi_2^b = .0763$, and seller 2's profits are $\Pi_2^s = .0101$.

We see here one of the main insights of our analysis. Compared to no merger, buyer 1 skews its investments to favor its internal seller. Seller 1 earns a positive price only when it faces excess demand. By increasing g_{11} and decreasing g_{12}, the buyer raises the probability that it will have value only for an input from seller 1, and hence that seller 1 will face excess demand. We can see this effect in seller 1's payoffs, which are increasing in g_{11} and decreasing in g_{21} :

$$\frac{\partial \Pi_1^s}{\partial g_{11}} = \frac{1}{2}\left[(1 - g_{12}) \, g_{21} \, (1 - g_{22})\right]$$

which is strictly positive. Similarly, $\frac{\partial \Pi_1^s}{\partial g_{12}} < 0$. Buyer 1 will take advantage of this opportunity to increase the demand for seller 1 and thereby increase their joint profits.

We also see the supply stealing and supply freeing effects and complementarities discussed

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[14] For each ownership structure, we solve the system of first order equations numerically using Maple. We use analytical methods to determine that the given solution is an equilibrium (the best reply functions are strictly concave). Details are available on request. To confirm the completeness of Maple's numerical findings, we conferred with the algebraic geometer John Little. He employed analytical methods (Cox, Little, and O'Shea (1991)) to determine that the number of solutions to the system of first-order equations agreed with Maple's output. We also ruled out any corner solutions.

in the introduction. By increasing g_{11} and decreasing g_{12} - that is, by skewing its investments - buyer 1 steals the supply of seller 1 and frees the supply of seller 2 from buyer 2. These effects change buyer 2's investment incentives. Because of the supply stealing effect, buyer 2 finds that investing in seller 1 is less profitable. Consider buyer 2's marginal return to its investment in seller 1, $\frac{\partial \Pi_2^b}{\partial g_{21}}$:

$$\frac{\partial \Pi_2^b}{\partial g_{21}} = \frac{1}{2} \left[g_{22} - g_{22} \left(1 - g_{12}(1 - g_{11}) \right) + \left(1 - g_{22} \right) \left(1 - g_{11}(1 - g_{12}) \right) \right] - g_{21}$$

We see that $\frac{\partial \Pi_2^b}{\partial g_{21}}$ is decreasing in g_{11} and increasing in g_{12}. Similarly, because of the supply freeing effect, buyer 2 finds that investing in seller 2 is more profitable; that is, the marginal return to investing in seller 2, $\frac{\partial \Pi_2^b}{\partial g_{22}}$, is increasing in g_{11} and decreasing in g_{12}. Thus, buyer 2 will skew its investments in favor of seller 2, even though the two firms are not merged.

Full merger. When both pairs of firms are merged, buyer 1 chooses investments to maximize $\Pi_1^b + \Pi_1^s$, and buyer 2 chooses investments to maximize $\Pi_2^b + \Pi_2^s$. As above, there is a unique equilibrium. Here

$$g_{11} = 0.2173 \qquad g_{12} = 0.1600$$
$$g_{21} = 0.1600 \qquad g_{22} = 0.2173$$

Each merged firm earns a profit of $\Pi_i^b + \Pi_i^s = 0.0870$.

Here again we see how merger affects investments. Relative to the other ownership structures, both buyers skew their investments to favor the internal seller. This skewing raises the probability that the internal seller earns a positive price. Holding the other firm's investments fixed, the merged firm can thus increase joint profits by skewing towards the internal seller. However, because of the supply stealing and freeing effects, the other merged buyer also skews its investments. Full merger involves greater skewing than any other ownership structure.

C. Equilibrium Ownership Structure

We now solve the first stage of the game to determine the equilibrium ownership structure. There is a unique equilibrium: full merger.

We first see that the no merger is not an equilibrium. Buyer 1 and seller 1 have higher joint

profits under partial merger.[15] This result would be obvious if buyer 2 chose the same investments whether or not buyer 1 and seller 1 were merged. Here, however, buyer 2 skews its investments towards seller 2 in response to the merger. The skewing reduces the merged firm's profit, but the change is not enough to deter buyer 1 and seller 1 from merging.

We next show that the partial merger is not an equilibrium. If buyer 1 and seller 1 are merged, then buyer 2 and seller 2 will also merge.[16] Despite the fact that buyer 1 skews its investments in response to the new merger, buyer 2 and seller 2 are not deterred from merging.

Finally, we conclude that the full merger ownership structure is an equilibrium, and therefore the unique equilibrium outcome. If buyer i and seller i are merged, buyer j and seller j will also merge. Buyer j's and seller j's joint profits are higher than in the partial merger case.

Here we see another insight of the analysis. The mutual skewing of investments hurts both merged firms $\frac{\partial\left[\Pi_1^b+\Pi_1^s\right]}{\partial g_{22}} < 0$ and $\frac{\partial\left[\Pi_1^b+\Pi_1^s\right]}{\partial g_{21}} > 0$. In fact, the joint profits are lower in the merger case than in the no merger case.[17] The game has the structure of a prisoners' dilemma. Both pairs of buyers and sellers earn a higher joint profits under no merger, but only full merger is an equilibrium.

D. Equilibrium Merger Reduces Welfare

Each merger reduces welfare through its effect on the investments. The welfare of the industry is the sum of the firms' profits: $\Pi_1^b + \Pi_1^s + \Pi_2^b + \Pi_2^s$. We find that, of the three ownership structures, no merger has the highest welfare and full merger has the lowest welfare.[18]

The equilibrium investments when no firms are merged are exactly the efficient investments.[19] This outcome is not a coincidence. When a buyer pays the lowest competitive price for inputs, the price is equal to the social opportunity costs of its obtaining a good. Therefore, a buyer's revenues always equal the welfare contribution of its investments.[20] Each buyer chooses investments to

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[15] Their joint profits, $\Pi_1^b + \Pi_1^s$, increase from .0881 to 0.0888.

[16] By merging, the firms increase their joint profits, $\Pi_2^b + \Pi_2^s$, from .0864 to 0.0870.

[17] The joint profits of buyer 1 and seller 1 in the no merger case are $\Pi_1^b + \Pi_1^s = 0.0881$. This is larger than their joint profits in full merger 0.0870.

[18] The welfare $\Pi_1^b + \Pi_1^s + \Pi_2^b + \Pi_2^s$ is 0.1762 with no merger, 0.1751 under partial merger, and 0.1740 under full merger.

[19] A straightforward numeric computation shows that $\Pi_1^b + \Pi_1^s + \Pi_2^b + \Pi_2^s$ is maximized over $(g_{11}, g_{12}, g_{21}.g_{22})$ at the no merger equilibrium.

[20] Formally, we say that the buyers' payoffs satisfy the Vickrey property. See our discussion in the proof of Proposition 3.

maximize welfare given the investments of the other buyer. To make this argument concrete, consider one possible realization of buyers' valuations: $v_{11} = \frac{1}{2}, v_{12} = \frac{1}{2}, v_{21} = \frac{1}{2}, v_{22} = 0$. In this event, in the efficient allocation, buyer 1 purchases from seller 2 and buyer 2 purchases from seller 1. The minimum competitive prices are $p_1 = 0$ and $p_2 = 0$, and each buyer earns payoffs of $\frac{1}{2}$. Total welfare is $\frac{1}{2} + \frac{1}{2} = 1$. Consider the change in welfare when buyer 1 is not present (i.e., makes no investments). Only buyer 2 would purchase an input, and welfare falls to $\frac{1}{2}$. The reduction in welfare is $\frac{1}{2}$, which is exactly equal to buyer 1's revenues in this event. When considering its investments, therefore, buyer 1's gains are exactly the gains in welfare.

When any firm is merged, investments are not efficient. A merged firm's incentives are not aligned with welfare because of the addition of the seller's revenues. The seller earns a positive price only when both buyers compete for its ouput; that is, when the seller faces excess demand. This competitive effect causes the merged firm to skew the investments inefficiently away from the external seller. As discussed above, the other buyer skews its investments in response, even if it is not merged. Thus, welfare falls whenever a firm merges. Equilibrium merger nevertheless occurs, because the merging firm earns a higher profit.

Hence we see that, contrary to Bork (1978), parallel vertical mergers can reduce welfare. In our model, merger does not distort incentives to sell inputs. Ex post, the allocation of inputs is efficient. But merged firms have an incentive to manipulate ex ante specific investments.

Of course, the stark result that we have here - merger always reduces welfare - is a consequence of our assumption that the party making the investment earns the full marginal surplus of an exchange. If we considered higher ex post competitive prices for inputs where the seller earns part of the marginal surplus, an unmerged buyer would underinvest in the specific asset. This outcome is the familiar hold-up problem, and merger might improve investment incentives. But we have the problem of the second-best. Because there are potentially other specific investments, merger creates other distortions. Merger creates investment incentives in favor of the internal seller and against the external seller. Whether merger improves welfare or decreases welfare would depend on the relative magnitude of the hold-up problem and the incentives to skew investments. We can, for example, parameterize the split of surplus so that buyers only earn a proportion q of the marginal surplus of exchange and sellers earn the remaining $(1 - q)$.[21] Above, $q = 1$ and

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[21]Formally, we define the price vector (p_1, p_2) as $\mathbf{p} \equiv q\mathbf{p}^{\min} + (1 - q)\mathbf{p}^{\max}$ where $q \in [0, 1]$ and $\mathbf{p}^{\min}$ and $\mathbf{p}^{\max}$

the buyer earns the full marginal surplus. We find for $q = 3/4$ merger still reduces welfare. For $q = 1/2$ the hold-up effect dominates, and mergers increase welfare.[22]

III. A General Model for Two Buyers and Two Sellers

We now show our results are robust to a more general model of specific investments and valuations of inputs.

A. Buyers, Sellers, and Production Technology

As above, there are two buyers and two sellers. We now suppose the specific investment g_{ij} is some positive number and for each $g_{ij} \geq 0$ the valuation $\tilde{v}_{ij}$ is a random variable with support in $[0, \overline{v}]$. The distribution function $F_{ij}(v_{ij}; g_{ij})$ is twice continuously differentiable with respect to g_{ij} and v_{ij}, where f_{ij} denotes the density function.[23] The distributions $F_{ij}(g_{ij}, \cdot)$ are independent for all i and j. We assume larger investments lead to higher valuations: for $g'_{ij} > g_{ij}$, the distribution $F_{ij}(\cdot; g'_{ij})$ first order stochastically dominates $F_{ij}(\cdot; g_{ij})$. We assume further that $f_{ij}(v_{ij}; g_{ij}) > 0$ for $v_{ij} \in [0, \overline{v}]$ and that $\partial F_{ij}(v_{ij}; g_{ij})/\partial g_{ij} < 0$ on an open interval of v_{ij} for each g_{ij}.[24] Let $\mathbf{g}_i = (g_{i1}, g_{i2})$ denote the vector of specific investments for buyer i and $\mathbf{G} = (\mathbf{g}_1, \mathbf{g}_2)$ denote the *investment pattern*. Each buyer i incurs an investment cost $C(\mathbf{g}_i) = c(g_{i1}) + c(g_{i2})$, where $c(\cdot) > 0$ and $c'(\cdot) > 0$. Let $\mathbf{v} = (v_{11}, v_{12}, v_{21}, v_{22})$ be a vector of realized valuations.

We analyze a three-stage game like that described above. In the first stage, firms make merger decisions. In the second stage, buyers make specific investments in sellers. In the third stage, buyers learn their valuations and exchange takes place.

Further discussion is necessary only for the third stage. We again consider competitive (core) outcomes. Allocations of inputs are efficient; buyer 1(2) purchases from seller 1(2) whenever

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are the lowest and highest competitive prices. Our analysis so far has assumed $\mathbf{p} = \mathbf{p}^{\min}$ or $q = 1$. The maximum competitve price vector is given by $p_i^{\max} = \frac{1}{2}$ when at least one buyer has value $\frac{1}{2}$ for input i. If neither buyer values input i, then $p_i^{\max} = 0$. In the next section we have more discussion of competitive prices.

[22] Another possibility is that sellers make investment decisions rather than buyers. When sellers earn the full marginal surplus of exchange, i.e., $q = 0$, we have similar results to our basic model. Mergers cause sellers to skew investments. Full merger is the unique equilibrium outcome and involves the lowest welfare. Calculations for all these cases are available upon request.

[23] Formally, we define the distribution $F_{ij}(v_{ij}, g_{ij})$ for $v_{ij} \in [0, \infty)$ and $g_{ij} \in [0, \infty)$. However, as noted, the support of $F_{ij}(\cdot, g_{ij})$ is always contained in $[0, \overline{v}]$.

[24] These positivity conditions guarantee strict inequalities in some of our results, but are not needed for most of them. The interval $[0, \overline{v}]$ may depend on g_{ij}.

$v_{11} + v_{22} \geq v_{12} + v_{21}$. Otherwise buyer 1(2) will purchase from seller 2(1). The price of exchange is the minimum price vector (p_1, p_2) that sustains an efficient allocation of goods.[25] The minimum prices are uniquely determined by the realization $\mathbf{v}$ of buyers' valuations. When a buyer i obtains a seller j 's good, buyer i's revenues are $v_{ij} - p_j$ and seller j's revenues are p_j.[26] A firm's expected third stage revenues are taken with respect to the distribution of possible realizations of $\mathbf{v}$. Let $R_i^b(\mathbf{G})$ denote the expected revenues for buyer i and $R_j^s(\mathbf{G})$ denote the expected revenues for seller j.

We derive here the expected revenues for buyer 1. Buyer 2's revenues are obtained similarly. Suppose $v_{11} + v_{22} \geq v_{12} + v_{21}$ so that in the efficient allocation buyer 1 purchases from seller 1. Buyer 1 pays a positive price only when there is competition for an input from seller 1. In this general model, competition occurs only when buyer 2 prefers an input from seller 1 to an input from seller 2: that is, $v_{21} \geq v_{22}$. The minimum prices that sustain the efficient allocation are then $p_1 = v_{21} - v_{22}$ and $p_2 = 0$. Seller 1's price is just high enough that buyer 2 earns more by purchasing from seller 2. Buyer 1's revenue is $v_{11} - p_1 = v_{11} - (v_{21} - v_{22})$. Suppose now that $v_{12} + v_{21} \geq v_{11} + v_{22}$ so that in the efficient allocation buyer 1 purchases from seller 2. Buyer 1 pays a positive price only when buyer 2 prefers an input from seller 2 to an input from seller 1; that is, $v_{22} \geq v_{21}$. The minimum prices here are $p_1 = 0$ and $p_2 = v_{22} - v_{21}$, and buyer 1's revenue is $v_{12} - p_2 = v_{12} - (v_{22} - v_{21})$. We can express buyer 1's expected revenues succinctly as:

$$R_1^b(\mathbf{G}) = \iiiint_{\mathbf{v} \geq \mathbf{0}} [\max\{v_{11} + v_{22}, v_{12} + v_{21}\} - \max\{v_{21}, v_{22}\}] f_{11} f_{12} f_{21} f_{22} dv_{11} dv_{12} dv_{21} dv_{22}.$$

We derive next the expected revenues for seller 1. Seller 2's revenue are obtained similarly. Seller 1 earns a positive price whenever it faces excess demand. As discussed above, there are only two cases where this occurs. In the first, buyer 1 obtains seller 1's input, even though buyer 2 prefers an input from seller 1 to an input from seller 2. That is: $v_{11} + v_{22} \geq v_{12} + v_{21}$ and $v_{21} \geq v_{22}$.

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[25] A price vector (p_1, p_2) is *competitive* for an efficient allocation if and only if : (1) If buyer i and seller j exchange a good, then $0 \leq p_j \leq v_{ij}$ and $p_j \leq p_i$ (2) If buyer i does not procure a good, then $p_j \geq v_{ij}$ and (3) if seller j does not sell a good, then $p_j = 0$. For each efficient allocation, the set of competitive price vectors forms a lattice. In particular, there is a minimum and maximum competitive price vector. If for some $\mathbf{v}$, several allocations are efficient, then the efficient allocations yield the same welfare and have the same set of competitive price vectors. For these results, see Shapley and Shubik (1972), Roth and Sotomayor (1990), or Kranton and Minehart (2000).

[26] As previously discussed, when a merged firm consumes its own input, the price is a transfer price that does not affect joint profits, but does insure that inputs are allocated efficiently.

Seller 1's price is $p_1 = v_{21} - v_{22}$.[27] In the second case, buyer 2 obtains seller 1's input, even though buyer 1 prefers an input from seller 1 to an input from seller 2. That is: $v_{12} + v_{21} \geq v_{11} + v_{22}$ and $v_{11} \geq v_{12}$. Seller 1 's price is $p_1 = v_{11} - v_{12}$.[28] We can express seller 1's revenues succinctly as:

$$R_1^s(\mathbf{G}) = \int_0^\infty \int_{v_{22}}^\infty \int_0^\infty \int_{v_{12}}^\infty \min\{v_{21} - v_{22}, v_{11} - v_{12}\} f_{11} f_{12} f_{21} f_{22} dv_{11} dv_{12} dv_{21} dv_{22}$$

A firm's expected profits in the game are its third stage revenues minus any second stage investment costs. Buyer i's profits are $\Pi_i^b(\mathbf{G}) \equiv R_i^b(\mathbf{G}) - C(\mathbf{g}_i)$ and seller j's profits are $\Pi_j^s(\mathbf{G}) \equiv R_j^s(\mathbf{G})$. If buyer i and seller i are merged, their joint profits are $\Pi_i^b(\mathbf{G}) + \Pi_j^s(\mathbf{G})$.

B. Comparative Statics on Equilibrium Investments

In this section we show second-stage equilibrium investments are more "skewed" the greater the number of merged firms.

We compare second-stage equilibrium investments for the three different ownership structures - no merger, partial merger, and full merger. A priori, it is not clear how to make such a comparison since there could be potentially many equilibrium investment patterns for each ownership structure.[29] We can, however, use the theory of monotone comparative statics to prove our results. As defined in Milgrom and Roberts (1990, p. 1255), a game is supermodular if each agent i's strategy set can be ordered so that increases in j's strategies cause the marginal return of i's strategy to rise, and, if the strategies are multidimensional, an increase in any component of an agent's strategy causes the marginal returns of the other components to rise. Games where strategies are strategic complements are typical examples. Games where strategies are strategic substitutes can also be supermodular when the "reverse order" is taken on the strategy sets.

In our game, investments by different buyers in different sellers are strategic complements and investments by different buyers in the same seller are strategic substitutes. For example, consider buyers' investments in different sellers, g_{11} and g_{22}. An increase in buyer 2's investment g_{22} causes the marginal return to buyer 1's investment g_{11} to rise; that is, $\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial g_{11} \partial g_{22}} \geq 0$ We find

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[27] Here, we necessarily have $v_{11} \geq v_{12}$ and $v_{21} - v_{22} = \min\{v_{21} - v_{22}, v_{11} - v_{12}\}$.

[28] Here, we necessarily have $v_{21} \geq v_{22}$ and $v_{11} - v_{12} = \min\{v_{21} - v_{22}, v_{11} - v_{12}\}$.

[29] We can see this possibility in our basic model of Section II. We assumed that buyers' valuations were $v_{ij} = \frac{1}{2}$ or $v_{ij} = 0$. If, instead, we had specified $v_{ij} = 1.5$ or $v_{ij} = 0$, then the second-stage continuation game has three equilibria when no firms are merged.

further that $\frac{\partial^2 \left[\Pi_1^b(\mathbf{G}) + \Pi_1^s(\mathbf{G})\right]}{\partial g_{11} \partial g_{22}} \geq 0$, so that the investments g_{11} and g_{22} are strategic complements for both a merged and unmerged buyer. As another example, consider buyers' investments in the same seller, g_{11} and g_{21}. We have $\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial g_{11} \partial g_{21}} \leq 0$ and $\frac{\partial^2 \left[\Pi_1^b(\mathbf{G}) + \Pi_1^s(\mathbf{G})\right]}{\partial g_{11} \partial g_{21}} \leq 0$ so that g_{11} and g_{21} are strategic substitutes for both a merged and unmerged buyer.

For independent buyers, these results are intuitive. They follow from supply stealing and supply freeing effects. As we saw in the example, when buyer 2 invests more in seller 2, it steals the supply of seller 2 from buyer 1. Since it demands a good from seller 2 more often, its investment also frees the supply of seller 1. Therefore, the marginal value of buyer 1's investment in seller 1 increases and the marginal value of investment in seller 2 decreases.

For merged buyers, these outcomes are less intuitive since a merged buyer also considers the profits of its merged seller. These latter effects may be quite different than for the buyer. For example, seller 1 receives a positive price only when *both* buyers prefer its input ($v_{11} > v_{12}$ and $v_{21} > v_{22}$), so an increase in g_{22} decreases the marginal value of g_{11} for seller 1. That is, $\frac{\partial^2 \Pi_1^s(\mathbf{G})}{\partial g_{11} \partial g_{22}} \leq 0$. We can still conclude, however, that for the merged firm the investments are complements; that is, $\frac{\partial^2 \Pi_1^M(\mathbf{G})}{\partial g_{11} \partial g_{22}} \geq 0$. When a merged firm obtains an input internally, the price is a transfer. The merged firm simply earns the value of the input. When g_{22} increases, buyer 1 procures the internal input from seller 1 more often, so the marginal return to g_{11} increases.

The next lemma shows that the second-stage investment game is supermodular for each ownership structure. We prove this lemma by establishing the signs of the cross partial derivatives of the merged and independent buyers' profit functions with respect to appropriate pairs of investments. The result follows from our first order stochastic dominance assumptions.[30]

Lemma 1. *For each ownership structure, the investment game is supermodular in the investment strategies $(g_{11}, -g_{12})$ for buyer 1 and $(-g_{21}, g_{22})$ for buyer 2.*

Proof: All proofs are provided in the Appendix.

We now consider characteristics of equilibrium investment patterns. We will consider mergers between buyer i and seller i. We say that an investment pattern $\mathbf{G}$ is *more skewed* than $\mathbf{G}'$ if and only if $g_{ii} \geq g'_{ii}$, and $g_{ij} \leq g'_{ij}$ for $i, j \in \{1, 2\}$. Our first result shows that for each ownership

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[30]Notice that we take the reverse order on the investments g_{12} and g_{21}. For example, to show supermodularity for buyer 1's own investments $(g_{11}, -g_{12})$ requires showing that $\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial g_{11} \partial(-g_{12})} \geq 0$, which is the same as showing that $\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial g_{11} \partial g_{12}} \leq 0$. That is, buyer 1's own investments are strategic substitutes.

structure there is an equilibrium and the set of equilibria is defined by a most and least skewed investment pattern. The proposition is a straightforward application of Milgrom and Roberts (1990) and follows directly from the supermodularity of the investment game.

Proposition 1. *For each ownership structure, there exists a (pure) Nash equilibrium of the second-stage investment game. Moreover, there exist a most skewed and a least skewed equilibrium investment pattern.*

We now show that investments are always more "skewed" as the number of merged firms increases. We begin with a lemma. We parameterize the ownership structure by $t \in \{N, P, F\}$ where N indicates no merger, P partial merger, and F full merger. The parameter t increases as the number of merged firms increases; i.e., we impose a partial ordering on t as follows: $N \prec P \prec F$. Firm i refers to either buyer i or the merged firm composed of buyer i and seller i, according to the ownership structure t. We have

Lemma 2. *The marginal return to firm 1's investment in g_{11} and in $-g_{12}$ is weakly increasing in t, holding the other investments fixed. The marginal return to firm 2's investment in g_{22} and in $-g_{21}$ is weakly increasing in t, holding the other investments fixed.*

The lemma is proved by showing that internal investment benefits a seller and external investment hurts a seller: $\frac{\partial \Pi_j^s}{\partial g_{ii}} > 0$ and $\frac{\partial \Pi_j^s}{\partial g_{ij}} < 0$. A merged firm i therefore earns a higher (lower) marginal return from internal (external) investment than does an unmerged firm i.[31]

Our second result shows that equilibrium investments become more skewed the greater the number of merged firms and is again a straightforward application of Milgrom and Roberts [1990] given Lemma 2.

Proposition 2. *Consider the most skewed equilibrium when no firms are merged. The most skewed equilibrium investment pattern when one firm is merged is more skewed. And the most skewed equilibrium investment pattern when both firms are merged is yet more skewed. Consider the least skewed equilibrium pattern when no firms are merged. The least skewed equilibrium investment pattern when one firm is merged is more skewed. The least skewed equilibrium when both firms are merged is yet more skewed.*

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[31] When buyer j merges, the marginal returns to firm i's investments do not change. This is consistent with the statement in the lemma that they weakly increase.

We can explain the relationship between merger and skewed investments by the supply stealing and supply freeing effects. The effects yield a feedback; when one buyer skews its investments more, the other will as well. When buyer 1 merges with seller 1, it has an incentive to decrease its investment in seller 2 and increase its investment in its internal seller, seller 1. These investments steal the supply of seller 1 and free the supply of seller 2. Although nothing has changed in buyer 2's holdings, the marginal return to buyer 2's investments in seller 1(2) has decreased (increased). Buyer 2 will then also skew its investments. The equilibrium involves more skewed investments for both buyers.[32]

C. Equilibrium Ownership Structures

Because we have not imposed much structure on the distributions of buyers' valuations, the three-stage merger game may in general have many different equilibria. More than one ownership structure might be an equilibrium, and for a given ownership structure, there are possibly many equilibrium investment patterns. Indeed, because the game has strategic complementarities, the existence of multiple equilibria seems a likely outcome in general.

A key question is whether we can expect merger to arise in equilibrium.[33] Equilibrium of the no merger ownership structure requires that each pair of firms has higher joint profits under no merger than under partial merger. Consider an investment pattern that is a second-stage equilibrium under no merger. Next suppose that buyer 1 and seller 1 merge, and that buyer 2's investments do not change. Then clearly, buyer 1 and seller 1 can choose investments in such a way that their joint profits are at least as high as before they merged. Indeed, buyer 1 and seller 1 can generally do strictly better than before.[34] Therefore, if buyer 2 did not change its investments in response to the merger, then no merger would not be an equilibrium. However, by Proposition 2, buyer 2 does change its investments, skewing them towards seller 2. The next lemma shows that this skewing response hurts the merged firm.

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[32]This reaction is also an example of the LeChatelier Principle. For a discussion of Le Chatelier's principle, see Milgrom and Roberts (1994), Theorem 6 and Section III.

[33]As we show in the next section, efficient investment patterns are second-stage equilibria for the no merger ownership structure, but not for other ownership structures. Because of this, merger reduces welfare.

[34]From lemma 2, the seller's first order conditions satisfy: $\frac{\partial \Pi_1^s}{\partial g_{11}} \geq 0$ and $\frac{\partial \Pi_1^s}{\partial g_{12}} \leq 0$. If the no merger equilibrium was an interior equilibrium, so that $\frac{\partial \Pi_1^b}{\partial g_{11}} = \frac{\partial \Pi_1^b}{\partial g_{12}} = 0$, and if for example, $\frac{\partial \Pi_1^s}{\partial g_{11}} > 0$, then the merged firm can do strictly better by changing the investments.

Lemma 3. *A merged firm's maximized profits are decreasing in any skewing by the other buyer.*

It follows that if buyer 2's response is sufficiently strong, then buyer 1 and seller 1 may not want to merge, so that no merger can be equilibrium.[35] Of course, merger would be an equilibrium if this response is muted.

D. Ownership Structures and Efficient Investments

We show here that merger leads to inefficient investments in the general game. We compare efficient investment patterns to the equilibrium investments under different ownership structures.

Efficient investment patterns balance the costs of the specific investments with the expected gains from exchange. By taking the efficient allocation of inputs for each realization of buyers' valuations $\mathbf{v}$, we can determine the maximal expected surplus from exchange, $H(\mathbf{G})$, for a given investment structure $\mathbf{G}$.[36] Let $W(\mathbf{G})$ denote the welfare of $\mathbf{G}$; that is, $H(\mathbf{G})$ minus the investment costs:

$$W(\mathbf{G}) \equiv H(\mathbf{G}) - C(\mathbf{g}_1) - C(\mathbf{g}_2)$$

We say an investment pattern $\mathbf{G}$ is *efficient* if and only if no other investment pattern yields strictly higher welfare.

We now show that merger reduces welfare in our game. When no firms are merged, the efficient investment structure is an equilibrium of the second stage investment game. However, when any firms are merged, the efficient investment structure is not an equilibrium.

We have

Proposition 3. *In the no merger ownership structure, every efficient investment pattern is an equilibrium of the second stage investment game. That is, independent buyers make socially optimal investments in links.*

The result holds because in no merger, buyers' investment incentives are aligned with welfare. A buyer's marginal return to its investment is exactly equal to its marginal social value, given

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[35] We can see the possibility of no merger in our basic model of Section II. We assumed that buyers' valuations were either $v_{ij} = \frac{1}{2}$ or $v_{ij} = 0$. If, instead, we had specified $v_{ij} = 1$ or $v_{ij} = 0$, then the merger game has two equilibria. No merger and full merger are both equilibrium ownership patterns. The no merger equilibrium involves efficient, skewed investments, and the full merger equilibrium involves inefficient investments that are yet more skewed. Details are available on request.

[36] We have $H(\mathbf{G}) = \iiint \max\{v_{11} + v_{22}, v_{12} + v_{21}\} \Pi f_{ij} dv_{ij}$.

other buyers' investments.[37] This outcome is a consequence of the competition for goods. When buyers pay the minimum competitive prices, the price a buyer pays is the incremental value the other buyer would place on obtaining the good. The purchasing buyer then earns the difference between its own value and the other buyer's foregone value - which is exactly the marginal return to welfare when it obtains the input. For example, consider the following realization of buyers' valuations: $v_{11} + v_{22} \geq v_{12} + v_{21}$ and $v_{21} > v_{22}$. In this case, in the efficient allocation, buyer 1(2) obtains a good from seller 1(2), but buyer 2 prefers an input from seller 1. As discussed above, seller 1's price is just high enough so that buyer 2 is willing to forego purchasing from seller 1; that is, $p_1 = v_{21} - v_{22}$. Buyer 1 's revenue is then $v_{11} - p_1 = (v_{11} + v_{22}) - v_{21}$. Welfare is $v_{11} + v_{22}$. Now consider social welfare when buyer 1 is not present (i.e., has no investments). Buyer 2 would purchase from seller 1 and welfare would be v_{21}. The change in welfare is $(v_{11} + v_{22}) - v_{21}$, which is exactly buyer 1's revenues in this event. Hence, when considering its investments, buyer 1 faces incentives that match social welfare.

Merged buyers' incentives, however, are not aligned with welfare. As we have shown previously, a merged buyer will skew its investments towards its internal seller. A merged buyer i chooses investments to maximize $\Pi_i^b(\mathbf{G}) + \Pi_i^s(\mathbf{G})$. As long as the seller's profits are strictly increasing in g_{ii} and decreasing in g_{ij}, buyer i will change its investments relative to the investments when not merged. We have[38]

Proposition 4. *When at least one pair of firms is merged, efficient investment patterns are not equilibrium outcomes. At an efficient investment pattern, a merged firm could increase its profit by either increasing the internal investment or by decreasing the external investment.*

IV. Greater Numbers of Buyers and Sellers

We consider here whether the main insights from the two-buyer-two-seller case generalize to $S \geq 2$ sellers and $B \geq 3$ buyers.[39]

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[37] As we show in the appendix, the buyer's profit function satisfies the property that $\Pi_i^b(\mathbf{G}) - \Pi_i^b(\mathbf{G}') = W(\mathbf{G}) - W(\mathbf{G}')$, where $\mathbf{G}$ and $\mathbf{G}'$ are any investment patterns that differ only on the investments of buyer i.

[38] The proposition assumes that the efficient investment pattern involves interior investments. If the efficient investment pattern involves corner solutions ($g_{ij} = 0$), then the pattern could be an equilibrium when some firms are merged.

[39] As previously, buyer i makes specific investments g_{ij} which determine distribution functions $F_{ij}(v_{ij}; g_{ij})$. The vector of specific investments for a buyer i is $\mathbf{g}_i = (g_{i1}, g_{i2}, ..., g_{iS})$, and $\mathbf{G} = (\mathbf{g}_1, \mathbf{g}_2, ..., \mathbf{g}_B)$ is the investment

Our main result, that merger reduces welfare, continues to hold. When no firms are merged, the efficient investment structure is an equilibrium of the second stage investment game. However, when any firms are merged, the efficient investment structure is not an equilibrium.

Proposition 5. *In the no merger ownership structure, every efficient investment pattern is an equilibrium of the second stage investment game. That is, independent buyers make socially optimal investments in links.*

As in our previous model, an umerged buyers' revenues are exactly its marginal contribution to welfare. Therefore, unmerged buyers choose investments efficiently, given the investments of the other buyers.

As for the pattern of investments with more than two buyers, we find new incentives for merged firms to invest in external suppliers. A merged firm will still want to increase the investments specific to its internal supplier, but might want to increase or decrease investments specific to external suppliers. With more than two buyers, there are potentially two outside buyers that can compete with each other for the capacity of a merged firm. A merged buyer may want to increase its investment in outside sellers in order to stimulate this competition. Suppose, for example, that buyer 1 is merged with seller 1. When buyer 1 increases its investment in seller 2, it steals seller 2's supply from, say, buyers 2 and 3. These two buyers may then both turn to seller 1 and compete more for its capacity. It is possible that the increased competition from outside buyers can more than offset the decrease in buyer 1's own demand for seller 1's capacity.[40]

We have[41]

Proposition 6. *When at least one pair of firms is merged, efficient investment patterns are not equilibrium outcomes. At an efficient investment pattern, a merged firm could increase its*

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pattern. Each buyer i incurs an investment cost $C(\mathbf{g}_i) = \sum_{j=1}^{S} c(g_{ij})$, and $\mathbf{v} = (v_{11}, v_{12}, v_{1S}, v_{21}, v_{22}, ..., v_{BS})$ is the vector of realized valuations. Further discussion is necessary only for the first and third stages of the game. In the first stage, we consider mergers between buyer i and seller i where $i \leq B$ and $i \leq S$. In the third stage, the price of exchange is the minimum price vector $(p_1, p_2, ..., p_S)$ that sustains an efficient allocation of goods, where p_j is the price received by seller j. Our definition of competitive prices is easily extended to this case.

[40]Because merger affects external investments in an ambiguous way, our skewing results (monotone comparative statics) of section B do not generalize to this model. However, we expect that in many examples, a merged buyer will reduce all or most of its external investments. The effect that leads a merged buyer to increase external investments is (loosely speaking) subtle.

[41]If the efficient investment pattern involves corner solutions ($g_{ij} = 0$), then the efficient pattern could be an equilibrium even when a firm is merged. The proposition assumes that the efficient investment pattern involves interior investments.

profits by increasing the internal investment. The effect on the merged firm of changing any of its external investments is ambiguous.

V. Other Extensions

We discuss here whether our results would change if we relax different assumptions of our model.

In our basic model of two buyers and two sellers, we considered section II.D how our results depend on the bargaining power of buyers and sellers. Those insights apply to our general model as well. In particular, our assumption of minimum competitive prices assigns all of the bargaining power to buyers. Alternatively, we might assume that buyers and sellers share bargaining power. Independent buyers would then no longer choose investments efficiently, and may invest too little in a seller as in the classical hold-up problem. Whether merger improves welfare or decreases welfare would depend on the relative magnitude of the hold-up problem and the incentives to skew investments.

The main insights would also extend to a model where different parties make investments. We have assumed that buyers make investments. Alternatively, we might assume that sellers make investments or that both buyers and sellers make investments. If sellers make investments, then independent sellers will choose investments efficiently if they earn all the marginal surplus of exchange. (Proof of this result is available from the authors upon request.) Sellers earn all the surplus when buyers and sellers exchange inputs for the maximum competitive prices. This outcome would correspond to $q = 0$ in our parameterization at the end of section II.D. Merged sellers, however, would also consider the downstream unit's revenues, and therefore not invest efficiently. Merger would again reduce welfare. If buyers and sellers both make investments, then coordination problems can arise. A seller might not invest in a buyer because the buyer is not investing in the seller, and vice versa. We might avoid this problem by using a solution concept such as pairwise stability (Jackson and Wolinsky 1996). However, because it is impossible to award the full marginal surplus of an exchange to both the buyer and the seller, investments are not likely to be made efficiently even by independent firms.

Our results rely on competition for sellers' inputs, which arise in our model due to the capacity constraints of sellers. The fact that a seller can produce only one unit of an input underlies the competition in the input market. When two buyers both prefer a particular seller's input, only

one will be able to obtain it. When buyers make specific investments, they anticipate this input market competition. The more a buyer invests in a seller, the more difficult it will be for another buyer to compete for the seller's input. This supply stealing effect (and the related supply freeing effect on the other seller) would not arise if sellers were not capacity constrained. However, if sellers have increasing marginal cost functions, versions of the supply stealing and supply freeing effects of specific investments should still obtain.

Finally, we consider how our results depend on the form of the costs for the specific investments. We have assumed that $C(\mathbf{g}_i) = c(g_{i1}) + c(g_{i2})$. More generally, a buyer's cost of investment could depend on the collective investments of all the buyers, and there could be cost interactions between a buyer's own investments. What matters for our results is that the investment game is supermodular.[42] With independent cost functions, the game is supermodular in part because the cross partial derivatives of the cost functions are zero. However, more general cost functions could also be compatible with supermodularity.

VI. Conclusion

This paper analyzes vertical merger in a multiple firm setting. Its innovation is to consider downstream firms that make specific investments in several suppliers at once. In this setting, we find that vertical merger can reduce welfare. A merged firm has the incentive to manipulate its investments to increase the revenues of its supply unit.

This paper has implications for the regulation of vertical mergers. Antitrust law has been concerned with the short-run effects of vertical mergers; that is, how merger directly affects the pricing and allocation of inputs [Hovenkamp (1994, pg. 331)]. This paper suggests that regulators consider the long-run effects of merger. Regulators have recognized that a merger can improve internal investments and lead to efficiency gains, but have not considered the general implications of merger for investments. We show that merger changes the incentives to invest in long-run specific assets which determine the relative value of inputs from different suppliers. Merged firms may make inefficient investments to raise the demand for their supply units. The result could be a loss in welfare and a loss in variety to final consumers.

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[42] In our model, the maximization problems of firms need not be convex. Instead, as shown above, the comparative statics results are obtained using supermodularity.

To make this argument concrete, let us return to the discussion of recent merger activity. In the introduction we discussed vertical mergers between pharmaceutical suppliers and pharmacy benefit managers (PBM's). These two types of firms match the buyers and sellers in our model; the pharmaceutical suppliers are the sellers, and the PBM's are the buyers - as they are intermediaries between the pharmaceutical suppliers and retailers/consumers. Like the buyers in our model, in the short-run the PBM's do not compete in the final goods market. Their client base is fixed; a consumer can obtain her pharmacy benefits only from the PBM that has contracted with her employer. The input market, the market for drugs, is competitive, and specific investments can increase the intrinsic value of a supplier's drugs (as a PBM investigates, for example, the efficacy and side effects of a particular treatment). According to our analysis, a vertical merger would increase a PBM's incentive to invest in assets or knowledge specific to its parent pharmaceutical supplier and reduce investments in assets specific to other suppliers. These investments raise the demand for the parent supplier's product, raising the supplier's revenue. The merged PBM would then (efficiently) favor its parent company's drugs on its formularies and in its health management protocols. The investments themselves, however, may be inefficient reducing the variety available to final consumers.

A similar process may be at work in cable television.[43,44] From the mid 1980's through the 1990's there was a wave of vertical mergers between cable television operators and content providers. Cable television operators, such as Time Warner, provide cable television service to subscribers' homes. They purchase programs from content providers, such as Cinemax and The Movie Channel. The cable television operators correspond to the buyers in our model, and the content providers correspond to the sellers. Like the buyers in our model, in the short-run, cable television operators do not compete in the final goods market. For the most part, consumers subscribe and can obtain programming only from a single operator at a time.[45] Cable television operators can make a variety of investments specific to content providers. They often

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[43]This discussion is drawn from information in Chipty (2001) and Waterman and Weiss (1997).

[44]Our framework may also apply to other industries. For example, in the 1990s, there were many mergers between companies providing information technology services and freight forwarding companies (Clark, 1990). In the 1980s and 1990s, Coca-Cola and Pepsi-Cola acquired and consolidated many of their bottlers. Cadbury-Schweppes responded by forming its own bottling company in 1998 (Moriguchi and Lane, 2000). In both industries, the vertical mergers facilitated specific investments between the merged firms, and in the case of the soft-drink industry, reduced the use of independent suppliers.

[45]Moreover, in many geographic markets in the United States, only one cable system operates.

provide capital to develop new programming and provide information so that these programs appeal to particular demographics. The operator can also promote a particular provider's channel. All of these investments raise the value of the provider's content to the operator. Our analysis suggests that a merged operator will have a greater incentive to invest in assets specific to its internal provider and lower incentive to invest in assets specific to other content providers. Empirically, Chipty (2001) and Waterman and Weiss (1997) find that integrated cable operators carry their own content providers' programming more often than that of other providers. Consumers subscribing to these operators have a smaller choice of programming.[46] Our analysis indicates this outcome may be efficient, *given* the specific investments of operators and content providers. However, the investments themselves may be inefficient and unduly exclude external suppliers.[47]

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[46] Regulators have been concerned that vertical mergers can lead to reduced variety. One aim of The Cable Television Consumer Protection and Competition Act of 1992, for example, was to ensure consumers' access to an array of programming (FCC (1992)).

[47] Chipty (2001) argues that mergers can increase overall welfare. The efficiency gains from merger (e.g., reduction in transactions cost, elimination of double marginalization) may outweigh the loss from reduced variety.

Appendix

In various proofs we (1) take derivatives under integral signs and (2) apply the integration by parts formula $\int x\,dy = xy \mid - \int y\,dx$. A note at the end of the Appendix shows that these operations are valid for the equations we analyze.

Proof of Lemma 1.

No firms are merged.

The second-stage investment game is supermodular if: (i) buyer i's investments (g_{ii} and $-g_{ij}$) are strategic complements with buyer j's investments and (ii) buyer i's investments (g_{ii} and $-g_{ij}$) are strategic complements with each other. Following Theorem 4 in Milgrom and Roberts (1990), these conditions can be expressed in terms of cross partials of profit functions. For buyer 1, supermodularity requires: (i) $\frac{\partial^2 \Pi_1^b}{\partial g_{11} \partial g_{22}} \geq 0, \frac{\partial^2 \Pi_1^b}{\partial g_{11} \partial(-g_{21})} \geq 0, \frac{\partial^2 \Pi_1^b}{\partial(-g_{12}) \partial(-g_{21})} \geq 0, \frac{\partial^2 \Pi_1^b}{\partial g(-g_{12}) \partial g_{22}} \geq 0$, and (ii) $\frac{\partial^2 \Pi_1^b}{\partial g_{11} \partial(-g_{12})} \geq 0$. Analogous conditions must hold for buyer 2.

We show here that g_{11} and g_{22} are strategic complements for buyer 1: $\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial g_{11} \partial g_{22}} \geq 0$. All other cross partial derivatives of the buyers' profit functions can obtained similarly. (Complete computations for this and other proofs are available from the authors on request.)

Buyer 1's profits are $\Pi_1^b(\mathbf{G}) = R_1^b(\mathbf{G}) - C(\mathbf{g}_1)$. Since $\frac{\partial^2 C(\mathbf{g}_1)}{\partial g_{11} \partial g_{22}} = 0$, the cross partial derivative of the profits is simply the cross partial derivative of the buyer's revenue: $\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial g_{11} \partial g_{22}} = \frac{\partial^2 R_1^b(\mathbf{G})}{\partial g_{11} \partial g_{22}}$.

We first evaluate $\frac{\partial R_1^b(\mathbf{G})}{\partial g_{11}}$. Differentiating $R_1^b(\mathbf{G})$ we have

$$\frac{\partial R_1^b(\mathbf{G})}{\partial g_{11}} = \int_0^\infty \int_0^\infty \int_0^\infty \left[\int_0^\infty (\max\{v_{11}+v_{22}, v_{12}+v_{21}\} - \max\{v_{21}, v_{22}\}) \frac{\partial f_{11}}{\partial g_{11}} dv_{11} \right] f_{12} f_{21} f_{22} dv_{12} dv_{21} dv_{22}$$

We evaluate the bracketed expression using integration by parts. Let $x = \max\{v_{11}+v_{22}, v_{12}+v_{21}\} - \max\{v_{21}, v_{22}\}$ and $dy = \frac{\partial f_{11}}{\partial g_{11}} dv_{11}$. We may apply the integration by parts formula $\int x\,dy = xy \mid - \int y\,dx$, where $y = \frac{\partial F_{11}}{\partial g_{11}}$ and $dx = \frac{\partial x}{\partial v_{11}} dv_{11}$. (Note that to obtain y from dy, we must integrate the function $\frac{\partial f_{11}(v_{11}, g_{11})}{\partial g_{11}}$ with respect to v_{11}, *not* with respect to g_{11}.) We have

$$\left[\int_0^\infty x \frac{\partial f_{11}}{\partial g_{11}} dv_{11} \right] = x \frac{\partial F_{11}}{\partial g_{11}} \mid_0^\infty - \int_0^\infty \frac{\partial x}{\partial v_{11}} \frac{\partial F_{11}}{\partial g_{11}} dv_{11}$$

The first term is equal to zero: By our assumption on the support of F_{ij}, for all g_{ij} $F_{ij}(0) = 0$ and $F(\overline{v}) = 1$ for $v \geq \overline{v}$. Hence, $\frac{\partial F_{11}}{\partial g_{11}} = 0$ at both $v_{11} = 0$ and $v_{11} = \infty$. Consider the second term. Consider the function $\frac{\partial x}{\partial v_{11}}$. For $v_{11} \geq v_{12} + v_{21} - v_{22}$, $\frac{\partial x}{\partial v_{11}} = \frac{d[\max\{v_{11}+v_{22}, v_{12}+v_{21}\} - \max\{v_{21}, v_{22}\}]}{dv_{11}} = 1$.

Otherwise, $\frac{\partial x}{\partial v_{11}} = \frac{d[\max\{v_{11}+v_{22},v_{12}+v_{21}\}-\max\{v_{21},v_{22}\}]}{dv_{11}} = 0$. We now have

$$\frac{\partial R_1^b}{\partial g_{11}} = -\int_0^\infty \int_0^\infty \int_0^\infty x(v_{12},v_{21},v_{22})f_{12}f_{21}f_{22}dv_{12}dv_{21}dv_{22}$$

where

$$x(v_{12},v_{21},v_{22}) = \int_{\max\{0,v_{12}+v_{21}-v_{22}\}}^\infty \frac{\partial F_{11}}{\partial g_{11}}dv_{11}.$$

We next derive $\frac{\partial^2 R_1^b(\mathbf{G})}{\partial g_{11}\partial g_{22}}$. Differentiating with respect to g_{22}, we have

$$\frac{\partial^2 R_1^b(\mathbf{G})}{\partial g_{11}\partial g_{22}} = -\int_0^\infty \int_0^\infty \left[\int_0^\infty x(v_{12},v_{21},v_{22})\frac{\partial f_{22}}{\partial g_{22}}dv_{22}\right]f_{12}f_{21}dv_{12}dv_{21}.$$

We evaluate the bracketed expression using integration by parts. We may apply the formula $\int xdy = xy \mid - \int ydx$ where $dy = \frac{\partial f_{22}}{\partial g_{22}}dv_{22}$ and $y = \frac{\partial F_{22}}{\partial g_{22}}$ and $dx = \frac{\partial x}{\partial v_{22}}dv_{22}$. We have

$$\left[\int_0^\infty x(v_{12},v_{21},v_{22})\frac{\partial f_{22}}{\partial g_{22}}dv_{22}\right] = x\frac{\partial F_{22}}{\partial g_{22}}\mid_0^\infty - \int_0^\infty \frac{\partial x}{\partial v_{22}}\frac{\partial F_{22}}{\partial g_{22}}dv_{22}$$

The first term is equal to zero because $\frac{\partial F_{22}}{\partial g_{22}} = 0$ at $v_{12} = 0$ and $v_{12} = \infty$. We show that the integrand of the second term, $\frac{\partial x}{\partial v_{22}}\frac{\partial F_{22}}{\partial g_{22}}$, is negative.

The derivative of x with respect to v_{22} is $\frac{\partial x}{\partial v_{22}} = \frac{\partial F_{11}(v_{12}+v_{21}-v_{22})}{\partial g_{11}} \leq 0$ if $v_{12} + v_{21} \geq v_{22}$ and $\frac{\partial x}{\partial v_{22}} = 0$ if $v_{12} + v_{21} < v_{22}$. Therefore, $\frac{\partial x}{\partial v_{22}} \leq 0$. By our assumption of first order stochastic dominance with respect to g_{22} (that is, for $g_{22} > g_{22}'$, $F_{22}(v_{22};g_{22}) \leq F_{22}(v_{22};g_{22}')$ for all v_{22}) we have $\frac{\partial F_{22}}{\partial g_{22}} \leq 0$. We conclude that $\frac{\partial x}{\partial v_{12}}\frac{\partial F_{12}}{\partial g_{12}}$ is positive. It follows that the bracketed expression $\left[\int_0^\infty x\frac{\partial f_{22}}{\partial g_{22}}dv_{22}\right]$ is negative, and $\frac{\partial^2 R_1^b(\mathbf{G})}{\partial g_{11}\partial g_{22}} \geq 0$. We have also derived the expression

$$\frac{\partial^2 R_1^b(\mathbf{G})}{\partial g_{11}\partial g_{22}} = \int_0^\infty \int_0^\infty \left[\int_0^{v_{12}+v_{21}} \frac{\partial F_{11}(v_{12}+v_{21}-v_{22})}{\partial g_{11}}\frac{\partial F_{22}(v_{22})}{\partial g_{22}}dv_{22}\right]f_{12}f_{21}dv_{12}dv_{21}.$$

Notice that $\frac{\partial x}{\partial v_{12}} \geq 0$ and $\frac{\partial x}{\partial v_{21}} \geq 0$. A similar integration by parts argument implies that $\frac{\partial^2 \Pi_1^b}{\partial g_{12}\partial g_{11}} \leq 0$ and $\frac{\partial^2 \Pi_1^b}{\partial g_{21}\partial g_{11}} \leq 0$.

One or more pairs of firms is merged.

When one or more pairs of firms are merged, the second-stage investment game is supermodular if the cross partial derivatives of the merged firm's profits satisfies the conditions for supermodularity. (We have just shown that an unmerged buyer's profits satisfy the supermodularity

conditions.) For a merged firm 1, it is sufficient that $\frac{\partial^2 \Pi_1^M}{\partial g_{11} \partial g_{22}} \geq 0, \frac{\partial^2 \Pi_1^M}{\partial g_{11} \partial(-g_{21})} \geq 0, \frac{\partial^2 \Pi_1^M}{\partial(-g_{12}) \partial(-g_{21})} \geq$ $0, \frac{\partial^2 \Pi_1^M}{\partial(-g_{12}) \partial g_{22}} \geq 0$, and $\frac{\partial^2 \Pi_1^M}{\partial g_{11} \partial(-g_{12})} \geq 0$ where $\Pi_1^M = \Pi_1^b(\mathbf{G}) + \Pi_1^s(\mathbf{G})$. Analogous conditions must hold for a merged firm 2.

We show here that $\frac{\partial^2 \left[\Pi_1^b(\mathbf{G}) + \Pi_1^s(\mathbf{G}) \right]}{\partial g_{11} \partial g_{22}} \geq 0$. All other cross partial derivatives of a merged firm can be obtained similarly.

For the buyer, we have already derived

$$\frac{\partial^2 \Pi_1^b(\mathbf{G})}{\partial g_{11} \partial g_{22}} = \int_0^\infty \int_0^\infty \left[\int_0^{v_{12}+v_{21}} \frac{\partial F_{11}(v_{12} + v_{21} - v_{22})}{\partial g_{11}} \frac{\partial F_{22}(v_{22})}{\partial g_{22}} dv_{22} \right] f_{12} f_{21} dv_{12} dv_{21}.$$

We consider seller 1 's profits:

$$\Pi_1^s(\mathbf{G}) = R_1^s(\mathbf{G}) = \int_0^\infty \int_{v_{22}}^\infty \int_0^\infty \int_{v_{12}}^\infty \min\{v_{21} - v_{22}, v_{11} - v_{12}\} f_{11} f_{12} f_{21} f_{22} dv_{11} dv_{12} dv_{21} dv_{22}$$

Differentiating, we have:

$$\frac{\partial \Pi_1^s}{\partial g_{11}} = \int_0^\infty \int_{v_{22}}^\infty \int_0^\infty \int_{v_{12}}^\infty \min\{v_{21} - v_{22}, v_{11} - v_{12}\} \frac{\partial f_{11}}{\partial g_{11}} dv_{11} f_{12} f_{21} f_{22} dv_{12} dv_{21} dv_{22}$$

Using integration by parts, we derive the expression:.

$$\frac{\partial \Pi_1^s}{\partial g_{11}} = \int_0^\infty \int_{v_{22}}^\infty \int_0^\infty x(v_{12}, v_{21}, v_{22}) f_{12} f_{21} f_{22} dv_{12} dv_{21} dv_{22}$$

where

$$x(v_{12}, v_{21}, v_{22}) = \left[-\int_{v_{12}}^{(v_{21}-v_{22})+v_{12}} \frac{\partial F_{11}}{\partial g_{11}} dv_{11} \right] \text{ and } v_{21} \geq v_{22}.$$

and we can now see that $\frac{\partial \Pi_1^s}{\partial g_{11}} \geq 0$ by our assumption that $\frac{\partial F_{11}}{\partial g_{11}} \leq 0$. Switching the order of integration in $\frac{\partial \Pi_1^s}{\partial g_{11}}$, we next derive the expression:

$$\frac{\partial^2 \Pi_1^s}{\partial g_{11} \partial g_{22}} = \int_0^\infty \int_0^\infty \left[\int_0^{v_{21}} x(v_{12}, v_{21}, v_{22}) \frac{\partial f_{22}(v_{22})}{\partial g_{22}} dv_{22} \right] f_{12} f_{21} dv_{12} dv_{21}$$

The derivative of x with respect to v_{22} is $\frac{\partial x}{\partial v_{22}} = \frac{\partial F_{11}(v_{21} - v_{22} + v_{12})}{\partial g_{11}}$. Using integration by parts where $y = \frac{\partial F_{22}(v_{22})}{\partial g_{22}}$ and $dx = \frac{\partial x}{\partial v_{22}} dv_{22}$ and noting that $y = 0$ at $v_{22} = 0$ and $x = 0$ at $v_{22} = v_{21}$,

28

we derive the expression:

$$\frac{\partial^2 \Pi_1^s}{\partial g_{11} \partial g_{22}} = -\int_0^\infty \int_0^\infty \left[\int_0^{v_{21}} \frac{\partial F_{11}((v_{21} - v_{22}) + v_{12})}{\partial g_{11}} \frac{\partial F_{22}(v_{22})}{\partial g_{22}} dv_{22} \right] f_{12} f_{21} dv_{12} dv_{21}$$

We combine the cross partials for the buyer and seller:

$$\frac{\partial^2 \Pi_1^b}{\partial g_{11} \partial g_{22}} + \frac{\partial^2 \Pi_1^s}{\partial g_{11} \partial g_{22}} = \int_0^\infty \int_0^\infty \left[\int_{v_{21}}^{v_{12}+v_{21}} \frac{\partial F_{11}((v_{21} - v_{22}) + v_{12})}{\partial g_{11}} \frac{\partial F_{12}(v_{12})}{\partial g_{12}} dv_{12} \right] f_{21} f_{22} dv_{21} dv_{22}$$

Given our assumption that $\frac{\partial F_{ij}(v_{ij}; g_{ij})}{\partial g_{ij}} \leq 0$ for all values of v_{ij}, it follows immediately that the cross partial for the merged firm is positive: $\frac{\partial^2 \Pi_1^M(\mathbf{G})}{\partial g_{11} \partial g_{22}} \geq 0$.

Proof of Proposition 2.

The result follows directly from Milgrom and Roberts (1990), Theorem 5. This theorem shows that for supermodular games, there exist smallest and largest serially undominated strategies. These strategies are pure strategies and constitute pure Nash equilibria. A corrollary to the Theorem is that there exists a smallest and largest pure strategy Nash equilibrium. In our framework, the smallest and largest equilibria correspond to the least and most skewed equilibrium for an ownership structure.

Proof of Lemma 2.

The marginal return to firm i's investment in g_{ii} is $\frac{\partial \Pi_i^b}{\partial g_{ii}}$ if buyer i is not merged and $\frac{\partial \Pi_i^b}{\partial g_{ii}} + \cdot \frac{\partial \Pi_i^s}{\partial g_{ii}}$ if buyer i is merged. Therefore, if $\frac{\partial \Pi_i^s}{\partial g_{ii}} \geq 0$, the marginal return to the merged firm i's investment in g_{ii} is larger than the return to the independent buyer i. Similarly, if $\frac{\partial \Pi_i^s}{\partial g_{ij}} \leq 0$ the marginal return to $-g_{ij}$ is larger for the merged firm. When $t = P$, buyer 1 and seller 1 are merged and payoffs include both buyer and seller revenues. If seller 1's payoffs satisfy $\frac{\partial \Pi_1^s}{\partial g_{11}} \geq 0$ and $\frac{\partial \Pi_1^s}{\partial g_{12}} \leq 0$, then the marginal return to g_{11} and to $-g_{12}$ have both increased. For the unmerged buyer 2, when buyer 1 merges, the marginal returns to g_{22} and $-g_{21}$ do not change (holding other investments fixed). Hence, we can say that these marginal returns weakly increase. When $t = F$, buyer 2 and seller 2 are also merged. As above, if $\frac{\partial \Pi_2^s}{\partial g_{22}} \geq 0$ and $\frac{\partial \Pi_2^s}{\partial g_{21}} \leq 0$, then the marginal return to the merged buyer 2's investments in g_{22} and $-g_{21}$ have both increased. For the other merged buyer 1, when buyer 2 merges, the marginal returns to g_{11} and $-g_{12}$ do not change (holding other investments fixed).

In the proof of Lemma 1, we showed that $\frac{\partial \Pi_1^s}{\partial g_{11}} \geq 0$. The other partial derivatives ($\frac{\partial \Pi_1^s}{\partial g_{12}} \leq$

$0, \frac{\partial \Pi_2^s}{\partial g_{22}} \geq 0$, and $\frac{\partial \Pi_2^s}{\partial g_{21}} \leq 0$) can be obtained similarly.

Proof of Proposition 2.

The result follows directly from Milgrom and Roberts (1990), Theorem 6. This theorem shows that for supermodular games, the smallest and largest serially undominated strategies are nondecreasing functions of a parameter that corresponds to our parameter t. By Lemma 1 and Lemma 2, our game is supermodular and satisfies the conditions of Theorem 6 and its corollaries.

Proof of Lemma 3.

The merged firm 1 is hurt when buyer 2 skews it's investments provided that $\frac{\partial \Pi_1^M}{\partial g_{22}} \leq 0$ and $\frac{\partial \Pi_1^M}{\partial g_{21}} \geq 0$, where $\Pi_1^M = \Pi_1^b + \Pi_1^s$. Similarly, the merged firm 2 is hurt when buyer 1 skews its investments provided that $\frac{\partial \Pi_2^M}{\partial g_{11}} \leq 0$ and $\frac{\partial \Pi_2^M}{\partial g_{12}} \geq 0$, where $\Pi_2^M = \Pi_2^b + \Pi_2^s$. We show here that $\frac{\partial \Pi_1^M}{\partial g_{22}} \leq 0$. The other conditions are shown similarly.

We first derive $\frac{\partial \Pi_1^b}{\partial g_{22}}$. This is equal to $\frac{\partial R_1^b(\mathbf{G})}{\partial g_{22}}$. We have

$$\frac{\partial \Pi_1^b}{\partial g_{22}} = \int_0^\infty \int_0^\infty \int_0^\infty \left[\int_0^\infty x(\mathbf{v}) \frac{\partial f_{22}}{\partial g_{22}} dv_{22} \right] f_{11} f_{12} f_{21} dv_{11} dv_{12} dv_{21}.$$

where $x(\mathbf{v}) = \max\{v_{11} + v_{22}, v_{12} + v_{21}\} - \max\{v_{21}, v_{22}\}$. We have that $\frac{\partial x}{\partial v_{22}} = 1$ if $v_{11} + v_{22} > v_{12} + v_{21}$ and $v_{21} > v_{22}$. And $\frac{\partial x}{\partial v_{22}} = -1$ if $v_{11} + v_{22} < v_{12} + v_{21}$ and $v_{21} < v_{22}$. Otherwise $\frac{\partial x}{\partial v_{22}} = 0$. (or we are at a discontinuity of the piecewise continuous function $\frac{\partial x}{\partial v_{22}}$).

Next, we consider $\frac{\partial \Pi_1^s}{\partial g_{22}}$. We have

$$\frac{\partial \Pi_1^s}{\partial g_{22}} = \int_0^\infty \int_{v_{22}}^\infty \int_0^\infty \int_{v_{12}}^\infty x(\mathbf{v}) \min\{v_{21} - v_{22}, v_{11} - v_{12}\} \frac{\partial f_{22}}{\partial g_{22}} dv_{11} f_{12} f_{21} f_{22} dv_{12} dv_{21} dv_{22}$$

where $x(\mathbf{v}) = \min\{v_{21} - v_{22}, v_{11} - v_{12}\}$. We have that $\frac{\partial x}{\partial v_{22}} = -1$ if $v_{11} + v_{22} > v_{21} + v_{12}$ and $v_{21} > v_{22}$. Otherwise $\frac{\partial x}{\partial v_{22}} = 0$ (or is discontinuous).

Integrating by parts and combining the two derivatives, we have:

$$\frac{\partial \Pi_1^b}{\partial g_{22}} + \frac{\partial \Pi_1^s}{\partial g_{22}} = \int_0^\infty \int_0^\infty \int_0^{v_{12}} \left[\int_{v_{21}}^{v_{21}+(v_{12}-v_{11})} \frac{\partial F_{22}}{\partial g_{22}} dv_{22} \right] f_{11} f_{12} f_{21} dv_{11} dv_{12} dv_{21}.$$

This shows that $\frac{\partial \Pi_1^M}{\partial g_{22}} \leq 0$, since by assumption $\frac{\partial F_{22}}{\partial g_{22}} \leq 0$.

Proof of Proposition 3.

We show below that a buyer's profits satisfy the Vickrey property, as in Kranton and Minehart

[2001] Proposition 1, p. 491. That is: let $\mathbf{G}$ be any investment pattern, and let $\mathbf{G}'$ be any other investment pattern that differs only on the investments of buyer i. Then $\Pi_i^b(\mathbf{G}) - \Pi_i^b(\mathbf{G}') = W(\mathbf{G}) - W(\mathbf{G}')$.

The proposition then follows, because we can now write buyer i's stage two investment optimization problem as

$$
\begin{aligned}
\max_{\mathbf{g}_i=(g_{i1},g_{i2})} \Pi_i^b(\mathbf{g}_i,\mathbf{g}_j) &= \max_{\mathbf{g}_i}\{\Pi_i^b(\mathbf{g}_i,\mathbf{g}_j) - \Pi_i^b(\mathbf{0},\mathbf{g}_j)\} \\
&= \max_{\mathbf{g}_i}\{W(\mathbf{g}_i,\mathbf{g}_j) - W(\mathbf{0},\mathbf{g}_j)\}.
\end{aligned}
$$

On the right hand side of each of the equalities above, the second term is constant with respect to $\mathbf{g}_i$. Therefore, the solution to the maximization problem is the same as the arg max of $W(\mathbf{g}_i,\mathbf{g}_j)$. That is, buyer i maximizes welfare given the investments of buyer j.

To finish the proof, we show that a buyer's profits satisfy the Vickrey property. We prove a slightly stronger property. Let $\mathbf{G}$ and $\mathbf{G}'$ differ only on the investments of buyer i. Recall that $H(\mathbf{G})$ denotes the maximal expected surplus from exchange. Let $H_{-i}(\mathbf{G})$ denote the maximal expected surplus from exchange when buyer i is not allowed to obtain any inputs. We show that $R_i^b(\mathbf{G}) = H(\mathbf{G}) - H_{-i}(\mathbf{G})$. The Vickrey property then follows because

$$
\begin{aligned}
\Pi_i^b(\mathbf{G}) - \Pi_i^b(\mathbf{G}') &= R_i^b(\mathbf{G}) - R_i^b(\mathbf{G}') - C(\mathbf{g}_i) + C(\mathbf{g}_i') \\
&= H(\mathbf{G}) - H(\mathbf{G}') - C(\mathbf{g}_i) + C(\mathbf{g}_i') \\
&= W(\mathbf{G}) - W(\mathbf{G}')
\end{aligned}
$$

The second equality above follows from the fact that $H_{-i}(\mathbf{G}) = H_{-i}(\mathbf{G}')$.

We show that $R_i^b(\mathbf{G}) = H(\mathbf{G}) - H_{-i}(\mathbf{G})$. It suffices to show that for each realized valuation $\mathbf{v}$ of buyers' valuations, buyer i's revenue equals the difference between the surplus of the efficient allocation and the maximal surplus that would arise if buyer i did not obtain an input. This is a property of the minimum competitive prices and is easily checked. For example, suppose that at the efficient allocation, buyer 1 obtains a good from seller 1 and there is excess demand for the capacity of seller 1. This happens when $v_{11} + v_{22} \geq v_{12} + v_{21}$ and $v_{21} > v_{22}$. Buyer 1's price is then $p_1 = v_{21} - v_{22}$, and buyer 1's revenue is $v_{11} - p_1 = (v_{11} + v_{22}) - v_{21}$. Here $(v_{11} + v_{22})$ is the surplus of the efficient allocation and v_{21} is the maximal surplus that would arise if buyer 1 did not obtain

an input. As another example, suppose again that buyer 1 obtains a good from seller 1, but that there is no excess demand for the capacity of seller 1. This happens when $v_{11} + v_{22} \geq v_{12} + v_{21}$ and $v_{21} \leq v_{22}$. Buyer 1's price is then $p_1 = 0$, and buyer 1's revenue is $v_{11} = (v_{11} + v_{22}) - v_{22}$. Here $(v_{11} + v_{22})$ is the surplus of the efficient allocation and v_{22} is the maximal surplus that would arise if buyer 1 did not obtain an input. A similar analysis applies when buyer 1 obtains a good from seller 2. We conclude that buyer 1's revenue is always the difference between the surplus of the efficient allocation and the maximial surplus that would arise if buyer 1 did not obtain an input. Taking expectations gives the result that $R_1^b(\mathbf{G}) = H(\mathbf{G}) - H_{-1}(\mathbf{G})$.

Proof of Proposition 4.

From Proposition 3, we know that efficient investments are an equilibrium of the second-stage investment game when no firms are merged. By assumption, the efficient investments are not corner solutions. It follows that $\frac{\partial \Pi_1^b}{\partial g_{1j}} = 0$ for $j = 1, 2$ when investments are at efficient levels. We will show that $\frac{\partial \Pi_1^s}{\partial g_{11}} > 0$. Then, the merged firm 1 has $\frac{\partial \Pi_1^M}{\partial g_{11}} = \frac{\partial \Pi_1^b}{\partial g_{11}} + \frac{\partial \Pi_1^s}{\partial g_{11}} > 0$. So the merged firm 1 could strictly increase profits by decreasing g_{11}, and this is not an equilibrium.

In the proof of Lemma 1, we derived the expression

$$\frac{\partial \Pi_1^s}{\partial g_{11}} = - \int_0^\infty \int_{v_{22}}^\infty \int_0^\infty \left[\int_{v_{12}}^{(v_{21}-v_{22})+v_{12}} \frac{\partial F_{11}}{\partial g_{11}} dv_{11} \right] f_{12} f_{21} f_{22} dv_{12} dv_{21} dv_{22}$$

This expression showed that $\frac{\partial \Pi_1^s}{\partial g_{11}}$ is weakly positive because $\frac{\partial F_{11}}{\partial g_{11}} \leq 0$. The strict inequality, $\frac{\partial \Pi_1^s}{\partial g_{11}} > 0$, now follows from our assumptions on the distributions: $f_{ij}(v_{ij}; g_{ij}) > 0$ for $v_{ij} \in [0, \overline{v}]$, and for each value of g_{ij}, there is an open interval in $[0, \overline{v}]$ on which $\partial F_{ij}(v_{ij}; g_{ij})/\partial g_{ij} < 0$. To see this, let $\frac{\partial F_{11}}{\partial g_{11}} < 0$ on the interval (a, b). When $v_{12} = a$ and $v_{21} - v_{22} = b - a$, the bracketed expression above reduces to $\left[\int_a^b \frac{\partial F_{11}}{\partial g_{11}} dv_{11} \right] < 0$. By continuity, the bracketed expression is strictly negative on an open neighborhood of $v_{11} = b$ and $v_{21} - v_{22} = b - a$, so that the integral $\frac{\partial \Pi_1^s}{\partial g_{11}}$ is strictly positive.

A similar argument establishes the result that $\frac{\partial \Pi_1^s}{\partial g_{12}}$ is strictly negative.

Proof of Proposition 5.

The result follows from the fact that a buyer's payoff satisfies the Vickrey property. The argument is the same as in the proof of Proposition 3, where the Vickrey property is defined.

Proof of Proposition 6.

At the efficient investments, we have $\frac{\partial \Pi_1^b}{\partial g_{11}} = 0$ because the efficient investments are an interior

equilibrium when no firms are merged. We will show that $\frac{\partial \Pi_1^s}{\partial g_{11}} > 0$. Then $\frac{\partial \Pi_1^b}{\partial g_{11}} + \frac{\partial \Pi_1^s}{\partial g_{11}} > 0$, so that the efficient investment pattern is not an equilibrium when buyer 1 and seller 1 are merged. We will also show that the sign of $\frac{\partial \Pi_1^s}{\partial g_{1k}}$ is ambiguous for $k > 1$.

The seller's profit function is

$$\Pi_1^s(\mathbf{G}) = \int_{\mathbf{v}} \cdots \int p_1(\mathbf{v}) f_{11} ... f_{BS} dv_{11} ... dv_{BS}$$

where $\mathbf{v} = (v_{11}, ..., v_{BS})$ is a realization of buyers valuations and $p_1(\mathbf{v})$ is the minimum competitive price vector. Differentiating, we have

$$\frac{\partial \Pi_1^s}{\partial g_{1k}} = \int ... \int_{v_{ij} \neq v_{1k}} \left[\int_0^\infty p_1(\mathbf{v}) \frac{\partial f_{1k}}{\partial g_{1k}} dv_{1k} \right] f_{11} ... f_{BS} dv_{11} ... dv_{BS}$$

Using integration by parts, we obtain:.

$$\frac{\partial \Pi_1^s}{\partial g_{1k}} = \int ... \int_{v_{ij} \neq v_{1k}} \left[p_1(\mathbf{v}) \frac{\partial F_{1k}(v_{1k}, g_{1k})}{\partial g_{1k}} |_0^\infty - \int_0^\infty \frac{\partial p_1(\mathbf{v})}{\partial v_{1k}} \frac{\partial F_{1k}}{\partial g_{1k}} dv_{1k} \right] f_{11} ... f_{BS} dv_{11} ... dv_{BS}$$

The first term in the bracketed epression above is zero. We have

$$\frac{\partial \Pi_1^s}{\partial g_{1k}} = \int ... \int_{v_{ij} \neq v_{1k}} \left[- \int_0^\infty \frac{\partial p_1(\mathbf{v})}{\partial v_{1k}} \frac{\partial F_{1k}}{\partial g_{1k}} dv_{1k} \right] f_{11} ... f_{BS} dv_{11} ... dv_{BS}$$

By assumption, we have $\frac{\partial F_{1k}}{\partial g_{1k}} \leq 0$. The sign of $\frac{\partial \Pi_1^s}{\partial g_{1k}}$ is therefore determined by the sign of $\frac{\partial p_1(\mathbf{v})}{\partial v_{1k}}$.

We will first show that $\frac{\partial p_1(\mathbf{v})}{\partial v_{11}} \geq 0$ and hence we conclude that $\frac{\partial \Pi_1^s}{\partial g_{11}} \geq 0$. (At the end of the proof, we argue that the inequality is strict: $\frac{\partial \Pi_1^s}{\partial g_{11}} > 0$.). To analyze the term $\frac{\partial p_1(\mathbf{v})}{\partial v_{11}}$, we will use the fact that the minimum competitive price $p_1(\mathbf{v})$ satisfies the Vickrey property that a buyer's payoff is the buyer's marginal contribution to social welfare. From this, we will find a useful expression for p_1.

Fix a realization of buyers valuations $\mathbf{v}$, and consider an efficient allocation. $A^*(\mathbf{v})$. The welfare of the allocation, $w(A^*(\mathbf{v}))$ is a sum of buyers' valuations, $\sum v_{ij}$, where each buyer i obtains a good from seller j. If seller 1 sells an input, then some buyer i obtains seller 1's input and $w(A^*(\mathbf{v})) = v_{i1} + \sum v_{hl}$, where each buyer h obtains a good from a seller $l \neq 1$. To compute buyer i's marginal contribution to surplus, we next assume that buyer i does not purchase an

input, and given this, consider an allocation $A^*_{-i}(\mathbf{v})$ that maximizes welfare. If seller 1 sells an input to some buyer j, then $w(A^*_{-i}(\mathbf{v})) = v_{j1} + \sum v_{mn}$ where each buyer m obtains a good from a seller $n \neq 1$. (If seller 1 does not sell an input in $A^*_{-i}(\mathbf{v})$, then $p_1(\mathbf{v}) = 0$ because there is no competition for seller 1's input.) Using the Vickrey property of buyers' payoffs, we write the buyer i's payoff as

$$v_{i1} - p_1 = w(A^*(\mathbf{v})) - w(A^*_{-i}(\mathbf{v})) = (v_{i1} + \sum v_{hl}) - (v_{j1} + \sum v_{mn}).$$

Solving for p_1, we have

$$p_1(\mathbf{v}) = v_{j1} + \sum v_{mn} - \sum v_{hl} \text{ where } n, l \neq 1$$

We want to find $\frac{\partial p_1(\mathbf{v})}{\partial v_{11}}$. If we derive the above expression, we obtain $\frac{\partial p_1}{\partial v_{11}} = 0$ if $j \neq 1$ and $\frac{\partial p_1}{\partial v_{11}} = 1$ if $j = 1$. This suggests that $\frac{\partial p_1}{\partial v_{11}}$ is weakly positive. There is a caveat, however. The allocations $A^*(\mathbf{v})$ and $A^*_{-i}(\mathbf{v})$ that underlie the expression for $p_1(\mathbf{v})$ may change when v_{11} changes. However, the space of valuations $\mathbf{v}$ is comprised of regions, such that on each region the allocations $A^*(\mathbf{v})$ and $A^*_{-i}(\mathbf{v})$ are constant. We have showed that $\frac{\partial p_1(\mathbf{v})}{\partial v_{11}}$ is a weakly positive constant on each region where seller 1 sells an input. If there are any additional regions where seller 1 does not sell an input, then $p_1(\mathbf{v}) = 0$ and $\frac{\partial p_1(\mathbf{v})}{\partial v_{11}} = 0$ on those regions. We have shown that $\frac{\partial p_1(\mathbf{v})}{\partial v_{11}} \geq 0$ for all $\mathbf{v}$ such that $p_1(\mathbf{v})$ is differentiable.

We next consider $\frac{\partial \Pi^s_1}{\partial g_{1k}}$ for $k \geq 2$. Again, we have that the sign of $\frac{\partial \Pi^s_1}{\partial g_{1k}}$ is determined by the sign of $\frac{\partial p_1(\mathbf{v})}{\partial v_{1k}}$. In the expression for $p_1(\mathbf{v})$ above, the term v_{1k} may occur in none, one, or both of the sums $\sum v_{mn}$ and $-\sum v_{hl}$. However, in each sum, v_{1k} occurs at most once. It follows that $\frac{\partial p_1(\mathbf{v})}{\partial v_{1k}} \in \{0, 1, -1\}$. Because $\frac{\partial p_1(\mathbf{v})}{\partial v_{1k}}$ can be either positive or negative we can not determine the sign of $\frac{\partial \Pi^s_1}{\partial g_{1k}}$. In particular, if $\frac{\partial p_1(\mathbf{v})}{\partial v_{1k}} = 1$ on a large enough region, it is possible that $\frac{\partial \Pi^s_1}{\partial g_{1k}} > 0$ and seller 1 may benefit from increase in the external investment g_{ik}.

To finish the proof, we must show the strict inequality: $\frac{\partial \Pi^s_1}{\partial g_{11}} > 0$. We have

$$\frac{\partial \Pi^s_1}{\partial g_{11}} = \int \cdots \int_{v_{ij} \neq v_{11}} \left[-\int_0^\infty \frac{\partial p_1(\mathbf{v})}{\partial v_{11}} \frac{\partial F_{11}}{\partial g_{11}} dv_{11} \right] f_{12} \ldots f_{BS} dv_{12} \ldots dv_{BS}$$

where $\frac{\partial p_1(\mathbf{v})}{\partial v_{11}} \in \{0, 1\}$. We will we done if we can show that $\frac{\partial p_1(\mathbf{v})}{\partial v_{11}} = 1$ and $\frac{\partial F_{11}(v_{11}, g_{11})}{\partial g_{11}} > 0$ on a

set of $\mathbf{v}$ with positive measure. This happens,for example, when some buyer k obtains seller 1's good in the efficient allocation, but if buyer k were not allowed to obtain a good, then it would be efficient for buyer 1 to obtain the good. It is straightforward to construct a set of such $\mathbf{v}$ with positive measure. We assume values of v_{11} in the open interval where $\frac{\partial F_{11}(v_{11},g_{11})}{\partial g_{11}} > 0$ and then construct open intervals of the other values v_{ij} consistent with such allocations. Because $f_{ij} > 0$ for all $v_{ij} \in [0, \overline{v}]$,the set will have positive measure.

Note on Differentiation and Integration by Parts

We can (1) take derivatives under integral signs and (2) apply the integration by parts formula $\int x dy = xy \mid - \int y dx$ provided the following conditions are met. The conditions are defined for integrals over compact intervals. Our integrals are all of this form, because of our assumption that the support of the distributions of the buyers' valuations is bounded. For notational simplicity, however, we will usually write the upper bound of the integrals as ∞.

(1) Consider an integral $I(g) = \int_a^b h(v,g) dv$ where $h(v,g)$ is a measurable function on $R = \{(v,g) | a \leq v \leq b, c \leq g \leq d\}$. If the integral $I(g)$ exists for all $g \in [c,d]$, and if the partial derivative $\frac{\partial h(v,g)}{\partial g}$ is continuous on R, then by Theorem 9-37 of Apostol (p. 196, 1957), the derivative $I'(g)$ exists for each $g \in [c,d]$ and is given by $I'(g) = \int_a^b \frac{\partial h(v,g)}{\partial g} dv$.

(2) Consider measurable functions $x(v)$ and $y(v)$ defined on an interval $a \leq v \leq b$. If x is continuous on $[a,b]$ and if y has bounded variation on $[a,b]$, then x is Riemann-integrable with respect to y, and y is Riemann-integrable with respect to x, and $\int_a^b x dy + \int_a^b y dx = xy|_a^b$. The result follows from Theorems 9-6 and 9-26 of Apostol (p. 194 and p. 211, 1957). A second sufficient condition for the integration by parts result is that y be continuous and x have bounded variation. Bounded variation is a mild condition. For example, piece-wise continuous functions have bounded variation..

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