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Some Second Thoughts on Monopolistic Distortions and Endogenous Growth

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Abstract

The most fundamental proposition about growth and competition is that there is a tradeoff between static welfare and long-term growth. This paper reconsiders this basic proposition in an increasing product variety endogenous growth model with competitive markets for “old” innovative products and for a traditional good. We shed light on some implications of monopolistic distortions which tend to be ignored by standard models. First, no growth may be better than some growth, since modest positive growth potentially requires sizeable static welfare losses. Second, the economy may converge to a steady state with zero growth, even though another (saddle-point stable) steady state with positive growth exists if the initial share of “cheap” competitive markets is sufficiently high, as this implies a relatively low demand for “expensive” innovative goods. Third, such a “no-growth trap” may happen in a world economy made up of several countries engaged in free trade with each other. The policy implications are that growth-enhancing policies may be misguided and that quick deregulation as well as quick trade liberalization can lead to stagnation in the long term.

JEL Classification: F15, F43, O31, O34, O41

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1 Introduction

The most fundamental proposition about growth and competition, taught in introductory economics courses, is that there is a tradeoff between static welfare and long-term growth: perfect competition brings about static efficiency but undermines the incentives to invest in the innovation of new goods, services, and processes (see, e.g., Blanchard, 2006, p. 256).¹ This paper highlights several important macroeconomic implications of this basic proposition. To do so, we consider the standard Grossman-Helpman (1991, Ch. 3) increasing variety endogenous growth model augmented to include erosion of monopoly power due to (exogenous) imitation and a non-innovative traditional sector. Neither of these two extensions is novel. Textbook expositions can be found, for instance, in Barro and Sala-i-Martin (2004, Section 6.2, pp. 305 ff.) and Grossman and Helpman (1991, Section 5.3, pp. 130 ff.), respectively.² However, the implications of the resulting monopolistic distortions for model dynamics and welfare are not fully worked out. We prove three results on the model's dynamics and welfare properties and derive corollaries which characterize second-best competition policies.

The first result is directly concerned with the tradeoff between static welfare and the incentives to innovate. In the Grossman-Helpman (1991, Ch. 3) model (i.e., without competitive markets), the equilibrium growth rate is lower than optimal, and no growth cannot be better than some growth. We show that, in our model, it can be. This is because there is a static welfare loss (of non-infinitesimal size) due to monopoly pricing in the innovative sector. If the incentives provided by monopoly profits bring about only a modest growth rate, it is preferable to dispense with growth altogether and implement static efficiency instead. The implication for economic policy is that so as to achieve a modest growth rate, it may not be worthwhile incurring the associated cost in terms of static welfare losses. This finding has not been emphasized in models of growth with competitive markets.

The second result says that the economy may get stuck in a “no-growth trap” (poverty trap): the unique perfect-foresight equilibrium possibly entails convergence to a steady state with zero growth, even though another (locally) saddle-point stable steady state with positive growth exists. This will happen if the initial share of competitive markets is sufficiently high.³ In that case, a potential innovator

¹Hellwig and Irmen (2001) point out that perfect competition per se does not rule out innovation-driven growth: positive profit and costly innovation are compatible with perfect competition in the presence of diminishing returns to scale and inframarginal rents (see also the discussion in Romer, 1990, pp. S75-S77).

²For models with exogenous imitation, see also Rustichini and Schmitz (1991), Kwan and Lai (2003), and Pelka (2005, Chapter 7). In Segerstrom (1991) and Walz (1995), imitation is endogenous. Perez-Sebastian (2000) shows that “growth miracles” can be explained in a model that interprets the process of imitation as the costly adaption of knowledge created abroad.

³The possibility of a no-growth trap is ignored in the papers with exogenous imitation mentioned in footnote 2 except

would compete with many relatively cheap products, so that it does not pay to innovate. This result is related to the literature on poverty traps, surveyed by Azariadis and Stachurski (2005). As for economic policy, it implies that quick deregulation, which turns many monopolistic markets (e.g., state monopolies) into perfectly competitive markets simultaneously may be very detrimental to long-term growth, since innovating becomes unattractive as the incumbent competitive producers attract the major part of the goods demand.

The second result is reminiscent of Tang and Wälde's (2001) finding that a two-country world economy may find itself in a no-growth trap if there are sufficiently many competitive markets due to a large initial overlap of products, invented before trade is opened up between the countries. Our third result is concerned with the open-economy version of our model and relates our model to Tang and Wälde's (2001). Adapting the analysis in Arnold (2007) appropriately, we prove that, under certain conditions, the world economy made up of several (identical) countries replicates the equilibrium of the hypothetical integrated economy (that would prevail if national borders did not exist). Together with the second result, it follows immediately that if the model parameters are such that the no-growth trap occurs in the integrated economy, then the no-growth trap is also an equilibrium of the world economy if there are sufficiently many competitive markets due to a large initial overlap of products. From a policy point of view, it follows that, like quick deregulation in a closed economy, quick trade liberalization can lead to stagnation in the long term: the opening up of free trade at a point in time when the overlap exceeds the threshold number of competitive markets, above which the (world) economy is stuck in a no-growth trap, leads to long-term stagnation.

In relating our model to the existing literature, several cautionary notes are in order. First, a huge and growing literature addresses the issue of competition between several firms in given markets and the relation between the intensity of competition and the pace of equilibrium growth. For instance, Aghion et al. (2001) demonstrate that more intense competition may spur growth in a model with innovation by both technological leaders and laggards, as it induces firms to try to escape fierce competition. Such effects are not present in our model, in which markets are either monopolies or perfectly competitive. Our motivation is that this is the easiest way of approaching the question of how distortions which stem from the fact that some markets are less competitive than others affect model dynamics.

Second, in their influential "Case Against Intellectual Property", Boldrin and Levine argue that "the case against monopoly rests less upon the welfare triangle from monopoly pricing than upon the rent-seeking activity used to get and keep a monopoly" (Boldrin and Levine, 2002, p. 211). This de-

in Pelka (2005, Chapter 7). The part of the present paper concerned with the possibility of a no-growth trap clarifies and extends Pelka's (2005, Chapter 7) analysis.

emphasis of monopoly distortions is at least debatable. At any rate, the two effects are complementary, and including the rent seeking problems emphasized by Boldrin and Levine would strengthen our conclusions.

Third, the terms deregulation and liberalization can be given different meanings. We call a change from monopoly to perfect competition deregulation (since our model is an increasing variety model, the market then remains competitive indefinitely). As an example of a different definition, see Büttner (2006). She considers a quality upgrading model in which some goods are publicly provided (without being upgraded) at monopoly prices and defines deregulation as a decrease in the number of such monopolies. Deregulation is then unambiguously conducive to growth. In the open economy version of our model, liberalization means a switch from no international trade at all to unrestricted free trade. A more general formulation would allow for finite iceberg costs and define trade liberalization as a decrease in the iceberg costs (see, e.g., Baldwin and Robert-Nicoud, 2007, or Gustafsson and Segerstrom, 2007) or introduce tariffs explicitly (as, e.g., in Dinopoulos and Segerstrom, 1999, or Baldwin and Forslid, 1999, 2000).

Fourth, it is well known that in models with quality upgrading as the source of growth, contrary to increasing variety models, growth can be too fast and zero growth can be preferable when equilibrium growth is positive (see, e.g., Grossman and Helpman, 1991, Ch. 4, pp. 103-106). To highlight that it is the monopolistic distortions which are responsible for our first result, we choose as our point of departure an expanding variety growth model, so that growth cannot be too fast in the absence of competitive markets.

Fifth, the Grossman-Helpman (1991, Ch. 3) model is a first-generation R&D model, which displays scale effects. Time series observations pose a great challenge to such models (see Jones, 1995a) and have led to the development of non-scale growth models, such as Jones (1995b), Young (1998), or Arnold (1998).⁴ A relatively general lesson of these models is that growth rates are much less responsive to changes in the model parameters than models with scale effects indicate.⁵ Our motivation for using a first-generation model is that this limits the number of state variables in such a way that we can carry out the (phase diagram) analysis of the model's global dynamic behavior which is necessary in order to identify a no-growth trap. This appears acceptable in view of the fact that the presence of a no-growth trap is a property of the model's qualitative dynamic behavior, which should not relate to

⁴Young (1998) emphasizes that an increase in the labor force may be absorbed by a sector of the economy that does not spur long-term growth. Jones (1995b) assumes diminishing returns to knowledge in the creation of new knowledge, in which case population growth is required to sustain long-term growth. Arnold (1998) replaces population growth with human capital accumulation.

⁵See, however, Howitt (1999).

the responsiveness of the steady-state growth rate to changes in model parameters.

The remainder of the paper is organized as follows. Section 2 introduces the model. The growth equilibrium is derived in Section 3. Section 4 proves our main results on growth and competition. Section 5 concludes.

2 Model

There is a continuum of mass one of identical households. Each household inelastically supplies L units of labor, the only primary factor of production. Their intertemporal utility is $U = \int_0^\infty e^{-\rho t} [\sigma \ln X + (1 - \sigma) \ln Y] dt$, where X and Y are the quantities consumed of two homogeneous goods, x and y (and $\rho > 0$, $0 < \sigma < 1$).⁶ Good x is produced using a set of intermediates, j , according to the production function $X = [\int_0^n x(j)^\alpha dj]^{1/\alpha}$, where $x(j)$ is the input of intermediate j , n is the “number” of producible intermediates, and $0 < \alpha < 1$. Each producible intermediate, j , is obtained one-to-one from labor. The “traditional” good y is also obtained one-to-one from labor: $Y = L_Y$, where L_Y is labor employed in the production of y (one may think of services with less scope for innovation than in manufacturing). Blueprints for new intermediates are invented in R&D according to $\dot{n} = nL_R/a$ (with $a > 0$), where L_R is employment in R&D (there are scale effects). As for market structure, we assume that all markets are perfectly competitive except for the markets for “new” intermediates. Immediately after the development of a new variety, the innovator is a monopolist (due to either patent protection or the fact that other agents are not yet able technologically to imitate the intermediate). Subsequently, in any short time interval dt the innovator loses his monopoly with probability ψdt (due to the loss of patent protection or of technological leadership), in which case the market becomes perfectly competitive. ψ (≥ 0) is called the rate of imitation. Consequently, letting n_m and n_c denote the “numbers” of monopolistic and competitive markets for intermediate goods, respectively, we have

$$\dot{n}_c = \psi n_m, \quad \dot{n} = n_c + n_m. \tag{1}$$

As mentioned in the Introduction, the presence of competitive markets (for “old” innovative goods and for the traditional good) is the only difference to Grossman and Helpman (1991, Ch. 3). Our main results, explained in the Introduction, go through for $\psi = 0$. The purpose of including $\psi > 0$, at the cost of some additional complexity, is two-fold. First, with $\psi = 0$, obviously, the number of competitive innovative goods markets converges to zero, which runs counter our focus on the role of competitive versus monopolistic markets. Second, by allowing for positive values, we can use ψ as a measure of

⁶The time argument is suppressed here and in what follows whenever this does not cause confusion.

the strength of intellectual property rights, which will be convenient in the policy experiments we consider.

3 Equilibrium

Using aggregate expenditure as the numéraire, utility maximization yields

$$p_X X = \sigma, \quad p_Y Y = 1 - \sigma, \quad r = \rho, \quad (2)$$

where p_X and p_Y are the prices of goods x and y , respectively, and r is the interest rate. Cost minimization in the x -sector yields the input coefficient $a(j) = p(j)^{-\epsilon} [\int_0^n p(j')^{1-\epsilon} dj']^{\epsilon/(1-\epsilon)}$ for good j , where $p(j)$ is the price of intermediate j and $\epsilon \equiv 1/(1-\alpha)$. Consequently, the unit production cost and, because of perfect competition, the price of good x is $p_X = [\int_0^n p(j)^{1-\epsilon} dj]^{1/(1-\epsilon)}$. The x -sector's demand for intermediate j is $x(j) = a(j)X$. The price elasticity of demand is ϵ ($< \infty$). Monopolists in the intermediate goods sector maximize profit, π , given these demand functions. Letting p_m and p_c denote the prices in monopolistic and competitive intermediate goods markets, respectively, and x_m the output of monopolistically supplied intermediates, we obtain the familiar pricing rules and ensuing profits:

$$p_m = \frac{w}{\alpha}, \quad p_c = w, \quad \pi = (1-\alpha)p_m x_m. \quad (3)$$

Substituting the pricing rules into the expression for the input coefficients, $a(j)$, the demands $x(j) = a(j)X$ can be rewritten as

$$x_m = \alpha^\epsilon (n_c + \alpha^{\epsilon-1} n_m)^{-\frac{1}{\alpha}} X, \quad x_c = (n_c + \alpha^{\epsilon-1} n_m)^{-\frac{1}{\alpha}} X, \quad (4)$$

where x_c denotes the output of competitively supplied intermediates. Moreover, substituting the pricing rules in (3) into $p_X = [\int_0^n p(j)^{1-\epsilon} dj]^{1/(1-\epsilon)}$ and using the fact that good y is obtained one-to-one from labor, we get the final goods' prices:

$$p_X = (n_c + \alpha^{\epsilon-1} n_m)^{-\frac{1}{\epsilon-1}} w, \quad p_Y = w. \quad (5)$$

Using the fact that, as of time t , a monopolist's probability of still being a monopolist at τ ($\geq t$) is $e^{-\psi(\tau-t)}$, the value of a monopoly is

$$v(t) \equiv \int_t^\infty \exp \left\{ - \int_t^\tau [r(s) + \psi] ds \right\} \pi(\tau) d\tau. \quad (6)$$

Imitation acts like additional discounting. Free entry into R&D requires

$$wa \geq nv, \quad \text{with equality if } \dot{n} > 0. \quad (7)$$

Finally, the labor market clearing condition reads:

$$L = a \frac{\dot{n}}{n} + n_c x_c + n_m x_m + Y. \quad (8)$$

Equations (1)-(8) comprise a system of 15 equations in 15 unknowns: n_c , n_m , n , p_X , X , p_Y , Y , r , p_m , w , p_c , π , x_m , x_c , and v .⁷ A vector of these 15 variables which solves (1)-(8) for all t is an equilibrium. Let $\theta \equiv n_c/n$ and $g = \dot{n}/n$ denote the proportion of intermediate goods markets which are competitive and the growth rate of the “number” of intermediates, respectively. Further, let $V \equiv 1/(nv)$. Using $Y = L_Y$, (2), (4), (5), (7), and these definitions, the labor market clearing condition (8) can be rewritten as

$$g = \max \left\{ 0, \frac{L}{a} - \sigma V \left[\frac{\theta(1 - \alpha^\epsilon) + \alpha^\epsilon}{\theta(1 - \alpha^{\epsilon-1}) + \alpha^{\epsilon-1}} + \frac{1 - \sigma}{\sigma} \right] \right\}. \quad (9)$$

Differentiating the definition of θ and using (1) yields

$$\dot{\theta} = (1 - \theta)\psi - \theta g. \quad (10)$$

Imitation tends to increase the proportion of competitive markets, growth tends to reduce it. From (2)-(5), we have

$$\pi = \frac{\sigma(1 - \alpha)}{[1 - \theta(1 - \alpha^{1-\epsilon})]n}.$$

Differentiating the definition of V and (6) with respect to time, eliminating \dot{v} , and using (2) and the equation for monopoly profit above, we obtain

$$\frac{\dot{V}}{V} = \frac{(1 - \alpha)\sigma}{1 - \theta(1 - \alpha^{1-\epsilon})}V - (\rho + \psi + g). \quad (11)$$

Given (9), equations (10) and (11) comprise an autonomous system of ordinary differential equations in θ and V . In the present section, we analyze this system of equations. The findings will be used in Section 4 to bring forth our main results. As mentioned above, it is possible to focus on the (easier) case $\psi = 0$.

From (9), $g > 0$ if, and only if,

$$V < \frac{L}{a} \frac{1 - \theta(1 - \alpha^{1-\epsilon})}{1 - \theta[1 - \alpha^{1-\epsilon} - (1 - \alpha)\sigma] - (1 - \alpha)\sigma} \equiv \tilde{V}(\theta) \quad (12)$$

and $g = 0$ otherwise. $\tilde{V}(\theta)$ is continuous and strictly decreasing for $\theta \in [0, 1]$, with $\tilde{V}(0) = (L/a)/[1 - (1 - \alpha)\sigma]$ and $\tilde{V}(1) = L/a$.

⁷The budget constraint $d(n_m v)/dt = r n_m v + w L - 1$ represents another equation in the same variables, but as usual in general equilibrium theory, one of the 16 equations can be obtained from the other 15, so that we have as many equations as unknowns.

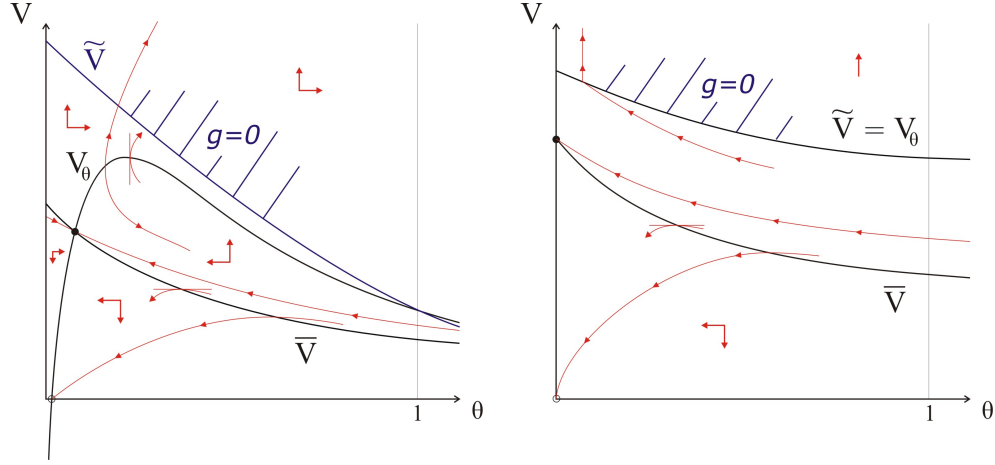


Figure 1: Dynamics in case 1 (left panel: $\psi > 0$, right panel: $\psi = 0$)

Consider first the $g = 0$ -region. According to (11), V is constant for $V = 0$ and for

$$V = (\rho + \psi) \frac{1 - \theta(1 - \alpha^{1-\epsilon})}{(1 - \alpha)\sigma} \equiv \bar{V}_0(\theta), \quad (13)$$

where $\bar{V}_0(0) = (\rho + \psi)/[(1 - \alpha)\sigma]$, $\bar{V}_0(1) = (\rho + \psi)/[(1 - \alpha)\alpha^{\epsilon-1}\sigma]$, and $\bar{V}_0'(\theta) > 0$. \dot{V} is positive for $V > \bar{V}_0(\theta)$ and negative for $V < \bar{V}_0(\theta)$. For $\psi > 0$, from (10) and $g = 0$, $\dot{\theta}$ is constant or increases depending on whether $\theta = 1$ or $\theta < 1$, respectively. $\psi = 0$, together with $g = 0$, implies $\dot{\theta} = 0$.

Here and in what follows, we distinguish three cases:

Case 1:

$$\frac{(1 - \alpha)\sigma \frac{L}{a}}{\rho + \psi} > \alpha^{1-\epsilon}. \quad (14)$$

In this case, $\bar{V}_0(1) < \tilde{V}(1)$. As $\bar{V}_0'(\theta) > 0 > \tilde{V}'(\theta)$ for all $\theta \in [0, 1]$, it follows that $\bar{V}_0(\theta) < \tilde{V}(\theta)$ and, hence, $\dot{V}/V > 0$ for all $\theta \in [0, 1]$. That is, trajectories in the $g = 0$ -region point to the north-east for $\psi > 0$ (see the left panel of Figure 1) and to the north for $\psi = 0$ (see the right panel of Figure 1).

Case 2:

$$\frac{(1 - \alpha)\sigma \frac{L}{a}}{\rho + \psi} < 1 - (1 - \alpha)\sigma. \quad (15)$$

Here, $\bar{V}_0(0) > \tilde{V}(0)$, so that the curve $\bar{V}_0(\theta)$ is located above the curve $\tilde{V}(\theta)$. V rises above and falls below $\bar{V}_0(\theta)$. Suppose $\psi > 0$. Then, the point $(\theta, V) = (1, \bar{V}_0(1))$ is a steady state. As can be seen from the left panel of Figure 2, for each $\theta \in [0, 1]$, there exists a unique path converging to this steady state. For $\psi = 0$, we have $(\dot{\theta}, \dot{V}) = (0, 0)$ for all $(\theta, \bar{V}_0(\theta))$. Hence, for any initial proportion of competitive markets, $\theta(0)$, there is a steady state $(\theta, V) = (\theta(0), \bar{V}_0(\theta(0)))$ (see the right panel of Figure 2).

Case 3:

$$1 - (1 - \alpha)\sigma \leq \frac{(1 - \alpha)\sigma \frac{L}{a}}{\rho + \psi} \leq \alpha^{1-\epsilon}. \quad (16)$$

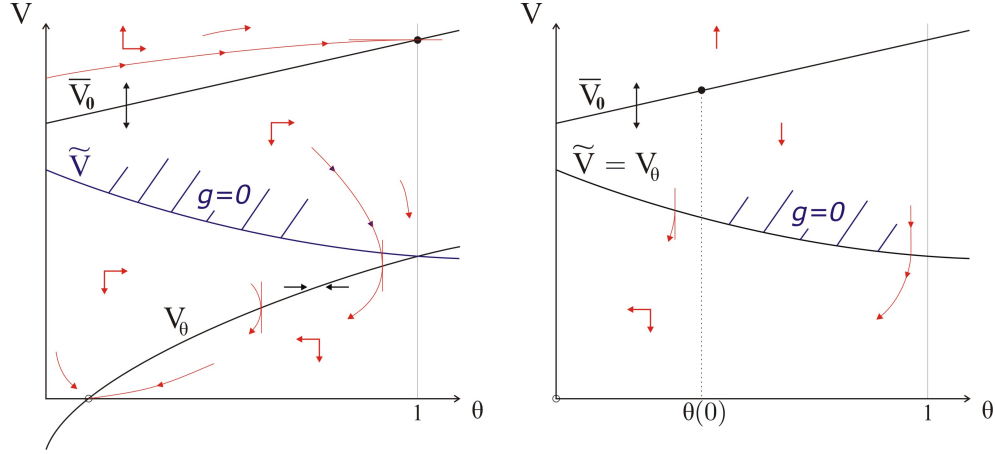


Figure 2: Dynamics in case 2 (left panel: $\psi > 0$, right panel: $\psi = 0$)

In this, intermediate, case the curves $\bar{V}_0(\theta)$ and $\tilde{V}(\theta)$ intersect for some $\theta \in [0, 1]$.⁸ As in case 2, V rises above $\bar{V}_0(\theta)$ and falls below the curve (see Figure 3).

Next, consider the region with positive growth (i.e., $g > 0$). From (9) and (11), V is constant if $V = 0$ or if

$$V = \left(\rho + \psi + \frac{L}{a} \right) \frac{1 - \theta(1 - \alpha^{1-\epsilon})}{1 - \theta[1 - \alpha^{1-\epsilon} - (1 - \alpha)\sigma]} \equiv \bar{V}(\theta). \quad (17)$$

$\bar{V}(0) = \rho + \psi + L/a$, $\bar{V}(1) = (\rho + \psi + L/a)/[1 + (1 - \alpha)\alpha^{\epsilon-1}\sigma]$, and $\bar{V}'(\theta) < 0$ for all $\theta \in [0, 1]$. V rises above the stationary locus and falls below. From (10), $\dot{\theta} = 0$ for

$$V = \left(\frac{L}{a} - \frac{1 - \theta}{\theta} \psi \right) \frac{1 - \theta(1 - \alpha^{1-\epsilon})}{1 - \theta[1 - \alpha^{1-\epsilon} - (1 - \alpha)\sigma] - (1 - \alpha)\sigma} \equiv V_\theta(\theta). \quad (18)$$

Using (12), we can rewrite $V_\theta(\theta)$ as

$$V_\theta(\theta) = \tilde{V}(\theta) - \frac{1 - \theta}{\theta} \psi \frac{1 - \theta(1 - \alpha^{1-\epsilon})}{1 - \theta[1 - \alpha^{1-\epsilon} - (1 - \alpha)\sigma] - (1 - \alpha)\sigma}. \quad (19)$$

For $\psi > 0$, from (18) and (19), we have $V_\theta(\theta) = 0$ for $(0 <) \theta = \psi/(\psi + L/a) (< 1)$ and for $\theta = 1/(1 - \alpha^{1-\epsilon}) (< 0)$. Moreover, $V_\theta(\theta) \rightarrow -\infty$ as $\theta \rightarrow 0$ from above, V_θ is continuous on $\theta \in (0, 1]$, $V'_\theta(1) = \psi - (L/a)\alpha^{\epsilon-1}(1 - \alpha)\sigma$, and $V_\theta(\theta) < \tilde{V}(\theta)$ for all $\theta \in (0, 1)$. For $\psi = 0$, on the other hand, $V_\theta(\theta) = \tilde{V}(\theta)$ for all θ and $V_\theta(1) = \tilde{V}(1) = L/a$.

Case 1: Suppose $\psi > 0$. Then, the case distinction (14) implies $\bar{V}(\theta) < \tilde{V}(\theta)$ for all $\theta \in [0, 1]$ (since $\bar{V}(0) < \tilde{V}(0)$ and $\bar{V}(\theta) = \tilde{V}(\theta)$ for some $\theta \in (0, 1]$ contradicts the case distinction). Moreover, $V_\theta(\theta) < \tilde{V}(\theta)$ for all $\theta \in (0, 1)$, and $V_\theta(1) = \tilde{V}(1)$ (i.e., $V_\theta(1) > \bar{V}(1)$). It follows that the stationary loci $\bar{V}(\theta)$

⁸Notice that the terminology is somewhat loose with regard to a common point at one of the boundaries, $\theta = 0$ or $\theta = 1$ (i.e., when one equality in (16) is strict).

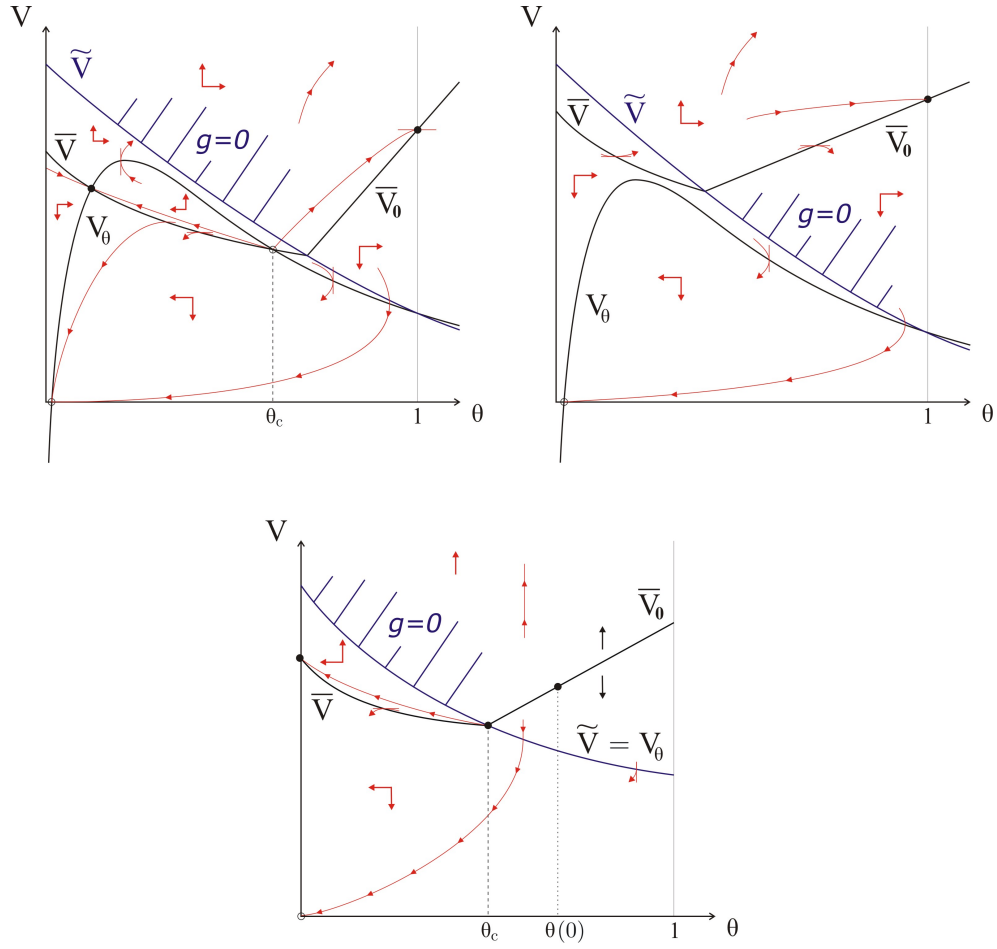


Figure 3: Dynamics in case 3 (upper panels: $\psi > 0$, lower panel: $\psi = 0$)

and $V_\theta(\theta)$ intersect an odd number of times on $(0, 1)$. The fact that $V'_\theta(1) < 0$ implies that $V_\theta(\theta)$ has an interior local maximum on $(0, 1]$. From (17) and (18), $\bar{V}(\theta) = V_\theta(\theta)$ for $\theta = 1/(1 - \alpha^{1-\epsilon})$ (< 0) and for those θ 's which satisfy the equality $\bar{V}(\theta)/[1 - \theta(1 - \alpha^{1-\epsilon})] = V_\theta(\theta)/[1 - \theta(1 - \alpha^{1-\epsilon})]$. This is a quadratic equation, with an even number of real-valued solutions. It follows that $\bar{V}(\theta)$ and $V_\theta(\theta)$ intersect exactly once in the interval $[0, 1]$, which proves that a unique steady state exists in the $g > 0$ -region. As can be seen from the left panel of Figure 1, the steady state is a saddle point. For $\psi = 0$, we have $\dot{\theta}/\theta = -g < 0$. $(\theta, V) = (0, \bar{V}(0))$ is the unique steady state in the $g > 0$ -region and is a saddle point (see the right panel of Figure 1). For each $\theta \in [0, 1]$, there exists a unique trajectory converging to the steady state both for $\psi > 0$ and for $\psi = 0$. Divergent paths can be ruled out adapting the arguments put forward by Grossman and Helpman (1991, p. 61): paths starting above the saddle path yield $V \rightarrow \infty$ and $\theta \rightarrow \theta' > 0$, where $\theta' = 1$ if $\psi > 0$ (see Figure 1). However, once the economy is in the $g = 0$ -region, $\pi n = \sigma(1 - \alpha)/[1 - \theta(1 - \alpha^{1-\epsilon})] \geq \sigma(1 - \alpha)\alpha^{\epsilon-1}$ and, from (6), $vn \geq \sigma(1 - \alpha)\alpha^{\epsilon-1}/(\rho + \psi)$. This contradicts $V \rightarrow \infty$. Paths starting below the saddle path converge to $(\theta, V) = (\psi/(\psi + L/a), 0)$ (see Figure 1). As $\pi \leq \sigma(1 - \alpha)/n$, we have $nv \leq \sigma(1 - \alpha)/(\rho + \psi)$, which contradicts $V \rightarrow 0$.

Case 2: In the $g = 0$ -region, $\dot{V}/V < 0$ below the curve $\bar{V}_0(\theta)$. A fortiori, from (11), $\dot{V}/V < 0$ in the $g > 0$ -region. As in case 1, if $\psi > 0$, the $\dot{\theta} = 0$ -locus diverges to $-\infty$ as $\theta \rightarrow 0$ from above and satisfies $V_\theta(1) = \tilde{V}(\theta)$. As can be seen from the left panel in Figure 2, all paths except the one converging to $(\theta, V) = (1, \bar{V}_0(1))$ violate perfect foresight analogously to the divergent paths in case 1. For $\psi = 0$, given a starting value $\theta(0)$, the only trajectory consistent with perfect foresight entails that the economy jumps to the steady state $(\theta(0), V_0(\theta(0)))$.

Case 3: In this intermediate case, the curves $\bar{V}_0(\theta)$ and $\tilde{V}(\theta)$ intersect for some $\theta \in [0, 1]$. From (11), the stationary locus for V is continuous on the border between positive and zero growth, $\tilde{V}(\theta)$. When the first inequality in (16) is strict, we have $\bar{V}(0) = \tilde{V}(0)$. For $\psi > 0$, by the arguments put forward in case 2, the number of intersections of $\bar{V}(\theta)$ and $V_\theta(\theta)$ in the $g > 0$ -region is two (see the upper left panel of Figure 3) or zero (see the upper right panel of Figure 3).⁹ In the former subcase (two intersections), let θ_c denote the abscissa value of the south-eastern intersection. Then, for each $\theta < \theta_c$, the unique trajectory consistent with perfect foresight converges to the north-western steady state, and for each $\theta > \theta_c$, the unique trajectory consistent with perfect foresight converges to $(\theta, V) = (1, \bar{V}_0(1))$. Similarly, in case of $\psi = 0$, let θ_c denote the θ -value at which the $\dot{V} = 0$ -locus intersects the $g = 0$ -boundary, $\tilde{V}(\theta)$. For $\theta(0) < \theta_c$, the economy converges to $(0, \bar{V}(0))$. By contrast, for $\theta(0) > \theta_c$, the

⁹The dynamics are similar in all respects important for our purposes if $V_\theta(\theta)$ is monotonically increasing, rather than taking on a maximum on $\theta \in (0, 1]$.

economy jumps to $(\theta(0), \bar{V}_\theta(\theta(0)))$ (see the lower panel of Figure 3).

4 Results

In this section, we use our findings about the model dynamics to prove three propositions about growth and welfare and derive corollaries addressing the issue of second-best competition policies mentioned in the Introduction.

The first result states that no growth may be better than some growth. To illustrate this, suppose it is possible to effectively protect monopolies indefinitely, so that $\psi = 0$. Suppose L is sufficiently large so that (14) or (16) holds for $\psi = 0$, i.e., case 1 or case 3 applies, and there is a steady state with positive growth. We know from Section 3 that if $\theta(0) = 0$, the economy settles down at a steady state with $\theta(t) = 0$ for all t (see the right panel of Figure 1 and the lower panel of Figure 3, respectively). From (9) and (11) with $\dot{\theta} = 0$, it follows that

$$g = (1 - \alpha)\sigma \frac{L}{a} - [1 - (1 - \alpha)\sigma]\rho. \quad (20)$$

Let L_+ denote the value for L such that $g = 0$ for $L \leq L_+$ and $g > 0$ for $L > L_+$ ($L = L_+$ implies that the first weak inequality in (16) holds with equality):

$$L_+ \equiv a\rho \left[\frac{1}{(1 - \alpha)\sigma} - 1 \right].$$

PROPOSITION 1 (“benefits of no growth”): *Suppose (14) or (16) holds for $\psi = 0$ (i.e., $L \geq L_+$). Then, there exists $L_c (> L_+)$ such that for $L \in (L_+, L_c)$, intertemporal utility with competitive prices, full employment, $\theta(t) = 1$, and $g(t) = 0$ for all $t \geq 0$ is higher than in the equilibrium with $\theta(t) = 0$ and $g(t) > 0$ given by (20) for all $t \geq 0$.¹⁰*

Proof: Let θ and g be constant. Furthermore, assume prices are set competitively in the y -sector and in n_c intermediate goods markets, while monopoly prices are charged in n_m intermediate goods markets. Then intertemporal utility, U , is

$$\rho U - \frac{1 - \alpha}{\alpha} \sigma \ln n(0) = \frac{\sigma}{\alpha} \ln [(n_c x_c)^\alpha \theta^{1-\alpha} + (n_m x_m)^\alpha (1 - \theta)^{1-\alpha}] + (1 - \sigma) \ln Y + \frac{1 - \alpha}{\alpha} \frac{\sigma}{\rho} g. \quad (21)$$

The important point to notice is that monopoly pricing in the intermediate goods sector causes the usual static welfare loss. To see this, notice that an allocation of labor across the intermediates and good y that maximizes static welfare requires symmetry across the intermediates (i.e., $x(j) = x$ for all

¹⁰In Grossman and Helpman’s (1991, Ch. 4) quality upgrading model, equilibrium growth is positive although zero growth would be preferable if $\rho a/L$ lies in the interval $(\log \lambda, \lambda - 1)$, where $\lambda (> 1)$ is the size of a quality jump.

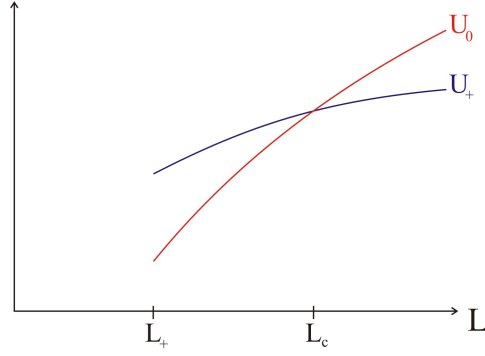


Figure 4: Comparing growth and no-growth equilibria

$j \in [0, n]$) and $nx/Y = \sigma/(1 - \sigma)$. For $\theta(t) = 1$, as $n_c(t) = n(t)$, the symmetry condition is satisfied. And from zero profit in x - and y -production (i.e., $n_c x_c = \sigma/w$ and $Y = (1 - \sigma)/w$, respectively), the allocation of labor is efficient. Next, consider the allocation with $\theta(t) = 0$ and $g(t) > 0$. Equations (2)-(5) and $\theta = 0$ imply $nx/Y = \alpha\sigma/(1 - \sigma)$. That is, markup pricing leads to too low a level of x -production relative to y .

Let U_0 denote the intertemporal utility level with $\theta(t) = g(t) = 0$ and U_+ the utility level obtained in the steady-state equilibrium with $\theta(t) = 1$ and $g(t) > 0$ given by (20). Let $L \rightarrow L_+$ from above. By the definition of L_+ , the growth rate, g , converges to zero. Given that this implies that the combined labor input in x - and y -production converges to L , the static welfare loss is of non-infinitesimal magnitude. Consequently, $U_0 > U_+$ as $L \rightarrow L_+$ from above.¹¹ The right-hand side of (21) is concave in $n_c x_c$ and in $n_m x_m$ but linear in g (which is itself linear in L , see (20)). So there is an $L_c > L_+$ such that $U_+ \geq U_0$ for $L \geq L_c$ (see Figure 4). Q.E.D.

A direct corollary of Proposition 1 is that no patent protection may be preferable to very strict patent protection. Suppose by giving up patent protection, the policymaker can raise the imitation rate from $\psi = 0$ to $\psi = \bar{\psi} (> 0)$. Consider the extreme case in which imitators learn instantaneously how to copy new innovations, so that $\bar{\psi} \rightarrow \infty$.

COROLLARY 1 (“benefits of preventing growth”): *For L slightly greater than L_c , $\theta(0) = 0$, and*

¹¹It is possible (albeit not necessary) to calculate the difference in welfare levels explicitly. Simple manipulations show that $\rho U_0 - [(1 - \alpha)/\alpha]\sigma \ln n(0) = \ln L + \sigma \ln \sigma + (1 - \sigma) \ln(1 - \sigma)$ and $\rho U_+ - [(1 - \alpha)/\alpha]\sigma \ln n(0) = \ln L + \sigma \ln \sigma + (1 - \sigma) \ln(1 - \sigma) + \sigma \ln \alpha - \ln[1 - (1 - \alpha)\sigma]$ as g goes to zero. So $\rho(U_0 - U_+) = \ln[1 - (1 - \alpha)\sigma] - \sigma \ln \alpha$ as g goes to zero. That $U_0 - U_+$ is strictly positive follows from the fact that it equals zero for $\sigma = 0$ and $\sigma = 1$ and is strictly concave. The fact that $U_0 - U_+ = 0$ for $\sigma = 1$ highlights that the presence of a traditional sector is essential for our argument. Monopolistic price setting per se does not cause a static distortion; distortions obtain when different goods have different markups (see Grossman and Helpman, 1991, p. 70).

$\bar{\psi} \rightarrow \infty$, *intertemporal utility is higher with $\psi = \bar{\psi}$ than with $\psi = 0$.*

Proof: As explained above, with $\psi = 0$, the economy settles down at a steady state with positive growth, $\theta = 0$, and, hence, with intertemporal utility U_+ (the first inequality in (16) is strict). As ψ rises sufficiently far, (15) becomes valid and case 2 applies. As illustrated by the left panel of Figure 2, zero growth prevails, and the economy converges towards $(1, \bar{V}_0(1))$. For $\bar{\psi} \rightarrow \infty$, the convergence process becomes infinitely short, so that θ quickly goes to unity and intertemporal utility is close to U_0 . Since, by Proposition 1, $U_0 > U_+$ for L slightly greater than L_c , giving up patent protection raises welfare. Q.E.D.

Numerical results, admittedly, suggest that some growth is better than no growth. However, it is possible to construct counterexamples with parameter values that should not be deemed unrealistic a priori. For instance, let $\rho = 0.04$, $\alpha = 5/8$ (which gives rise to the standard 60% markup), $a = 1$, and $n(0) = 1$. We let $\sigma = 0.999$ or $\sigma = 0.5$ and choose L such that without imitation (i.e., if $\psi = 0$) $g = 0.5\%$, which implies 0.3% growth in the x -sector's real output (i.e., $L = 0.0801$ or $L = 0.2$, respectively). For $\sigma = 0.999$, we get $U_0 = -63.3097 < -63.0515 = U_+$. For $\sigma = 0.5$, on the other hand, $U_0 = -57.5646 > -57.9446 = U_+$. So the economy with $\sigma = 0.5$ (but not the economy with σ close to unity) would be willing to give up long-term manufacturing output growth of 0.3% in exchange for static efficiency. That is, raising the imitation rate from zero to infinity is beneficial to this economy's representative consumer.

Our second proposition states that the economy may get stuck in a “no-growth” trap (even though there exists a steady state with positive growth) due to too much competition initially.

PROPOSITION 2 (“no-growth trap”): *Suppose (16) holds (case 3). Suppose further that either $\psi > 0$ and $\bar{V}(\theta)$ and $V_\theta(\theta)$ intersect twice in the $g > 0$ -region, or else $\psi = 0$. Then, $g(t) > 0$ for all $t \geq 0$ if $\theta(0) < \theta_c$, while there is $t_c \geq 0$ such that $g(t) = 0$ for all $t \geq t_c$ if $\theta(0) > \theta_c$.*

Proof: This simply rephrases the results of the analysis of case 3 in Section 3. If $\psi > 0$, $t_c (> 0)$ is the point in time at which the trajectory converging to $(1, \bar{V}_0(1))$ crosses the $g = 0$ -boundary. $t_c = 0$ for $\psi = 0$ (see the upper left and the lower panels of Figure 3, respectively). Q.E.D.

The intuition for Proposition 2 is: the lower its competitors' prices, the lower the share of aggregate demand that accrues to a potential innovator. If too many competitors supply at competitive prices, it does not pay to innovate, even though it would pay if the competitors' products were more expensive. As an example, let $\rho = 0.02$, $\psi = 0.01$, $\alpha = 0.6$, $a = 1$, $\sigma = 1$, and $L = 0.15$. This gives rise to case 3, and the stationary loci for V and θ intersect twice in the $g > 0$ -region, at $\theta = 0.4352$ and $\theta = 0.7404 \equiv \theta_c$. The growth rate corresponding to the steady state with $\theta = 0.4352$ is $g = 1.30\%$,

which implies 0.87% growth of the x -sector. So this economy fails to reach a steady state with 0.87% manufacturing output growth if the initial proportion of competitive markets exceeds 74.04%.

The implication of Proposition 2 for competition policy is that quick deregulation of monopolies may do more harm than good, as it makes it harder for a potential innovator to compete with incumbent producers. To illustrate this, consider an emerging economy with state monopolies in θ_m intermediate goods markets initially. Assume $\theta_m < \theta_c$. Without deregulation, $\theta(0) = \theta_m$, and the economy converges to the saddle-point stable steady state, in which growth is positive. Suppose, by contrast, the government deregulates some of the monopolies, in which case they instantaneously become perfectly competitive. That is, the government determines a starting value $\theta(0)$ in the interval $[\theta_m, 1]$ ($\ni \theta_c$). From Proposition 2, we have:

COROLLARY 2 (“perils of quick liberalization”): *Let the conditions of Proposition 2 be satisfied. If markets are deregulated such that $\theta(0) > \theta_c$, then there is a $t_c > 0$ such that $g(t) = 0$ for all $t \geq t_c$, while $g(t) > 0$ for all $t \geq 0$ without deregulation.*

In the example above, if $\theta_m = 74\%$, then deregulating a further 0.1% of the markets means giving up 0.87% long-term manufacturing growth.

Tang and Wälde (2001) show that a no-growth trap is possible in the two-country open economy version of our model with $\sigma = 1$ and $\psi = 0$. Proposition 2 is strongly reminiscent of their finding. We now turn to the m -country open economy version of our model. We show that under certain conditions the m -country economy behaves exactly identically to the hypothetical integrated economy that occurs in the absence of national borders (i.e., the restrictions on labor movements they imply). To do so, we generalize the analysis in Arnold (2007) (which assumes $\psi = 0$). The Tang-Wälde (2001) result is then obtained as a corollary to this replication theorem.¹² Consider a world economy made up of m (≥ 2) countries of the type introduced in Section 2 (i.e., with identical parameter values everywhere). Variables referring to individual countries, i , are distinguished by a superscript i ($= 1, \dots, m$). Variables without the superscript are world aggregates. We assume that knowledge spillovers in R&D are international in scope, so that the R&D technologies become $\dot{n}^i = nL_R^i/a$. An important issue is which products can be produced where. We start with the assumptions least conducive to the possibility of replication: non-imitated goods have to be produced where they were invented, and imitation is also “local”, in that in short time intervals, dt , a fraction $(L^i/L)\psi dt$ of the n_m goods not yet imitated before becomes producible in country i : $\dot{n}_c^i = (L^i/L)\psi n_m$. We say that replication of the equilibrium of the integrated economy (ignoring national borders that inhibit movements of labor across borders) is feasible if this allocation is an equilibrium of the world economy

¹²Our result also generalizes Tang and Wälde (2001), to the cases $\sigma > 0$ and $m > 2$.

(with national borders) as well.

PROPOSITION 3 (“replication”): *If*

$$L^i - n_c^i(t)x_c(t) - n_m^i(t)x_m(t) \geq 0, \quad \text{for all } i = 1, \dots, m, t \geq 0, \quad (22)$$

then replication is feasible.

Proof: Ignoring national borders, the equilibrium obeys equations (1)-(8) in Section 3 (where $L \equiv \sum_{i=1}^m L^i$). We have to show that this set of equations is also satisfied in the world economy with national borders. Equation (1) follows from adding up $\dot{n}_c^i = (L^i/L)\psi n_m$ for all $i = 1, \dots, m$. The conditions for utility maximization in (2) are unaffected by the presence of national borders. Since the cost minimization problem is also unchanged, so are the input coefficients, $a(j)$, the zero profit condition $p_X = [\int_0^n p(j)^{1-\epsilon} dj]^{1/(1-\epsilon)}$, the demands for intermediates $x(j) = a(j)X$, and, therefore, the pricing rules and ensuing profit in (3) and the expressions for x_m and x_c in (4) (where X is the world production of good x , x_m is the output of any monopolistically supplied intermediate, and x_c is the output level of any competitively supplied intermediate). Evidently, the equations for pricing of the final goods, the value of an innovation, and free entry into R&D, (i.e., (5)-(7), respectively) hold true in equilibrium. Finally, labor market clearing in country i requires

$$L^i = a \frac{\dot{n}^i}{n} + n_c^i x_c + n_m^i x_m + Y^i. \quad (23)$$

Assumption (22) ensures that for each country, i , given n_c^i and n_m^i , there exist $\dot{n}^i \geq 0$ and $Y^i \geq 0$ such that (23) is satisfied. Adding up (23) for all $i = 1, \dots, m$ yields (8). Q.E.D.

Propositions 2 and 3 can be jointly used to prove a generalized version of the Tang-Wälde (2001) theorem on the existence of a no-growth trap due to the opening up of international trade between several countries. To do so, assume that at time $t = 0$, m countries with free international flows of knowledge between them start to engage in trade with each other. Suppose that, while still in autarky (i.e., before time $t = 0$), the producers do not take into account the possibility of future trade liberalization, so that they do not have an incentive to avoid the invention of identical intermediates in different countries (“duplication”). Let n denote the total “number” of different intermediates producible somewhere in the world economy and n_d the “number” of duplicated intermediates at the point in time when trade is liberalized. From Propositions 2 and 3, we obtain:

COROLLARY 3 (generalization of Tang and Wälde, 2001): *Let the conditions of Proposition 2 be satisfied, and (22) holds. Then the no-growth trap described in Proposition 2 is an equilibrium of the world economy if $n_d/n > \theta_c$.*

From a policy point of view, this corollary implies that, like quick deregulation in a closed economy, quick trade liberalization can lead to stagnation in the long term: if countries decide to liberalize trade at a point in time when $n^d/n \geq \theta_c$, growth will come to a halt.¹³

Under the maintained assumptions, replication may fail due to the fact that a country, i , starts out with a disproportionately large number of blueprints. If, for instance, $n_m^i x_m > L^i$, then replication is not feasible, as country i does not have enough resources to manufacture the integrated equilibrium outputs of the intermediates with a domestic monopolist. On the other hand, as $L^i - n_c^i(t)x_c(t) - n_m^i(t)x_m(t) = L_R^i + Y_i$, (22) is satisfied with strict equality in a steady state with L_R^i and Y_i positive, it follows that if the world economy is close to its steady state initially, then (22) will hold. Moreover, the problem vanishes altogether under assumptions more conducive to the possibility of replication. To see this, assume that intermediates invented in one country can be manufactured in a different country, i , either within multinational firms or via international patent licensing. Assume further that once imitation is possible in one country, i , it is possible in each country, $i = 1, \dots, m$ (“simultaneous imitation”). Then, all production activities are “footloose”. Equation (23) is satisfied, for instance, for $\dot{n}^i/\dot{n} = n_c^i/n_c = n_m^i/n_m = Y^i/Y = L^i/L$ (and for many other allocations of productive activities across countries as well). This proves:

COROLLARY 4: *Let the conditions of Proposition 2 be satisfied. Then, with simultaneous imitation and either multinational firms or international patent licensing, the no-growth trap described in Proposition 2 is an equilibrium of the world economy if $n_d/n > \theta_c$.*

5 Conclusion

This paper is concerned with the question of how competition with “cheap”, i.e., old or traditional, goods affects the incentives to enter markets with new, innovative products. It shows that no growth may be better than some growth and that both a closed economy and a world economy made up of several countries engaged in free trade with each other may get stuck in a no-growth trap. As a result, growth-enhancing policies may be misguided, and quick deregulation as well as quick trade liberalization possibly lead to avoidable stagnation in the long term.

¹³To maintain growth, one has to wait for a point in time where $n_d/n < \theta_c$, or one has to choose a non-stationary path of the imitation rate, ψ , which prevents too much competition. A thorough analysis would require announcement effects and/or explicitly making ψ non-stationary.

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